

## NTV and kinetic MHD research using POCA

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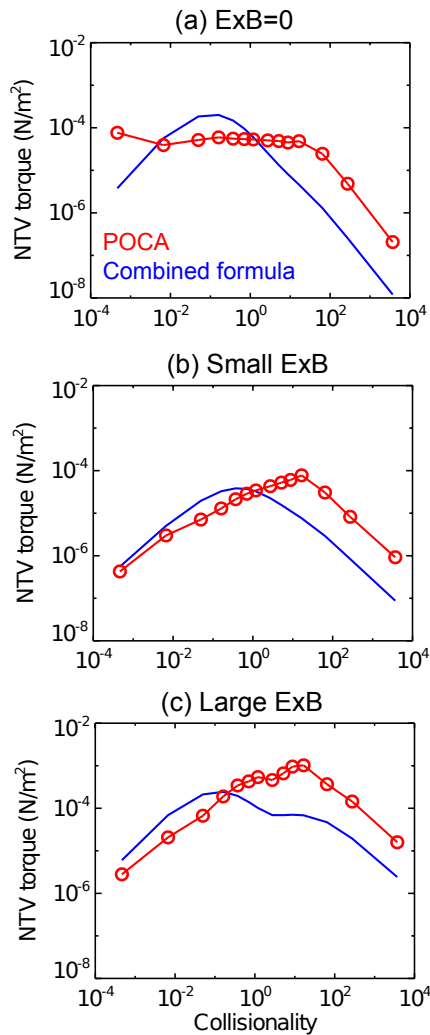
Joint Meeting for NCC  
October 1, 2013

### Outline

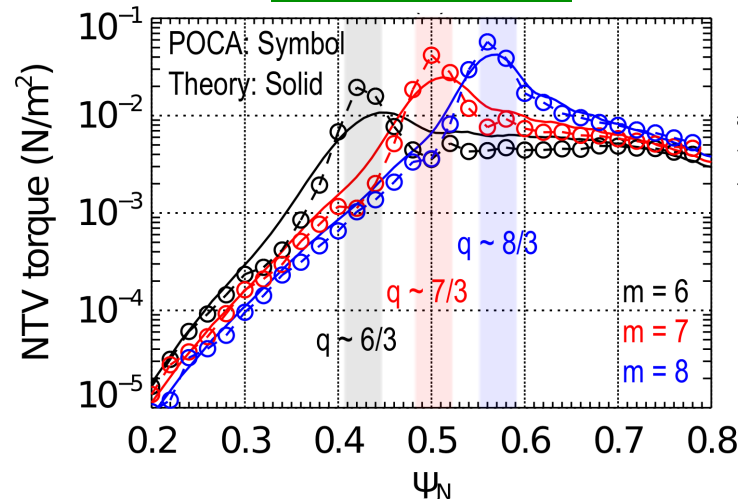
- Validation of NTV physics with particle simulation
- Prediction of magnetic braking in NSTX-U
- Possible application to kinetic potential energy
- Future work

# NTV physics has been validated using a guiding-center particle simulation with POCA

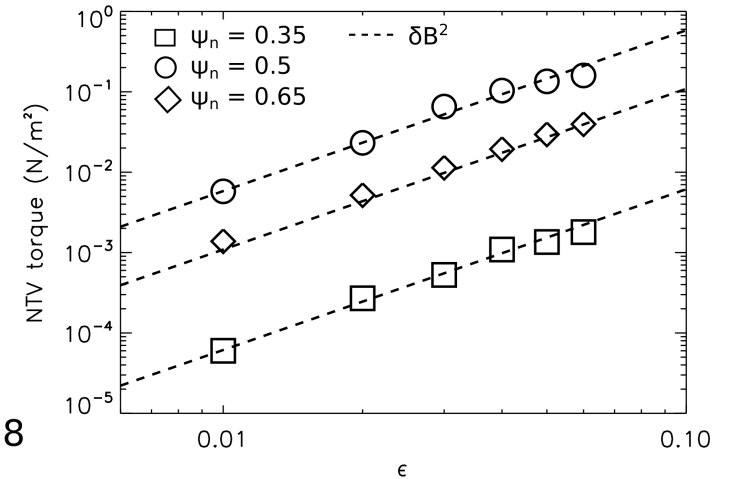
## Collisionality dependency



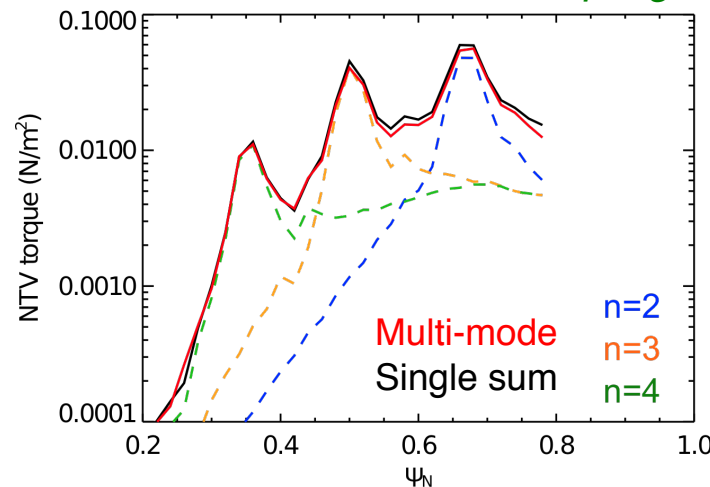
## Field resonance



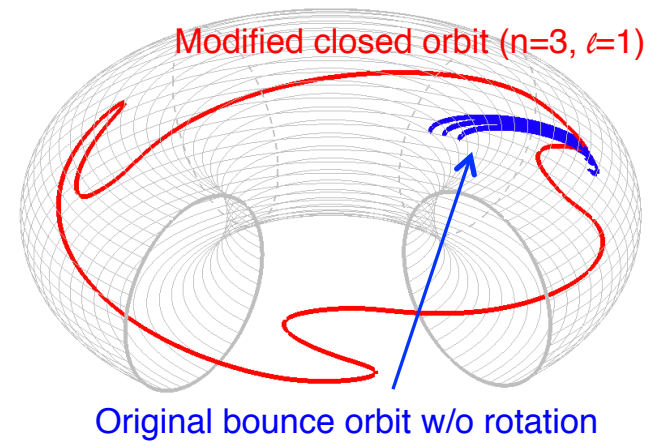
## Quadratic $\delta B$ dependency



## Toroidal mode de-coupling

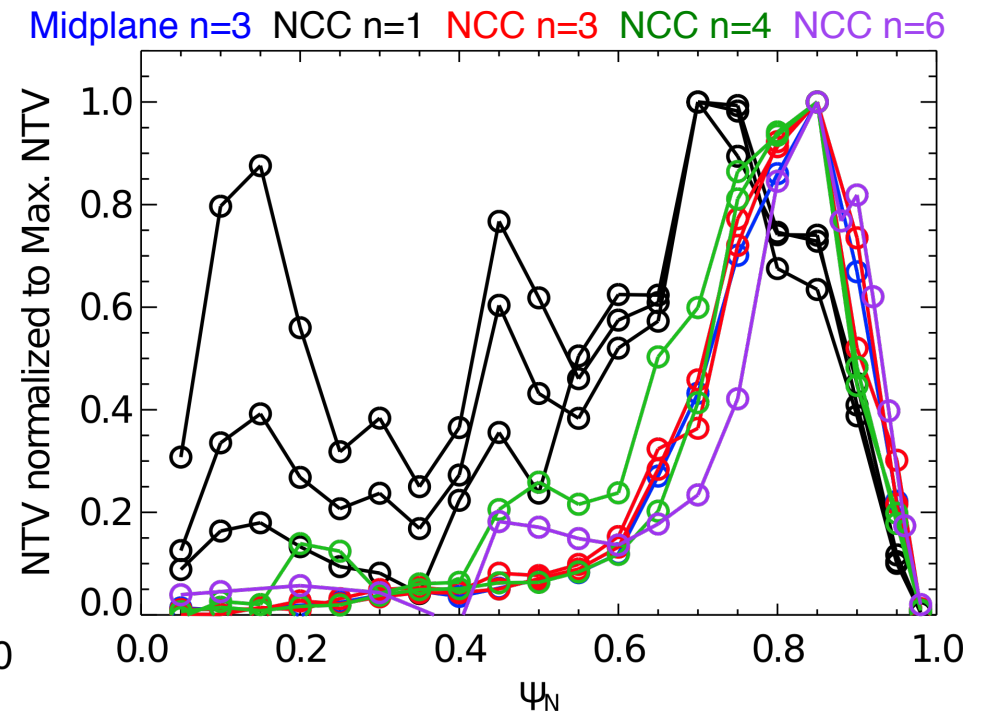
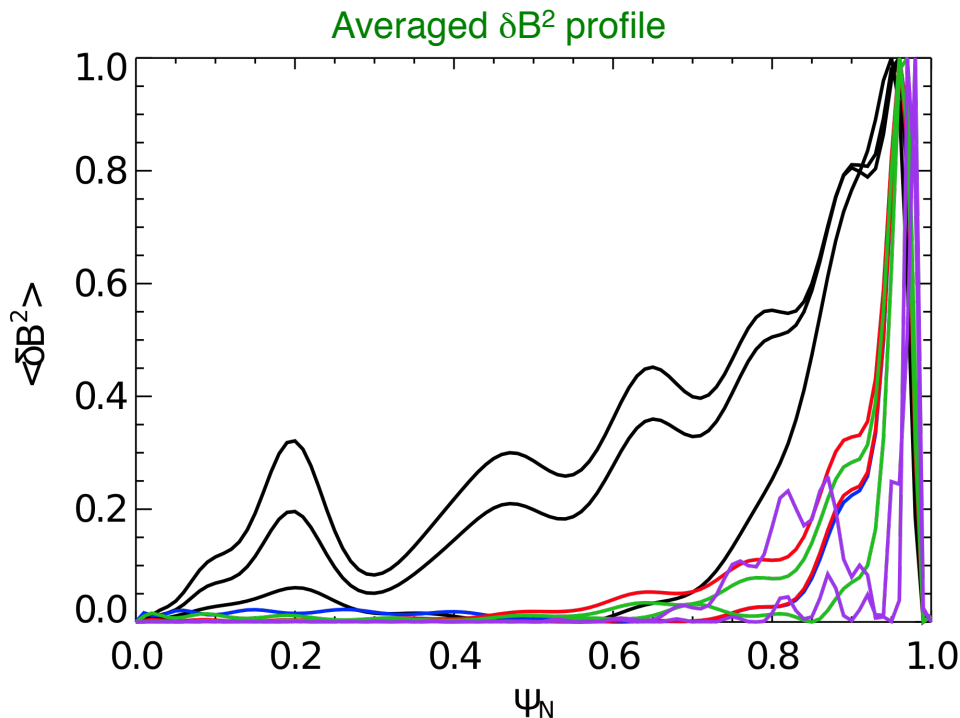


## Bounce-harmonic resonance



# Various braking profiles can be achieved in NSTX-U

- POCA predicts variability of braking profiles by Midplane and NCC
  - (Full and/or partial) NCC can provide various braking profiles by  $n=1 \sim 6$ , depending on phases: **Consistent to  $\delta B$  profiles**
  - Polynomial degree for fitting  $\delta B$  should be carefully selected in high  $n$  ( $=4,6$ ) cases for better radial resolution in NTV calculations
  - Change target equilibrium and kinetic profiles



## $\delta W_K$ can be calculated using POCA in NSTX-U

- Perturbed magnetic field is given as

$$\delta B_{mn}(\psi_n) = \sum_m a_{mn}(\psi_n) \cos(m\vartheta - n\varphi) + b_{mn}(\psi_n) \sin(m\vartheta - n\varphi)$$

- POCA calculates NTV by

$$T_\varphi = \left\langle \frac{\delta P}{B} \frac{\partial B}{\partial \varphi} \right\rangle \longrightarrow T_\varphi = \sum_m n \left\langle \frac{\delta P}{B} \left[ a_{mn} \sin(m\vartheta - n\varphi) - b_{mn} \cos(m\vartheta - n\varphi) \right] \right\rangle$$

- Kinetic potential energy drives NTV force

$$T_\varphi = 2in\delta W_K \quad [J.-K. Park, POP (2011)]$$

- $\delta W_K$  calculation with particle simulation is under test

$$\delta W_K = -\frac{1}{2} \sum_m \left\langle \frac{\delta P}{B} \left[ a_{mn} \cos(m\vartheta - n\varphi) + b_{mn} \sin(m\vartheta - n\varphi) \right] \right\rangle$$

# Future Work

- Particle simulation will be useful to extensively study NTV transport in NSTX-U
  - Predict broad/steep braking profiles depending on the toroidal mode and coil phase
  - Require detailed study on high toroidal mode for local braking
  - Change target equilibrium and kinetic profiles
- Kinetic potential energy calculation will be tested
  - Compare  $\delta W_K$  with MARS-K, MISK, IPEC-PENT, etc.
  - Physics verification and validation, application to NSTX-U
- Field line tracing with vacuum and ideal plasma response is available

# Back up

# POCA solves Fokker-Planck equation with guiding-center orbit equations

- Guiding-center motion is described by Hamiltonian equations of motion

$$\begin{aligned} \dot{\theta} &= -\frac{2\pi}{q} \frac{1}{g+u} \left\{ \frac{q^2 B^2}{m} \rho_{\parallel} \left[ \rho_{\parallel} \frac{\partial g(\psi)}{\partial \psi} - u(\psi) \right] - g(\psi) \left[ \left( \frac{q^2 \rho_{\parallel}^2 B^2}{m} + \mu B \right) \frac{1}{B} \frac{\partial B}{\partial \psi} + q \frac{\partial \Phi}{\partial \psi} \right] \right\} && \text{[R.B. White, PFL B (1990)]} \\ &&& \text{[M. Sasinowski and A.H. Boozer, POP (1997)]} \\ \dot{\psi} &= -\frac{2\pi}{q} \frac{1}{g+u} \left\{ i(\psi) \left[ \left( \frac{q^2 \rho_{\parallel}^2 B^2}{m} + \mu B \right) \frac{1}{B} \frac{\partial B}{\partial \psi} + q \frac{\partial \Phi}{\partial \psi} \right] - \left( \rho_{\parallel} \frac{\partial i(\psi)}{\partial \psi} + 1 \right) \frac{q^2 B^2}{m} \rho_{\parallel} \right\} \\ \dot{\psi} &= -\frac{2\pi}{q} \frac{1}{g+u} \left[ g(\psi) \left\{ \left( \frac{q^2 \rho_{\parallel}^2 B^2}{m} + \mu B \right) \frac{1}{B} \frac{\partial B}{\partial \theta} + q \frac{\partial \Phi}{\partial \theta} \right\} - i(\psi) \left\{ \left( \frac{q^2 \rho_{\parallel}^2 B^2}{m} + \mu B \right) \frac{1}{B} \frac{\partial B}{\partial \phi} + q \frac{\partial \Phi}{\partial \phi} \right\} - \frac{q}{2\pi} V(t) i(\psi) \right] \\ \dot{\rho}_{\parallel} &= \frac{2\pi}{q} \frac{1}{g+u} \left[ \left( \rho_{\parallel} \frac{\partial g(\psi)}{\partial \psi} - u(\psi) \right) \left\{ \left( \frac{q^2 \rho_{\parallel}^2 B^2}{m} + \mu B \right) \frac{1}{B} \frac{\partial B}{\partial \theta} + q \frac{\partial \Phi}{\partial \theta} \right\} - \left( \rho_{\parallel} \frac{\partial i(\psi)}{\partial \psi} + 1 \right) \left\{ \left( \frac{q^2 \rho_{\parallel}^2 B^2}{m} + \mu B \right) \frac{1}{B} \frac{\partial B}{\partial \phi} + q \frac{\partial \Phi}{\partial \phi} \right\} - \frac{q}{2\pi} V(t) \left( \rho_{\parallel} \frac{\partial i(\psi)}{\partial \psi} + 1 \right) \right] \end{aligned}$$

- $\delta f$  is calculated from Fokker-Planck equation

[M.N Rosenbluth, PFL (1972)]  
[M. Sasinowski and A.H. Boozer, POP (1997)]

- Fokker-Planck equation is written as

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \vec{v} \cdot \frac{\partial f}{\partial \vec{x}} + \frac{\vec{F}}{m} \cdot \frac{\partial f}{\partial \vec{v}} = C(f), \quad f = f_M \exp(\hat{f}) \approx f_M (1 + \hat{f}) \longrightarrow \frac{d \ln f_M}{dt} + \frac{d \hat{f}}{dt} = C_m(f) \equiv \frac{C(f)}{f}$$

- Fokker-Planck equation is reduced to

$$\frac{d \hat{f}}{dt} - C_m(f) = -\vec{v} \cdot \nabla \psi \frac{\partial \ln f_M}{\partial \psi} - \vec{F} \cdot \frac{\partial \ln f_M}{\partial \vec{v}}$$

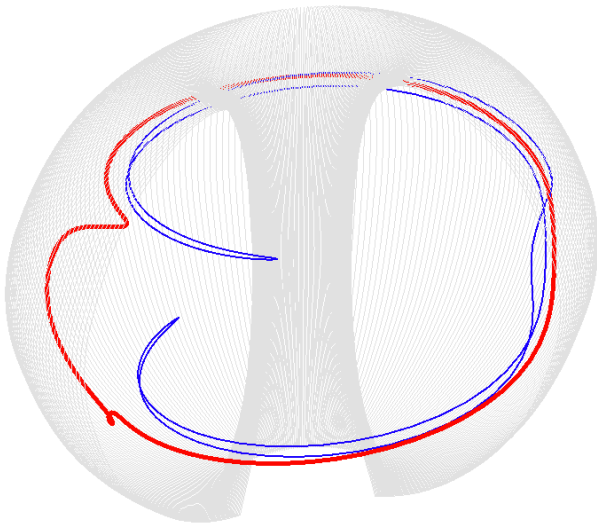
- Using local Maxwellian,  $\delta f$  can be obtained from

$$f_M = \frac{N}{(\sqrt{\pi} v_t)^3} \exp\left(-\frac{U - e\Phi}{T}\right) \longrightarrow \Delta \hat{f} = -\left[ \frac{1}{n} \frac{\partial n}{\partial \psi} + \left( \frac{3}{2} - \frac{E}{T} \right) \frac{1}{T} \frac{\partial T}{\partial \psi} \right] \Delta \psi - \frac{e}{T} \frac{d\Phi}{d\psi} \Delta \psi + 2v \frac{u}{v} \lambda \Delta t$$

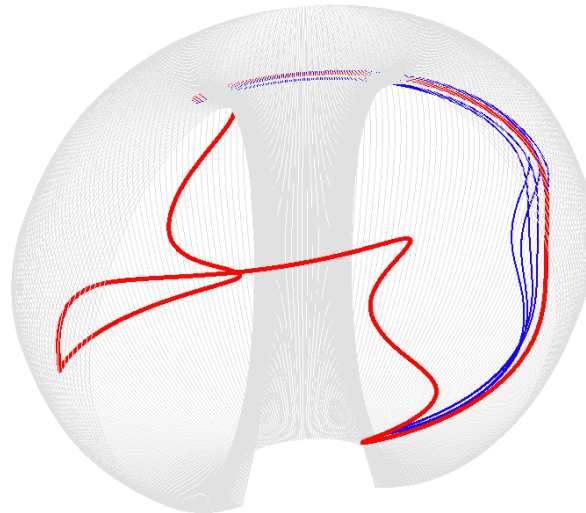
# Closed orbits by BH resonances can be found in NSTX

- Bounce-harmonic resonance almost always exist in perturbed tokamaks
  - BH resonance always exist in the finite ExB due to Maxwellian energy distribution, and on every surface due to multi-harmonic magnetic perturbations
  - Modified closed orbits, theoretically predicted and numerically reproduced in the simple configuration, can be also found in the complicated NSTX configuration
  - Identical features in orbit-closing by resonance

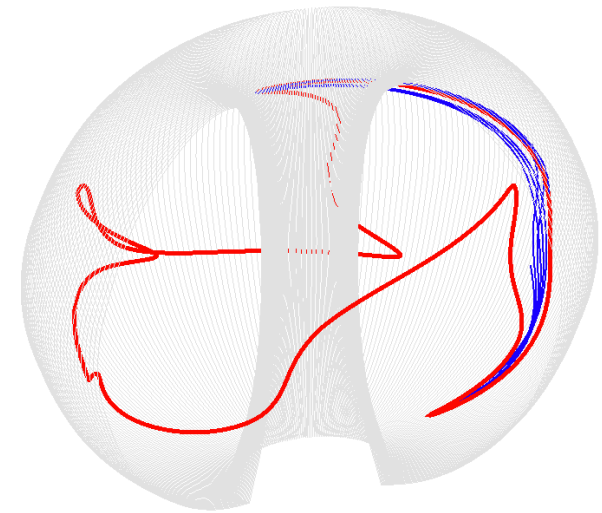
Closed orbit ( $n=1, \ell=1$ )



Closed orbit ( $n=2, \ell=1$ )



Closed orbit ( $n=3, \ell=1$ )



————— Original bounce orbits w/o rotation



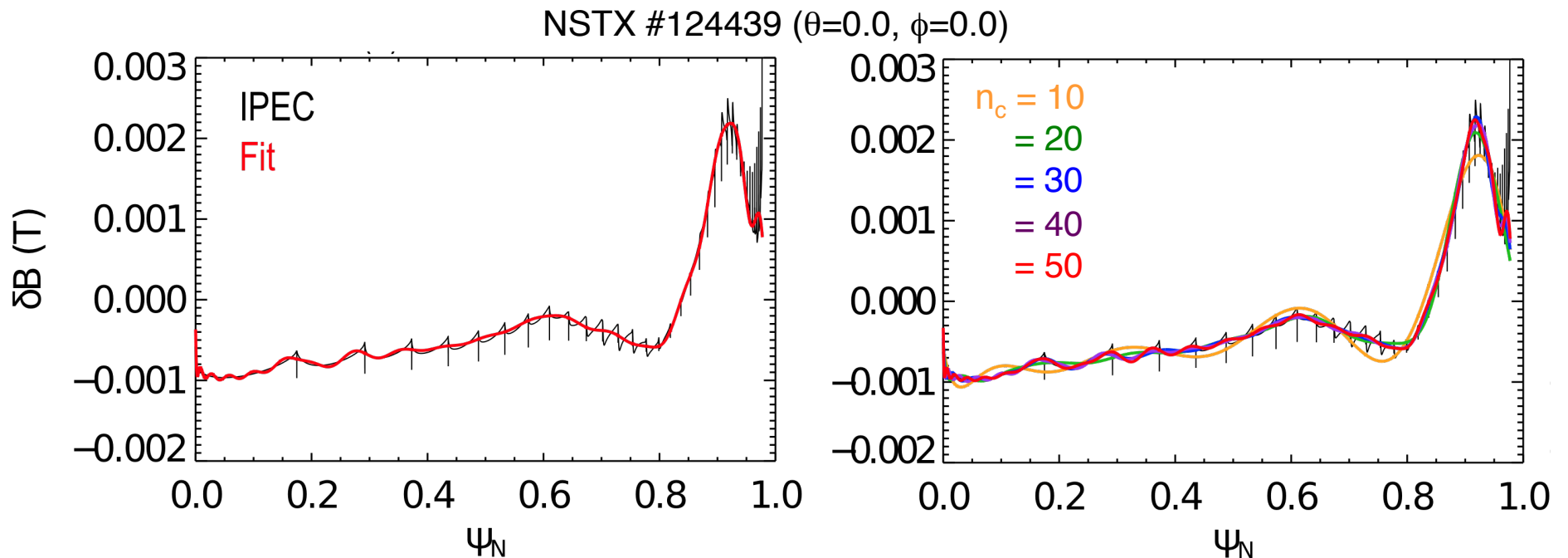
# POCA is being actively applied to experimental analysis

- Perturbed magnetic field spectrum provided by IPEC
  - Original IPEC output contains nonphysical peaks at the rational surfaces
  - Fitting technique (i.e. Chebyshev polynomials) will be used in POCA as

$$\delta B_{mn}(\psi_n) = \sum_m a_{mn}(\psi_n) \cos(m\theta - n\phi) + b_{mn}(\psi_n) \sin(m\theta - n\phi) \quad \leftarrow \text{IPEC}$$

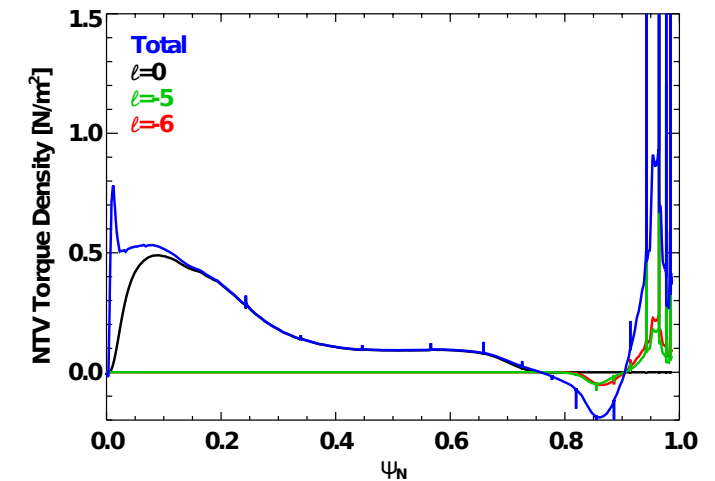
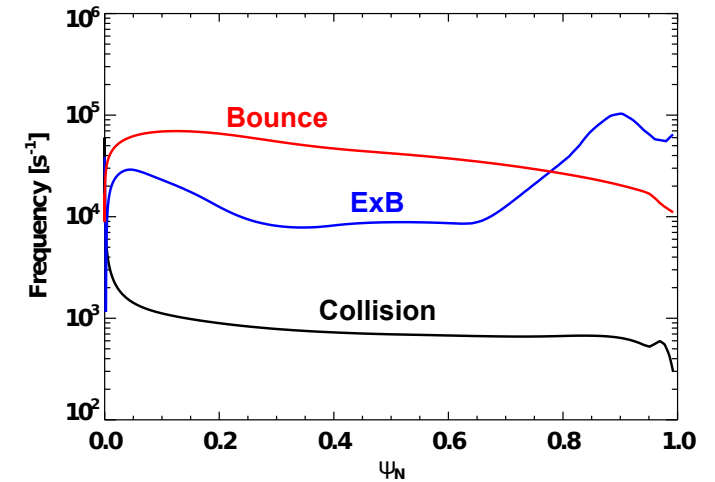
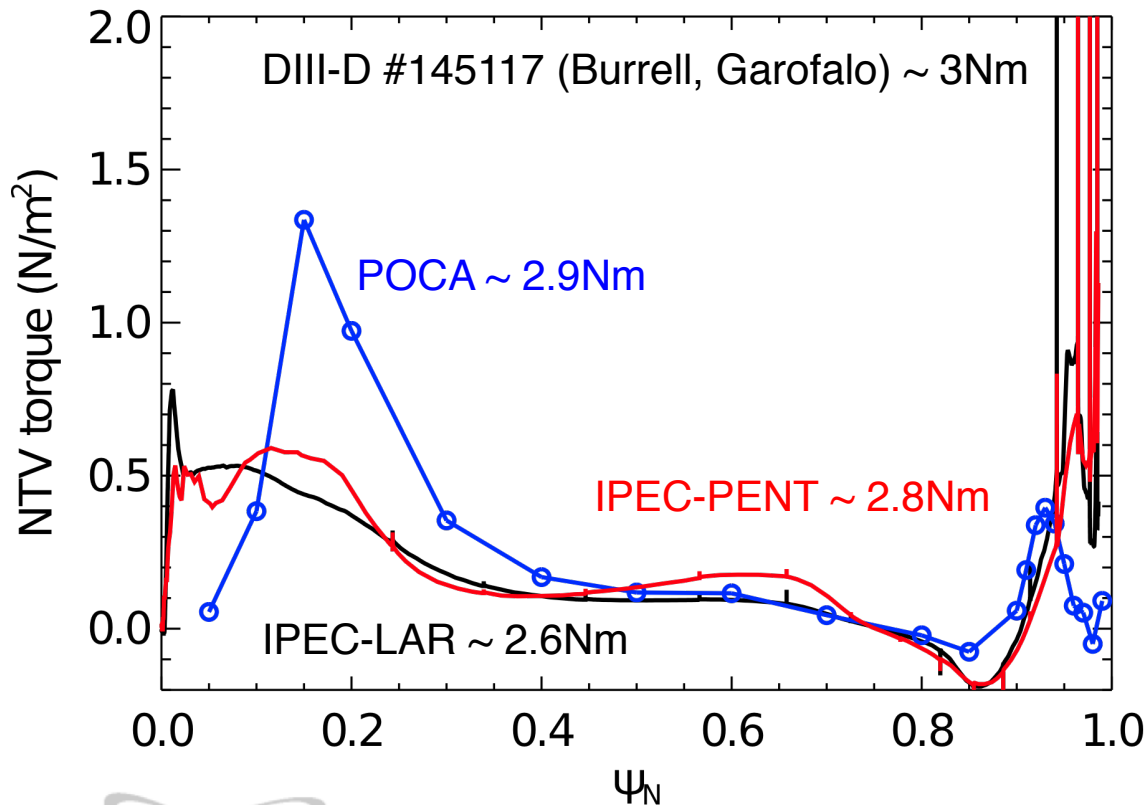
$$\delta B_{mn}(\psi_n) = \sum_m \left[ \sum_j^{n_c} a_j \cos(j \cos^{-1}(x)) \cos(m\theta - n\phi) + b_j \cos(j \cos^{-1}(x)) \sin(m\theta - n\phi) \right] \quad \rightarrow \text{POCA}$$

- Fitting follows overall features of IPEC  $\delta B$ , and effectively smoothes the peaks



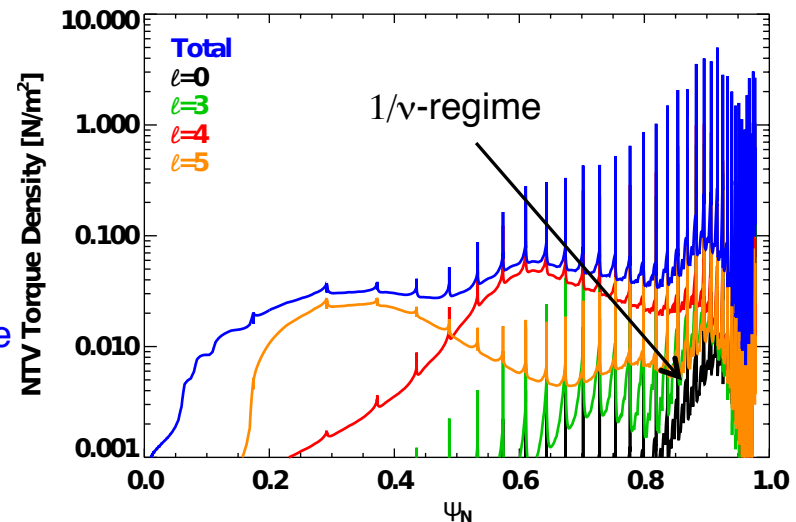
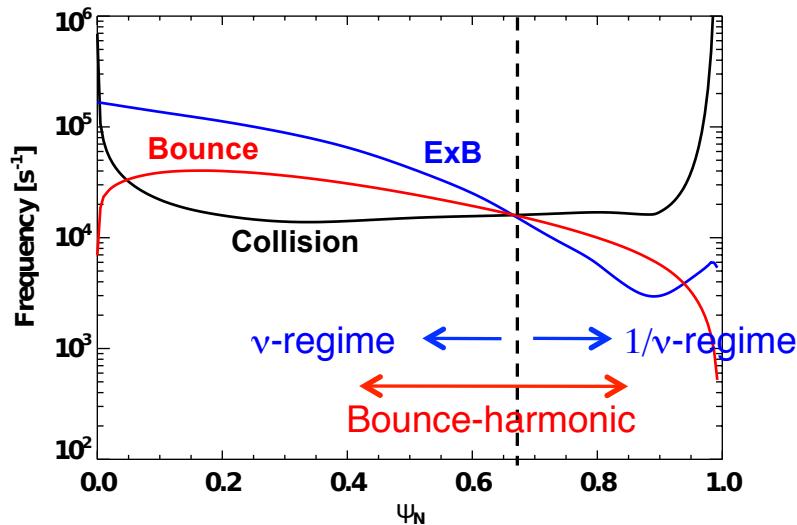
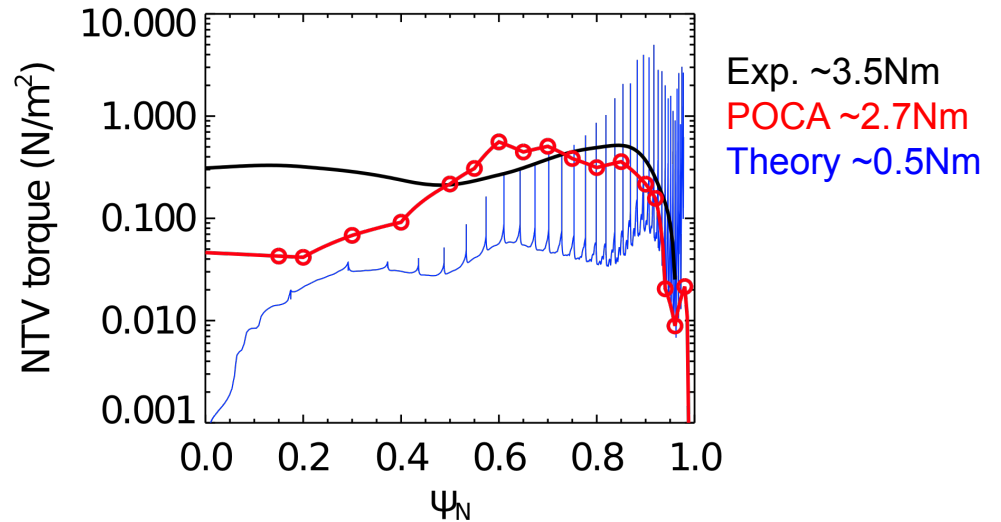
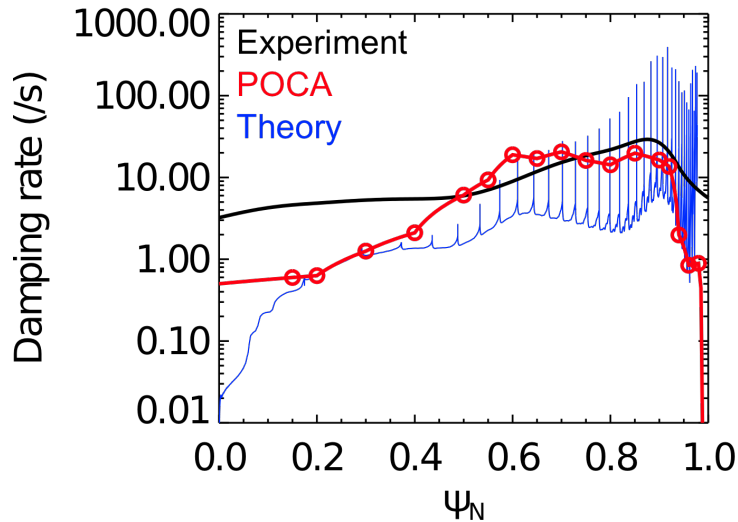
# Consistent NTV profile and total NTV was obtained in DIII-D n=3 magnetic braking

- Application of POCA to DIII-D n=3 magnetic braking (in QH mode experiments) using IPEC  $\delta B$  gives a consistent NTV



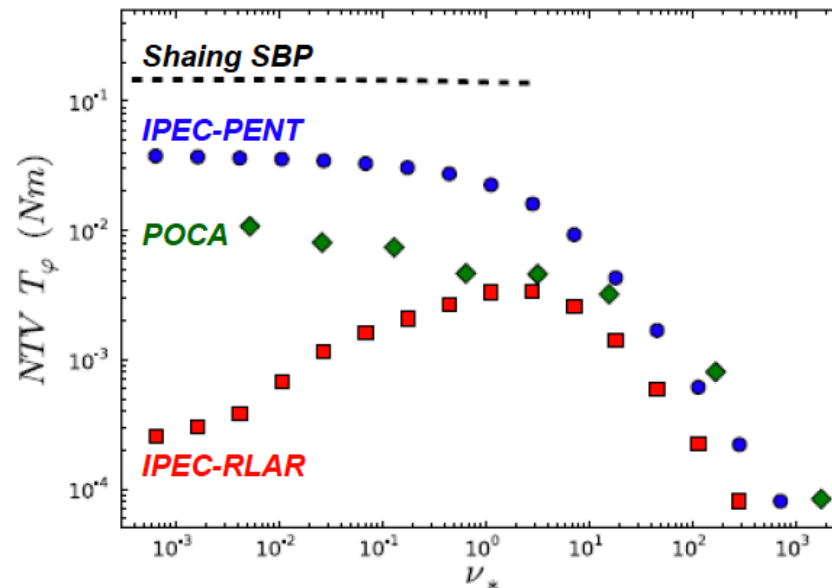
# Prediction of NTV in NSTX can be improved with POCA

- Bounce-harmonic resonances can dominate NTV transport



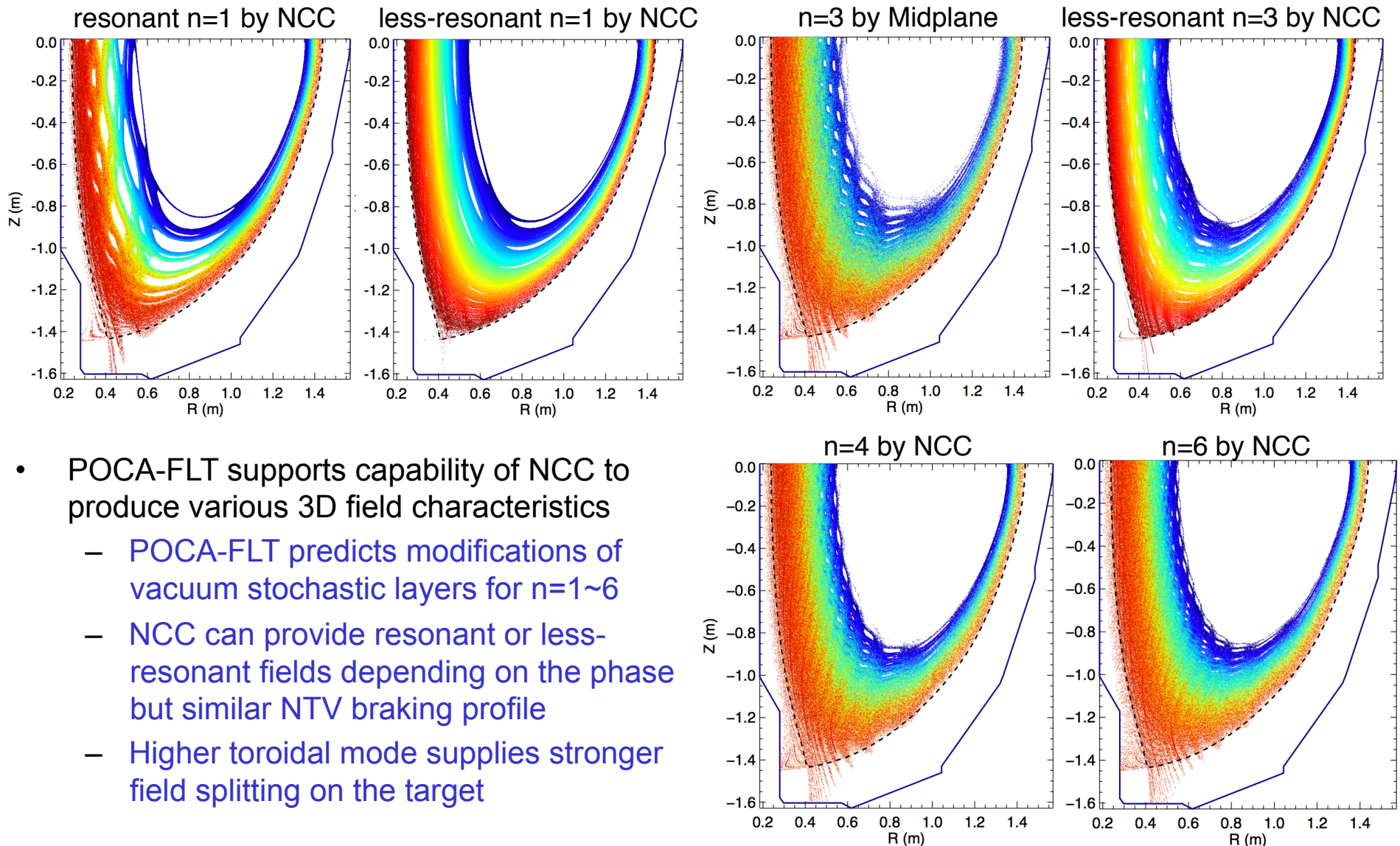
# Semi-analytic numerical routines in IPEC have been compared to POCA

- IPEC previously used a reduced large-aspect-ratio NTV formulation with approximated collision operator : IPEC-RLAR
- Now new formulation and implementation for NTV with general geometry have been completed : IPEC-PENT
- Computational cost : IPEC-RLAR < IPEC-PENT < POCA
- Accuracy : POCA > IPEC-PENT > IPEC-RLAR (unacceptable for SBP)



[Logan, To be submitted to POP (2013)]

# Field Line Tracing Simulations for NCC



- POCA-FLT supports capability of NCC to produce various 3D field characteristics
  - POCA-FLT predicts modifications of vacuum stochastic layers for  $n=1\sim 6$
  - NCC can provide resonant or less-resonant fields depending on the phase but similar NTV braking profile
  - Higher toroidal mode supplies stronger field splitting on the target

# General Perturbed Equilibrium Code (GPEC) is on progress

- Perturbed equilibrium codes are efficient to study 3D field physics in tokamaks with non-axisymmetric perturbations
  - IPEC solves ideal force balance with ideal constraints
  - GPEC will solve non-ideal force balance with arbitrary jump conditions, which will be matched with inner-layer solver
  - POCA will use 3D perturbations from IPEC, and provide anisotropic pressure tensor to GPEC

