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A simple model of blobs in the X-point configuration*

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Numerous discussions with

R. Cohen

M. Umansky

S. Zweben

J. Terry

are gratefully acknowledged

OUTLINE

- Introduction and motivation
- Flux-tube mapping near the X-point
- General kinematics
- Effects of sheath resistance on blob velocity
- Effects of the divertor plate orientation
- What determines the shape of the blobs?
- Resistive ballooning
- Blobs in the private flux region
- Summary

The "blobology" is becoming an important part of the edge physics, with a large number of papers appearing every year

Some important theory papers:

S.I. Krasheninnikov. "On scrape-off layer plasma transport." Phys. Lett. **A 283**, 368 (2001).

D.A. D'Ippolito, J.R. Myra, S.I. Krasheninnikov. "Cross-field blob transport in tokamak scrape-off-layer plasmas." Phys. Plasmas, **9**, 222 (2002).

S.I. Krasheninnikov, D.D. Ryutov, G. Yu. "Large plasma pressure perturbations and radial convective transport in a tokamak", Journal of Plasma and Fusion Research (Japan), **6**, 139 (2004).

G.Q. Yu, S.I. Krasheninnikov, P.N. Guzdar. Two-dimensional modelling of blob dynamics in tokamak edge plasmas Phys. Plasmas, **13**, 042508 (2006)

O.E. Garcia, N.H. Bian, W. Fundamentski. "Radial interchange motions of plasma filaments" Phys. Plasmas, **13**, 082309 (2006).

J. R. Myra, D. A. D'Ippolito, D. P. Stotler, et al., "Blob birth and transport in the tokamak edge plasma: Analysis of imaging data" Phys. Plasmas **13**, 092509 (2006)

J.R. Myra, D.A. Russell, D.A. D'Ippolito. "Collisionality and magnetic geometry effects on tokamak edge turbulent transport. 1. A two-region model with application to blobs." Phys. Plasmas, **13**, 112502 (2006).

We consider isolated blobs (ambient plasma density is negligible) and concentrate on the "global" dynamics

We base our analysis on the papers

R.H. Cohen, D.D. Ryutov, "Dynamics of an isolated blob in the presence of the X-point", *Contrib. Plasma Phys.,* **46,** 678, August 2006

D.D. Ryutov. "The dynamics of an isolated plasma filament at the edge of a toroidal device." Phys. Plasmas, **13**, 122307, December 2006

R.H. Cohen, B. LaBombard, D.D. Ryutov, J.L. Terry, M.V. Umansky, X.Q. Xu, S. Zweben. "Theory and Fluid Simulations of Boundary-Plasma Fluctuations." Nucl. Fusion, **47,** 612, July 2007.

D.D. Ryutov, R.H. Cohen. "Geometrical effects in plasma stability and dynamics of coherent structures in the divertor." To appear in Contrib. Plasma Phys., **48**, 2008

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D.D. Ryutov, R.H. Cohen. "Geometrical effects in plasma stability and dynamics of coherent structures in the divertor." To appear in Contrib. Plasma Phys., **48**, 2008

Nice experimental pictures of isolated blobs can be found in:

J. L. Terry et al. "Observations of the turbulence in the scrape-off-layer of Alcator C-Mod and comparisons with simulation." Phys. Plasmas **10**, 1739 (2003)

O. Grulke, J. L. Terry, B. LaBombard, and S. J. Zweben. "Radially propagating fluctuation structures in the scrape-off layer of Alcator C-Mod." Phys. Plasmas **13**, 012306 (2006)

S. J. Zweben et al. "Structure and motion of edge turbulence in the National Spherical Torus Experiment and Alcator C-Mod ."Phys. Plasmas **13**, 056114 (2006)

Basic model

A fluxtube filled with a low-beta plasma

The fluxtube radius $a \gg \rho_\mathrm{i}$

Length $l >>> a$

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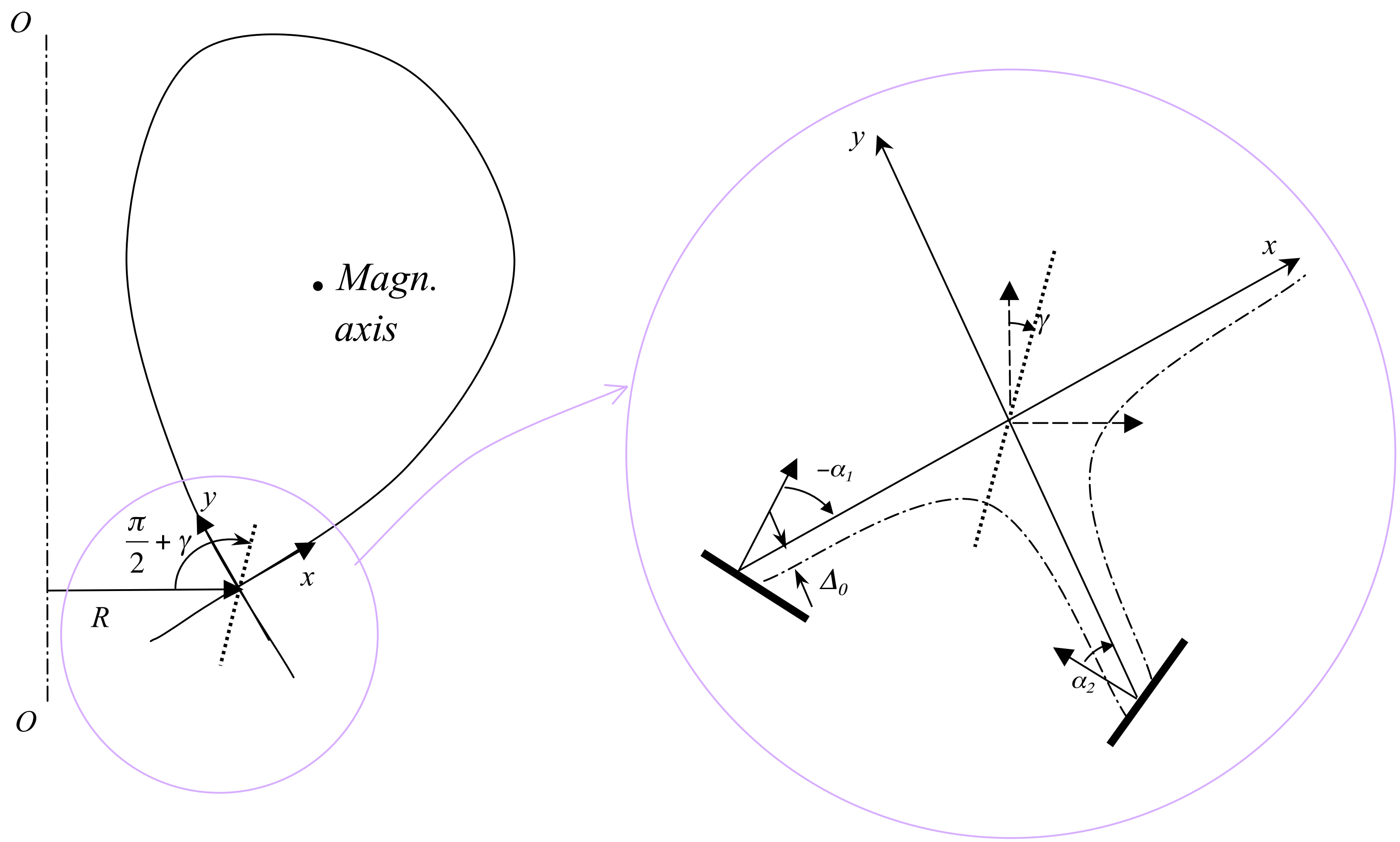
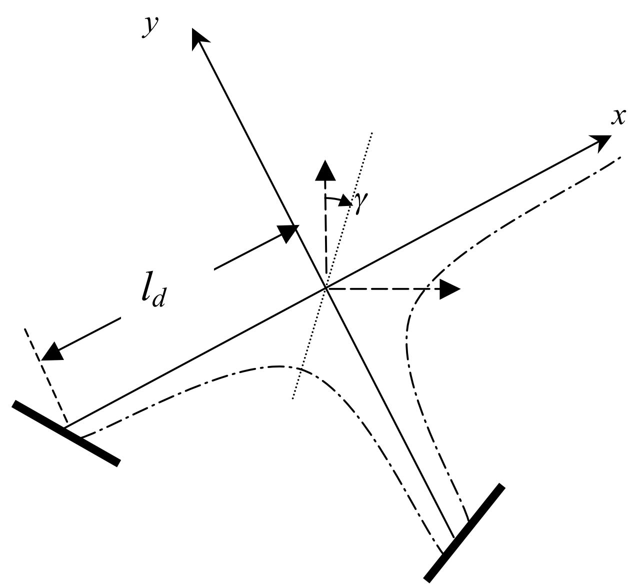
We concentrate on the extremely important role of the effects occurring in the vicinity of the X-point

We consider the geometry of a poloidal divertor



Geometrical parameters: γ (angle between the bisector between two branches of the separatrix); α_1, α_2 (angles between the normal to the divertor plates and the poloidal field in the strike point)

The model of the magnetic field*



$x = x_0/E,\ y = y_0E$

$E = \exp(z - z_0)/L$

$B_x = -C\cdot x$; $B_y = C\cdot y$; $B_z = \text{const} = B_T$

$C = B_P/l_d$;

B_p is the poloidal field at the strike point ($B_p \ll B_T$)

Field line equation:

$$-\frac{dx}{x} = \frac{dy}{y} = \frac{dz}{L}; \quad L \equiv \frac{B_T}{B_P} l_d$$

$x = x_0 \exp -(z - z_0)/L$; $y = y_0 \exp(z - z_0)/L$

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*D.Farina, R.Pozzoli, D.D. Ryutov. "Effect of the magnetic field geometry on the flute-like perturbations near the divertor X-point". Nuclear Fusion, **33**, 1315 (1993).

Consider a bunch of field lines forming a flux tube.

When moving along the fluxtube, one finds that the initially circular cross-section becomes an ellips, with the axes directed along x and y.

$$x = \frac{x_0}{E}; \quad y = y_0 E$$

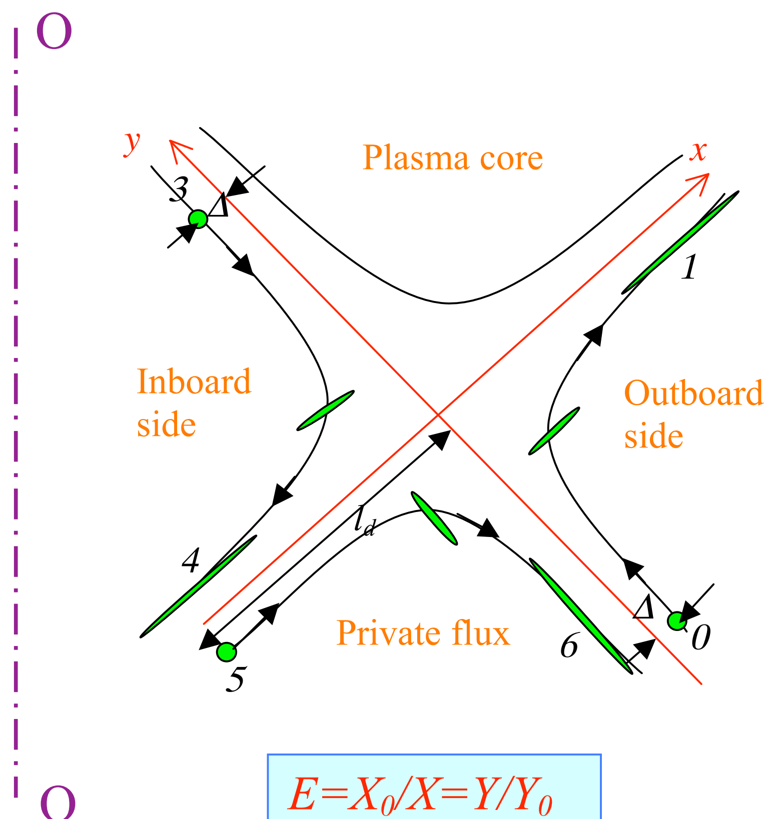
$$x_0 = X_0 + \delta x_0; \quad y_0 = Y_0 + \delta y_0$$

$$\frac{\delta x_0^2}{a^2} + \frac{\delta y_0^2}{a^2} = 1$$

$$x = X + \delta x; y = Y + \delta y$$

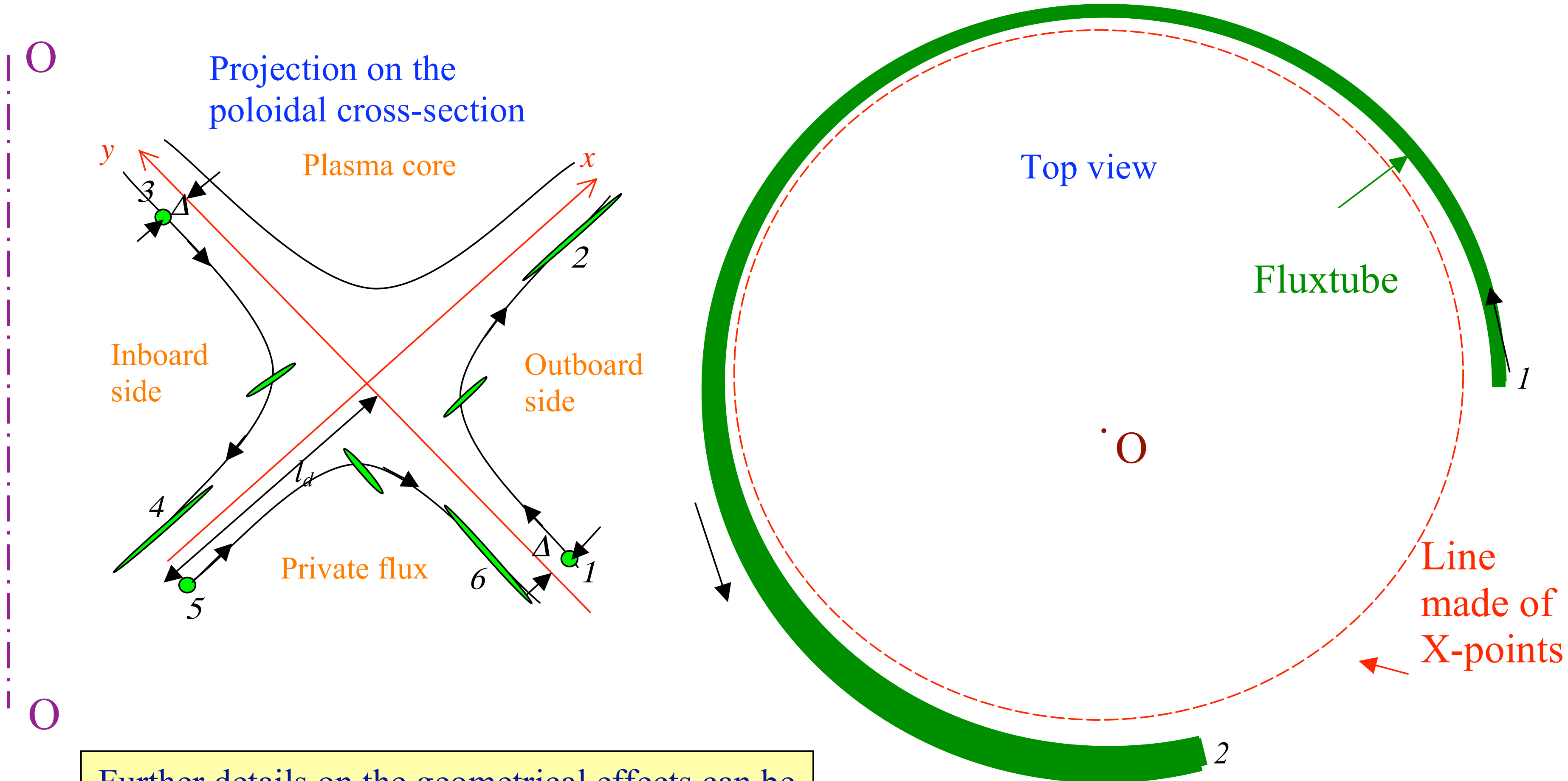
$$\frac{\delta x^2}{(a/E)^2} + \frac{\delta y^2}{(aE)^2} = 1$$

The parameter *E* is the ratio of the major semi-axis to the initial radius and called “elongation.”



$$E = X_0/X = Y/Y_0$$

The flux tube is long in the z direction!



Further details on the geometrical effects can be found in: D.D. Ryutov. Phys. Plasmas, **13**, 122307, 2006

Consider a bunch of field lines forming a flux tube.

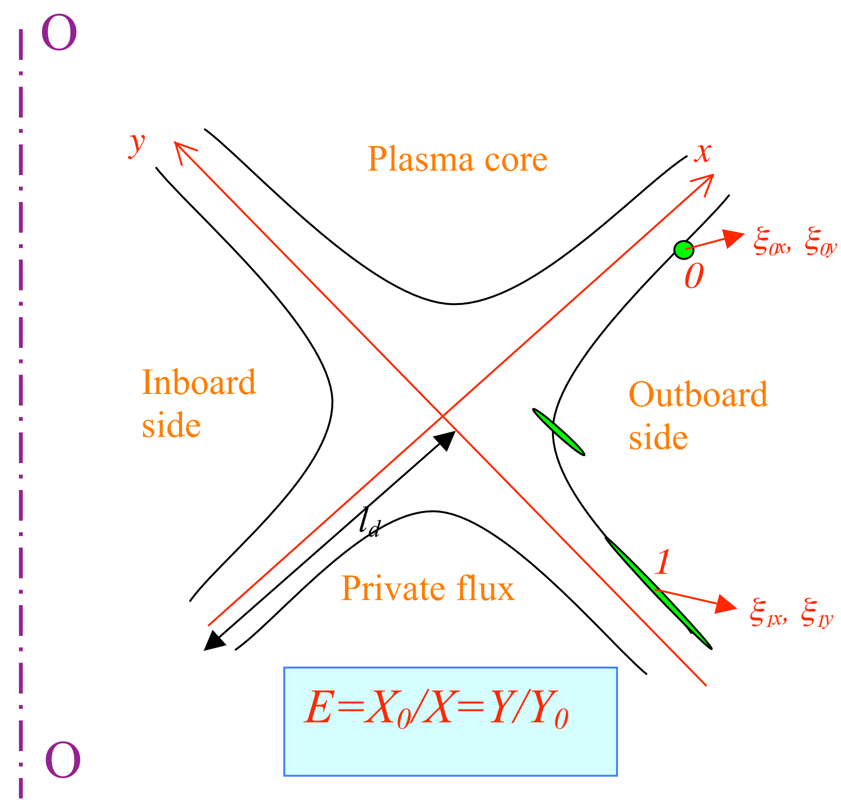
Consider a displacement of a center line of a certain fluxtube

$$X = \frac{X_0}{E}; \quad Y = Y_0 E$$

$$X_0 \to X_0 + \xi_{0x}; \quad Y_0 \to Y_0 + \xi_{0y}$$

$$X_1 \to X_1 + \xi_{1x}; \quad Y_1 \to Y_1 + \xi_{1y}$$

$$\xi_{1x} = \frac{\xi_{0x}}{E}; \quad \xi_{1y} = \xi_{0y} E$$



One can isolate the global motion by a technique similar to the virial theorem (D.D. Ryutov. Phys. Plasmas, **13**, 122307, 2006)

$$\int \boldsymbol{j}_\perp dS = -\frac{c\ddot{\xi} \times \boldsymbol{B}}{B^2}\int \rho dS + 2c\left(\frac{\boldsymbol{B} \times \nabla B}{B^3}\right)\int p dS$$

$$\nabla \cdot \boldsymbol{j}_\perp = -B\frac{\partial}{\partial l}\frac{j_\parallel}{B}$$

Muliply this equation by $\boldsymbol{r}_\perp$ and perform integration over dS to obtain:

$$\frac{1}{B}\int \boldsymbol{j}_\perp dS = \frac{\partial}{\partial l}\left(\frac{1}{B}\int \boldsymbol{r}_\perp j_\parallel dS\right)$$

Integrate end-to end and impose boundary conditions on the parallel current. In particular, for freely "dangling" ends, one obtains*:

$$\int \frac{ds}{B}\int \boldsymbol{j}_\perp dS = 0$$

Displacement $\boldsymbol{\xi}_\perp$ at an arbitrary point along the field line can be expressed via displacement $\boldsymbol{\xi}_{\perp 0}$ at some reference point (e.g., one of the ends). This then allows one to obtain an equation defining $\boldsymbol{\xi}_{\perp 0}(t)$

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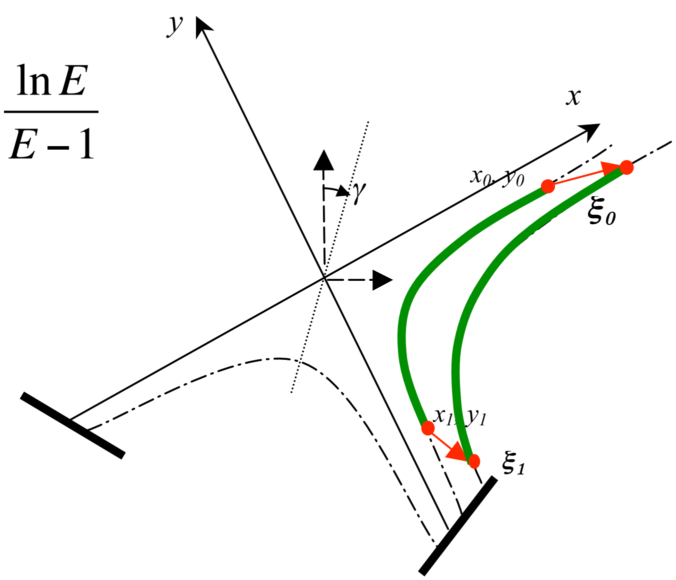
* The smallness of the flux tube cross-section compared to all other dimensions was used

Apply this approach to the blob in the divertor area

$$\ddot{\xi}_{0x} = \frac{2p\cos\left(\frac{\pi}{4}-\gamma\right)}{\rho R}\frac{E\ln E}{E-1} \qquad \ddot{\xi}_{0y} = -\frac{2p\cos\left(\frac{\pi}{4}+\gamma\right)}{\rho R}\frac{\ln E}{E-1}$$

$E = y_1/y_0$



- The ends move predominantly in the poloidal direction

- The normal acceleration is by a factor of *E* smaller than one might expect for the curvature-driven acceleration

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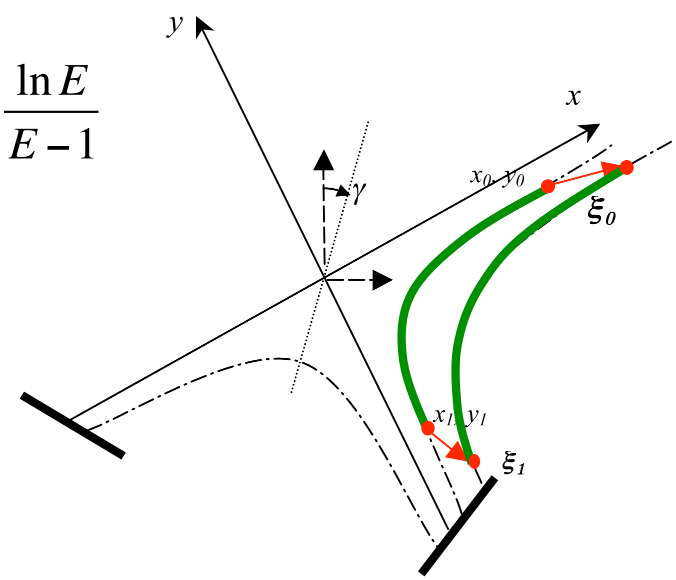

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Without accounting for the shear the observed dynamics can look very puzzling!

Include the contact with the divertor plate (sheath resistance)

The boundary condition:

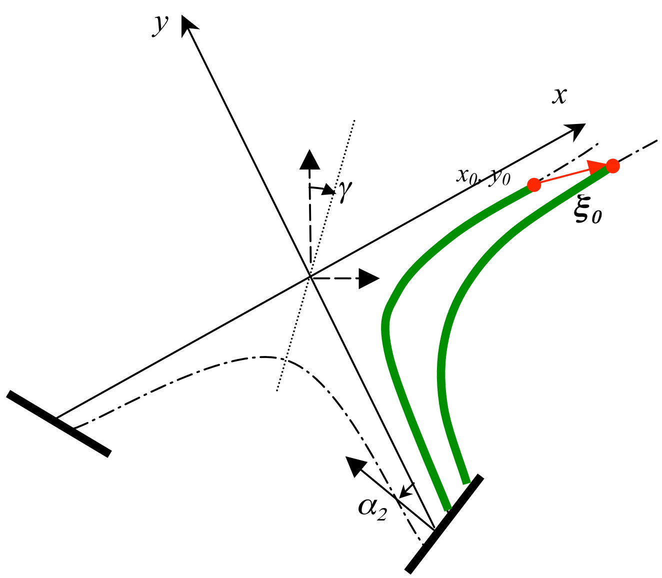
$$\frac{c}{B}\left(\frac{\partial p_e}{\partial x}-\frac{\partial p_e}{\partial y}\tan\alpha_2\right)=\frac{B_p}{B}\left(j_{\parallel}-j_{sat}\frac{e(\varphi-\varphi_f)}{T_e}\right).$$

Find the velocity

$$\dot{\xi}_{0x}=\frac{w}{E_d}\left(\cos\left(\frac{\pi}{4}-\gamma\right)\ln E_d-\frac{p_{ed}}{\overline{p}}\frac{R\tan\alpha_2}{l_d}\right)$$

$$\dot{\xi}_{0y}=-E_d w\left(\cos\left(\frac{\pi}{4}+\gamma\right)\ln E_d-\frac{p_{ed}}{\overline{p}}\frac{R}{l_d}\right)$$

$$w\sim\frac{cT_{ed}}{eBa}\frac{\rho_{id}}{a}\frac{L}{R}\frac{\overline{p}}{p_d}$$



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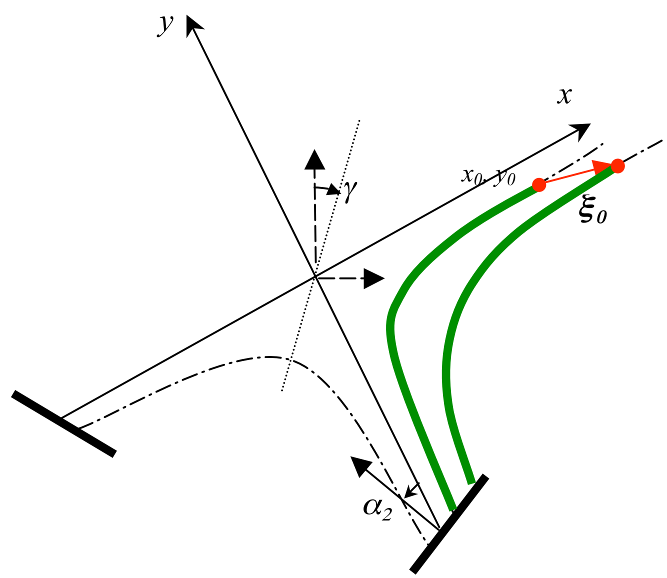

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$$w\sim\frac{cT_{ed}}{eBa}\frac{\rho_{id}}{a}\frac{L}{R}\frac{\overline{p}}{p_d}$$

The change of the tilt may produce a reversal of the blob motion with respect to the curvature drive

For the blobs extending from the equatorial plane to the X-point region, there is a minimum size in the equatorial plane

$$a_0 > \rho_i E_d$$

One more "selection rule" could be a strong dependence of the blob velocity on a,

$$v \sim 1/a^2$$

(slower blobs have difficulty in separating themselves from the main SOL and "dissolve" faster than are ejected)

If plasma electrical conductivity is low, resistive ballooning may cause detachment from the divertor plate

$$j_{\parallel} \approx \frac{cp}{BR}\int_{end}^{z}\left(\frac{1}{a_x}+\frac{1}{a_y}\right)dz$$

$$j_{\parallel} \approx \frac{cp}{BRa_{xd}}\int_{end}^{z}\frac{dz}{E} = \frac{cpL}{BRa_{xd}}\left(\frac{1}{E}-\frac{1}{E_{end}}\right)$$

$$\delta\varphi_{res}\Big|_{end}^{div} \approx \eta\int_{end}^{div} j_{\parallel}dz \approx \eta L\int_{end}^{div} j_{\parallel}\frac{dE}{E} \approx \frac{cpL^2\eta}{BRa_{xd}}$$

This is to be compared with the potential difference that drives the motion,

$$\varphi-\varphi_f = \frac{T_e}{e}\frac{j_{\parallel d}}{j_{sat}} \sim \frac{T_e}{ej_{sat}}\frac{cpL}{BRa_{xd}}$$

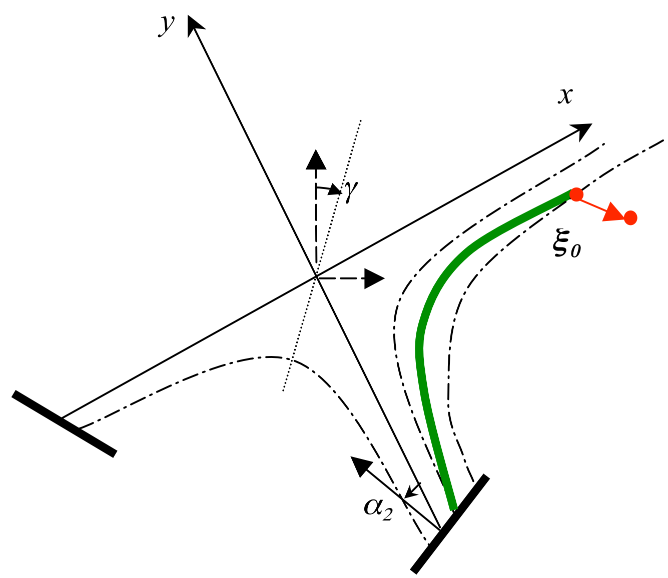
The resistive potential drop is significant if $\frac{T_e}{ej_{sat}} < \eta L$, or

$$\frac{\lambda_{ei}}{L} >> \sqrt{\frac{m_e}{m_i}}$$

If resistivity is high, the blobs in the main SOL are decoupled from the divertor zone

$$\frac{\partial}{\partial z} E \frac{\partial \dot{\xi}_x}{\partial z} = \frac{4 D_M \beta}{R a^2} \cos\left(\frac{\pi}{4} - \gamma\right)$$

$$\frac{\partial}{\partial z} \frac{1}{E} \frac{\partial \dot{\xi}_y}{\partial z} = \frac{4 D_M \beta}{R a^2} \cos\left(\frac{\pi}{4} + \gamma\right)$$



Boundary conditions:

$$divertor: \quad \dot{\xi}_x, \dot{\xi}_y = 0$$

$$midplane: \frac{\partial \dot{\xi}_x}{\partial z}, \frac{\partial \dot{\xi}_y}{\partial z} = 0$$

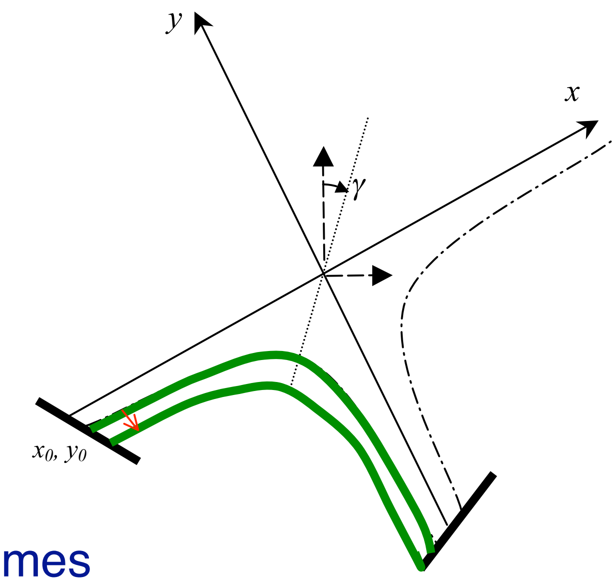
The blob velocity in the midplane can be evaluated as

$$\dot{\xi}_\perp \sim F(E) \frac{L^2 D_M \beta}{R a^2}$$

Blobs can exist in the private flux region

$$\dot{\xi}_{1x} = -w\left(\cos\left(\frac{\pi}{4}-\gamma\right)\ln E^* + \frac{R}{l_d}\left(\tan\alpha_2 + 1\right)\right);$$

$$\dot{\xi}_{2y} = w\left(\cos\left(\frac{\pi}{4}+\gamma\right)\ln E^* + \frac{R}{l_d}\left(\tan\alpha_1 - 1\right)\right)$$



One end may move towards the plate, whereas the other move away from the plate.
A disconnection is possible*.

The situation is very reach in possible outcomes

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*See discussion in R.H. Cohen et al, Nucl. Fus., **47**, 2007.

Summary

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Predictions are made in the form allowing for their direct comparison with experiments