Physics Design Bases of a Spherical Torus with a Plasma Center Column

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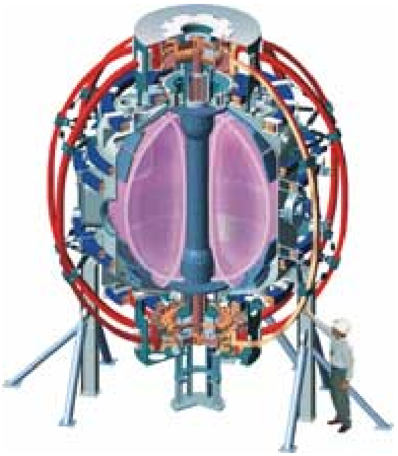
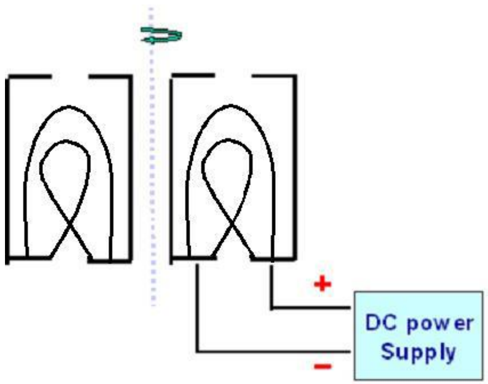
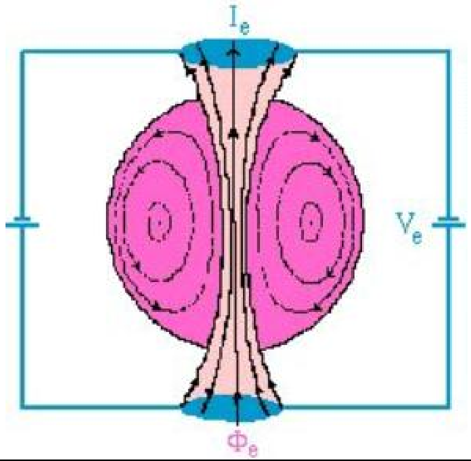
What is a ST-PCC?







ST: NSTX

ST-CHI: NSTX, HIT

ST-PCC: Proto-Sphera

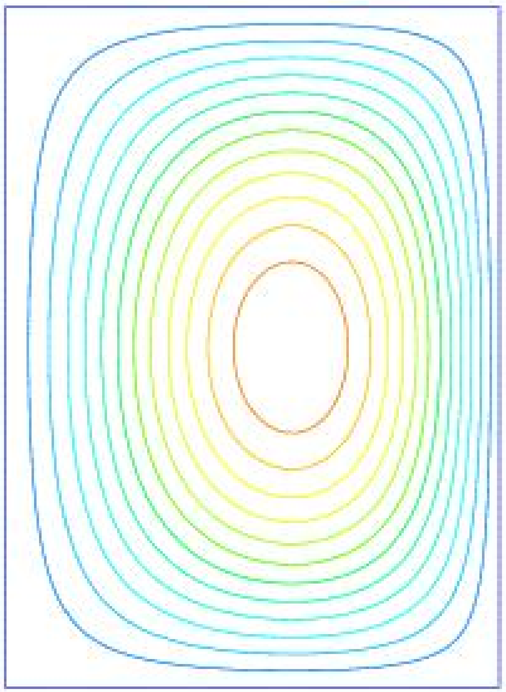
- Replace center post by plasma center column (PCC).
  - Like the flux-core spheromak. Differences?
- Biased Screw Pinch acts as TF coils → Tokamak q.
- Naturally formed by driven-relaxation.
- Beta ramp up and sustainment by auxilliary heating and current drive.

What is a classical Spheromak?

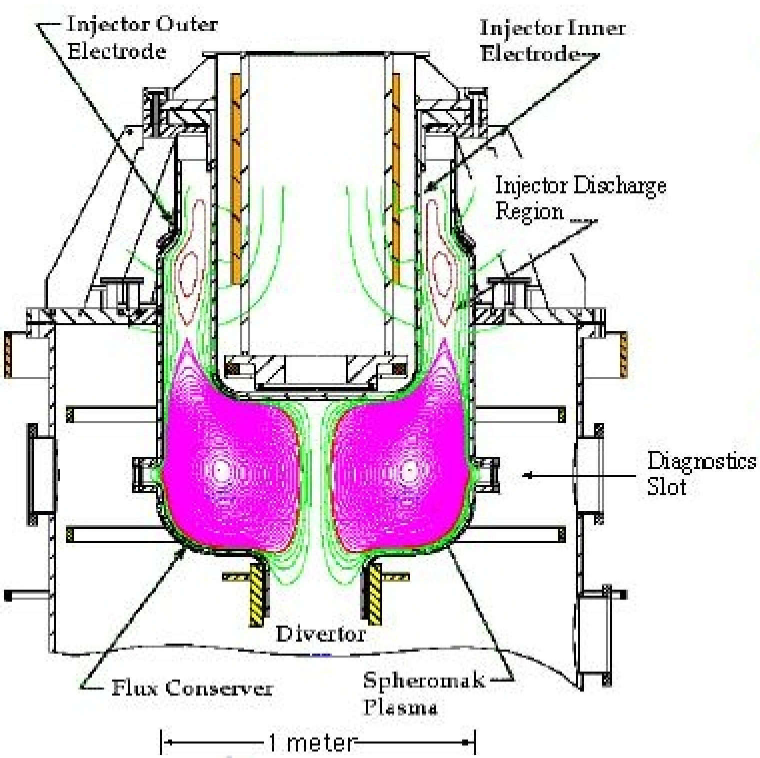
- Defined by Rosenbluth and Bussac as the first Chandrasekhar-Kendall mode,

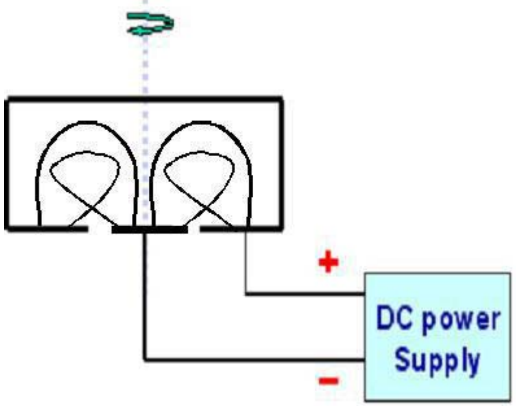
$$\nabla \times \mathrm{B}_1 = k_1 \mathrm{B}_1, \quad \mathrm{B}_1 \cdot \hat{\mathrm{n}}|_{\partial\Omega} = 0.$$

- Characteristics:
  - Singly connected separatrix.
    - Rarely satisfied.
  - q < 1.
    - Requires oblate chamber.
  - Negative magnetic shear.
    - Most robust feature.



What is an actual Spheromak?

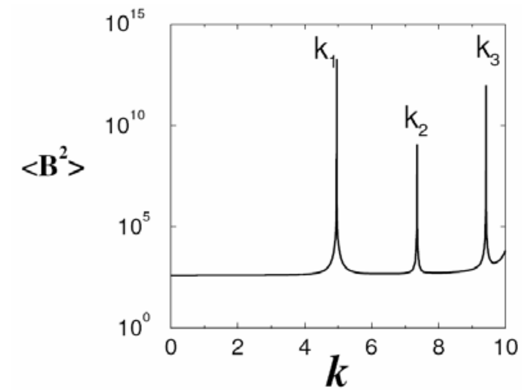




- Requires open flux intercepting electrodes to facilitate helicity injection.
  - gun and flux-core Spheromaks.
- Resonance at first CK mode prevents a classical Spheromak.
  - Interior magnetic separatrix.
- Current on both open and closed fluxes provides freedom to manipulate q profile.
- Robust formation, flux amplification factor ~ 2, electron Te ~ 400 eV in decaying Spheromak. (CTX and SSPX)

General Solution to Taylor state plasma

- Axismmetric: $\mathbf{B} = G(\chi)\nabla\varphi + \nabla\varphi \times \nabla\chi.$
- Singly connected domain: $G(\chi) = -k\chi,$
- Force-free Grad-Shafranov eq.: $\Delta^*\chi + k^2\chi = 0, \;\; \chi|_{\partial\Omega} = \chi_0.$
- Flux decomposition: $\chi = \chi_v + \sum \alpha_i \chi_i,$
- Vacuum flux: $\Delta^*\chi_v = 0, \;\; \chi_v|_{\partial\Omega} = \chi_0,$
- CK eigemodes: $\Delta^*\chi_i + k_i^2\chi_i = 0, \;\; \chi_i|_{\partial\Omega} = 0.$
- General solutions: $\alpha_i = \frac{k^2}{k_i^2 - k^2}\langle\chi_v\chi_i\rangle$
- Jensen-Chu resonances.



ST-PCC

- Flux amplification: $\mathcal{A} = \frac{\chi_c}{\chi_I}$. $\mathcal{A} \approx \alpha_1 = \frac{k^2}{k_1^2 - k^2} \langle \chi_v \chi_1 \rangle$.
- Aspect ratio of toroidal pinch: $A = \frac{R_o}{R_o - R_s}$,
- Plasma elongation: $e \equiv b/a$,

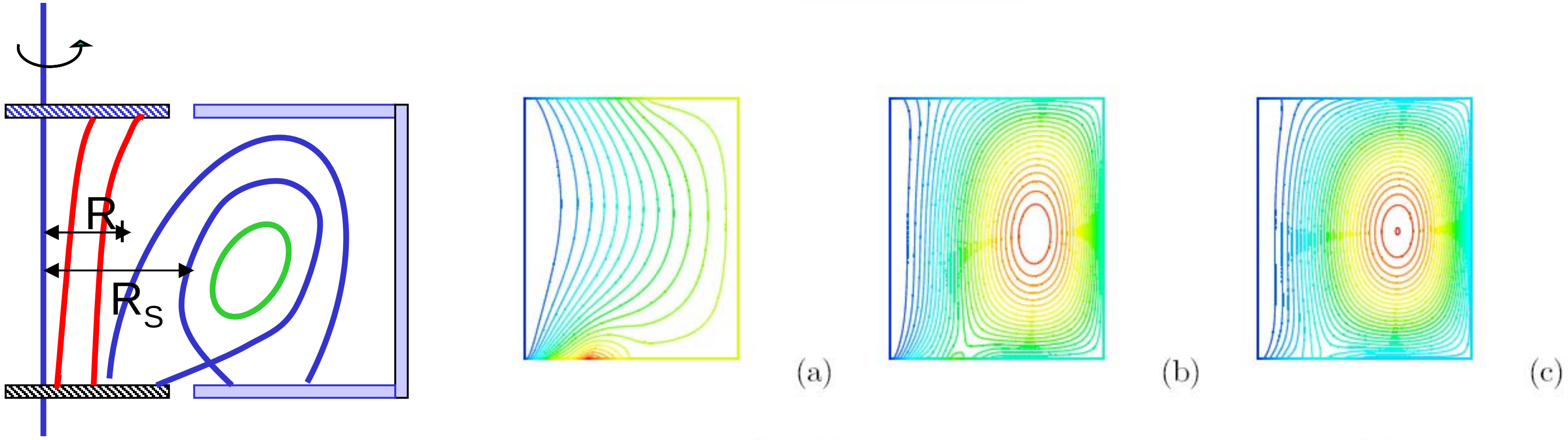


FIG. 1: Flux contours in the (R, Z) cross section. $R = 0$ axis is to the left. (a) vacuum bias poloidal flux; (b) poloidal flux at flux amplification factor one; (b) poloidal flux at flux amplification factor 2.5.

ST-PCC: aspect ratio

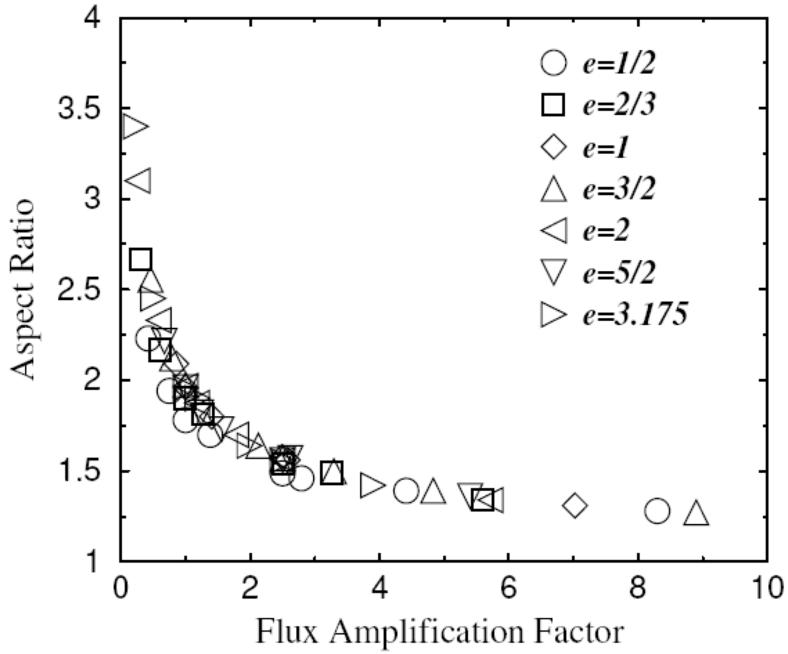


FIG. 2: Plasma aspect ratio as a function of the flux amplification factor.

- Flux amplification factor determines aspect ratio.
- Flux amp. factor ~ 1-2.5 → aspect ratio 1.5-2.
- More compact configuration (e.g. aspect ratio 1.25) requires exceedingly large flux amp. (e.g. ~10).
- Weak dependence on plasma shaping and bias flux → critical freedom to tailoring q profile characteristics.

What determines the q of ST-PCC?

- Field decomposition: $$\mathbf{B} = \mathbf{B}_v + \sum_i \alpha_i \mathbf{B}_i,$$
- Role of bias flux: $$\mathbf{B}_v = -k\chi_v \nabla\varphi + \nabla\varphi \times \nabla\chi_v,$$
- CK modes: $$\mathbf{B}_i = -k\chi_i \nabla\varphi + \nabla\varphi \times \nabla\chi_i.$$
- Two primary factors (B_v, B_1) in deciding q.

$$\frac{\mathbf{B}_v \cdot \frac{\partial \mathbf{x}}{\partial \varphi}}{\alpha_1 \mathbf{B}_1 \cdot \frac{\partial \mathbf{x}}{\partial \varphi}} = \frac{\chi_v}{\chi_1}\frac{1}{\alpha_1} = \frac{\chi_v}{\chi_1}\frac{k_1^2 - k^2}{k^2}\frac{1}{\langle \chi_v \chi_1 \rangle} \approx \frac{\chi_v}{\chi_1}\mathcal{A}(k).$$

- Higher order CK modes less important:

$$\frac{\alpha_i \mathbf{B}_i \cdot \frac{\partial \mathbf{x}}{\partial \varphi}}{\alpha_1 \mathbf{B}_1 \cdot \frac{\partial \mathbf{x}}{\partial \varphi}} = \frac{\chi_i}{\chi_1}\frac{\alpha_i}{\alpha_1} = \frac{\chi_i}{\chi_1}\frac{k_1^2 - k^2}{k_i^2 - k^2}\frac{\langle \chi_v \chi_i \rangle}{\langle \chi_v \chi_1 \rangle}.$$

Elongation and q of primary CK

- In a cylinder with height b and radius a,

$$k_1^2 = k_r^2 + \frac{\pi^2}{b^2},$$

k_r is the first root of $J_0'(k_r a) = 0$, i.e., $\tilde{k}_r = k_r a = 3.8317$.

- Edge q is a function of elongation alone,

$$q(\chi = 0) = \frac{k}{2\pi}\int_0^b \lim_{r\to 0}\frac{J_0'(k_r r)}{\frac{\partial}{\partial r}[rJ_0'(k_r r)]}dz = \frac{kb}{4\pi} = \frac{1}{4}\sqrt{\left(\frac{\tilde{k}_r}{\pi}\right)^2\left(\frac{b}{a}\right)^2 + 1}.$$

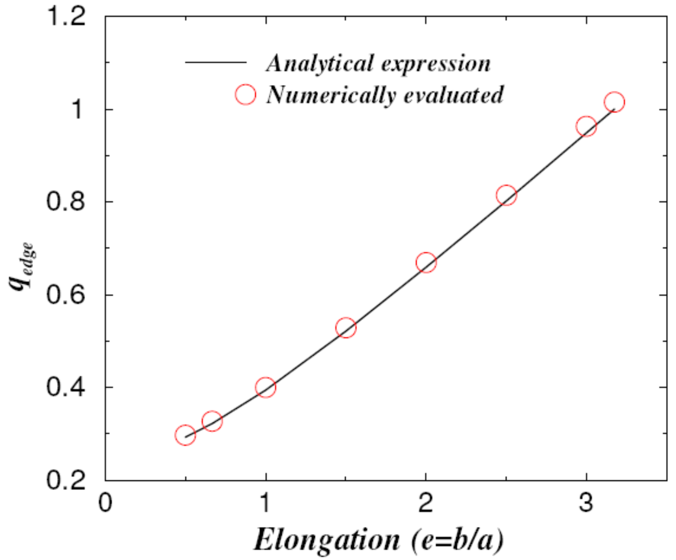


FIG. 3: Edge q of the primary CK mode as a function of elongation.

Elongation and q of primary CK

- Reversed magnetic shear.
- Both q and shear increase with elongation.

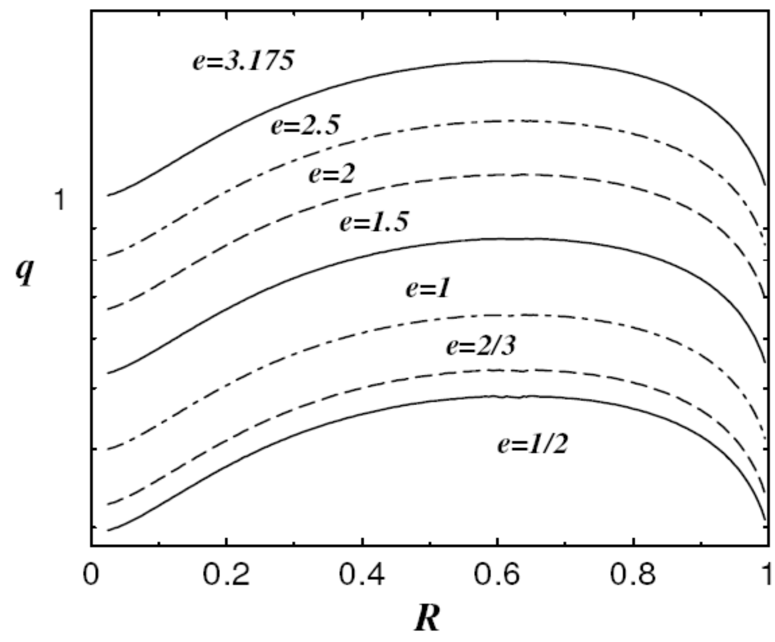
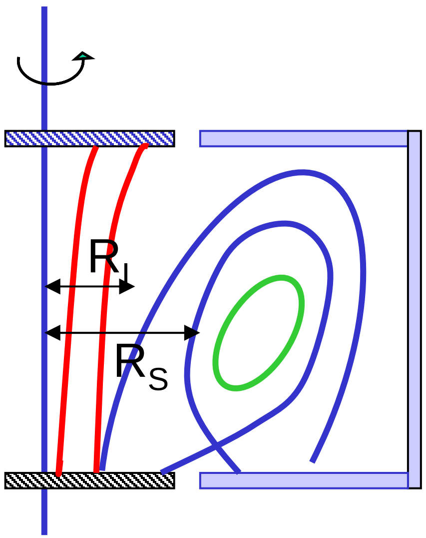


FIG. 4: q of the primary CK mode for different elongation is plotted as a function of radius R along the mid-plane.

Standard ST-PCC (q>1)

- Design space: flux amplification factor 1-3; aspect ratio 1.5-2; elongation 2-3.



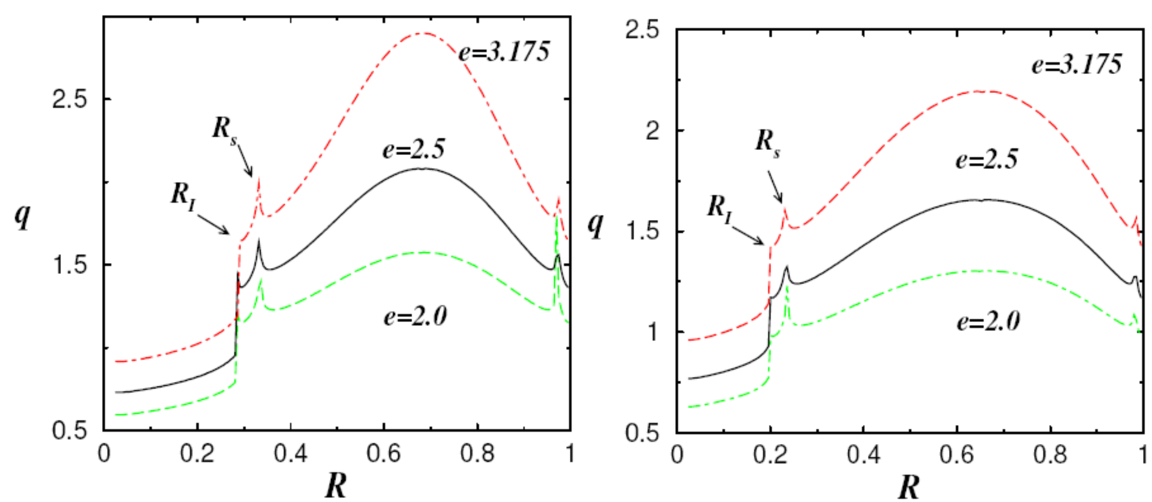
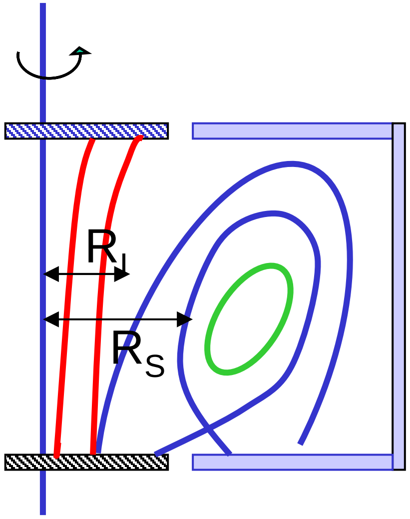


FIG. 5: q is plotted along the mid-plane. Left: flux amplification factor is one. Right: flux amplification factor is 2.5.

Ultra-low-q ST-PCC

- Design space: flux amplification factor 1-3; aspect ratio 1.5-2; elongation ≤ 1.
  - Same as flux-core spheromak?



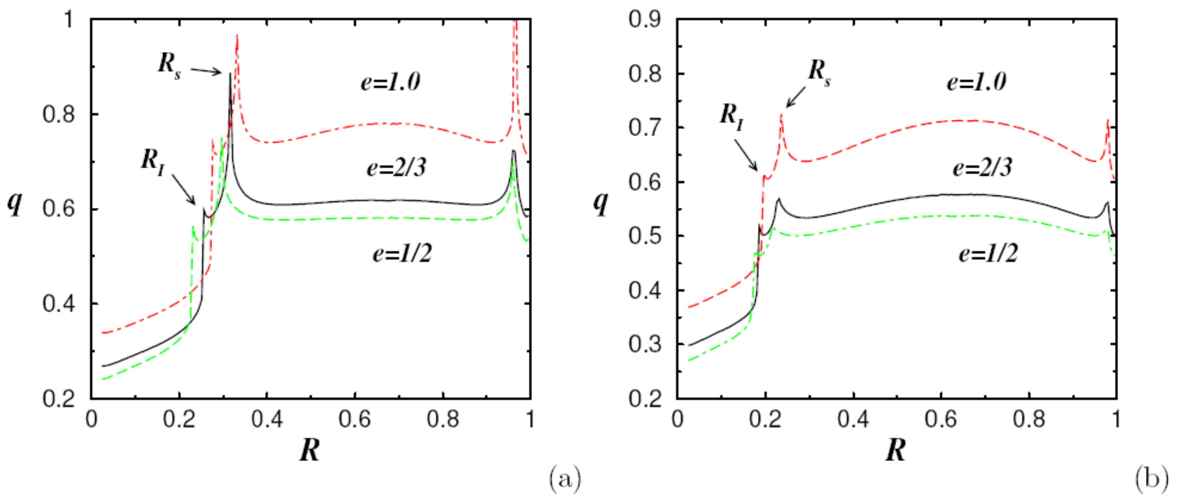
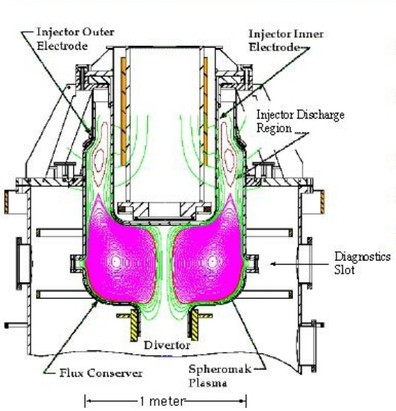


FIG. 6: q is plotted along the mid-plane. Left: flux amplification factor is one. Right: flux amplification factor is 2.5.

Spheromak and ULQ ST-PCC



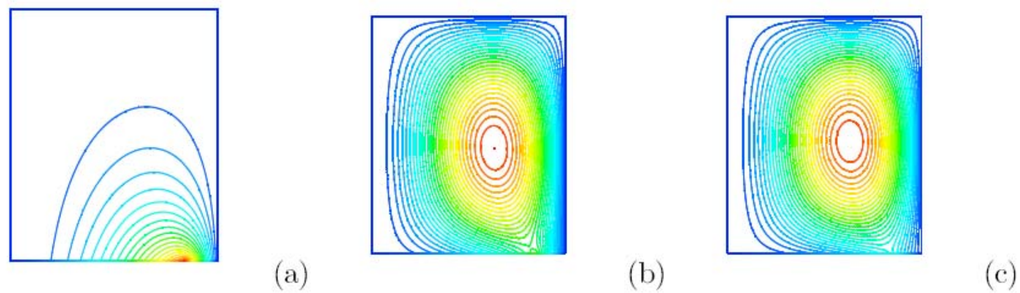
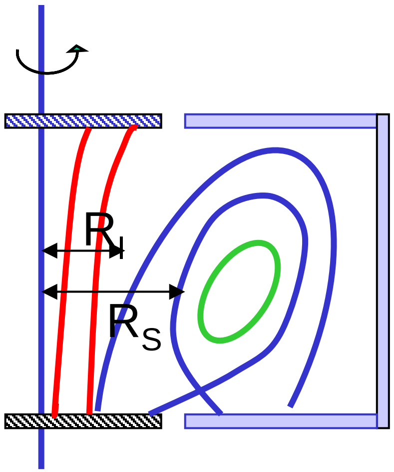
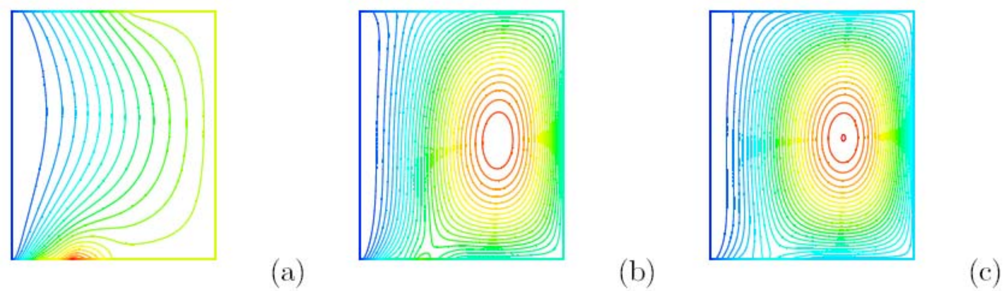


FIG. 7: Flux contours in the (R, Z) cross section. $R = 0$ axis is to the left. (a) vacuum bias poloidal flux; (b) poloidal flux at flux amplification factor one; (b) poloidal flux at flux amplification factor 2.5.





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ULQ ST-PCC and Spheromak comparison

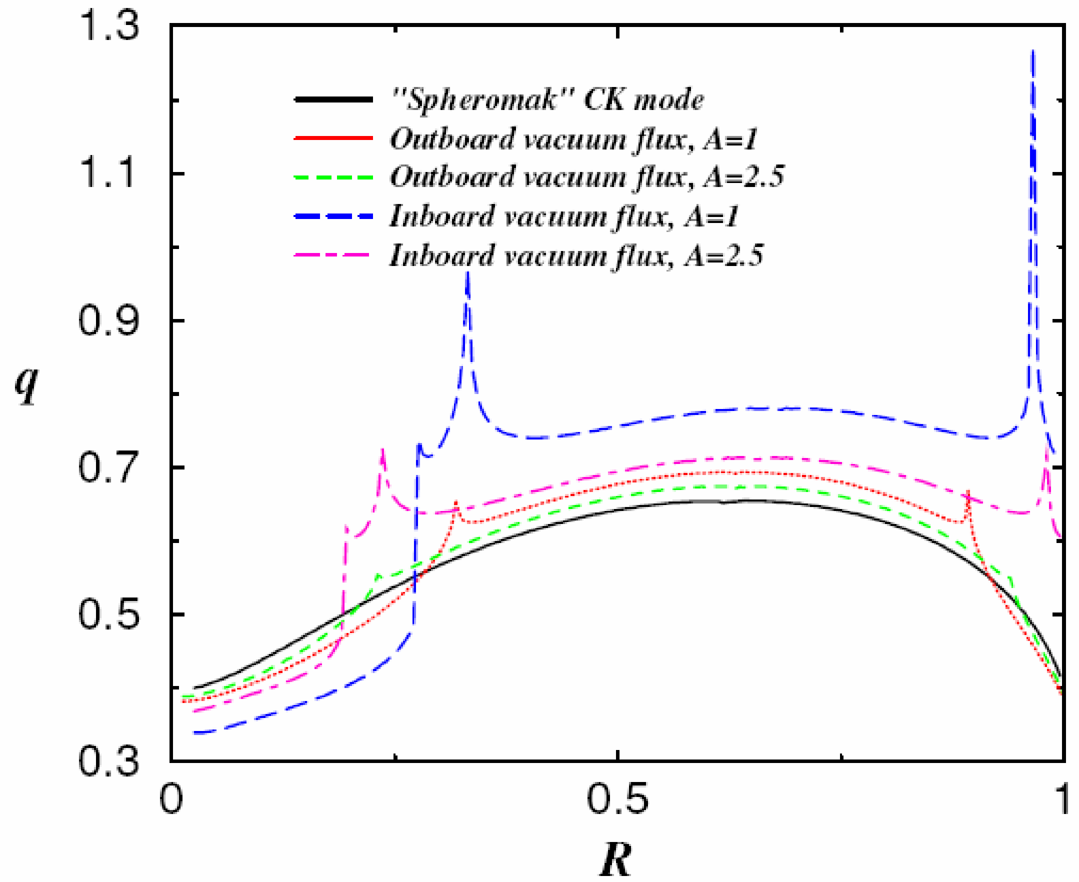


FIG. 8: $q(R)$ in the mid-plane is plotted for two sets of bias flux arranegment.

How to design a Spheromak Exp.?

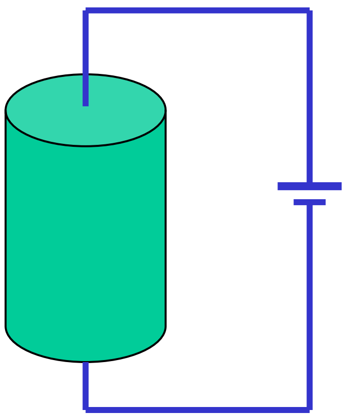
- Both ST-PCC and Spheromak have interior magnetic separatrix and current-carrying open flux.
  - ST-PCC: maximize the effect of PCC current.
  - Spheromak: minimize the effect of open flux current.
- Subtle role of bias flux:

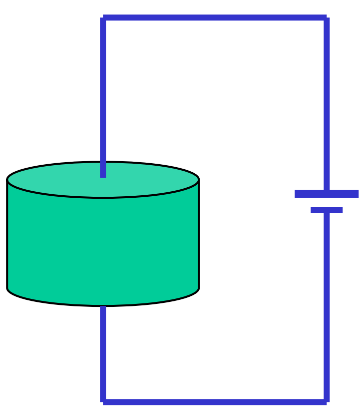
$$\mathrm{B}_v = -k\chi_v \nabla\varphi + \nabla\varphi \times \nabla\chi_v,$$

- Spheromak design strategy:
  - Localize the bias flux to a gun.
  - Move the gun to the outboard as far out as possible.

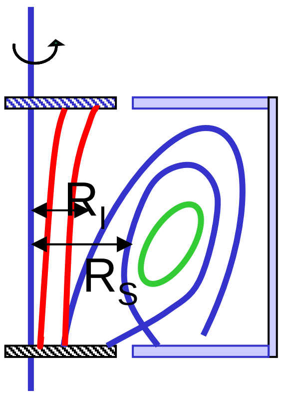
Overview of the design space

- ST-PCC: Elongation 2-3; Flux Amp. Factor 1-2.5; Aspect Ratio 1.5-2; q>1.
- ULQ ST-PCC: Elongation ≤ 1; Flux Amp. Factor 1-2.5; Aspect Ratio 1.5-2; q<1 mostly.
- Gun Spheromak: Elongation ≤ 1; Flux amp. Factor 1-2.5; gun positioned outboard.
- Flux-core Spheromak: either at extremely large Flux Amp. Factor or flipped spheromak state (past first Jensen-Chu resonance). Neither is likely accessible experimentally.



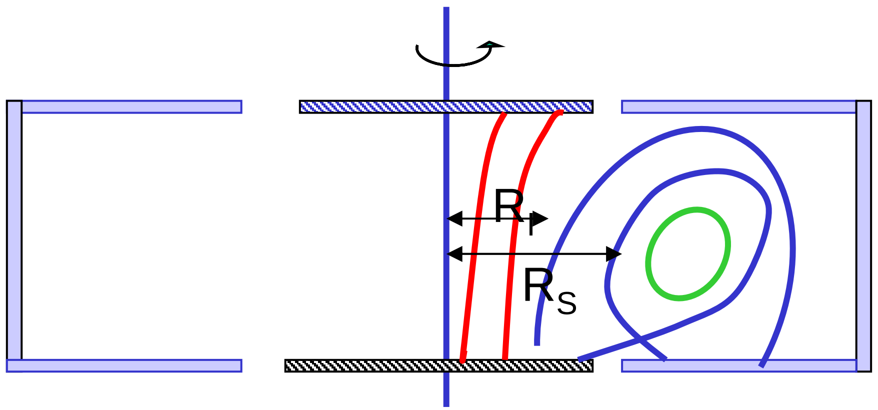


Additional Considerations



- ST-PCC prefers to have two sets of voltage bias system, one for PCC, and other for CHI injector flux
  - Relax either by SP kink of PCC or Open flux kink of CHI injector flux, smaller voltage bias on ST plasma.
  - Naturally single null ST.
  - CHI current drive on SOL reduces current gradient between sustained ST and PCC plasma.
- PCC electrodes need to sustain a large current, engineering challenge.
- Larger flux amplification factor → less PCC current. Too large → stability concerns. Experimental scan needs access to high flux amplification regime (overcome the present limitation of factor two?).

Conclusions and Future Plans



- ST-PCC with aspect ratio 1.5, elongation 2-3, and reversed magnetic shear, is supported by a presently observed flux amplification factor of two.
- ULQ ST-PCC with similar characteristics is obtained at reduced elongation ≤ 1.
- Plan I: kink (internal and external), tilt, and vertical instability will be investigated for a high beta and even higher flux amplification factor ST-PCC. More sophisticated plasma shaping.
- Plan II: an experimental design for a CE experiment. (Hsu, Intrator, and Wurden)