U.S. DEPARTMENT OF ENERGY | Office of Science

NSTX-U

Hybrid MHD/particle simulation study of sub-cyclotron Alfvén eigenmodes in NSTX

Jeff Lestz, Elena Belova, Nikolai Gorelenkov (PPPL)

Shawn Tang, Neal Crocker (UCLA)

NSTX-U Monday Meeting
PPPL
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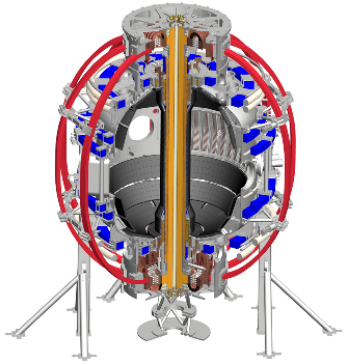
PPPL



Outline

- Introduction
- Simulation results
- Comparison to experimental database
- Summary and future work

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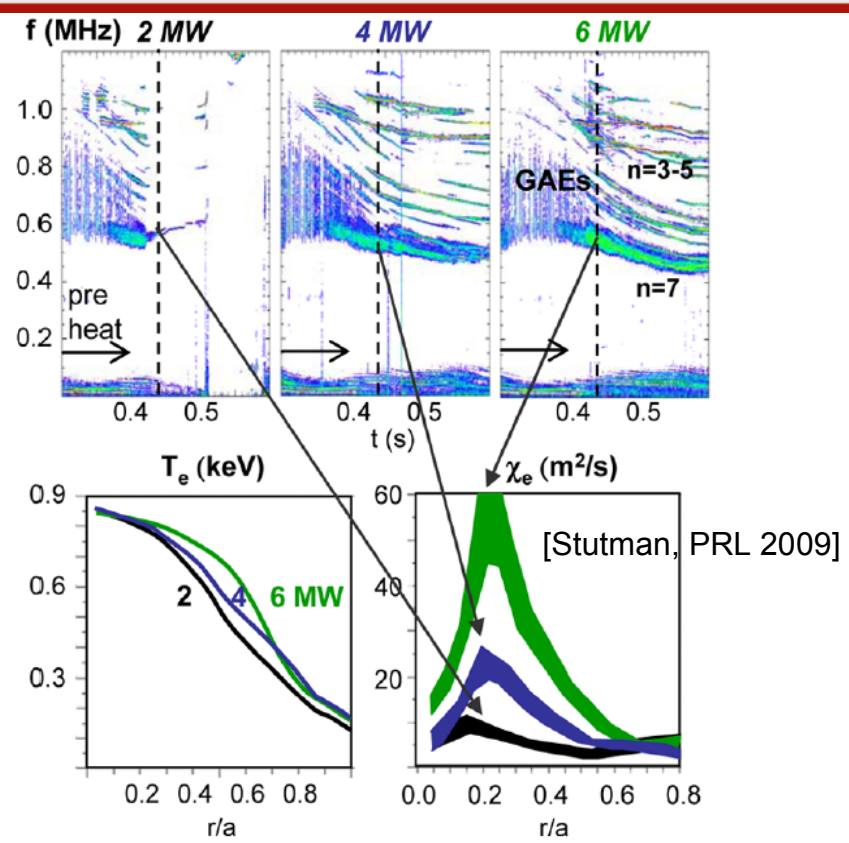
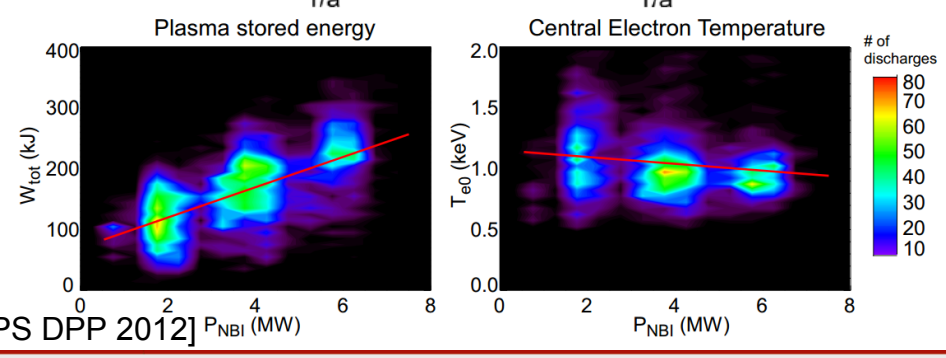
CAE/GAE May Limit ST Performance

- Many beam-heated NSTX discharges exhibit anomalously flat T_e profile.
  - Correlates with increased beam power, strong CAE/GAE activity
- Vital to understand preferential conditions for exciting (or suppressing) these allegedly deleterious modes.
  - Anomalously low T_e imperils future ST development

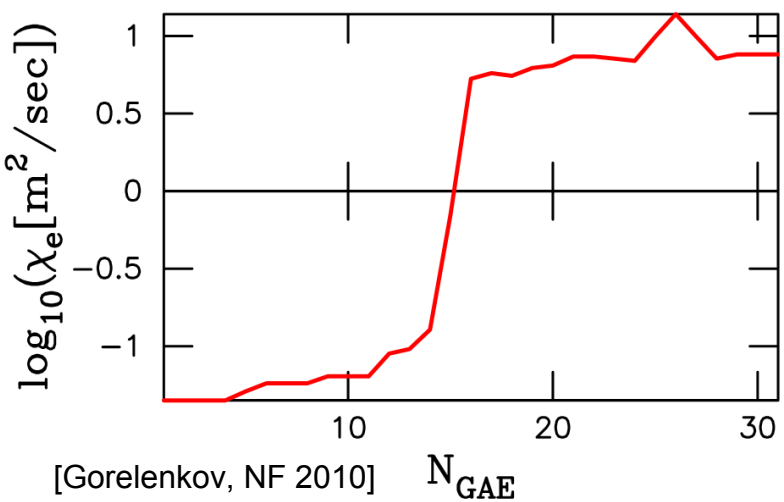




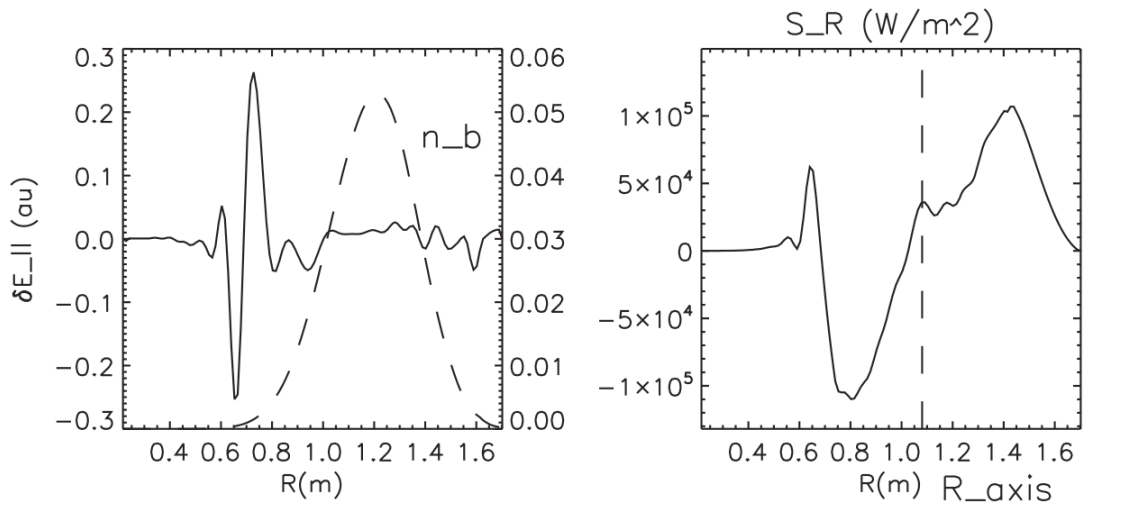
[Tritz, APS DPP 2012]

Proposed Explanations

- Enhanced electron transport due to orbit stochasticity induced by many overlapping GAE
  - Diffusion of ~10-50 m^2/s needed to match inferred experimental rate
- Energy channeling via mode conversion from core CAE to edge KAW
  - Rough estimate predicts power deposition as ~0.5 MW effect *per eigenmode*



[Gorelenkov, NF 2010]



[Belova, PRL 2015]

HYbrid MHD/Particle Code (HYM)

- Allows investigation of kinetic effects on MHD modes in toroidal geometry
- Single fluid MHD thermal plasma + particle fast ions
- Full-orbit kinetic ions in delta-f numerical scheme
- 3D geometry, initial value code
- Linear and nonlinear capabilities
  - Linear: linearized fluid equations + evolve weights along equilibrium particle trajectories
- Self consistently solves for equilibrium including energetic particle effects
- Typical run at NERSC: ~100 CPU x 10-20 hrs = 1-2k CPUHrs
  - Total hours used for this study: ~1.7 million CPUHrs

Fast Ion Distribution Function

Equilibrium distribution function $F_0 = F_1(v)\, F_2(\lambda)\, F_3(p_\varphi, v)$

$$F_1(v) = \frac{1}{v^3 + v_*^3}, \quad \text{for} \quad v < v_0$$

$$F_2(\lambda) = \exp(-(\lambda - \lambda_0)^2 / \Delta\lambda^2)$$

$$F_3(p_\phi, v) = \frac{(p_\phi - p_0)^\beta}{(R_0 v - \psi_0 - p_0)^\beta}, \quad \text{for} \quad p_\phi > p_0$$

where $v_0 =$ 2 - 6 v_A, $v_* = v_0/2$, $\lambda = \mu B_0/\varepsilon$ – pitch angle parameter, $\lambda_0 =$ 0.1 - 0.9

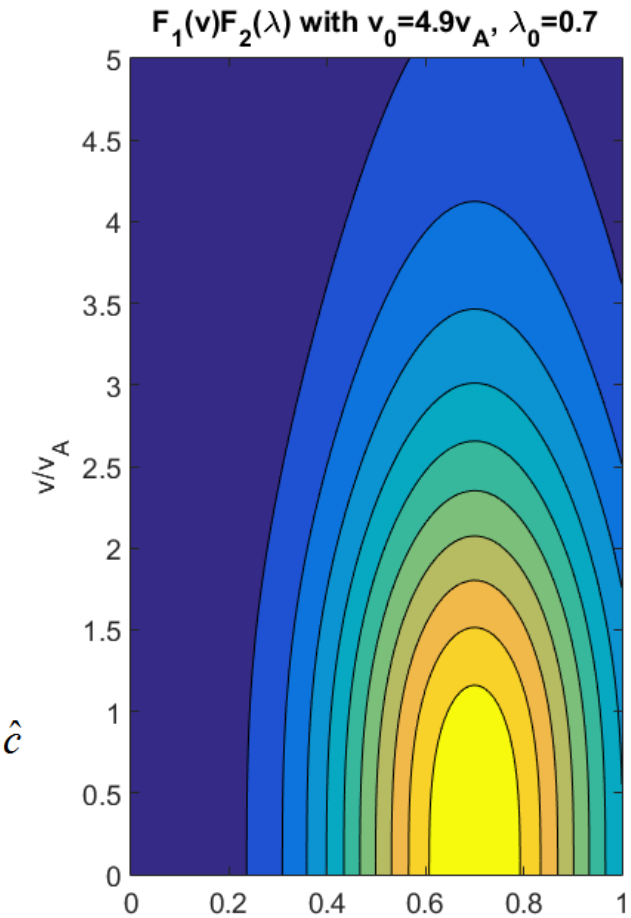
$\Delta\lambda =$ 0.3

and $\mu = \mu_0 + \mu_1$ includes first-order corrections [Littlejohn'81]:

$$\mu = \frac{(\mathbf{v}_\perp - \mathbf{v}_\mathbf{d})^2}{2B} - \frac{\mu_0 v_\parallel}{2\mathrm{B}}[\hat{b}\cdot\nabla\times\hat{b} - 2(\hat{a}\cdot\nabla\hat{b})\cdot\hat{c}]$$

$\mathbf{v}_d$ is magnetic gradient and curvature drift velocity, $\hat{c} = \mathbf{v}_\perp / v_\perp$, $\hat{a} = \hat{b}\times\hat{c}$

Parameters are chosen to match TRANSP beam profiles.



$$\lambda = \mu B_0/E \approx \frac{v_\perp^2}{v^2} = 1 - \chi^2$$

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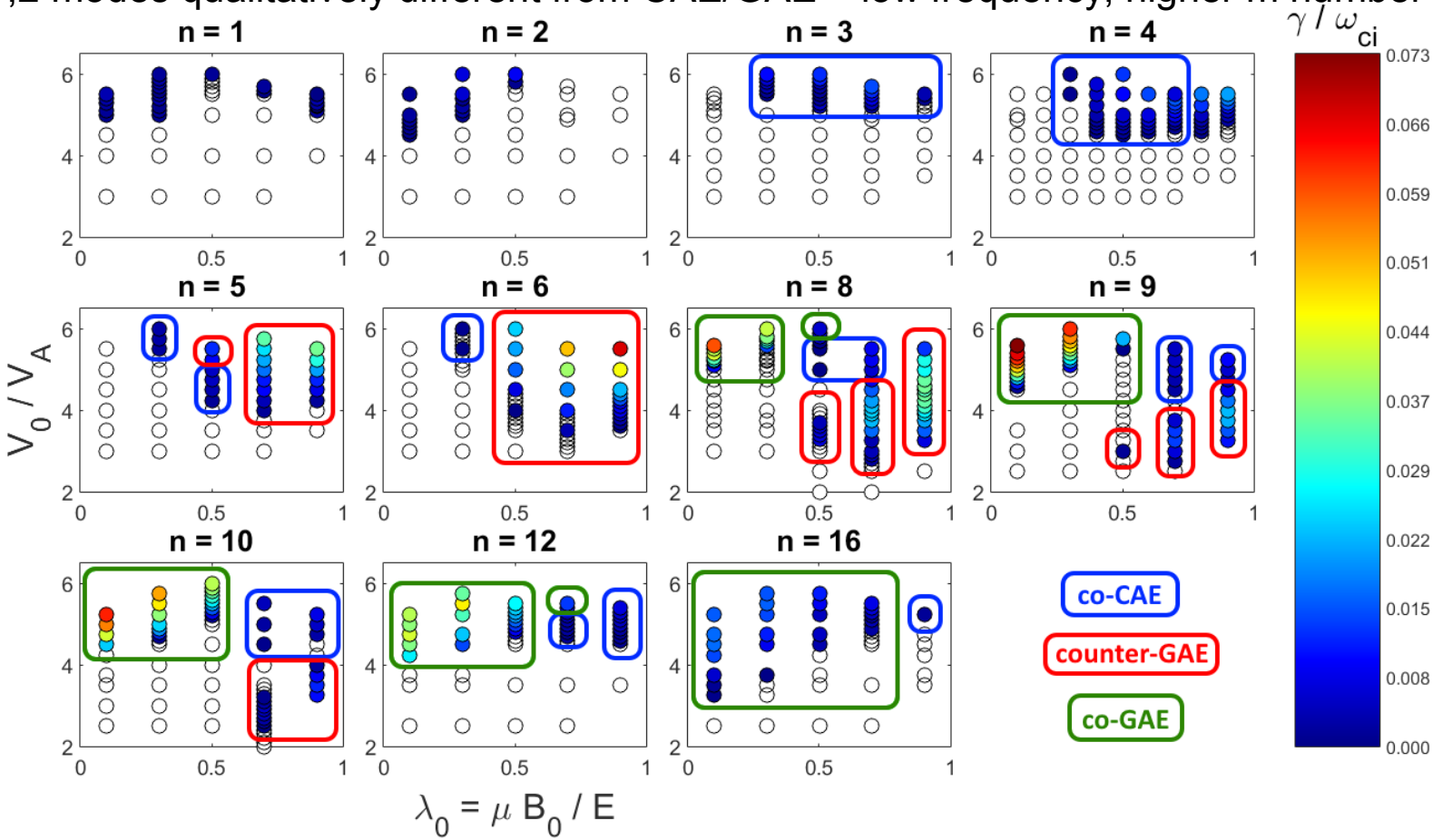
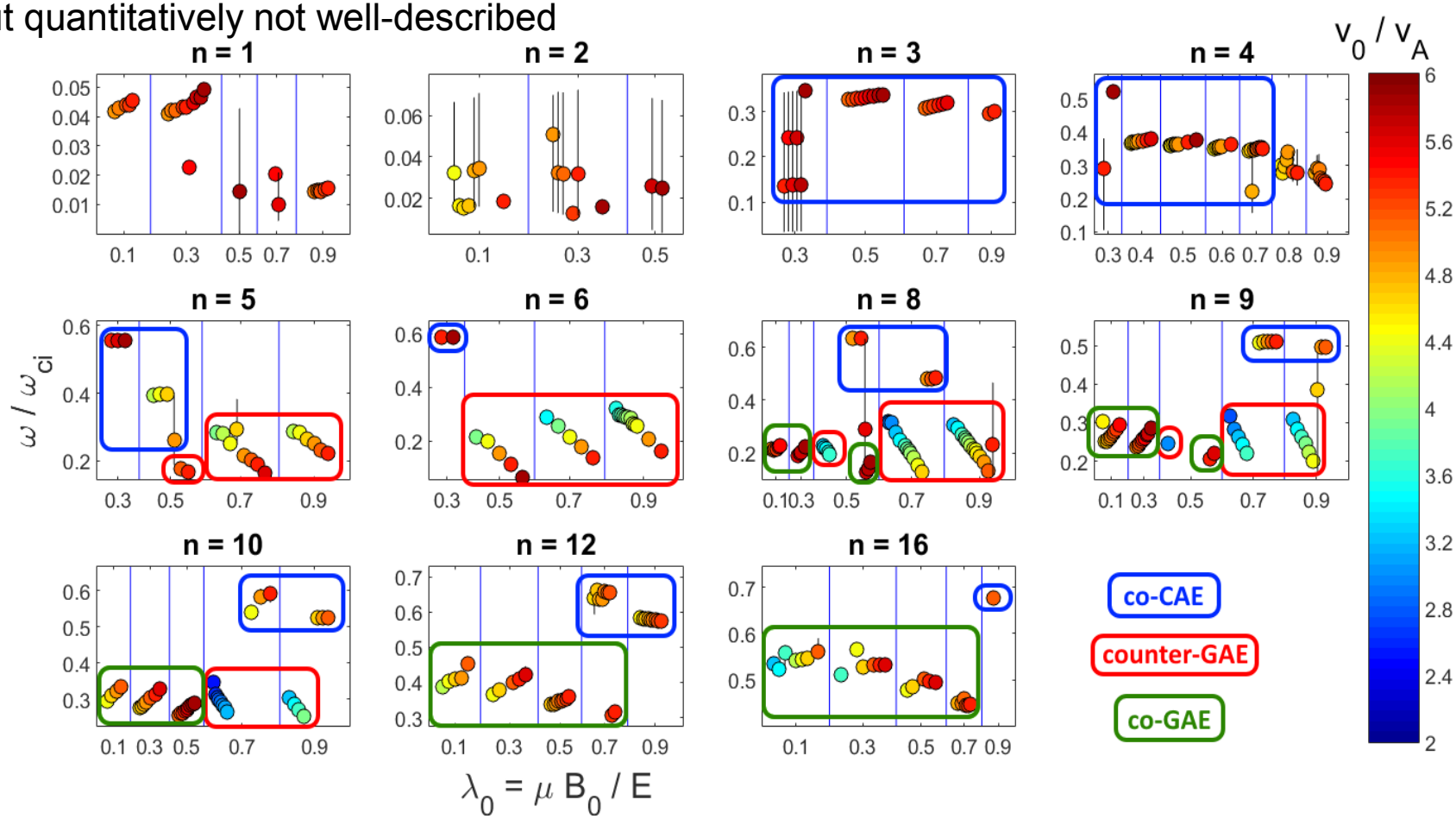
Variety of Unstable Modes Found with Diverse Parameters

- Most unstable are n=8-10 co-GAE, then n=5-8 cntr-GAE, then n=3-4 co-CAE
  - Colored circles: linear growth rate of most unstable mode
  - White circles: no unstable mode of any type
- n = 1,2 modes qualitatively different from CAE/GAE – low frequency, higher m number



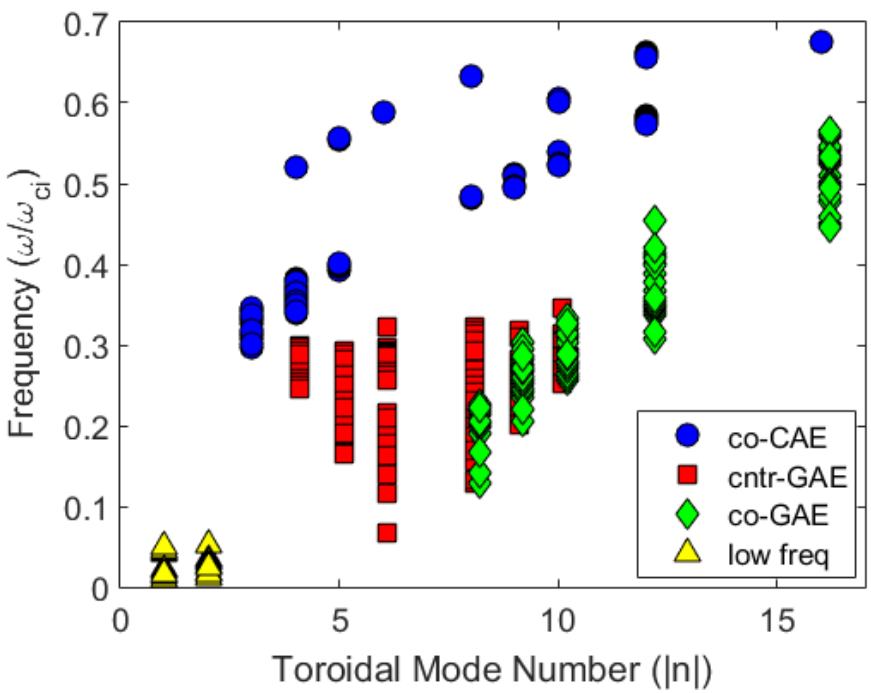
Mode Frequencies Shift with λ_0, v_0

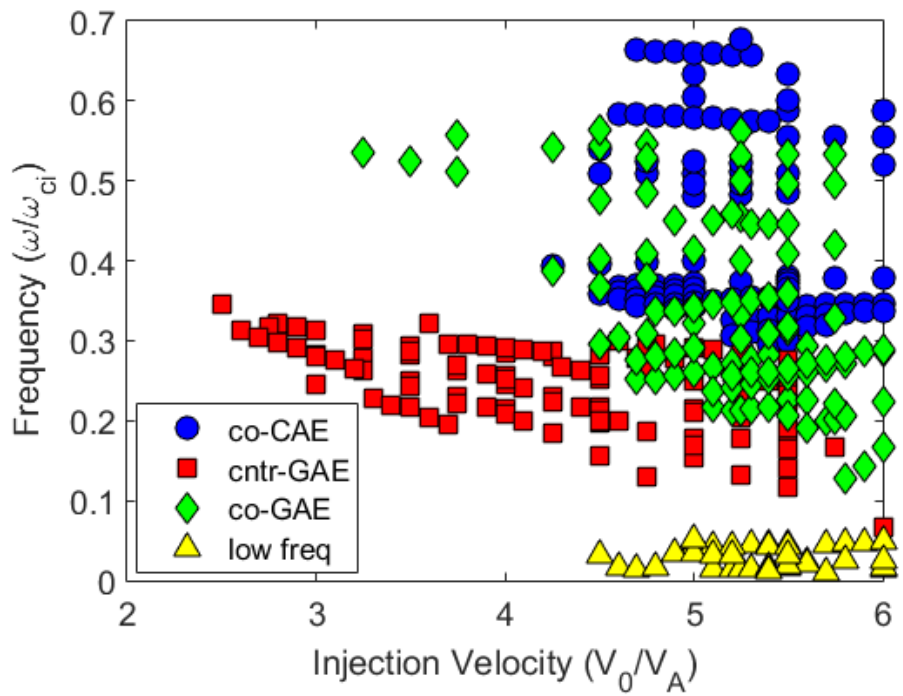
- Cntr-GAE frequency decreases substantially with increasing v_0, co-GAE shows opposite effect
- Co-CAE (and low-n mode) frequencies increase, but much more modestly than GAE
- Qualitatively match expected trends from dispersion + resonance conditions, but quantitatively not well-described



Mode Spectrum Depends Strongly on n & v_0

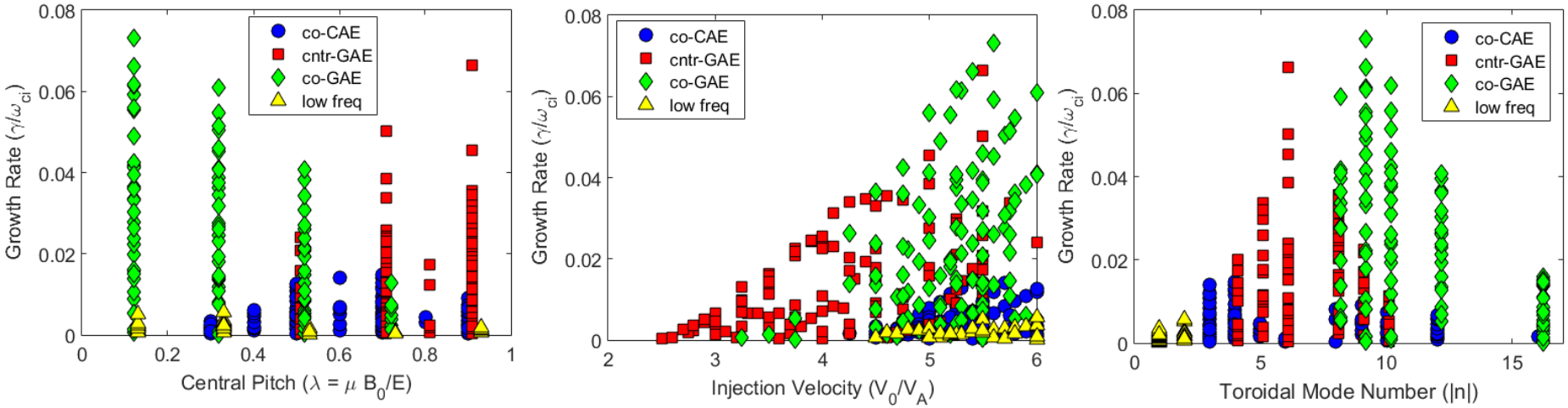
- At each n, CAE and GAE modes appear at distinct frequencies
- Co-GAE seen in simulations at large |n| (>7) are yet to be thoroughly investigated experimentally
- Mode spectrum becomes much more rich as V/V_A increases past 4.5





GAE Typically More Unstable than CAE

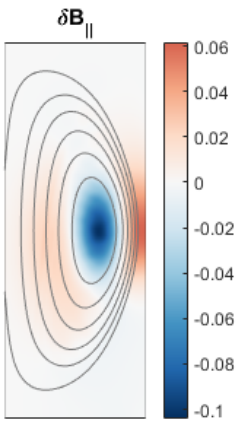
- Largest growth rates occur for n ~ 6 – 10
- GAE mostly co-rotating when driven by low lambda population, counter-rotating for large lambda
- In all modes, gamma increases with v (with few exceptions)
- Almost exclusively cntr-GAE for $2.5 < V/V_A < 4$
- co-CAE are strictly more stable than GAE for $V/V_A < 4$
  - Implications for T_e flattening mechanism

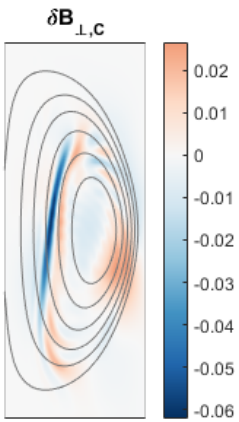


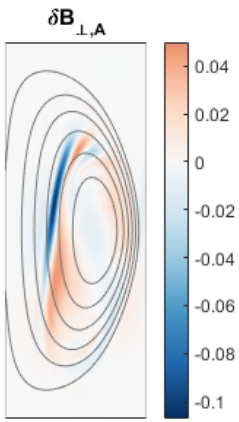
Co-CAE Mode Converts to KAW

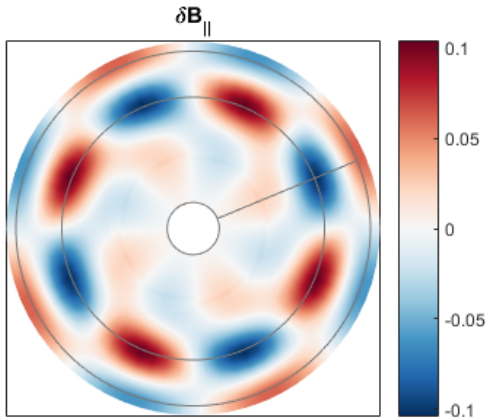
- CAE typically m = 0.5 – 2, peaking on axis
- KAW structure visible in $\delta B_\perp$ fluctuation on HFS

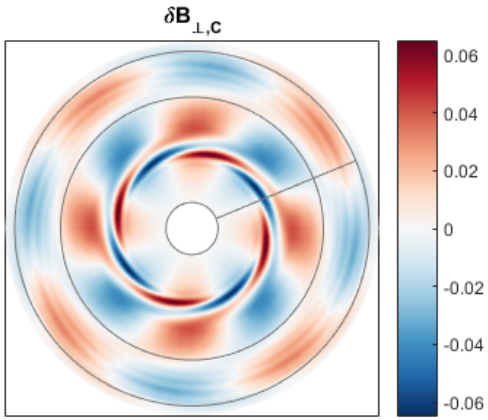
n=4
λ_0=0.7
v_0=5.0

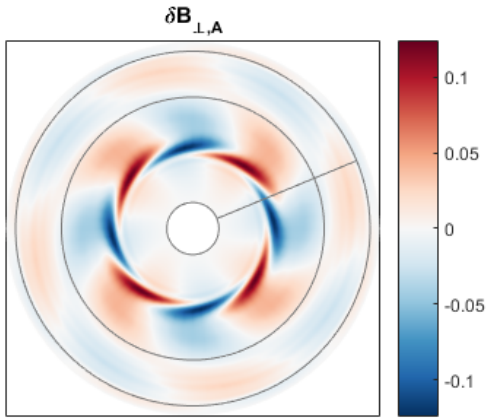








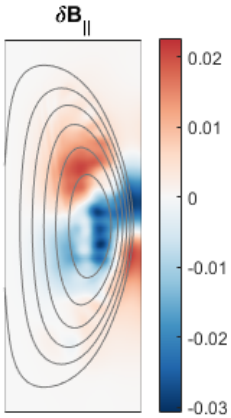


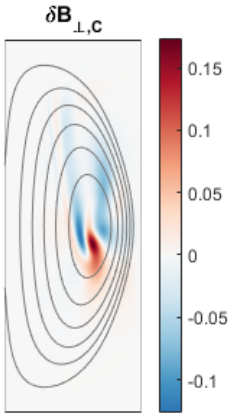


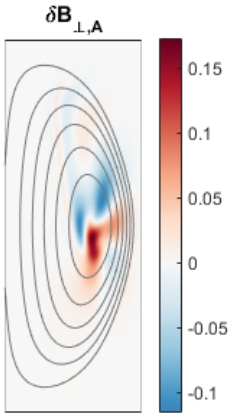
Counter-GAE are Core-Localized

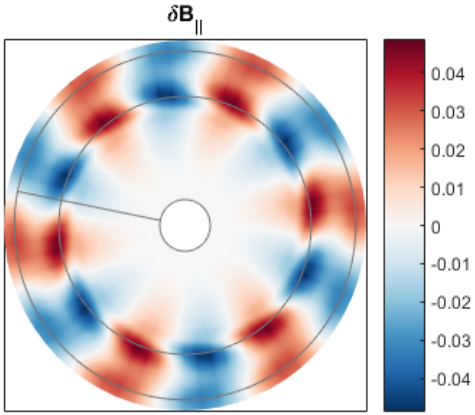
- Typically m = 1.5 – 2.5
- Often has large compressional component near edge

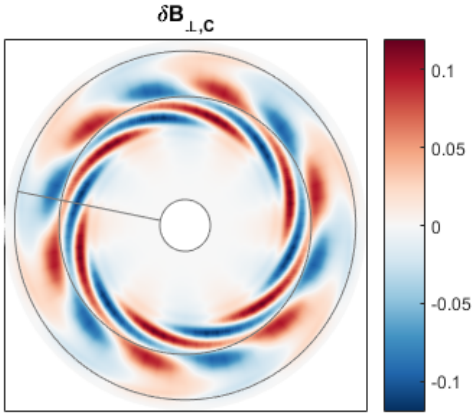
n=6
λ_0=0.7
v_0=4.0

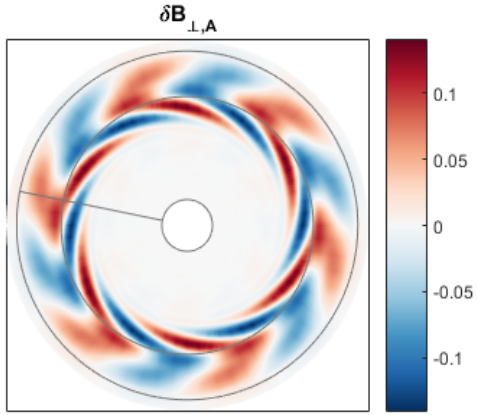








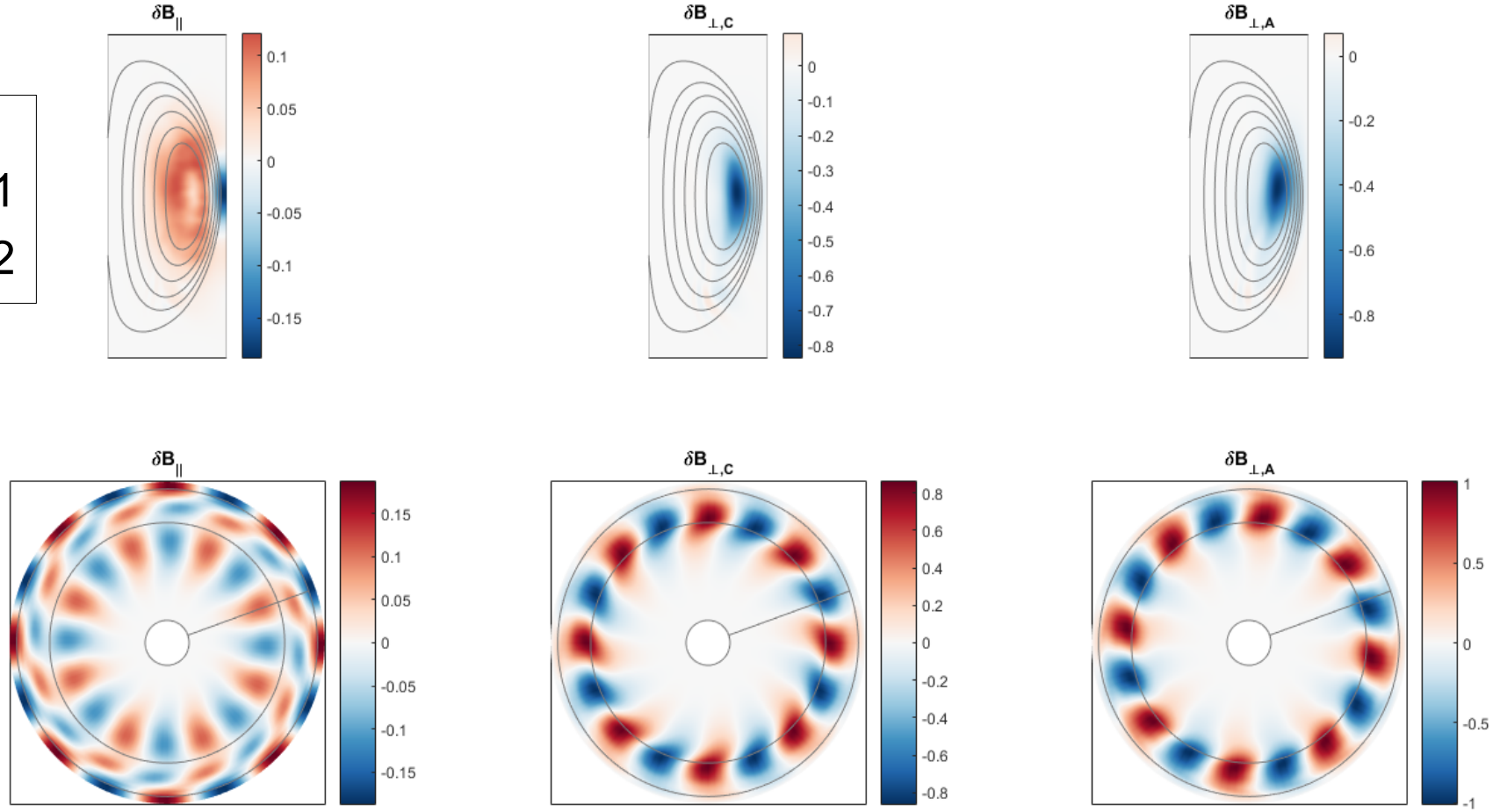




Co-GAE are also Core-Localized

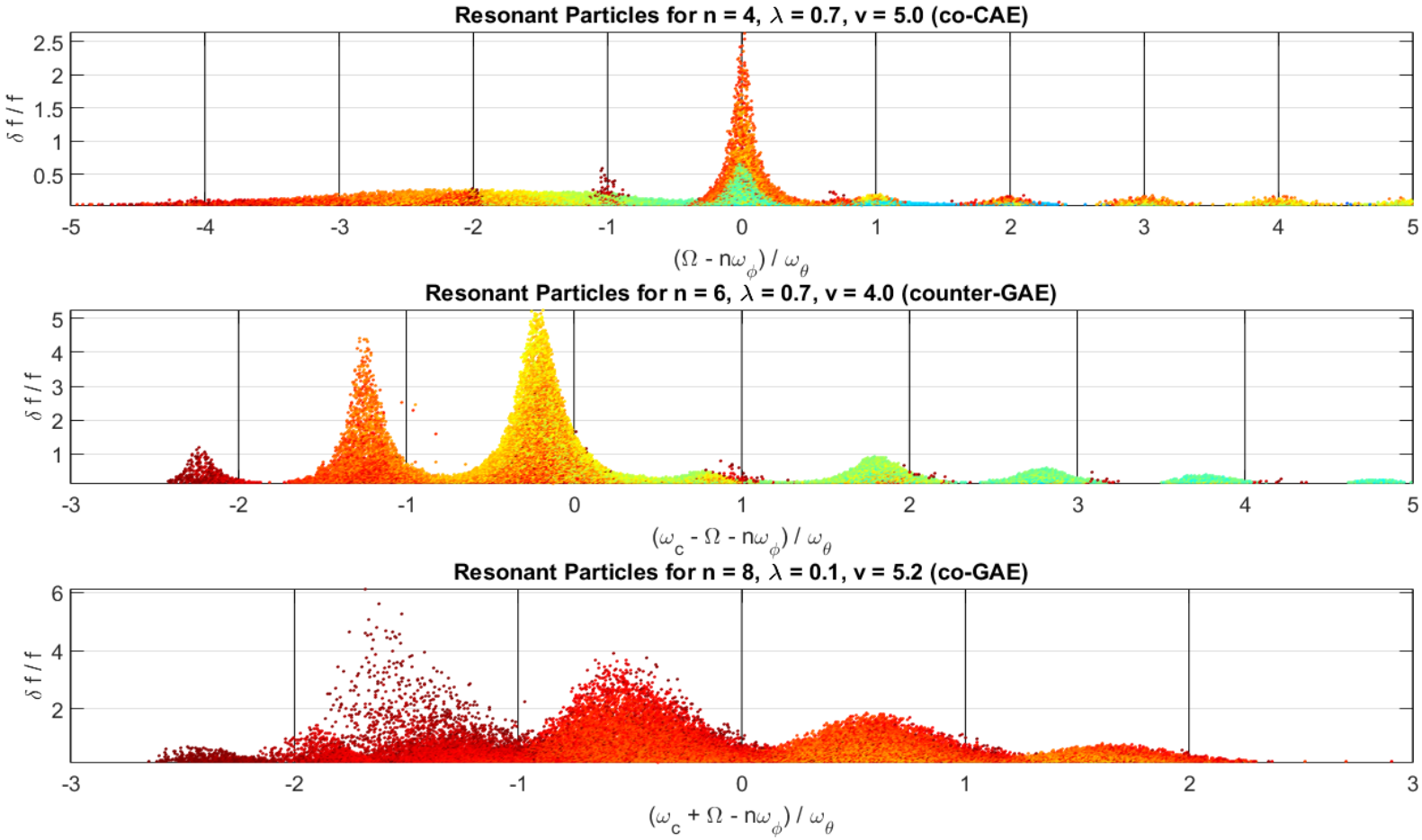
- Typically very low m = 0.5 – 1
- Not commonly observed in experiment?

n=8
λ_0=0.1
v_0=5.2



δf Weights Reveal Wave-Particle Resonances

- co-CAE (and low-n mode) has ordinary resonance: $\Omega - n\omega_\varphi - p\omega_\theta = 0$ with $p = 0,\pm1$
- cntr/co-GAE have Doppler-shifted cyclotron resonance: $\pm\omega_c - \Omega + n\omega_\varphi + p\omega_\theta) = 0$ with $p = 0,\pm1,\pm2$
  - GAE resonances often shifted from integers → time averaging effect or physics?



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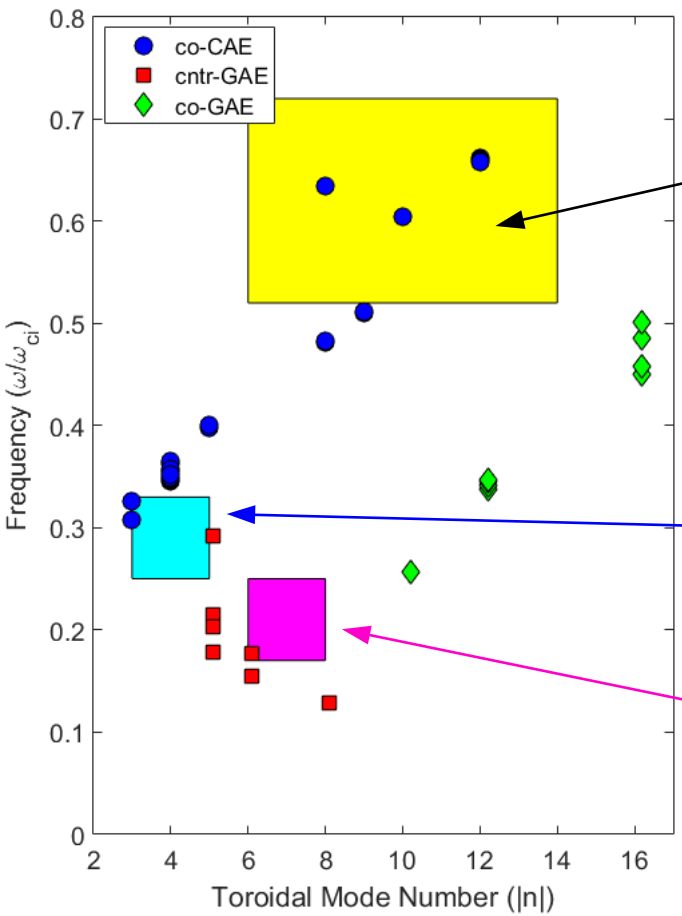
Experimental vs Simulated Mode Spectrum for NSTX Shot 141398

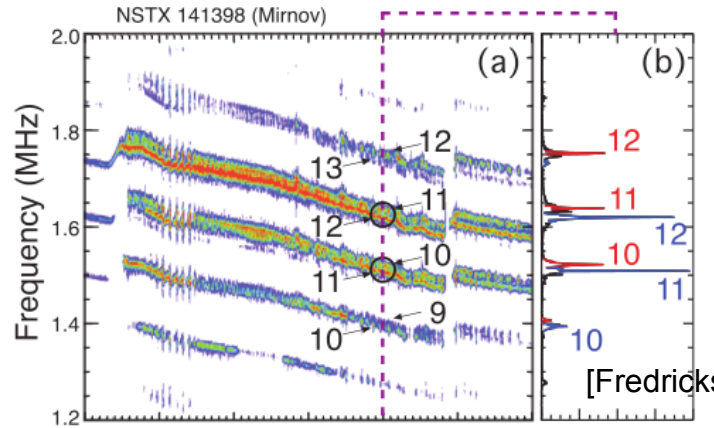
- From TRANSP, realistic parameters for discharge 141398 are λ_0 = 0.5 – 0.7, v_0 = 4.75 – 5.25
- Co-CAE agree with high frequency observations, <u>disagree on direction</u> at lower frequency range
- Cntr-GAE spectrum in qualitative agreement with experimental observations



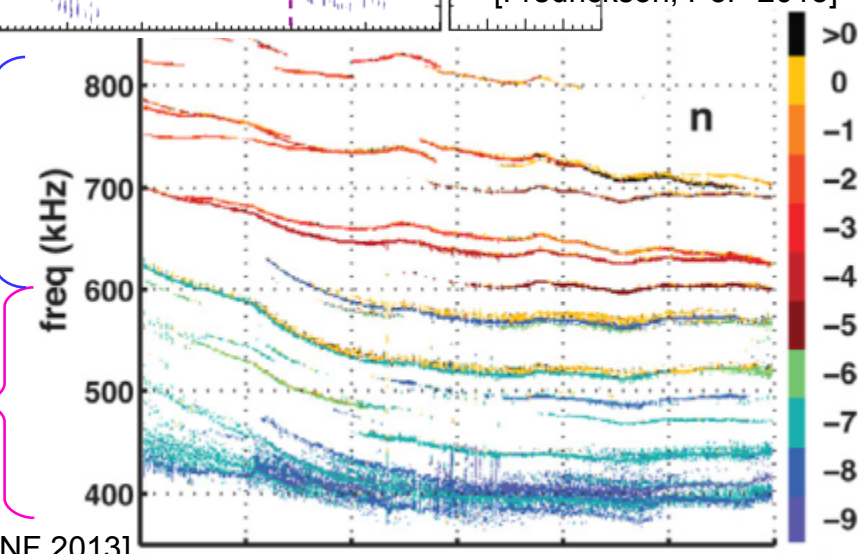
co-CAE

cntr-CAE

cntr-GAE



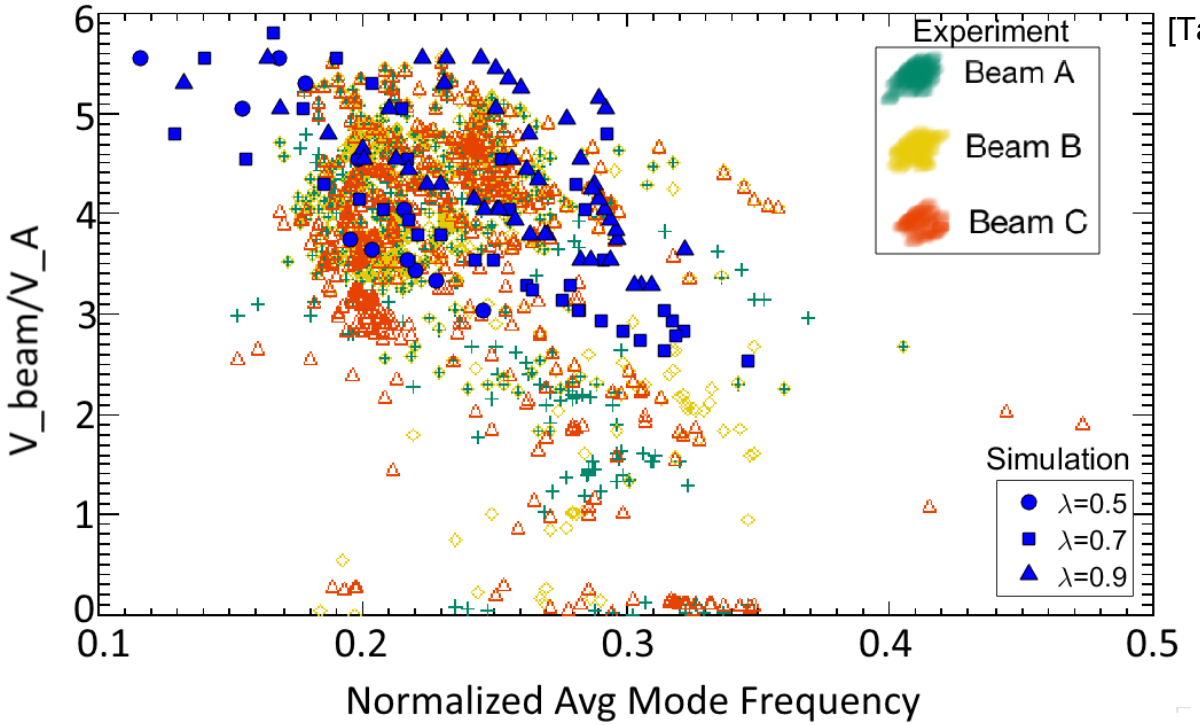
[Fredrickson, PoP 2013]



[Crocker, NF 2013]

Mode Spectrum Dependence on V_0/V_A

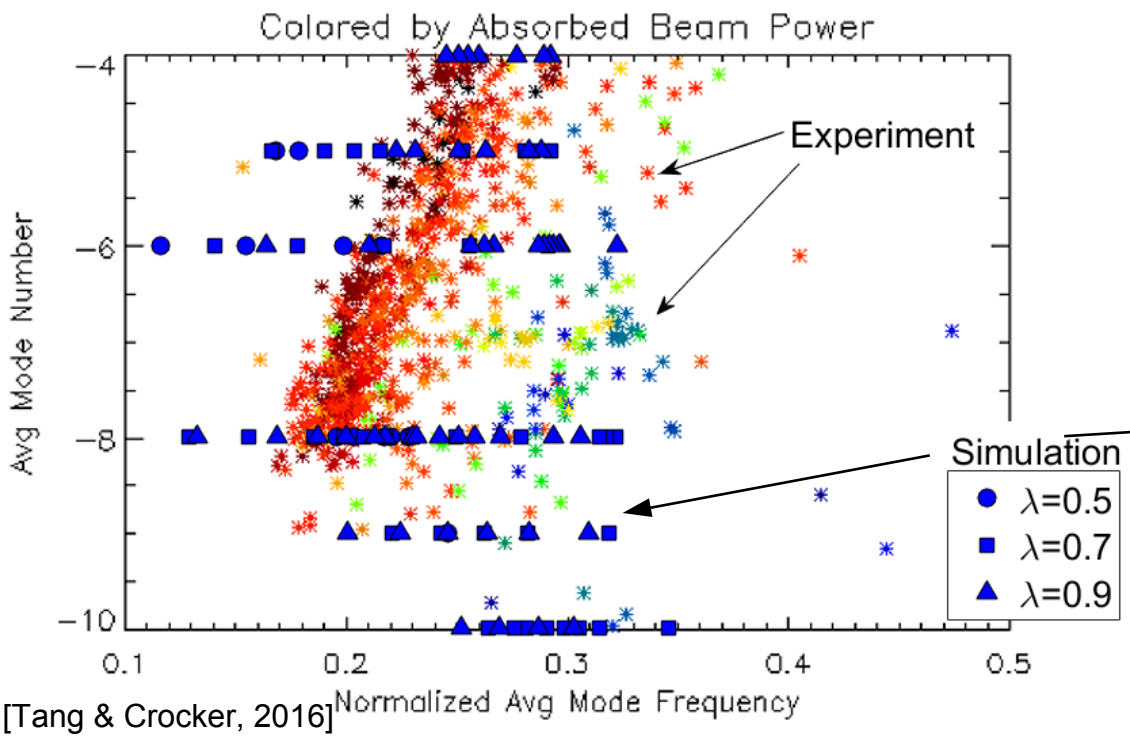
- Extensive experimental survey of NSTX discharges with high frequency AE activity has been compiled by S. Tang and N. Crocker. For 173 discharges, analyzed 50ms time windows for mode properties.
- Red/Yellow/Green points: experimental amplitude-weighted average mode number for times satisfying:
  - (1) T_e > 500 eV, (2) <ω> > 200 Hz, and (3) -10 < <n> < -4
- Blue marks: individual simulated cntr-GAE modes
- Some qualitative agreement, but very preliminary, requires more direct comparison
  - Individual mode comparison required to compare against simulation-predicted stability boundaries in V_0/V_A

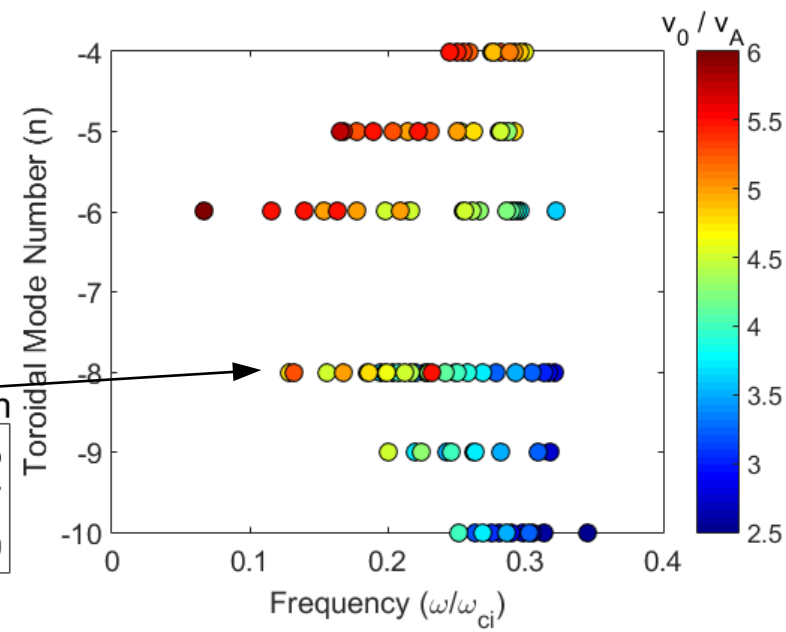


[Tang & Crocker, 2016]

Mode Spectrum Dependence on n

- Red/Yellow/Green points: experimental amplitude-weighted average mode number for time slices satisfying 3 filters:
  - (1) $T_e \geq 500$ eV, (2) $\langle\omega\rangle \geq 200$ Hz, and (3) $-10 \leq \langle n\rangle \leq -4$
- Blue marks: individual simulated cntr-GAE modes
- Reasonable agreement for $|n| < 7$, and correct trends for all n for constant v
- Also requires more direct comparison



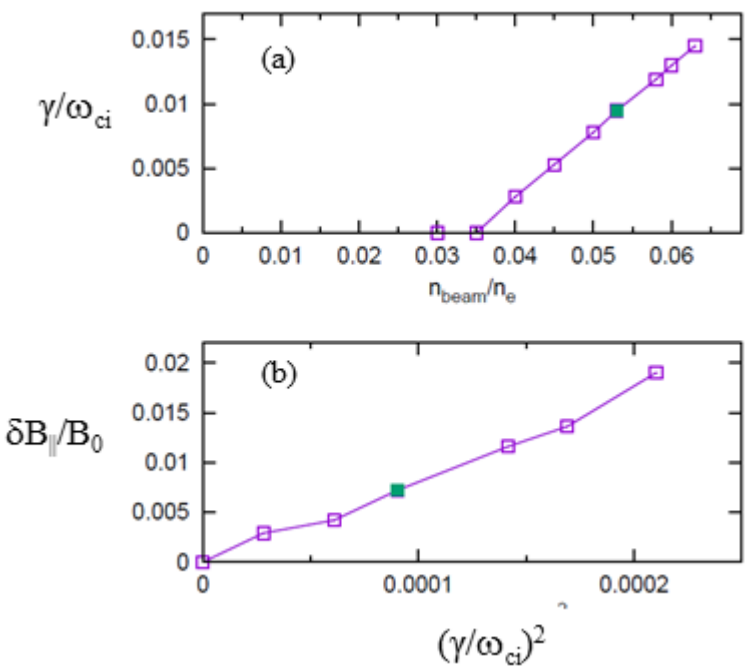


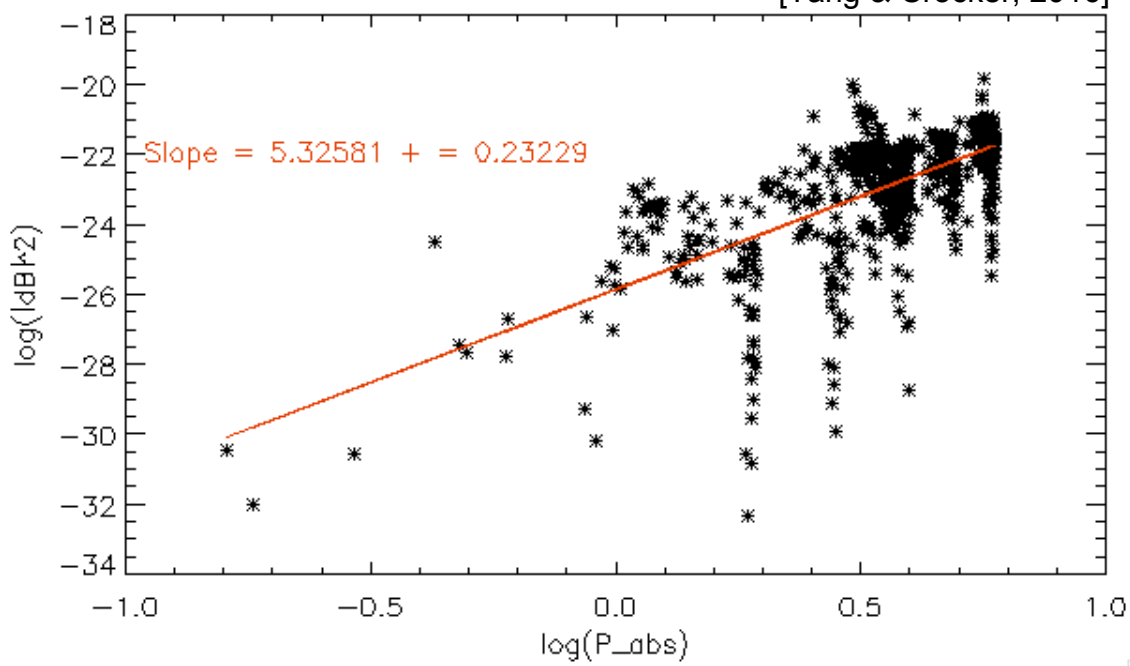
[Tang & Crocker, 2016]

Modes Saturate with γ^2

- Nonlinear simulations of n=4 CAE reveal $\delta b \propto \gamma^2 \propto P_b^2$
- Experimental database shows $\delta b \propto P_b^{2.6}$ when fitting all CAE/GAE modes
- Results consistent with saturation via particle trapping
- Power to KAW $\propto \gamma \delta b^2 \propto \gamma^5 \propto P_b^5$
  - Very strong dependence implies energy channeling should be strong effect in high beam power NSTX-U discharges

[Tang & Crocker, 2016]





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Summary and Conclusions

- A large set of 3D hybrid simulations (HYM code) were conducted for a wide range of beam parameters ($2 \leq V_b/V_A \leq 6$, $0.1 \leq \lambda_0 \leq 0.9$)
- GAE are more unstable than CAE for $V_b/V_A < 4$
- Modes saturate nonlinearly with $\gamma^2 \left(\propto P_b^2\right)$, similar to experimental observations and predicting strong CAE to KAW energy channeling ($\propto P_b^5$) for NSTX-U discharges at high beam power
- Frequency spectrum agrees qualitatively with theory description, but not quantitatively
- Preliminary comparisons of the mode spectrum with experiment show plausible agreement, though not definitive
- Two areas of disagreement with experiment identified
  1) Direction of propagation of most unstable CAE at moderate frequency
  2) Existence and large growth rate of co-GAE

Ongoing and Future Work

- Improve physics model in HYM and equilibrium solver
- More realistic and flexible forms of fast ion distribution
  - Especially multi-beam distributions
  - Ideal: interface directly with TRANSP when comparing with specific discharge
- More detailed experimental comparison
  - Current comparisons are indirect
  - V_b/V_A stability thresholds and their connection to T_e flattening
- Comparison with analytic theory
- Compare relative importance of enhanced electron diffusion and energy channeling in various scenarios

Backup Slides

Review of CAE/GAE Mode Properties

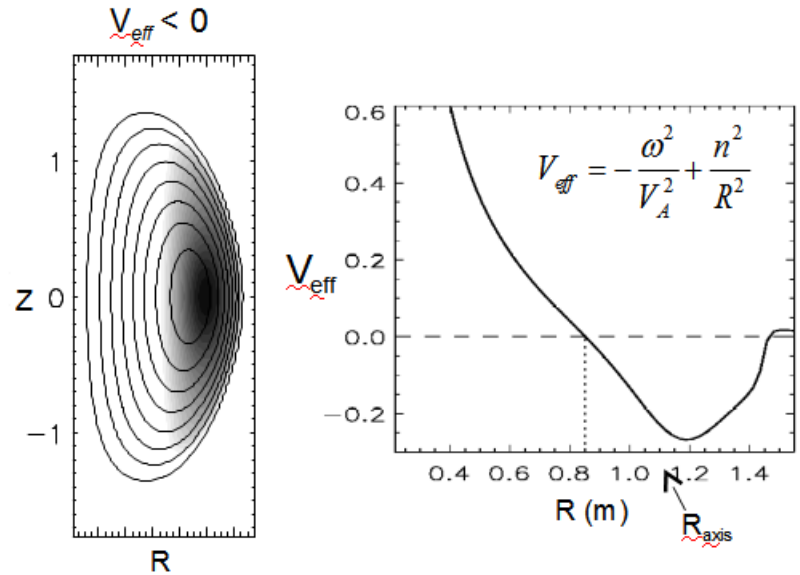
- Common Properties
  - Typical frequency in range $0.1\omega_{ci} < \omega < \omega_{ci}$
  - Core-localized MHD modes
  - May be driven unstable by resonant energetic particles
    - Regular $\omega - k_{||}v_{||} = 0$ or Doppler-shifted cyclotron $\omega - k_{||}v_{||} = \omega_{ci}$

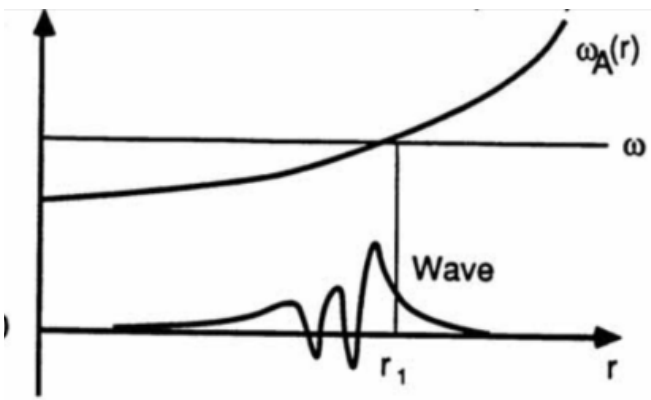
- Compressional AE (CAE)
  - Compressional polarization
    - $\delta B \sim \delta B_{||} >> \delta B_{\perp} \sim 0$
  - Dispersion: $\omega \sim k\, v_A$
  - Mode converts to KAW at Alfvén resonance $\omega=\omega_A(r_0)$

- Global AE (GAE)
  - Shear polarization
    - $\delta B \sim \delta B_{\perp} >> \delta B_{||} \sim 0$
  - Dispersion: $\omega < (k_{||}v_A)_{max}$
  - Exists below extremum of Alfvén continuum (e.g. low magnetic shear)

Compressional (CAE) and Kinetic (KAW) Modes

- Compressional polarization $\delta \mathbf{B} \cdot \mathbf{B}_0 \sim 1$, fast magnetosonic mode from before
- Dispersion: $\omega \sim V_A k$
- Obeys approximate wave equation with effective potential
- Where $V_{eff} > 0$ wave is evanescent
  - Typically localizes mode structure near core
- Alfven resonance location at $V_{eff}=0$
  - CAE mode converts to KAW here

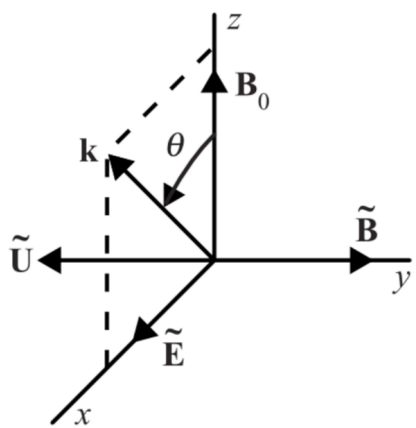


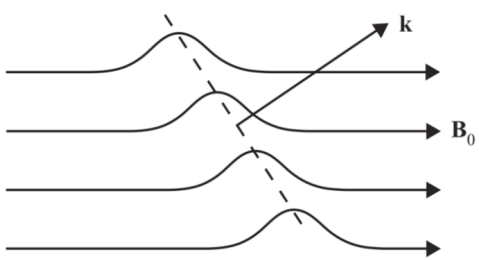


- KAW is shear wave solution from including ion FLR effects
- Efficiently Landau damps on electrons due to short scale length

Global Alfven Eigenmode (GAE)

- Shear polarization $\boldsymbol{\delta B \cdot B_0} \sim 0$, "extremum type" mode
- Minimum in Alfven Continuum created by shear-reversed q-profile or extremum in $V_A(R) \sim B(r)/n(r)$ e.g. since $B_\varphi \sim 1/R$
- Extremum typically occurs near core → GAE is radially localized near this location
- $\omega \sim V_A k_{||} - \Delta\omega$, where $\Delta\omega << \omega$
- Typically somewhat broader mode structure than CAE
- Part of shear branch: intuition is shear Alfven mode from uniform plasma





HYM Physical Equations

Background plasma - fluid:

$$\rho \frac{d\mathbf{V}}{dt} = -\nabla p + (\mathbf{j} - \mathbf{j}_b) \times \mathbf{B} - n_b (\mathbf{E} - \eta \mathbf{j})$$

$$\mathbf{E} = -\mathbf{V} \times \mathbf{B} + \eta \mathbf{j}$$

$$\mathbf{B} = \mathbf{B}_0 + \nabla \times \mathbf{A}$$

$$\partial \mathbf{A} / \partial t = -\mathbf{E}$$

$$\mathbf{j} = \nabla \times \mathbf{B}$$

$$\partial p^{1/\gamma} / \partial t = -\nabla \cdot (\mathbf{V} p^{1/\gamma})$$

$$\partial \rho / \partial t = -\nabla \cdot (\mathbf{V} \rho)$$

Fast ions – delta-F scheme:

$$\frac{d\mathbf{x}}{dt} = \mathbf{v}$$

$$\frac{d\mathbf{v}}{dt} = \mathbf{E} - \eta \mathbf{j} + \mathbf{v} \times \mathbf{B}$$

$w = \delta F / F$ - particle weight

$$\frac{dw}{dt} = -(1 - w) \frac{d(\ln F_0)}{dt}$$

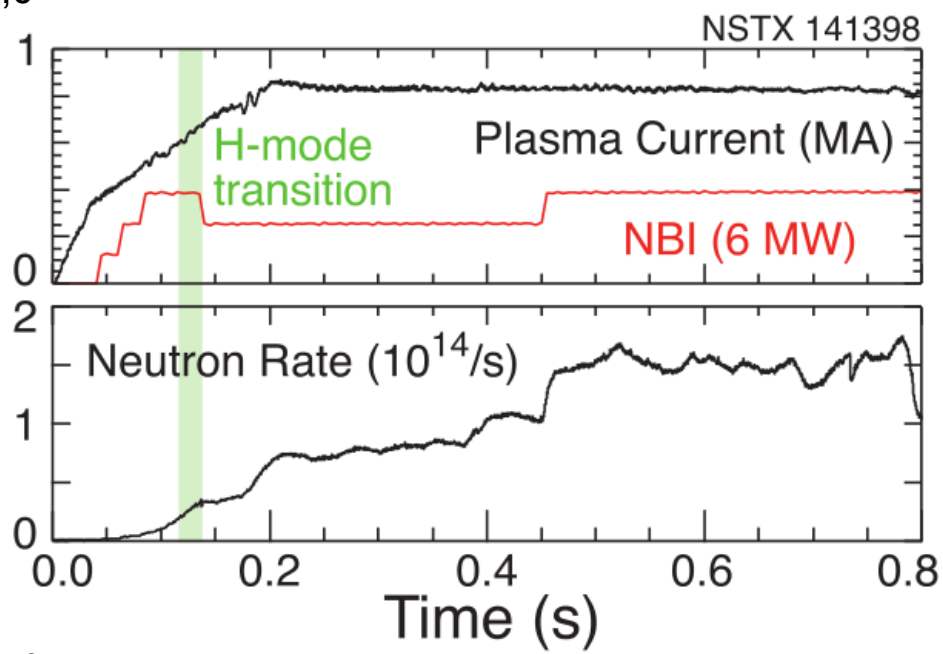
$$F_0 = F_0(\varepsilon, \mu, p_\phi)$$

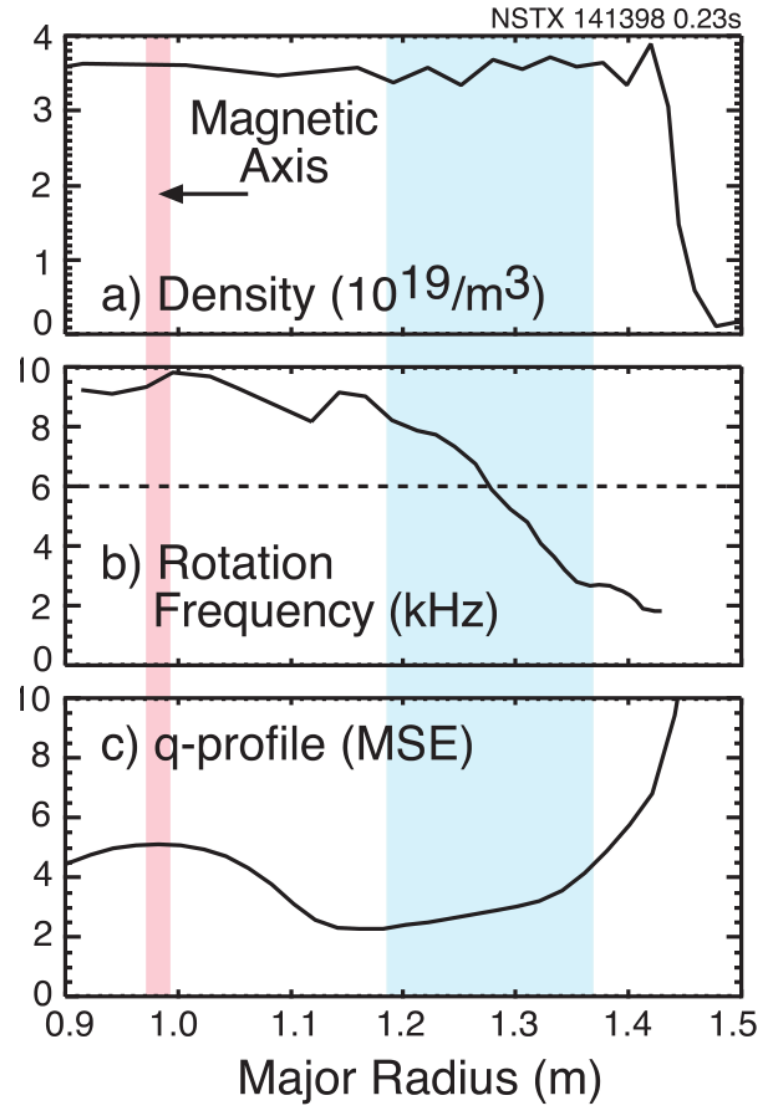
ρ, **V** and p are thermal plasma density, velocity and pressure, n_b and $\boldsymbol{j}_b$ are beam ion density and current, and $n_b << n_e$ – is assumed.

NSTX H-mode Shot 141398

- $n_e = 6 \times 10^{19}\ m^{-3}$
- $B_{tor,0}$=0.325 T
- I_p = 0.8 MA
- $f_{ci,0}$=2.5 MHz

- 90 keV NBI at 6 MW
- $v_0 = 4.9\ v_A$
- $\lambda_0 = 0.7$
- $n_b/n_e \sim 5\%$

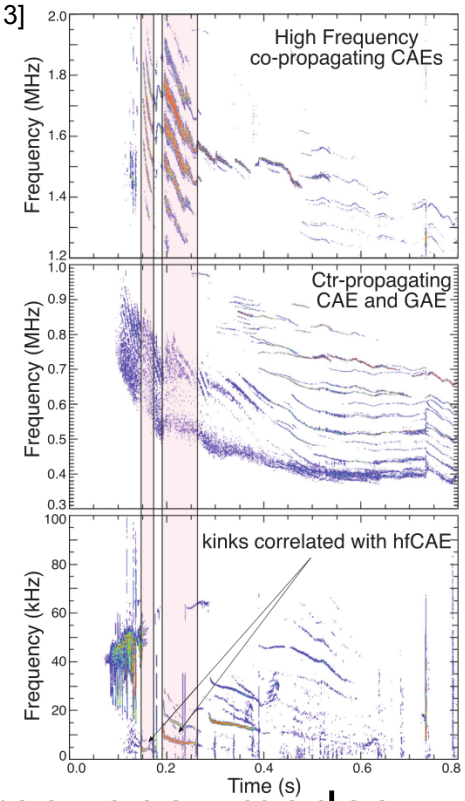




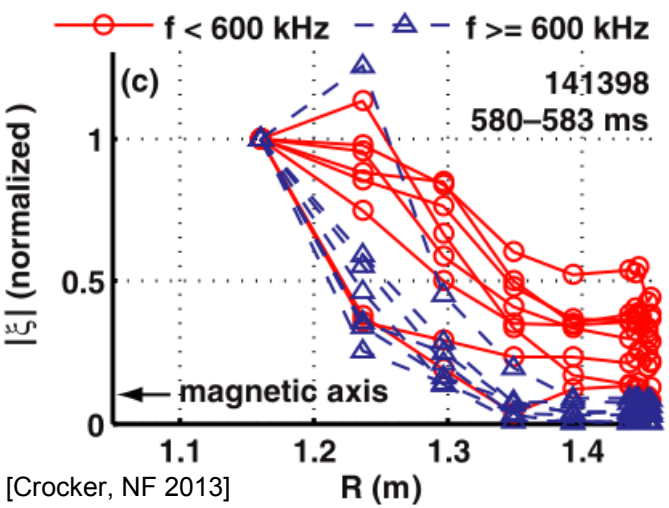
[Figures from Fredrickson, PoP 2013]

CAE & GAE Observed in Experiment

[Fredrickson, PoP. 2013]



- Fredrickson observes 3 groups of modes
  - Co-propagating CAE
    - $n=6\text{-}14,\ 0.5\omega_{ci} < \omega < 0.75\omega_{ci}$
  - Counter-propagating CAE & GAE
    - $0.15\omega_{ci} < \omega < 0.35\omega_{ci}$
  - Kinks correlated with high frequency co-CAE
    - $\omega \sim .005\omega_{ci}$



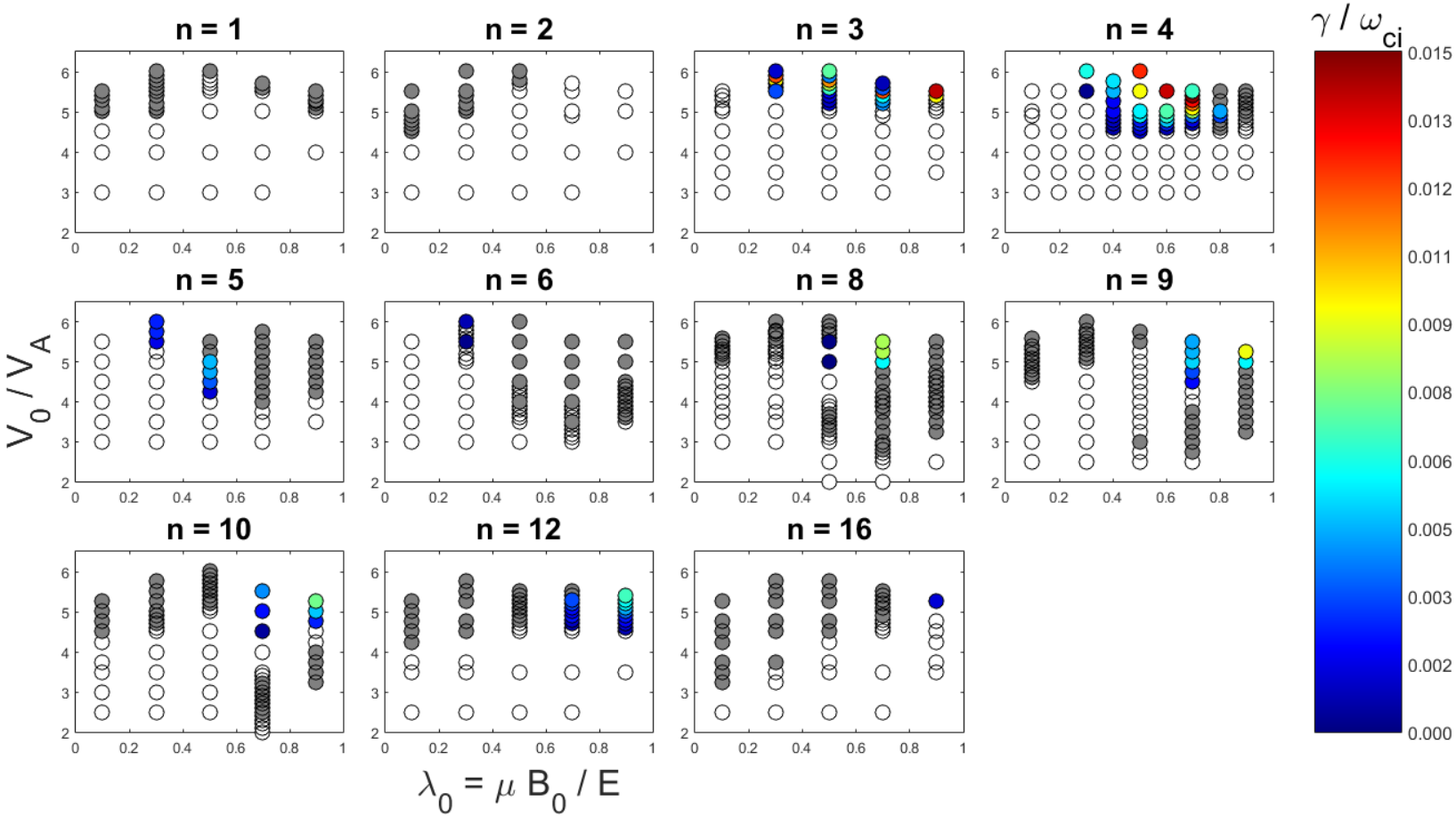
[Crocker, NF 2013]

- Crocker studies moderate frequency modes
  - $\omega > 0.25\omega_{ci}$ CAE
    - Mostly $-3 < n < -5$, more core-localized
  - $\omega < 0.25\omega_{ci}$ GAE
    - Mostly $-6 < n < -8$, broad mode structure

Co-CAE Stable for $v_0/v_A<4$

- Frequencies : $\omega \sim 0.3 - 0.7\omega_{ci} = 2\pi(700 - 1700)$ kHz
- Growth rates : $\gamma/\omega = 0.0004 - 0.04$, $\gamma_{max} = 0.015\omega_{ci} = 230$ kHz
- Most unstable modes have n=3,4 large central pitch $\lambda_0 \geq 0.5$
- CAE stable (or less unstable than other modes) for $v_0/v_A < 4$

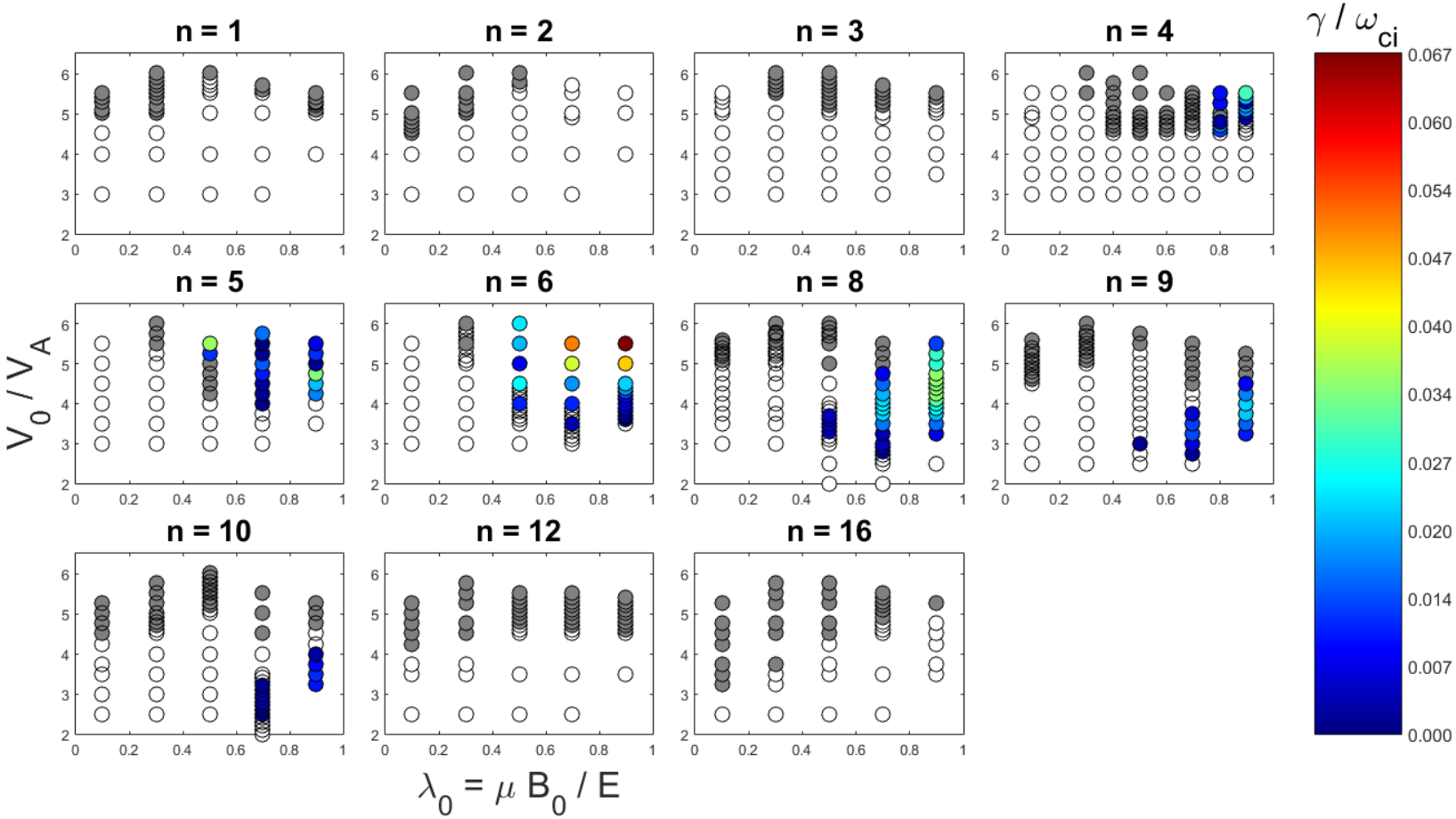
- Colored circles: CAE is most unstable mode
- Gray circles: Different mode has larger growth rate
- White circles: No unstable modes of any type



Cntr-GAE Unstable Even for Low V_0

- Frequencies : $\omega \sim 0.13 - 0.35\omega_{ci} = 2\pi(300 - 850)$ kHz
- Growth rates : $\gamma/\omega = 0.0009 - 0.41$, $\gamma_{max} = 0.067\omega_{ci} = 1050$ kHz
- Most unstable modes have n=5 – 8, large central pitch $\lambda_0 \geq 0.5$
- Counter-GAE unstable for v_0/v_A as low as 2.5

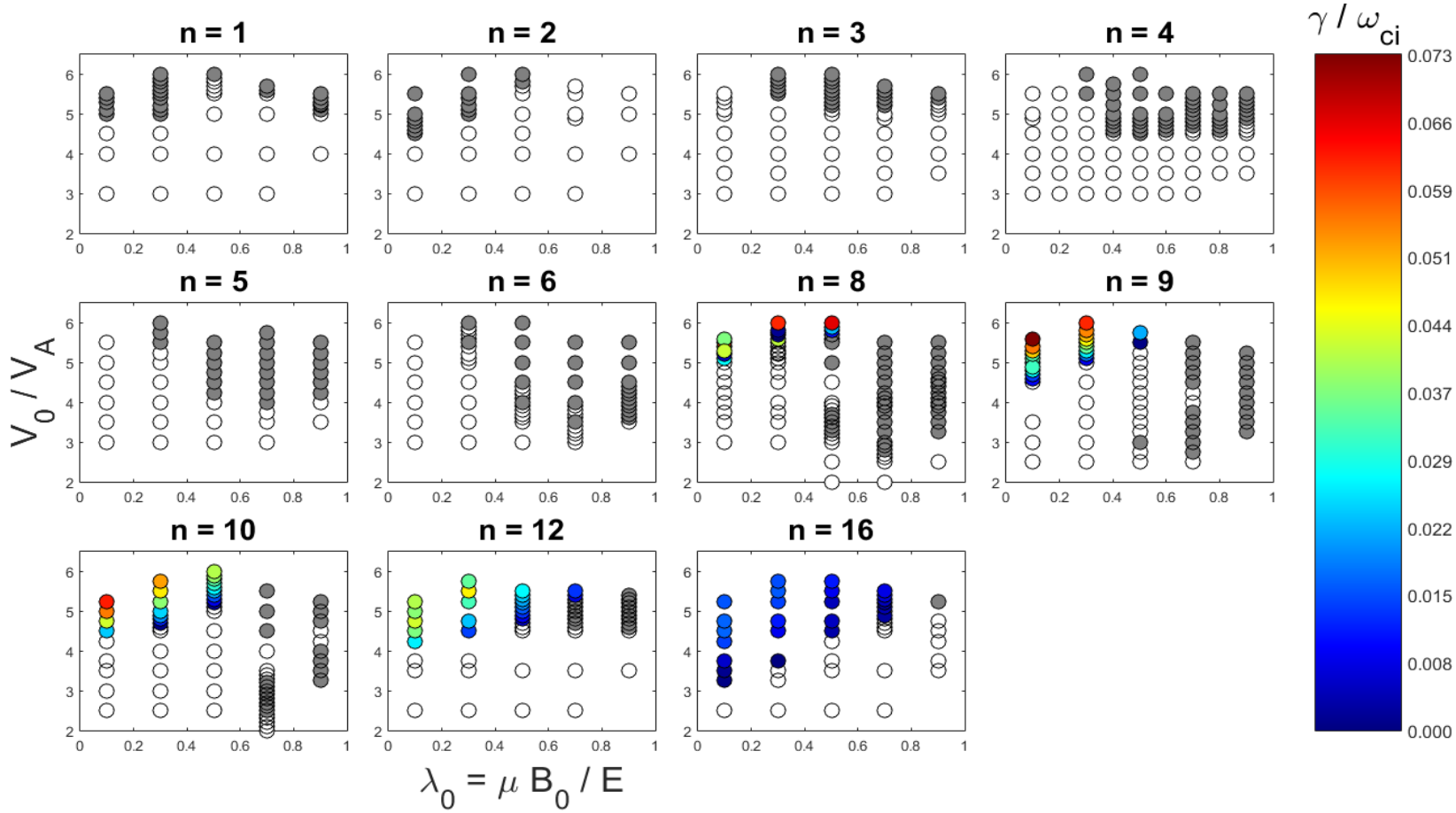
- Colored circles: cntr-GAE is most unstable mode
- Gray circles: Different mode has larger growth rate
- White circles: No unstable modes of any type



Co-GAE are Very Unstable

- **Frequencies : $\omega \sim 0.13 - 0.56\omega_{ci} = 2\pi(300 - 1400)$ kHz**
- **Growth rates : $\gamma/\omega = 0.0005 - 0.26$, $\gamma_{max} = 0.073\omega_{ci} = 1150$ kHz**
- **Most unstable modes have n=8 – 10, <u>small</u> pitch $\lambda_0 \leq 0.5$**
- **Co-GAE most unstable for $v_0/v_A \sim 5 - 6$**

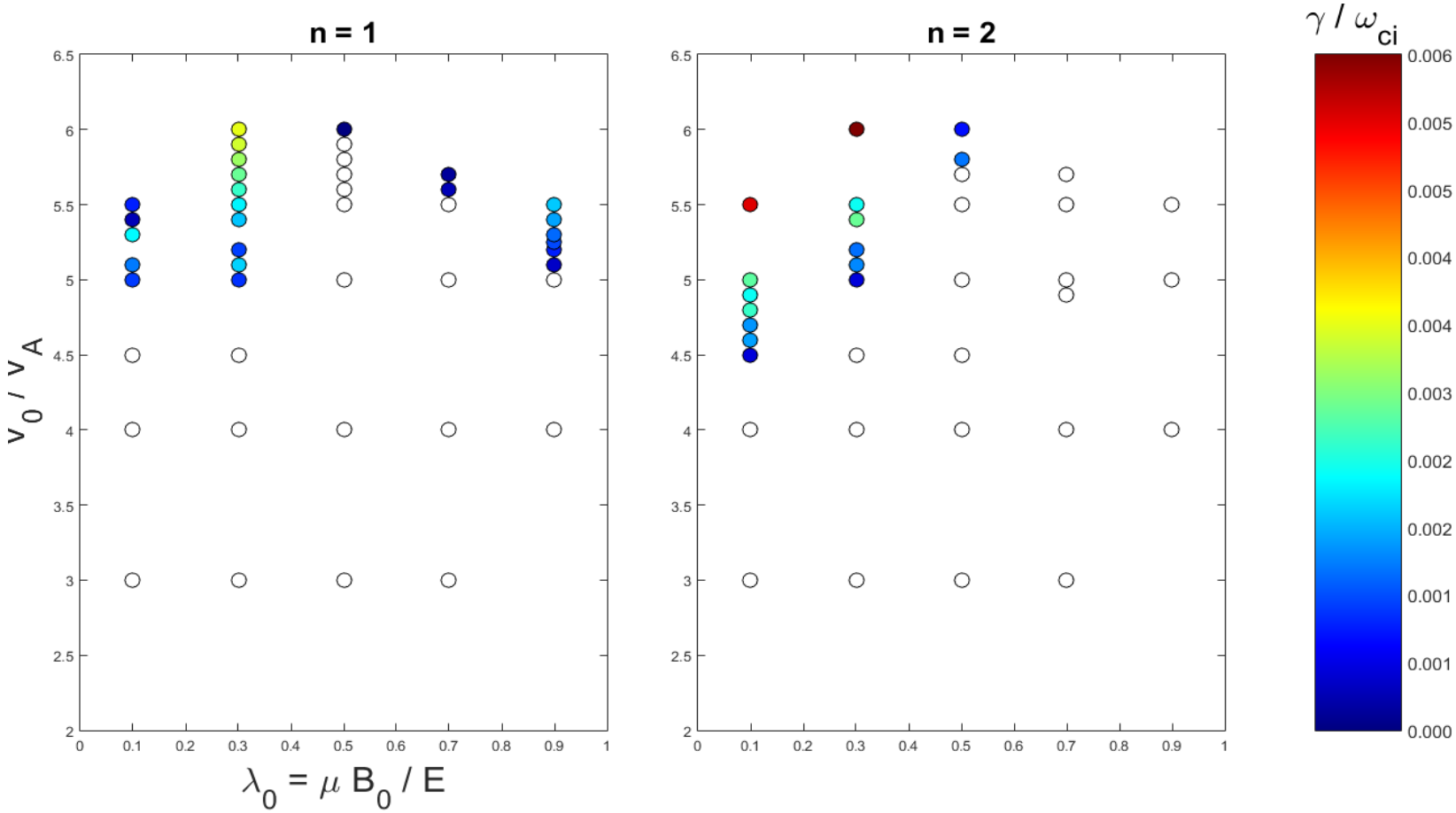
- Colored circles: co-GAE is most unstable mode
- Gray circles: Different mode has larger growth rate
- White circles: No unstable modes of any type



Unexpected Low-Frequency Modes

- **Frequencies : $\omega \sim 0.01 - 0.05\omega_{ci} = 2\pi(25 - 125)$ kHz**
- **Growth rates : $\gamma/\omega = 0.013 - 0.37$, $\gamma_{max} = 0.073\omega_{ci} = 90$ kHz**
- **Require large v_0 and either very high or low λ_0**

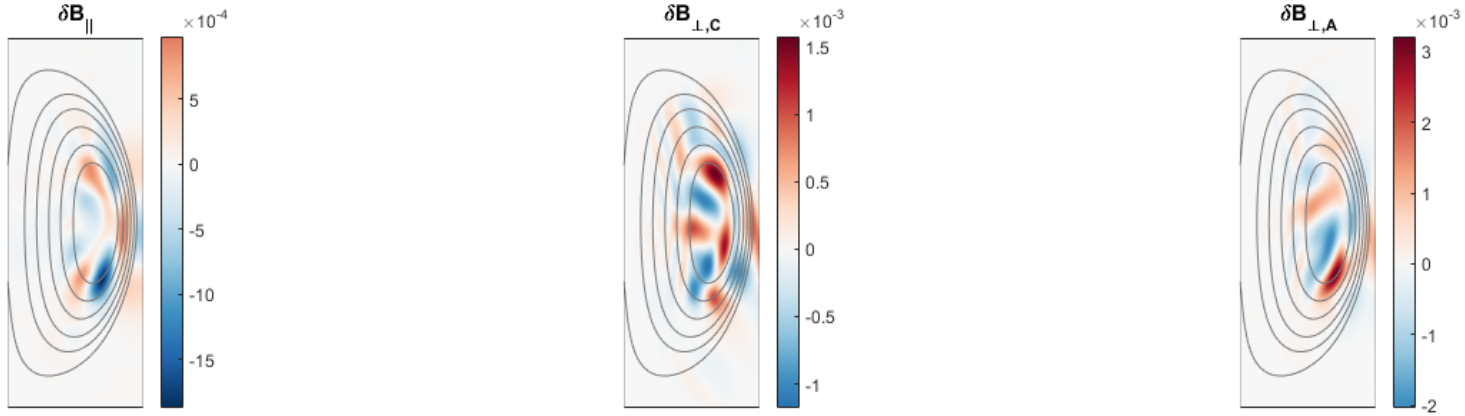
- Colored circles: low n mode is most unstable mode
- Gray circles: Different mode has larger growth rate
- White circles: No unstable modes of any type

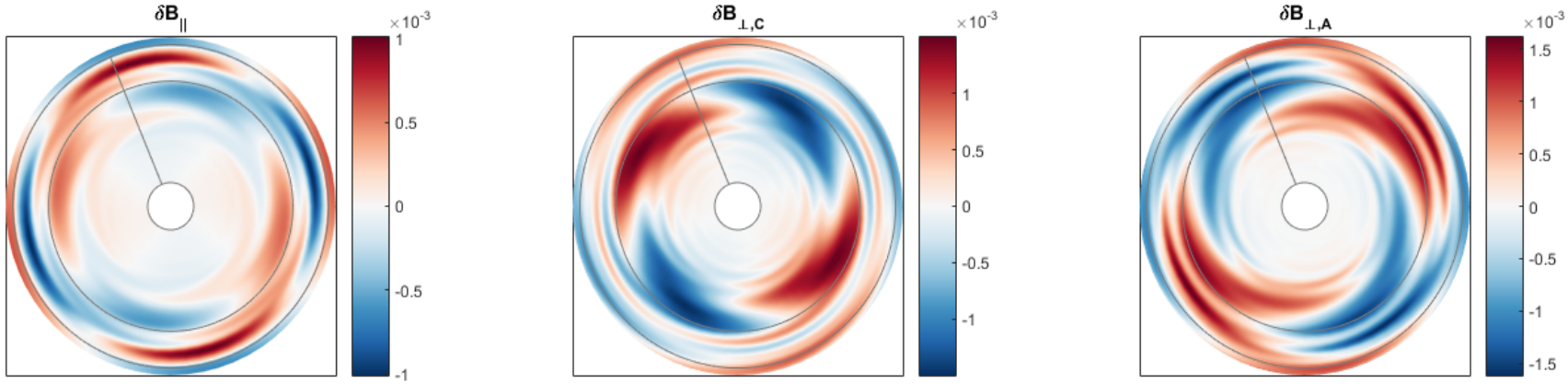


Low Frequency Modes are Qualitatively Different

- Typically m ≥ 3 (as large as m~6)
- What are they? TAE/fishbone/EPM/???

n=2
λ_0=0.3
v_0=5.4





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