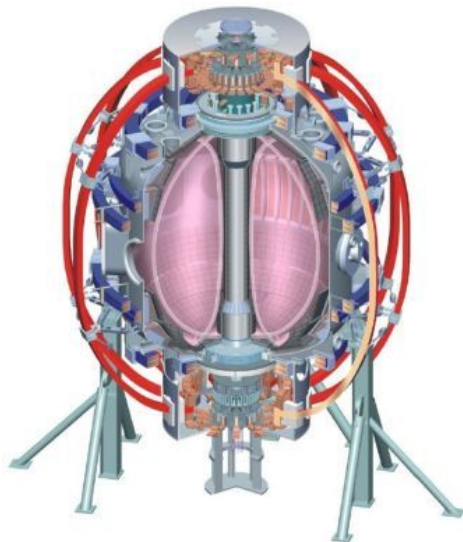


Physics and Control Considerations for a Steady State High- β Spherical Torus

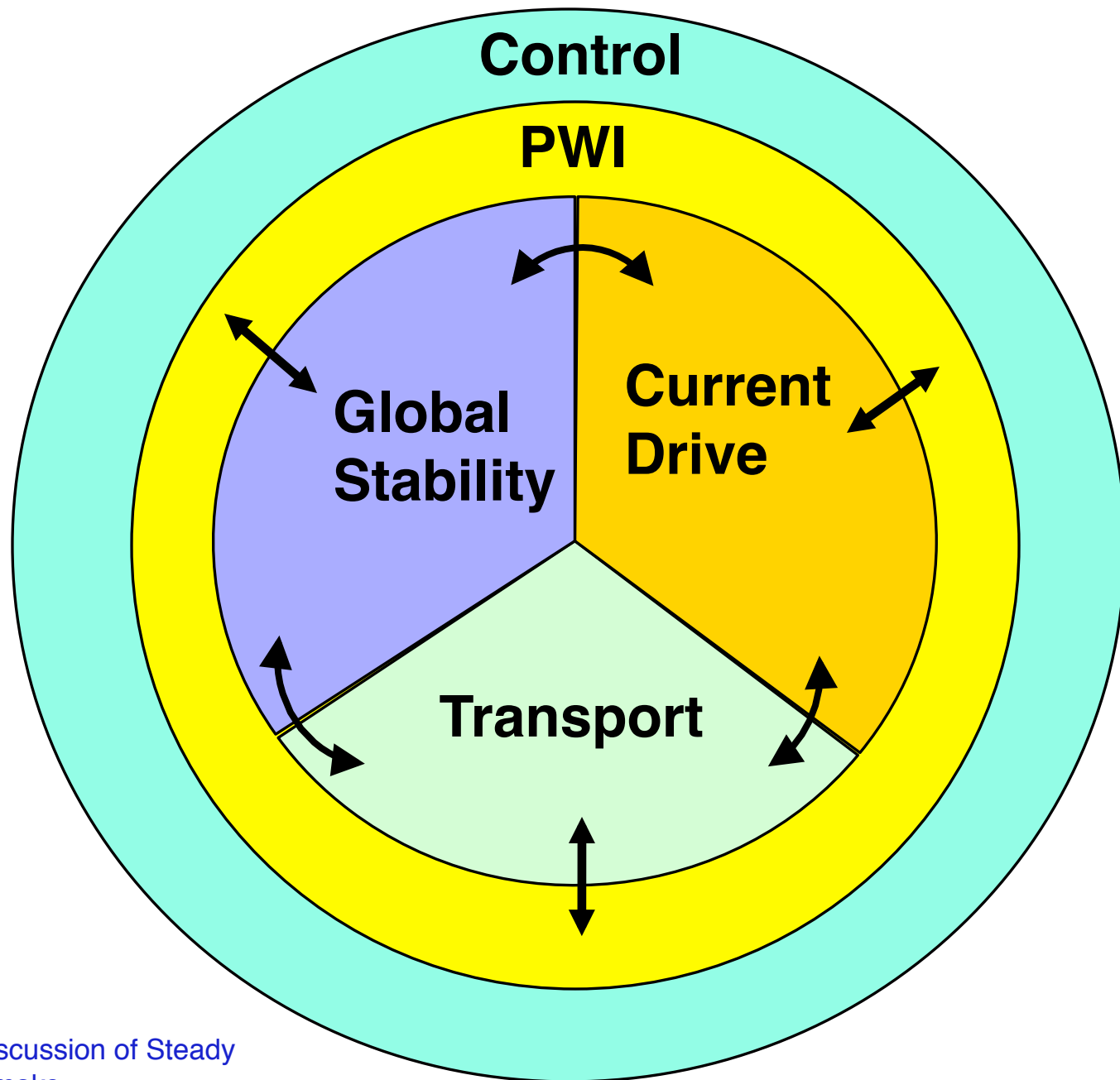
Stefan Gerhardt
PPPL

*College W&M
 Colorado Sch Mines
 Columbia U
 CompX
 General Atomics
 INL
 Johns Hopkins U
 LANL
 LLNL
 Lodestar
 MIT
 Nova Photonics
 New York U
 Old Dominion U
 ORNL
 PPPL
 PSI
 Princeton U
 Purdue U
 SNL
 Think Tank, Inc.
 UC Davis
 UC Irvine
 UCLA
 UCSD
 U Colorado
 U Illinois
 U Maryland
 U Rochester
 U Washington
 U Wisconsin*



*Culham Sci Ctr
 U St. Andrews
 York U
 Chubu U
 Fukui U
 Hiroshima U
 Hyogo U
 Kyoto U
 Kyushu U
 Kyushu Tokai U
 NIFS
 Niigata U
 U Tokyo
 JAEA
 Hebrew U
 Ioffe Inst
 RRC Kurchatov Inst
 TRINITI
 KBSI
 KAIST
 POSTECH
 ASIPP
 ENEA, Frascati
 CEA, Cadarache
 IPP, Jülich
 IPP, Garching
 ASCR, Czech Rep
 U Quebec*

A View of High- β ST Operation (Essentially the Same as for Conventional Aspect Ratio)



See T. Luce, Phys. Plasmas **18** For Discussion of Steady State Conventional Aspect Ratio Tokamaks

Overview

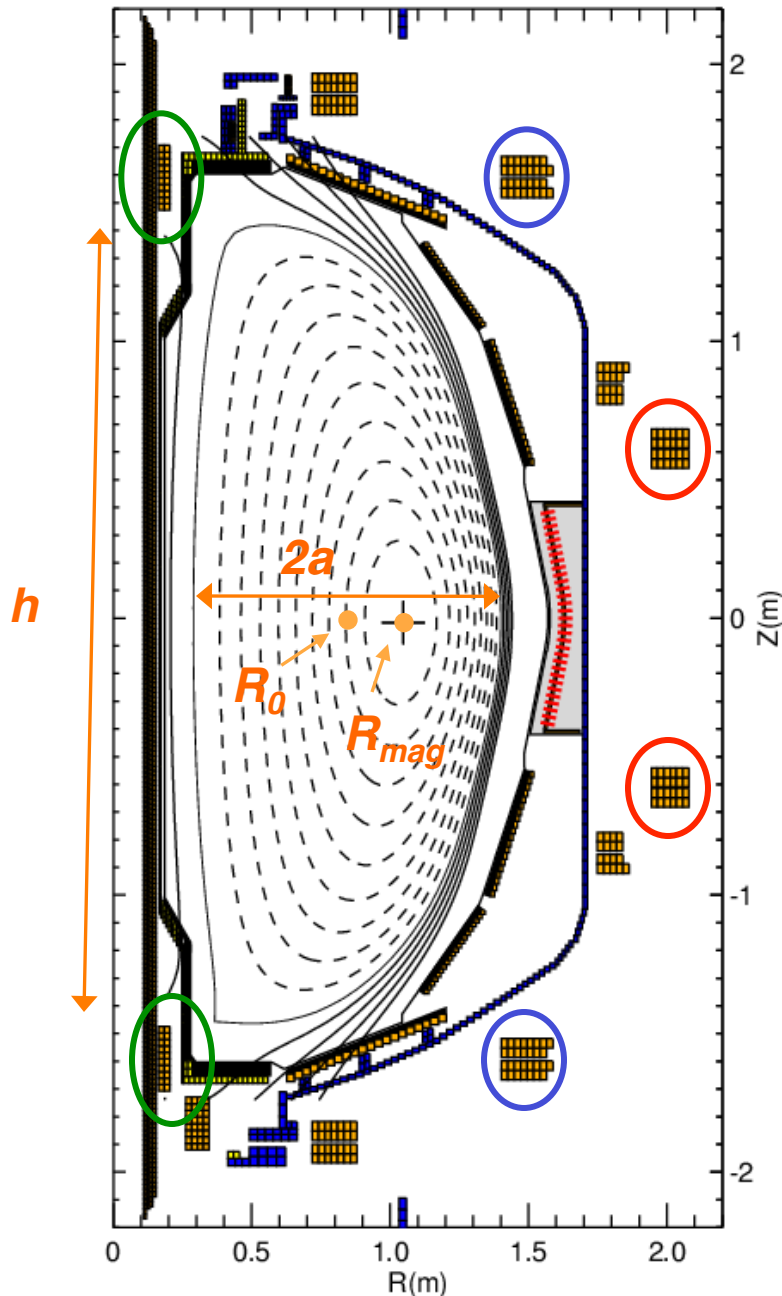
- Take a narrow view: focus on global stability, current drive, confinement...and their control.
 - Neglect critical issues of pedestal & divertor physics, PMI, energetic particle physics, plasma startup.
- Outline:
 - Basics: Historical & technical background of the ST.
 - Optimization to high β .
 - Current drive requirement.
 - How transport impacts the above.
 - Control considerations (time permitting).

Give you some sense of the interplay between, and optimization of, parameters called β_N , β_T , β_P , I_j , F_P , $A=1/\varepsilon$, κ , δ , H

Brief History of the ST

- 1980s
 - Peng and Strickler (NF, 1986) synthesized many of the important advantages of the ST in a single reference:
 - Natural high elongation, improved average field curvature & better utilization of the TF coil
- 1990s:
 - START device demonstrated high β with at low-A and strong shaping.
 - Theory studies demonstrated the existence of very attractive high- β equilibria.
- 2000s:
 - NSTX and MAST demonstrated sustained high-performance plasmas.
 - ST designs for a component test facility and a reactor appear.
- 2010s:
 - NSTX and MAST are both approved for major upgrades.

Major Components of An ST Like NSTX



Coils

Vertical Field: Anti-parallel to plasma current, counteracts tendency for the plasma column to radially expand, controls the radius of the plasma.

$$F_R \sim J_T B_Z$$

Radial Field: Anti-parallel to plasma current, controls the elongation, vertical position.

Divertor: Parallel to the plasma current, “pull” an X-point, increase the elongation.

Solenoid: Current ramps through the shot, providing an inductive voltage to drive current (goal of research: eliminate this inductive current drive).

$$\kappa = \frac{h}{2a}$$

$$A = 1/\epsilon = \frac{R_0}{a}$$

$$\beta_P = \frac{2\mu_0 \langle p \rangle}{B_P^2}$$

$$\beta_T = \frac{2\mu_0 \langle p \rangle}{B_{T,0}^2}$$

Must Have Strong Shaping and High β_N For an Efficient Reactor

- Fusion power scales as $n^2 T^2 \sim B_T^2 \beta_t^2$
 - B_T is limited by field at the coil, so we must maximize β_t .
- Bootstrap current scales as $f_{BS} = C_{BS} \varepsilon^{1/2} \beta_P$.
 - Want to maximize the self driven current, so maximize β_P .
- What is the requirement for them both to be large?

- Consider definition of poloidal β :

$$\beta_P = \frac{2\mu_0 \langle p \rangle}{B_P^2} = \frac{2\mu_0 \langle p \rangle}{(\mu_0 I / l_P)^2}$$

- Approximate expression of the poloidal circumference l_P :

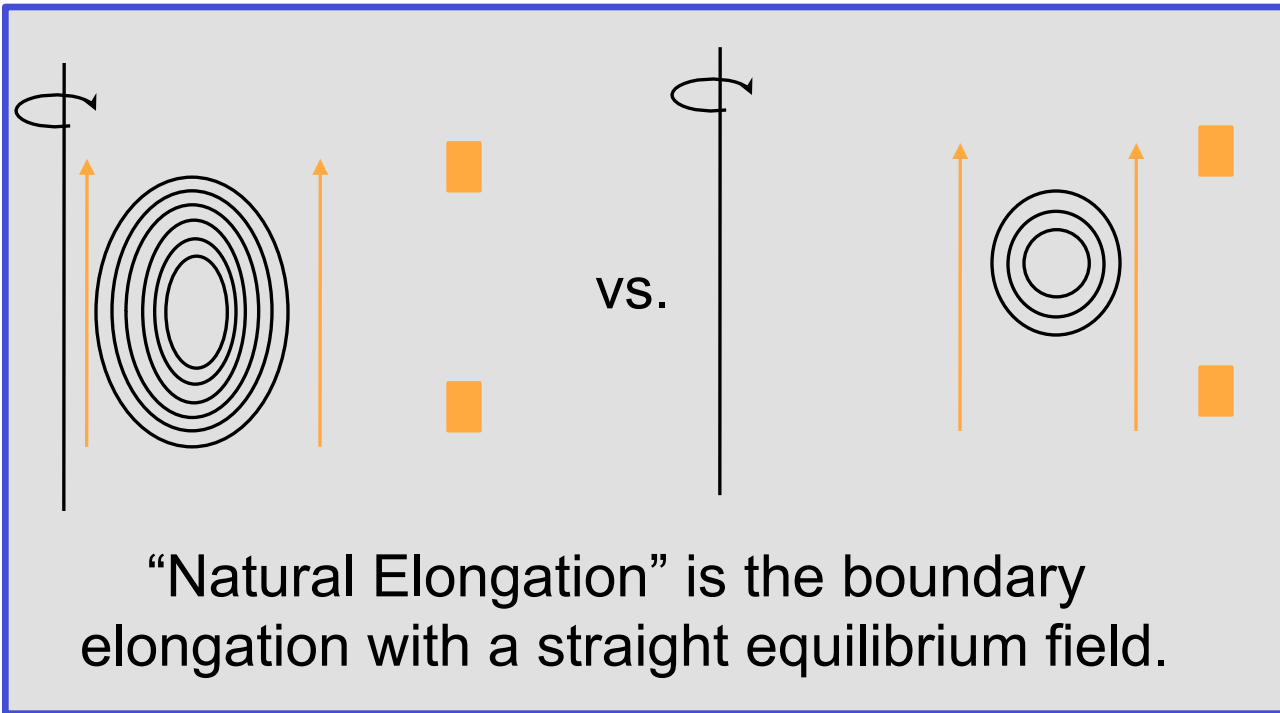
$$l_P \approx 2\pi a \sqrt{\frac{1 + \kappa^2}{2}} \quad \Rightarrow \quad \begin{cases} \beta_P = \frac{2\mu_0 \langle p \rangle}{B_P^2} = \frac{2\mu_0 \langle p \rangle}{(\mu_0 I)^2} (2\pi a)^2 (1 + \kappa^2) \\ q \sim \frac{B_T}{B_P} \sim \frac{B_T}{I_P} \sqrt{1 + \kappa^2} \end{cases}$$

- Multiply β_P and β_T to get:

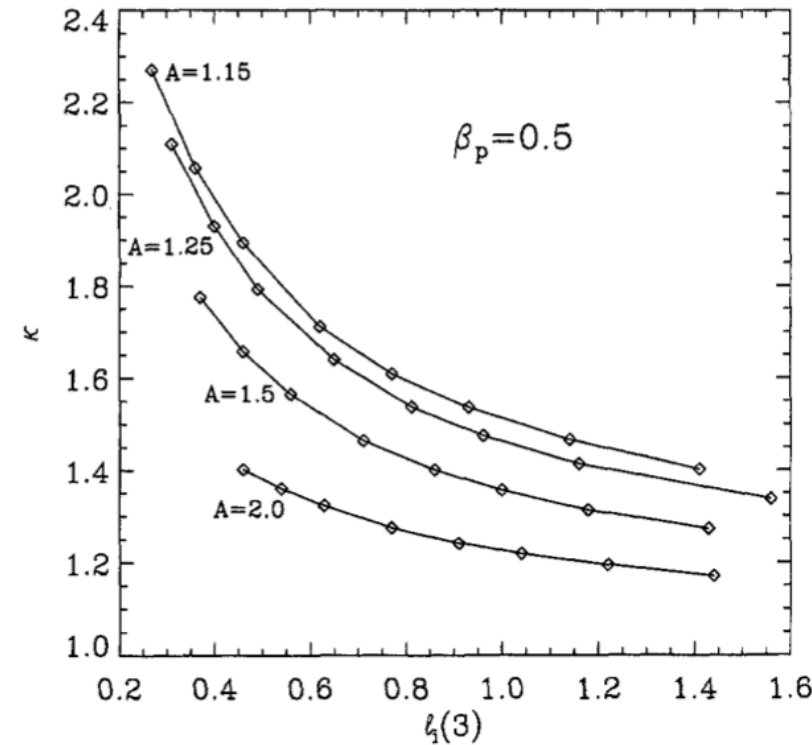
$$\beta_T \beta_P = 25 \frac{1 + \kappa^2}{2} \left(\frac{\beta_N}{100} \right)^2, \quad \text{where } \beta_N = \frac{\beta_T}{I_P / a B_T}$$

- *Must have simultaneously high elongation and β_N .*

ST Plasmas are “Naturally Elongated”and it Depends on the Current Profile

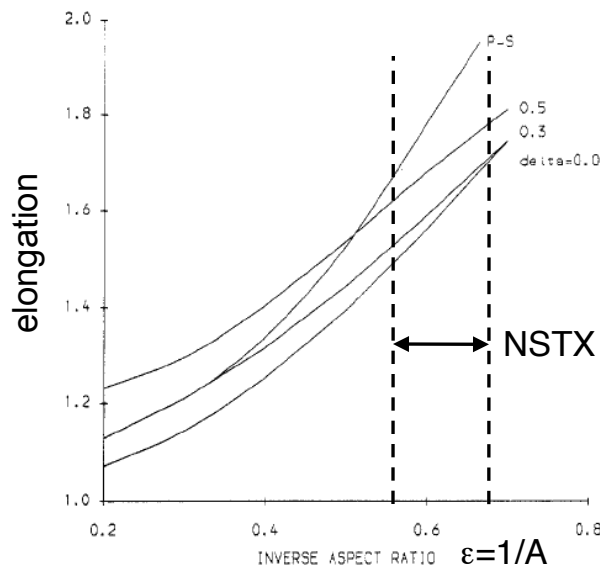


J.E. Menard, et al, Nuclear Fusion 37



l_i indicates “peakedness” the current profile.
“low- l_i ” means broad current profile.

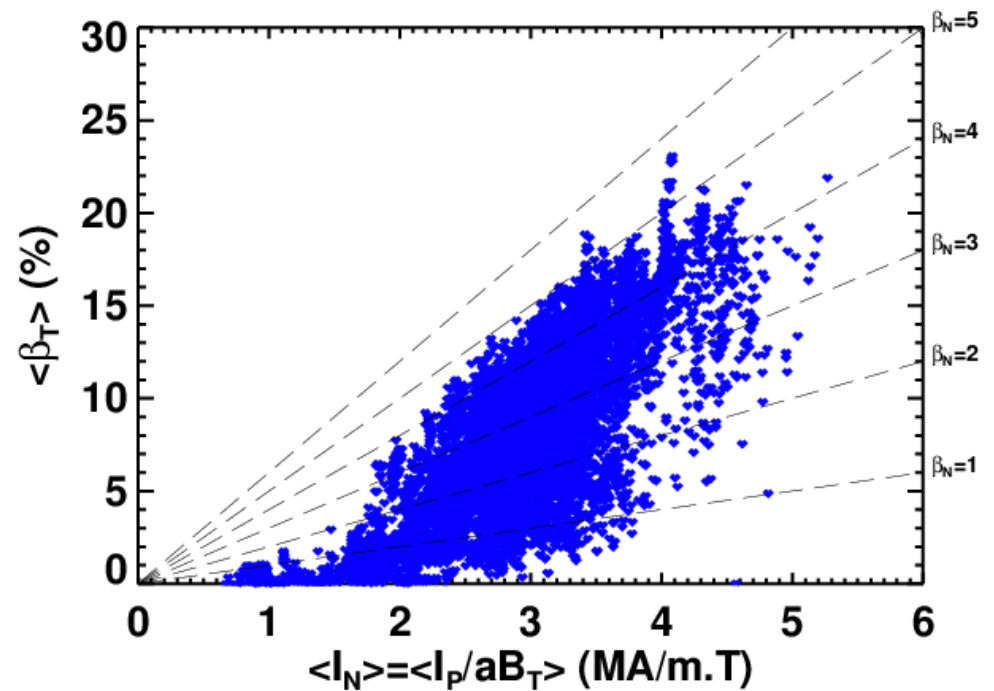
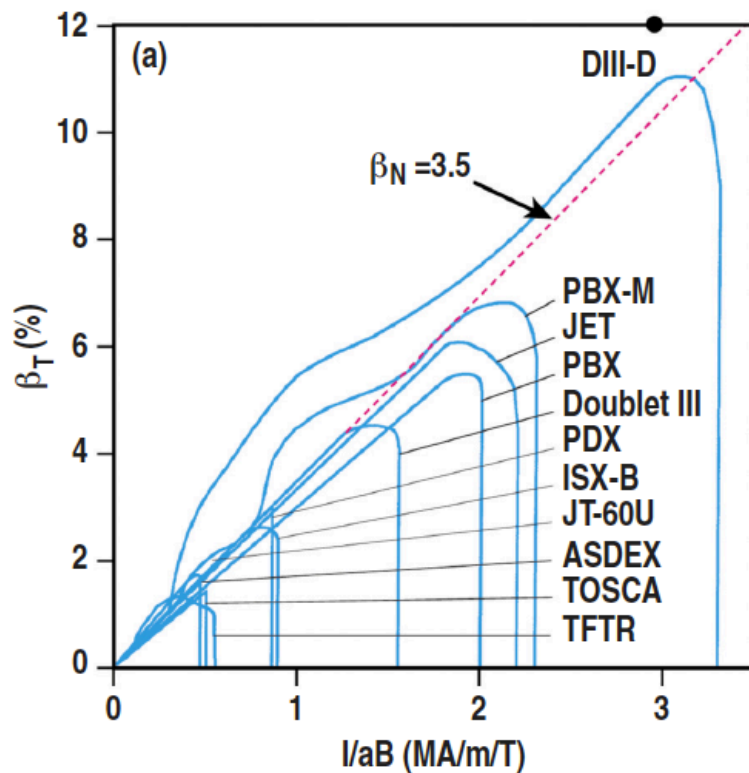
$$l_i(1) = \frac{l_p^2 \iiint B_P^2 dV}{V(\mu_0 I_P)^2}$$



Roberto & Galvao,
Nuclear Fusion 32

To Lowest Order, β_N Controls Ideal Stability

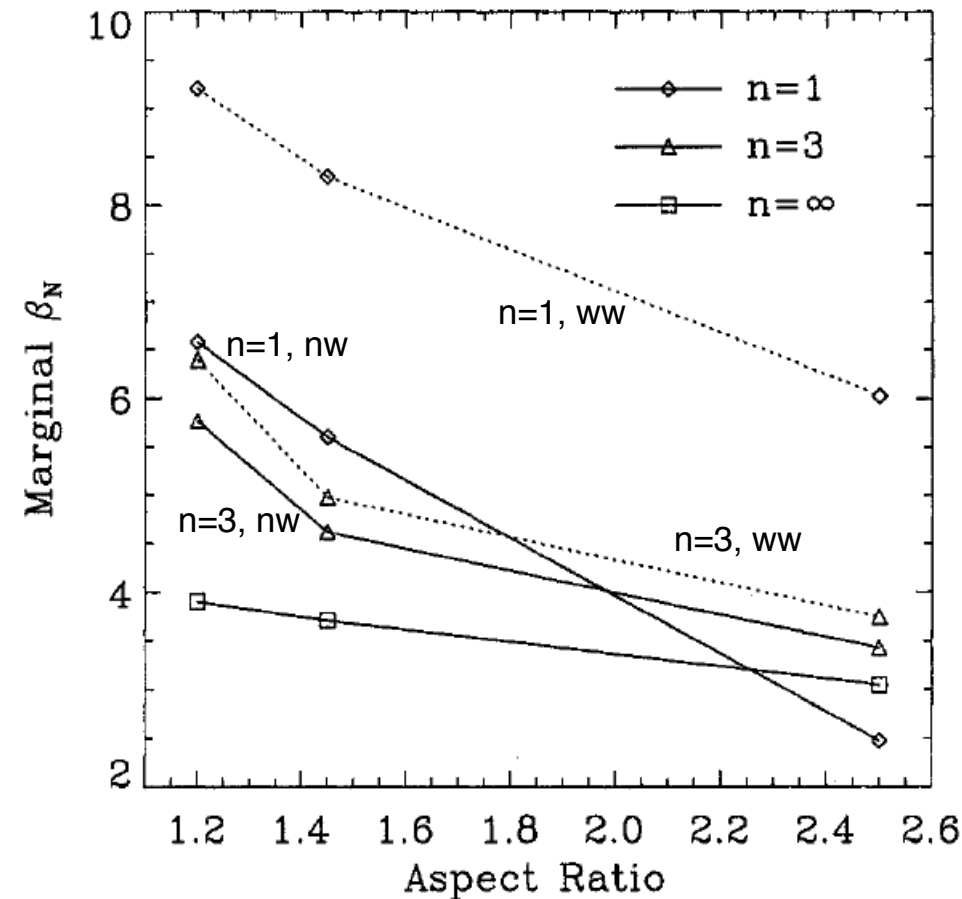
- Troyon, others, showed that the β_T limit scales as I_p/aB_T .
 - Based on calculations of low-n kink stability, high-n ballooning stability.
 - Said another way, if you want high- β_T , must increase I_p/aB_T .



The β_N Limit Can Be Modified by Changes to the Plasma Shape....

J.E. Menard, et al, Nuclear Fusion 37

- Three important degrees of freedom in axisymmetric plasma boundary shaping.
 - **Aspect ratio (A):** Better utilization of the good-curvature, high-TF region, increased safety factor.

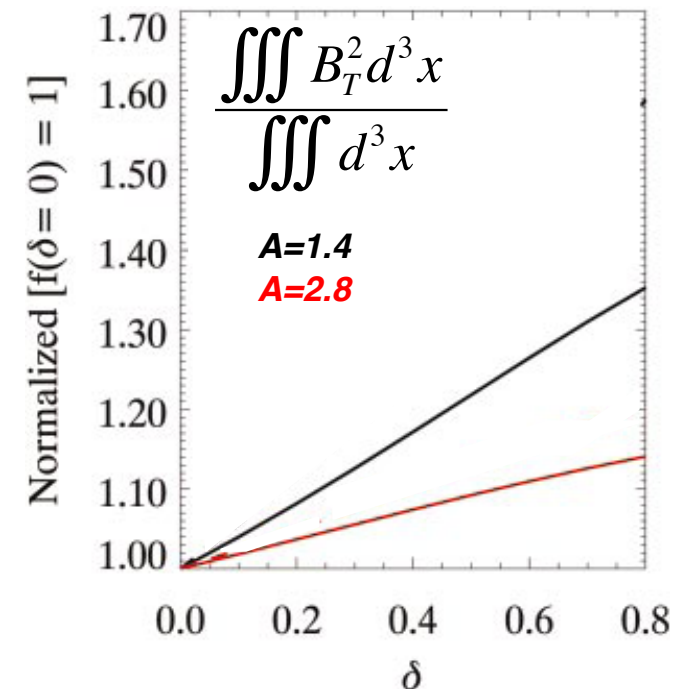
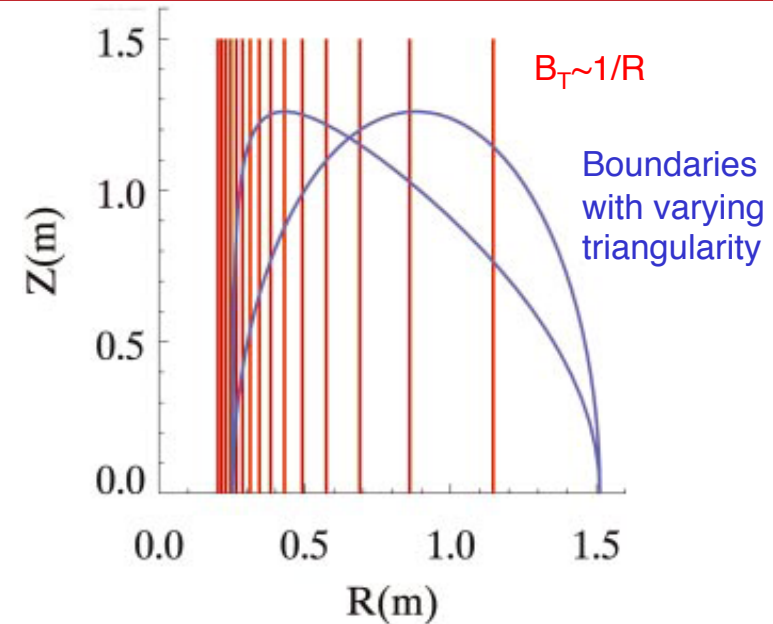


Solid: No Wall (nw)

Dashed: Conformal Wall 0.4a
from Plasma (ww)

The β_N Limit Can Be Modified by Changes to the Plasma Shape....

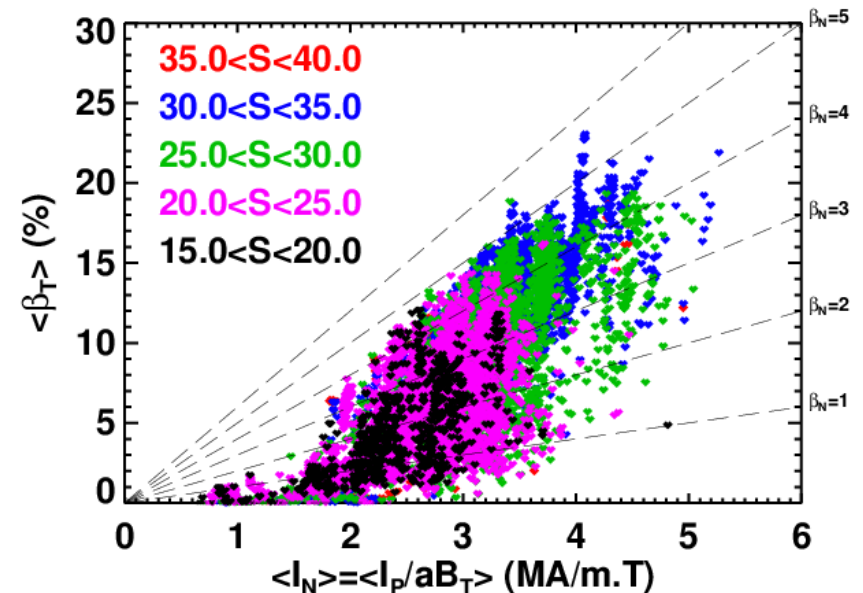
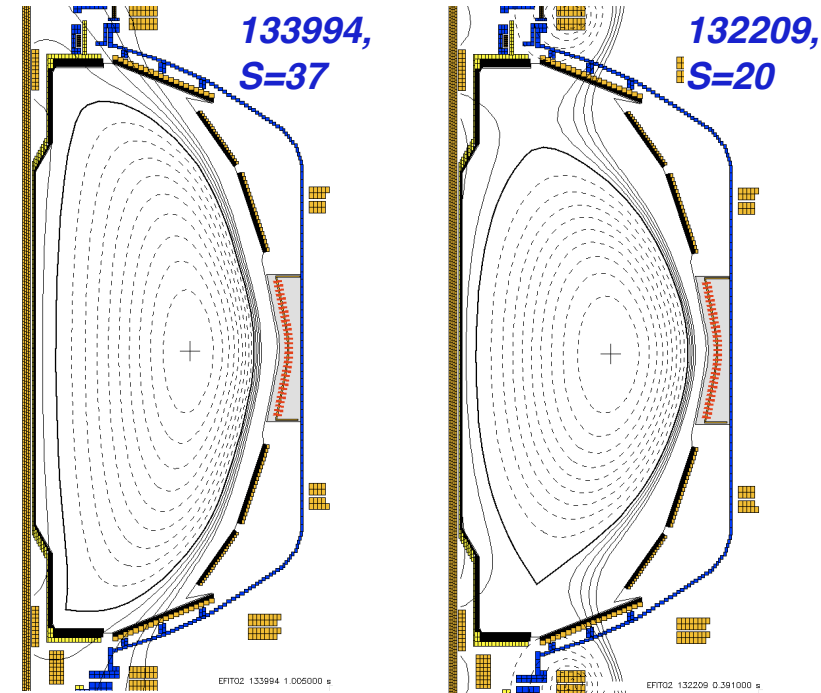
- Three important degrees of freedom in axisymmetric plasma boundary shaping.
 - **Aspect ratio (A):** Better utilization of the good-curvature, high-TF region, increased safety factor.
 - **Elongation (κ):** Increasing elongation increases q , improving the *kink stability* & increasing *bootstrap currents*.
 - **Triangularity (δ):** Increasing the triangularity causes fields lines to spend more time in the *high-field good-curvature* region.



The β_N Limit Can Be Modified by Changes to the Plasma Shape....

- Three important degrees of freedom in axisymmetric plasma boundary shaping.
 - **Aspect ratio (A)**: Better utilization of the good-curvature, high-TF region, increased safety factor.
 - **Elongation (κ)**: Increasing elongation increases q , improving the *kink stability* & increasing *bootstrap currents*.
 - **Triangularity (δ)**: Increasing the triangularity causes fields lines to spend more time in the *high-field good-curvature* region.
 - Many more degrees of freedom are possible with non-axisymmetric shaping...*this is a stellarator!*
- “Shape Parameter” S encapsulates shape effects.

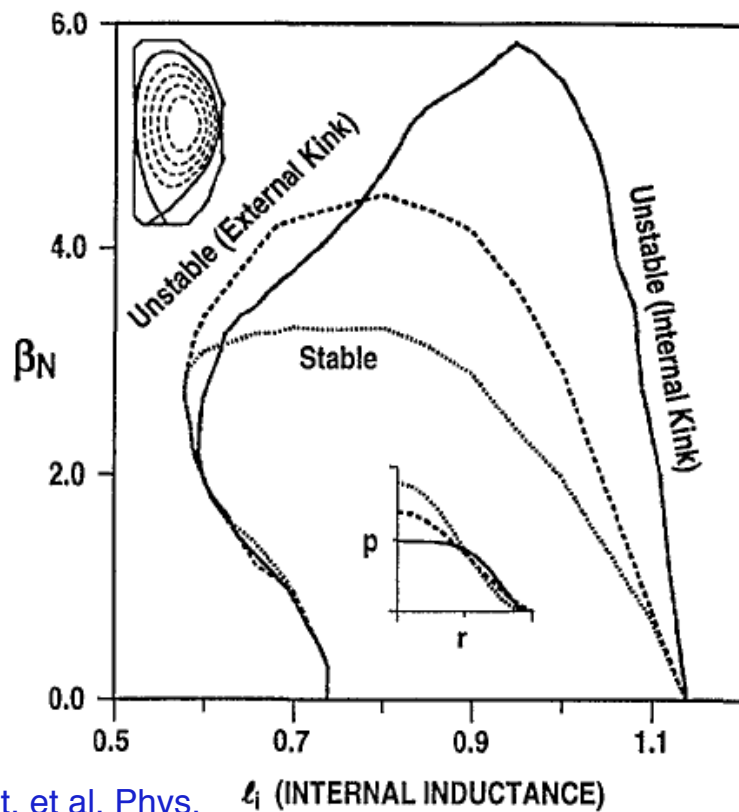
$$S = \frac{q_{95} I_P}{a B_T} \propto \varepsilon (1 + \kappa^2) f(\kappa, \delta, \varepsilon, \dots)$$



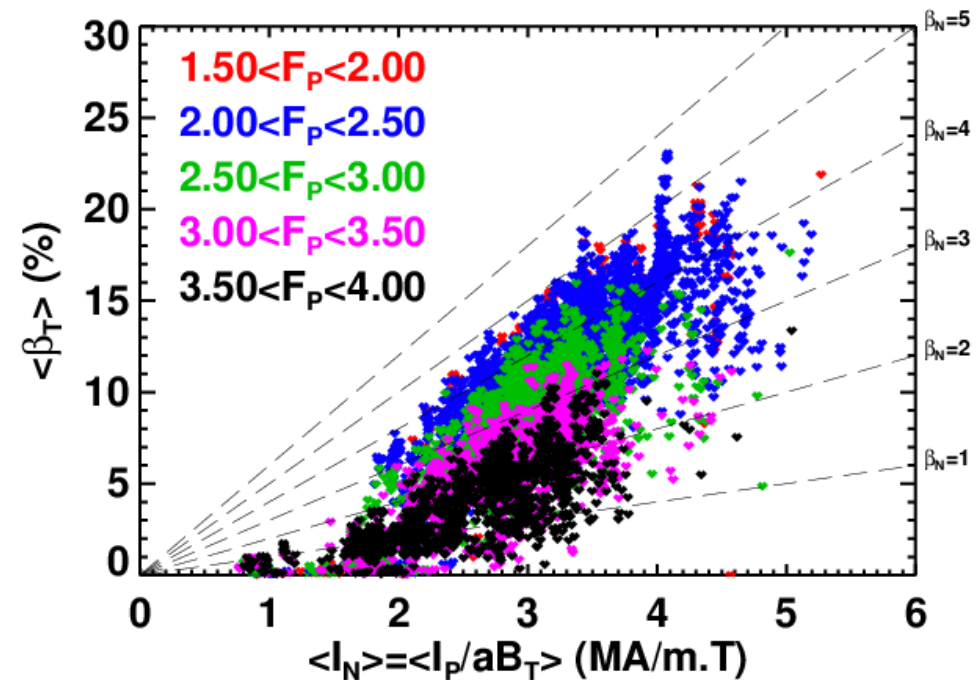
The β_N Limit Can Be Modified by Changes to the Plasma Shape....and the Profiles

- Broad pressure profiles align the pressure gradient with region of large magnetic shear.
 - Highest stability possible with broadest profiles.
- Instability predicted at both high & low I_i .
 - High- I_i -> Internal kinks, low- I_i -> external kinks.

$$F_P = \frac{p_0}{\frac{1}{V} \iiint p d^3V}$$



$\langle \beta_N \rangle$ vs. $\langle F_P \rangle$ in NSTX
 Low pressure peaking required for high- β_N .



Theoretical Studies in the Late 1990s Showed the Promise of the ST

Both cases stable to kink and ballooning instabilities, ~99% bootstrap driven
Strong shaping and broad pressure profiles are key.

J.E. Menard, et al, Nuclear Fusion 37

$\beta_T=45\%$

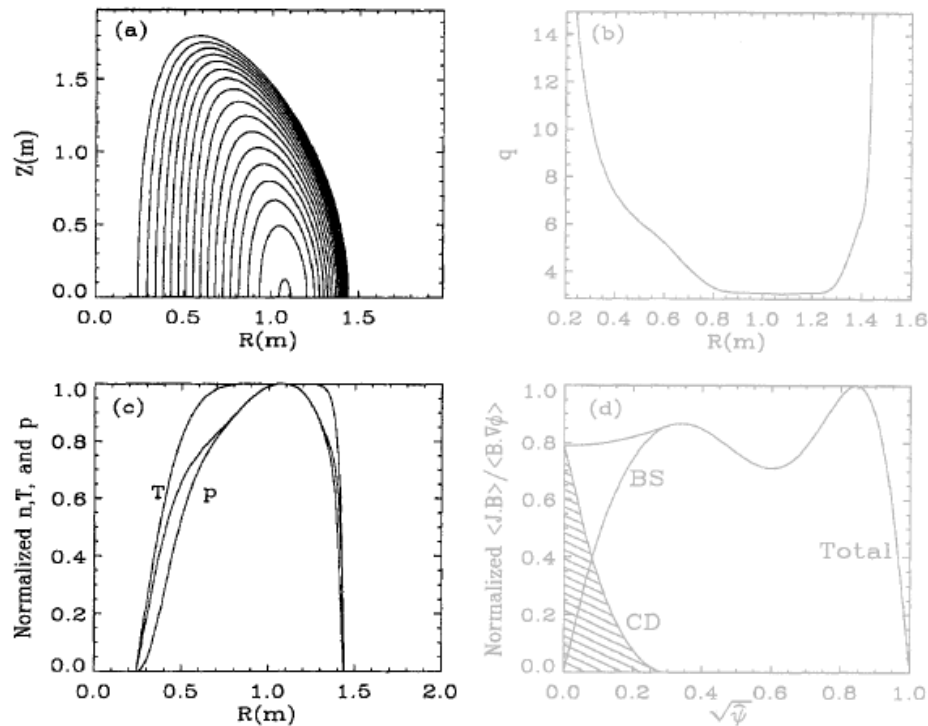


FIG. 17. Details of an $A = 1.40$, $\kappa = 3.0$, $\delta = 0.45$ equilibrium with $\beta = 45\%$ and $f_{\nabla p} = 99.3\%$ that is stable to ballooning and $n = 1-4$ kink modes with a conducting wall: (a) poloidal flux contours, (b) safety factor profile, (c) pressure, temperature and density profiles, (d) total current, bootstrap current and external current drive profiles.

R.L. Miller, et al, Physics of Plasmas 4

$\beta_T=54\%$

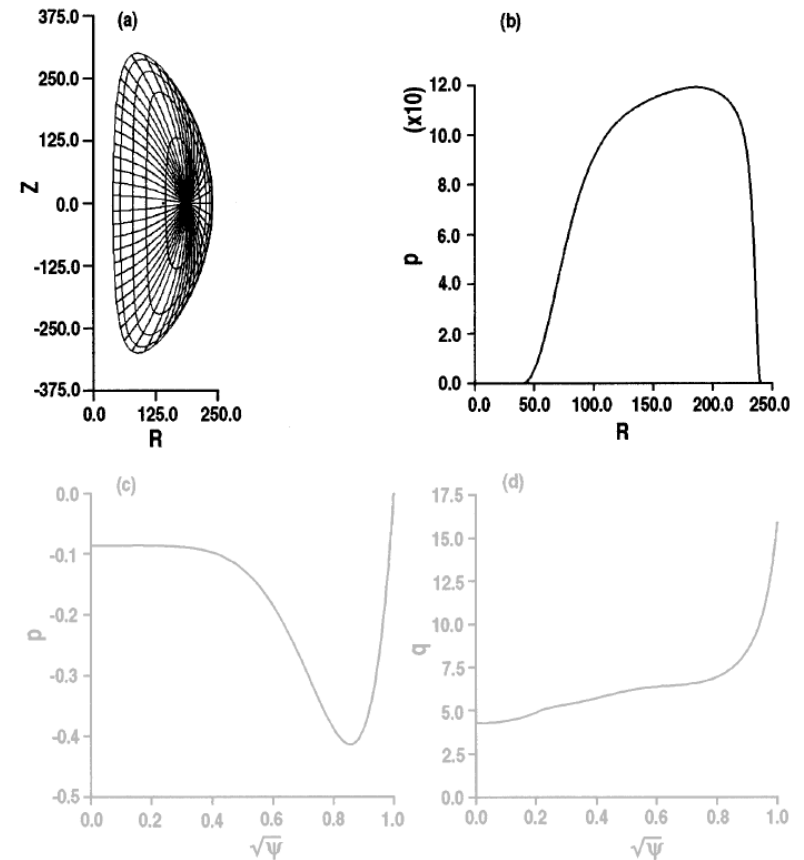


FIG. 1. High beta equilibrium: $\kappa=3.0$, $\delta=0.5$, $L_P/L_T=0.5$. (a) Flux contours; (b) pressure profile across the midplane as a function of major radius; (c) p' as a function of $(\psi)^{1/2}$; (d) q as a function of $(\psi)^{1/2}$.

START Studies Demonstrated Highest-Ever β_T in a Tokamak-Like Device.

M. Gryaznevich, et al, PRL 1998

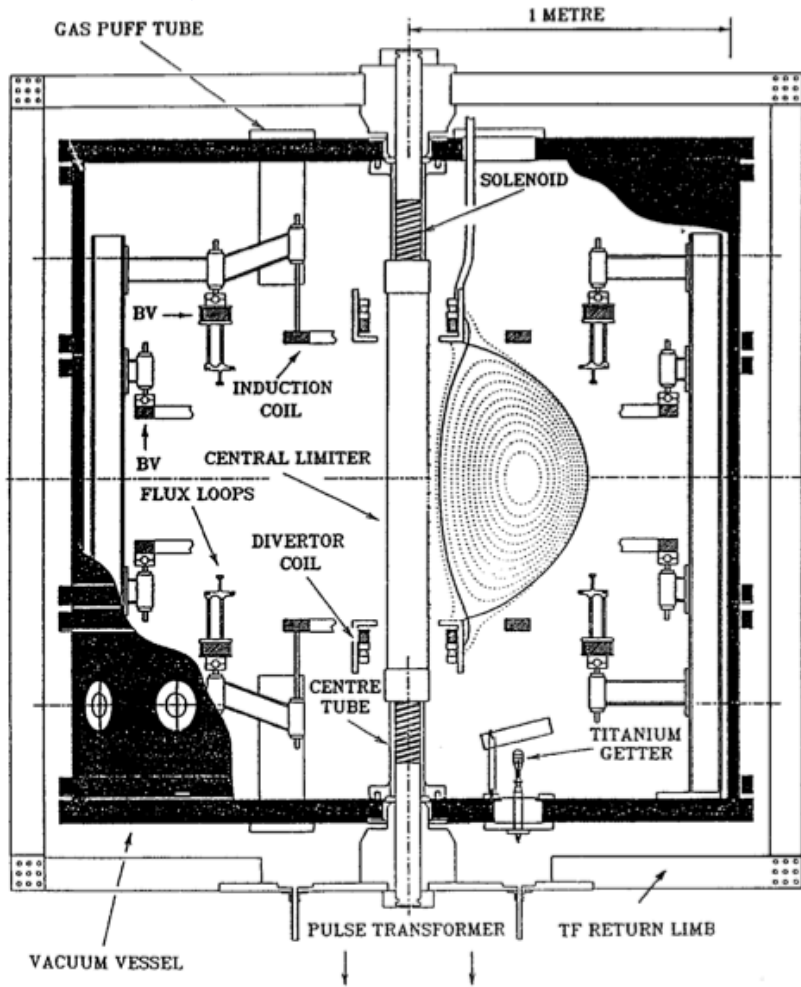


FIG. 1. Cross section of the START device (April, 1997) showing an equilibrium reconstruction of high- β shot #32993.

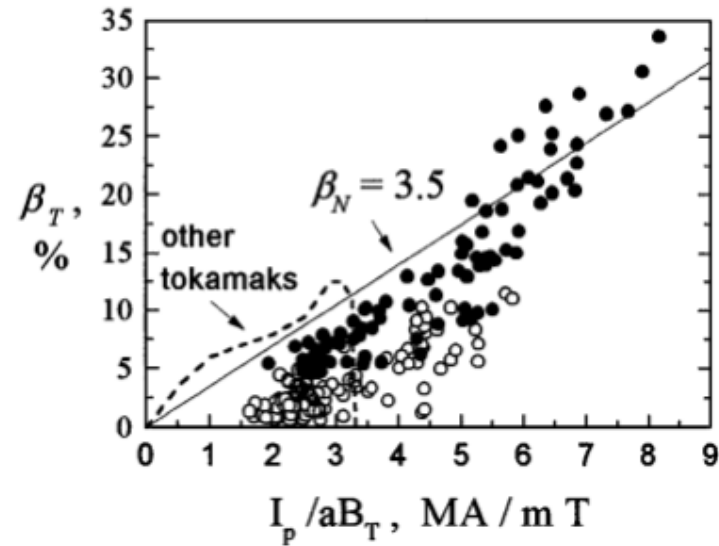
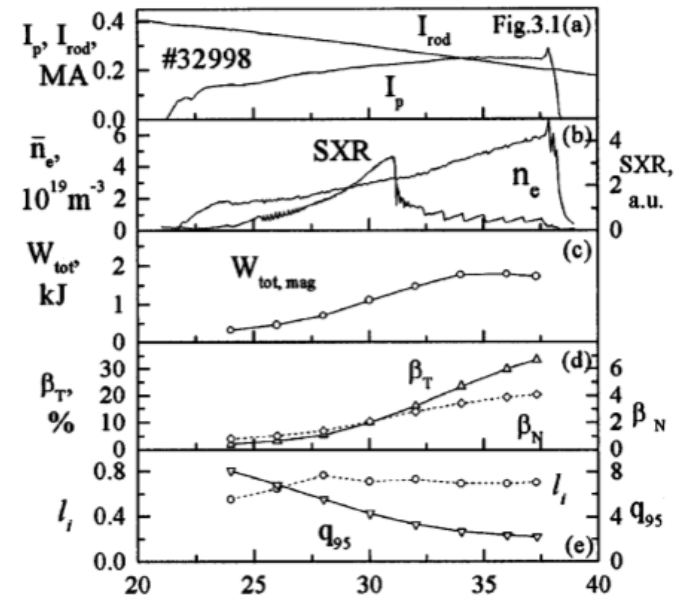


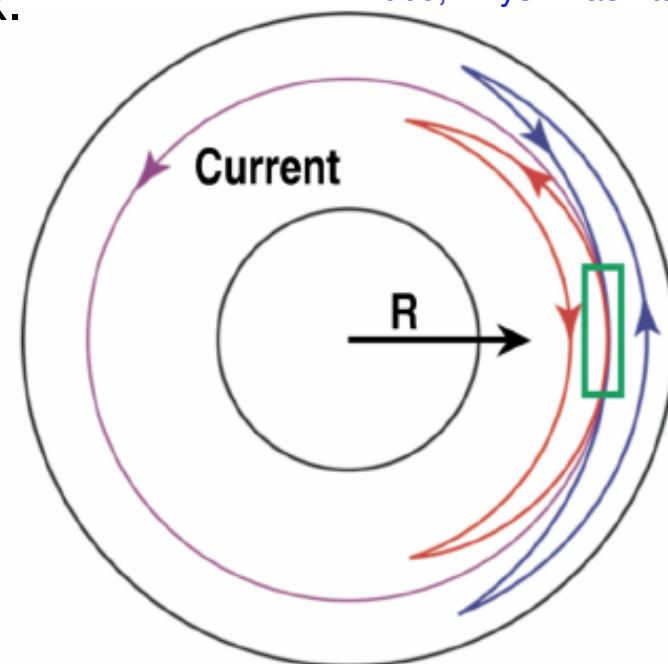
FIG. 2. β_T vs I_p/aB_T in START; high- β operation limits for other tokamaks are shown for comparison.



Plasmas Have the Ability To Drive Their Own Toroidal Current

- Consider the trapped particle orbits in the green box.
- Net counter-clockwise current if:
 - There is a density gradient:
 - More particles on the red orbit.
 - There is a temperature gradient:
 - Particles on red orbit more faster.
- Trapped particles do not make net current.
 - But collisions with passing particles transfer the asymmetry...and a net current flows.

T. Luce, Phys. Plasmas 18



Simple Expression For Current Density

$$J_{BS} \approx \sqrt{\epsilon} \frac{c}{B_P} \frac{dP}{dr}$$

Convert to Toroidal Current

$$I_{BS} \sim \iint \sqrt{\epsilon} \frac{c}{B_P} \frac{dP}{dr} r dr d\theta \sim \sqrt{\epsilon} \frac{caP}{B_P}$$

Convert to Fraction of Total Current

$$f_{BS} \equiv \frac{I_{BS}}{I_P} \approx \sqrt{\epsilon} \frac{caP}{I_P B_P} \sim \sqrt{\epsilon} \frac{cP}{B_P^2} = C_{BS} \sqrt{\epsilon} \beta_P$$

More General Expression:

$$\langle j_{\parallel} B \rangle = \sigma_{neo} \langle E_{\parallel} B \rangle - I(\psi) p_e (\mathcal{L}_{31} A_1 + \mathcal{L}_{32} A_2 + \mathcal{L}_{34} A_4),$$

with

$$A_1 = \frac{1}{p_e} \frac{\partial p_e}{\partial \psi} + \frac{1}{p_e} \frac{\partial p_i}{\partial \psi},$$

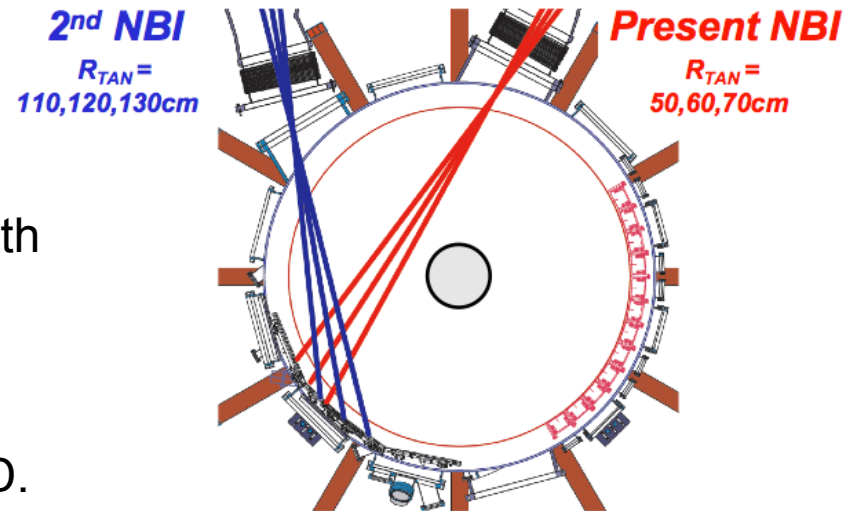
$$A_2 = \frac{1}{T_e} \frac{\partial T_e}{\partial \psi}, \quad A_2^i = \frac{1}{T_i} \frac{\partial T_i}{\partial \psi},$$

$$A_4 = \alpha \frac{1 - R_{pe}}{R_{pe}} A_2^i,$$

O. Sauter, et al,
Phys. Plasmas 6

Neutral Beam Current Drive Typically Invoked to Supplement the Bootstrap Current

- Neutral beams are the work-horse heating source for present and proposed STs.
 - Can also inject momentum and drive current.
- Current is due to the circulating fast ions.
- Collisions would generally bring electrons along with ions, cancelling the current.
 - Impurity ions and electron trapping impede this canceling current.
- Remember, reduces plant efficiency to drive NBCD.



Lin-Liu & Hinton,
Phys Plasmas 4

$$\langle j_{\parallel} B \rangle = \underbrace{\langle j_{\parallel, f} B \rangle}_{\text{Fast ion current}} \left[1 - \frac{Z_b}{Z_{eff}} \left(1 - G(Z_{eff}, \varepsilon) \right) \right]$$

- Fast ion current.
- Computed via Monte Carlo methods for NSTX.
- Increases when the slowing down time is longer:

$$I_{NBCD} \propto C_{NBCD} \frac{\bar{T}_e^{3/2}}{\bar{n}_e} P_{inj}$$

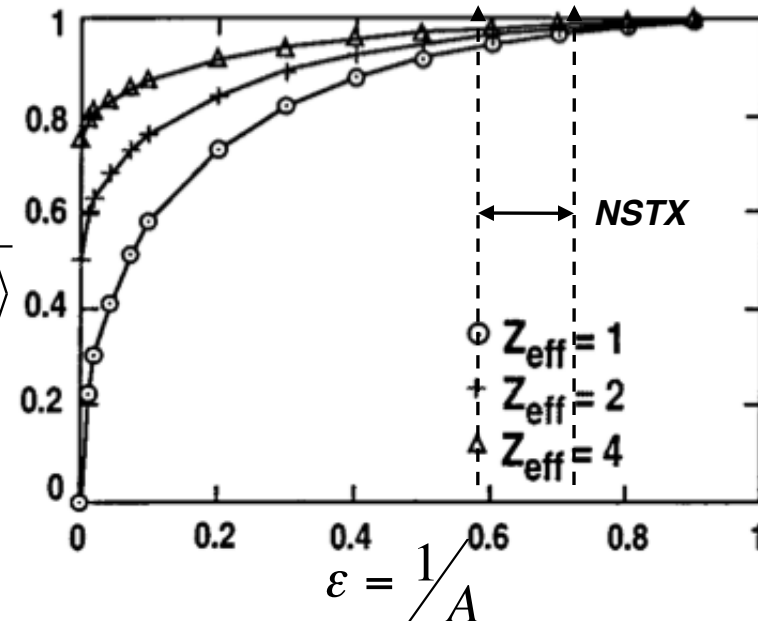
- Correction due to electron response, including neoclassical effects.
- Computed analytically, for instance:

$$G(Z_{eff}, \varepsilon) = \frac{x}{D} \left[\begin{array}{l} (0.754 + 2.21Z_{eff} + Z_{eff}^2) \\ + x(0.348 + 1.243Z_{eff} + Z_{eff}^2) \end{array} \right]$$

$$D = 1.414Z_{eff} + Z_{eff}^2 + x(0.754 + 2.657Z_{eff} + 2Z_{eff}^2) \\ + x^2(0.348 + 1.243Z_{eff} + Z_{eff}^2)$$

$$x = f_t / (1 - f_t)$$

$$\frac{\langle j_{\parallel} B \rangle}{\langle j_{\parallel, f} B \rangle}$$



NSTX Studies Show Good Agreement Between Calculated Current Sources and Total Current Profile

Current Profile Reconstructed from...

Pressure-Driven Currents: Bootstrap, Pfirsch-Schluter+Diamagnetic

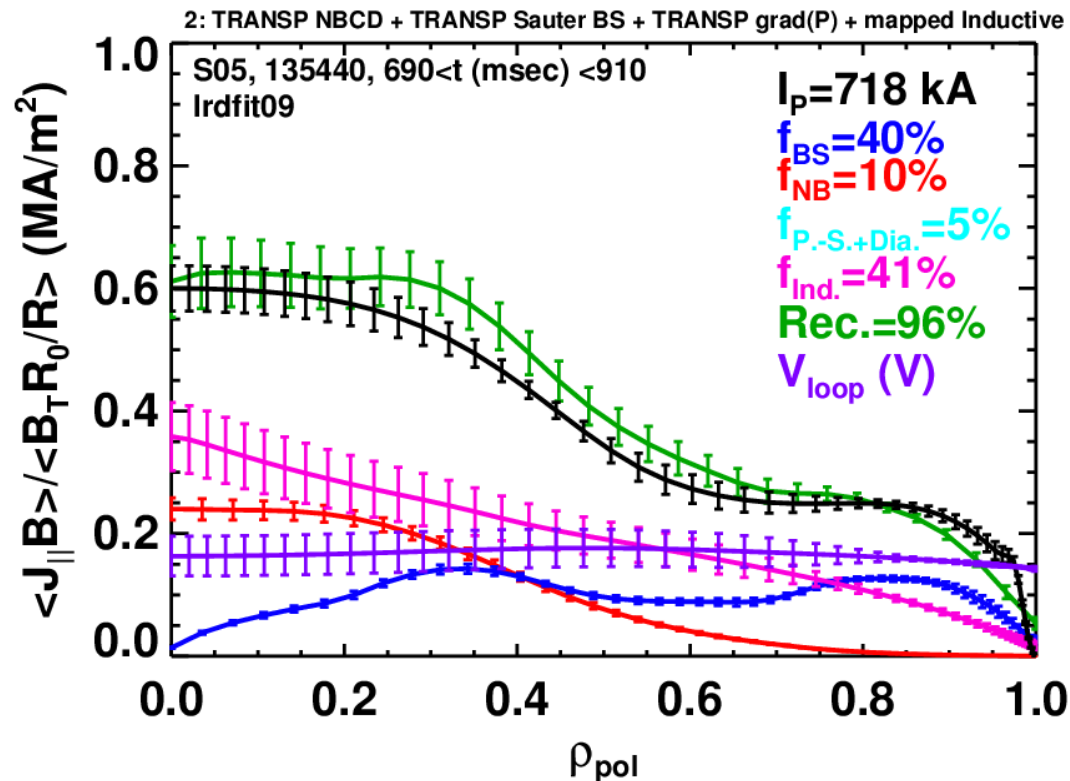
Inductive current: time derivatives of reconstructed equilibria + neoclassical resistivity

Neutral Beam Current Drive from NUBEAM, with classical beam physics

Compare to...

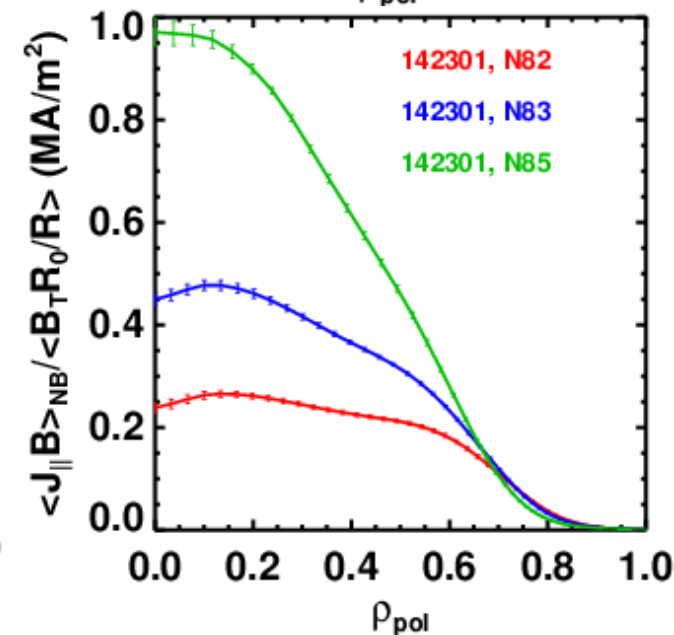
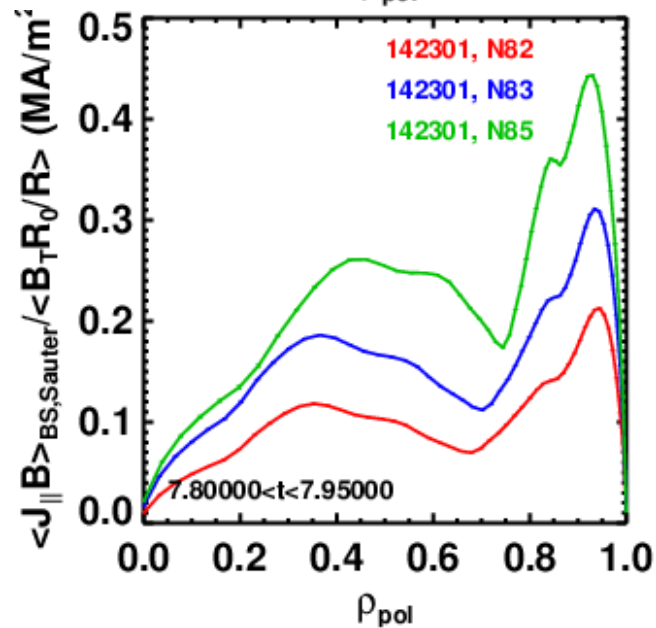
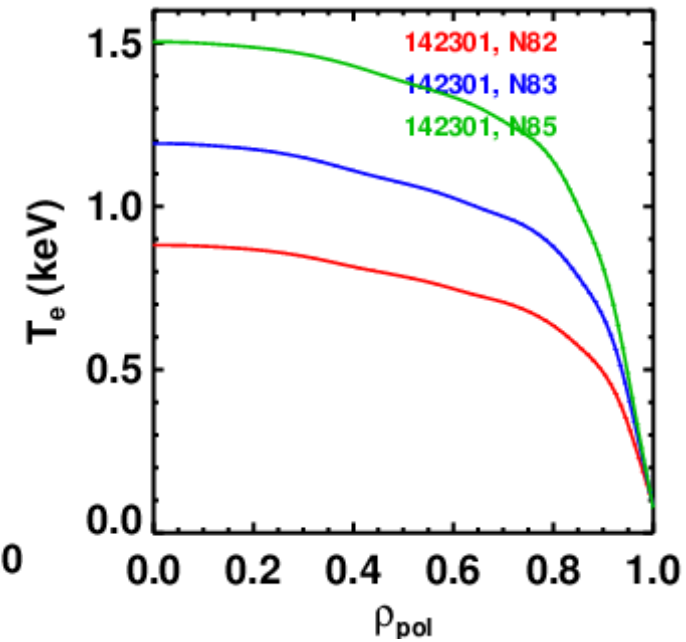
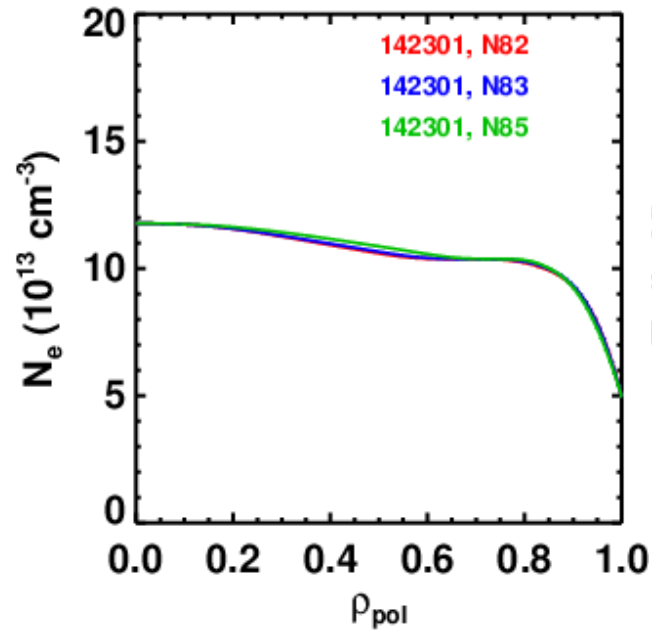
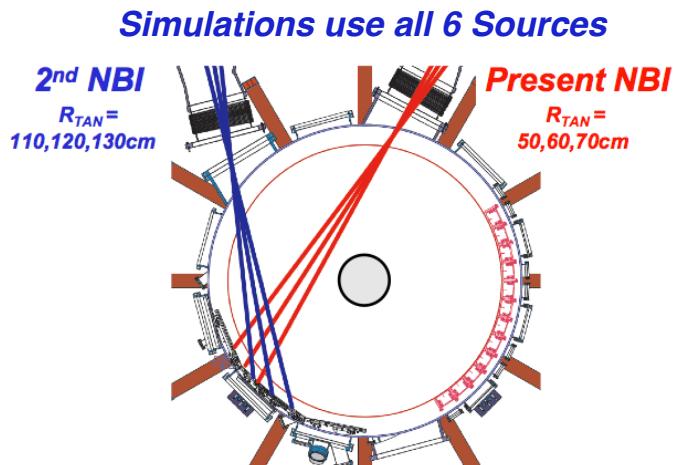
Reconstructions constrained by MSE and T_e isotherm constraint

Choose time with no EP MHD or low-frequency kink/tearing

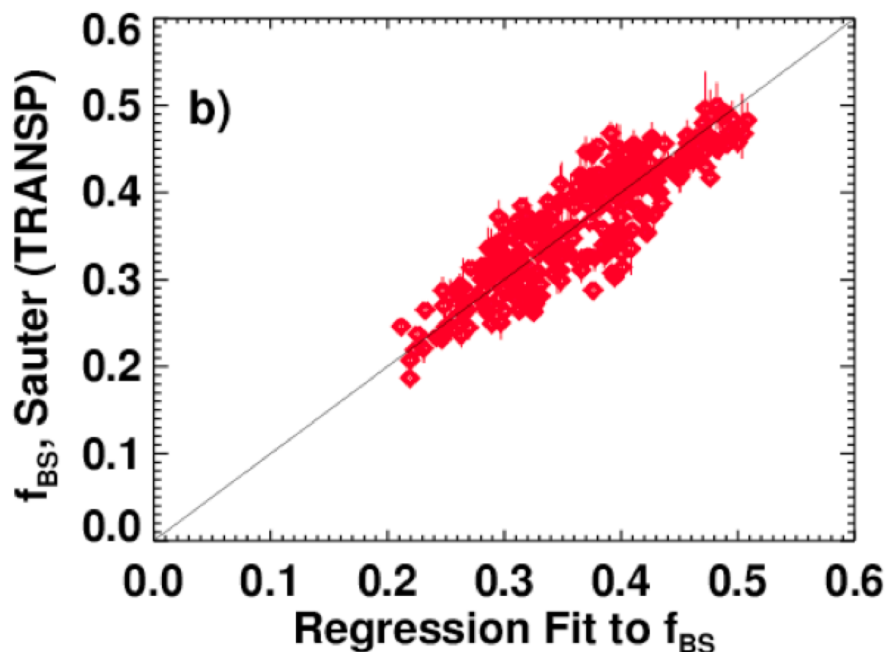
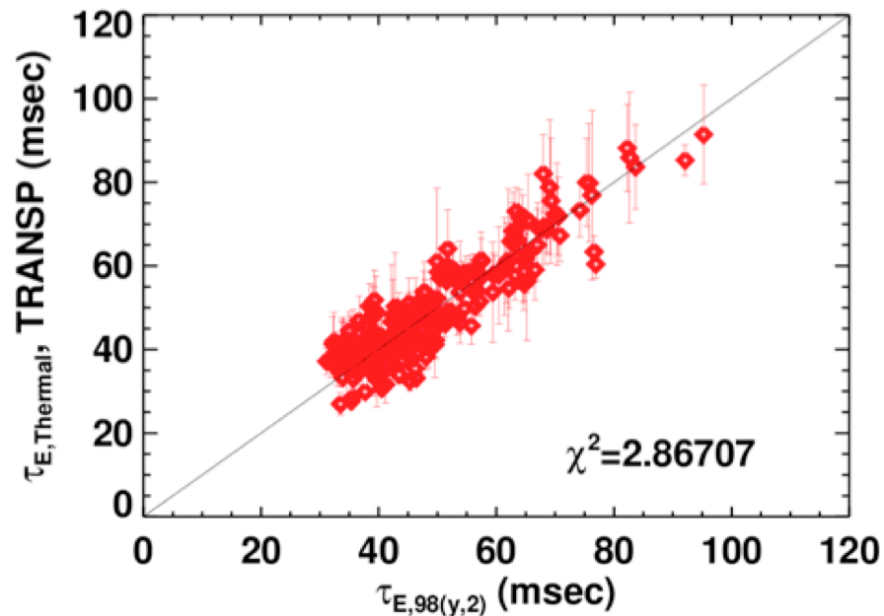


Non-Inductive Current Profiles Depend on the Electron Temperature (and others profiles as well).

- Fix the electron density, Z_{eff} and scale T_e & T_i .
- Calculations from TRANSP
 - Sauter model for Bootstrap currents
 - NUBEAM for NBCD
- Beam currents tend to be centrally localized for these parameters.
- Bootstrap currents tend to be edge localized in H-mode \rightarrow low- I_i



Plasma Transport Determines The Achievable Level Of Bootstrap Current (when stability is not violated)



Experimental Confinement Scaling

$$\tau_{E, ITER-98} \propto I_P^{0.93} B_T^{0.15} n_e^{0.41} P_L^{-0.69}$$

$$H_{98} = \frac{\tau_{E, achieved}}{\tau_{E, ITER-98}}$$

Bootstrap Fraction From Confinement Scaling

$$f_{BS} \propto \beta_{P, th} \propto \frac{P \tau_{E, th}}{I_P^2}$$

$$f_{BS} \propto \frac{P I_P^{0.93} B_T^{0.15} n_e^{0.41} P^{-0.69}}{I_P^2} \approx \frac{n_e^{0.4} B_T^{0.2} P^{0.3}}{I_P}$$

Bootstrap Fraction From Direct Regression

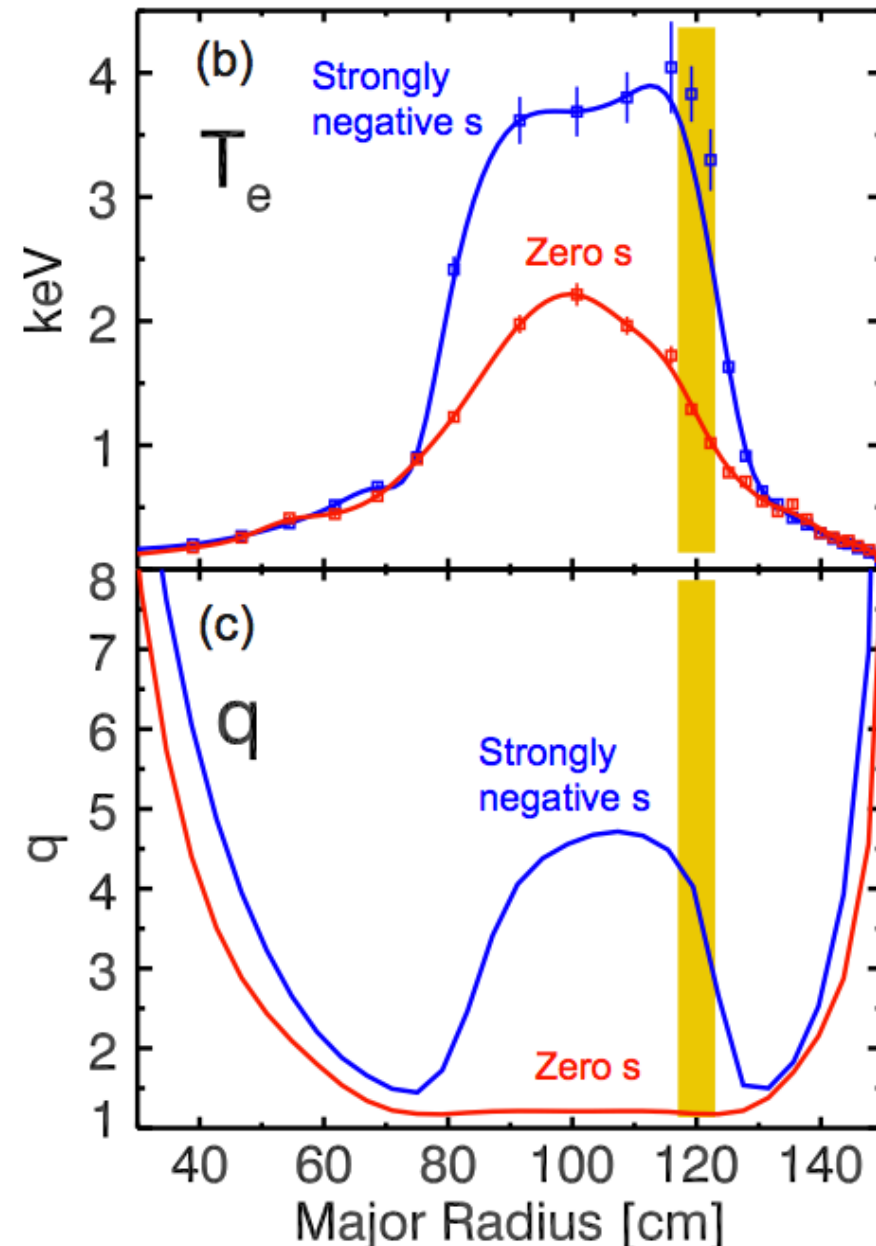
$$f_{BS} \approx \frac{n_e^{0.4} B_T^{0.25} P^{0.2}}{I_P}$$

S.P. Gerhardt, submitted to Nuclear Fusion

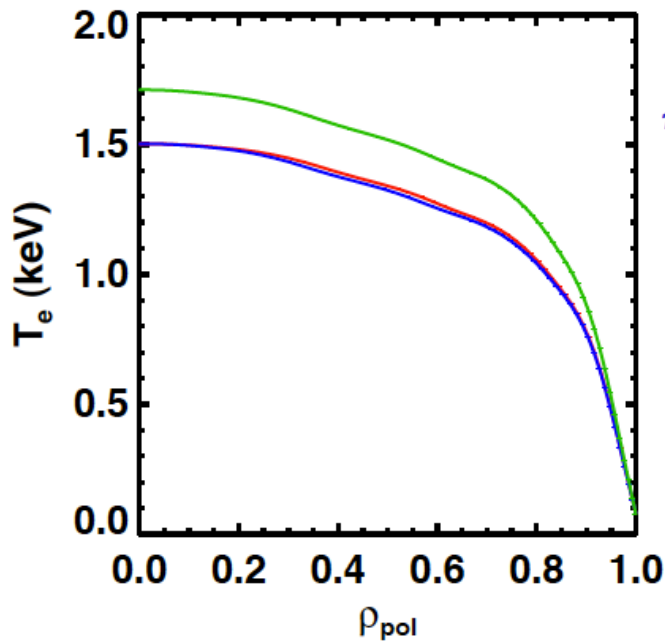
Internal Transport Barrier Formation is a More Complicated Interaction Between the Thermal and Current Profiles

- Confinement depends on magnetic shear.
 - Peak electron temperature typically occurs near location of most negative magnetic shear (in NSTX).
- Strong gradient can produce strong bootstrap current.
 - Modifies the q -profile...necessary to have current profile carefully “aligned” with the pressure profile.
- Resulting configuration has a higher pressure peaking factor.
 - is the ideal MHD stability acceptable in this scenario?
 - Double tearing can also be unstable.

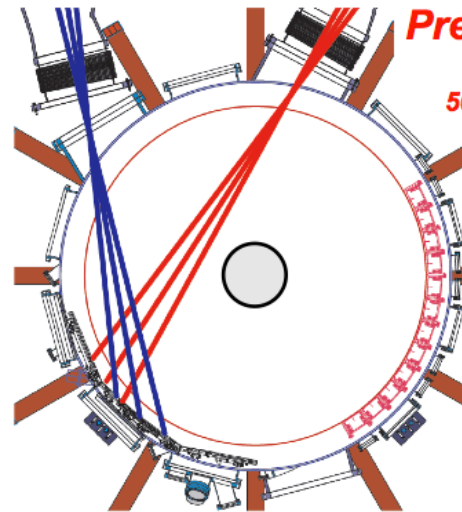
H. Yuh, Phys. Plasmas 16



Changing Beam Sources Can Modify the Current Profile Shape



2nd NBI
 $R_{TAN} =$
 110,120,130cm

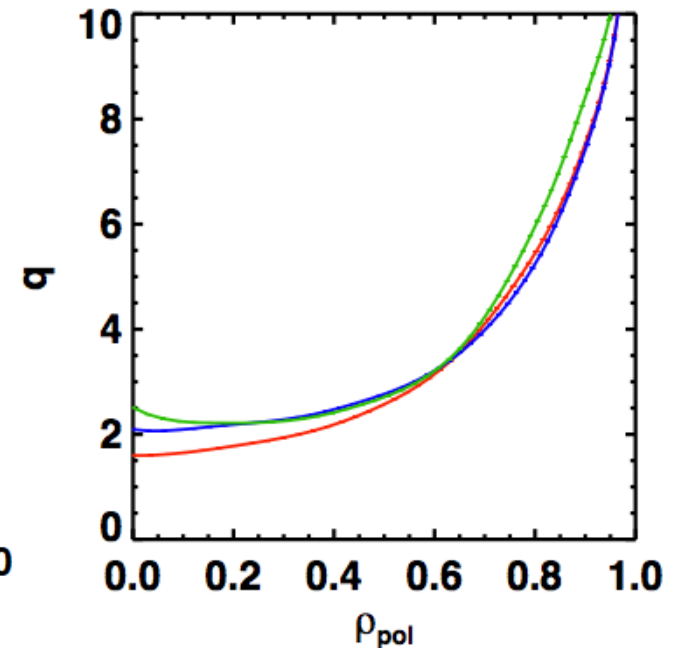
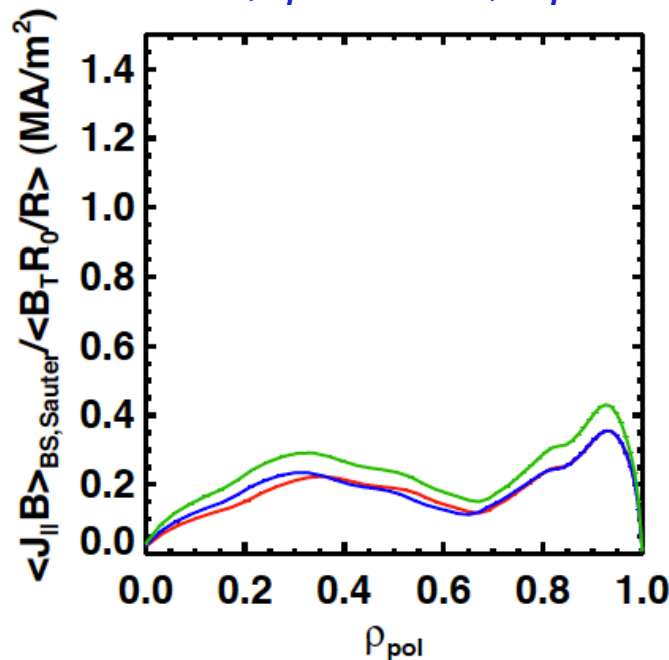
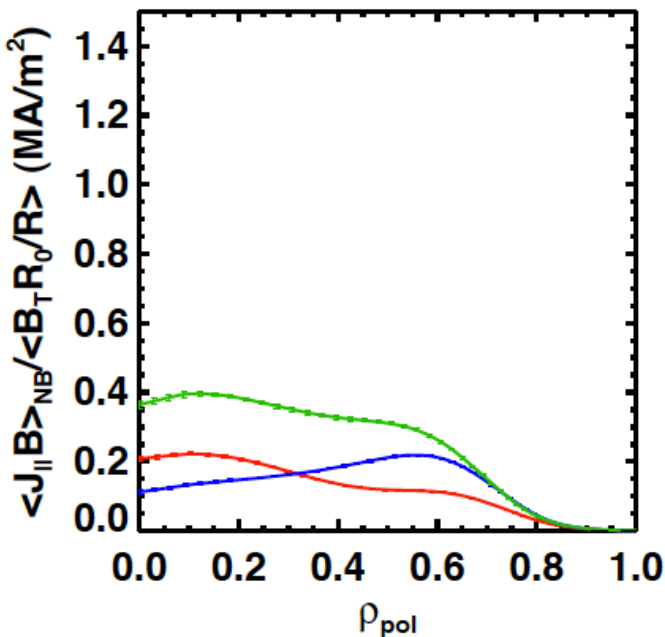


Present NBI
 $R_{TAN} =$
 50,60,70cm

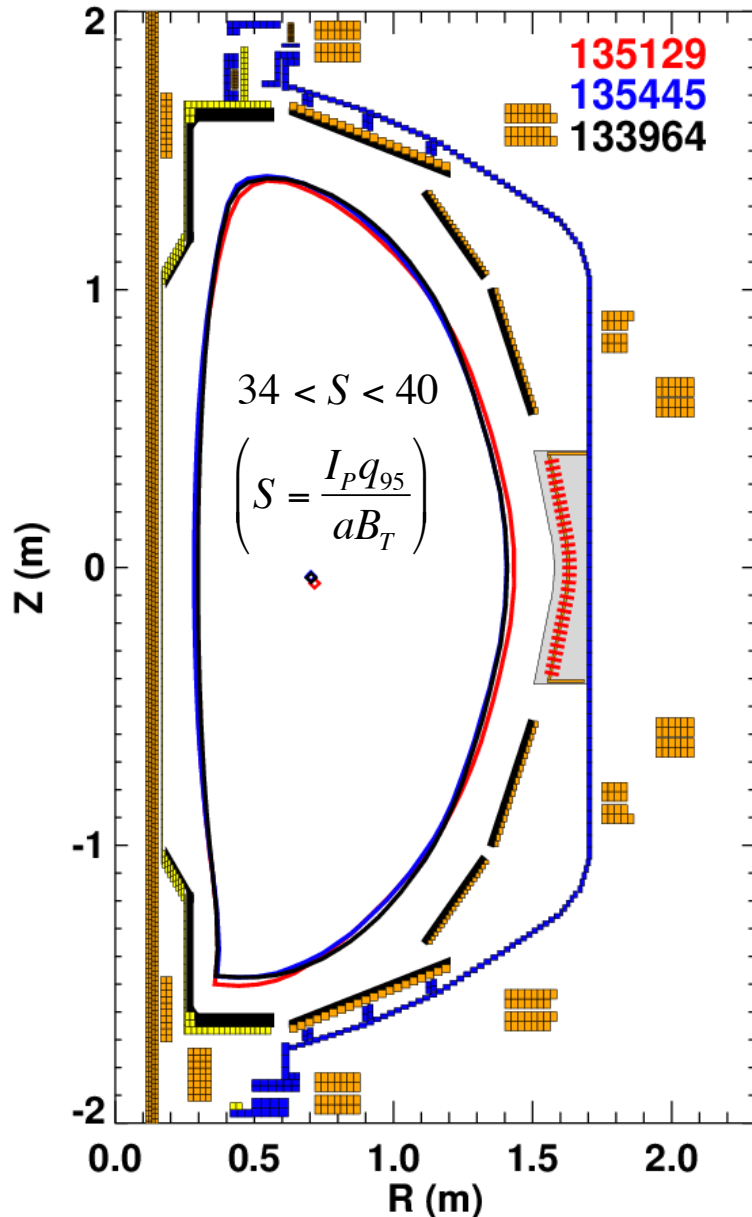
50, 60, 70, 130
50, 60, 120, 130
All 6 Sources

- **Higher power case has higher T_e , and better NBCD efficiency.**
- **Keeps q_{min} above 2, avoiding possibility of 2/1 tearing.**

$A=1.8, I_p=1300$ kA, $B_T=1.0$

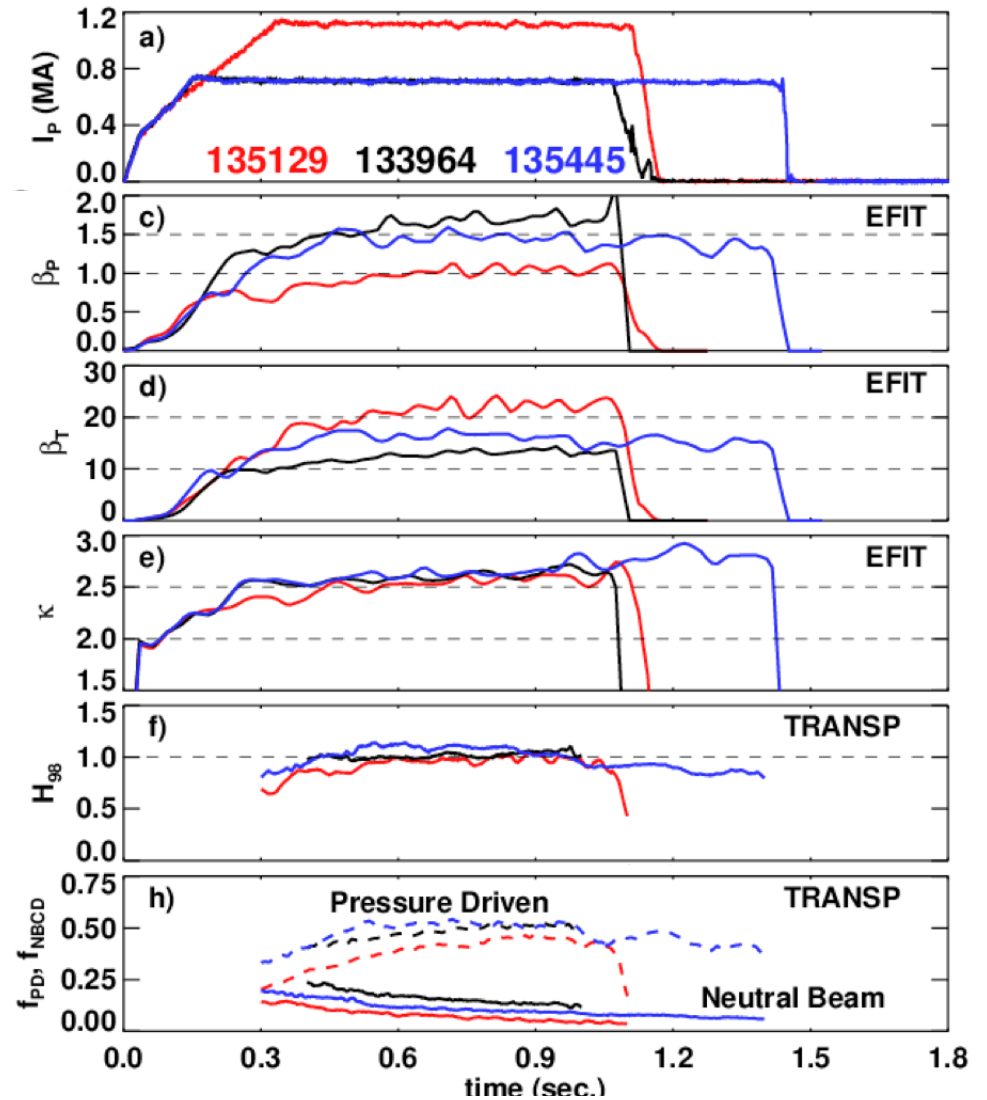


Best NSTX Discharges Combine Strong Shaping, Good Confinement, and Large Non-Inductive Currents



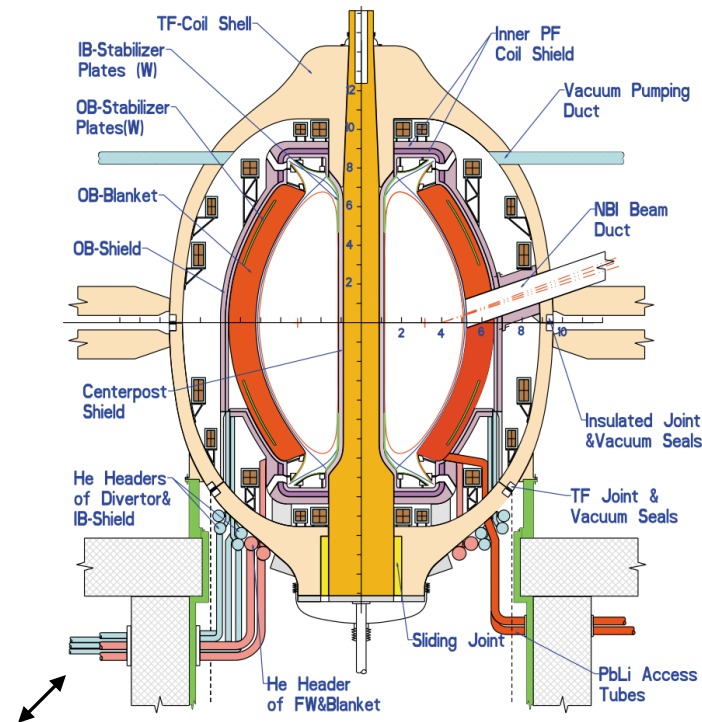
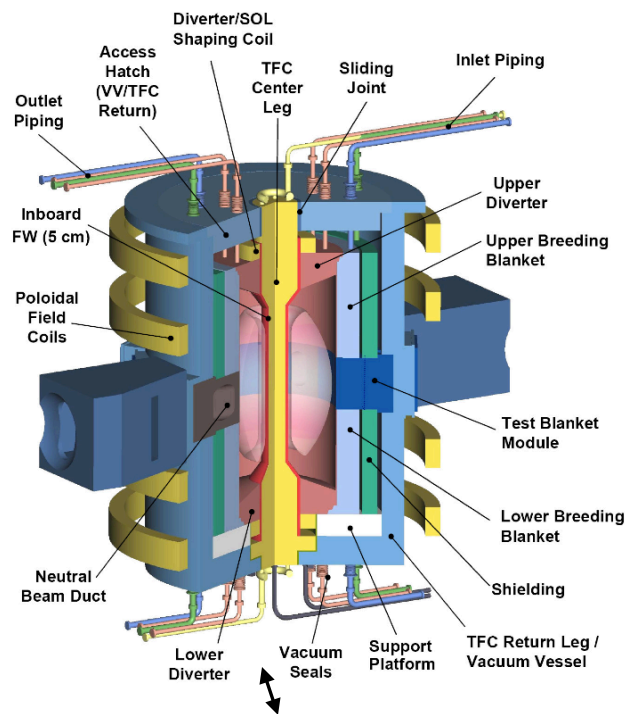
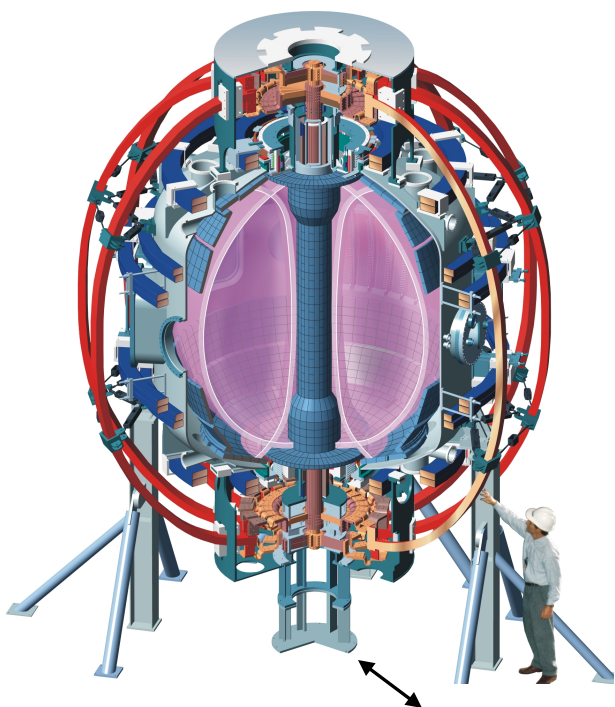
Aspect Ratios 1.45-1.55

Optimized For:
High sustained β_T high non-inductive fraction longest-pulse



S.P. Gerhardt, submitted to Nuclear Fusion

Large Steps Still Required For Next-Step Devices



	NSTX ¹	ORNL CTF ²	ARIES-ST ³
R_0 (m)	0.85	1.2	3.2
I_P/B_T (MA/T)	1.0/0.55	8.2/1.13	29/2.1
β_N	5.0	3.8	7.4
κ	2.8	3.1	3.5
f_{BS} (%)	50-60	50	96
P_{fusion} (MW)	0	75	3000

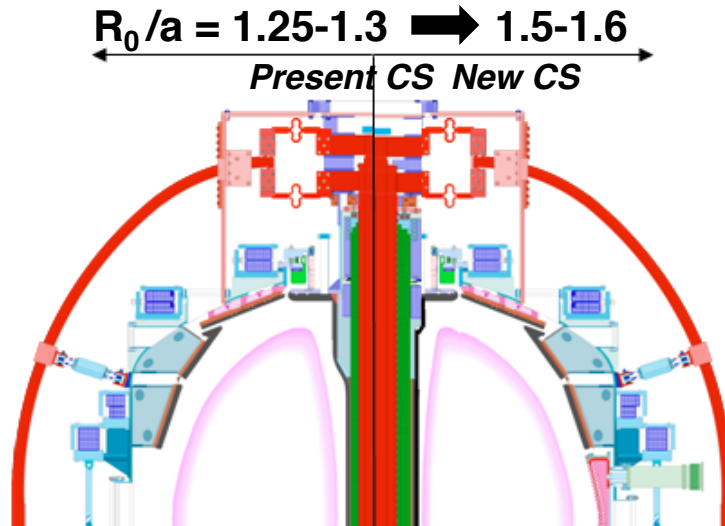
[1] S.P. Gerhardt, et al., EXS/P2-08, IAEA FEC 2010.

[2] Y.K.M. Peng, et al., FT/P3-14, IAEA FEC 2008. WL= 1 MW/m²

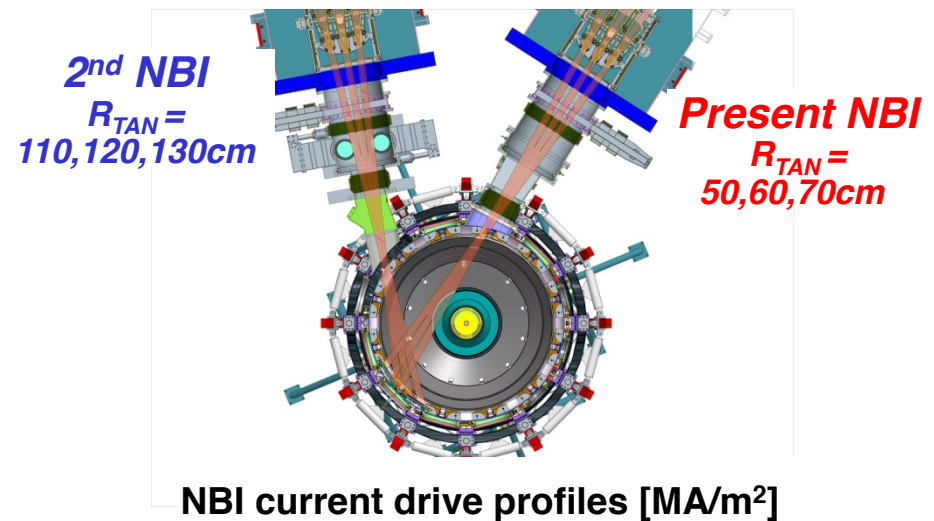
[3] F. Najmabadi, et al., Fusion Engineering and Design, 2003.

NSTX (and MAST) Are Planning Substantial Upgrades

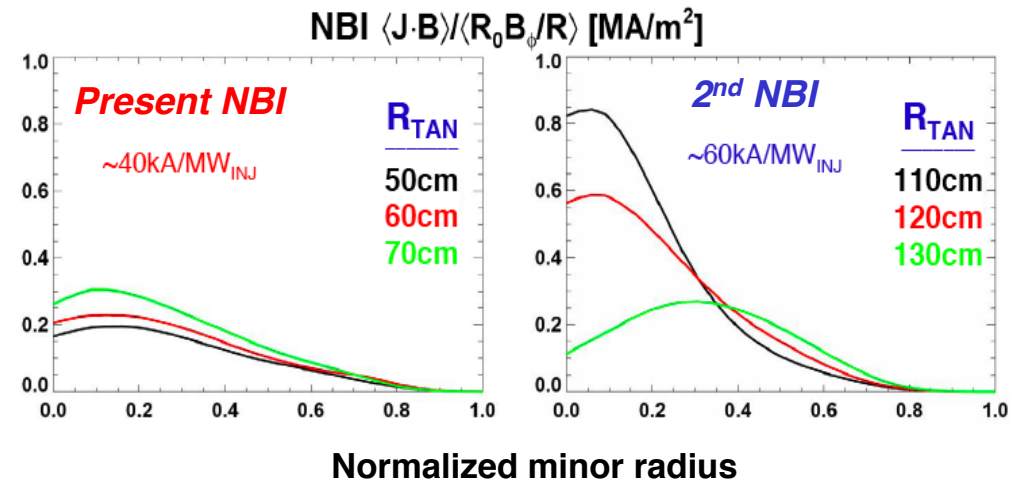
New center stack for 1T, 2MA, 5s



2nd NBI with 5 MW, 5s at larger R_{TAN}



- New center column:
 - 1 MA -> 2MA,
 - 0.5T ->1T, 1.4 sec ->5.0 sec.
- 1 additional beamline with three sources.
- Study transport, stability, current drive
 - in low-collisionality plasmas with 100% non-inductive current drive.



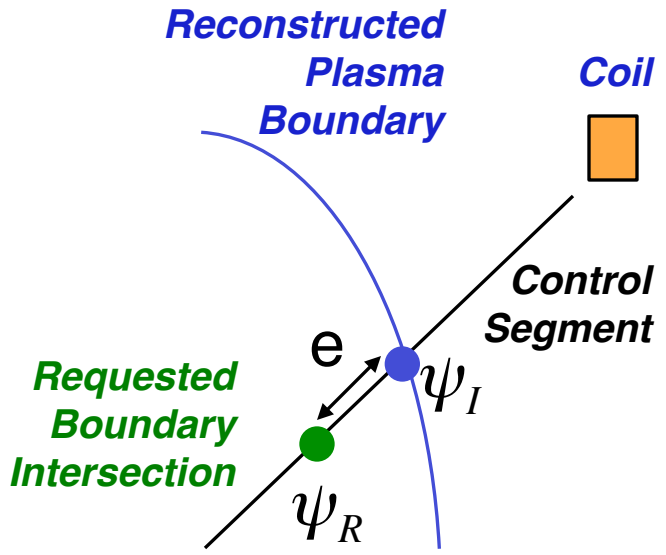
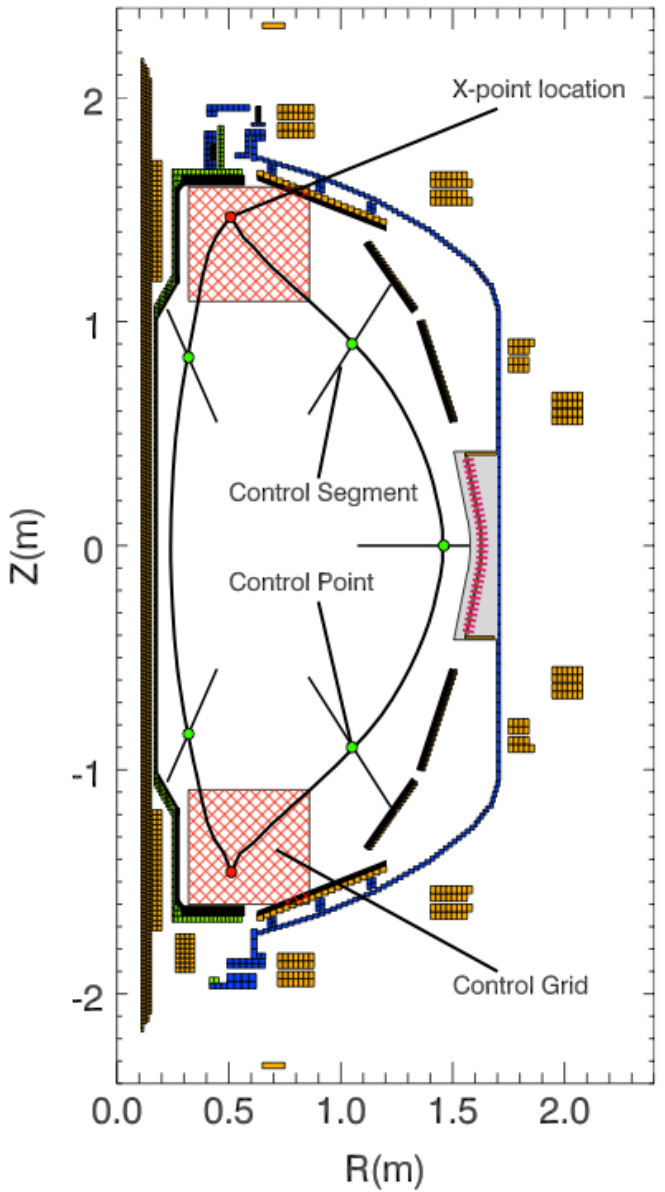
Up to 2 times higher NBI current drive efficiency, and current profile control

Summary

- Sustained, economical tokamak fusion power requires simultaneous high β_N and κ .
- ST offers a potential route to maximizing these parameters.
 - Strong shaping, profile optimization lead to further improvements.
- Bootstrap and neutral beam current drive are the common non-inductive currents sources in present and future STs.
 - Theory of these current drive sources is (mostly) verified.
- Transport determines the level of these non-inductive currents, as well as the profile magnitudes and shapes...
 - ...which in turn determine the stability.
- Best NSTX discharges optimize this physics for high-performance, but large steps are required for next-step devices.
 - NSTX and MAST upgrades will help bridge the gap.

Control The Plasma Shape in With Realtime Grad-Shafranov Equation Solutions

Specify intersections of Control Segments with Desired Boundary



- Compute a solution to the Grad-Shafranov equation solution.
- Compute the flux on control segment at intersection with boundary (ψ_I).
- Compute the error

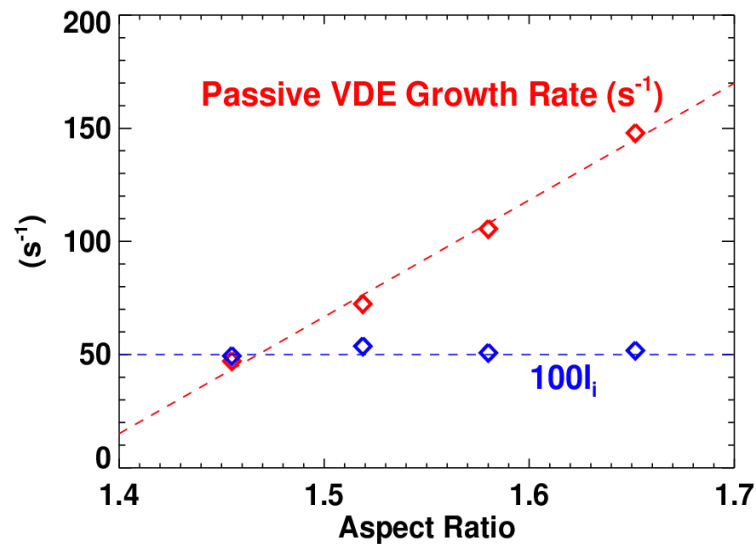
$$e = \psi_R - \psi_I$$

- Adjust coil voltage as

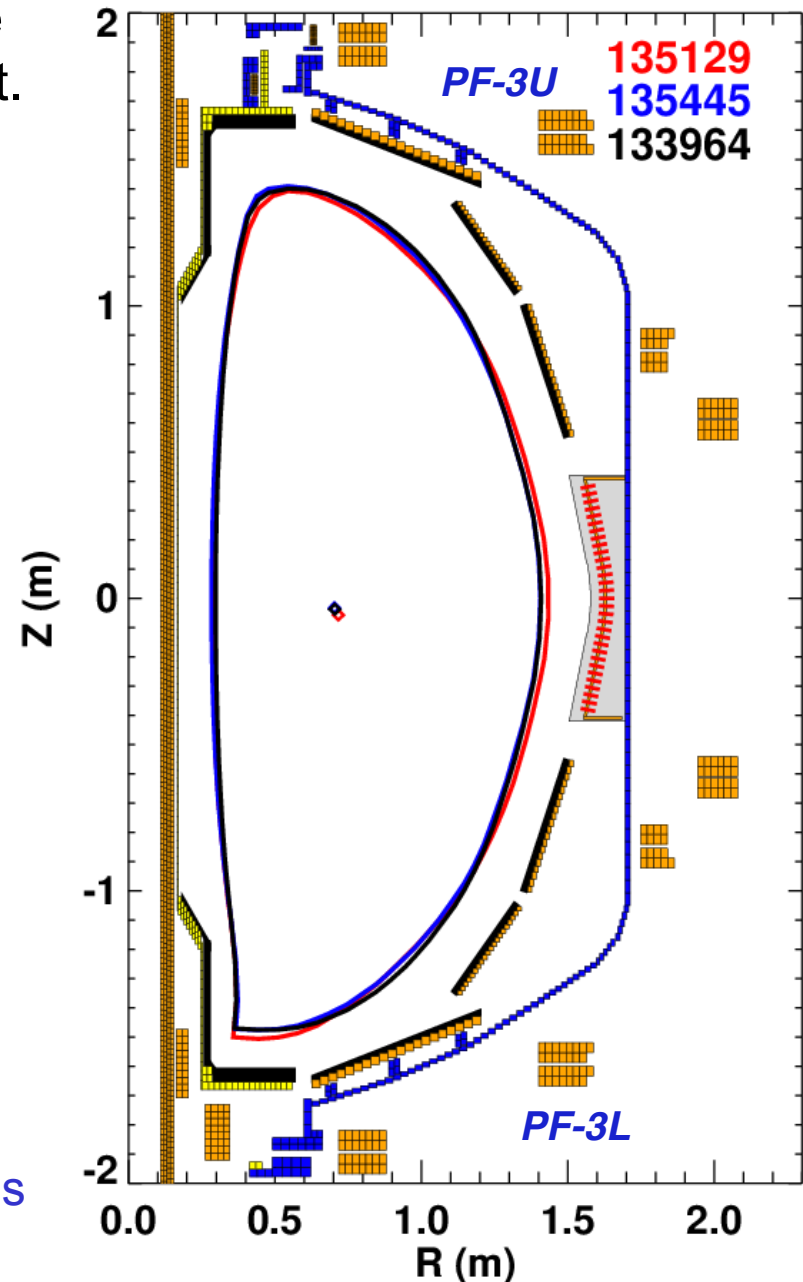
$$V_{coil} = Pe + D \frac{de}{dt} + I \int edt$$

Simplest “Fast Feedback” System in NSTX: Vertical Position Control (I)

- Large natural elongation of the ST stabilizes the “Vertical Displacement Event” (VDE)...to a point.
 - VDE growth rate increases with A .



- Fast vertical position control is required to keep plasma centered in vessel.
- Internal inductance plays an important role. Higher- I_i means:
 - The natural elongation is reduced.
 - Current is farther from the coils, making them less effective for control.



Simplest “Fast Feedback” System in NSTX: Vertical Position Control (II)

- VDE is too fast to be captured by the shape controller.
 - Use an additional feedback system, based on the plasma velocity.
- Two flux-loops are usually sufficient for control.

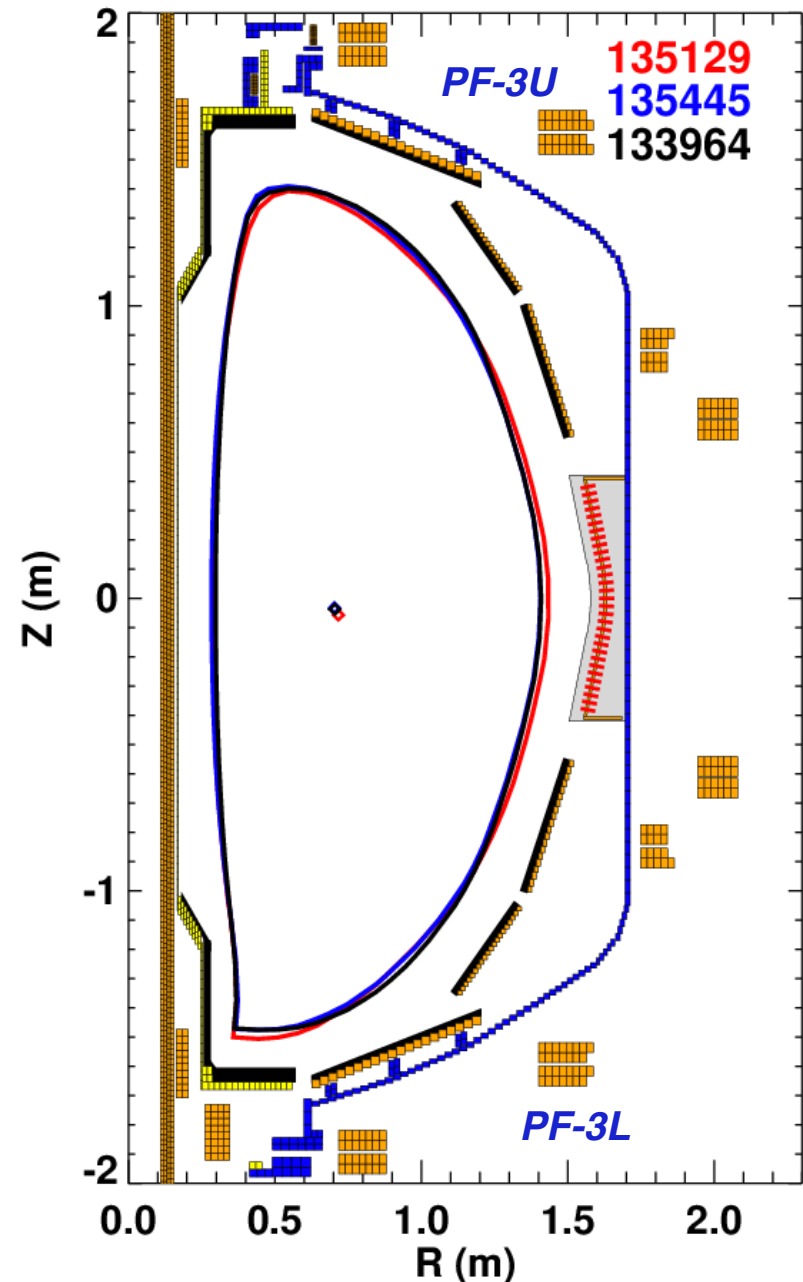
$$Z_P \approx C(\psi_U - \psi_L)$$

$$\frac{dZ_P}{dt} \approx C \left(\frac{d\psi_U}{dt} - \frac{d\psi_L}{dt} \right) = C(V_U - V_L)$$

- Apply a voltage to the “Radial Field” coil proportional inferred velocity.

$$V_{PF-3U} = V_{PF-3U,ISOFLUX} - D(V_U - V_L)$$

$$V_{PF-3L} = V_{PF-3U,ISOFLUX} + D(V_U - V_L)$$



Many Other Quantities are Under Feedback Control in NSTX...But Is this Feasible in Future Devices?

- Other examples of control in NSTX:
 - Slowly-growing $n=1$ kinks (resistive wall modes) are suppressed.
 - Magnetic detection of kink-perturbation using 48 sensors, feedback with midplane coils.
 - Key for providing reliable operation at high- β ...big research program at NSTX.
 - The plasma β_N is regulated.
 - Use rtEFIT to measure β_N , modulate the heating sources.
 - The plasma rotation will (soon) be controlled .
 - Measure V_f with realtime CHERS, control with NB torque changes and magnetic braking.
- Next-step devices pose many challenges for control:
 - Can the appropriate real-time measurements be made in a neutron environment?
 - e. g., neutron streaming limits available sizes of ports?
 - Will we have the actuators available?
 - e. g., internal coils for non-axisymmetric fields may not be available?
- This is a key issue going to next-step devices.

NSTX Shows a β_N Limit, Not a β_T Limit

- Show β_N and β_T averaged over the pulse duration, and plotted against the pulse duration.
 - But, what sets the β_N limit? Why is it 5 and not 3?

