Heat and Particle Pinches from Beam-Ion Friction

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Motivation

- Present models for neutral beam injection (NBI) in major transport codes routinely include the following effects on the thermal plasma:
  - Prompt source of thermal electrons
  - Delayed source of thermal ions (after slowing down)
  - Energy source to thermal ions and electrons during slowing down
  - Net toroidal torque to thermal ions when injection is unbalanced toroidally
  - Non-inductive current drive
- But anomalies result when these models are used to analyze some present plasmas with strongly reduced turbulence and unbalanced NBI:
  - Negative effective ion thermal conductivity with co-NBI in NSTX
  - Ion thermal conductivity below standard neoclassical in ITBs
- Additional effects of unbalanced NBI, estimated in various theoretical treatments since the early ’70s, suggest that physics missing in the analysis codes might provide an explanation:
  - Neoclassical particle and energy fluxes driven by the parallel friction and heat friction between fast ions on thermal ions
  - Viscous heating from poloidal rotation of thermal ions driven by the fast ions
- These are possibly stronger in NSTX with ~80keV NBI in ~1-2keV plasmas

Neoclassical Parallel Force Balances with NBI

S.P. Hirshman and D.J. Sigmar, *Nucl. Fusion* **21** (1981) 1079
W.A. Houlberg, K.C. Shaing, S.P. Hirshman and M.C. Zarnstorff, *Phys. Plasmas* **4** (1997) 3230

- **In NCLASS we use the Hirshman-Sigmar formulation of neoclassical theory for a multiple species plasma.**
- **The flux surface averaged parallel force and heat force balance equations for thermal species j come from the odd velocity moments of the Boltzmann equation on a timescale t>>τ_{jj}:**

$$\langle \vec{B}\cdot\vec{\nabla}\cdot\vec{\Pi}_j\rangle = \langle \vec{F}_{1,j}\cdot\vec{B}\rangle + \langle \vec{F}^b_{1,j}\cdot\vec{B}\rangle + e_j n_j \langle \vec{E}\cdot\vec{B}\rangle \qquad \boxed{\text{Force}}$$

$$\langle \vec{B}\cdot\vec{\nabla}\cdot\vec{\Theta}_j\rangle = \langle \vec{F}_{2,j}\cdot\vec{B}\rangle + \langle \vec{F}^b_{2,j}\cdot\vec{B}\rangle \qquad \boxed{\text{Heat force}}$$

Viscous | **Thermal Friction** | **Beam Friction** | **Ohmic**

- **The beam friction terms provide a parallel force on the thermal species:**
  - **That is analogous to the thermal friction terms (between two thermal species)**
  - **That acts to drive fluxes just as the force from the parallel electric field drives fluxes (Ware pinch)**

The Thermal Friction and Viscous Forces are Functions of the Parallel and Poloidal Flows

- **Classical parallel friction forces between thermal species:**

$$\langle \vec{F}_{\alpha,j} \cdot \vec{B} \rangle = \sum_k \sum_\beta \ell^{jk}_{\alpha\beta} \hat{u}_{\parallel\beta,k}$$

Thermal friction coefficients

$$\hat{u}_{\parallel 1,j} \equiv \langle \vec{u}_j \cdot \vec{B} \rangle$$

Parallel flow

$$\hat{u}_{\parallel 2,j} \equiv \frac{2}{5} \frac{\langle \vec{q}_j \cdot \vec{B} \rangle}{p_j}$$

Parallel heat flow

Friction forces are a function of the parallel flows of all species

- **Neoclassical parallel viscous forces for thermal species:**

$$\langle \vec{B} \cdot \vec{\nabla} \cdot \vec{\Pi}_j \rangle = \langle B^2 \rangle \sum_\beta \hat{\mu}_{1\beta,j} \, \hat{u}_{\theta\beta,j}$$

$$\langle \vec{B} \cdot \vec{\nabla} \cdot \vec{\Theta}_j \rangle = \langle B^2 \rangle \sum_\beta \hat{\mu}_{2\beta,j} \, \hat{u}_{\theta\beta,j}$$

Thermal viscosity coefficients

$$\hat{u}_{\theta 1,j} \equiv \frac{\langle \vec{u}_j \cdot \vec{\nabla}\theta \rangle}{\langle \vec{B} \cdot \vec{\nabla} B \rangle}$$

Poloidal flow

$$\hat{u}_{\theta 2,j} \equiv \frac{2}{5} \frac{1}{p_j} \frac{\langle \vec{q}_j \cdot \vec{\nabla}\theta \rangle}{\langle \vec{B} \cdot \vec{\nabla} B \rangle}$$

Poloidal heat flow

Viscous forces are a function of the poloidal flows of own species

- **Kinetic theory provides the viscosity and friction coefficients**

The Radial Force Balances Relate the Poloidal and Toroidal Flows to T', p', and Φ'

- **The radial force balance equation for each odd velocity moment (α=1,2,3) relates the flows within a surface to gradients of flux surface quantities (driving forces):**

$$\begin{aligned}
\langle B^2\rangle \hat{u}_{\theta\alpha,j} &= \hat{u}_{\|\alpha,j} + S_{\theta\alpha,j} \qquad \alpha = 1,2,3 \\
S_{\theta 1,j} &= \frac{2\pi R B_t}{\Psi'}\left(\frac{p_j'}{e_j n_j} + \Phi'\right) \\
S_{\theta 2,j} &= \frac{2\pi R B_t}{\Psi'}\frac{kT_j'}{e_j}
\end{aligned}$$

- **This allows us to eliminate the toroidal flows and solve a matrix of equations for the poloidal flows for each charge state j**
- **The addition of NBI friction and heat friction terms leads to a modification to the poloidal flows (and the toroidal flows through the radial force balances), and consequently the other neoclassical transport properties**

Neoclassical Particle and Heat Fluxes with NBI

- The banana plateau (BP) fluxes are related to the poloidal flows:

$$\Gamma_j^{\mathrm{BP}} = -\frac{2\pi R B_t}{\Psi' e_j} \sum_\beta \hat{\mu}_{1\beta,j} \left(\hat{u}^{nc}_{\theta\beta,j} + \hat{u}^{b}_{\theta\beta,j}\right)$$

$$q_j^{\mathrm{BP}} = -\frac{2\pi R B_t k T_j}{\Psi' e_j} \sum_\beta \hat{\mu}_{2\beta,j} \left(\hat{u}^{nc}_{\theta\beta,j} + \hat{u}^{b}_{\theta\beta,j}\right)$$

Flows of thermal species driven by NBI modify BP

  – Importance relative to standard neoclassical is governed by the ratio of induced poloidal flows

- The Pfirsch-Schlüter (PS) fluxes are related directly to the forces:

$$\Gamma_j^{\mathrm{PS}} = \frac{2\pi R B_t}{\Psi' e_j} \left[1/\langle B^2\rangle - \langle B^{-2}\rangle\right] \left[\langle \vec{F}_{1,j} \cdot \vec{B}\rangle + \langle \vec{F}^b_{1,j} \cdot \vec{B}\rangle\right]$$

$$q_j^{\mathrm{PS}} = \frac{2\pi R B_t k T_j}{\Psi' e_j} \left[1/\langle B^2\rangle - \langle B^{-2}\rangle\right] \left[\langle \vec{F}_{2,j} \cdot \vec{B}\rangle + \langle \vec{F}^b_{2,j} \cdot \vec{B}\rangle\right]$$

NBI friction and heat friction modify PS

  – Importance relative to standard neoclassical governed by the ratio of forces

- All of the NBI terms are inward with co-injection, outward with counter-injection (testable by co-/counter-NBI comparisons)

The Radial Force Balances with NBI

- Radial force balance equations in terms of parallel and poloidal flows:

$$\langle B^2\rangle\left[\hat{u}^{nc}_{\theta 1,j} + \hat{u}^{b}_{\theta 1,j}\right] = \left[\hat{u}^{nc}_{\parallel 1,j} + \hat{u}^{b}_{\parallel 1,j}\right] + \frac{2\pi R B_t}{\Psi'}\left[\frac{p'_j}{e_j n_j} + \Phi'\right] \quad \boxed{\text{Force}}$$

$$\langle B^2\rangle\left[\hat{u}^{nc}_{\theta 2,j} + \hat{u}^{b}_{\theta 2,j}\right] = \left[\hat{u}^{nc}_{\parallel 2,j} + \hat{u}^{b}_{\parallel 2,j}\right] + \frac{2\pi R B_t}{\Psi'}\frac{kT'_j}{e_j} \quad \boxed{\text{Heat force}}$$

Poloidal flow induced by NBI
(typically only use neoclassical value)

Parallel flow induced by NBI
(typically use measured total toroidal flow)

- NBI effects on poloidal rotation should be considered when:
  - Determining the radial electric field using the radial force balance with theoretical models for the poloidal rotation
  - Comparing measured poloidal rotation with theoretical models

Viscous Heating with NBI

- **The neoclassical viscous heating is enhanced as the square of the poloidal flow velocities:**

$$
\begin{aligned}
P^{\mathrm{BP}}_{\mu,j} &= \hat{u}_{\theta 1,j}\langle \vec{B}\cdot\vec{\nabla}\cdot\vec{\Pi}_j\rangle \\
&= \left[\hat{u}^{nc}_{\theta 1,j} + \hat{u}^{b}_{\theta 1,j}\right]\langle B^2\rangle \sum_{\beta} \hat{\mu}_{1\beta,j}\left[\hat{u}^{nc}_{\theta\beta,j} + \hat{u}^{b}_{\theta\beta,j}\right]
\end{aligned}
$$

Poloidal flow from NBI modifies poloidal viscous heating

- **In addition to this are heating terms from classical (small) and toroidal viscosity (e.g., Stacey's gyroviscosity, turbulence induced viscosity, …)**
- **Viscous heating is likely important for:**
  - **ITBs with strong pressure gradients**
  - **Strong toroidal rotation (NSTX)**

NBI Friction and Heat Friction Terms

S.P. Hirshman and D.J. Sigmar, *Nucl. Fusion* **21** (1981) 1079
J.P. Wang, M. Azumi, K. Tani, J.D. Callen, *Nucl. Fusion* **34** (1994) 231

- **The general expressions for the NBI friction terms are expressed in terms of Laguerre polynomials:**

$$
\begin{aligned}
\vec{F}^{b}_{1,j} &= \int d^3v\, m_j \vec{v} L_0^{(3/2)}(x_j^2) C_{jb}[f_j, f_b] \\
\vec{F}^{b}_{2,j} &= -\int d^3v\, m_j \vec{v} L_1^{(3/2)}(x_j^2) C_{jb}[f_j, f_b] \\
L_0^{(3/2)}(x_j^2) &= 1 \\
L_1^{(3/2)}(x_j^2) &= \frac{5}{2} - x_j^2 \\
x_j &= v/v_{tj} \\
v_{tj} &= \left(\frac{2kT_j}{m_j}\right)^{1/2}
\end{aligned}
$$

Coulomb Collision Operator

J.P. Wang, M. Azumi, K. Tani, J.D. Callen, *Nucl. Fusion* **34** (1994) 231

- **The Coulomb collision operator can be written in terms of the Fokker-Planck coefficients, which are in turn expressed in terms of the Rosenbluth potentials**

$$
\begin{aligned}
C_{jb}[f_j, f_b] &= -\vec{\nabla}_v \cdot \left[\vec{A}_{jb} f_j - \frac{1}{2}\vec{\nabla}_v \cdot \left(\vec{D}_{jb} f_j\right)\right] \\
\vec{A}_{jb} &= \left(1 + \frac{m_j}{m_b}\right) \gamma_{jb} \vec{\nabla}_v h_b \\
\vec{D}_{jb} &= \gamma_{jb} \vec{\nabla}_v \vec{\nabla}_v g_b \\
\gamma_{jb} &= \frac{4\pi}{(4\pi\epsilon_0)^2} \frac{e_j^2 e_b^2 \ln \Lambda_j}{m_j^2} \\
h_b &= \int d^3v' \frac{f_b(v')}{|\vec{v} - \vec{v}'|} \\
g_b &= \int d^3v' f_b(v') |\vec{v} - \vec{v}'|
\end{aligned}
$$

Test Particle Distribution with Flows

J.P. Wang, M. Azumi, K. Tani, J.D. Callen, *Nucl. Fusion* **34** (1994) 231

- **For the thermal particle distribution we keep the flow and heat flow perturbations from the Maxwellian:**

$$
\begin{aligned}
f_j(v) &= \left[1 + \frac{2\vec{v}\cdot\vec{u}_j}{v_{tj}^2} L_0^{(3/2)}(x_j^2) - \frac{2\vec{v}\cdot\vec{q}_j}{v_{tj}^2 p_j}\frac{2}{5} L_1^{(3/2)}(x_j^2)\right] f_{0j} \\
f_{0j} &= \frac{n_j}{\pi^{3/2} v_{tj}^3} e^{-x_j^2}
\end{aligned}
$$

- **Except for the work of Wang, et al, the the flow corrections are usually ignored for the thermal ions.**

Thermal Electron-NBI Terms

J.P. Wang, M. Azumi, K. Tani, J.D. Callen, *Nucl. Fusion* **34** (1994) 231

- **For the thermal electrons we assume v >> v', therefore the only information needed for the beam ions is the total fast ion density, and the friction is expressed in terms of the electron flow and heat flow:**

$$
\begin{aligned}
\vec{F}^b_{1,e} &= -\frac{m_e n_b}{\tau_{eb}}\left(\vec{u}_e - \vec{u}_b - \frac{3}{5}\frac{\vec{q}_e}{p_e}\right) \\
\vec{F}^b_{2,e} &= \frac{m_e n_b}{\tau_{eb}}\left(\frac{3}{2}\vec{u}_e - \frac{3}{2}\vec{u}_b - \frac{13}{10}\frac{\vec{q}_e}{p_e}\right) \\
\frac{1}{\tau_{jb}} &= \frac{4}{3\pi^{1/2}}\frac{m_j n_j \gamma_{jb}}{v_{tj}^2}
\end{aligned}
$$

- **Taking the flux surface average of the force parallel to B and using the normalized flows yields:**

$$
\begin{aligned}
\langle \vec{F}^b_{1,e} \cdot \vec{B} \rangle &= -\frac{m_e n_b}{\tau_{eb}}\left(\hat{u}_{\parallel 1,e} - \hat{u}_{\parallel 1,b} - \frac{3}{2}\hat{u}_{\parallel 2,e}\right) \\
\langle \vec{F}^b_{2,e} \cdot \vec{B} \rangle &= \frac{m_e n_b}{\tau_{eb}}\frac{3}{2}\left(\hat{u}_{\parallel 1,e} - \hat{u}_{\parallel 1,b} - \frac{13}{6}\hat{u}_{\parallel 2,e}\right)
\end{aligned}
$$

Thermal Ion-NBI Terms

Theory Under Development

- **Existing theoretical models have limitations for quantitative applications:**
  - **The work of Wang, et al., assumes an isotropic distribution in the rest frame of the fast ions, which is not appropriate for unbalanced NBI.**
  - **Hinton and Kim approximate the parallel force using the toroidal torque, and don't include the heat friction.**
  - **All the theoretical estimates assume simplified forms for the fast ion distribution function.**
  - **…**
- **Work on evaluating the ion integrals (tedious) using NSTX geometry and NBI distributions is in progress:**
  - **Improve the approximate theoretical estimates for the NBI friction and heat friction terms for input to NCLASS**
  - **Apply to NSTX using NCLASS in FORCEBAL and TRANSP**

Summary of NBI Driven Neoclassical Effects

- There is an extensive literature on the theory for NBI driven neoclassical fluxes and viscous heating
- Various of these analyses have shown the effects are comparable to the standard neoclassical effects
- Much of that literature considers individual effects and approximations that can be treated more comprehensively
- These effects should be most visible in experiments with unbalanced NBI when turbulence induced transport is suppressed
  - NSTX cases where negative effective ion thermal conductivity is inferred
- The primary work for a quantitative study would be to evaluate the parallel NBI friction and heat friction in TRANSP for input to NCLASS

Other Extensions to Neoclassical Theory for NSTX

- The low aspect ratio and toroidal field in NSTX limit the applicability of other approximations used in standard neoclassical theory:
  - Small banana width compared with system size
    - » Addressed by potato orbit models near the axis (which vary in their predictions of whether thermal ion transport is enhanced, decreased or unaffected)
  - Small gyroradius compared to banana width
    - » Classical and neoclassical effects become comparable in NSTX and theory/models might have to be revised
- Our approach is to watch for indications in NSTX of deviations from standard neoclassical theory, and extend the theory and models as necessary.
- The most vulnerable elements of standard neoclassical theory in NSTX are:
  - Ion thermal transport
  - Impurity transport
  - NBI current drive
  - Bootstrap current