Progress of the non-Maxwellian extension of the full-wave TORIC v.5 code in the high harmonic and minority heating regimes

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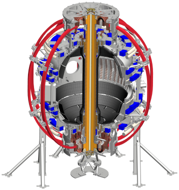

Outline

- Motivation
- TORIC v.5: brief code description
- Non-Maxwellian extension of TORIC v.5 in HHFW heating regime
  - Test I: Numerical vs. analytical Maxwellian full hot dielectric tensor
  - Test II: TORIC wave solution: numerical vs. analytical Maxw. case
  - P2F code: from a particles list to a continuum distribution function
  - Test I: TORIC wave solution: particle list + P2F for a Maxw. case
- Initial applications
  - Bi-Maxwellian distribution
  - Slowing-down distribution
  - from a NUBEAM particles list (preliminary & still in progress)
- Conclusions
- Future steps

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Motivation

- Experiments show that the interactions between fast waves and fast ions can be so strong to significantly modify the fast ion population from neutral beam injection (NBI)
  - The distribution function modifications will, generally, result in finite changes in the amount and spatial location of absorption
  - In NSTX, fast waves (FWs) can modify and, under certain conditions, even suppress the energetic particle driven instabilities, such as toroidal Alfvén eigenmodes (TAEs) and global Alfvén eigenmodes (GAEs) and fishbones See Fredrickson et al NF 2015

- Similarly, the non-Maxwellian effects play an important role in the interaction between FWs and ion minority species in the IC minority heating scheme

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TORIC v.5 code

- The TORIC v.5 code solves the wave equation for the electric field $\mathbf{E}$:

$$\nabla \times \nabla \times \mathbf{E} - \frac{\omega^2}{c^2}\boldsymbol{\varepsilon} \cdot \mathbf{E} = 4\pi i \frac{\omega}{c^2}\mathbf{J}^A$$

  - TORIC-HHFW: High Harmonic Fast Wave regime
    - Full hot-plasma dielectric tensor employed
    - The $k^2_\perp$ value in the argument of the Bessel functions is obtained by solving the local dispersion relation for FWs
  - TORIC: IC minority regime
    - FLR corrections only up to the $\omega = 2\omega_{ci}$
    - Non-Maxw. extension completed and tested but not shown here

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- TORIC v.5 uses a Maxwellian plasma dielectric tensor

$$\boldsymbol{\varepsilon} \equiv \mathbf{I} + \frac{4\pi i}{\omega} \boldsymbol{\sigma} = \mathbf{I} + \boldsymbol{\chi}$$

▸ More TORIC info

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Extra slides TORIC-IC

Standard "Maxwellian" procedure to run TORIC v.5

$$\boldsymbol{\varepsilon} \equiv \mathbf{I} + \frac{4\pi i}{\omega}\boldsymbol{\sigma} = \mathbf{I} + \boldsymbol{\chi}$$

Susceptibility tensor $\boldsymbol{\chi}[f_0(\mathbf{x};\mathbf{v})]$, is a functional of f_0, which, in general, is **non-Maxwellian**

INPUT:

- Density & Temp. for each species
- Magnetic equilibrium

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- $\boldsymbol{\chi}\,(f = f_{\mathrm{Maxw.}}) \Longleftrightarrow$ Analytical expression
- Thermal species $\Longrightarrow$ NSTX-U data
- Non-thermal species (fast ions) $\Longrightarrow$ NUBEAM

$$T_{\mathrm{FI}} = \frac{2}{3}\frac{E}{n_{\mathrm{FI}}}(\text{effective temperature})$$

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Fast ions energy

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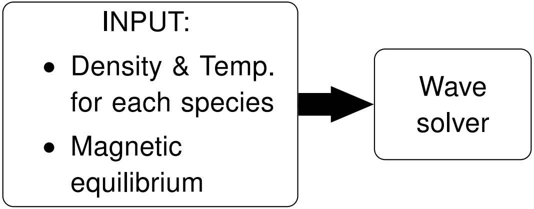

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INPUT:

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→ Wave solver →

OUTPUT:

- Wave electric field
- Pow. density profiles for each species
- Total absorbed pow. for each species

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The susceptibility for a hot plasma with an arbitrary distribution function (Eq. 48 in Stix's book page 255)

Local coordinate frame $(\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}})$ with $\hat{\mathbf{z}} = \hat{\mathbf{b}}$ and $\mathbf{k} \cdot \hat{\mathbf{y}} = 0$ (Stix)

$$\begin{aligned} \boldsymbol{\chi}_{\mathrm{s}} &= \frac{\omega_{\mathrm{ps}}^2}{\omega} \int_0^{+\infty} 2\pi v_\perp \mathrm{d}v_\perp \int_{-\infty}^{+\infty} \mathrm{d}v_\parallel \hat{\mathbf{z}}\hat{\mathbf{z}} \frac{v_\parallel^2}{\omega} \left(\frac{1}{v_\parallel} \frac{\partial f}{\partial v_\parallel} - \frac{1}{v_\perp} \frac{\partial f}{\partial v_\perp} \right)_{\mathrm{s}} + \\ &+ \frac{\omega_{\mathrm{ps}}^2}{\omega} \int_0^{+\infty} 2\pi v_\perp \mathrm{d}v_\perp \int_{-\infty}^{+\infty} \mathrm{d}v_\parallel \sum_{n=-\infty}^{+\infty} \left[\frac{v_\perp U}{\omega - k_\parallel v_\parallel - n\Omega_{\mathrm{cs}}} \mathbf{T}_n \right] \end{aligned}$$

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$$U \equiv \frac{\partial f}{\partial v_\perp} + \frac{k_\parallel}{\omega}\left(v_\perp \frac{\partial f}{\partial v_\parallel} - v_\parallel \frac{\partial f}{\partial v_\perp} \right) \qquad \text{and}$$

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and

$$\mathbf{T}_n = \begin{pmatrix} \frac{n^2 J_n^2(z)}{z^2} & \frac{i n J_n(z) J_n'(z)}{z} & \frac{n J_n^2(z) v_\parallel}{z v_\perp} \\ -\frac{i n J_n(z) J_n'(z)}{z} & (J_n'(z))^2 & -\frac{i J_n(z) J_n'(z) v_\parallel}{v_\perp} \\ \frac{n J_n^2(z) v_\parallel}{z v_\perp} & \frac{i J_n(z) J_n'(z) v_\parallel}{v_\perp} & \frac{J_n^2(z) v_\parallel^2}{v_\perp^2} \end{pmatrix}, \quad z \equiv \frac{k_\perp v_\perp}{\Omega_{\mathrm{cs}}}$$

Numerical evaluation of χ needed for arbitrary distribution function: χ is pre-computed

- The “best” approach for a complete extension of the code is to implement directly the general expression for $\boldsymbol{\chi}$ (previous slide)
  - Plemelj’s formula $\rightarrow \frac{1}{\omega-\omega_0 \pm i0} = \wp\frac{1}{\omega-\omega_0} \mp i\pi\delta(\omega-\omega_0)$

- Integrals in the expression for $\boldsymbol{\chi}$ are computed numerous times in `TORIC-HHFW` so **an efficient evaluation is essential**

- Precomputation of χ:
  - A set of N_ψ files is constructed, each containing the principal values and residues of χ for a single species on a uniform $(v_\parallel, B/B_{\min}, N_\perp)$ mesh, for a specified flux surface ψ_j
  - The distribution, $f(v_\parallel, v_\perp)$, is specified in functional form at the minimum field strength point $B(\theta) = B_{\min}$ on ψ_j
  - An interpolator returns the components of χ

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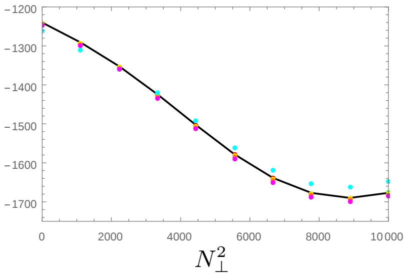
Good agreement between numerical and analytical evaluation of the full hot dielectric tensor

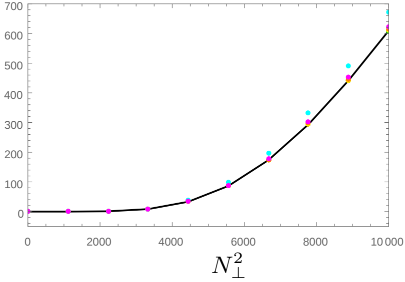
	$N_{v_\parallel}$	$N_{v_\perp}$
● (cyan)	100	50
● (magenta)	200	100
● (orange)	324	150
● (yellow)	650	300
● (red)	1300	600
● (green)	2600	1200

Parameters:
$f = 30 \times 10^6$ Hz; $n_{\mathrm{dens}} = 5 \times 10^{13}$ cm^{-3},
$N_\parallel = 10$, $B = 0.5$ T, $T_i = 20$ keV
$N_{\mathrm{harmonics}} = 10$
Ion species: Deuterium
Black curve: analytical solution

Re(ε_{xx})



Im(ε_{xx})



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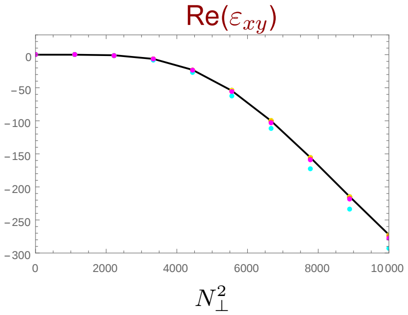

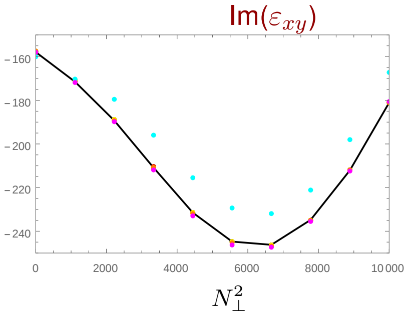

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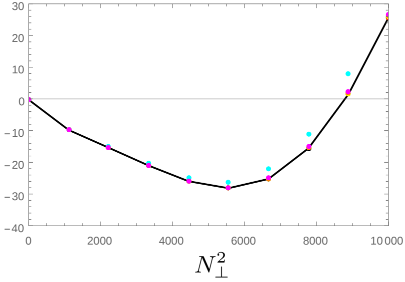
Re(ε_{xz})



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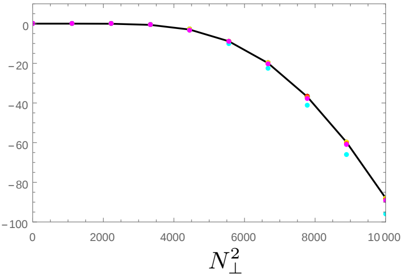

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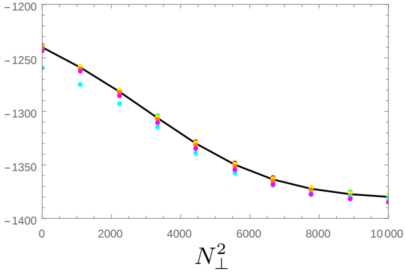
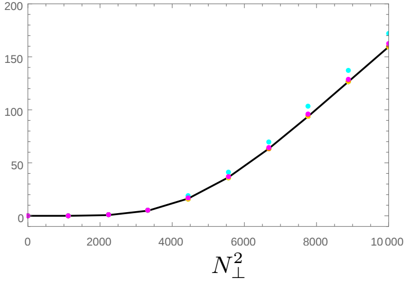


Im(ε_{yy})

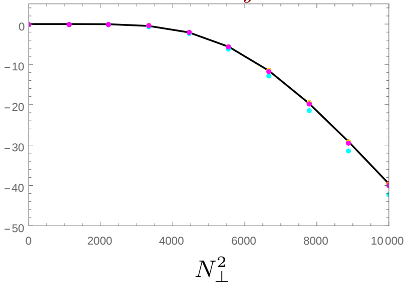


Good agreement between numerical and analytical evaluation of the full hot dielectric tensor

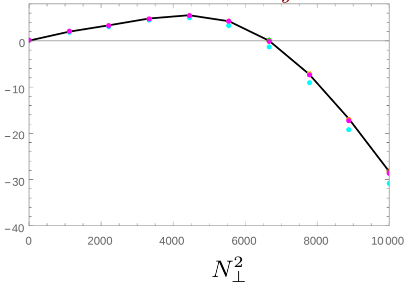
Parameters:
$f = 30 \times 10^6$ Hz; $n_{\mathrm{dens}} = 5 \times 10^{13}$ cm^{-3},
$N_{\|} = 10$, $B = 0.5$ T, $T_i = 20$ keV
$N_{\mathrm{harmonics}} = 10$
Ion species: Deuterium
Black curve: analytical solution

	$N_{v_{\|}}$	$N_{v_{\perp}}$
	100	50
	200	100
	324	150
	650	300
	1300	600
	2600	1200

Re(ε_{yz})



Im(ε_{yz})

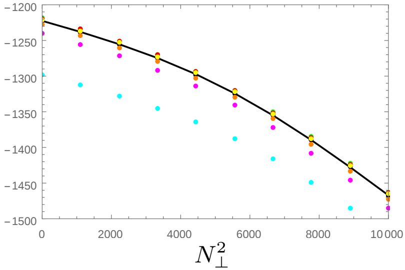


Good agreement between numerical and analytical evaluation of the full hot dielectric tensor

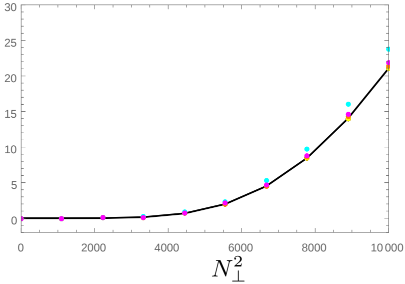
Parameters:
$f = 30 \times 10^6$ Hz; $n_{\mathrm{dens}} = 5 \times 10^{13}$ cm^{-3},
$N_{\parallel} = 10$, $B = 0.5$ T, $T_i = 20$ keV
$N_{\mathrm{harmonics}} = 10$
Ion species: Deuterium
Black curve: analytical solution

	$N_{v_{\parallel}}$	$N_{v_{\perp}}$
● (cyan)	100	50
● (magenta)	200	100
● (orange)	324	150
● (yellow)	650	300
● (red)	1300	600
● (green)	2600	1200

Re(ε_{zz})



Im(ε_{zz})



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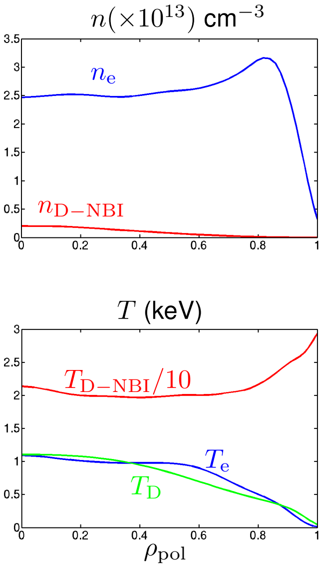
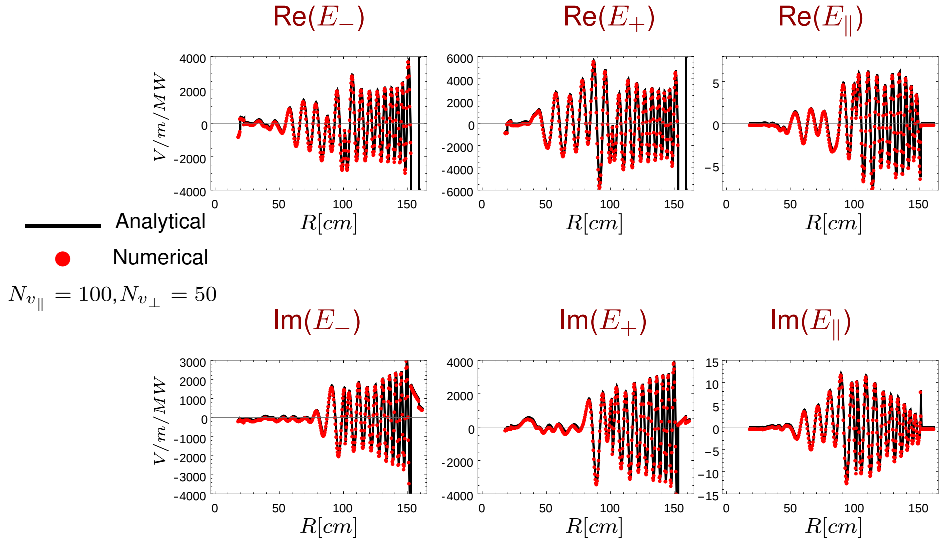
NSTX case

Main parameters:

- TRANSP Run ID: 134909B01
- Plasma species: electron, D, D-NBI
- $B_{\rm T} = 0.53$ T
- $I_{\rm p} = 868$ kA
- $T_{\rm e}(0) = 1.09$ keV
- $n_{\rm e}(0) = 2.47 \times 10^{13}$ cm^{-3}
- $T_{\rm D}(0) = 1.1$ keV
- $T_{\rm D-NBI}(0) = 21.37$ keV
- $n_{\rm D-NBI}(0) = 2.01 \times 10^{12}$ cm^{-3}
- `TORIC` resolution: $n_{\rm mod} = 31$, $n_{\rm elm} = 200$



Excellent agreement between numerical and analytical evaluation of HHFW fields in the midplane

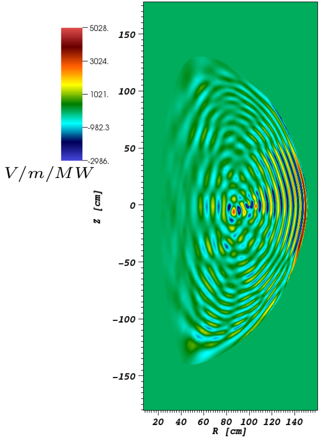


Excellent agreement between numerical and analytical evaluation of the 2D HHFW fields

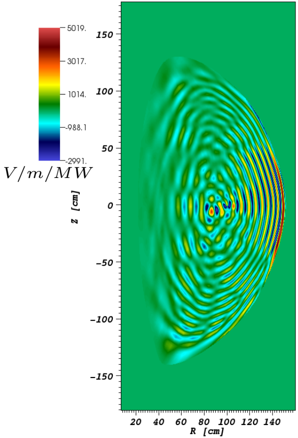
TORIC resolution: $n_{\rm mod} = 31$, $n_{\rm elm} = 200$

Resolution used for χ: $N_{v_{||}} = 100$ and $N_{v_\perp} = 50$

Maxw. analytical: Re(E_-)



Maxw. numerical: Re(E_-)

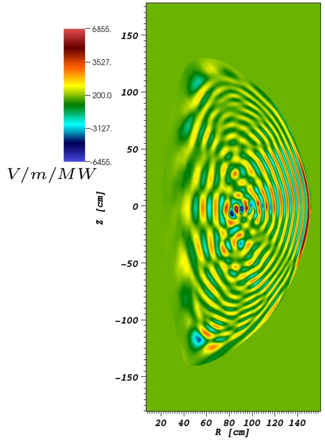


Excellent agreement between numerical and analytical evaluation of the 2D HHFW fields

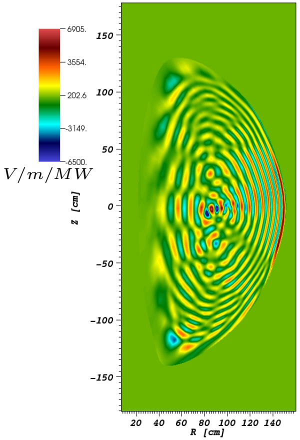
TORIC resolution: $n_{\mathrm{mod}} = 31$, $n_{\mathrm{elm}} = 200$

Resolution used for χ: $N_{v_{\|}} = 100$ and $N_{v_{\perp}} = 50$

Maxw. analytical: Re(E_+)



Maxw. numerical: Re(E_+)



Excellent agreement between numerical and analytical evaluation of the 2D HHFW fields

TORIC resolution: $n_{\mathrm{mod}} = 31$, $n_{\mathrm{elm}} = 200$

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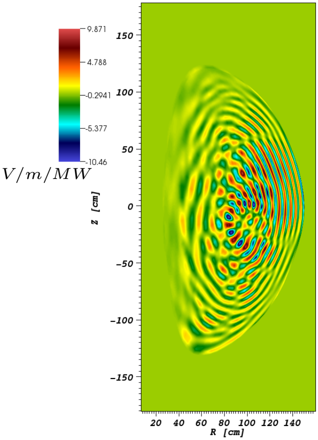
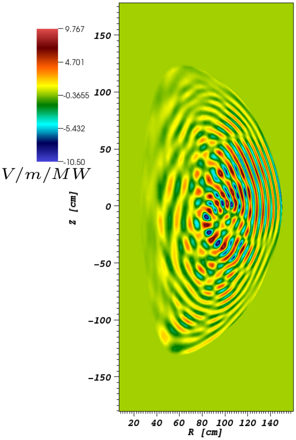
Maxw. analytical: Re($E_{\|}$)



Maxw. numerical: Re($E_{\|}$)



Excellent agreement in terms of absorbed power

TORIC resolution: $n_{\mathrm{mod}} = 31$, $n_{\mathrm{elm}} = 200$
Resolution used for χ: $N_{v_{\parallel}} = 100$ and $N_{v_{\perp}} = 50$

Absorbed fraction	Maxw. analytical	Maxw. numerical
D		
D-NBI		
Electrons		

Excellent agreement in terms of absorbed power

TORIC resolution: $n_{\mathrm{mod}} = 31$, $n_{\mathrm{elm}} = 200$
Resolution used for χ: $N_{v_\parallel} = 100$ and $N_{v_\perp} = 50$

Absorbed fraction	Maxw. analytical	Maxw. numerical
D	0.22 %	
D-NBI		
Electrons		

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Absorbed fraction	Maxw. analytical	Maxw. numerical
D	0.22 %	0.22 %
D-NBI		
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Absorbed fraction	Maxw. analytical	Maxw. numerical
D	0.22 %	0.22 %
D-NBI	73.88 %	
Electrons		

Excellent agreement in terms of absorbed power

TORIC resolution: $n_{\mathrm{mod}} = 31$, $n_{\mathrm{elm}} = 200$
Resolution used for χ: $N_{v_\parallel} = 100$ and $N_{v_\perp} = 50$

Absorbed fraction	Maxw. analytical	Maxw. numerical
D	0.22 %	0.22 %
D-NBI	73.88 %	73.58 %
Electrons		

Excellent agreement in terms of absorbed power

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Resolution used for χ: $N_{v_\parallel} = 100$ and $N_{v_\perp} = 50$

Absorbed fraction	Maxw. analytical	Maxw. numerical
D	0.22 %	0.22 %
D-NBI	73.88 %	73.58 %
Electrons	25.90 %	

Excellent agreement in terms of absorbed power

TORIC resolution: $n_{\mathrm{mod}} = 31$, $n_{\mathrm{elm}} = 200$
Resolution used for χ: $N_{v_\parallel} = 100$ and $N_{v_\perp} = 50$

Absorbed fraction	Maxw. analytical	Maxw. numerical
D	0.22 %	0.22 %
D-NBI	73.88 %	73.58 %
Electrons	25.90 %	26.21 %

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P2F code: from a particles list to a distribution function

- P2F code was developed by D. L. Green (ORNL)
- P2F takes a particle list and creates a 4D $(R, z; v_{\|}, v_{\perp})$ distribution function for use in a continuum code like TORIC
- At present it has essentially three modes:
  - The first really is a straight up 4D histogram giving a noisy distribution
  - The second uses Gaussian shape particles in velocity space to give smooth velocity space distributions at each point in space
  - The third is to distribute each particle along its orbit according to the percentage of bounce time giving even better statistics
- Tested P2F code starting with a particles list representing a Maxwellian:
  - Excellent agreement between the input kinetic profiles and the corresponding ones obtained from the distribution generated by P2F code

P2F test

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▸ P2F test

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TORIC + non-Maxw. + P2F code: test on Maxwellian case

Procedure:

1. generate particle list representing a Maxwellian
2. run P2F to obtain a distribution function, f
3. pre-compute χ with f above
4. run TORIC with pre-computed χ
5. compare TORIC run with standard TORIC

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$n_\mathrm{e}(\rho = 0) = 2.5 \times 10^{13}$ cm^{-3}

$n_\mathrm{e}(\rho = 1) = 2.5 \times 10^{12}$ cm^{-3}

$T_\mathrm{e}(\rho = 0) = 1$ keV; $T_\mathrm{e}(\rho = 1) = 0.1$ keV

$n_\mathrm{FI}(\rho = 0) = 2.0 \times 10^{12}$ cm^{-3}

$n_\mathrm{FI}(\rho = 1) = 2.0 \times 10^{11}$ cm^{-3}

$T_\mathrm{FI}(\rho = 1) = 20$ keV; $T_\mathrm{e}(\rho = 1) = 5$ keV

Parabolic profiles for n_e, T_e, and n_FI

$$T_\mathrm{FI}(\rho) = (T_{\mathrm{FI},0} - T_{\mathrm{FI},1}) \left(1 - \rho^2\right)^5 + T_{\mathrm{FI},1}$$

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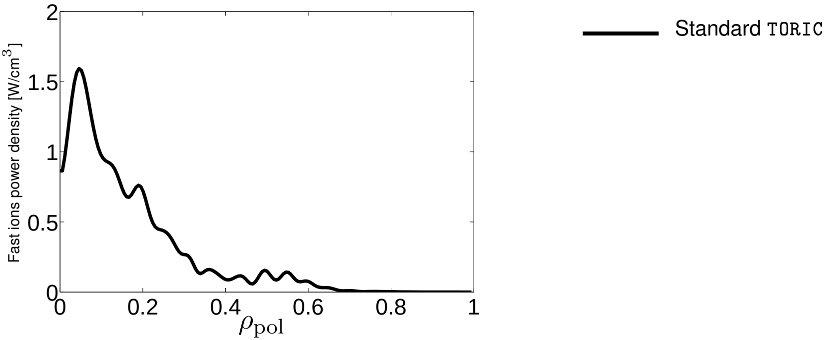

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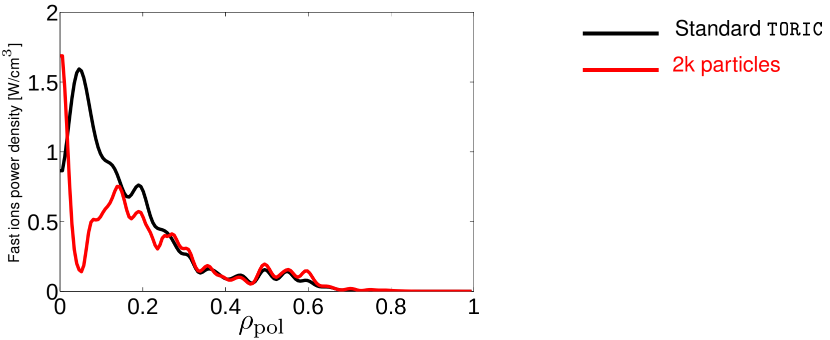

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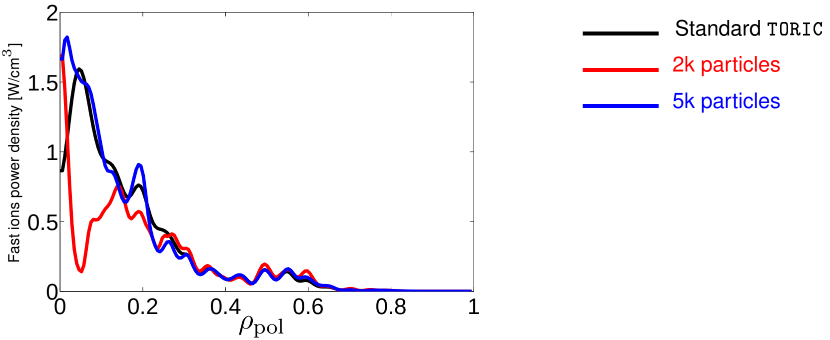

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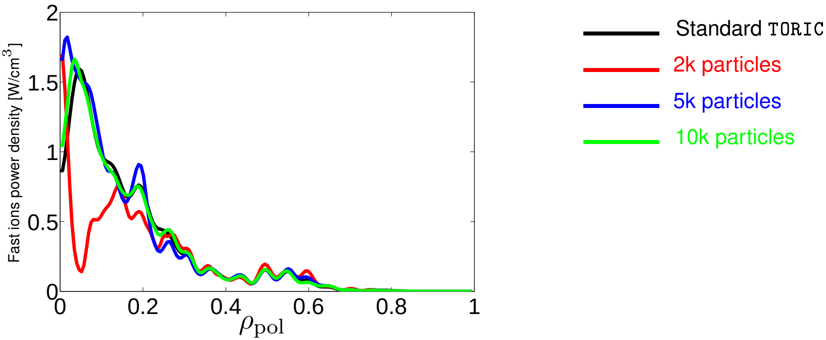

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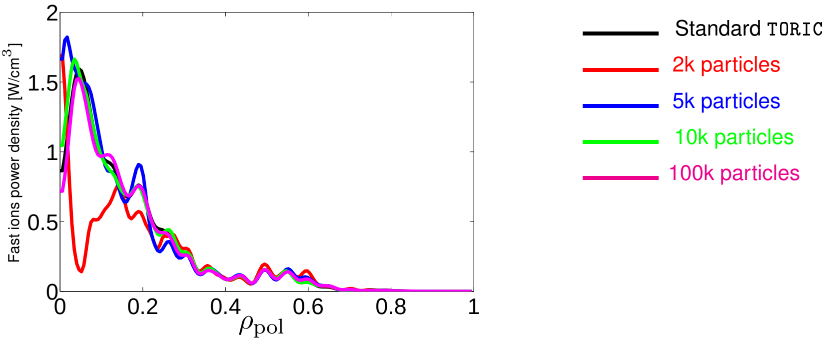

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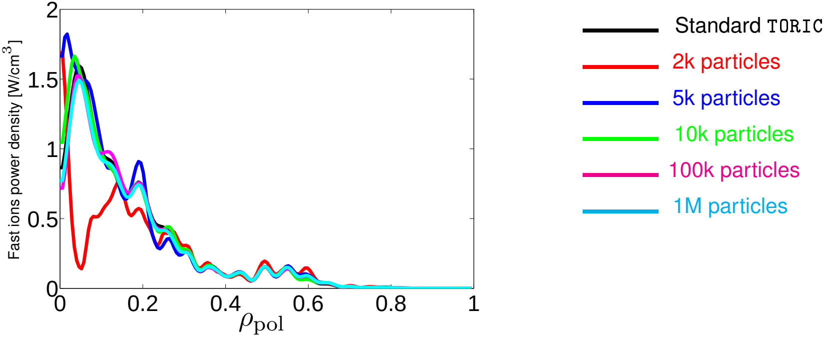

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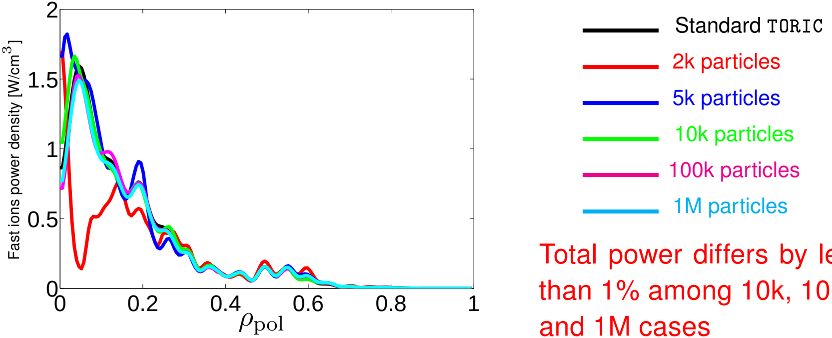

Total power differs by less than 1% among 10k, 100k, and 1M cases

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Bi-Maxwellian distribution

$$f_{\mathrm{D}}(v_{\parallel}, v_{\perp}) = (2\pi)^{-3/2}(v_{\mathrm{th},\parallel} v_{\mathrm{th},\perp}^2)^{-1} \exp[-(v_{\parallel}/v_{\mathrm{th},\parallel})^2 - (v_{\perp}/v_{\mathrm{th},\perp})^2]$$

with $v_{\mathrm{th},\parallel} = \sqrt{2C_{\parallel}T(\psi)/m_{\mathrm{D}}}$, $v_{\mathrm{th},\perp} = \sqrt{2C_{\perp}T(\psi)/m_{\mathrm{D}}}$, with constants $C_{\parallel}$ and $C_{\perp}$

- For $C_{\perp} = 1$ and $C_{\parallel} = \{.5, 1., 3., 5.\}$, $P_{\mathrm{D-NBI}}$, varied by less than 1%
  - for small $C_{\parallel}$, the absorption profile tends to be localized to the resonant layers

- For $C_{\parallel} = 1$ and $C_{\perp} = \{.5, 1., 3., 5.\}$, the corresponding $P_{\mathrm{D-NBI}} = \{70.06, 73.56, 62.84, 48.48\}$

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$$f_{\mathrm{D}}(v_\|, v_\perp) = (2\pi)^{-3/2}(v_{\mathrm{th},\|}v_{\mathrm{th},\perp}^2)^{-1}\exp[-(v_\|/v_{\mathrm{th},\|})^2 - (v_\perp/v_{\mathrm{th},\perp})^2]$$

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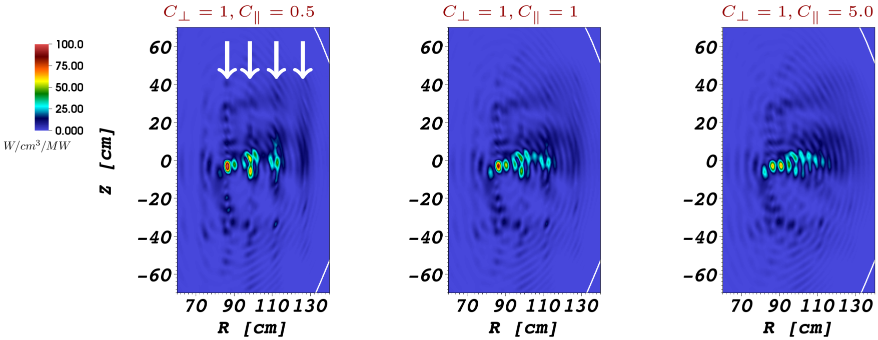

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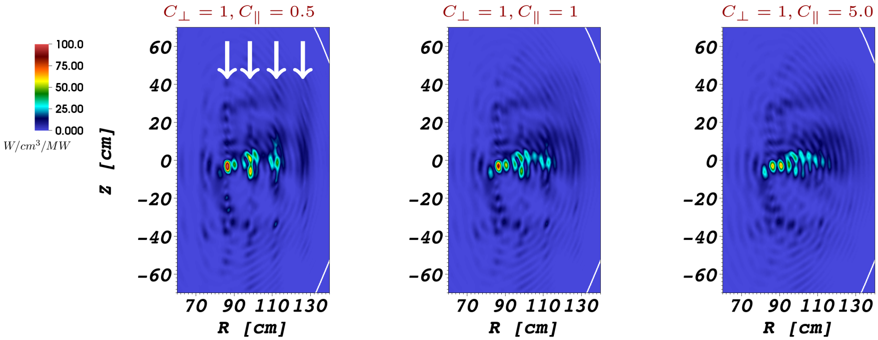

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Slowing-down distribution

$$f_{\rm D}(v_\parallel, v_\perp) = \begin{cases} \frac{A}{v_{\rm c}^3}\frac{1}{1+(v/v_{\rm c})^3} & \text{for } v < v_{\rm m}, \\ 0 & \text{for } v > v_{\rm m} \end{cases} \qquad v_{\rm m} \equiv \sqrt{2E_{\rm D-NBI}/m_{\rm D}}$$

$$A = 3/[4\pi \ln(1+\delta^{-3})], \quad \delta \equiv \frac{v_{\rm c}}{v_{\rm m}}, \quad v_{\rm c} = 3\sqrt{\pi}(m_{\rm e}/m_{\rm D}) Z_{\rm eff} v_{\rm th}^3, \quad Z_{\rm eff} \equiv \sum_{\rm ions} \frac{Z_{\rm i}^2 n_{\rm i}}{n_{\rm e}}$$

For $Z_{\rm eff} = 2$ and $E_{\rm D-NBI} = 30, 60, 90, 120$ keV $\Longrightarrow P_{\rm D-NBI} = \{77.84\%, 75.85\%, 70.97\%, 64.71\%\}$

- Similar behavior when varied $C_\perp$ in the bi-Maxwellian case
- Fast ions absorption should decrease with something like $T_{\rm fast\ ions}^{-3/2}$ (?)

Slowing-down distribution

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- Fast ions absorption should decrease with something like $T_{\rm fast\ ions}^{-3/2}$ (?)

Slowing-down distribution

$$f_D(v_\parallel, v_\perp) = \begin{cases} \frac{A}{v_c^3}\frac{1}{1+(v/v_c)^3} & \text{for } v < v_m, \\ 0 & \text{for } v > v_m \end{cases} \qquad v_m \equiv \sqrt{2E_{D-NBI}/m_D}$$

$$A = 3/[4\pi \ln(1+\delta^{-3})], \quad \delta \equiv \frac{v_c}{v_m}, \quad v_c = 3\sqrt{\pi}(m_e/m_D)Z_{eff}v_{th}^3, \quad Z_{eff} \equiv \sum_{ions}\frac{Z_i^2 n_i}{n_e}$$

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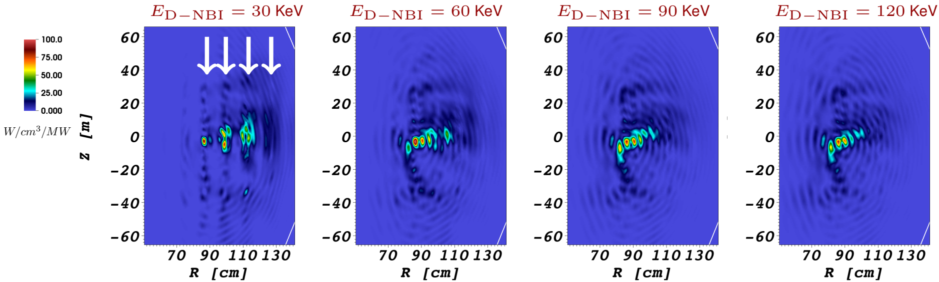

Outline

- Motivation
- TORIC v.5: brief code description
- Non-Maxwellian extension of TORIC v.5 in HHFW heating regime
  - Test I: Numerical vs. analytical Maxwellian full hot dielectric tensor
  - Test II: TORIC wave solution: numerical vs. analytical Maxw. case
  - P2F code: from a particles list to a continuum distribution function
  - Test I: TORIC wave solution: particle list + P2F for a Maxw. case
- Initial applications
  - Bi-Maxwellian distribution
  - Slowing-down distribution
  - from a NUBEAM particles list (preliminary & still in progress)
- Conclusions
- Future steps

NUBEAM particles list (tests in progress)

NSTX shot 117929
$P_{\mathrm{HHFW}} = 2.9$ MW
$P_{\mathrm{NBI}} = 2$ MW
$I_{\mathrm{P}} = 300$ kA
TAE & GAE suppressed
particles number = 3344

- Need to start a case adding low P_{HHFW} and then increase it

NUBEAM particles list (tests in progress)

Procedure:

1. get NUBEAM particle list
2. run P2F to obtain a distribution function, f
3. pre-compute χ with f above
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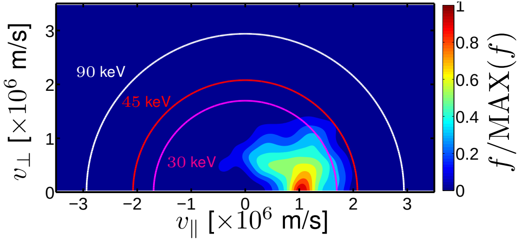
$R \sim 0.9$ m, $Z \sim 0$ m



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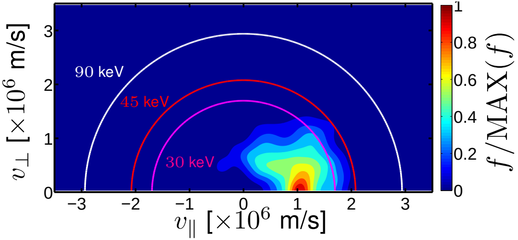

Abs. fraction	f Maxw.	f **non**-Maxw.
Electrons	50.85 %	79.40 %
D-NBI	49.13 %	20.57 %

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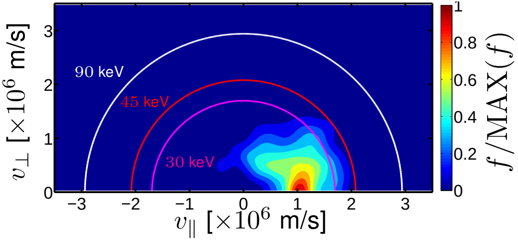

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Conclusions

- A generalization of the full wave `TORIC v.5` code in the high harmonic and minority heating regimes has been implemented to include species with arbitrary velocity distribution functions
- Implementation of the full hot dielectric tensor reproduces the analytic Maxwellian case
- Non-Maxwellian extension of `TORIC` in HHFW regime reproduces previous simulations with both a specified functional form of the distribution functions and a particle list
- For a bi-Maxwellian distribution, the fast ions absorbed power is insensitive to variations in $T_{\|}$, but varies with changes in $T_{\perp}$
- For slowing down distribution, the fast ions absorbed power varies with changes in E_{NBI}
  - $P_{\mathrm{D-NBI}}$ decreases with increasing E_{NBI}
- First attempts to apply `TORIC` generalization with a NUBEAM particle list
  - preliminary results with arbitrary distribution functions appears significantly different than Maxw. case results
  - still additional tests/checks needed

Future steps

For HHFW regime:

- Add options to read a distribution function from CQL3D Fokker-Planck code for HHFW heating regime
  - Use the distribution function from CQL3D to include, for instance, finite orbit effects
  - Possible comparison with FIDA data as done previously by D. Liu & R. Harvey
- Further NSTX/NSTX-U applications/tests using a NUBEAM particle list and comparison with slowing down distribution function
- Attempts to apply this extension in a self-consistent way with the NUBEAM module
  - Need first some tests to the kick-operator implemented in NUBEAM

For IC minority regime (in collaboration with J. Lee, J. Wright, and P. Bonoli)

- the quasilinear diffusion coefficients has been recently derived and implemented in TORIC v.5 (work done by Jungpyo Lee from MIT)
  - Necessary to couple TORIC v.5 and CQL3D
  - Able to iterate the extension of TORIC v.5 with the quasilinear coefficients routine and CQL3D
  - Tests underway on the evaluation of the quasilinear diffusion coefficients

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TORIC code: additional info

- Spectral ansatz $\mathbf{E}(\mathbf{r},t) = \sum_{m,n} \mathbf{E}^{mn}(\psi) e^{i(m\theta + n\phi - \omega t)}$

  $m \rightarrow$ poloidal mode number; $n \rightarrow$ toroidal mode number

- For each toroidal component one has to solve a (formally infinite) system of coupled ordinary differential equations for the physical components of $E^{mn}(\psi)$, written in the local field-aligned orthogonal basis vectors.

- The Spectral Ansatz transforms the θ-integral of the constitutive relation into a convolution over poloidal modes.

- Due to the toroidal axisymmetry, the wave equations are solved separately for each toroidal Fourier component.

- A spectral decomposition defines an accurate representation of the “local" parallel wave-vector $k_{\parallel}^{m} = (m\nabla\theta + n\nabla\phi) \cdot \hat{\mathbf{b}}$

- The ψ variation is represented by Hermite cubic finite elements

- Principal author M. Brambilla (IPP Garching, Germany)

◂ back

The susceptibility for a hot plasma with a Maxwellian distribution function can be evaluated analitically

back

$$\chi_{\mathrm{s}} = \left[\hat{\mathbf{z}}\hat{\mathbf{z}} \frac{2\omega_{\mathrm{p}}^2}{\omega k_\parallel v_{\mathrm{th}}^2} \langle v_\parallel \rangle + \frac{\omega_{\mathrm{p}}^2}{\omega} \sum_{n=-\infty}^{+\infty} e^{-\lambda} \mathbf{Y}_n(\lambda) \right]_s$$

where

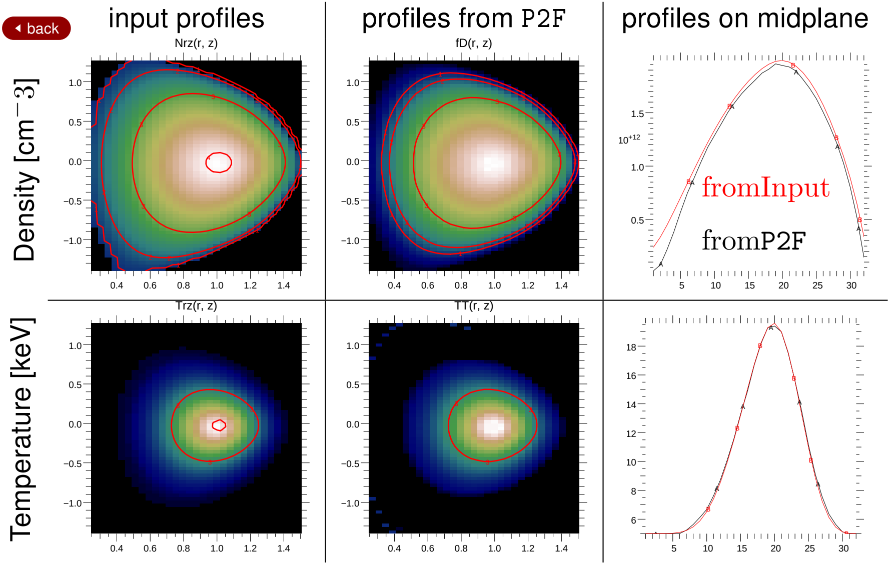
$$\mathbf{Y}_n = \begin{pmatrix} \frac{n^2 I_n}{\lambda} A_n & -in(I_n - I_n')A_n & \frac{k_\perp}{\omega_c}\frac{nI_n}{\lambda} B_n \\ in(I_n - I_n')A_n & \left(\frac{n^2}{\lambda} I_n + 2\lambda I_n - 2\lambda I_n'\right) A_n & \frac{ik_\perp}{\omega_c}(I_n - I_n')B_n \\ \frac{k_\perp}{\omega_c}\frac{nI_n}{\lambda} B_n & -\frac{ik_\perp}{\omega_c}(I_n - I_n')B_n & \frac{2(\omega - n\omega_c)}{k_\parallel v_{\mathrm{th}}^2} I_n B_n \end{pmatrix}$$

$$A_n = \frac{1}{k_\parallel v_{\mathrm{th}}} Z_0(\zeta_n), \quad B_n = \frac{1}{k_\parallel}(1 + \zeta_n Z_0(\zeta_n)), \quad Z_0(\zeta_n) \equiv \text{plasma dispersion func.}$$

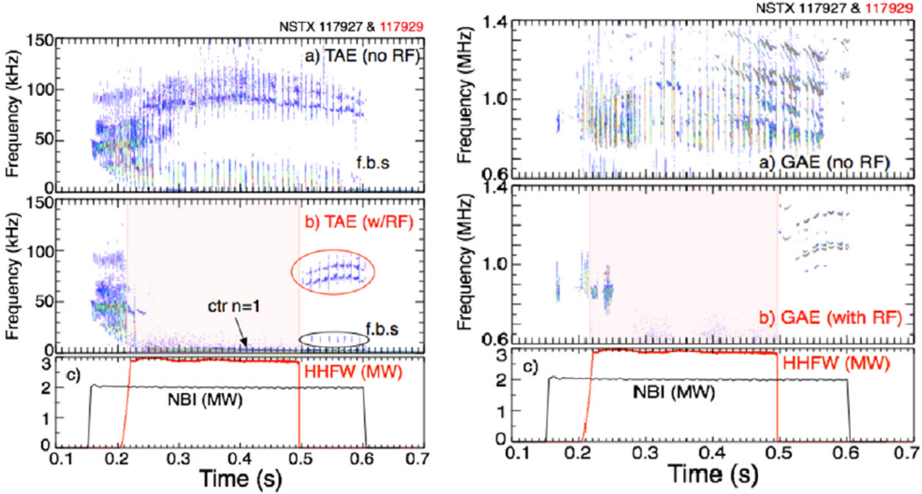
$$\zeta_n \equiv \frac{\omega - n\omega_c}{k_\parallel v_{\mathrm{th}}}, \quad \lambda \equiv \frac{k_\perp^2 v_{\mathrm{th}}^2}{2\Omega_c^2}$$

P2F code: test on Maxwellian case

back



NSTX shot 117929 from Fredrickson et al. NF 2015



back

Finite elements to use to compute the resonant integrals

- We need to evaluate integrals of the form

$$I_k = \int \mathrm{d}v \frac{C(v)}{v - v_k}.$$

- Since I_k is a smooth function of v_k , evaluate on a uniform mesh $v_k = k\Delta v$, and interpolate
- Express smooth integrand $C(v)$ in terms of (linear) finite elements $C(v) = \sum_j c_j T_j$, with T_j centered at v_j

Then

$$I_k = \sum_j \int \mathrm{d}v \frac{c_j T_j}{v - v_k} = \sum_j c_j K_{j-k} = \sum_j c_{j+k} K_j$$

where the kernel is given by

$$K_j = \int_{-1}^{1} \mathrm{d}v \frac{1 - |v|}{v + j\Delta v} = \begin{cases} \ln\left(\frac{j+1}{j-1}\right) - j\ln\left(\frac{j^2}{j^2-1}\right), & |j| > 1, \\ \pm\ln 4, & j = \pm 1, \\ i\pi, & j = 0. \end{cases}$$

Beyond Maxwellian

FLR non-Maxwellian susceptibility in a local coordinate (Stix) frame $(\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}})$, with $\hat{\mathbf{z}} = \hat{\mathbf{b}}$, $\mathbf{k} \cdot \hat{\mathbf{y}} = 0$, to second order in $k_\perp v_\perp/\omega_\mathrm{c}$

$$
\begin{aligned}
\chi_{xx} &= \frac{\omega_{\mathrm{p,s}}^2}{\omega}\left[\frac{1}{2}\left(A_{1,0}+A_{-1,0}\right)-\frac{\lambda}{2}\left(A_{1,1}+A_{-1,1}\right)+\frac{\lambda}{2}\left(A_{2,1}+A_{-2,1}\right)\right] \\
\chi_{xy} &= -\chi_{yx} = i\frac{\omega_{\mathrm{p,s}}^2}{\omega}\left[\frac{1}{2}\left(A_{1,0}-A_{-1,0}\right)-\lambda\left(A_{1,1}-A_{-1,1}\right)+\frac{\lambda}{2}\left(A_{2,1}-A_{-2,1}\right)\right] \\
\chi_{xz} &= +\chi_{zx} = -\chi_{yx} = \frac{\omega_{\mathrm{p,s}}^2}{\omega}\left(\frac{1}{2}\frac{k_\perp}{\omega}\right)\left[\left(B_{1,0}+B_{-1,0}\right)-\lambda\left(B_{1,1}+B_{-1,1}\right)+\frac{\lambda}{2}\left(B_{2,1}+B_{-2,1}\right)\right] \\
\chi_{yy} &= \frac{\omega_{\mathrm{p,s}}^2}{\omega}\left[2\lambda A_{0,1}+\frac{1}{2}\left(A_{1,0}+A_{-1,0}\right)-\frac{3\lambda}{2}\left(A_{1,1}+A_{-1,1}\right)+\frac{\lambda}{2}\left(A_{2,1}+A_{-2,1}\right)\right] \\
\chi_{yz} &= -\chi_{zy} = i\frac{\omega_{\mathrm{p,s}}^2}{\omega}\left(\frac{k_\perp}{\omega}\right)\left[B_{0,0}-\lambda B_{0,1}-\frac{1}{2}\left(B_{1,0}+B_{-1,0}\right)-\lambda\left(B_{1,1}+B_{-1,1}\right)\right. \\
&- \left.\frac{\lambda}{4}\left(B_{2,1}+B_{-2,1}\right)\right] \\
\chi_{zz} &= \frac{2\omega_\mathrm{p}^2}{k_\parallel w_\perp^2}\left[(1-\lambda)B_{0,0}+\int_{-\infty}^{+\infty}\mathrm{d}v_\parallel\int_0^{+\infty}\mathrm{d}v_\perp v_\perp\frac{v_\parallel}{\omega}f_0(v_\parallel, v_\perp)\right] \\
&+ \frac{\lambda}{2}\frac{\omega_\mathrm{p}^2}{\omega}\left[2\frac{\omega-\omega_\mathrm{c}}{k_\parallel w_\perp^2}B_{1,0}+2\frac{\omega+\omega_\mathrm{c}}{k_\parallel w_\perp^2}B_{-1,0}\right]
\end{aligned}
$$

$$\lambda \equiv \frac{1}{2}\left(\frac{k_\perp v_\perp}{\omega_\mathrm{c}}\right)^2$$

back

Beyond Maxwellian (2)

Evaluations of the FLR susceptibility requires computation of two functions $A_{n,j}$ $B_{n,j}$, for $n = -2 \ldots 2$, $j = 0, 1$, which are $v_\perp$ moments of resonant integrals of $f_0(\psi, \frac{B}{B_{\rm min}}, v_\parallel, v_\perp)$

$$\left\{ \begin{array}{c} A_{n,j} \\ B_{n,j} \end{array} \right\} = \int_{-\infty}^{\infty} \mathrm{d}v_\parallel \left\{ \begin{array}{c} 1 \\ v_\parallel \end{array} \right\} \frac{1}{\omega - k_\parallel v_\parallel - n\omega_{\rm c}} \int_0^{+\infty} 2\pi v_\perp \mathrm{d}v_\perp H_j(v_\parallel, v_\perp)$$

with

$$H_0(v_\parallel, v_\perp) = \frac{1}{2}\frac{k_\parallel w_\perp^2}{\omega}\frac{\partial f_0}{\partial v_\parallel} - \left(1 - \frac{k_\parallel v_\parallel}{\omega}\right) f_0(v_\parallel, v_\perp)$$

$$H_0(v_\parallel, v_\perp) = \frac{1}{2}\frac{k_\parallel w_\perp^2}{\omega}\frac{\partial f_0}{\partial v_\parallel}\frac{v_\perp^4}{w_\perp^4} - \left(1 - \frac{k_\parallel v_\parallel}{\omega}\right) f_0(v_\parallel, v_\perp)\frac{v_\perp^2}{w_\perp^2}$$

and

$$w_\perp^2 \equiv \int_{-\infty}^{\infty} \mathrm{d}v_\parallel \int_0^{+\infty} 2\pi v_\perp \mathrm{d}v_\perp^2 f_0(v_\parallel, v_\perp)$$

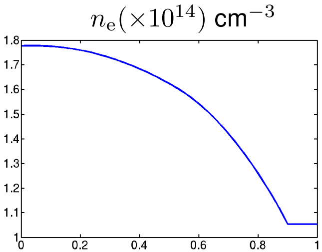
back

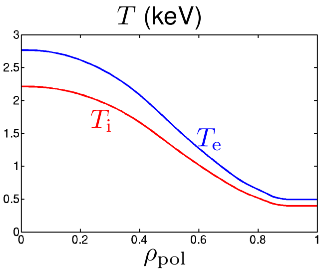
Alcator C-Mod case

◂ back

Main parameters:

- Plasma species: electron, D, and minority H (4%)
- $B_{\mathrm{T}} = 5$ T
- $I_{\mathrm{p}} = 1047$ kA
- $q(0)$ = 0.885
- q at plasma edge = 4.439
- $T_{\mathrm{e}}(0) = 2.764$ keV
- $n_{\mathrm{e}}(0) = 1.778 \times 10^{14}$ cm^{-3}
- $T_{\mathrm{D,H}}(0) = 2.212$ keV
- `TORIC` resolution: $n_{\mathrm{mod}} = 255$, $n_{\mathrm{elm}} = 480$



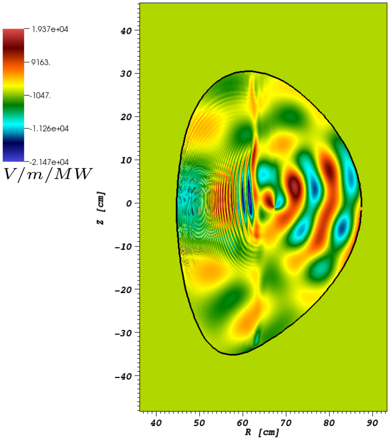


Excellent agreement between numerical and analytical evaluation of the electric field

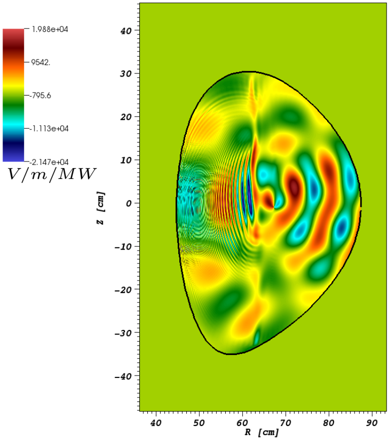
TORIC resolution: $n_{\rm mod} = 255$, $n_{\rm elm} = 480$

back

Maxw. analytical: Re(E_-)



Maxw. numerical: Re(E_-)



Excellent agreement between numerical and analytical evaluation of the electric field

◂ back

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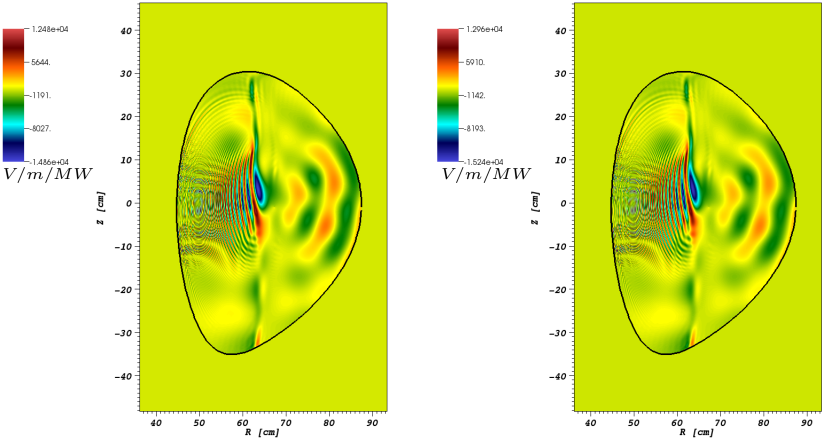
Maxw. analytical: $\mathrm{Re}(E_+)$ Maxw. numerical: $\mathrm{Re}(E_+)$



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◂ back

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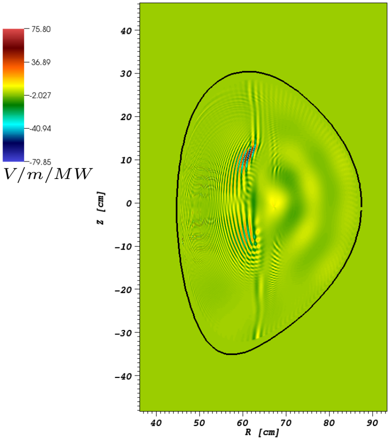
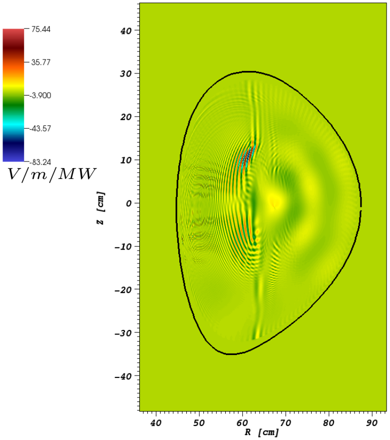
Maxw. analytical: Re($E_{||}$)



Maxw. numerical: Re($E_{||}$)



Excellent agreement in terms of absorbed power

TORIC resolution: $n_{\mathrm{mod}} = 255$, $n_{\mathrm{elm}} = 480$

Absorbed fraction	Maxw. analytical	Maxw. numerical
2nd Harmonic D	10.18	10.28
Fundamental H	69.95	68.81
Electrons - FW	11.35	11.91
Electrons -IBW	8.53	9.00

back

Bi-Maxwellian distribution

back

$$f_{\rm H}(v_\parallel, v_\perp) = (2\pi)^{-3/2}(v_{\rm th,\parallel} v_{\rm th,\perp}^2)^{-1} \exp[-(v_\parallel/v_{\rm th,\parallel})^2 - (v_\perp/v_{\rm th,\perp})^2]$$

with $v_{\rm th,\parallel} = \sqrt{2C_\parallel T(\psi)/m_{\rm H}}$, $v_{\rm th,\perp} = \sqrt{2C_\perp T(\psi)/m_{\rm H}}$, with constants $C_\parallel$ and $C_\perp$

- For $C_\parallel = 1$ and $C_\perp = \{.5, 1., 3., 5.\}$, $P_{\rm H}$, varied by less than 2%
- For $C_\perp = 1$ and $C_\parallel = \{.5, 1., 3., 5.\}$, the corresponding $P_{\rm H} = \{61.27\%, 70.50\%, 90.46\%, 94.18\%\}$
  - for small $C_\parallel$, the absorption profile is localized to the resonant layer
  - for large $C_\parallel$, the absorption profile is significantly broadened radially

$C_\perp = 1, C_\parallel = 0.5$ $\quad$ $C_\perp = 1, C_\parallel = 1$ $\quad$ $C_\perp = 1, C_\parallel = 5.0$