



# Surprises in gyrokinetic turbulence simulations: Some recent **GENE** results

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Acknowledgements:  
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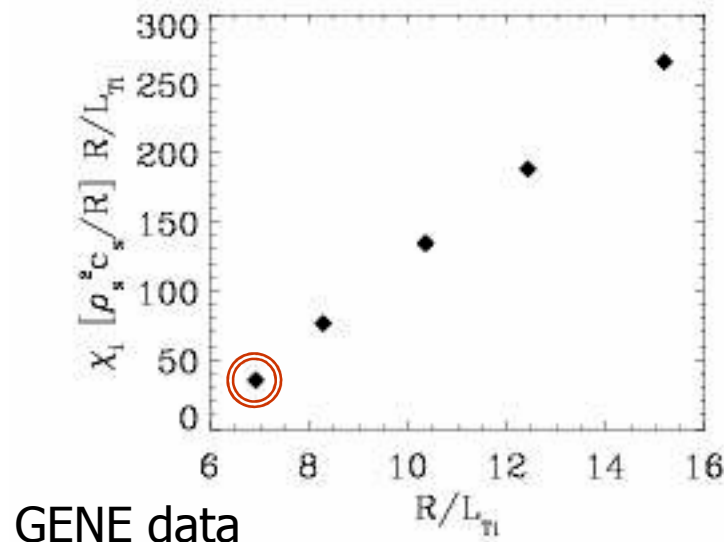
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# Adiabatic ITG turbulence in a simple tokamak



## Reference case for core turbulence simulations:

- “Cyclone base case” – also serves as standard paradigm of turbulence
- idealized physical parameters; adiabatic electrons; s- $\alpha$  model equilibrium



## Key findings:

- saturation via zonal flows
- ion heat flux is offset-linear
- nonlinear upshift of threshold

What about other transport channels, modes, and scales?  
How generic is the adiabatic ITG s- $\alpha$  scenario?

# Overview



- The tool: GENE
- The nature of (pure) TEM turbulence
- Nonlinear ITG-TEM(-ETG) interactions
- The role of magnetic geometry (NSTX, NCSX)



The tool: GENE

# The nonlinear gyrokinetic equations

$$f = f(\mathbf{X}, v_{\parallel}, \mu; t)$$

Advection/Conservation equation

$$\frac{\partial f}{\partial t} + \dot{\mathbf{X}} \cdot \frac{\partial f}{\partial \mathbf{X}} + \dot{v}_{\parallel} \frac{\partial f}{\partial v_{\parallel}} = 0$$

$$\dot{\mathbf{X}} = v_{\parallel} \mathbf{b} + \frac{B}{B_{\parallel}^*} \left( \frac{v_{\parallel}}{B} \bar{\mathbf{B}}_{1\perp} + \mathbf{v}_{\perp} \right)$$

$$\mathbf{v}_{\perp} \equiv \frac{c}{B^2} \bar{\mathbf{E}}_1 \times \mathbf{B} + \frac{\mu}{m\Omega} \mathbf{b} \times \nabla (B + \bar{B}_{1\parallel}) + \frac{v_{\parallel}^2}{\Omega} (\nabla \times \mathbf{b})_{\perp}$$

$$\dot{v}_{\parallel} = \frac{\dot{\mathbf{X}}}{mv_{\parallel}} \cdot (e\bar{\mathbf{E}}_1 - \mu \nabla (B + \bar{B}_{1\parallel}))$$

$\mathbf{X}$  = gyrocenter position

$v_{\parallel}$  = parallel velocity

$\mu$  = magnetic moment

Appropriate field equations

$$\frac{n_1}{n_0} = \frac{\bar{n}_1}{n_0} - (1 - \|I_0^2\|) \frac{e\phi_1}{T} + \|x I_0 I_1\| \frac{B_{1\parallel}}{B}$$

$$\nabla_{\perp}^2 A_{1\parallel} = -\frac{4\pi}{c} \sum \bar{J}_{1\parallel}$$

$$\frac{B_{1\parallel}}{B} = -\sum \epsilon_{\beta} \left( \frac{\bar{p}_{1\perp}}{n_0 T} + \|x I_1 I_0\| \frac{e\phi_1}{T} + \|x^2 I_1^2\| \frac{B_{1\parallel}}{B} \right)$$

# Current physics features of GENE

## Treatment of particle dynamics

- Arbitrary number of gyrokinetic particle species, passing and trapped
- Can be active (feedback via field equations) or passive
- Non-Maxwellian (beam-type) equilibrium distributions
- Electromagnetic effects are included

## Collisions

- Collisions between any pair of species are kept
- Pitch angle scattering *and* energy scattering are retained
- Momentum and energy conserving terms are implemented

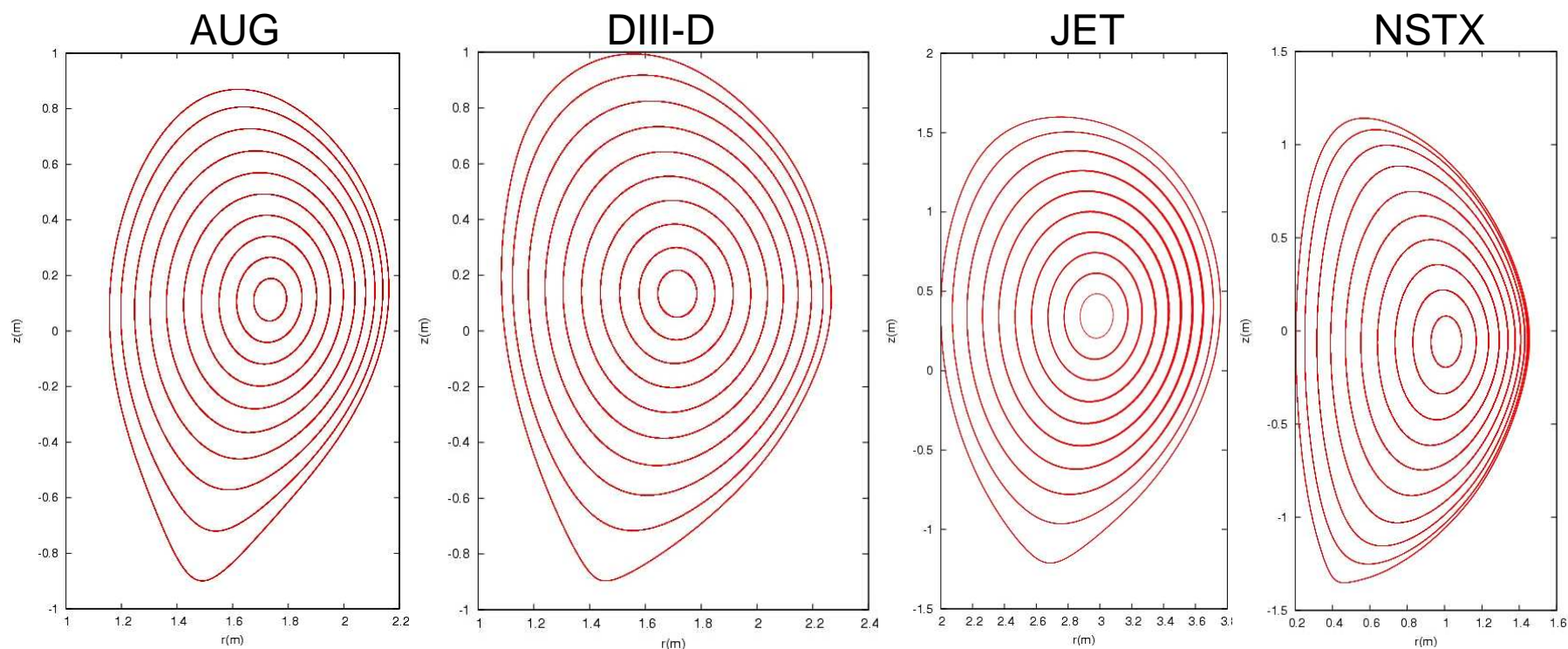
## General geometry

- Interface to CHEASE MHD equilibrium code
- Interface to other MHD codes: TRACER

# The TRACER code

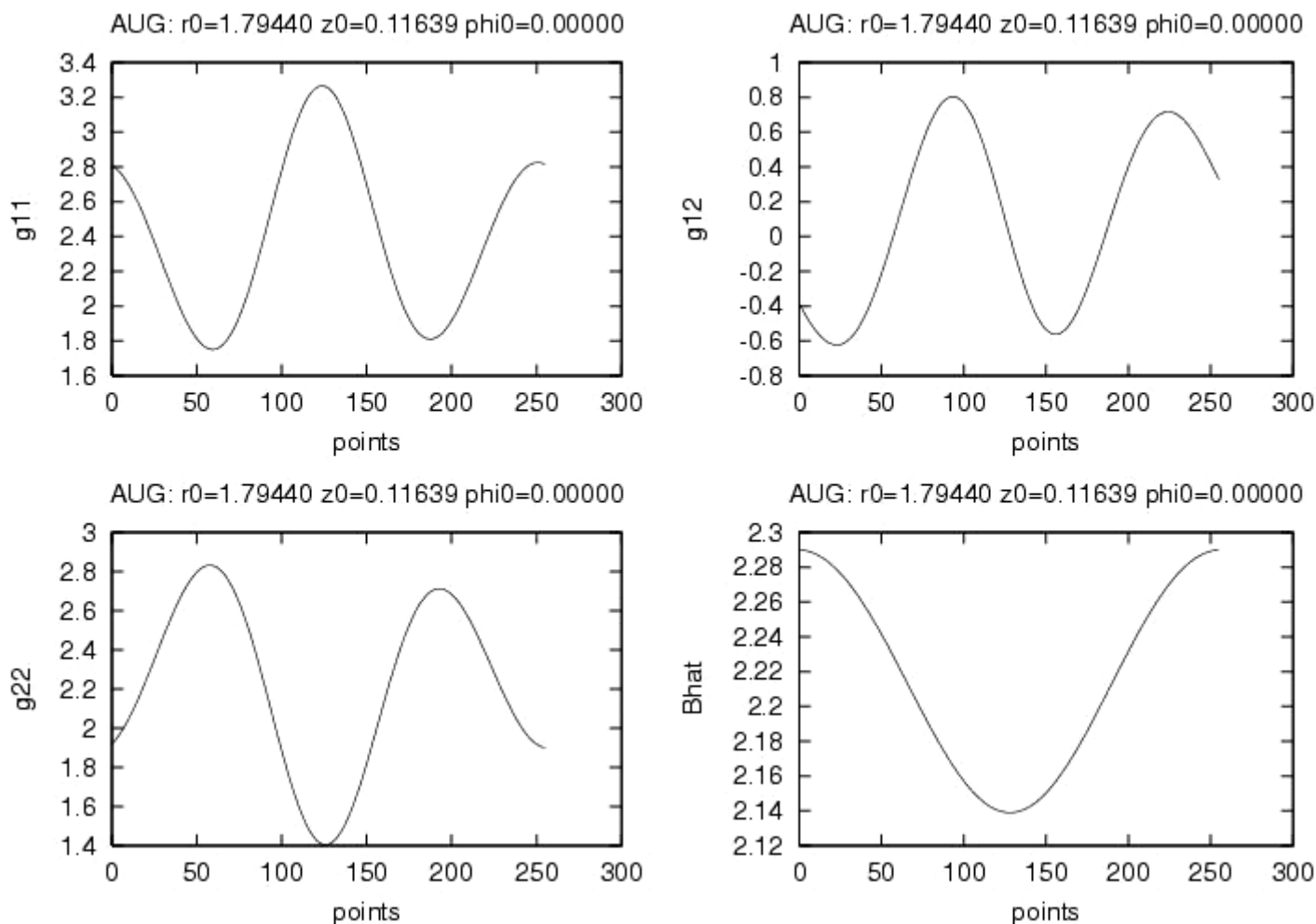
## Description

- Numerical generation of a **Clebsch system** via field line tracing
- No assumptions on the existence or properties of flux surfaces
- Flexibility in construction of flux surface label
- Coupling to plasma parameter databases



# The TRACER code (cont'd)

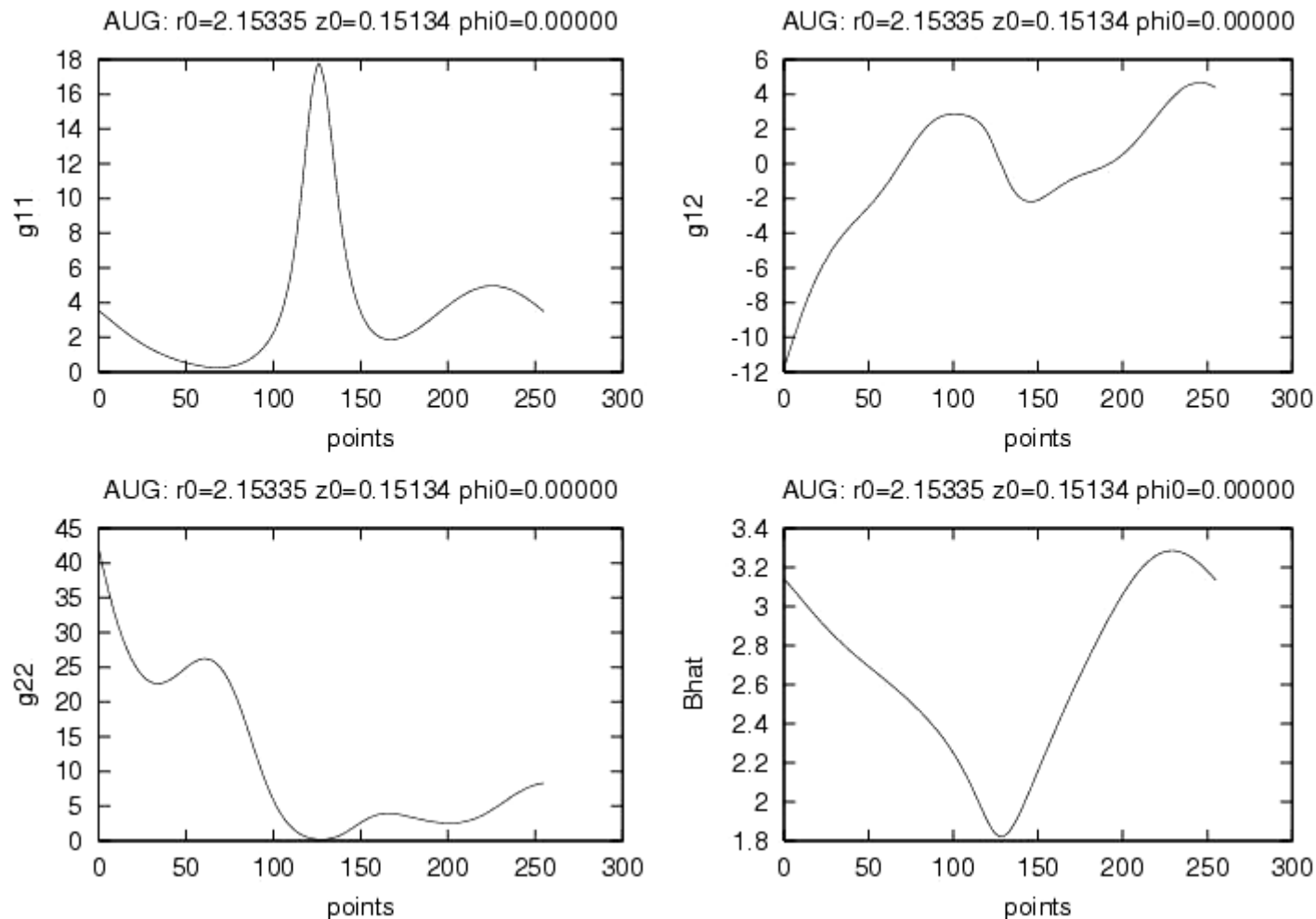
Some metric coefficients for AUG at  $\rho_{\text{tor}} = 0.1$





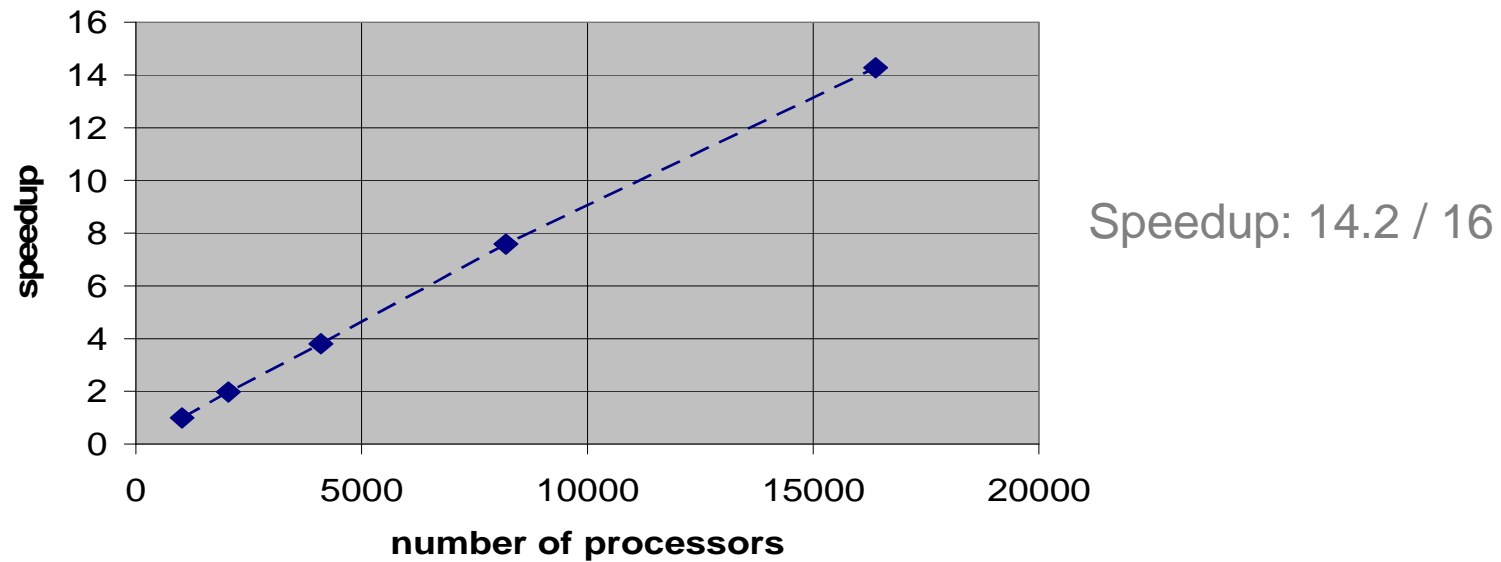
# The TRACER code (cont'd)

Some metric coefficients for AUG at  $\rho_{\text{tor}} = 0.95$



# Hyperscaling of GENE

- GENE runs very efficiently on a large number of parallel platforms
- Example: IBM BlueGene/L @ Watson Research Center



**Strong scaling (fixed problem size) – from 1k to 16k cores**

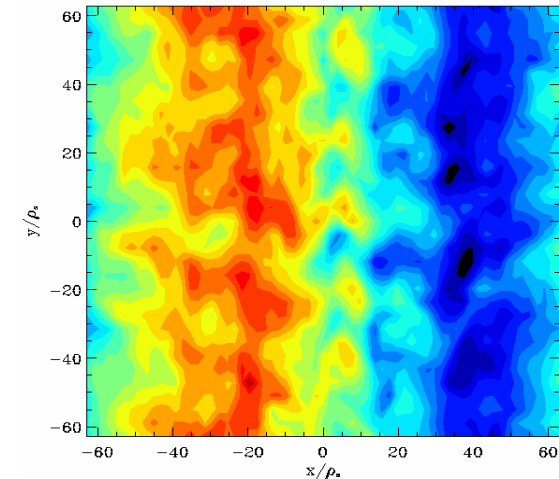
# The nature of (pure) TEM turbulence

# Questions concerning TEM turbulence

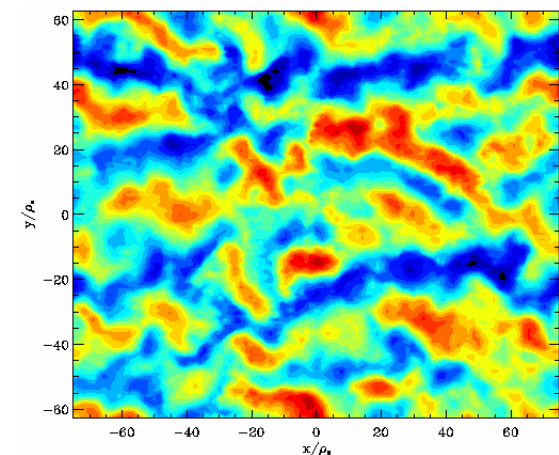


- Established result for ITG turbulence: Nonlinear saturation via zonal flows
- In plasmas with strong electron heating, TEM turbulence can be dominant: What is the nonlinear saturation mechanism?
- When ITG modes and TEMs coexist, how do they interact nonlinearly?
- In the following, we will tend to concentrate on collisionless, temperature gradient driven TEM turbulence away from linear thresholds

$\Phi$  ITG



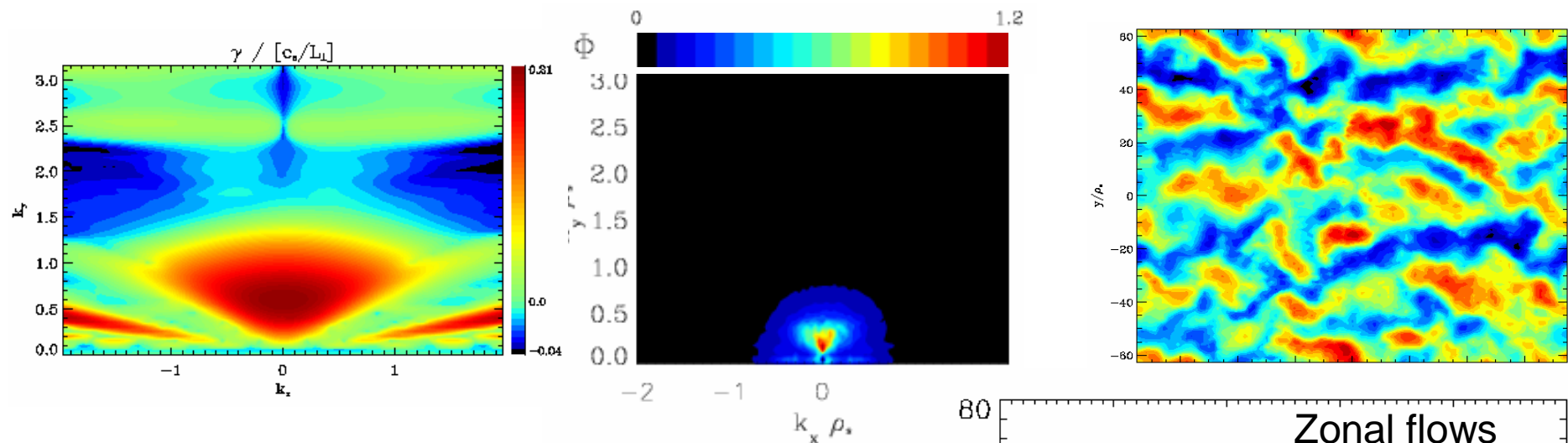
$\Phi$  TEM



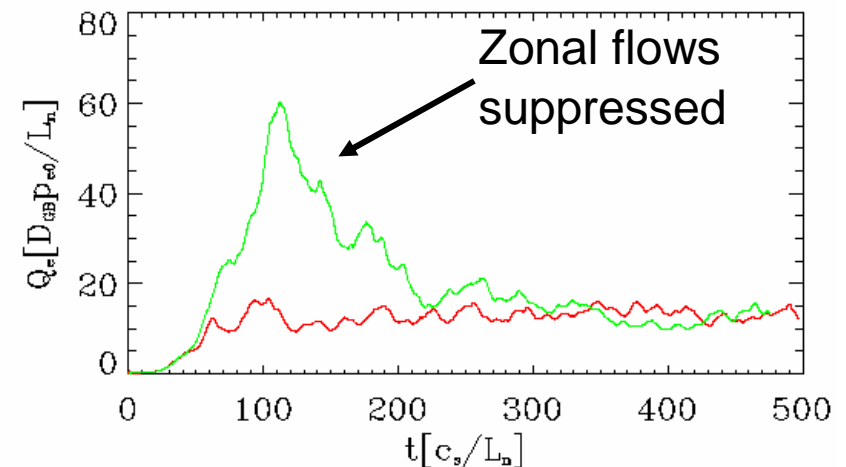
# Characteristics of TEM turbulence

In the saturated phase, TEM turbulence often exhibits:

- radially elongated structures (“streamers”; remnants of linear modes), nonlinear spectrum reflects linear growth rate spectrum



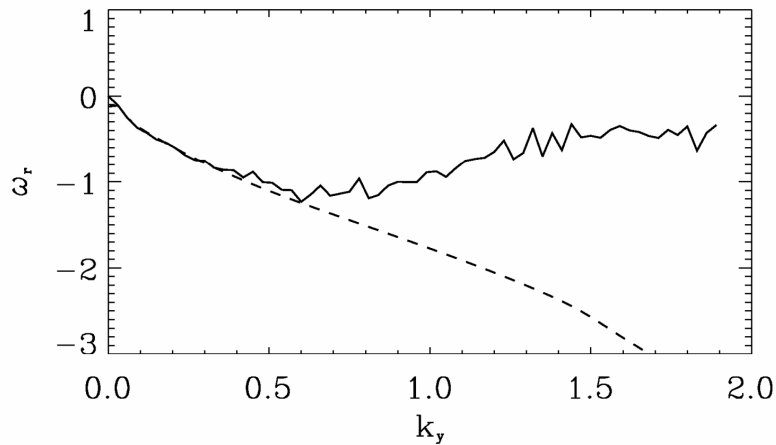
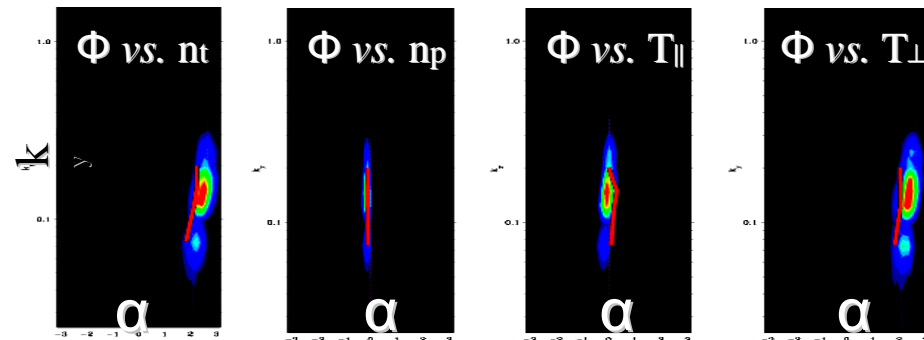
- no dependence of transport level on (intrinsically nonlinear) zonal flows [Dannert & Jenko, PoP 2005]



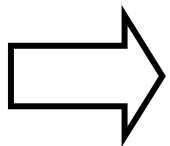
# Characteristics of TEM turbulence (cont'd)



- no significant shift of cross phases w.r.t. linear ones [Dannert & Jenko, PoP 2005]



- nonlinear frequencies close to linear ones for low  $k_y$  values



Description of the nonlinear system as linear modes in a turbulent bath?

# Short-hand notation of gyrokinetics

- Gyrokinetic Vlasov-Maxwell system:

$$\frac{\partial g}{\partial t} = \mathcal{L}g + \mathcal{N}l[g]$$

where

- $g$ : modified distribution function (state vector) depending on  $k_x, k_y, z, v_{\parallel}, \mu$  coordinates and species label
  - $\mathcal{L}$  : linear integro-differential operator
  - $\mathcal{N}l[g]$  : (quadratic) ExB nonlinearity
- Linear physics determined by eigenspectrum of  $\mathcal{L}$  (eigenvalue solvers)
  - Saturation provided by  $\mathcal{N}l[g]$
  - In the following, for simplicity, we use s- $\alpha$  geometry (with  $\alpha=0$ ); ETG modes are linearly stable

# Quasilinear ansatz

- Assumption  $\mathcal{N}l[g] \sim g$  leads to an effective linear equation

$$\frac{\partial g}{\partial t} = \mathcal{L}g + \mathcal{X}g$$

- $\mathcal{N}l[g]$  and  $g$  are fluctuating quantities; to get an estimate for the complex proportionality constant  $\mathcal{X} = \mathcal{X}(k_x, k_y, z, spec)$ , we minimize the model error  $\langle |\mathcal{N}[g] - \mathcal{X}g|^2 \rangle$
- The resulting expression  $\mathcal{X} = \langle g^* \mathcal{N}[g] \rangle / \langle |g|^2 \rangle$  is evaluated in numerical simulations of TEM turbulence

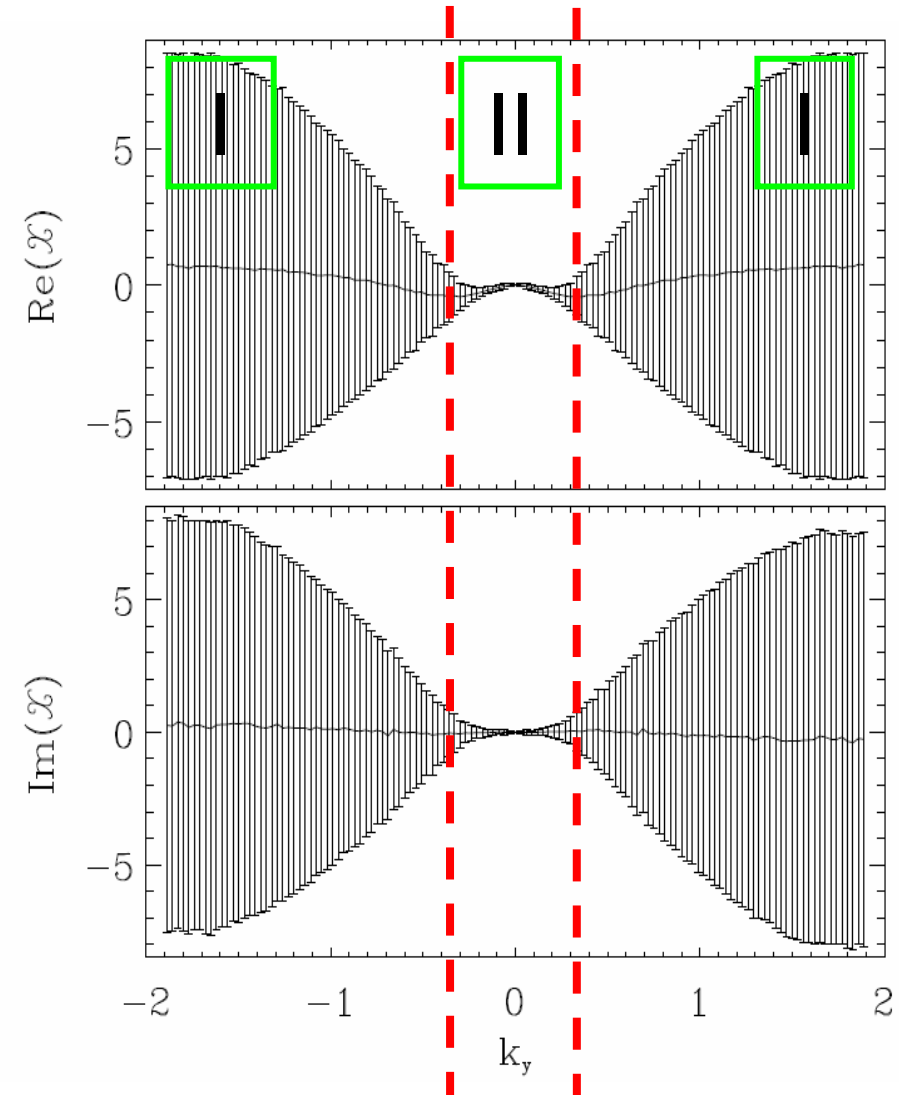
(  $\langle \rangle$  : average over velocity space and time)



# Structure of the nonlinearity



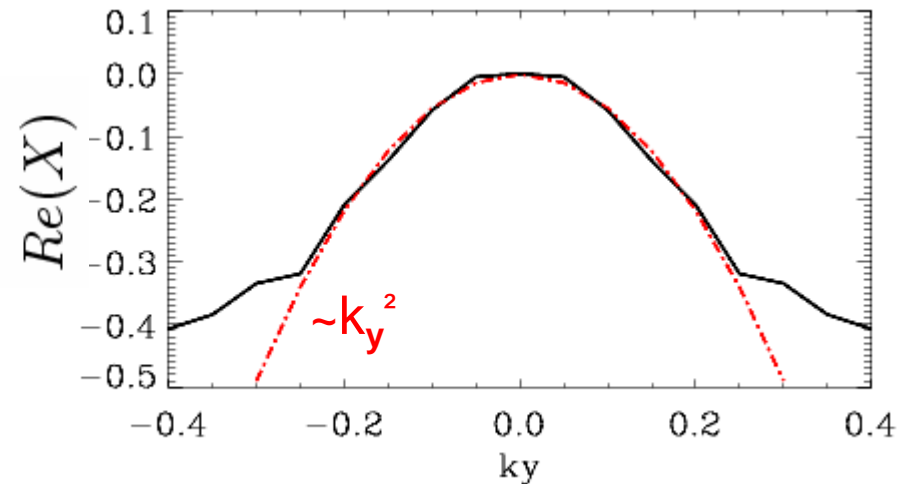
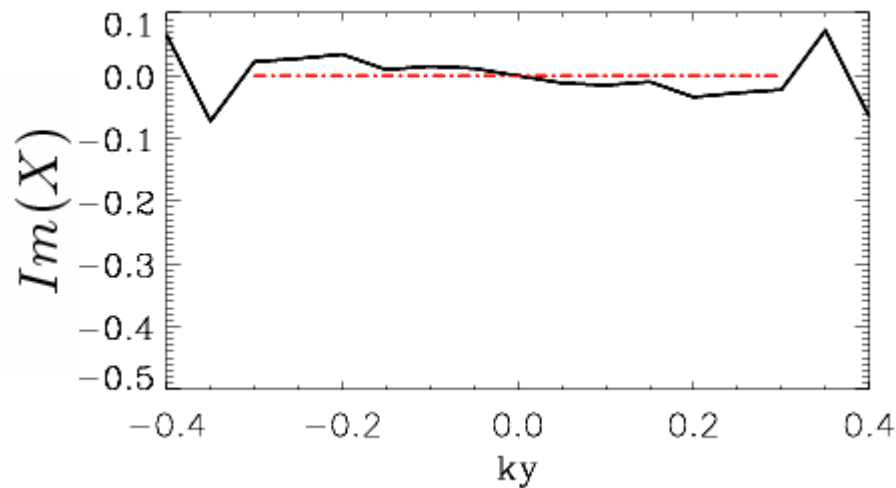
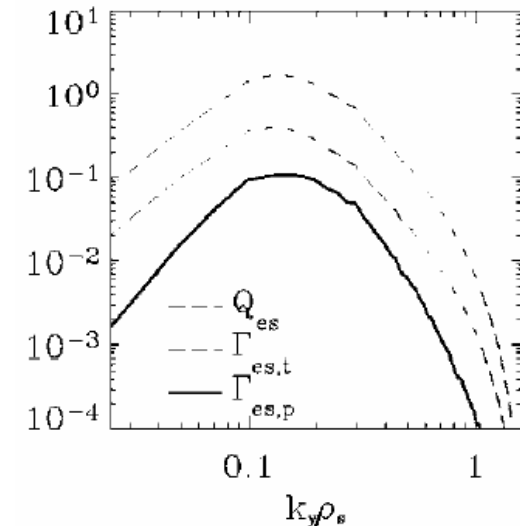
- Numerical result ( $k_x=0$ ): two distinguishable regions
  - Region **I**  $|k_y|>1$ : dominated by fluctuations
  - Region **II** ,  $|k_y|<0.3$ : clear structure in  $X$ , the model error  $\langle |\mathcal{N}[g] - \mathcal{X}g|^2 \rangle$  is small



# Region II: Transport relevant $k_y$ range

- The  $k_y$  range where fluctuations are small coincides with the  $k_y$  range **relevant for transport**
- Result:  $\text{Im}(X)$  is negligible,  $\text{Re}(X)$  is a parabola

$$\mathcal{N}l[g] \simeq D(-k_{\perp}^2)g = D\nabla_{\perp}^2 g$$

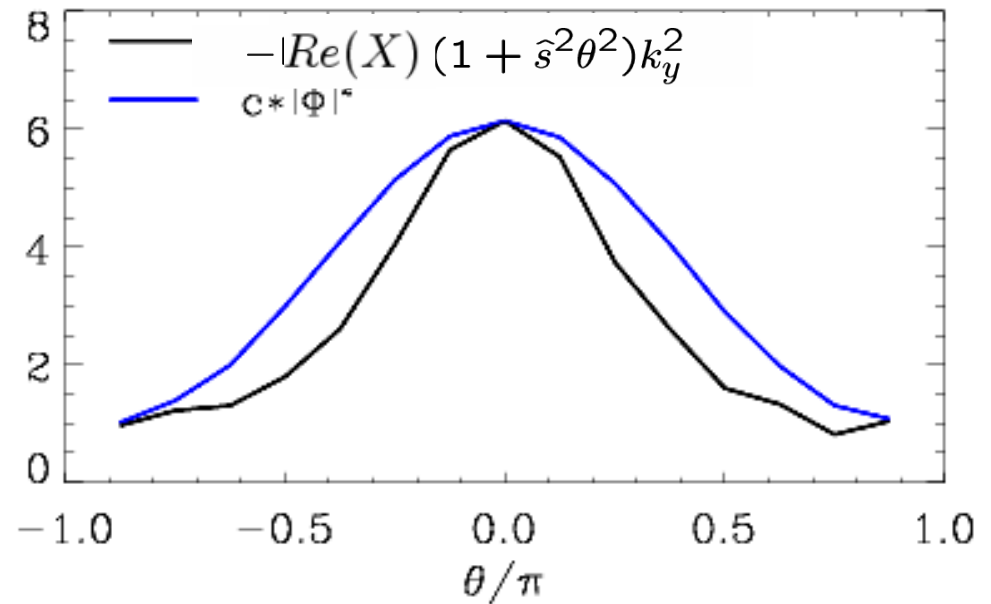


Cp. Resonance Broadening Theory (Dupree), MSR formalism (Krommes), Dressed Test Mode Approach (Itoh) in long wavelength, low frequency limit

# Region II: Parallel structure of diffusivity



- Dependence on parallel coordinate:  $\approx |\Phi|^2$



- Integration with parallel weighting yields

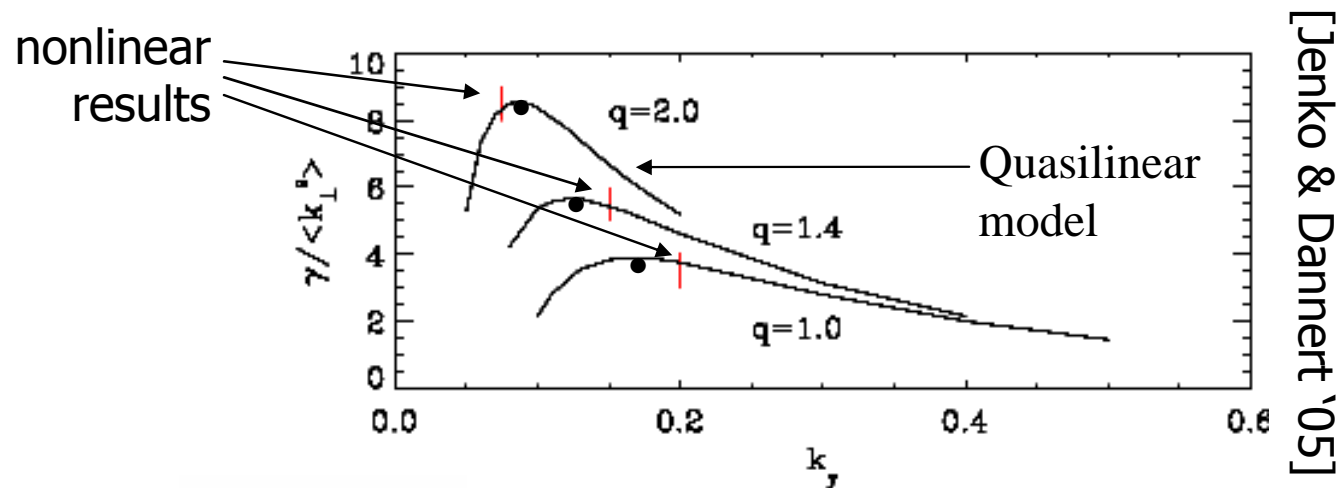
effective wave number  $\langle k_{\perp}^2 \rangle := \int d\theta D(\theta) k_{\perp}^2 \simeq c \int d\theta |\Phi^2(\theta)| k_{\perp}^2$

- Quasilinear equation:  $\frac{\partial g}{\partial t} = \mathcal{L}g + \mathcal{N}l[g] \simeq (i\omega_r + \gamma - D_0 \langle k_{\perp}^2 \rangle)g$

- Stationarity implies  $D_0 \sim \frac{\gamma}{\langle k_{\perp}^2 \rangle}$

# Quasilinear transport model

- Fick's law  $Q \sim D_0 \frac{R}{L_{Te}}$  gives  $Q_e \propto \max_{k_y} \left[ \frac{\gamma}{\langle k_{\perp}^2 \rangle} \right] \frac{R}{L_{Te}}$
- Application: q dependence of TEM-induced transport



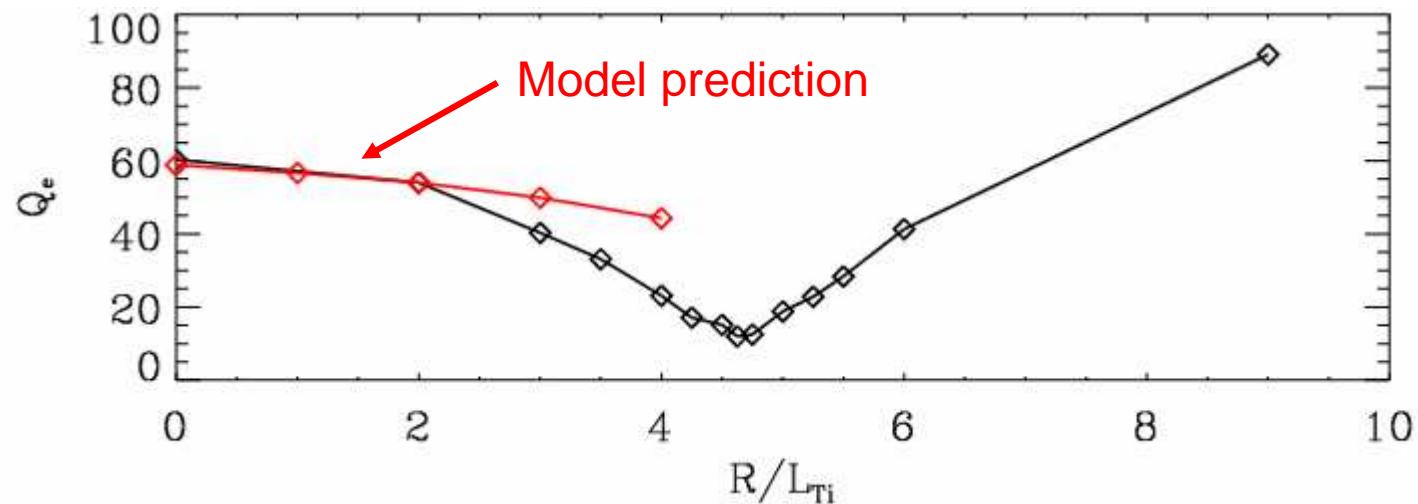
- Scaling:  $Q_e \propto q^{\nu}$
- The quasilinear model captures the q-dependence seen in nonlinear simulations (here  $\nu \approx 1.7$ ) and in experiments ( $\nu = 1 - 2$ )



# Nonlinear ITG-TEM interactions

# TEM-ITG turbulence transition

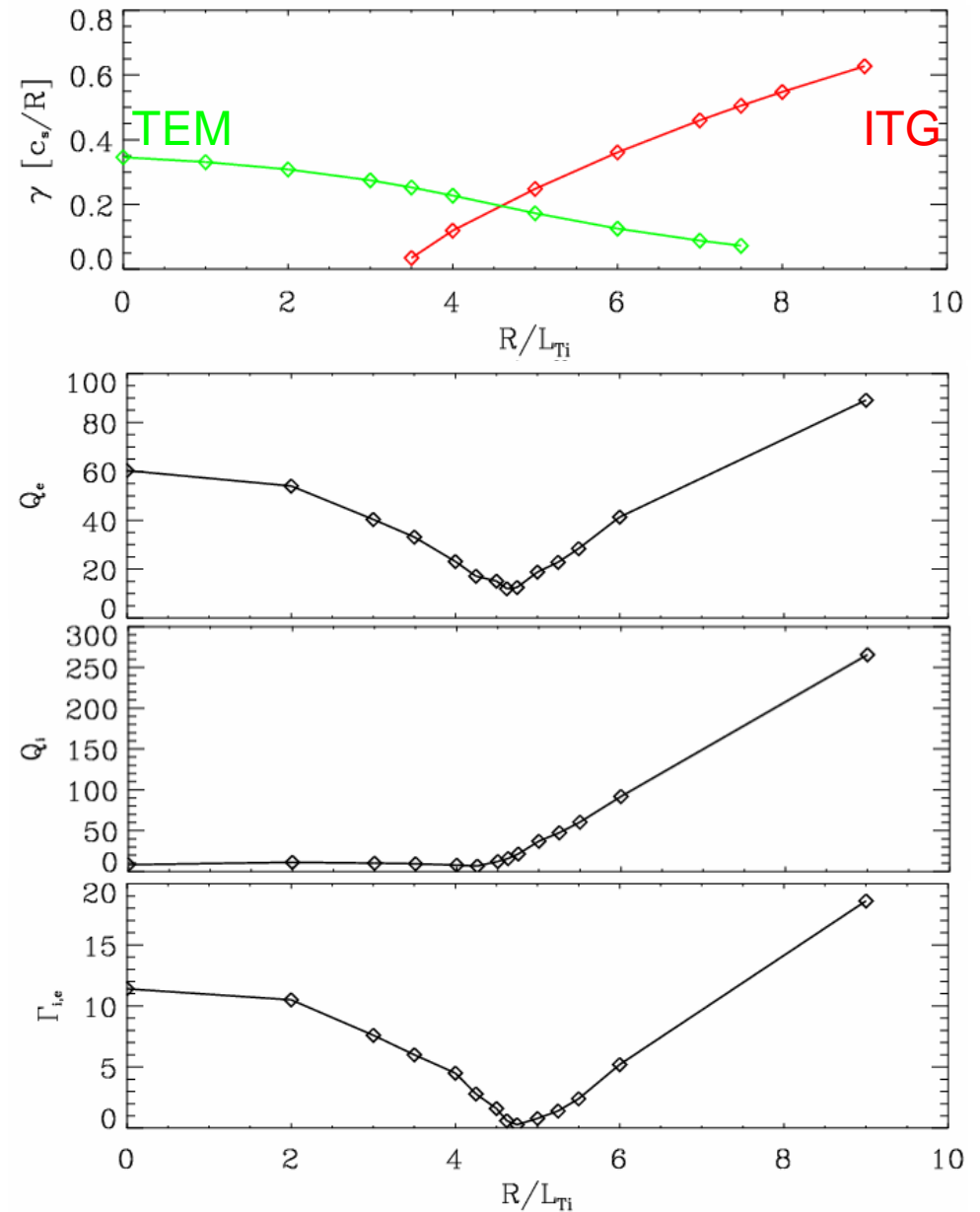
- In the following,  $R/L_{Te}$  is held constant at 4.5
- Increasing  $R/L_{Ti}$  finally leads to a breakdown of the quasilinear model and transition to ITG dominated turbulence



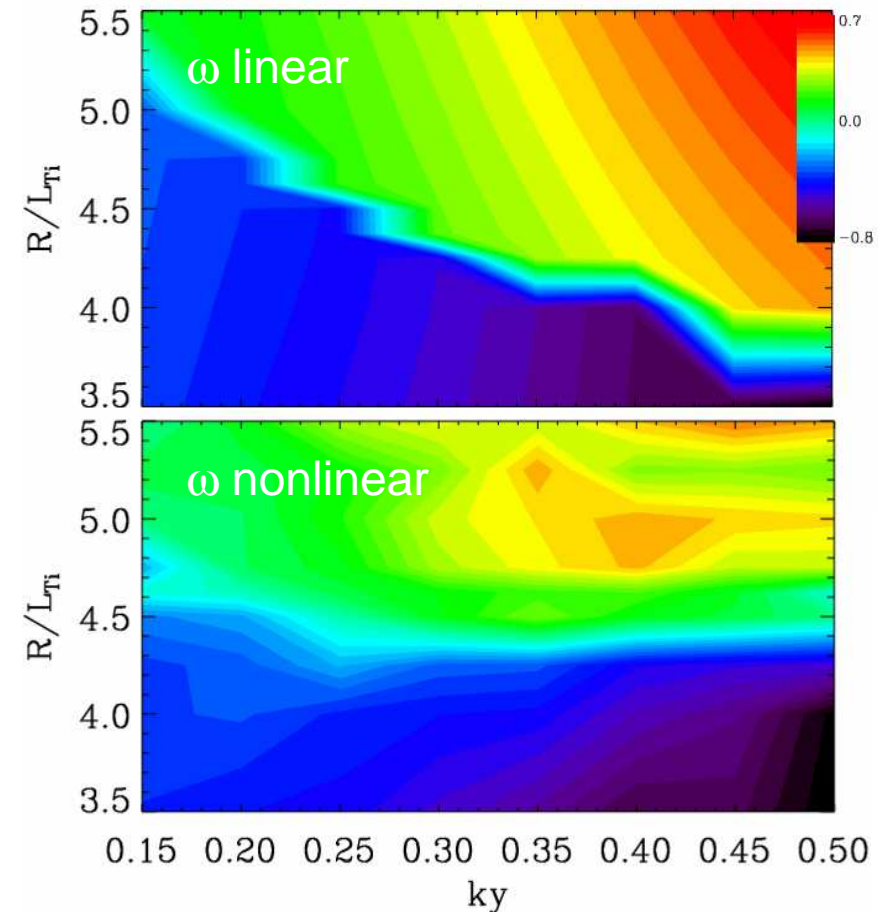
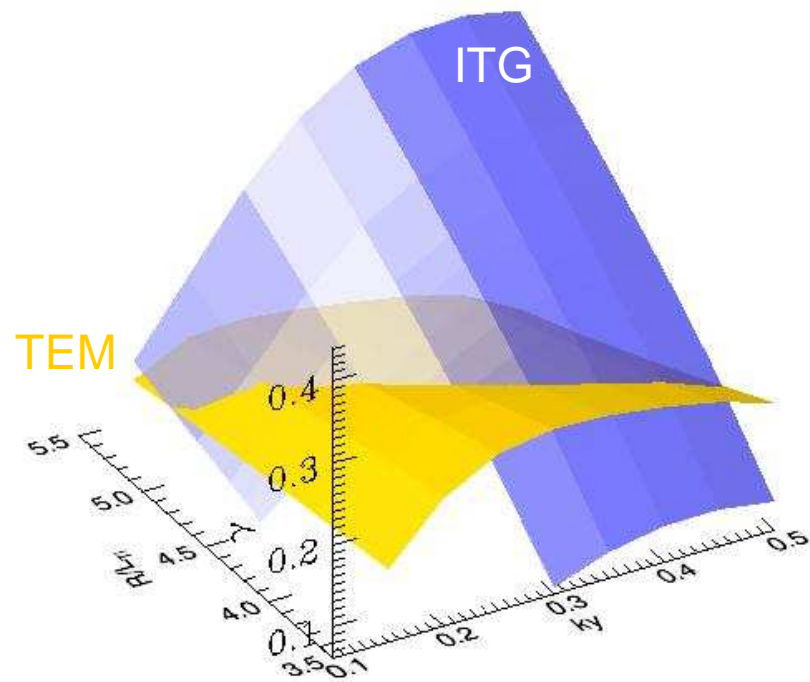
# TEM-ITG turbulence transition (cont'd)



- Linear growth rates ( $k_y=0.25$ ): smooth transition
- Subdominant modes are present (GENE as eigenvalue solver)
- With the additional instability, heat and particle fluxes are suppressed instead of increased
- ITG branch: Nonlinear upshift of critical  $R/L_{Ti}$



# Nonlinear frequency spectrum

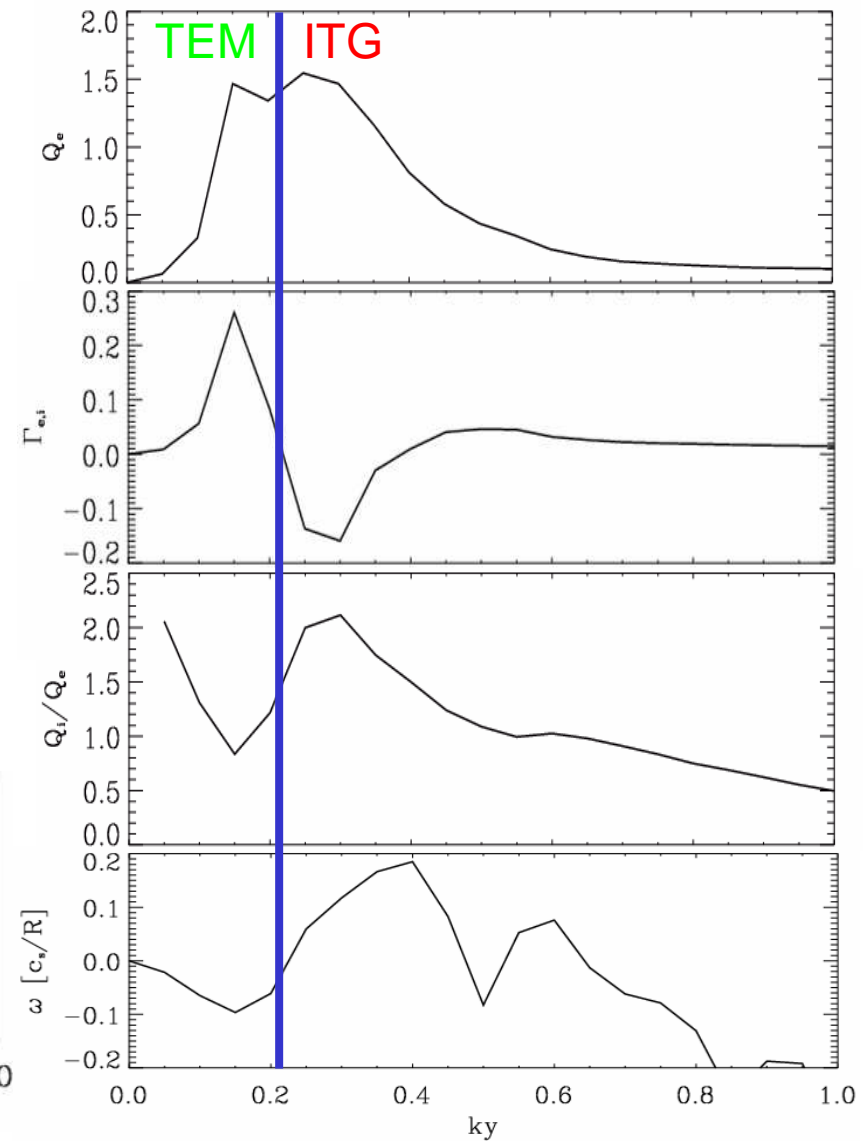
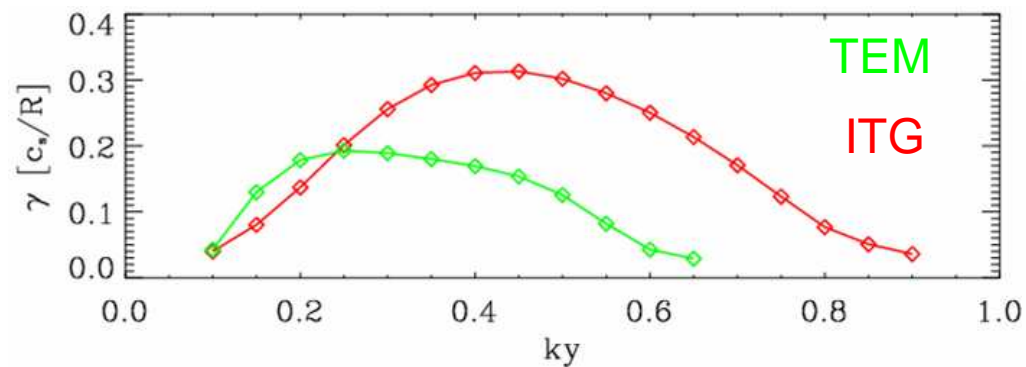


- Even though the linear growth rates can differ significantly, the  $k_y$  dependence of critical  $R/L_{Ti}$  is strongly flattened
- The nonlinear frequencies show rather abrupt transition in  $R/L_{Ti}$ ; there is only a relatively small  $R/L_{Ti}$  interval with both signs



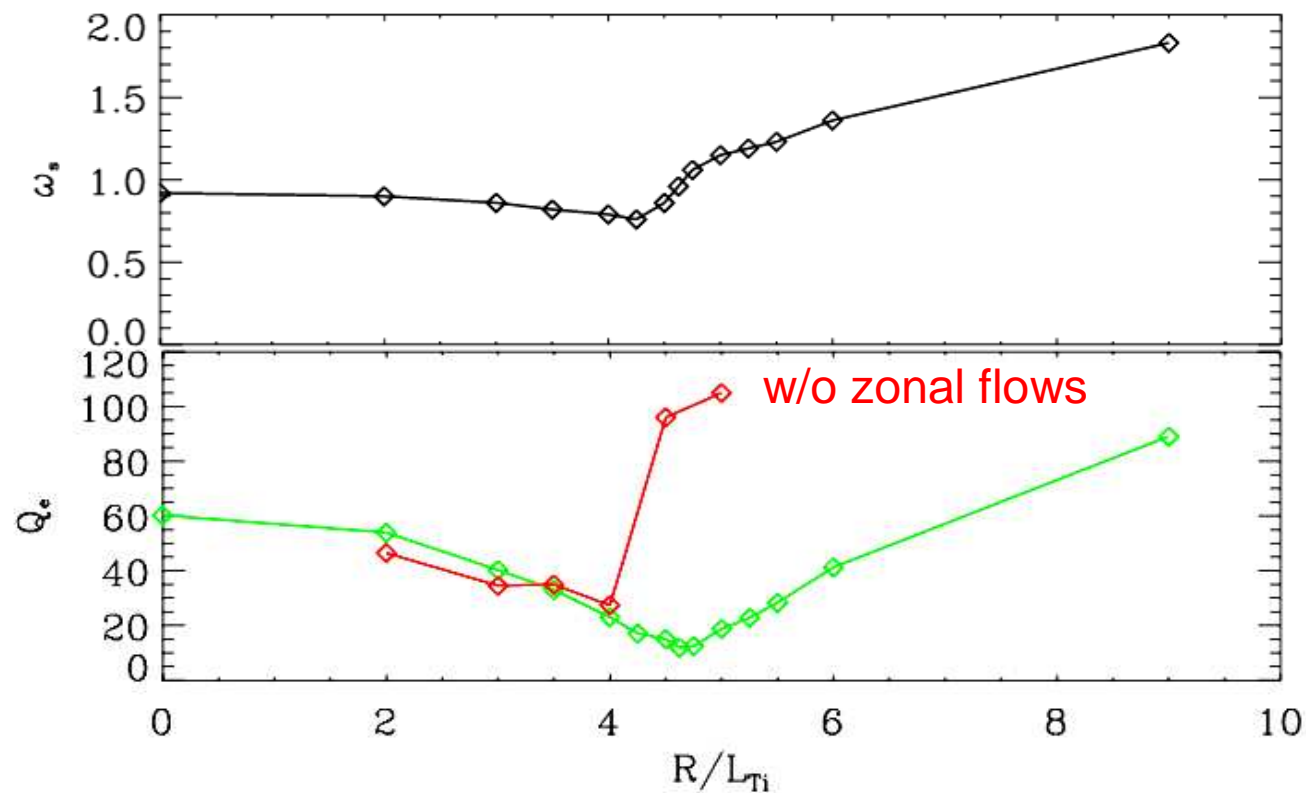
# Coexistence near critical $R/L_{Ti}$

- In the (narrow) transition region ( $R/L_{Ti}=4.6$ ): Different kinds of turbulence coexist at different  $k_y$
- Turbulence properties (nonlinear frequency, transport ratio, ..) are consistent at a given  $k_y$
- The spectral distribution of TEM and ITG turbulence roughly corresponds to the linear result



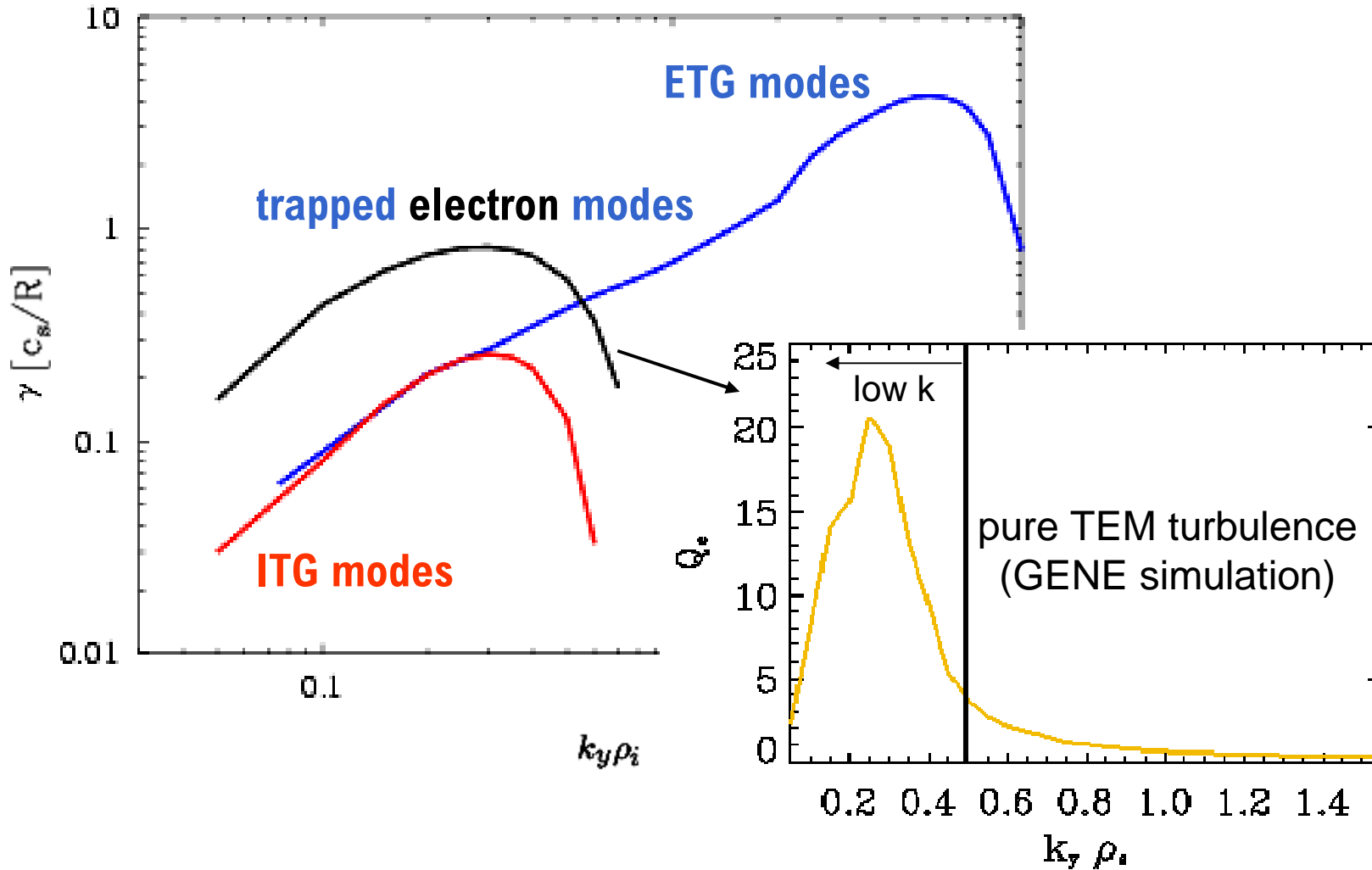
# Zonal flow behavior

- TEM-ITG transition changes the role of zonal flows ( $k_y=k_{||}=0$ )
- Relatively sharp transition seen in the value of the ExB shearing rate  $\omega_s$  and in simulations where zonal flows have been artificially suppressed



# Nonlinear ITG-TEM-ETG interactions

# Plasma microturbulence: Multiple scales

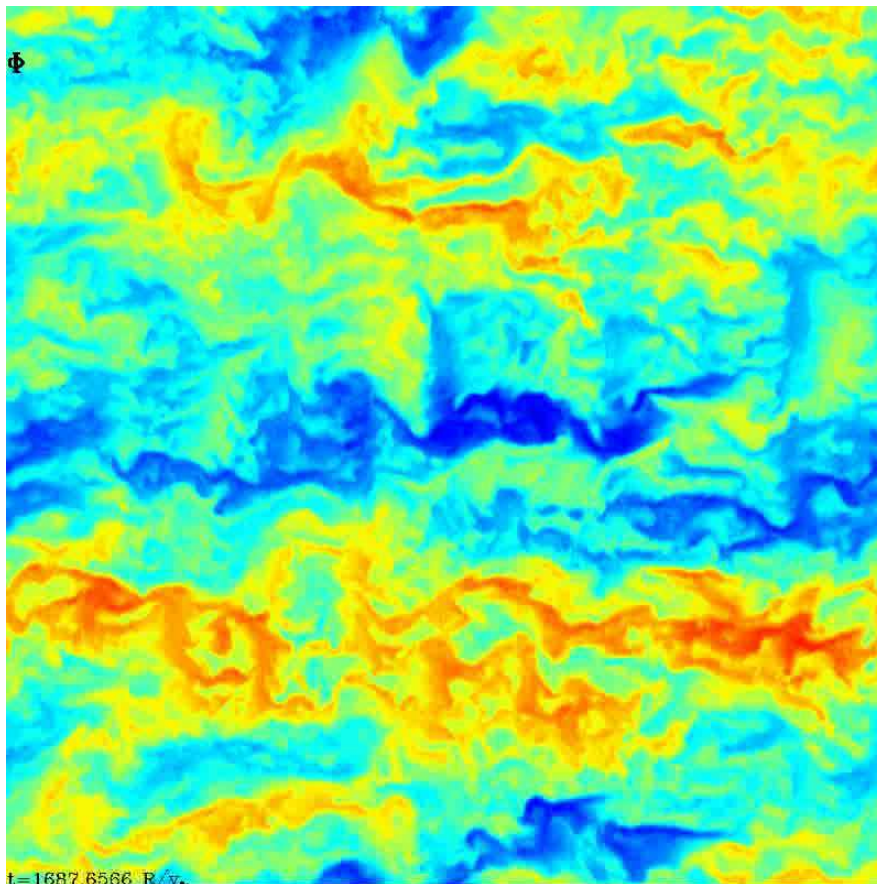


**Question to theory: What is the role of high wavenumbers?**

# TEM-ETG turbulence ( $\Phi$ contours)

Here: electrostatic, collisionless, s- $\alpha$  model equilibrium;  
Cyclone-like parameters, reduced mass ratio

Case I: ITG is turned off



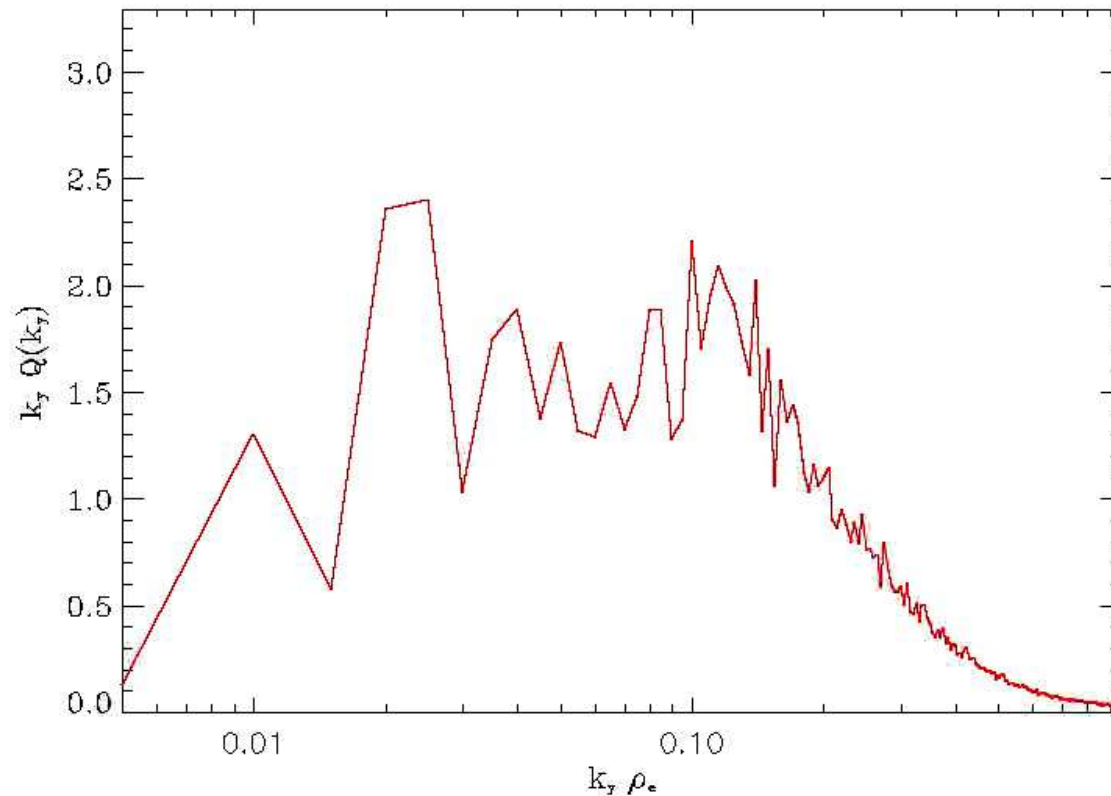
~ 100,000 CPUh / run

box size:  
64 ion gyroradii

resolution:  
~2 electron gyroradii

ETG streamers and  
TEM streamers coexist

# TEM-ETG turbulence (transport spectrum)



t=1646.71 R/v<sub>Te</sub>

$$Q = \sum_{k_y} Q(k_y) \propto \sum_{k_y} k_y Q(k_y) \Delta(\log k_y)$$

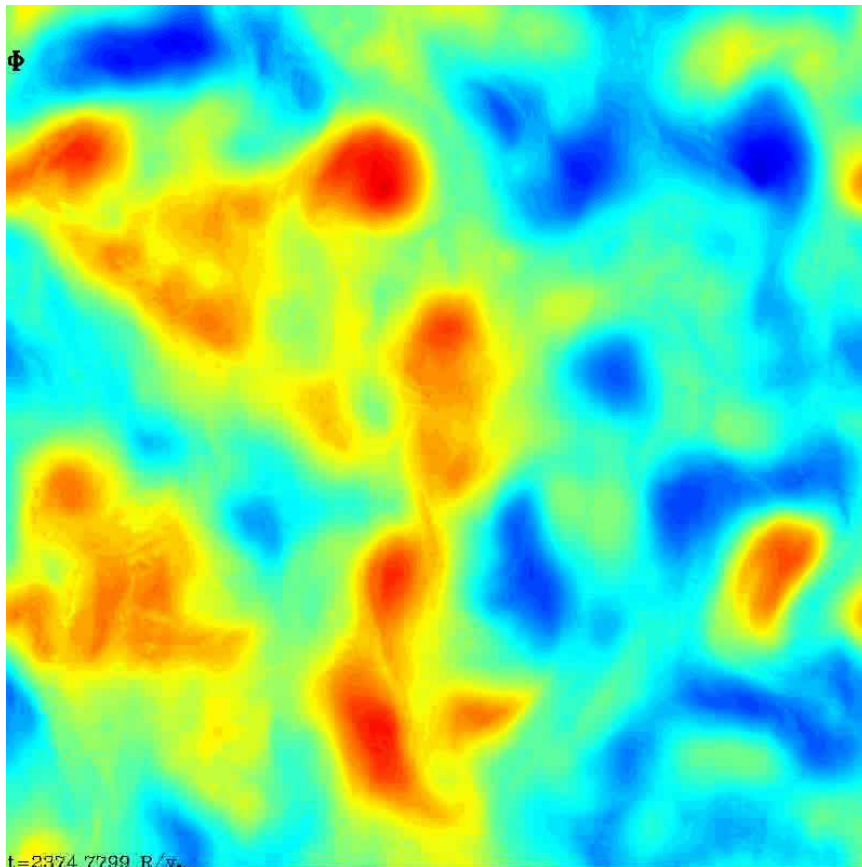
ETG transport level is in line with pure ETG simulations  
**75% of the electron heat transport is in the  $k\rho_i > 0.5$  regime**

# ITG/TEM-ETG turbulence ( $\Phi$ contours)

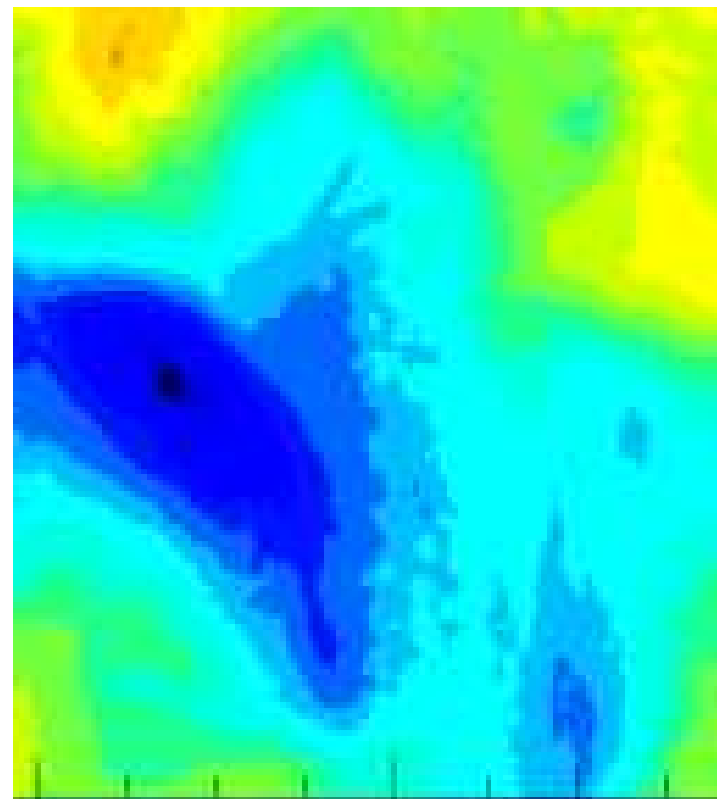


Note: For  $R/L_{Ti} = 6.9$ , one obtains  $\chi_i \sim 50 \text{ m}^2/\text{s}$  (!) and a fairly small ETG fraction; therefore, we use  $R/L_{Ti} = 5.5$

Case II: ITG is dominant



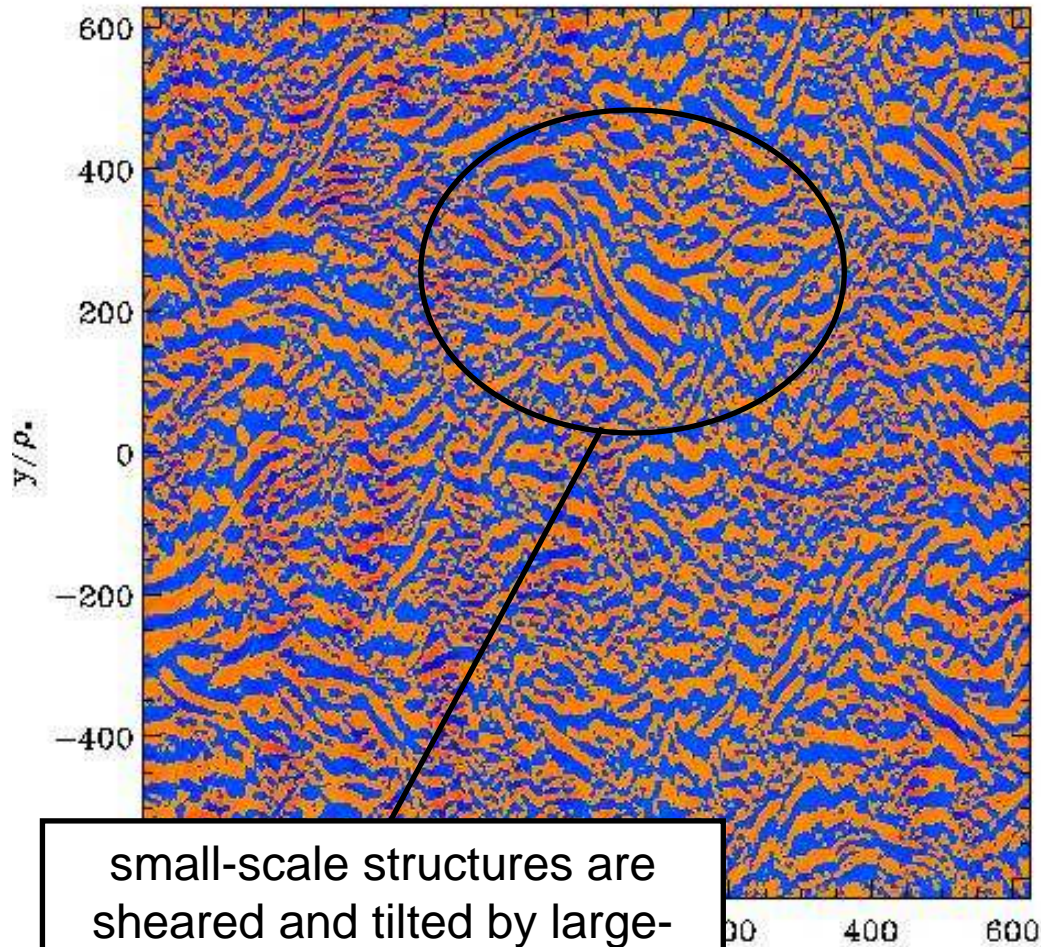
small-scale streamers are subject to large-scale vortex shearing



# ITG/TEM-ETG turbulence (cont'd)

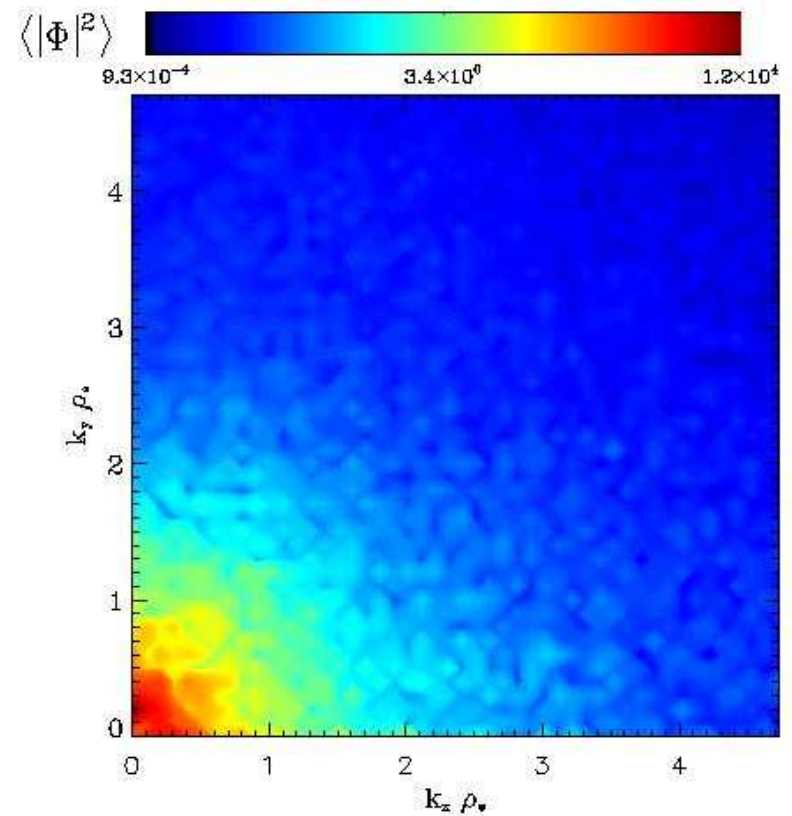


**Filter:** Set modes with  $k_y \rho_s < 2$  to zero



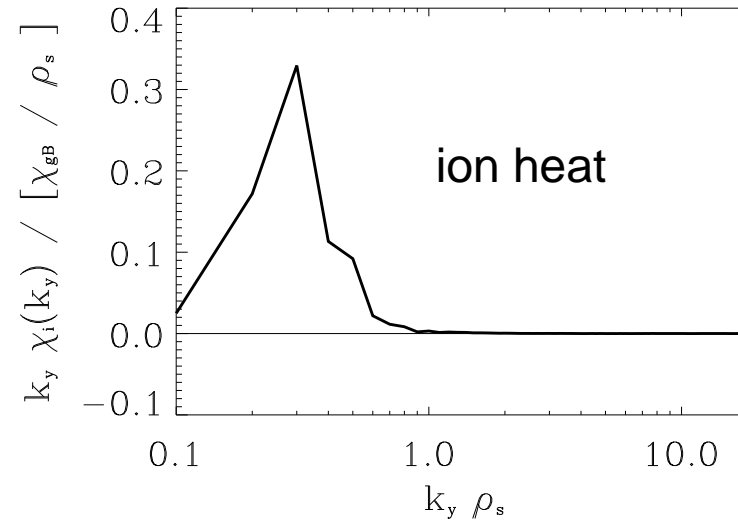
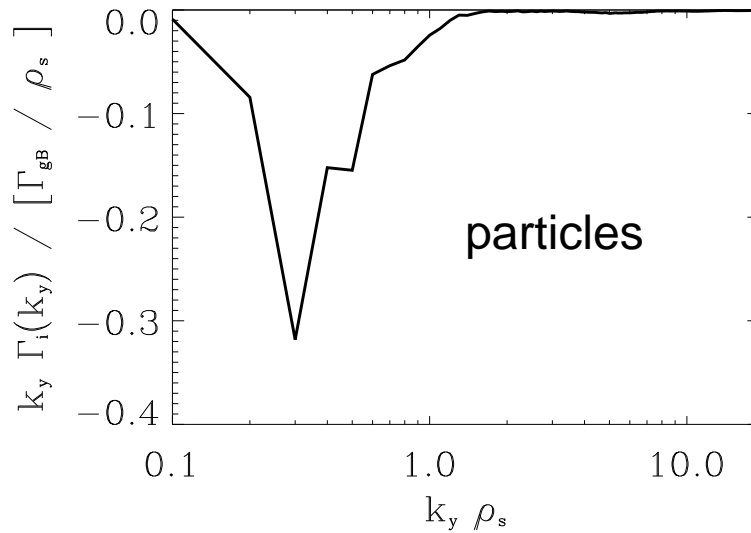
small-scale structures are sheared and tilted by large-scale vortices

Isotropic density spectrum



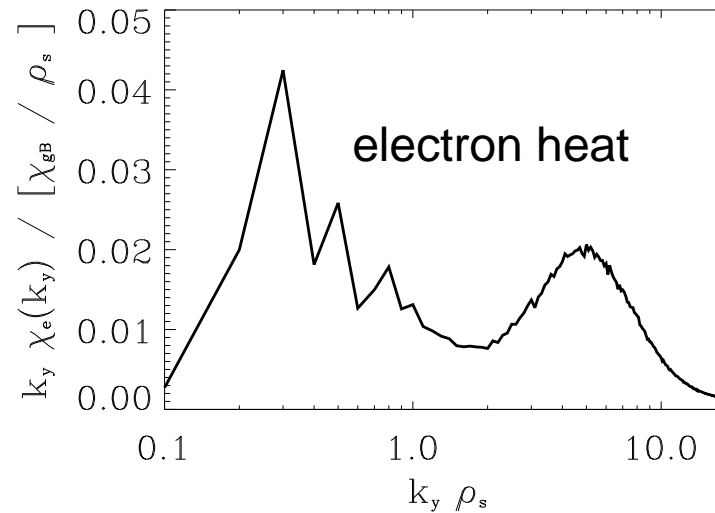


# ITG/TEM-ETG turbulence (cont'd)



**Ion heat flux is still too large, but > 50% of the electron heat transport is in the  $k_{\perp i} > 0.5$  regime**

ETG even more pronounced in presence of ExB shear: NSTX



# The role of magnetic geometry



# $\mu$ tearing modes in the high-beta spherical tokamak NSTX

# $\mu$ tearing modes: A brief overview

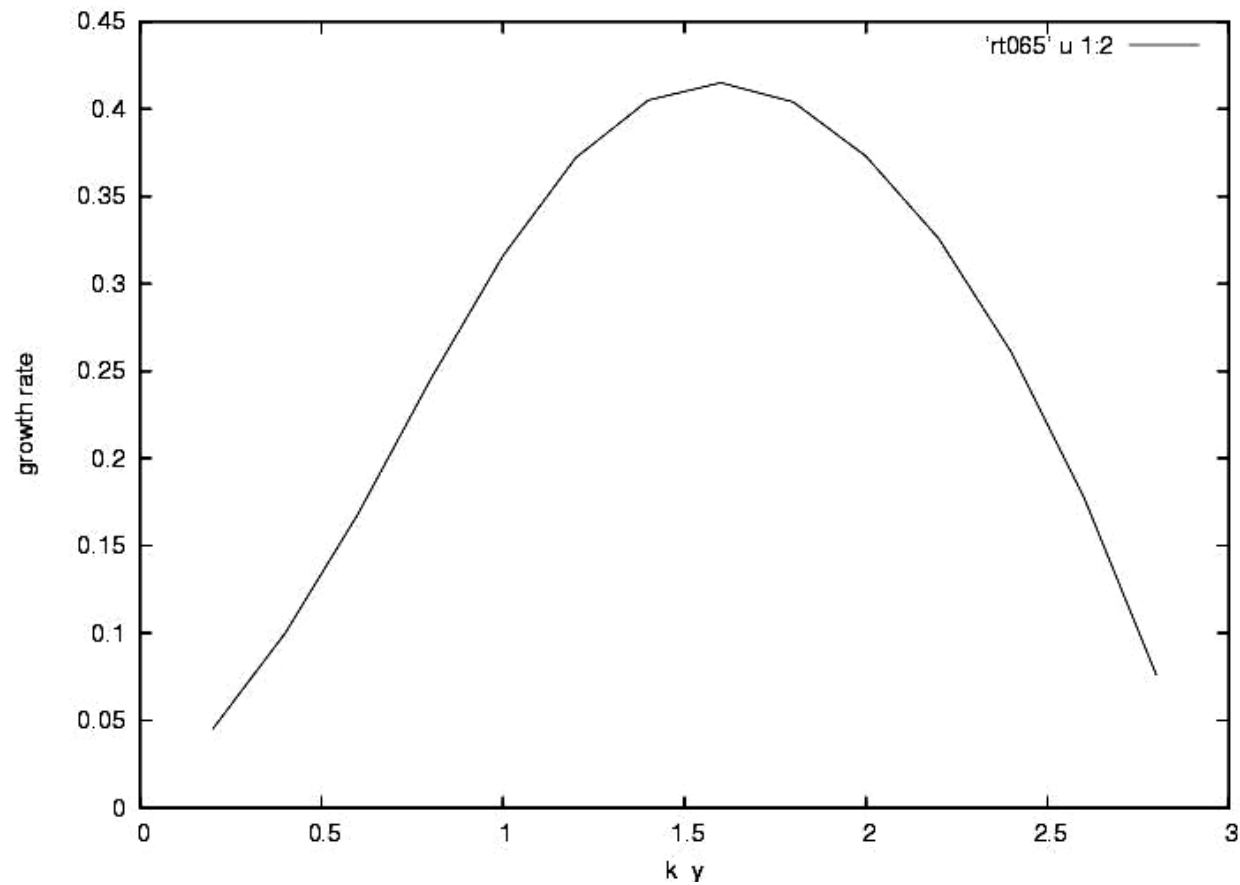


- Finite beta modes with “tearing parity”; may induce substantial electron heat transport due to magnetic flutter
- First wave of theoretical papers in the 1970s and 1980s; have received renewed interest in the last few years
- Generally thought to be unimportant in (the core of) tokamaks, but found to be relevant in the core of NSTX (M. Redi *et al.*)
- Nonlinear simulations are extremely challenging, and even the linear properties are not yet fully understood
- Test case: NSTX #120967 ( $t=0.49\text{s}$ ,  $r/a=0.65$ )

# NSTX #120967 ( $t=0.49\text{s}$ , $r/a=0.65$ )



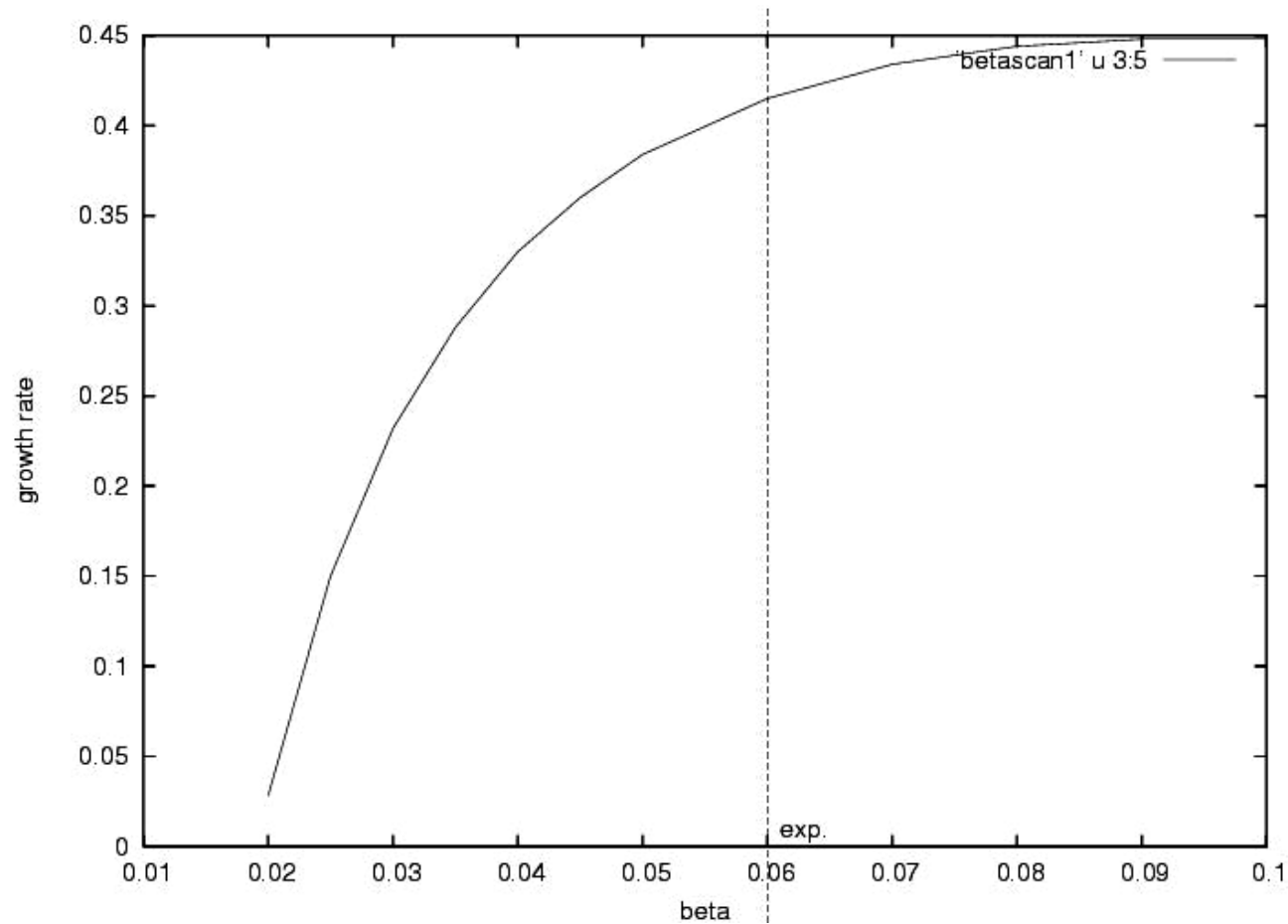
Dominant microinstability:  $\mu$ tearing mode



# $\mu$ tearing modes in NSTX: $\gamma(\beta)$

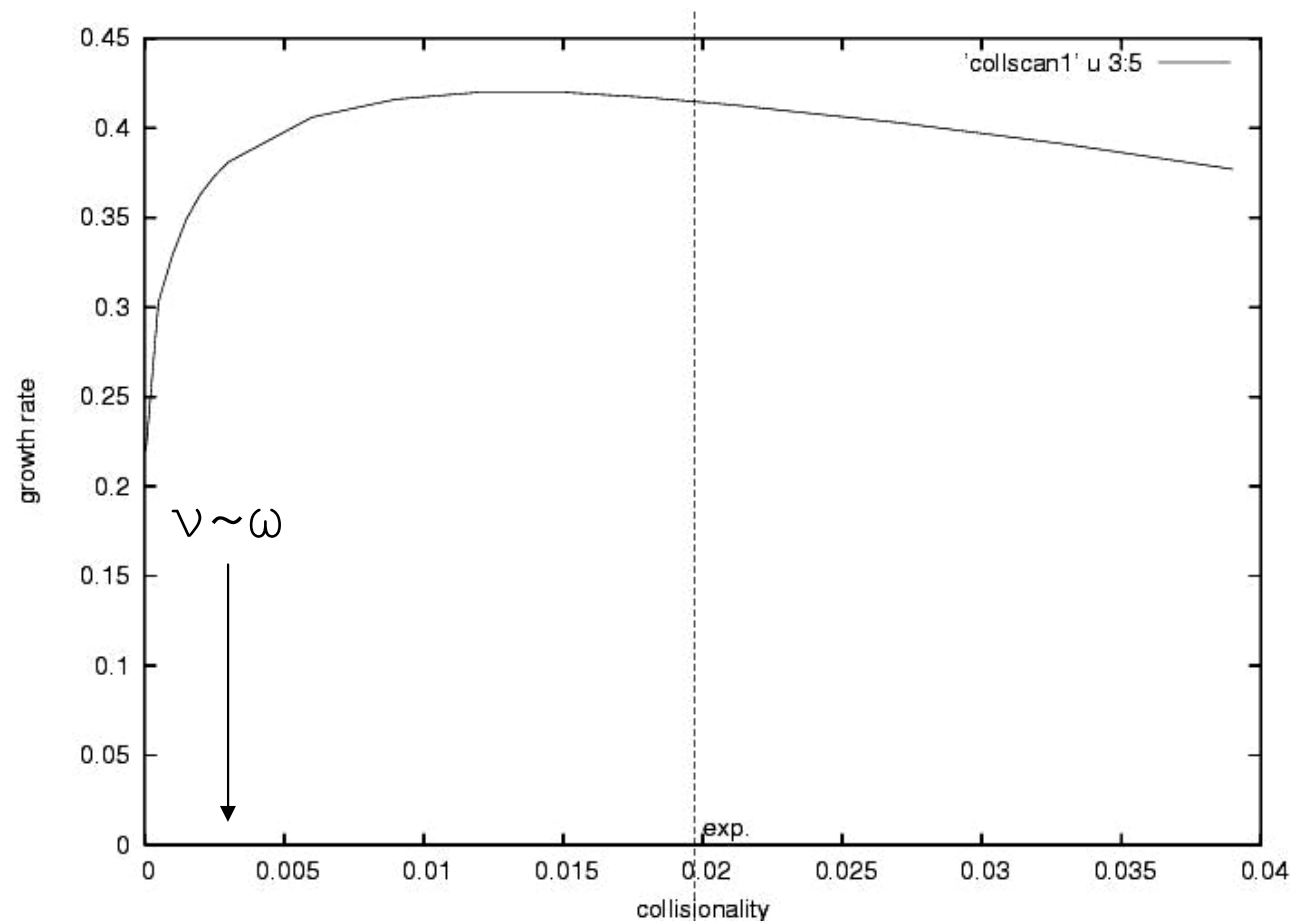


Critical beta value is exceeded by a factor of 3.



# $\mu$ tearing modes in NSTX: $\gamma(v)$

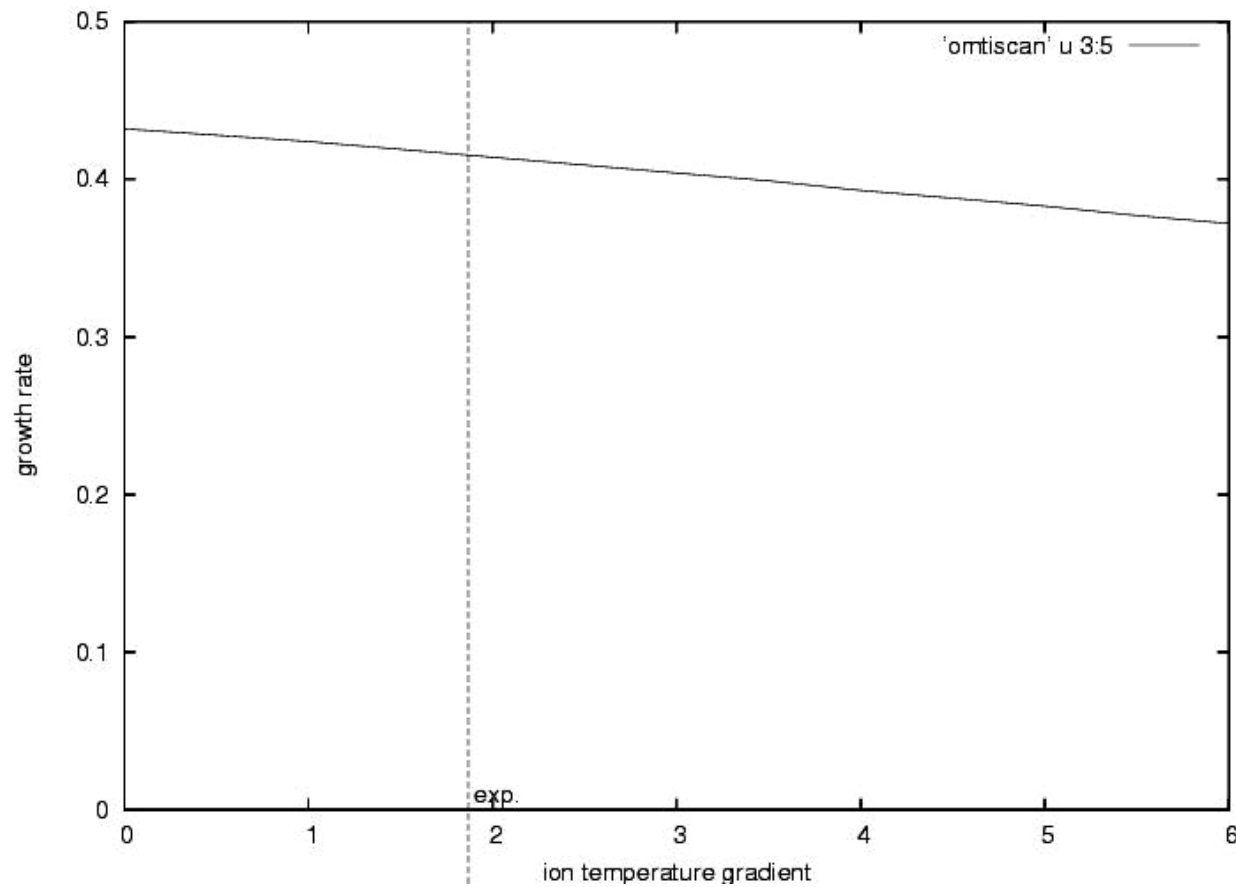
The modes are deep in the collisional regime.



# $\mu$ tearing modes in NSTX: $\gamma(R/L_{Ti})$



Weak dependence on the ion temperature gradient.

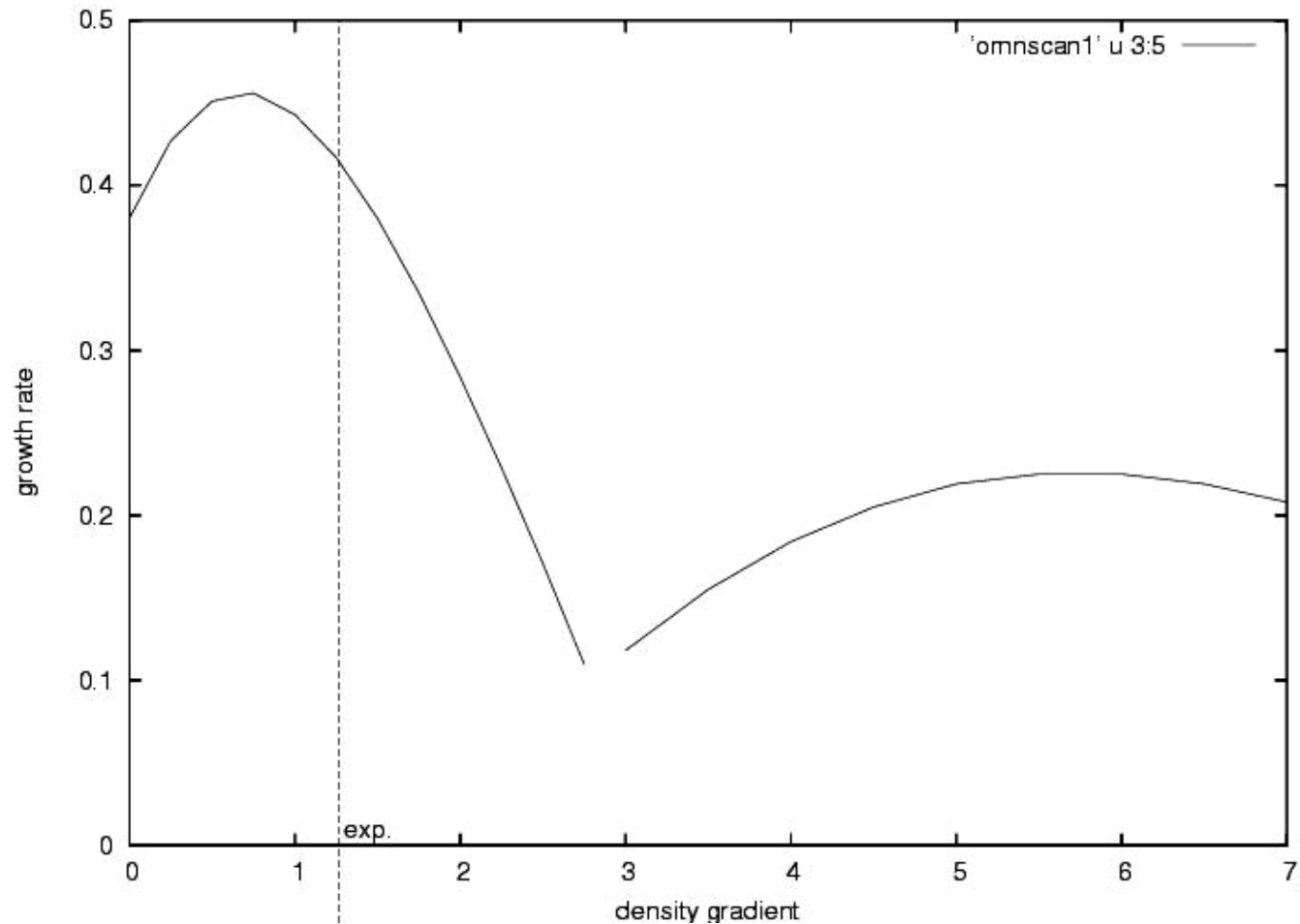




# $\mu$ tearing modes in NSTX: $\gamma(R/L_n)$



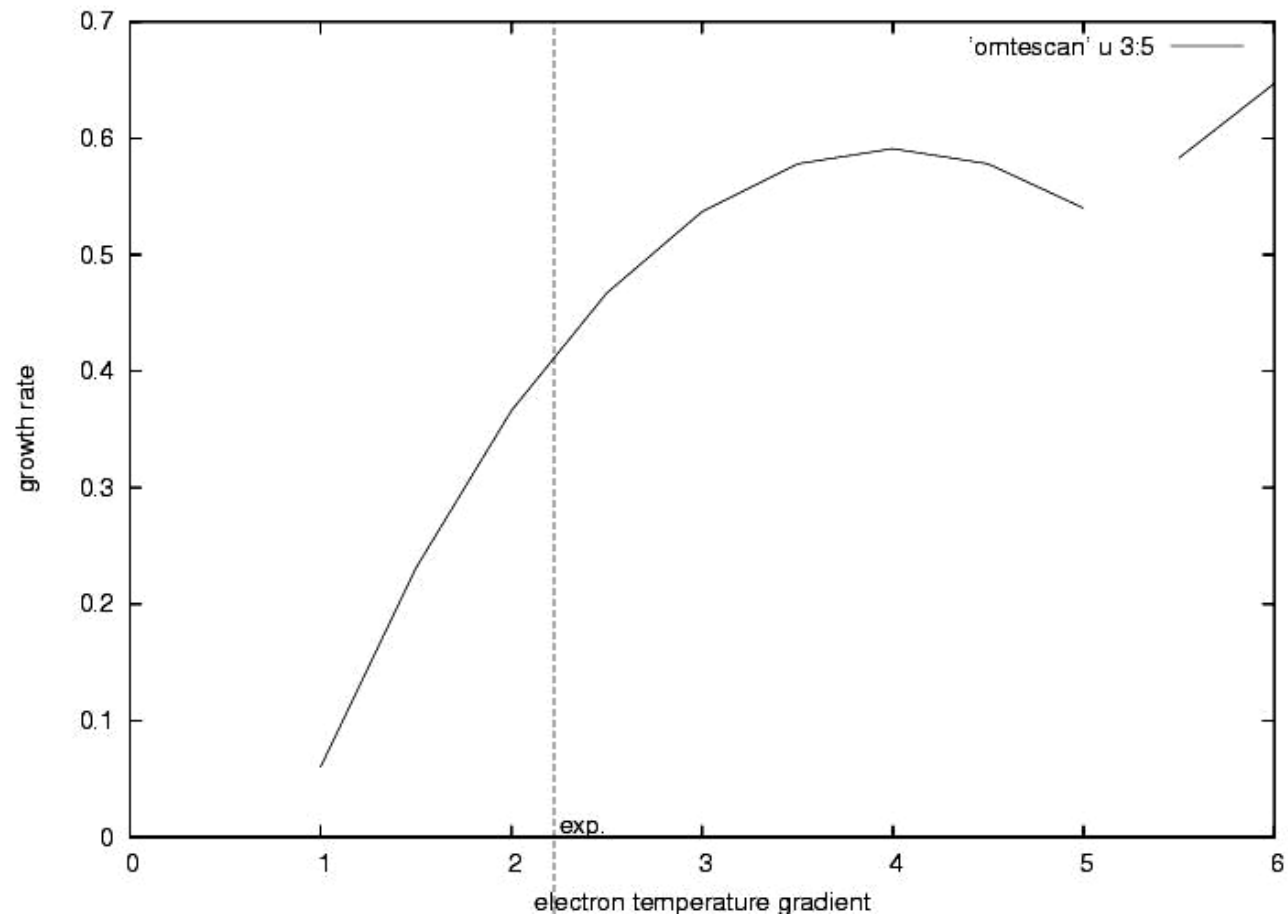
Density gradient dependence is rather complex.



# $\mu$ tearing modes in NSTX: $\gamma(R/L_{Te})$



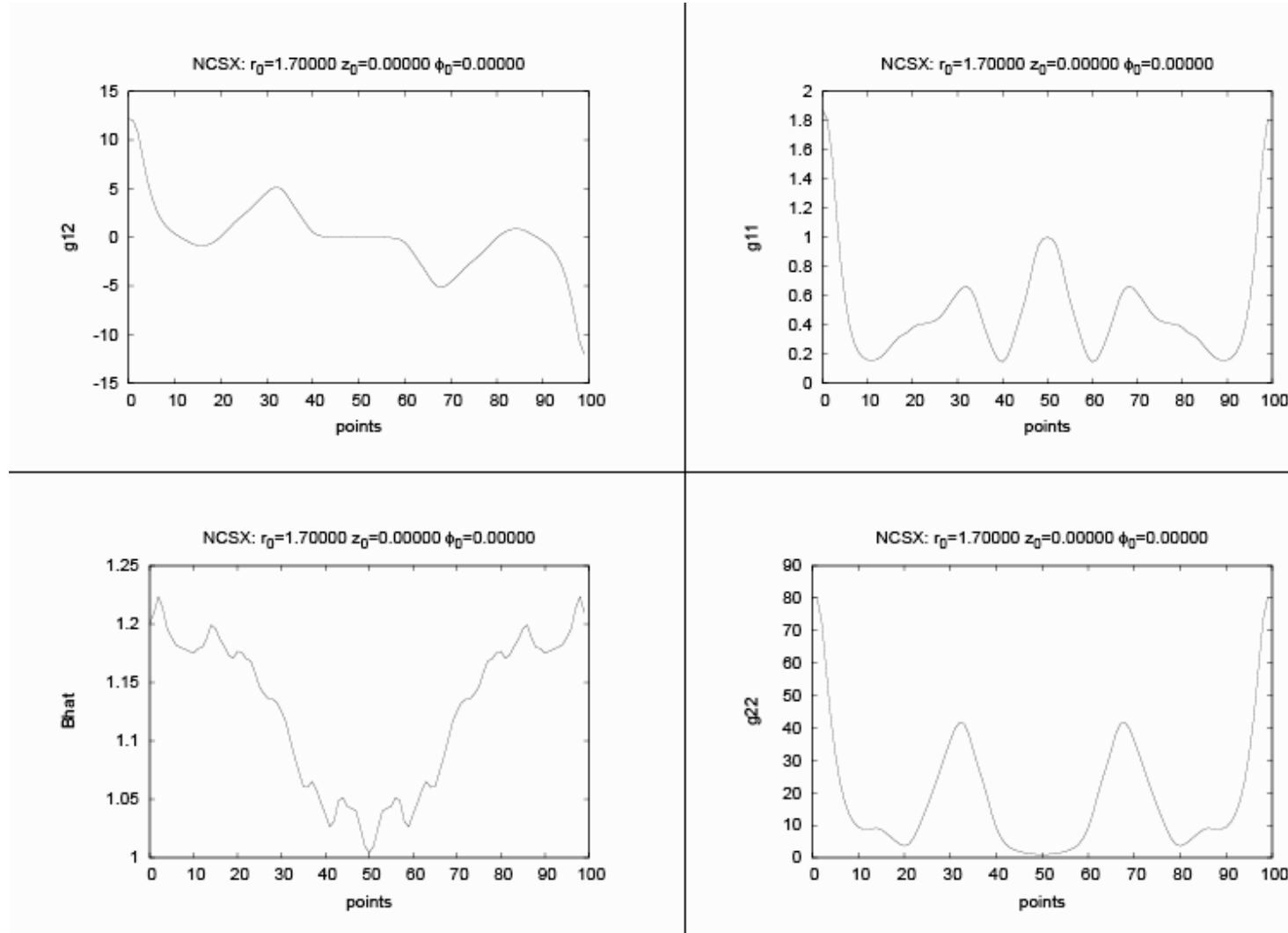
Critical electron temperature gradient is clearly exceeded.





# Adiabatic ITG turbulence in the NCSX stellarator

# Geometric coefficients from TRACER

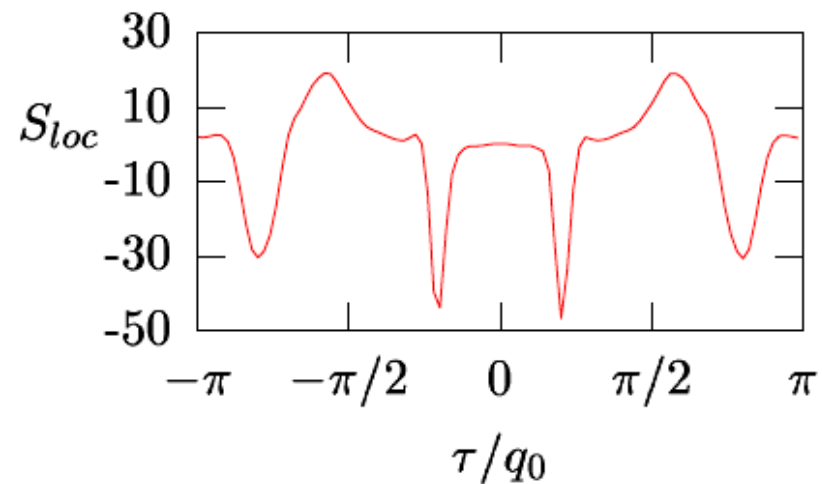
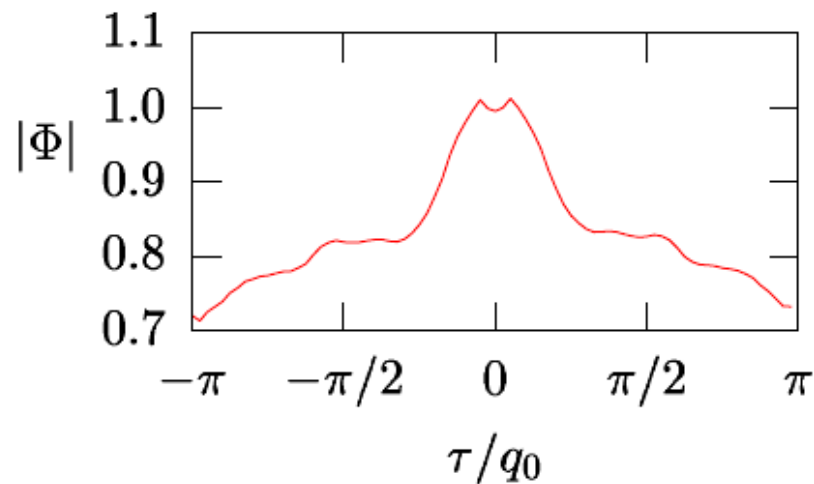


# Nonlinear ITG parallel mode structure



NCSX exhibits strong ballooning controlled by strong local shear regions

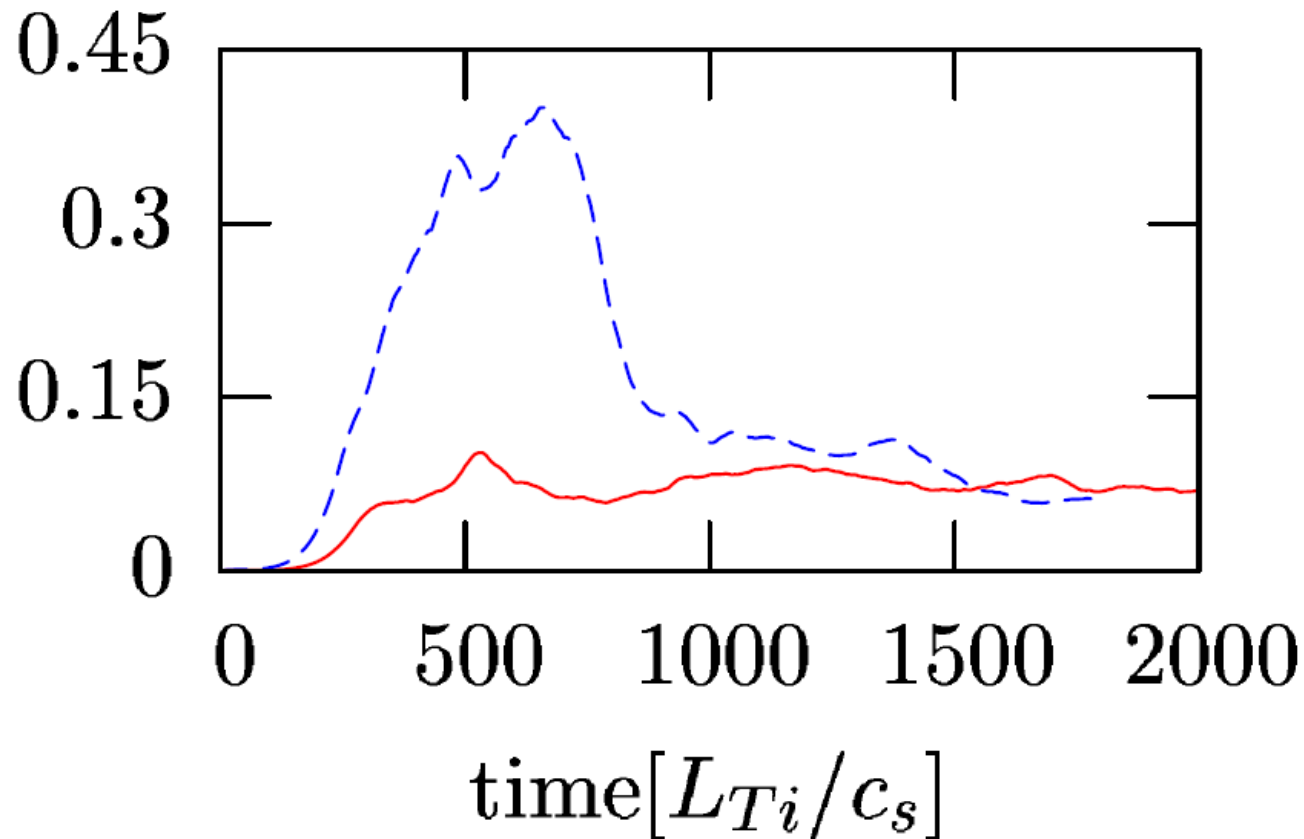
Example (*nonlinear* GENE simulation):



# Zonal flow effects



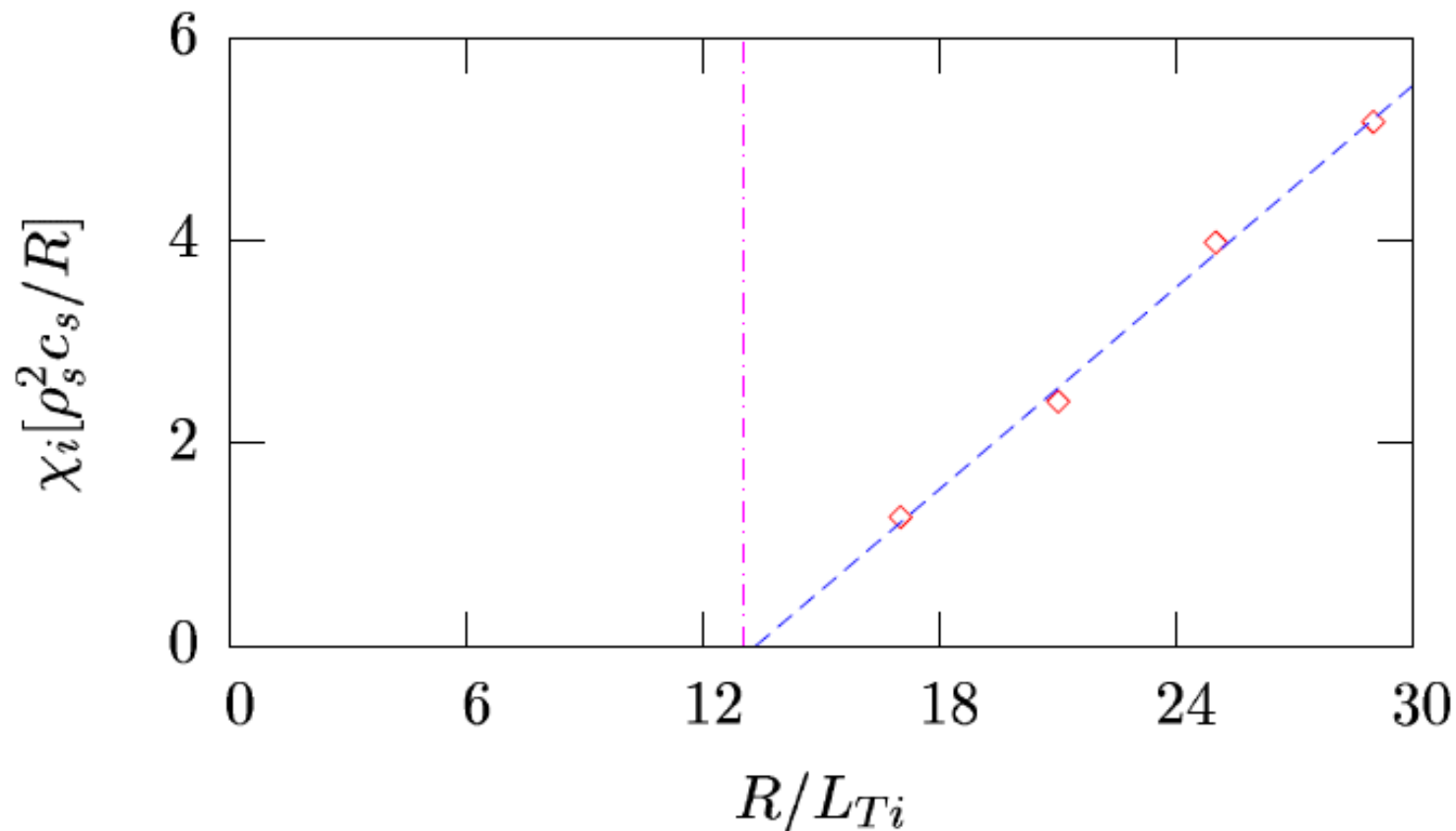
Simulation with suppressed zonal flows yields the same (!) transport level



# Flux-gradient relationship (adiabatic ITG modes)



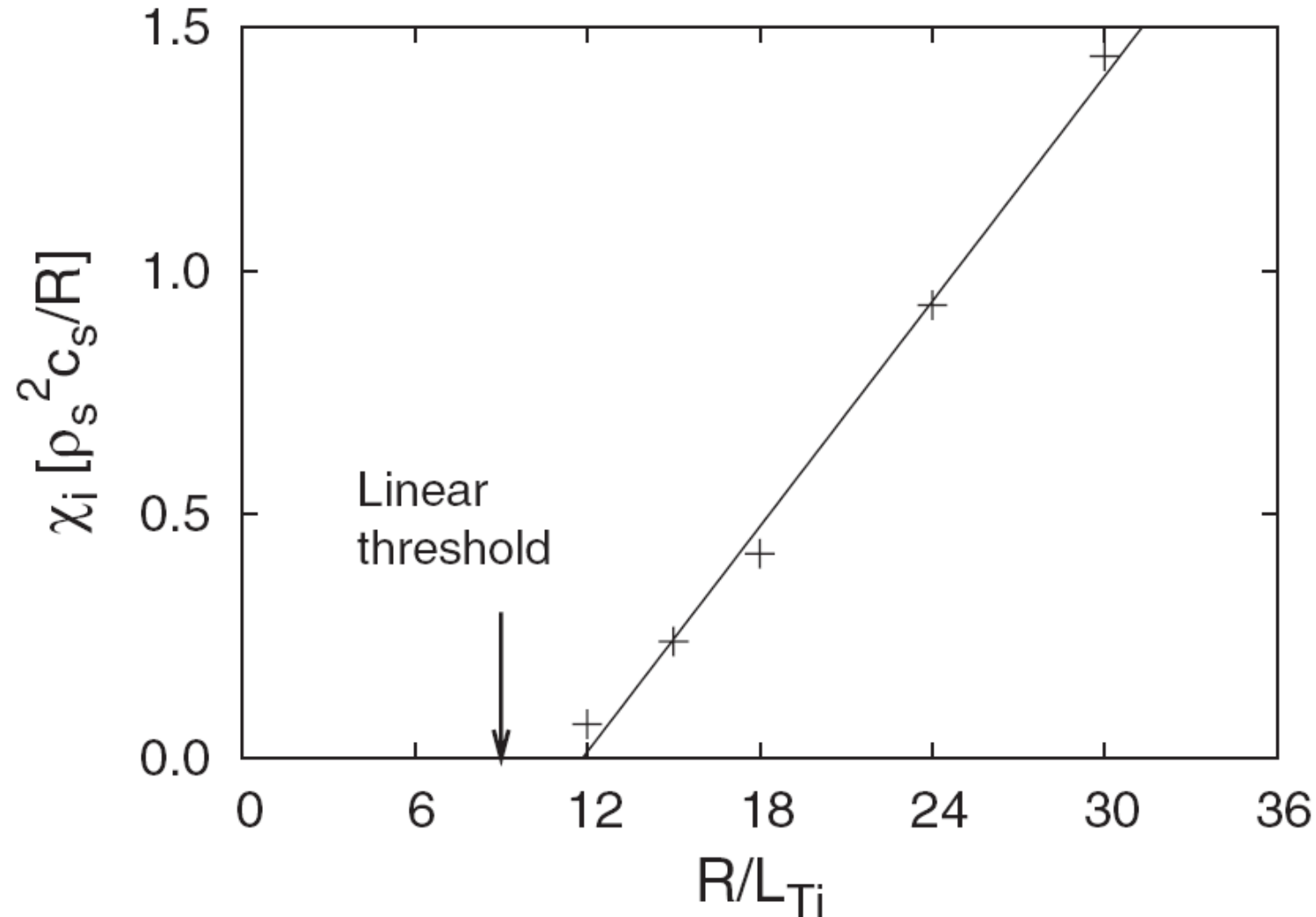
- Offset-linear scaling for  $\chi$  – not  $Q$
- Moderate transport levels (profile stiffness)
- No Dimits shift, but large threshold



# Comparison: ITG turbulence in W7-X



P. Xanthopoulos *et al.*, PRL **99**, 035002 (2007)





# Conclusions



Our theoretical understanding of plasma microturbulence is still rather fragmentary, and the adiabatic ITG scenario is not universal

*GENE simulations show:*

- Nonlinear TEM saturation due to perpendicular particle diffusion: F. Merz and F. Jenko, PRL **100**, 035005 (2008)
- Coexistence with ITG modes only in a transitional regime; significant nonlinear, TEM-induced upshift of critical  $R/L_{Ti}$
- Scale separation between ion and electron heat transport for realistic plasma parameters
- Interesting magnetic geometry effects (e.g., NSTX, NCSX)



More information and papers:

[www.ipp.mpg.de/~fsj](http://www.ipp.mpg.de/~fsj)