Burn control in fusion reactors via isotopic fuel tailoring

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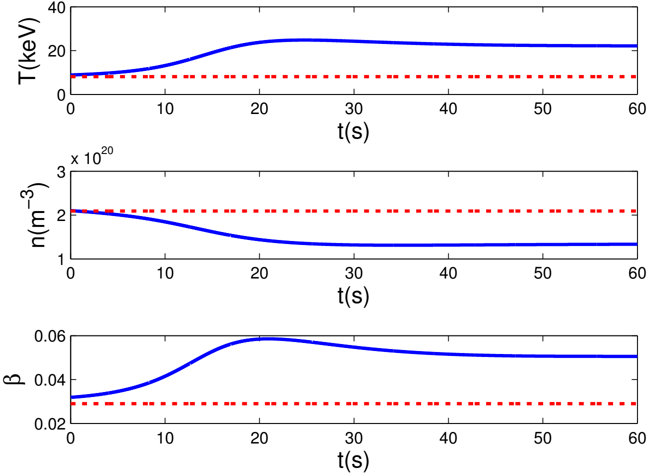
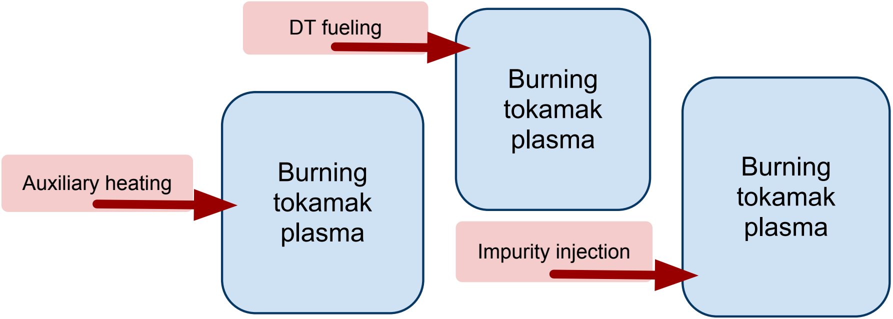
The Need for Burn Control

- For tokamak fusion power to become an economical source of energy, reactors must operate for extended periods of time with a high power gain
- The gain is determined by the burn condition, i.e. the plasma density and temperature
- Some of the more economically and technologically attractive operating points may be unstable
  - Small decreases in temperature can lead to quenching
  - Small increases can cause the system to move to a stable, but undesired, high-temperature equilibrium
- Even when operating at stable equilibria, system performance during transients and disturbances could be undesirable without control
- **Active burn control could enable operation at unstable operating points and could improve overall reactor performance.**

An Unstable Equilibrium

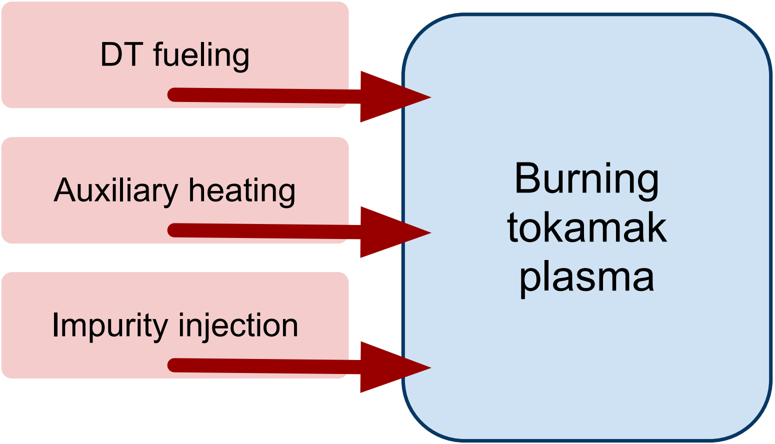


Approaches Used in Previous Studies



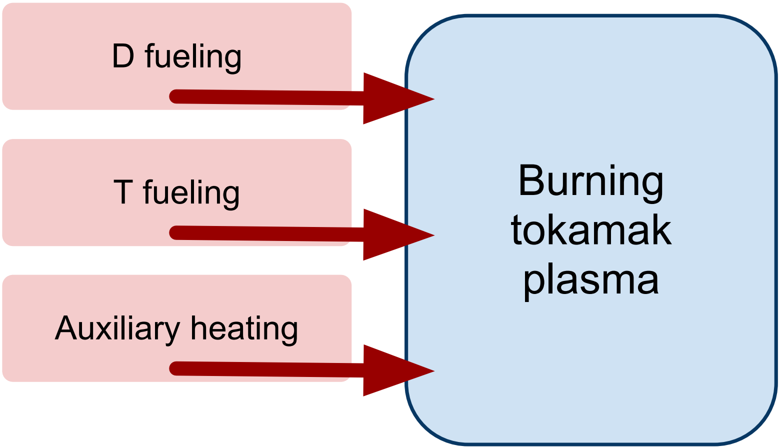
Many previous works **utilize only one of the available actuators**. Most also use **linearized models** to generate control laws. The stable operating range of these controllers is therefore limited.

Approach Used in Our Previous Work



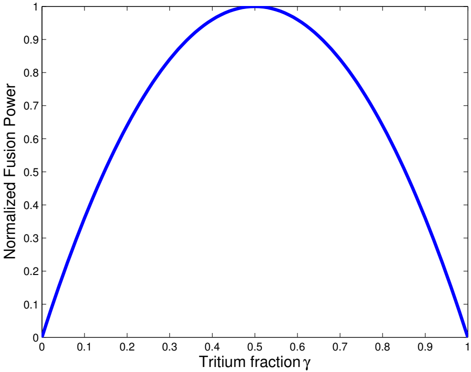
Our previous work **combined actuation methods** and used a **nonlinear control design** to extend the region of attraction of the control scheme. However, the use of impurity injection could cause accumulation of impurities

Approach Used in This Work



In this work we continue the use of **nonlinear control design** techniques, while **avoiding impurity injection** by adjusting the deuterium and tritium fuel sources separately, a technique called **isotopic fueling**.

Isotopic Fueling



By adjusting the relative density of tritium and deuterium in the plasma, denoted as $\gamma = n_T/(n_D + n_T)$, the fusion power output can be modulated.

Model Used for Controller Design

$$\text{Alpha particles: } \frac{dn_\alpha}{dt} = -\frac{n_\alpha}{\tau_\alpha} + S_\alpha \quad (1)$$

$$\text{Deuterium: } \frac{dn_D}{dt} = -\frac{n_D}{\tau_D} - S_\alpha + S_D \quad (2)$$

$$\text{Tritium: } \frac{dn_T}{dt} = -\frac{n_T}{\tau_T} - S_\alpha + S_T \quad (3)$$

$$\text{Energy: } \frac{dE}{dt} = -\frac{E}{\tau_E} + Q_\alpha S_\alpha - P_{rad} + P_{aux} \quad (4)$$

$$\text{Reaction rate: } S_\alpha = \gamma (1-\gamma) n_H^2 \langle \sigma \nu \rangle \quad (5)$$

$$\text{Radiation losses: } P_{rad} = A_b Z_{eff} n_e^2 \sqrt{T} \quad (6)$$

Desired Equilibrium

An equilibrium can be obtained by choosing three parameters, for example: $\bar{T}, \bar{n}, \bar{\gamma}$, and solving:

$$\text{Alpha particles: } 0 = -\frac{\bar{n}_\alpha}{\tau_\alpha} + \bar{S}_\alpha \tag{7}$$

$$\text{Deuterium: } 0 = -\frac{\bar{n}_D}{\tau_D} - \bar{S}_\alpha + \bar{S}_D \tag{8}$$

$$\text{Tritium: } 0 = -\frac{\bar{n}_T}{\tau_T} - \bar{S}_\alpha + \bar{S}_T \tag{9}$$

$$\text{Energy: } 0 = -\frac{\bar{E}}{\tau_E} + Q_\alpha \bar{S}_\alpha - \bar{P}_{rad} + \bar{P}_{aux} \tag{10}$$

Stabilization of Energy Subsystem

Defining $\tilde{E} = E - \bar{E}$, we can obtain

$$\frac{d\tilde{E}}{dt} = -\frac{\tilde{E}}{\tau_E} - \frac{\bar{E}}{\tau_E} + Q_\alpha S_\alpha - P_{rad} + P_{aux} \quad (11)$$

we can cancel the terms in red and stabilize the subsystem by satisfying the following condition:

$$Q_\alpha S_\alpha - P_{rad} + P_{aux} = \frac{\bar{E}}{\tau_E} \quad (12)$$

This condition can be satisfied by modulating

- P_{aux} directly
- P_{rad} through impurity injection
- S_α through the tritium fraction

In this work we avoid using impurity injection and instead use modulation of the tritium fraction.

Isotopic Fuel Tailoring Controller Design

Desired tritium fraction

We first set $P_{aux} = 0$ and seek a tritium fraction $\gamma = \gamma^*$ that satisfies condition (12).

$$Q_\alpha \gamma^*(1-\gamma^*) n_H^2 \langle \sigma \nu \rangle - P_{rad} = \frac{\bar{E}}{\tau_E} \tag{13}$$

- Either two real solutions or two complex solutions
- If real, we take the solution satisfying $\gamma^* \leq 0.5$
- If complex, auxiliary heating is needed to satisfy the condition, so we set $\gamma^* = 0.5$ and move on

Heating control law

We use the desired tritium ratio to calculate

$$P_{aux} = \frac{\bar{E}}{\tau_E} - Q_\alpha \gamma^*(1-\gamma^*) n_H^2 \langle \sigma \nu \rangle + P_{rad} \tag{14}$$

Isotopic Fuel Tailoring Controller Design

γ is a virtual input, so it has dynamics. We define the error

$$\hat{\gamma} = \gamma - \gamma^* \tag{15}$$

which is governed by

$$\dot{\hat{\gamma}} = \frac{1}{n_H}\left[-\frac{n_T}{\tau_T} - S_\alpha + S_T - n_H\dot{\gamma}^* - \gamma\left(-\frac{n_T}{\tau_T} - \frac{n_D}{\tau_D} - 2S_\alpha + S_D + S_T\right)\right] \tag{16}$$

The error between the actual and desired tritium ratio enters into the energy dynamics:

$$\frac{d\tilde{E}}{dt} = -\frac{\tilde{E}}{\tau_E} + \hat{\gamma}\phi \tag{17}$$

Finally, the dynamics of the plasma density deviations can be written as

$$\begin{aligned}\dot{\tilde{n}} &= -3\frac{n_\alpha}{\tau_\alpha} - 2\frac{n_T}{\tau_T} - 2\frac{n_D}{\tau_D} - (Z_I+1)\frac{n_I}{\tau_I} \\ &\quad - S_\alpha + 2S_D + 2S_T + (Z_I+1)S_I\end{aligned} \tag{18}$$

Isotopic Fuel Tailoring Controller Design

We take the Lyapunov functional candidate

$$V = \frac{k_1^2 \tilde{E}^2 + k_2^2 \hat{\gamma}^2 + \tilde{n}^2}{2}$$

where $k_1 \approx 10^{15}$ and $k_2 \approx 10^{20}$ (recall that $\tilde{E} \approx 10^5$, $\hat{\gamma} \approx 10^{-1}$, and $\tilde{n} \approx 10^{20}$). We then compute the time derivative of V as

$$\begin{aligned} \dot{V} &= k_1^2 \tilde{E} \dot{\tilde{E}} + k_2^2 \hat{\gamma} \dot{\hat{\gamma}} + \tilde{n}_H \dot{\tilde{n}}_H \\ &= \frac{k_2^2 \hat{\gamma}}{n_H} \left[\frac{k_1^2 n_H \tilde{E} \phi}{k_2^2} - \frac{n_T}{\tau_T} - S_\alpha + S_T - n_H \dot{\gamma}^* \right. \\ &\quad \left. -\gamma \left(-\frac{n_T}{\tau_T} - \frac{n_D}{\tau_D} - 2S_\alpha + S_D + S_T \right) \right] \\ &\quad + \tilde{n} \left[-3 \frac{n_\alpha}{\tau_\alpha} - 2 \frac{n_T}{\tau_T} - 2 \frac{n_D}{\tau_D} - (Z_I + 1) \frac{n_I}{\tau_I} \right. \\ &\quad \left. -S_\alpha + 2S_D + 2S_T + (Z_I + 1) S_I \right] - \frac{k_1^2 \tilde{E}^2}{\tau_E} \end{aligned} \qquad (19)$$

Isotopic Fuel Tailoring Controller Design

Tritium fueling control law

$$S_T = -\frac{k_1^2 n_H \tilde{E}\phi}{k_2^2} + \frac{n_T}{\tau_T} + S_\alpha + n_H\dot{\gamma}^* - K_\gamma\hat{\gamma} + \gamma\left(-\frac{3}{2}S_\alpha + \frac{3}{2}\frac{n_\alpha}{\tau_\alpha} + \frac{(Z_I+1)}{2}\frac{n_I}{\tau_I} - \frac{(Z_I+1)}{2}S_I - \frac{1}{2}K_n\tilde{n}\right) \quad (20)$$

Deuterium fueling control law

$$S_D = \frac{1}{2}\left[3\frac{n_\alpha}{\tau_\alpha} + 2\frac{n_D}{\tau_D} + 2\frac{k_1^2 n_H \tilde{E}\phi}{k_2^2} - S_\alpha - K_n\tilde{n} + 2K_\gamma\hat{\gamma} - 2n_H\dot{\gamma}^* - \gamma\left(-3S_\alpha + 3\frac{n_\alpha}{\tau_\alpha} + (Z_I+1)\left(\frac{n_I}{\tau_I} - S_I\right) - K_n\tilde{n}\right)\right] \quad (21)$$

where $K_\gamma > 0$ and $K_n > 0$

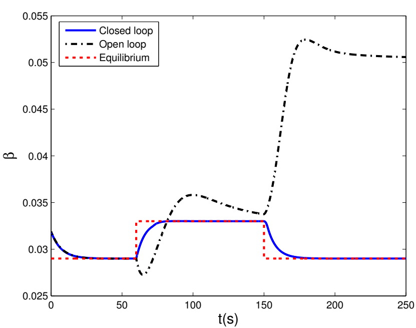
Isotopic Fuel Tailoring Controller Design

The choice of control laws (20) and (21) reduces (19) to

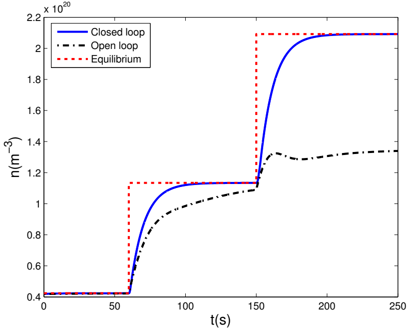
$$\dot{V} = -\frac{k_1^2 \tilde{E}^2}{\tau_E} - K_n \tilde{n}^2 - \frac{k_2^2}{n_H} K_\gamma \hat{\gamma}^2$$

- This shows that the control laws guarantee asymptotic stability of the energy, plasma density and requested tritium ratio.
- Since no linearization of the model was used, the control laws globally stabilize the model used for design
- The scheme works for both ignited and sub-ignited operating points
- The controller depends parametrically on the equilibrium, so it can be used to go from one operating point to another

Simulation Testing - Sub-ignition

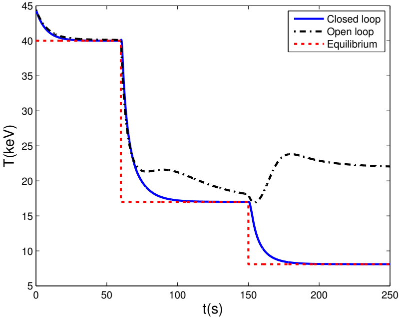


(a) Plasma β

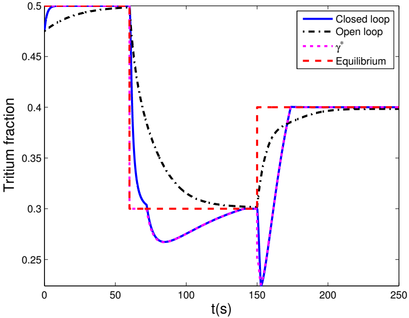


(b) Plasma density

Simulation Testing - Sub-ignition

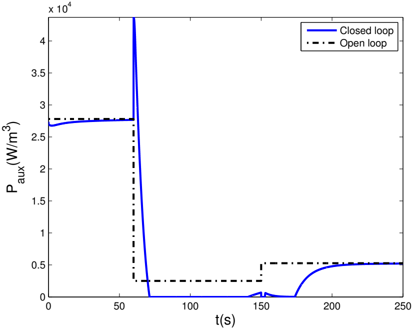


(c) Temperature

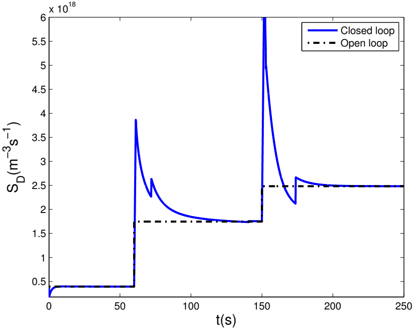


(d) Tritium fraction

Simulation Testing - Sub-ignition

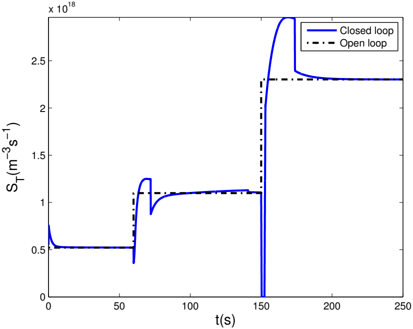


(e) Auxiliary heating

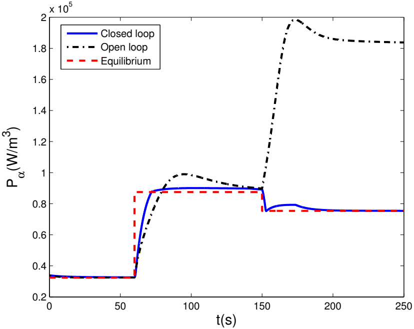


(f) Deuterium fueling

Simulation Testing - Sub-ignition

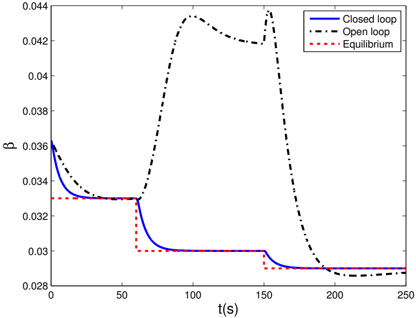


(g) Tritium fueling

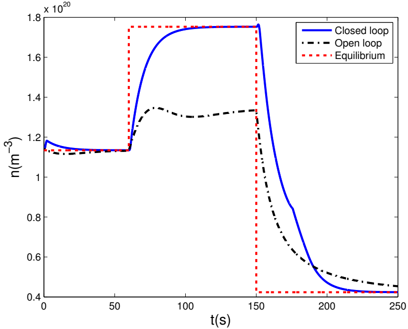


(h) α-heating

Simulation Testing - Ignition

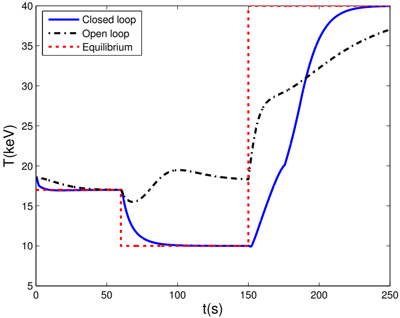


(i) Plasma β

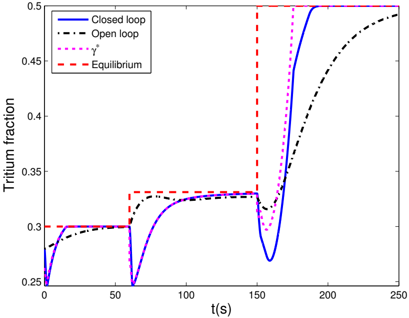


(j) Plasma density

Simulation Testing - Ignition

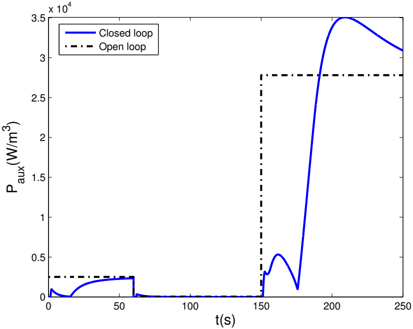


(k) Temperature

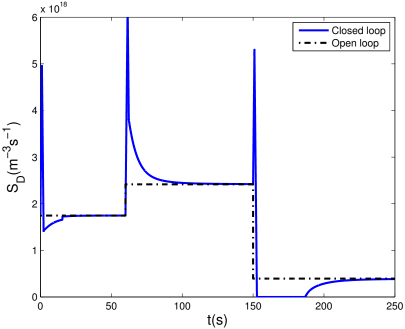


(l) Tritium fraction

Simulation Testing - Ignition

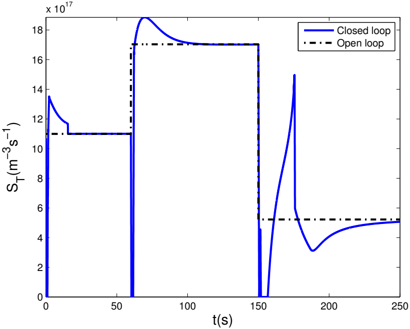


(m) Auxiliary heating

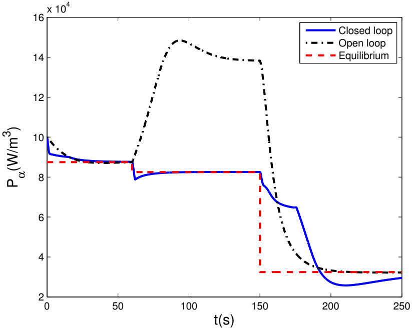


(n) Deuterium fueling

Simulation Testing - Ignition



(o) Tritium fueling



(p) α-heating

Conclusions and Future Work

- A multi-input nonlinear fusion burn controller has been proposed
- Impurity injection is avoided by use of isotopic fuel tailoring
- The controller depends parametrically on the operating point and works for ignited and sub-ignited equilibria
- Simulation results show good performance
  - Stabilization of unstable equilibria
  - Improved transient performance when moving between equilibria
  - Successful transition between ignited and sub-ignited points
- Future work will focus on dealing with model uncertainty and spatially distributed effects