Clarification of the new XGC0-based bootstrap current formula for tight aspect ratio

Robert Hager and C.S. Chang
PPPL

Special thanks to: Rob Andre, Luca Guazzotto

EPSi
Edge Physics Simulation

PPPL
PRINCETON
PLASMA PHYSICS
LABORATORY

Bootstrap current large in edge pedestal

- Bootstrap current in edge pedestal is large, hence important
- Hard to measure → need formulas to estimate it
- Simplifying assumptions in existing formulas valid for core plasma → problems for edge pedestal and separatrix
- Koh et al. (Phys. Plasmas **19**, 2012) → new XGC0-based formula to improve Sauter's formula in pedestal/separatrix region
- E. Belli reported disagreement between NEO results and the improved formula for tight aspect ratio (APS 2013)

→ **Verification of Koh's findings is the purpose of this work**

Outline

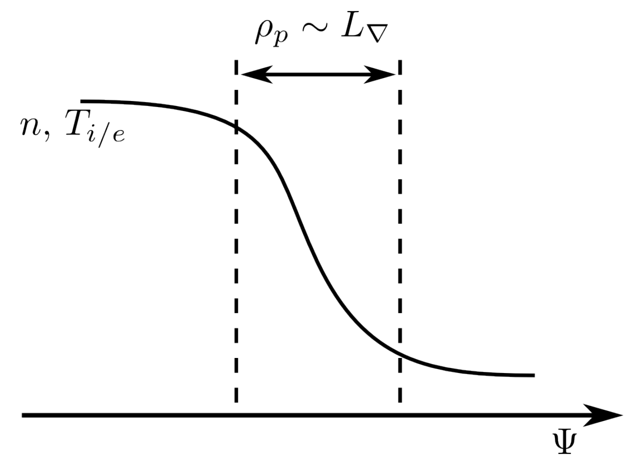
- Introduction
  - The XGC codes
  - Special physics in the edge
  - Koh’s bootstrap current formula
- Verification study of Koh’s formula
- Summary and conclusions

The XGC codes

- XGC: whole-volume full-f particle-in-cell codes for the simulation of fusion plasmas
  - XGC0: guiding-center neoclassical physics
  - XGC1: gyrokinetic turbulence code
  - XGCa: axisymmetric version of XGC1 (GK neoclassical code)
  - Include X point and scrape-off layer
  - 5D phase space (3D in configuration space)
  - Full-f
  - Rich physics: impurities, neutrals (with DEGAS2), heating/cooling/torque…
- XGC0 is used for Koh's formula and this study
  - Monte Carlo collisions with intra-/inter-species conservation

What makes the plasma edge special?

- Radial orbit width ~ L_P
  - Small orbit width expansion (used in most codes, incl. Sauter and NEO) breaks down
  - Separatrix effect and orbit loss physics
- Strong ExB flows
- Small passing particle region
- Geometry effect at small aspect ratio→ special field line topology



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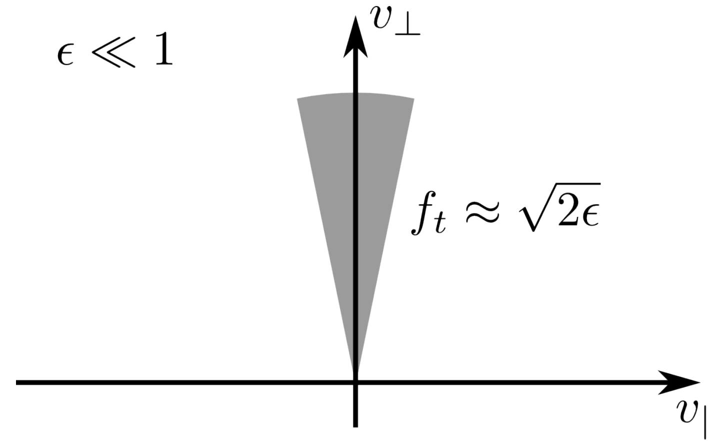

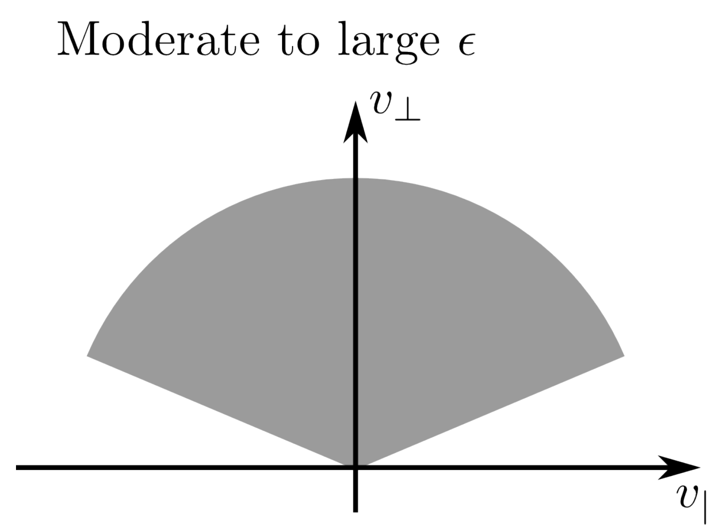

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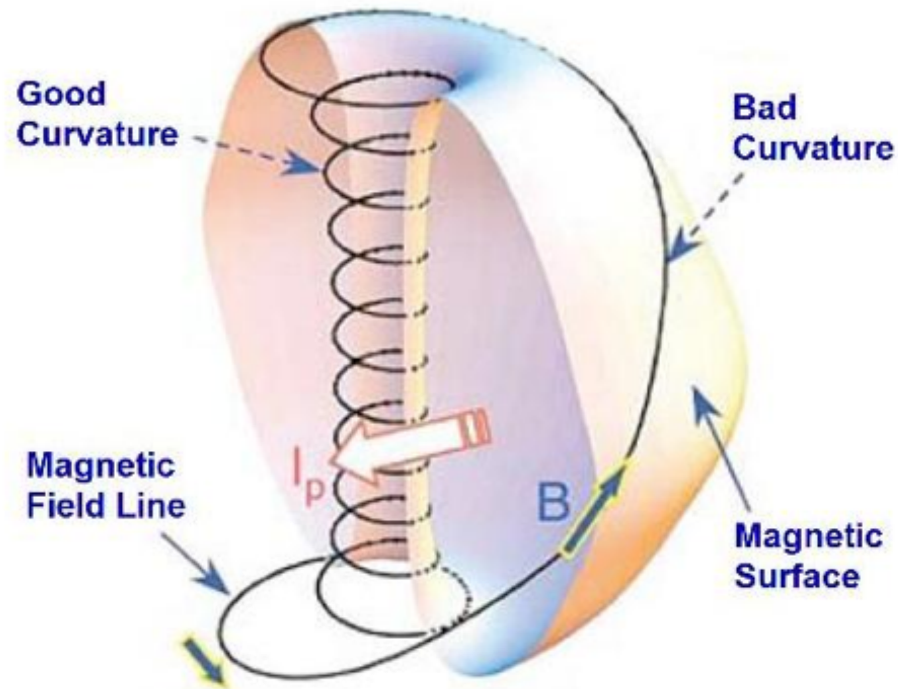
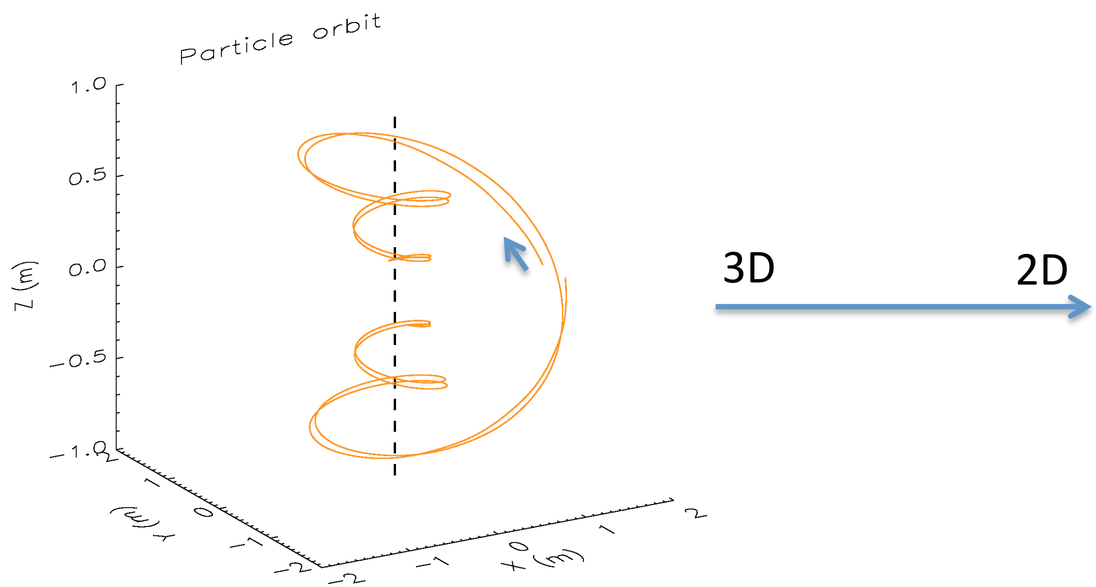
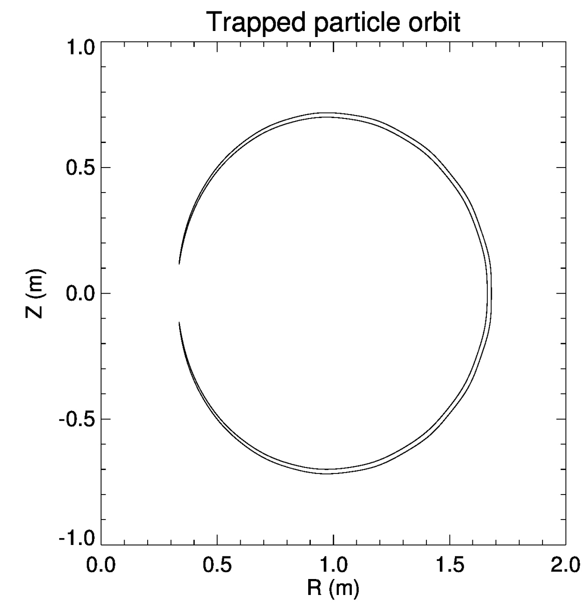


Trapped-Passing boundary layer physics is more important at tight aspect ratio

- Barely trapped particles not confined in toroidal direction





→ Solubility condition is used in 2D equation to implement approximately toroidal dynamics (e.g., trapped particles do not contribute to bootstrap current)

→ 2D orbits fine for collisionless physics

→ With collisions: these particles “forget” that they are trapped → toroidal current

→ 3D (x-space) code needed to treat particle orbits correctly to show enhanced bootstrap current → Removal of the solubility condition is crucial

XGC0-based bootstrap current formula for edge pedestal

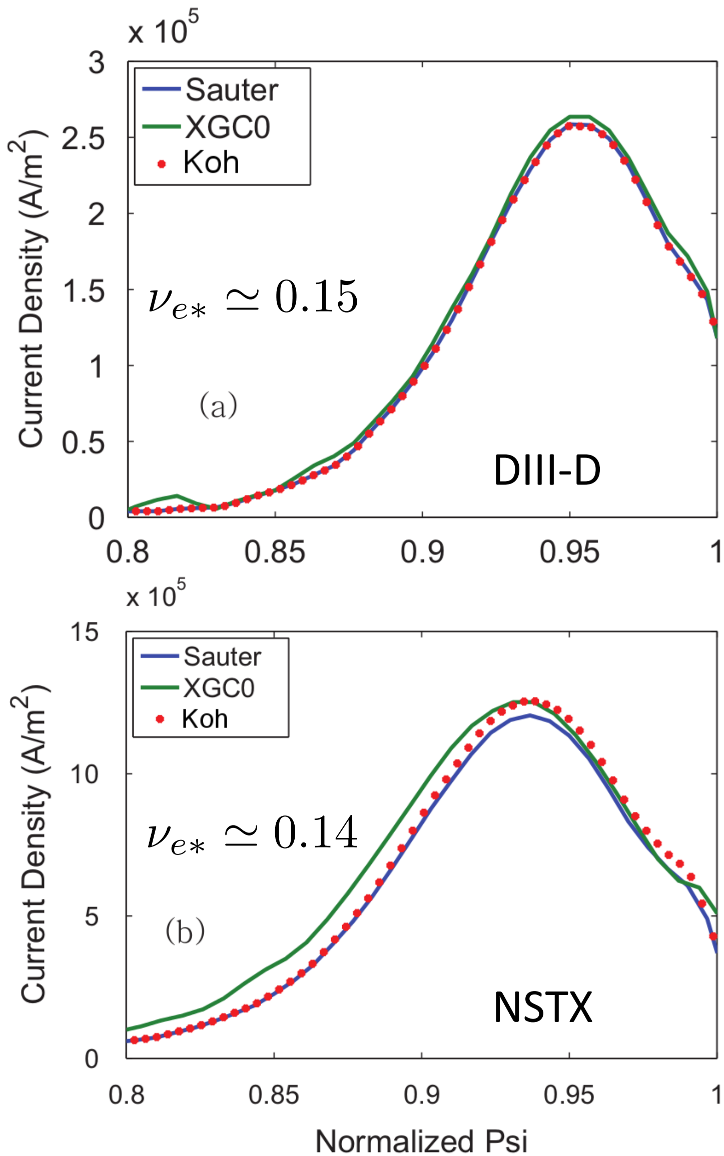
Koh et al., Phys. Plasmas 19, 072505 (2012)

- Retains structure of Sauter's formula because of the reasonable agreement at low ν_{e*} or ε → simplification of the application

$$\langle \boldsymbol{J}_b \cdot \boldsymbol{B} \rangle = I p_e \left(L_{31} \frac{p}{p_e} \frac{\mathrm{d} \ln p}{\mathrm{d}\Psi} + L_{32} \frac{\mathrm{d} \ln T_e}{\mathrm{d}\Psi} + L_{34} \alpha \frac{T_i}{Z T_e} \frac{\mathrm{d} \ln T_i}{\mathrm{d}\Psi} \right)$$

- Coefficients L_{3x} modified
  - Based on numerical results of XGC0
  - Based upon >100 simulation cases
- **Optimized for edge plasma**
  - Orbit width comparable to gradient scale length
  - Moderate to large trapped particle fraction
  - Arbitrary aspect ratio → spherical tokamaks
  - Effect of separatrix
  - Allows ~5% fitting error
- Formula is made to reduce to Sauter for lower ϵ or weaker ν_{e*}

Verification: Good agreement with Sauter at low collisionality or large aspect ratio

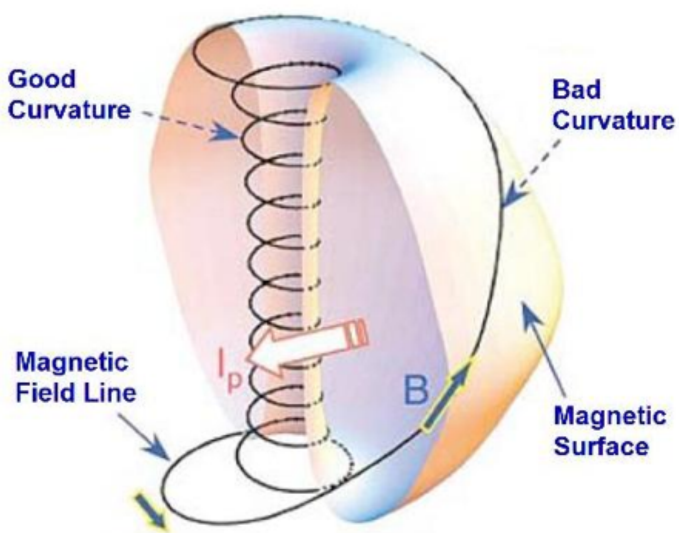


Sauter's high confidence regime

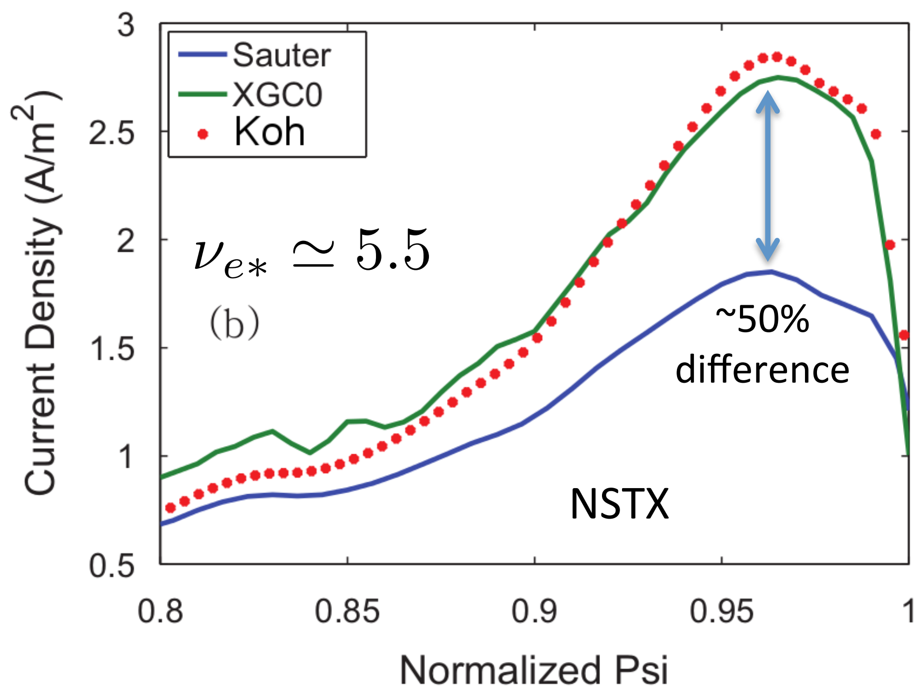
Koh et al., Phys. Plasmas 19, 072505 (2012)

XGC0 does not agree with Sauter's formula (and other existing formulas or simulations) for NSTX

- XGC0 finds that bootstrap current in pedestal significantly larger than Sauter in collisional regime
- Koh et al.'s explanation: Field line pitch + collisionality



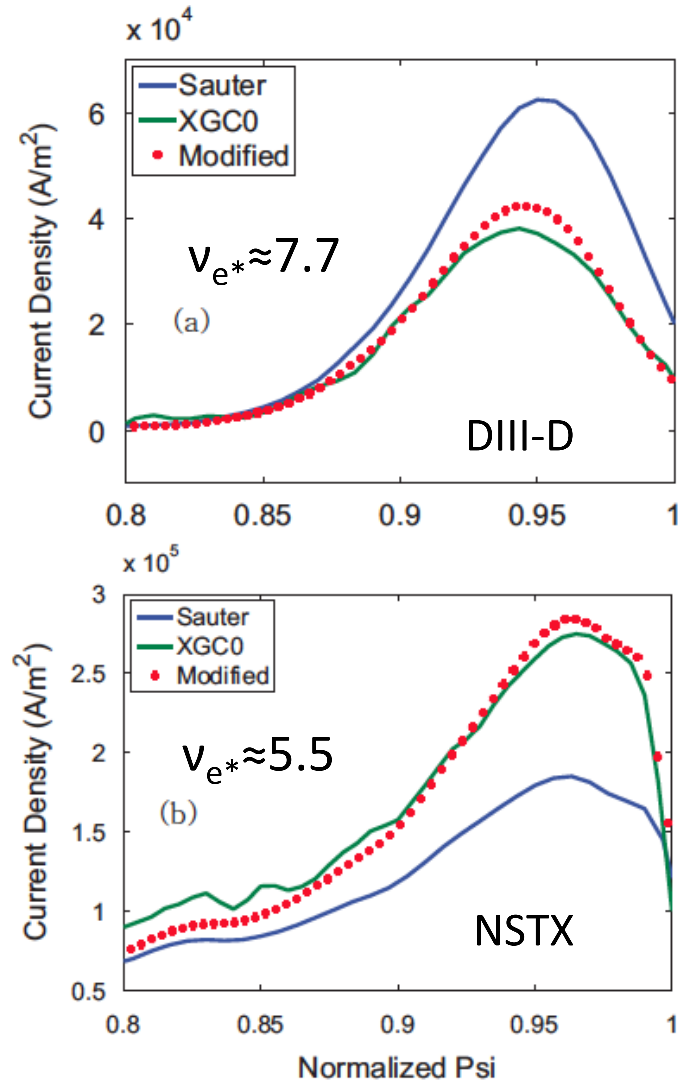
NSTX



Koh et al., Phys. Plasmas 19, 072505 (2012)

Edge effect appears in the opposite directions between conventional and tight aspect ratio tokamaks

- There is a boundary layer in ε and ν_{e*} space across which the trapped-passing particle physics change sharply
- In conventional aspect ratio edge, strong collisions destroy the passing particle dynamics and reduce j_b
- In tight aspect ratio edge, collisions make many of the trapped particles forget that they are trapped and enhance j_b



Koh et al., Phys. Plasmas 19, 072505 (2012)

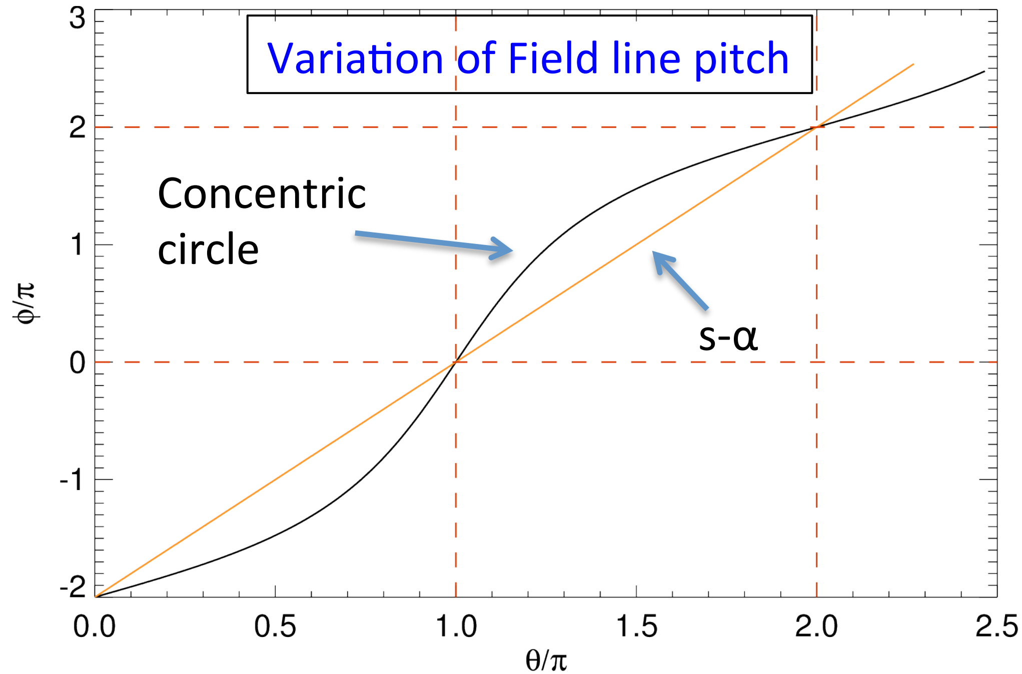
Verification of tight aspect ratio effect on enhancement of j_{boot}

- Set up **simple** test to check Koh et. al.'s explanation
- Hypothesis: some trapped particles contribute to bootstrap current at tight aspect ratio due to extreme variation of field line pitch

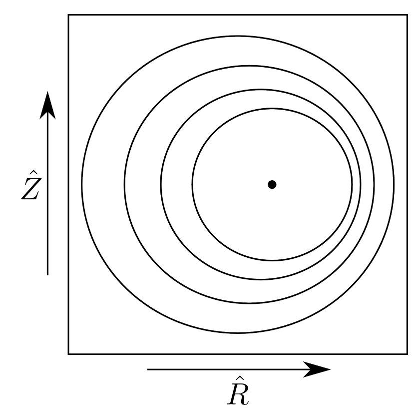
$$\frac{d\varphi}{d\theta} \equiv \frac{\boldsymbol{B}\cdot\nabla\varphi}{\boldsymbol{B}\cdot\nabla\theta} \qquad q \equiv \left\langle \frac{d\varphi}{d\theta} \right\rangle$$

→ Compare

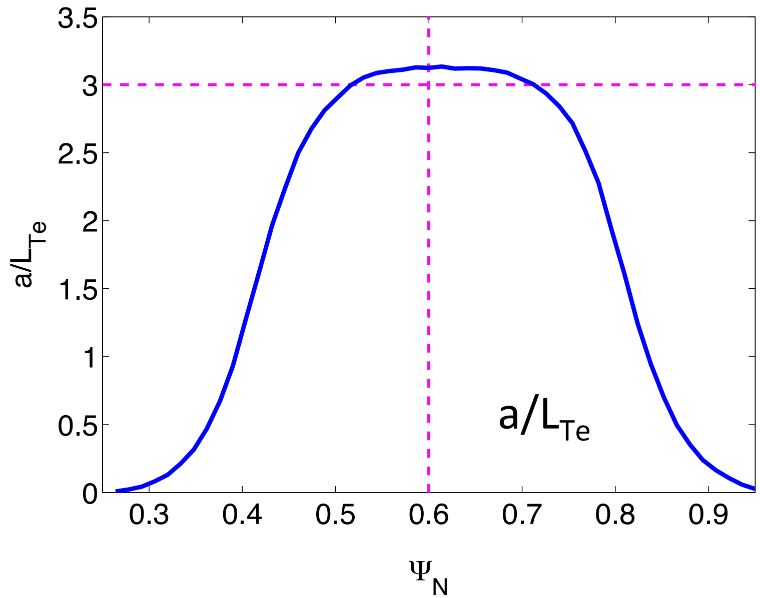
- s-α: **constant field line pitch** → "normal" j_{boot}
- Concentric circle → enhanced j_{boot}

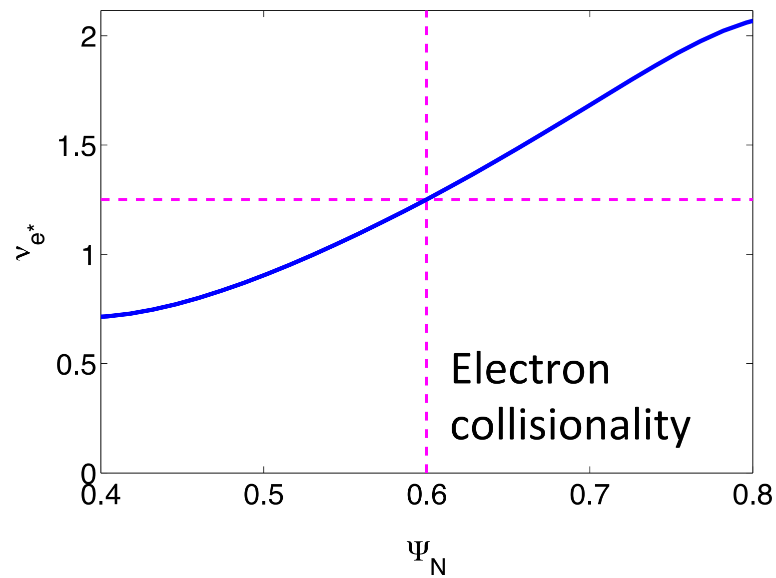


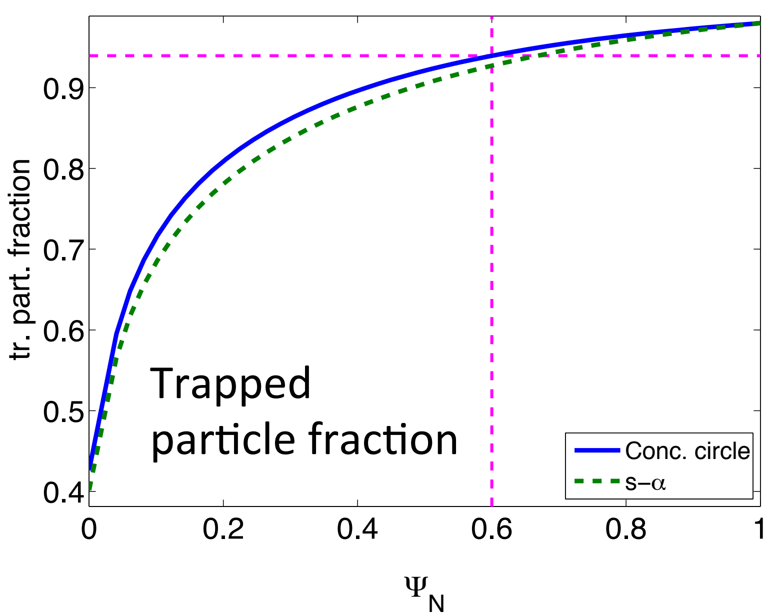
Variation of field-line pitch is even more pronounced in Grad-Shafranov equilibrium!

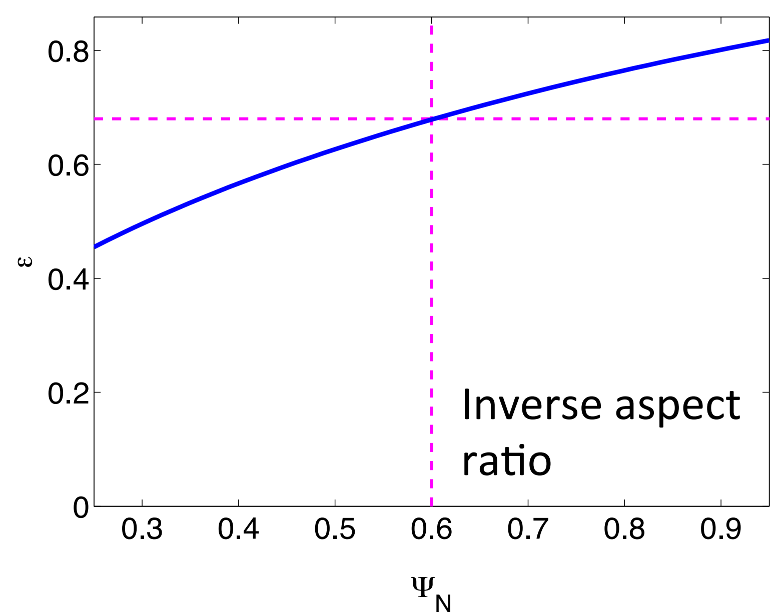


Simulation setup (in response to NEO's request)



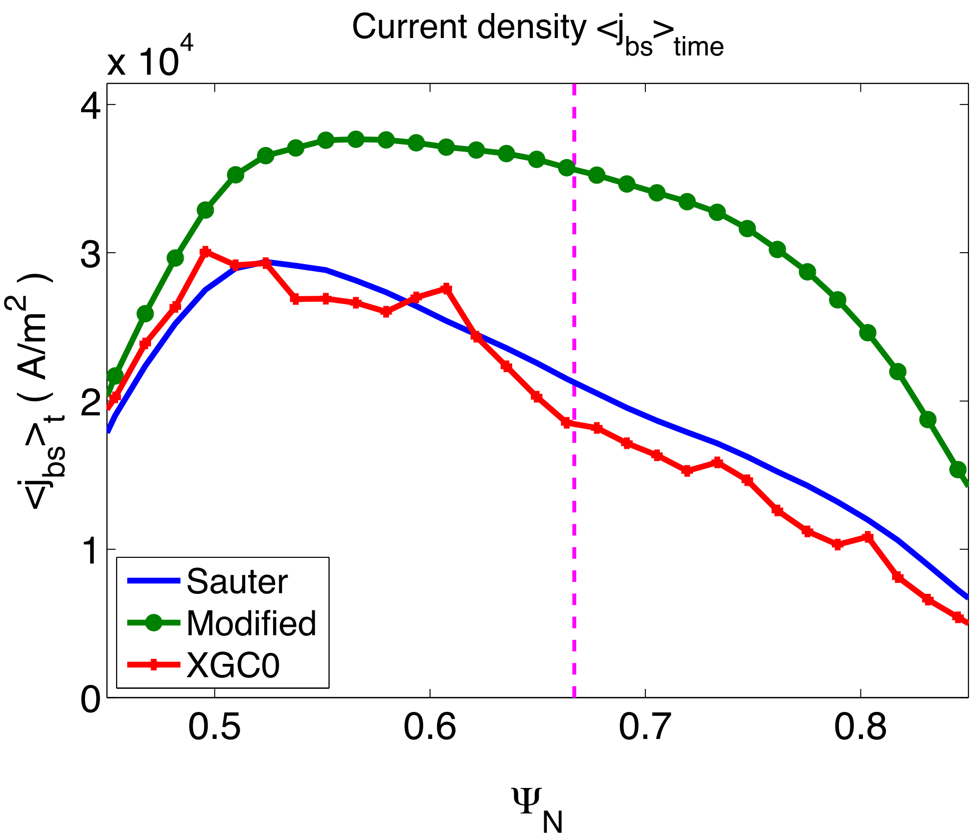




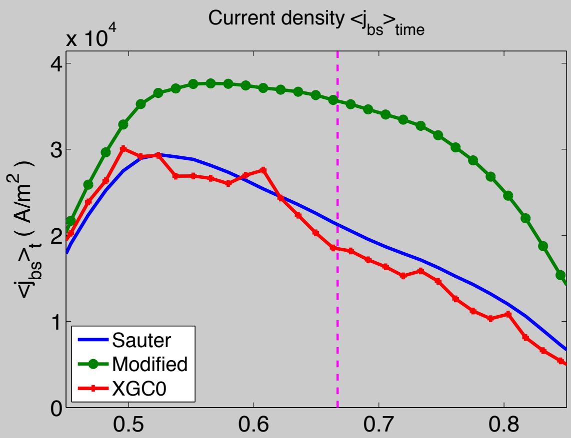


Results

- s-α: Numerical XGC0 result agrees reasonably with Sauter (and NEO)

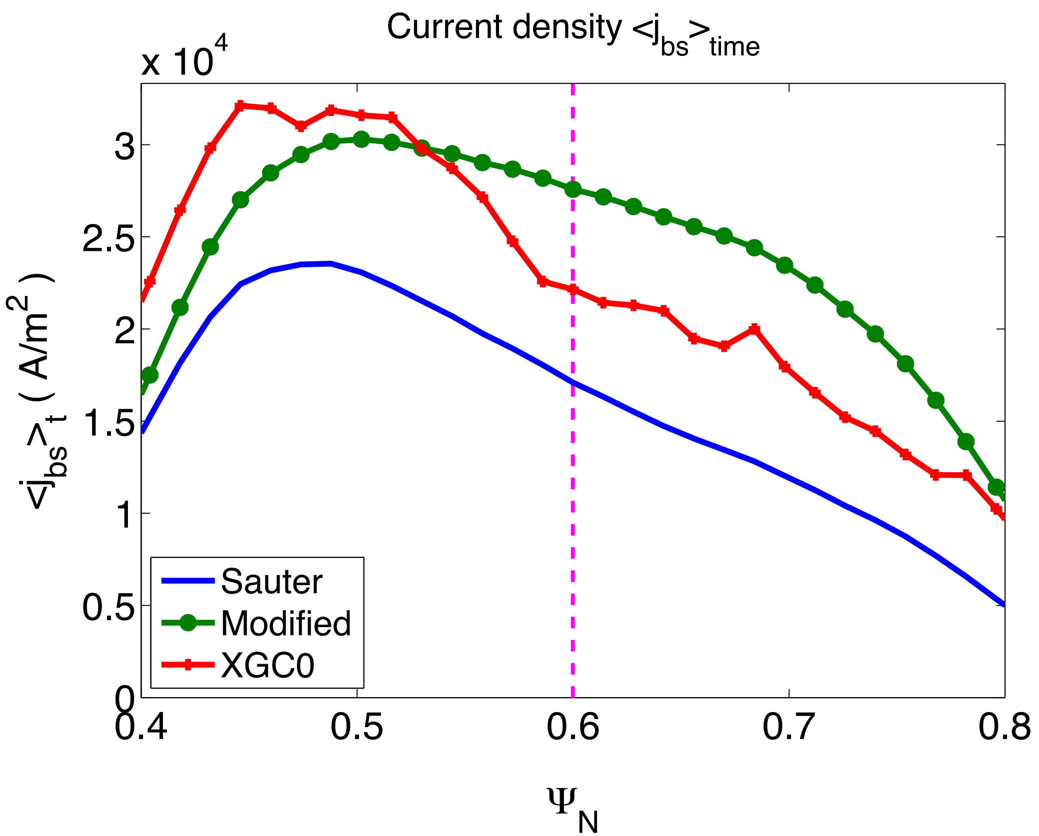


s-α

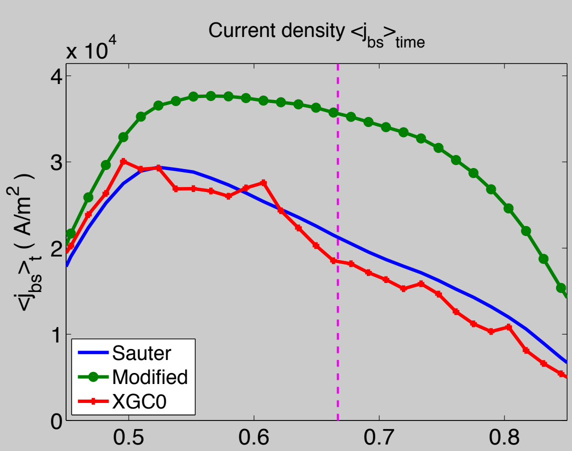


Results

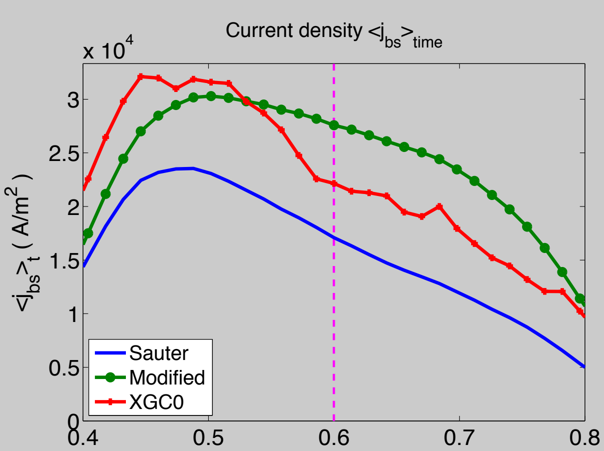
- s-α: Numerical XGC0 result agrees reasonably with Sauter (and NEO)
- Concentric circle: j_b deviates from Sauter towards Koh
- NEO does not see this geometry difference (E. Belli, private comm.)



s-α



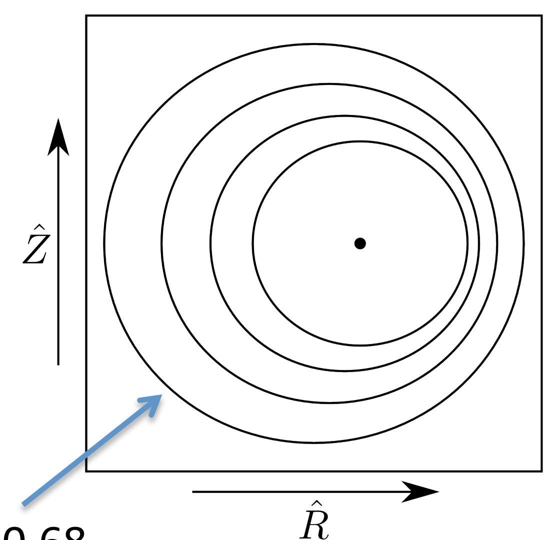
Concentric circle



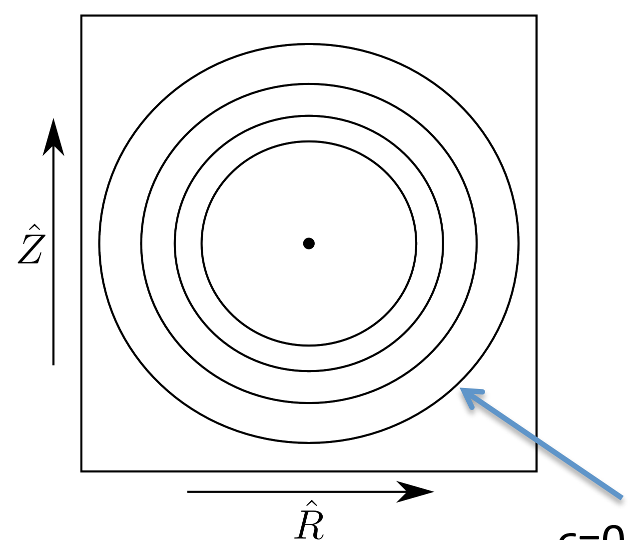
Verification of tight aspect ratio effect on enhanced j_{boot}

- More sophisticated approach
  → three simulations, same (pedestal) profiles:
  - One reference surface at same position in all 3 cases
  - "Circular" Grad-Shafranov geometry from Isolver (R. Andre)
  - Concentric circular
  - s-α geometry

Grad-Shafranov



ϵ=0.68



Concentric circular and s-α

ϵ=0.68

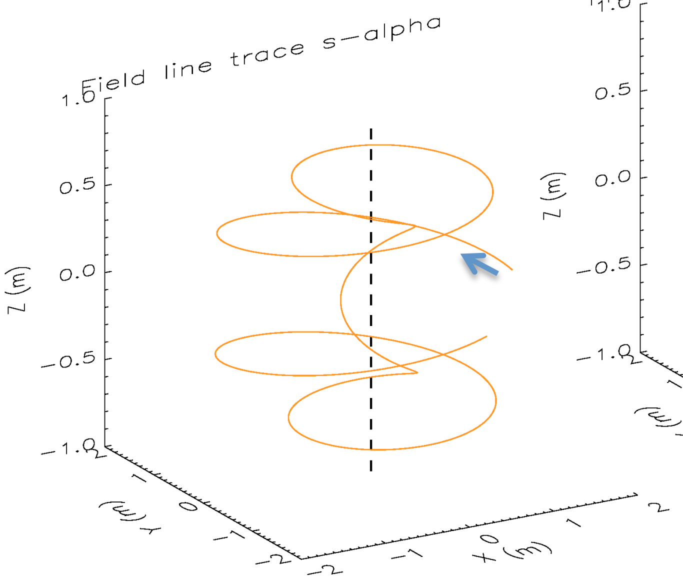
Field-line traces on reference surface

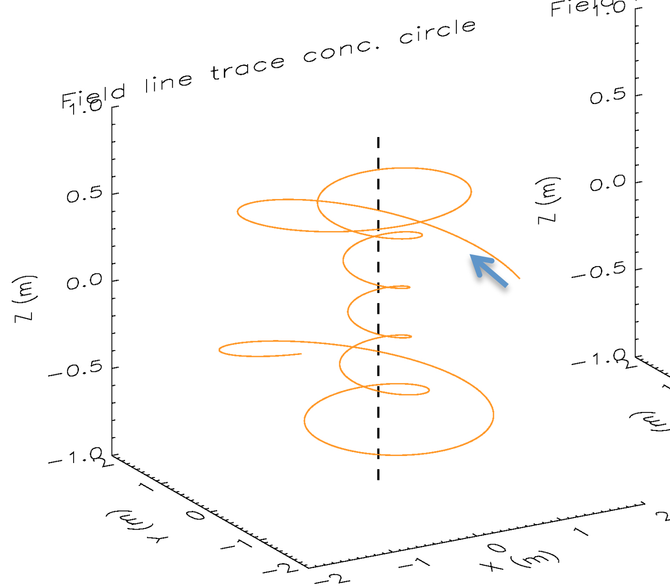
Degree of realism

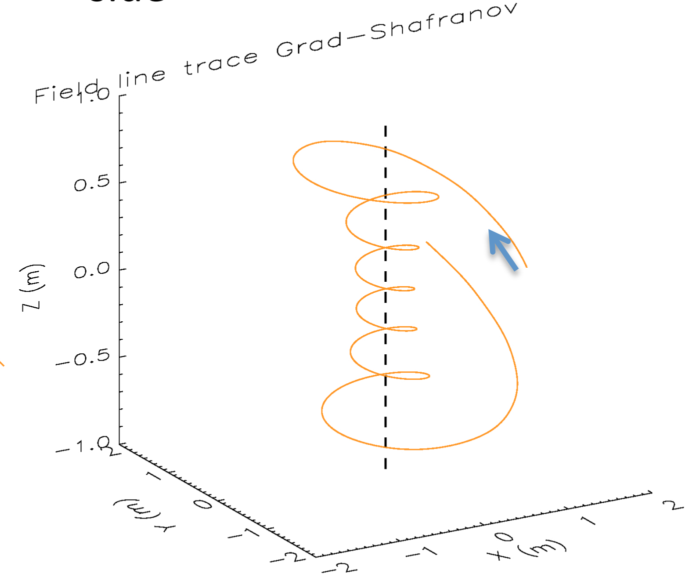
Grad-Shafranov
Variation of field line pitch more pronounced due to Shafranov-shift,
Field line mostly on high-field side

Concentric circle
Large field line pitch on the high-field side

s-α
Constant field line pitch







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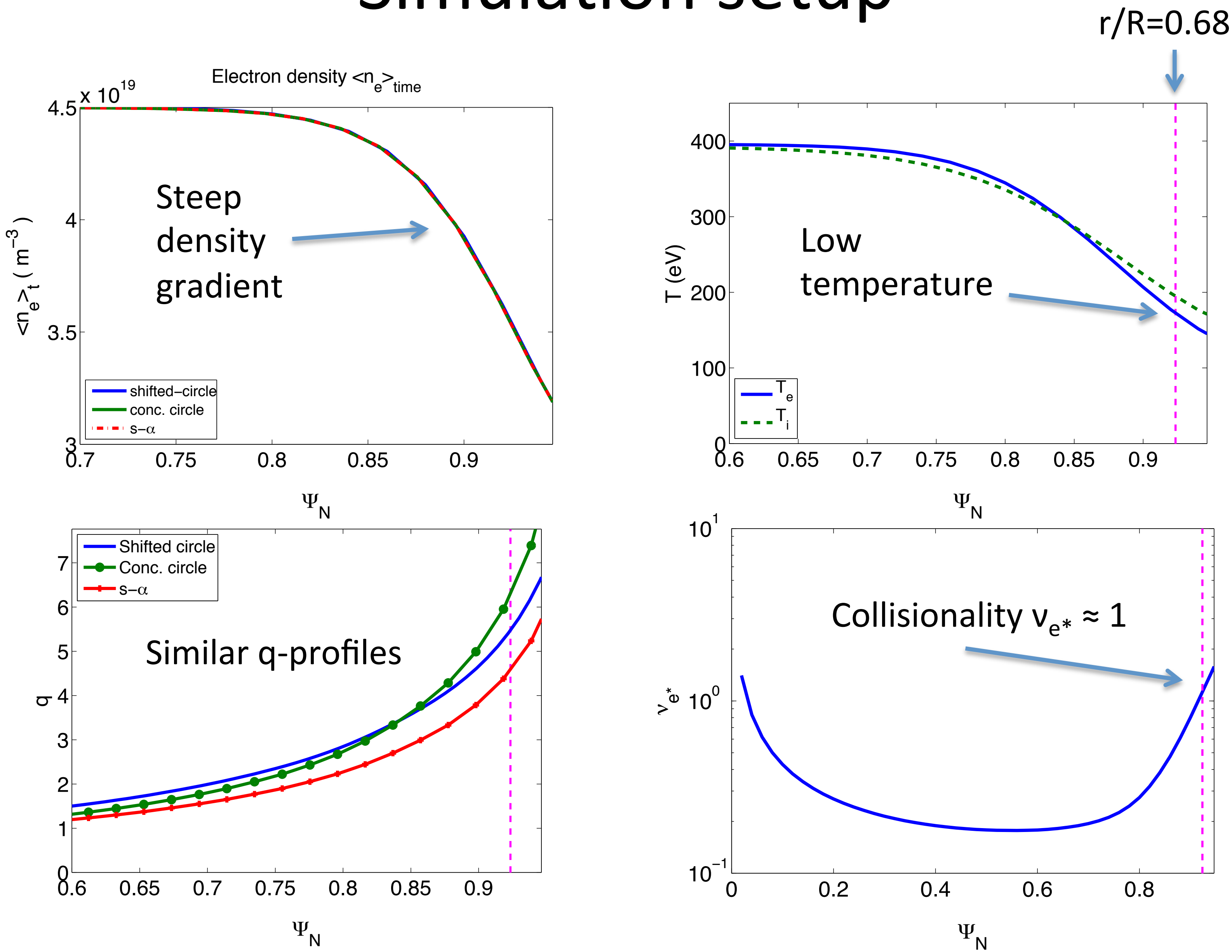

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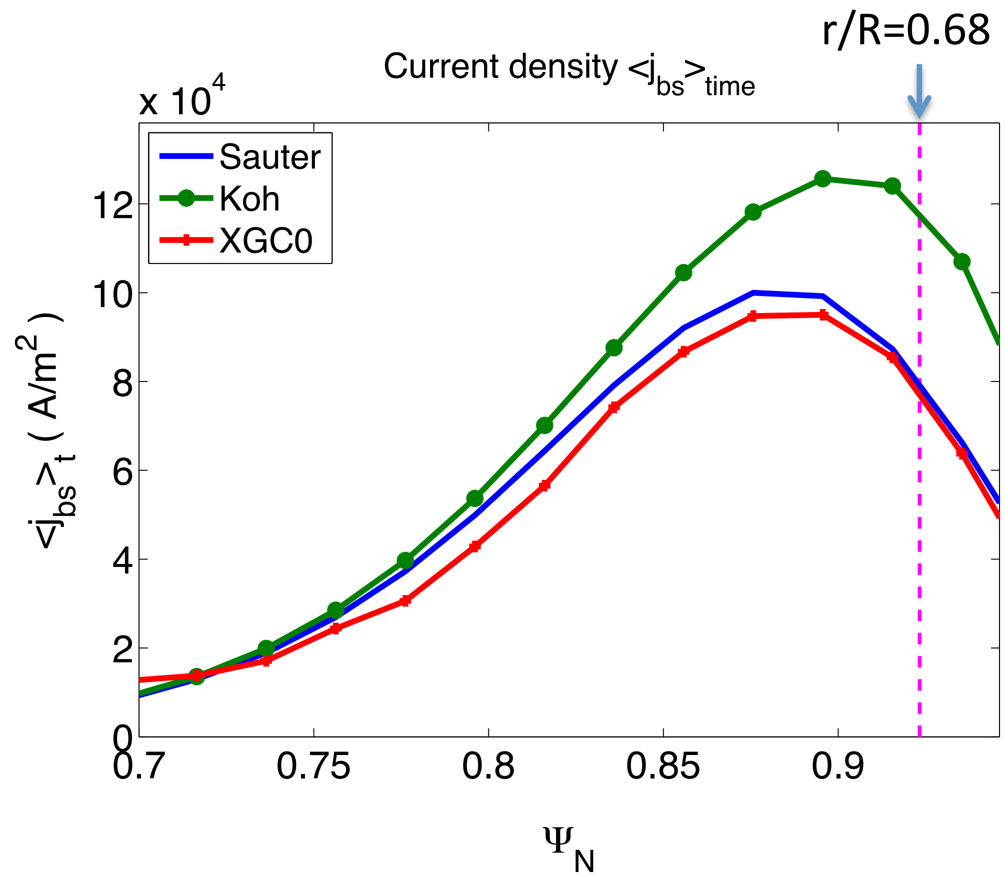
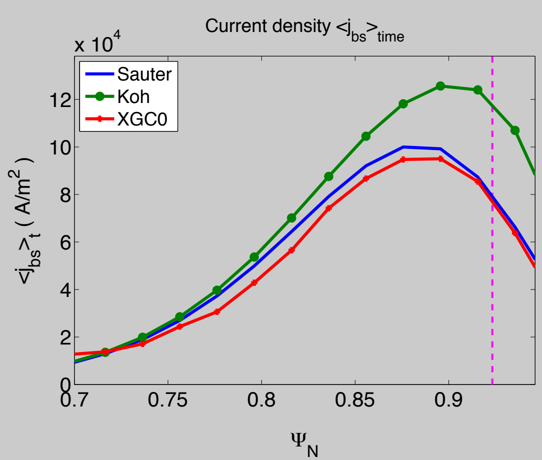


s-α



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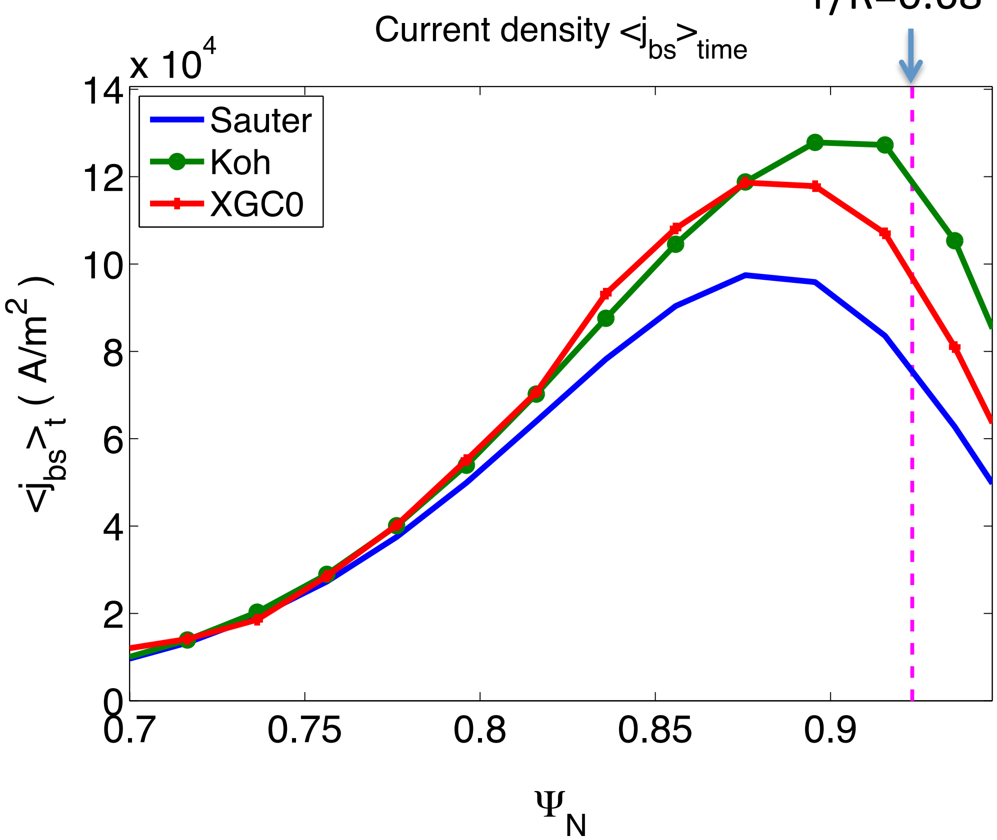
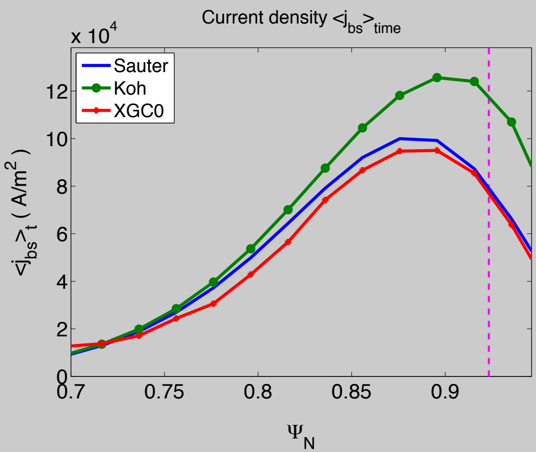
- s-α: Numerical XGC0 result agrees reasonably with Sauter (and NEO)
- Concentric circle: XGC0 deviates toward Koh

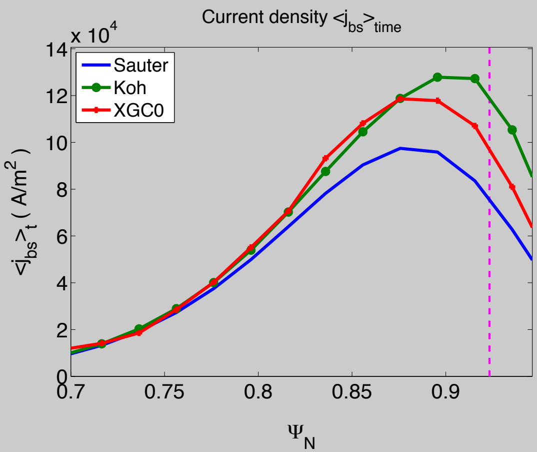
r/R=0.68



s-α



Concentric circle



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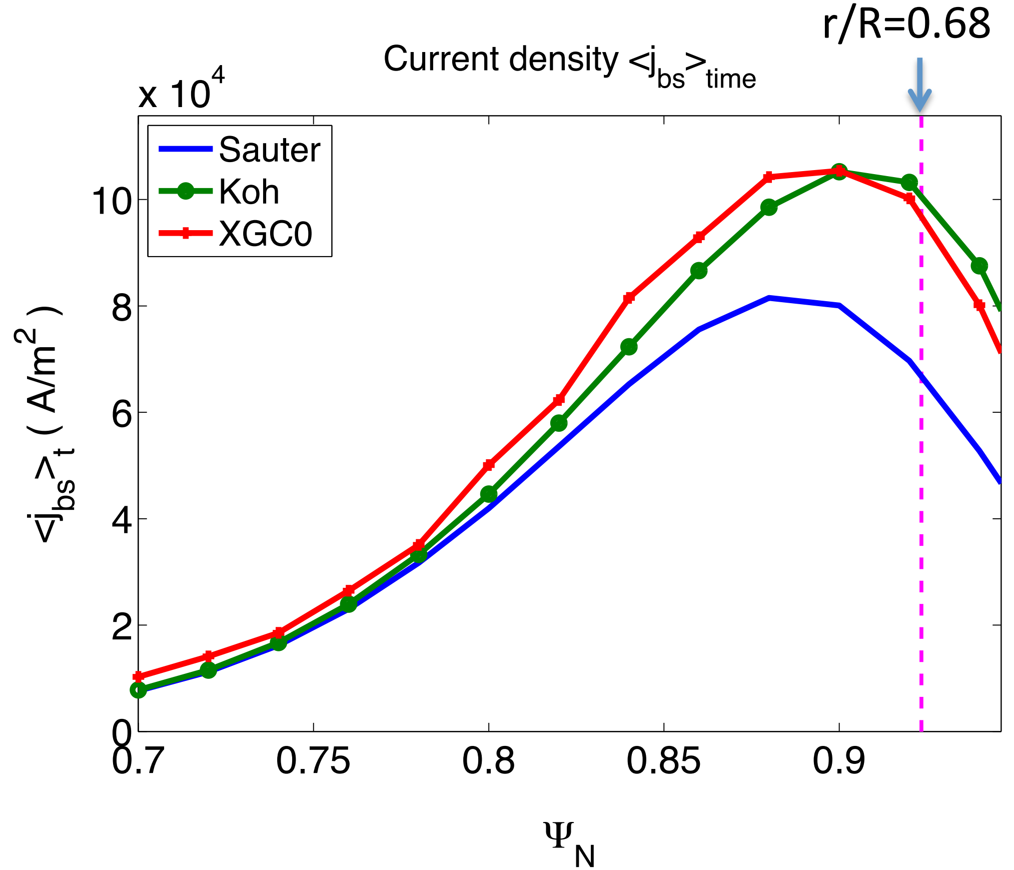
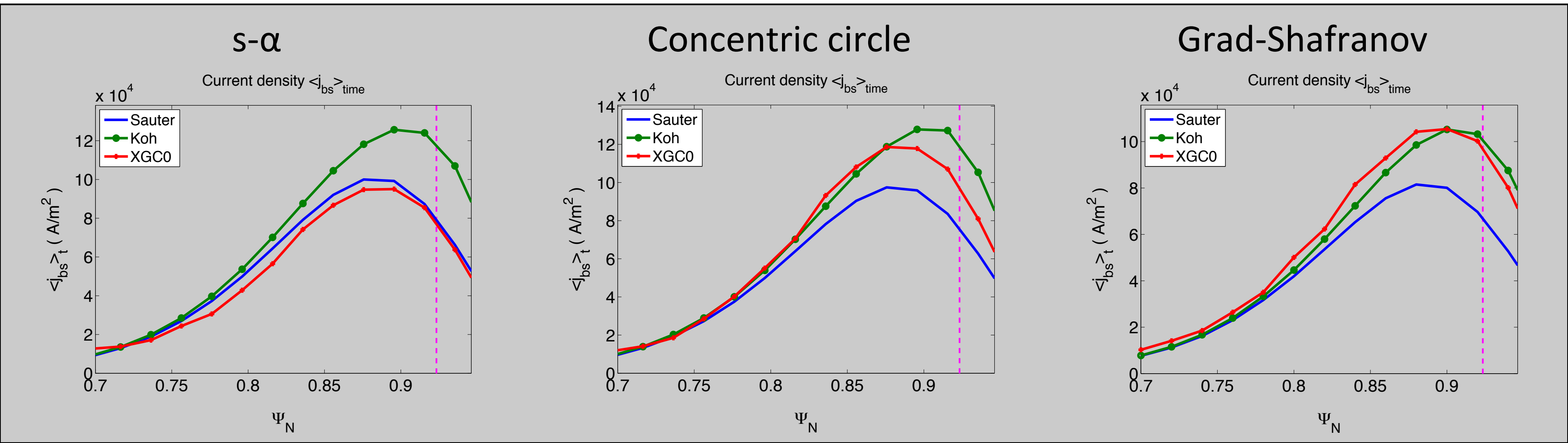
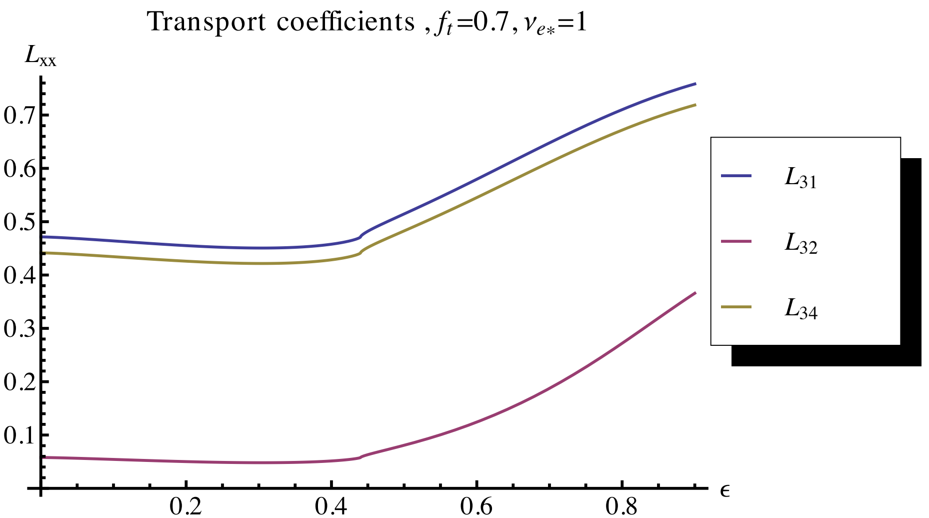
- Concentric circle: XGC0 deviates toward Koh
- Grad-Shafranov circle: XGC0 agrees reasonably with Koh





The transition of edge physics across ϵ=0.44 at high ν_{e*} is from real physics

- Koh made this transition quite sharp (jump in 1st derivative), in an attempt to produce the simplest transition formula



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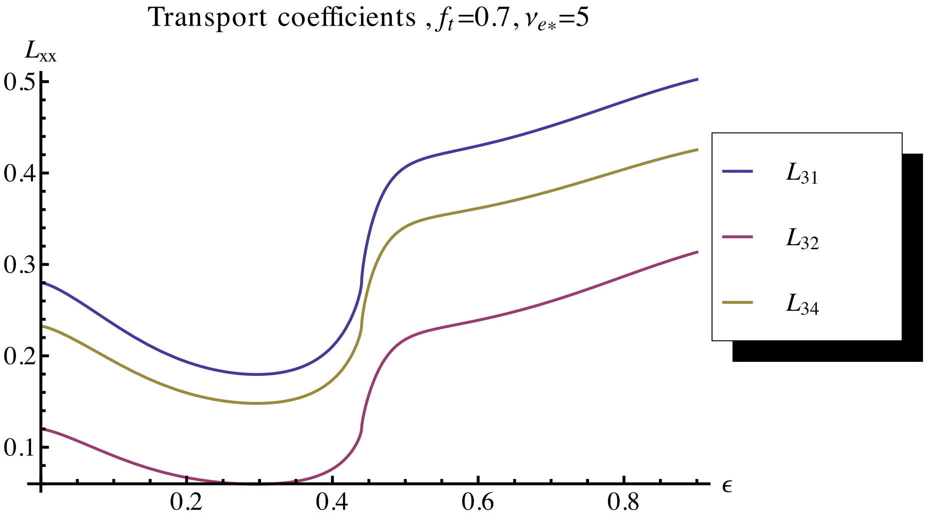

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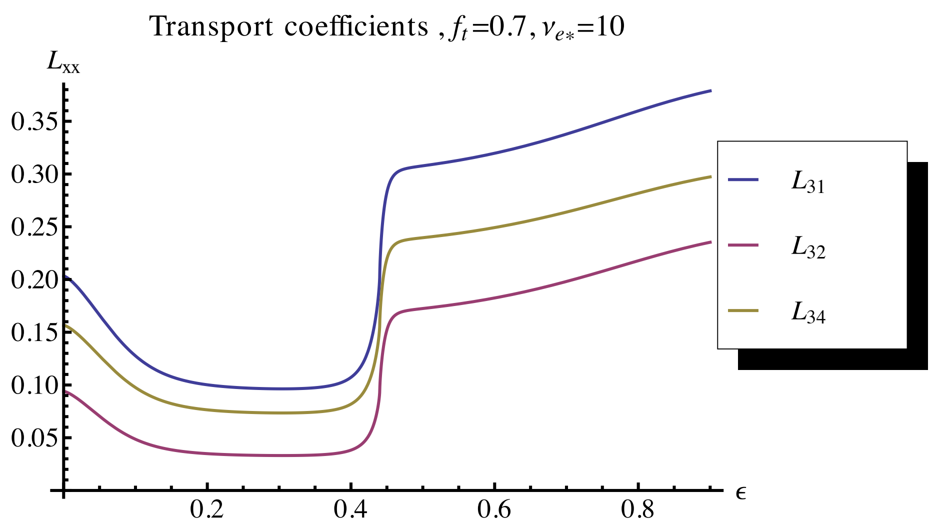


- We can improve the transition to be milder in the formula
- But, no machines with highly collisional edge pedestal at ϵ=0.44 exist: This sharp transition will not be seen in practical tokamaks

Sharp transition due to unfortunate choice of fitting function

- Koh's formula reproduces enhanced bootstrap current
- Jump unfortunate but no show-stopper

$$\langle \boldsymbol{J}_b \cdot \boldsymbol{B} \rangle = I p_e \left(L_{31} \frac{p}{p_e} \frac{\mathrm{d} \ln p}{\mathrm{d}\Psi} + L_{32} \frac{\mathrm{d} \ln T_e}{\mathrm{d}\Psi} + L_{34} \alpha \frac{T_i}{Z T_e} \frac{\mathrm{d} \ln T_i}{\mathrm{d}\Psi} \right)$$

$$L_{31} = \alpha_1 f^{31}_{t,eff} + \alpha_2 (f^{31}_{t,eff})^2 + \alpha_3 (f^{31}_{t,eff})^3 + \alpha_4 (f^{31}_{t,eff})^4$$

Koh's modification

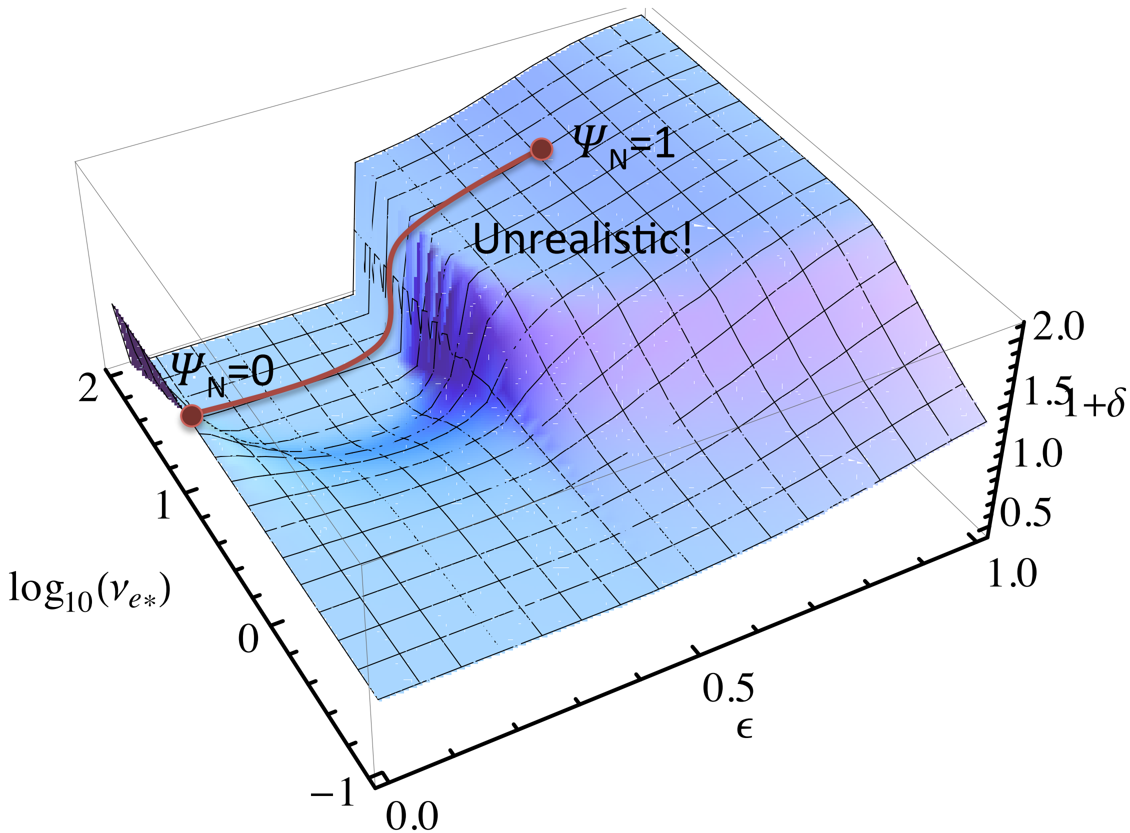
$$f^{31}_{t,eff}(f_t, \epsilon, \nu_{e*}) \longrightarrow f^{31}_{t,eff} \left[1 + \delta(\epsilon, \nu_{e*}) \right]$$

$$\delta \propto \tanh \left(3.2 \beta(\epsilon) \frac{(\epsilon^{3/2} \nu_{e*})^{1.4}}{Z^{\alpha(Z)}} \right), \ \beta(\epsilon) = \Re((\epsilon - 0.44)^{0.7})$$

- Width of tanh becomes small for high collisionality

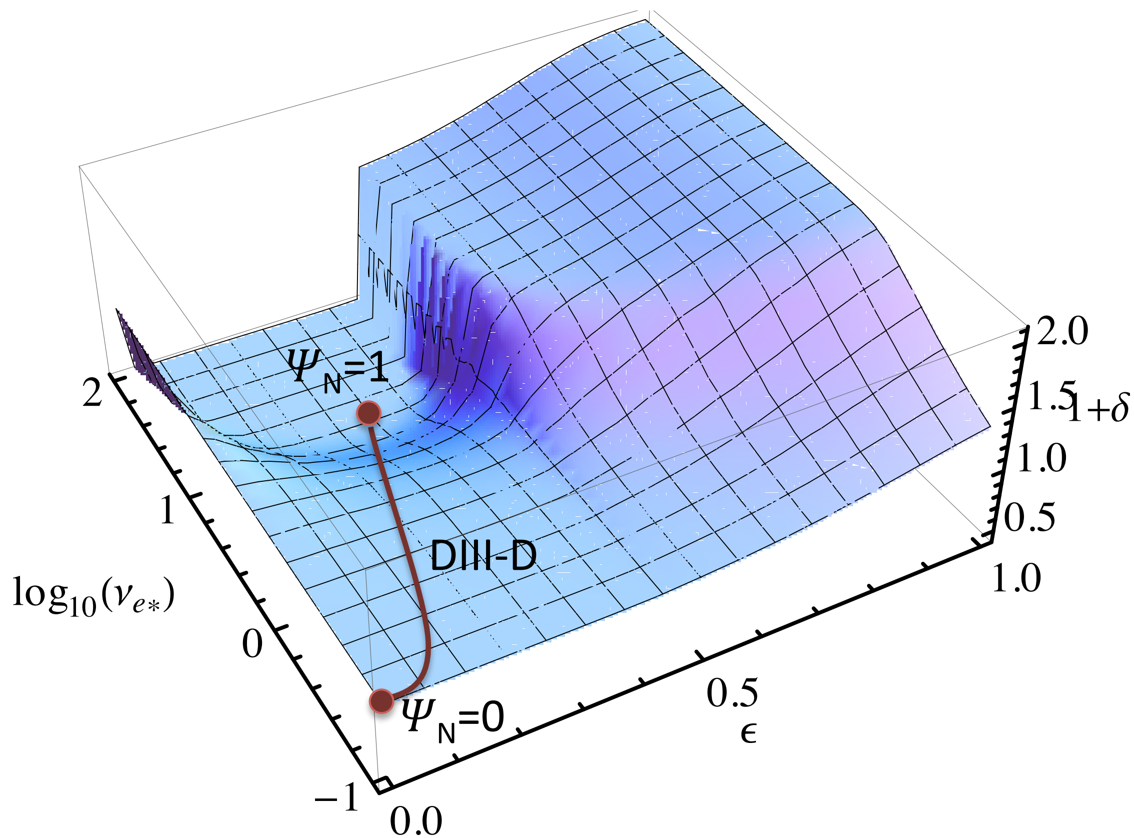
Jump hard to hit for practical conditions

- Only unrealistic condition can hit the jump



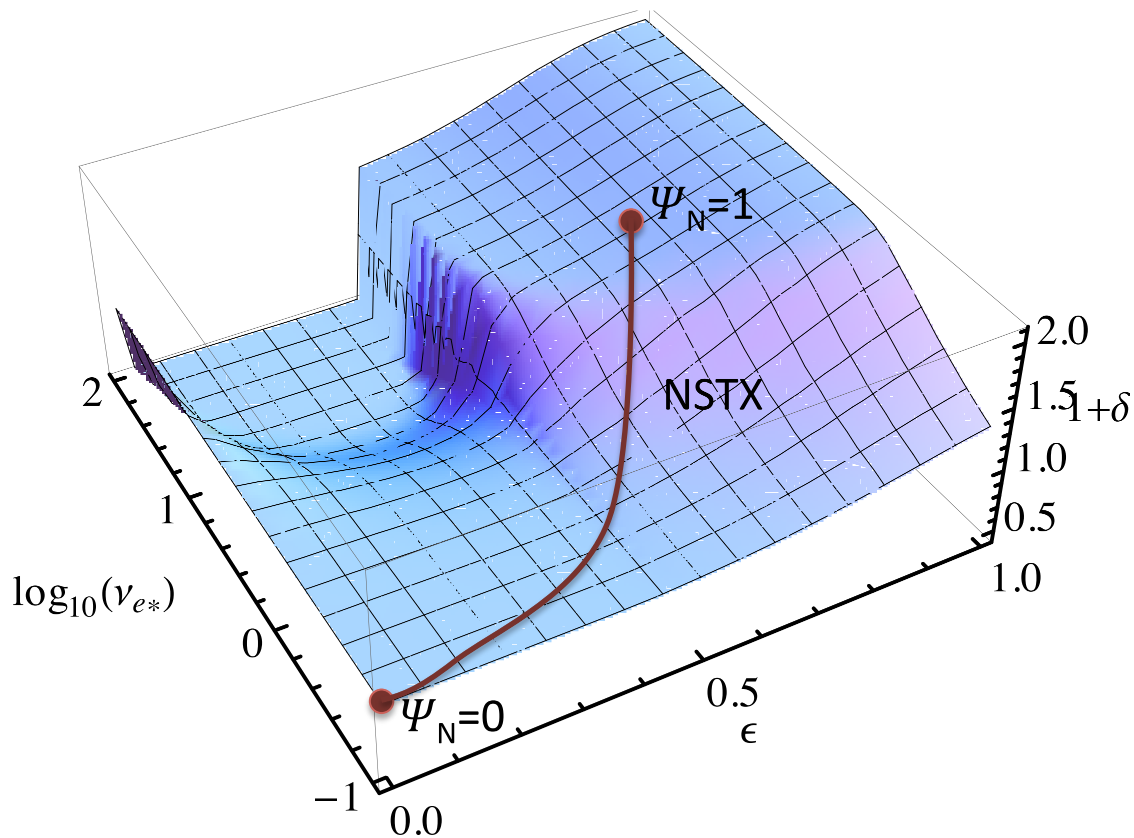
Jump hard to hit for practical conditions

- In real experiment, everything varies with Ψ_N: aspect ratio, collisionality, pressure…
- Koh's very sharp formula is not for unrealistic situations
- Realistic parameters give milder transition results

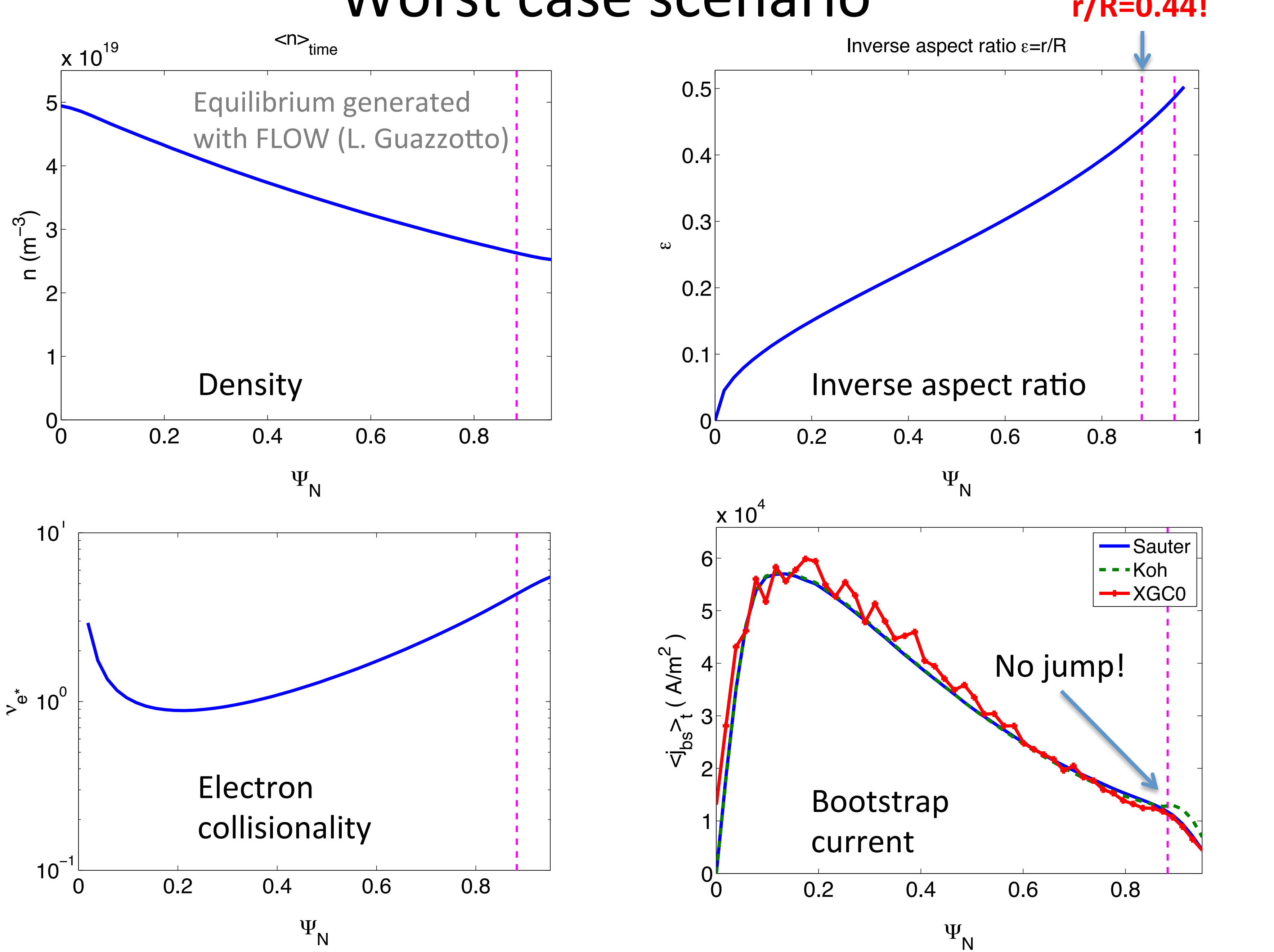


Jump transition is hard to hit for practical conditions

- In real experiment, everything varies with Ψ_N: aspect ratio, collisionality, pressure…
- Koh's very sharp formula is not for unrealistic situations
- Realistic parameters give milder transition results.
- We can improve the transition formula to be not so jumpy.



Worst case scenario



Simple fix for “jump” in formula

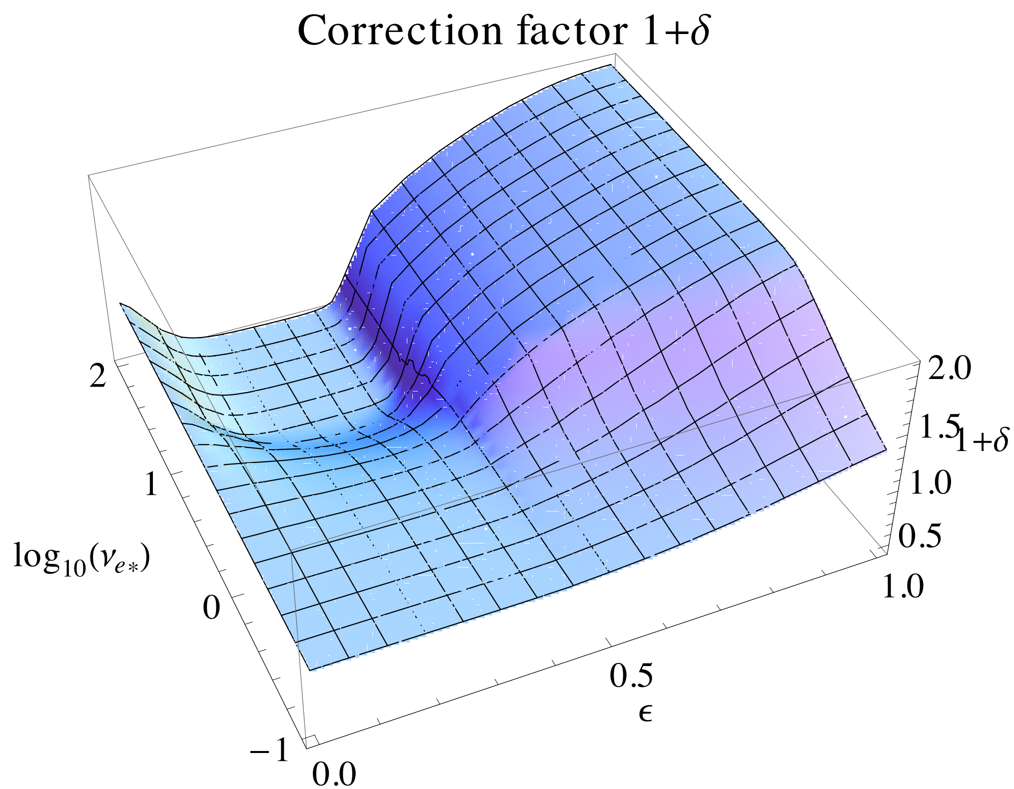
- Width of transition region?
  - → Set minimal width for tanh-function!
  - Ad-hoc improvedment based on XGC0 results:

$$\delta \propto \tanh\left(3.2\beta(\epsilon)\frac{(\epsilon^{3/2}\nu_{eff})^{1.4}}{Z^{\alpha(Z)}}\right)$$

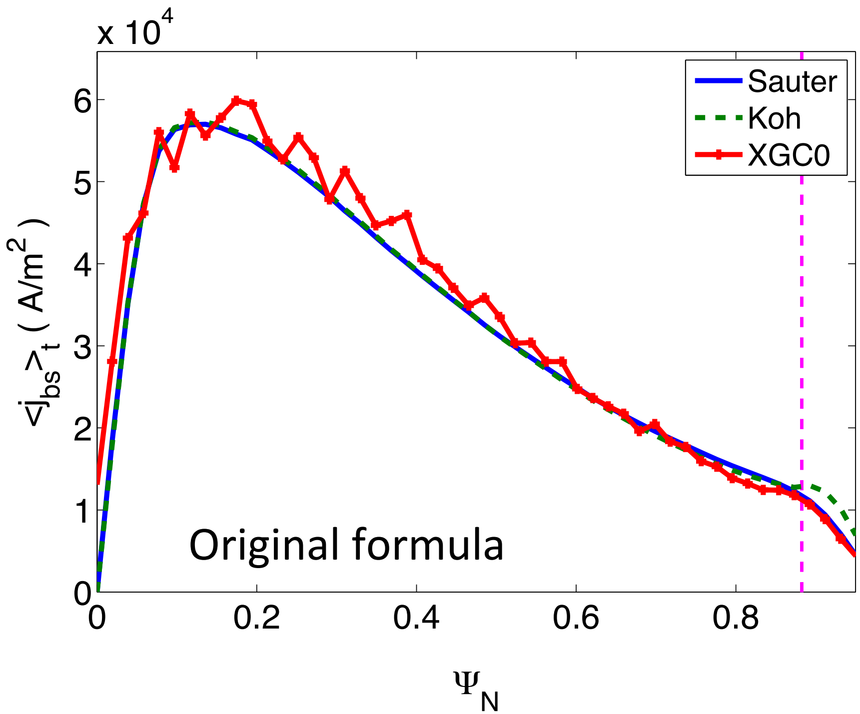
$$\nu_{eff}(\epsilon < 0.44) = \min[\nu_{e*}, 12]$$

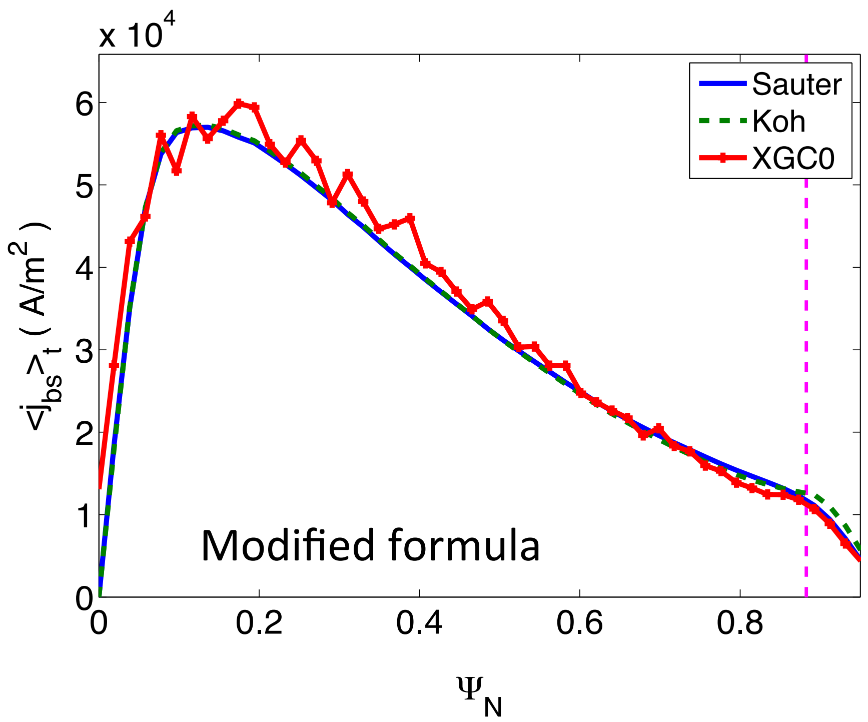
$$\nu_{eff}(\epsilon > 0.44) = \min[\nu_{e*}, 2]$$

- To be included in the new manuscript



Comparison between original and modified formula In the worst case scenario





Summary

- Koh et al's formula optimized for edge pedestal plasma
- Reproduces XGC0 results with reasonable accuracy in pedestal
- Enhanced bootstrap current in tight aspect ratio tokamaks
  - Field line pitch + collisionality Koh et al., Phys. Plasmas 19, 072505 (2012)
  - Confirmed by XGC0 comparing s-α , concentric circular and Grad-Shafranov circular geometry
  - **Realistic geometry crucial for accurate bootstrap current in pedestal**
- Verification of the collision operator: XGCa with nonlinear FPL collisions queued on Hopper
- **We welcome cross verification with other codes on these results**
  - **BUT: improved treatment of the strong field line pitch variation of a spherical tokamak (appearing as analytic solubility condition) is crucial in 2D configuration space!!!**

Material for discussion

Local approximation used by Sauter and NEO breaks down in edge pedestal

E. Belli, J. Candy, Plasma Phys. Control. Fusion **50** (2008) 095010

2.5. Expansions to second order

Noting that $v_\parallel \sim v_{ti}$, $|\boldsymbol{v}_D| \sim \rho_{*i}\epsilon v_{ti}$ and $|\boldsymbol{v}_E| \sim \rho_{*i} v_{ti} a\, \partial_r(e\Phi/T_{0i})$, where $\rho_{*i} = \rho_i/a$ is the ratio of gyroradius to system size of the primary ions and $\epsilon = r/R_0$ is the inverse aspect ratio, we may work out the usual hierarchy of equations by expanding the full equation as a series in the small parameter ρ_{*i}.

$$f_a = f_{0a} + f_{1a} + f_{2a} + \cdots, \quad (33)$$

$$\Phi = \Phi_0 + \Phi_1 + \Phi_2 + \cdots, \quad (34)$$

$$S_a = S_{2a} + \cdots, \quad (35)$$

where $\Phi_k \sim \mathcal{O}(\rho_{*i}^k)$, etc. For the $E \times B$ drift velocity, we introduce the notation

$$\boldsymbol{v}_E^{(k)} \doteq \frac{c}{B}\boldsymbol{b} \times \nabla\Phi_k \sim \rho_{*i}^{k+1}. \quad (36)$$

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E. Belli, J. Candy, Plasma Phys. Control. Fusion **54** (2012) 015015

Plasma Phys. Control. Fusion **54** (2012) 015015

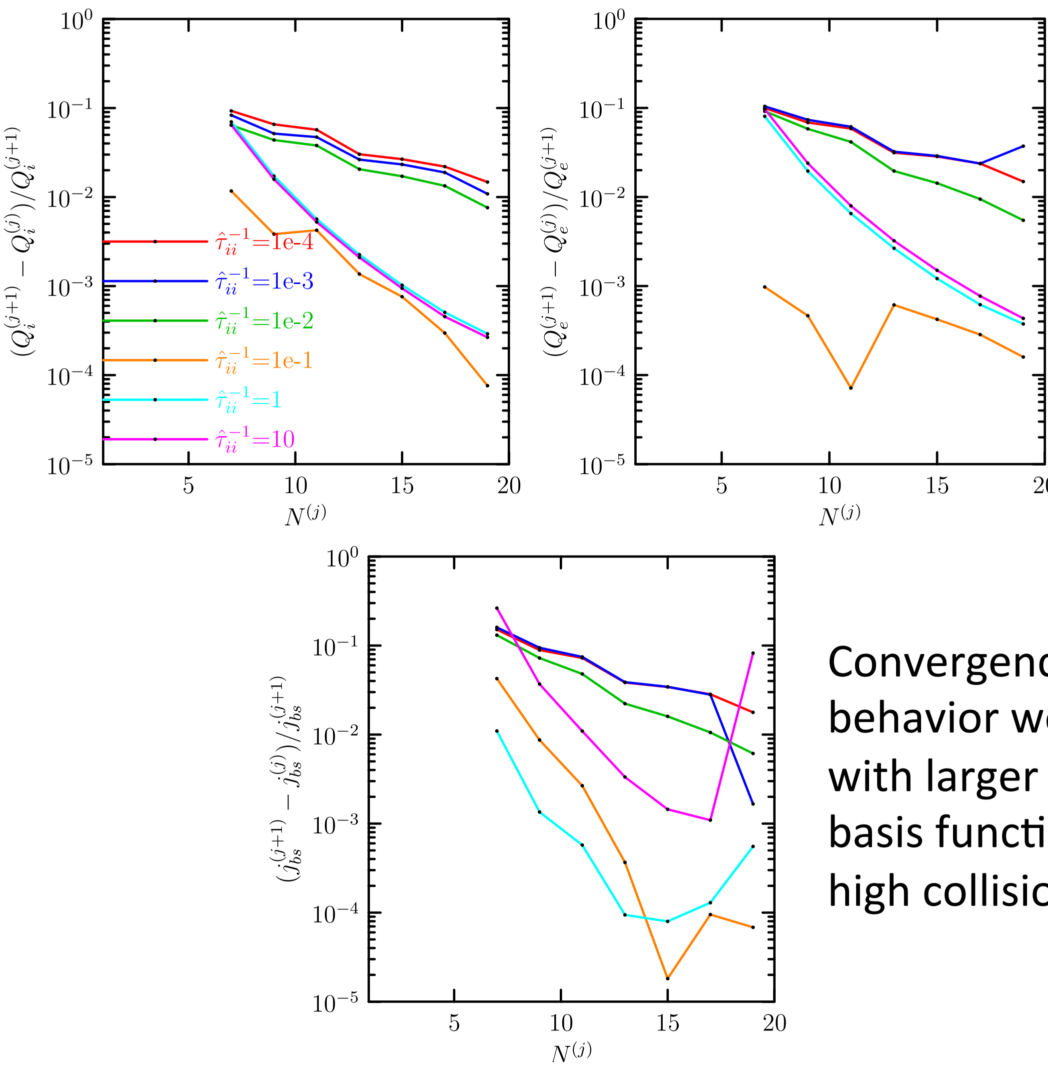
by just the term proportional to $W_a^{(1)}$, which reduces to the usual drift velocity.

We note that the hierarchal expansion in $\rho_{*\mathrm{i}}$ assumed here is valid throughout most of the core of the plasma. If the equilibrium-scale temperature and density gradient factors or the rotational shear becomes large, such that ρ_a/L_{1a}, ρ_a/L_{2a} and/or ρ_a/L_{3a} are larger than $\mathcal{O}(\rho_{*\mathrm{i}})$, e.g. in the H-mode pedestal, then the perturbative formalism breaks-down and is not valid, as physically the transport becomes non-local.

Tight aspect ratio magnetic geometry effect: Solubility condition and convergence of bootstrap current in NEO

Plasma Phys. Control. Fusion **54** (2012) 015015 E A Belli and J Candy

E. Belli, J. Candy, Plasma Phys. Control. Fusion **54** (2012) 015015



Convergence behavior worsens with larger set of basis function at high collisionality?

Figure 5. Convergence of the ion energy flux, electron energy flux and bootstrap current with number of grid points for a case with kinetic electrons and using the full Fokker–Planck collision operator implemented with the Laguerre-(1/2+3/2) method for the energy basis.

Disagreement with NEO unlikely due to collision operator?

Deviations between NEO's collision operator and simplified operator only for extremely high collisionality!

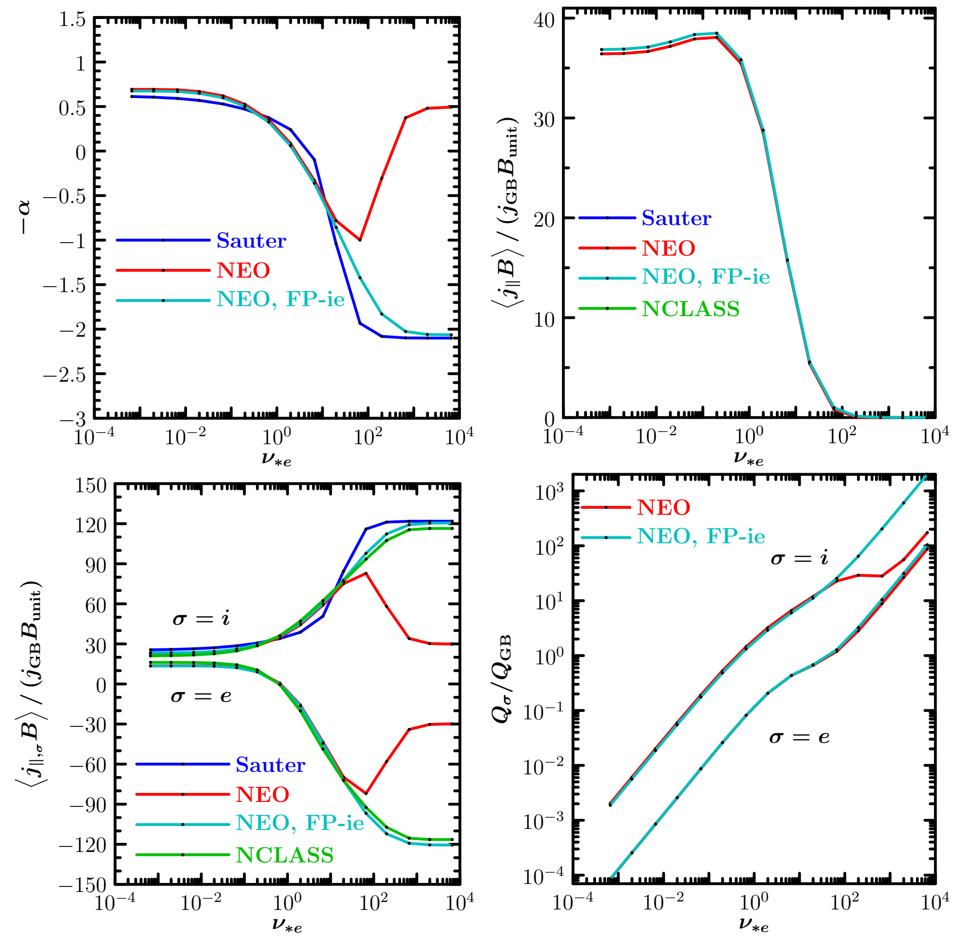
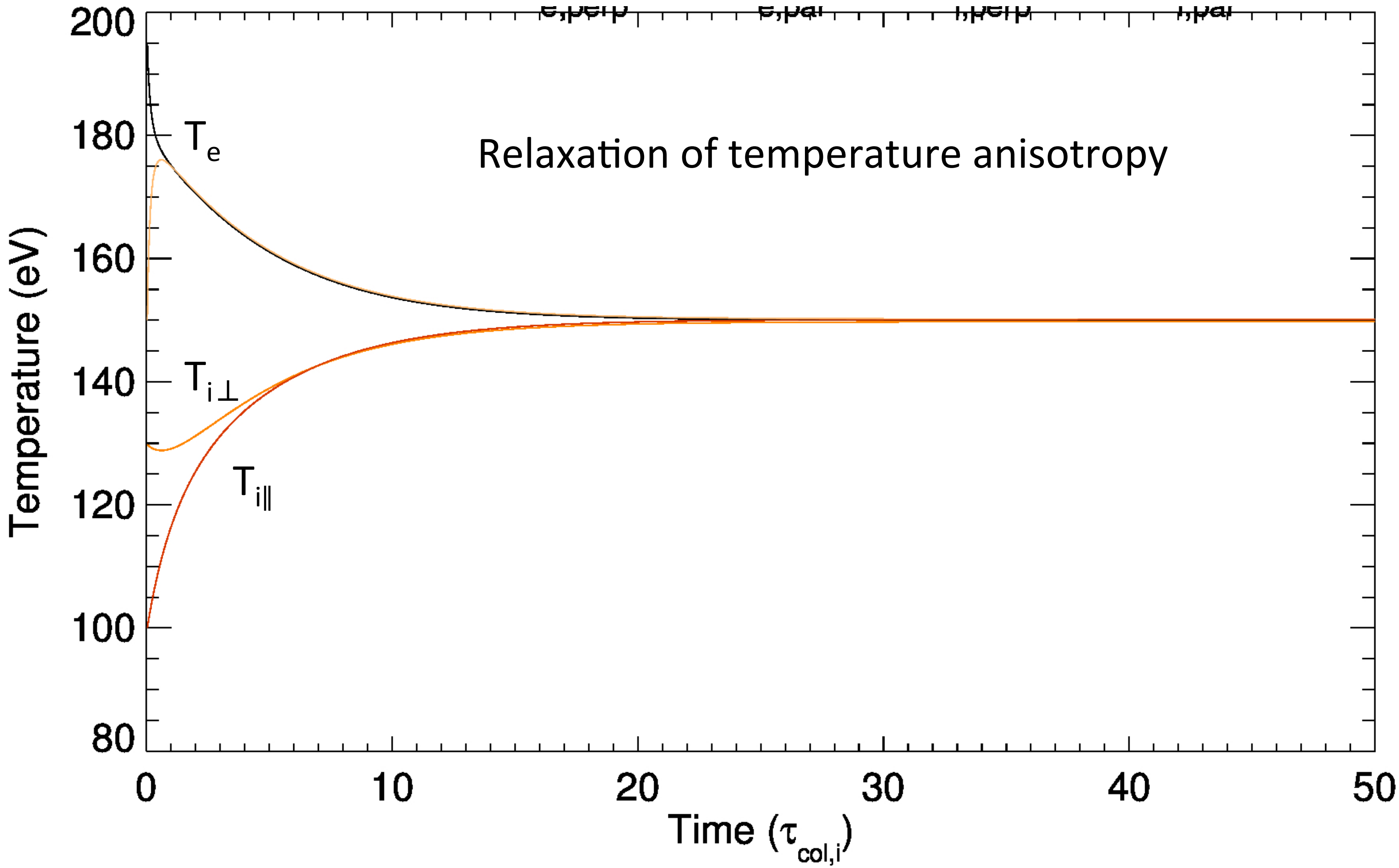


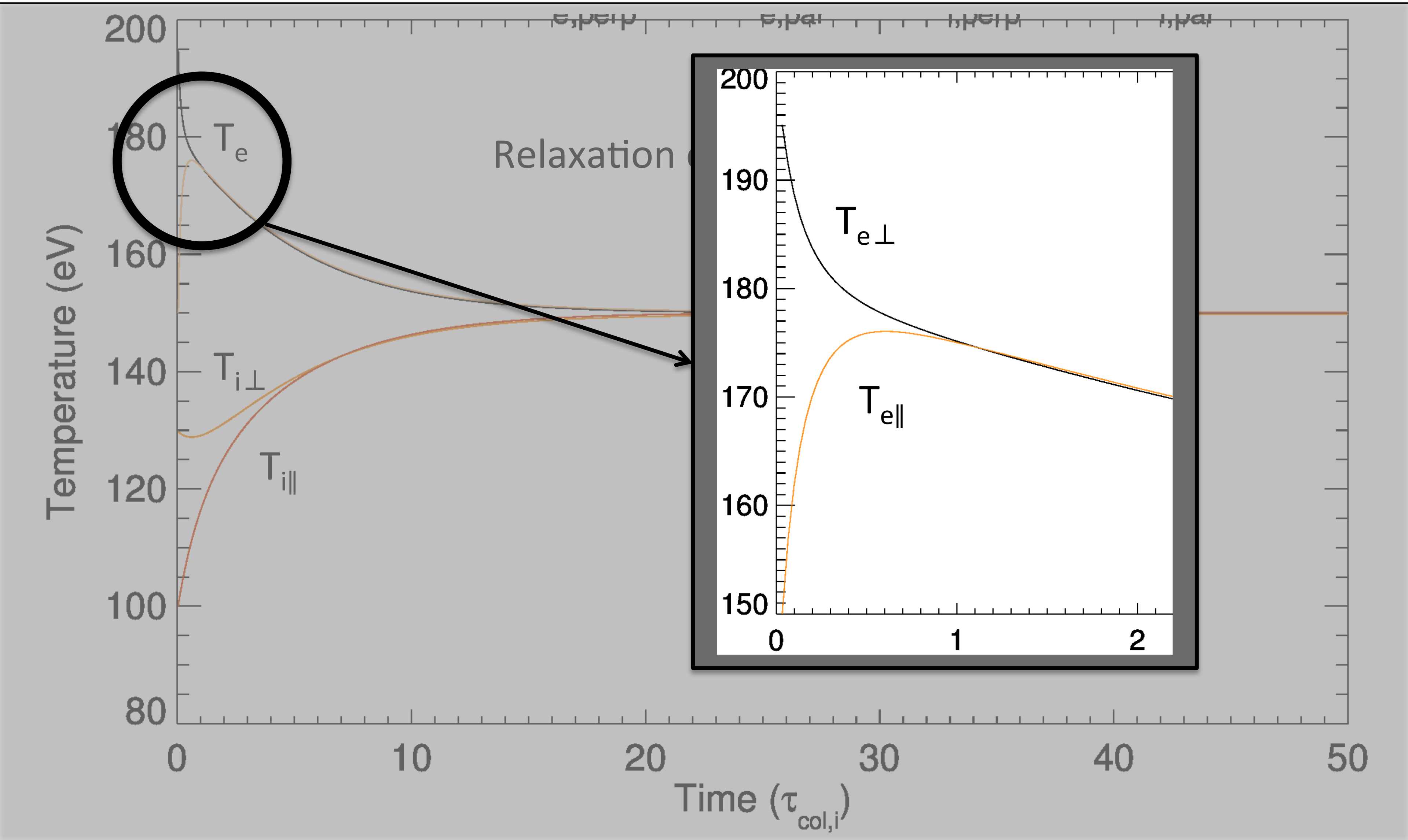
Figure 8. Ion flow coefficient, bootstrap current, and ion and electron parallel currents and energy fluxes versus electron effective dimensionless collision frequency ν_{*e} for the GA standard case, comparing the NEO simulation results with full (NEO) and reduced (FP-ie) ion-electron collisional coupling. NEO is the only simulation code with the full Fokker-Planck collision operator and complete cross-species collisional coupling, which allows for accurate simulations at high collisionality.

E A Belli, J Candy, O Meneghini and T H Osborne, submitted for review to NSTX group, 2013

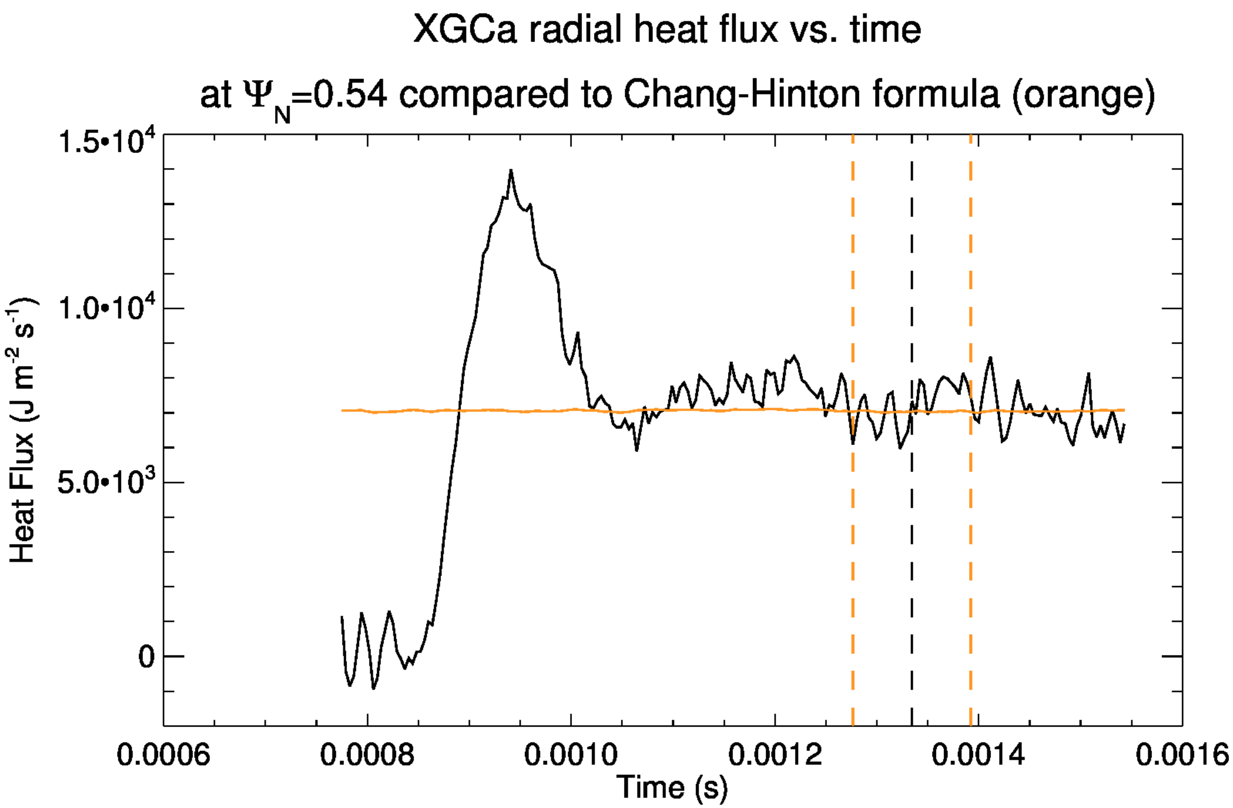
Nonlinear Fokker-Planck-Landau collisions (E. Yoon) with XGC1/a



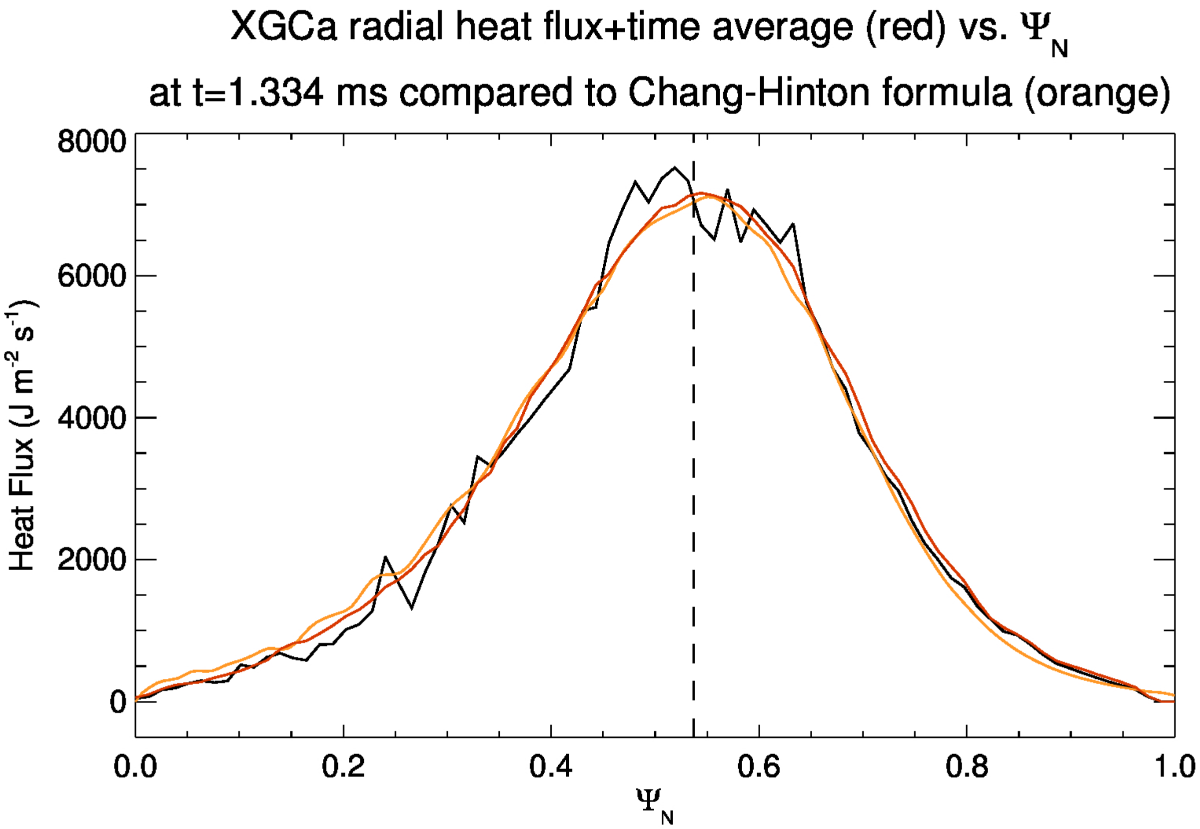
Nonlinear Fokker-Planck-Landau collisions (E. Yoon) with XGC1/a



Nonlinear Fokker-Planck-Landau collisions (E. Yoon) with XGC1/a



Nonlinear Fokker-Planck-Landau collisions (E. Yoon) with XGC1/a



Differences between NEO and XGC0

NEO

- Time-independent continuum code
- 4D phase space
- δf-code
- Local approximation $a/\rho \ll 1$
- Solution: Laguerre and Legendre polynomials → Choice of basis?
- Uses solubility condition for toroidal particle dynamics info.
- Full linearized Fokker-Planck (relevant only for $\nu_{e*} \gtrsim 100$, E. Belli)

XGC0

- Time-dependent particle code
- 5D phase space
- Full-f code
- Global (including SOL)
- Fully numerical particle simulation: no approximate solubility condition
- Solution is statistical sampling of phase space
- Simplified linearized Fokker-Planck