Robust Control of the Spatial Current Profile in the DIII-D Tokamak

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Overview - Need for Current Profile Control

- In order for ITER to be successful we must develop ability to operate tokamak for sufficiently long plasma discharges.
  - Limited ability to generate magnetic flux needed to sustain a purely inductive plasma current means plasma current will primarily have to be generated by non-inductive means.
- Extensive research has been conducted to find so-called advanced tokamak operating scenarios characterized by a high fusion gain, good confinement, MHD stability, and a non-inductively driven plasma current with a dominant fraction coming from the bootstrap current.
- If these performance objectives are achieved, cost and size of fusion reactors could be greatly reduced.
- One possible advanced operating scenario is related to setting up a suitable toroidal current density profile in the machine.
- Current profile control philosophy at DIII-D.
  - Create desired current profile during ramp-up and early flat-top phases of plasma current evolution and maintain this target profile throughout remainder of discharge.
  - System outputs: q profile on normalized flux spatial domain.
  - System inputs: Total plasma current, neutral beam injection, radio frequency wave injection, plasma line-averaged density.

Motivation for Model-based Control

- During initial phase of discharge, feedback control of $q(0, t)$ and $q_{min}(t)$ has been demonstrated at DIII-D [1].
  - Change plasma conductivity through electron heating.
  - Employed controller requests a power level to actuator, either ECH or NBI.
    - Preprogrammed feedforward value plus error in q times a proportional gain.
- q profile is obtained in real-time from motional Stark effect (MSE) diagnostic measurement.
- If sampling rate of q profile is reduced, non-model-based controller has been observed to become unstable.
- This behavior, along with strong coupling between magnetic and kinetic plasma profiles and high dimensionality of problem, motivates design of a model-based controller that takes into account dynamics of entire q profile in response to available actuators.
  - Has the potential for improved performance.

[1] J. Ferron et al., "Feedback Control of the Safety Factor Profile Evolution during Formation of an Advanced Tokamak Discharge," *Nuclear Fusion*, 2006.

Motivation for First Principles Based Modeling

- Linear plasma response modeling
  - At JET, linear, dynamic plasma response models were identified by performing system identification experiments [2].
  - Linear models only valid near the equilibrium from which they were identified.
  - To extend control scheme designed from plasma response models to other tokamaks, new system identification experiments have to be conducted.
- First principles based modeling
  - Derived from Gauss's law, Ampere's law, Faraday's law, Ohm's law, and an equilibrium momentum balance for L-mode discharges [3].
  - Adaptable to various tokamaks and equilibrium configurations.
  - Include nonlinear coupling between magnetic and kinetic plasma profiles.
  - Explicitly describe the temporal and spatial evolution of the current profile.
  - Control strategies for various tokamaks can be synthesized from one model.
- Control strategy: feedforward + feedback.
  - Feedforward is computed off-line [4,5] and feedback is computed on-line.

[2] D. Moreau et al., "A Two-time-scale Dynamic-model Approach for Magnetic and Kinetic Profile Control in Advanced Tokamak Scenarios on JET," *Nuclear Fusion*, 2008.
[3] Y. Ou et al., "Towards Model-Based Current Profile Control at DIII- D," *Fusion Engineering and Design*, 2007.
[4] C. Xu et al., "Ramp-Up Phase Current Profile Control of Tokamak Plasmas via Nonlinear Programming," *IEEE Transactions on Plasma Science*, 2010.
[5] Y. Ou et al., "Design and Simulation of Extremum-Seeking Open-Loop Optimal Control of Current Profile in the DIII-D Tokamak," *Plasma Physics and Controlled Fusion*, 2008.

Current Profile Evolution Model

- Plasma current mainly driven by induction during the ramp-up and early flat-top phases of discharge.
- Simplified scenario-oriented models for electron temperature, non-inductive current density, and plasma resistivity are identified for L-mode discharges.
- By employing these simplified models, evolution of poloidal magnetic flux is given by magnetic diffusion equation [3]

$$\frac{\partial \psi}{\partial t} = f_1(\hat{\rho}) u_1(t) \frac{1}{\hat{\rho}} \frac{\partial}{\partial \hat{\rho}} \left(\hat{\rho} f_4(\hat{\rho}) \frac{\partial \psi}{\partial \hat{\rho}} \right) + f_2(\hat{\rho}) u_2(t) \qquad (1)$$

with boundary conditions

$$\left. \frac{\partial \psi}{\partial \hat{\rho}} \right|_{\hat{\rho}=0} = 0 \qquad \left. \frac{\partial \psi}{\partial \hat{\rho}} \right|_{\hat{\rho}=1} = -k_3 u_3(t). \qquad (2)$$

Note: $u_1(t)$, $u_2(t)$, and $u_3(t)$ are control actuators which are nonlinear functions of:

- Total plasma current, total non-inductive power, line-averaged density.

Poloidal Flux Gradient Profile Evolution Model

- Using constant relationship between ρ and Φ, $\pi B_{\phi,0}\rho^2 = \Phi$, and definition of $\hat{\rho} = \rho/\rho_b$, safety factor is written as

$$q(\hat{\rho},t) = -\frac{d\Phi}{d\Psi} = -\frac{d\Phi}{2\pi d\psi} = -\frac{\frac{\partial\Phi}{\partial\rho}\frac{\partial\rho}{\partial\hat{\rho}}}{\frac{\partial\psi}{\partial\hat{\rho}}} = -\frac{B_{\phi,0}\rho_b^2\hat{\rho}}{\partial\psi/\partial\hat{\rho}}. \quad (3)$$

- q profile is dependent on $\partial\psi/\partial\hat{\rho}$ and is chosen to be controlled variable

$$\theta(\hat{\rho},t) = \partial\psi/\partial\hat{\rho}(\hat{\rho},t). \quad (4)$$

- Expanding (1) using the chain rule, inserting (4) into this expanded equation, and differentiating resulting equation with respect to $\hat{\rho}$, PDE governing evolution of $\theta(\hat{\rho},t)$ is

$$\frac{\partial\theta}{\partial t} = \left[h_0(\hat{\rho})\theta'' + h_1(\hat{\rho})\theta' + h_2(\hat{\rho})\theta\right]u_1(t) + h_3(\hat{\rho})u_2(t) \quad (5)$$

with boundary conditions

$$\theta(0,t) = 0 \qquad \theta(1,t) = -k_3u_3(t). \quad (6)$$

Model Reduction (PDE to System of ODEs)

- Construct a reduced-order model suitable for control design: governing PDE is discretized in space.
- After applying spatial derivative approximations of $O(\Delta\hat{\rho})^2$ to (5) and taking into account boundary conditions (6), we obtain a matrix representation for reduced-order model

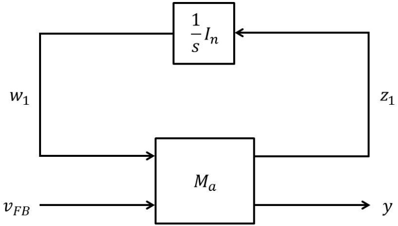
$$\dot{\alpha}(t) = \Gamma\alpha(t)v_1(t) + \Omega v_2(t) + \Pi v_3(t). \quad (7)$$

  Note: $[v_1(t), v_2(t), v_3(t)]^T = [u_1(t), u_2(t), u_1(t)u_3(t)]^T$ is control input.
- Perturbation variables: $x(t) = \alpha(t) - \alpha_{FF}(t)$ and $v_{FB}(t) = v(t) - v_{FF}(t)$.
- After linearizing (7) around feedforward trajectories, time variant state-space dynamic model for deviation dynamics $x(t)$

$$\begin{aligned} \dot{x} &= A(t)x + B(t)v_{FB} \\ y &= Ce + Dv_{FB} \end{aligned} \quad (8)$$

  $A(t) = \Gamma v_{1_{FF}}(t)$, $B(t) = [\Gamma\alpha_{FF}(t), \Omega, \Pi]$, $C = I_n$, $D = 0$, and $v_{FB} = [v_{1_{FB}}, v_{2_{FB}}, v_{3_{FB}}]^T$.

Linear State-Space System Represented as a LFT



Transfer function $G(s)$ represented as a LFT.

- By defining the matrix

$$M_a = \begin{bmatrix} A & B \\ C & D \end{bmatrix}, \tag{9}$$

system transfer function $G(s)$ of a linear state-space system can be written as a linear fractional transformation (LFT) as

$$G(s) = F_u\left(M_a, \frac{1}{s}I_n\right) = C(sI_n - A)^{-1}B + D \tag{10}$$

where n is the number of states of the system.

Time Varying Parameter Modeling

- Time varying parameters $v_{1_{FF}}(t)$ and $\alpha_{FF}(t)$ in the system matrices of (8) are modeled as a time varying uncertainty as

$$v_{1_{FF}}(t) = \gamma_v \Big(1 + \beta_v \delta_v(t)\Big) \qquad \alpha_{i_{FF}}(t) = \gamma_\alpha^i \Big(1 + \beta_\alpha^i \delta_\alpha^i(t)\Big) \tag{11}$$

  where $i = 1, 2, \ldots, n$.
- By employing (11), the deviation dynamic model (8) is expressed as

$$\dot{x}_k = \sum_{j=1}^{n} \left[A_{0_{k,j}} + \delta_v A_{1_{k,j}}\right] x_j + \left[B_{0_k} + \sum_{i=1}^{n} \delta_\alpha^i B_{i_k}\right] v_{FB} \tag{12}$$

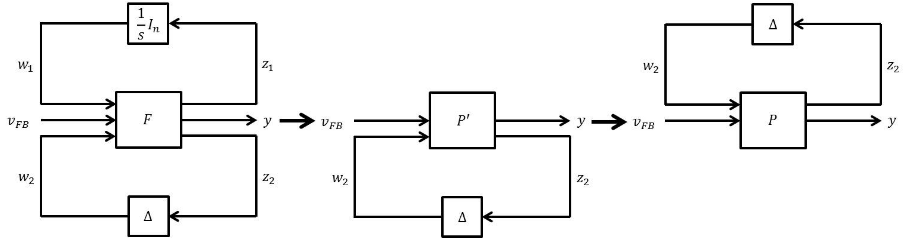
  where $k = 1, 2, \ldots, n$ and

$$A_{0_{k,j}} = \gamma_v \Gamma_{k,j} \qquad A_{1_{k,j}} = \gamma_v \beta_v \Gamma_{k,j}$$

$$B_{0_k} = \left[\sum_{i=1}^{n} \gamma_\alpha^i \Gamma_{k,i}, \Omega_k, \Pi_k\right] \qquad B_{i_k} = \left[\left(\gamma_\alpha^i \beta_\alpha^i\right)\Gamma_{k,i}, 0, 0\right] \tag{13}$$

- By defining total uncertainty vector δ as $\delta = [\delta_v, \delta_\alpha^1, \ldots, \delta_\alpha^n] \in \mathbb{R}^{n+1}$, matrix M_a, defined in (9), is written as a general affine state-space uncertainty.
  - By exploiting structure of state matrices, uncertainty is formulated into a LFT.

Model in Robust Control Framework



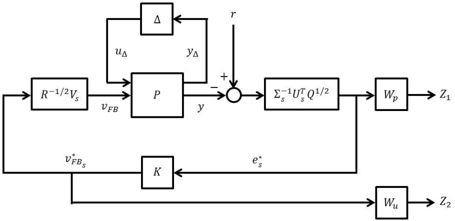
Block diagram manipulation to obtain generalized plant P.

- The transfer function $G(s)$ of the uncertain state-space model is next expressed as

$$G(s) = F_u\left(M_a, \frac{1}{s}I_n\right) = F_u\left(F_l(Q, \Delta), \frac{1}{s}I_n\right) = F_l\left(F_u\left(Q, \frac{1}{s}I_n\right), \Delta\right)$$
$$= F_l(P', \Delta) = F_u(P, \Delta). \quad (14)$$

  For convention purposes, it is necessary to move the uncertainty to create an upper LFT.
- Goal is to synthesize a feedback controller to stabilize closed-loop system in presence of uncertain parameters.
  - Referred to as robust control.

Feedback Controller Synthesis



Mixed-sensitivity H_∞ control problem.

- Static gain matrix of nominal model used to identify relevant channels.
- Mixed-sensitivity H_∞ problem

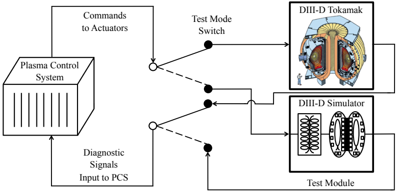
$$\min_K \left\| \begin{matrix} W_p S_{DC_O} \\ W_u K S_{DC_O} \end{matrix} \right\|_\infty, \qquad \forall \omega. \tag{15}$$

- Multi-input-multi-output feedback controller is expressed as

$$\begin{aligned} \dot{x}_c &= A_c x_c + B_c \Sigma_s^{-1} U_s^T Q^{1/2} e \\ v_{FB} &= R^{-1/2} V_s C_c x_c + R^{-1/2} V_s D_c \Sigma_s^{-1} U_s^T Q^{1/2} e. \end{aligned} \tag{16}$$

where A_c, B_c, C_c, D_c are state space matrices of K.

Simserver Architecture



Simserver architecture.

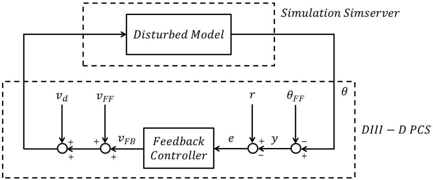
- Simserver architecture is a valuable simulation environment used for testing algorithms running in DIII-D Plasma Control System (PCS).
  - Incorporates a tokamak simulation model that is used to test PCS in realistic closed-loop simulations.
  - Simulation model accepts control inputs from PCS and then generates simulated diagnostics.
- Simulation used to determine effectiveness of controllers and correctness of their real-time implementation before experimental tests are run [6].

[6] M. Walker et al., “Advances in Integrated Plasma Control on DIII-D,” *Fusion Engineering and Design*, 2007.

Simulation Conditions

- Nominal initial poloidal flux gradient profile is extracted from DIII-D shot # 129412 at an experimental time of $t = 0.5$ seconds.
- It is difficult to achieve a perfect matching of nominal initial poloidal flux gradient profile during tokamak operation.
- Magnetic diffusion equation does not capture all of the physical phenomena that effect the poloidal flux gradient profile evolution.
- Therefore perturb initial poloidal flux gradient profile and perturb nominal electron temperature and non-inductive current density models by 10%.
  - Referred to as disturbed model.
- Simulation conditions provide means to test feedback controller in a realistic operating scenario.
  - Mismatch between plant and model and between actual and assumed initial conditions.

DIII-D Profile Control Algorithm Configuration

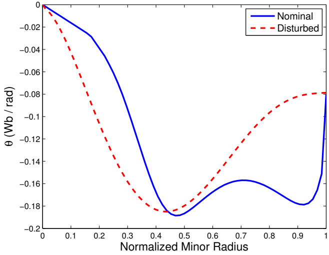


Simserver configuration with DIII-D PCS real-time code.

- A general framework for real-time feedforward + feedback control of magnetic and kinetic plasma profiles implemented in DIII-D PCS.
- Feedback portion of controller implemented as a discrete time state-space system with a sampling time of 20 milliseconds.
  - Interfaced with rtEFIT code [7] for magnetic profile control and rtCER code [8] for kinetic profile control.
- This PCS configuration provides the ability to test feedback controller in reference tracking and disturbance rejection simulations and experiments.

[7] J. Ferron et al., "Real Time Equilibrium Reconstruction for Tokamak Discharge Control," *Nuclear Fusion*, 1998.
[8] D. Piglowski et al., "Enhancements in the Second Generation DIII-D Digital Plasma Control System," *Fusion Engineering and Design*, 2007.

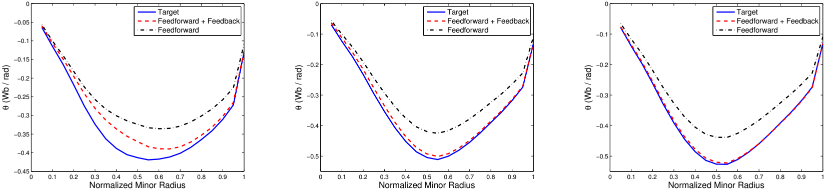
Simulation Study - Execution



Initial poloidal flux gradient profile $\theta(\hat{\rho})$ profile.

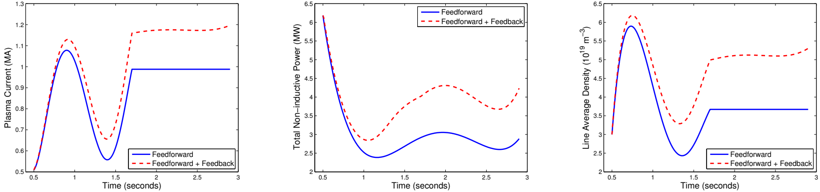
- Nominal model and initial condition and optimal feedforward control inputs [4,5] used to generate target poloidal flux gradient profile evolution $\theta_{tar}(\hat{\rho}, t)$.
- Feedforward control trajectories v_{FF} (generate $\theta_{FF}(\hat{\rho}, t)$), perturbed initial poloidal flux gradient profile, disturbed magnetic diffusion equation model, and no input disturbances v_d used in a closed-loop Simserver simulation.
- Reference vector set according to $r(\hat{\rho}, t) = \theta_{tar}(\hat{\rho}, t) - \theta_{FF}(\hat{\rho}, t)$.

Simulation Study - Reference Successfully Tracked



(a) (b) (c)

Poloidal flux gradient profile $\theta(\hat{\rho})$ at time (a) $t = 1.7$, (b) $t = 2.3$, and (c) $t = 2.9$ seconds.



(a) (b) (c)

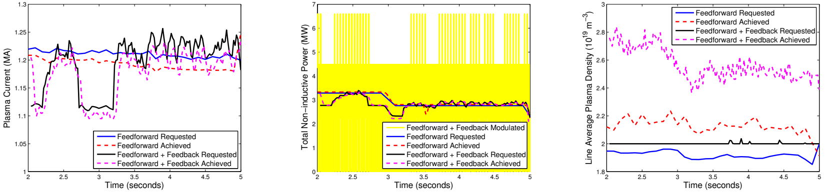
Control trajectory comparison: (a) plasma current (MA), (b) total non-inductive power (MW), and (c) line average density (10^{19} m^{-3}).

Experiment - Overview

- Experimentally test reference tracking (during ramp-up and early flat-top phases of discharge) and disturbance rejection (during flat-top phase of discharge) capabilities of feedback controller in DIII-D.
- It is important to note that requests made by combined feedforward + feedback controller are references to dedicated physical actuators.
  - Plasma current - a PID loop regulates ohmic coil voltage so plasma current, measured by a Rogowski loop, follows desired waveform generated by feedforward + feedback algorithm.
  - Line-averaged density - a PID loop regulates gas puffing and pumping to make line-averaged density measured by a CO_2 interferometer follow prescribed trajectory.
  - Total non-inductive power - directly controlled by power supplies.
- During experiments, MSE beam used to obtain q profile measurements in real-time is modulated on for 10 milliseconds then off for 10 milliseconds which translates to a 20 millisecond sampling time for feedback controller.

Experimental Setup - Disturbance Rejection

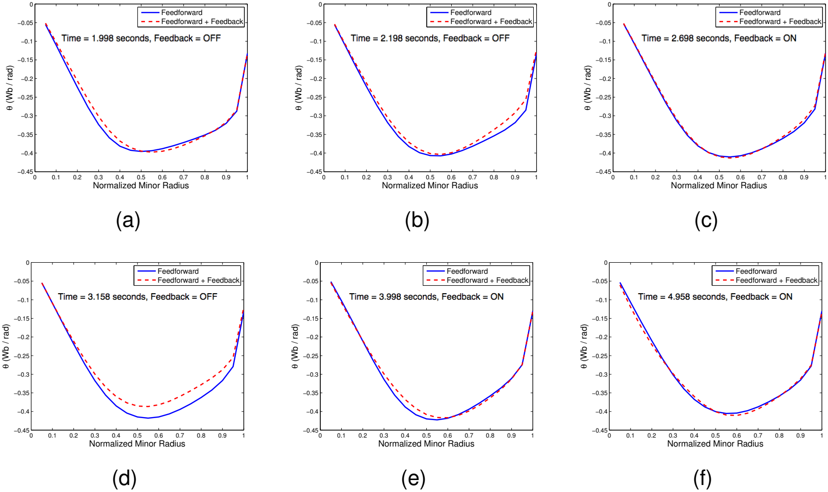
- Poloidal flux gradient profile evolution $\theta(\hat{\rho}, t)$ from DIII-D shot # 145477 (feedforward) is chosen as target profile evolution.
- In DIII-D shot # 146153 (feedforward + feedback), feedforward actuator trajectories from DIII-D shot # 145477 are applied during experimental time interval $t = [0.5, 2]$ seconds (ramp-up and early flat-top phases).
  - During experimental time interval $t = [2, 5]$ seconds (flat-top phase), a disturbance in control input u_3 of -0.1 MA is added to feedforward actuator trajectories from DIII-D shot # 145477.



(a) (b) (c)

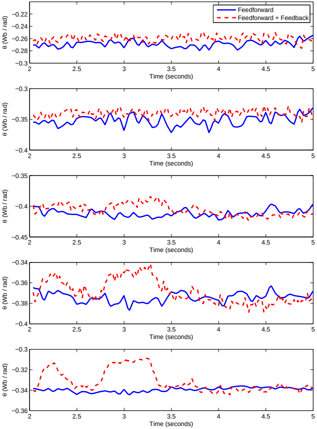
Control trajectory comparison: (a) plasma current (MA), (b) total non-inductive power (MW), and (c) line average density (10^{19} m^{-3}).

Experimental Profiles - Disturbance Rejection



Poloidal flux gradient profile $\theta(\hat{\rho})$ at time (a) $t = 1.998$, (b) $t = 2.198$, (c) $t = 2.698$, (d) $t = 3.158$, (e) $t = 3.998$, and (f) $t = 4.958$ seconds.

Experiment - Disturbance Successfully Rejected

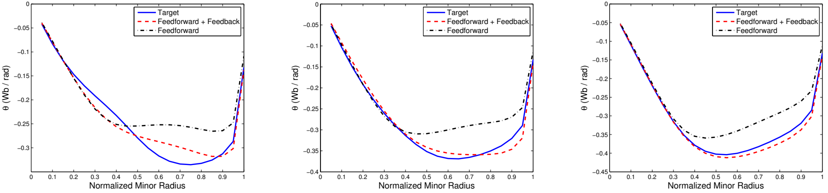


Time trace of poloidal flux gradient profile θ at normalized radii $\hat{\rho}$=0.3, 0.4, 0.6, 0.8, and 0.9 respectively (top to bottom) for disturbance rejection experiment.

Experimental Setup - Reference Tracking

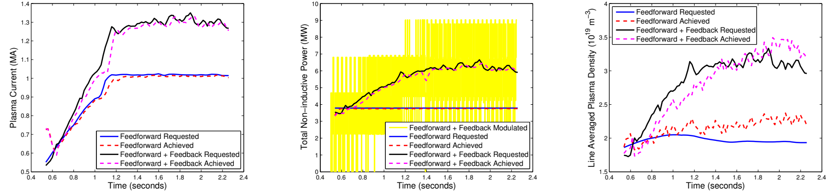
- A poloidal flux gradient profile evolution $\theta(\hat{\rho}, t)$ from a DIII-D discharge where feedforward control only was applied is chosen as target profile evolution.
- Another feedforward control discharge was used to generate a second poloidal flux gradient profile evolution in DIII-D shot # 146411.
- Finally, in DIII-D shot # 146458 (feedforward + feedback), feedforward actuator trajectories from DIII-D shot # 146411 (feedforward) are combined with feedback controller (16) to track target poloidal flux gradient profile evolution.
- For the DIII-D discharges used to test feedback controller, ramp-up phase is associated with the time $t = [0.5, 1.2]$ seconds, and early flat-top phase corresponds to time $t = [1.2, 2.25]$ seconds.

Experimental Profiles - Reference Tracking



(a) (b) (c)

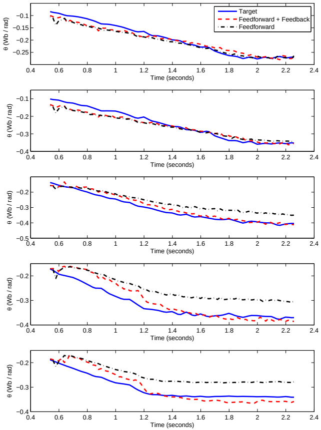
Poloidal flux gradient profile $\theta(\hat{\rho})$ at time (a) $t = 1.218$, (b) $t = 1.618$, and (c) $t = 2.258$ seconds.



(a) (b) (c)

Control trajectory comparison: (a) plasma current (MA), (b) total non-inductive power (MW), and (c) line average density (10^{19} m^{-3}).

Experiment - Reference Successfully Tracked



Time trace of poloidal flux gradient profile θ at normalized radii $\hat{\rho}$=0.3, 0.4, 0.6, 0.8, and 0.9 respectively (top to bottom) for reference tracking experiment.

Conclusions and Future Work

- Robust feedback controller was synthesized to control poloidal flux gradient profile evolution in DIII-D.
- Controller was successfully implemented in the DIII-D PCS, interfaced with the available real-time measurements, and tested experimentally during both the ramp-up and flat-top phases of L-mode discharges.
- Working towards developing a control-oriented model of poloidal flux profile evolution valid in H-mode plasma discharges that incorporates the effects of bootstrap current.
- Also of interest would be development of first principles based, control-oriented models of kinetic plasma profiles, such as the electron temperature.
  - Subsequent synthesis of feedforward + feedback control schemes to regulate both magnetic and kinetic profiles around desired target profiles simultaneously in H-mode discharges.