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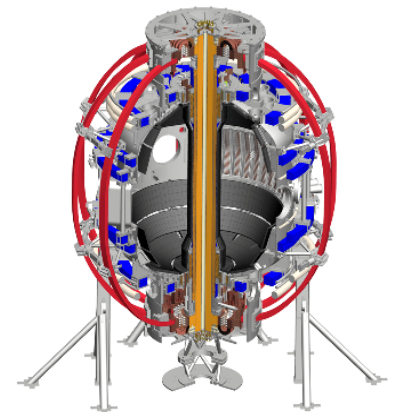


# Computation of AE stability and saturation through a reduced fast ion transport model in TRANSP

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NSTX-U Monday Meeting  
Room B318 - PPPL  
01/09/2017

[http://nstx.pppl.gov/DragNDrop/Draft\\_Paper\\_Review/MPodesta\\_kick\\_model\\_AEstability.pdf](http://nstx.pppl.gov/DragNDrop/Draft_Paper_Review/MPodesta_kick_model_AEstability.pdf)



# Including effects of Alfvénic modes (AEs) can be important for reliable time-dependent simulations

- NUBEAM/TRANSP now include a reduced, physics-based model for energetic particles (EP): *kick model*
  - Enables more accurate computation of NB heating, current-drive
  - Enables “numerical experiments” leveraging on TRANSP/NUBEAM capabilities
- Analysis with TRANSP + kick model has been quite successful to interpret experimental data
  - Thermal profiles from exp’t, mode properties from NOVA+exp’t
  - Tested on a variety of NSTX/NSTX-U, DIII-D scenarios
- > *Can the model be used for predictive AE+EP runs?*
  - > *What are the pros/cons of the kick model approach?*
  - > *What are the limitations?*

# Some definitions...

- *Predictive*: all quantities are predicted, including thermal profiles, q-profile, unstable spectrum, etc.
  - *Semi-predictive*
  - *Semi-interpretive*
- } Typical range of kick model simulations
- *Interpretive*: all quantities are known (thermal profiles, mode spectrum vs time) -> how is EP population responding?

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# Outline

- Methodology for interpretive vs predictive analysis
  - Inferring linear mode stability
  - Inferring mode saturation amplitude
- Example from NSTX #141711
  - Predictions of AE linear stability, saturation, EP behavior
  - Comparison with experiment
- Future work

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# Power balance is the key for kick model analysis in NUBEAM/TRANSP

Consider evolution of mode energy,  $E_w$

$$\frac{\partial E_w}{\partial t} = P_{EP} - 2\gamma_{damp} E_w$$

power from EPs to the mode      damping rate

Introduce 'growth rate' in analogy with damping rate:

$$\gamma_{gr}(E_w) \doteq \frac{P_{EP}(E_w)}{2E_w} \Rightarrow \frac{\partial E_w}{\partial t} = 2[\gamma_{gr} - \gamma_{damp}] E_w$$

- In the limit of vanishing mode amplitude (*linear* phase)

$$\gamma_{lin} \doteq \lim_{E_w \rightarrow 0} \frac{P_{EP}(E_w)}{2E_w} \quad \leftarrow \text{linear growth rate}$$

- In the limit of vanishing  $dE_w/dt$ , finite  $E_w$  (*saturated* phase)

$$\gamma_{gr}(E_w) - \gamma_{damp} \rightarrow 0 \Rightarrow E_w \rightarrow E_w^{sat} \quad \leftarrow \text{saturation amplitude}$$

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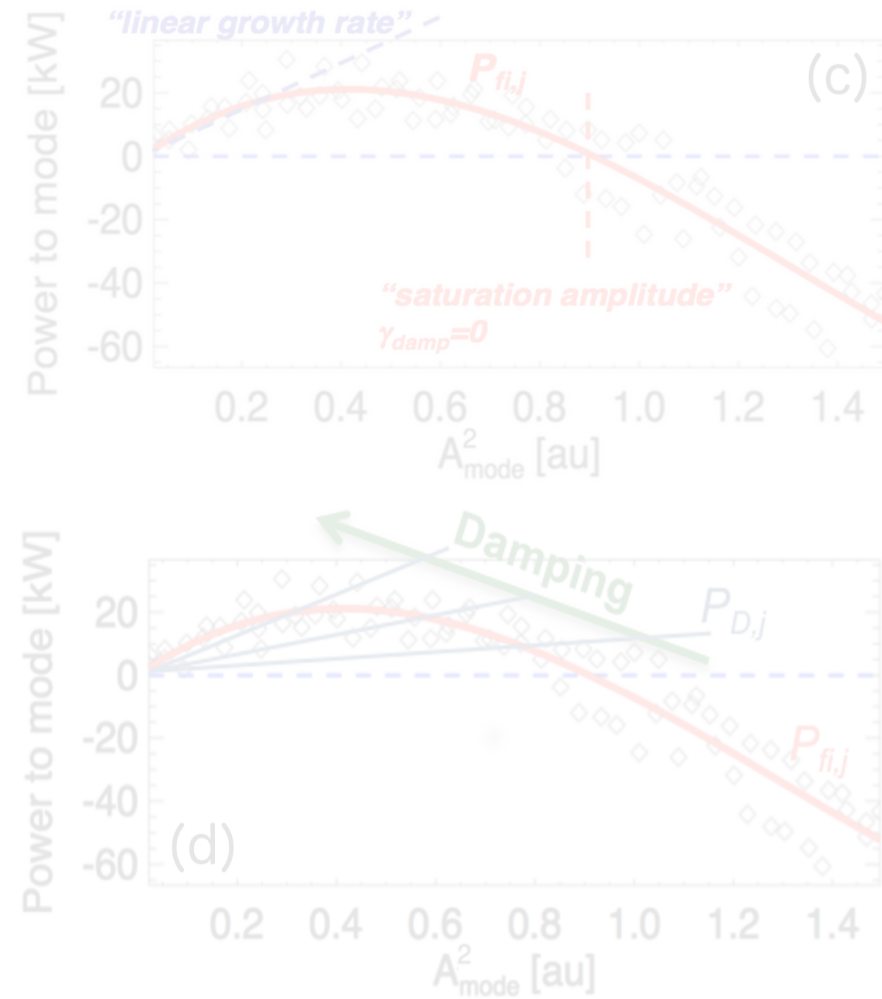
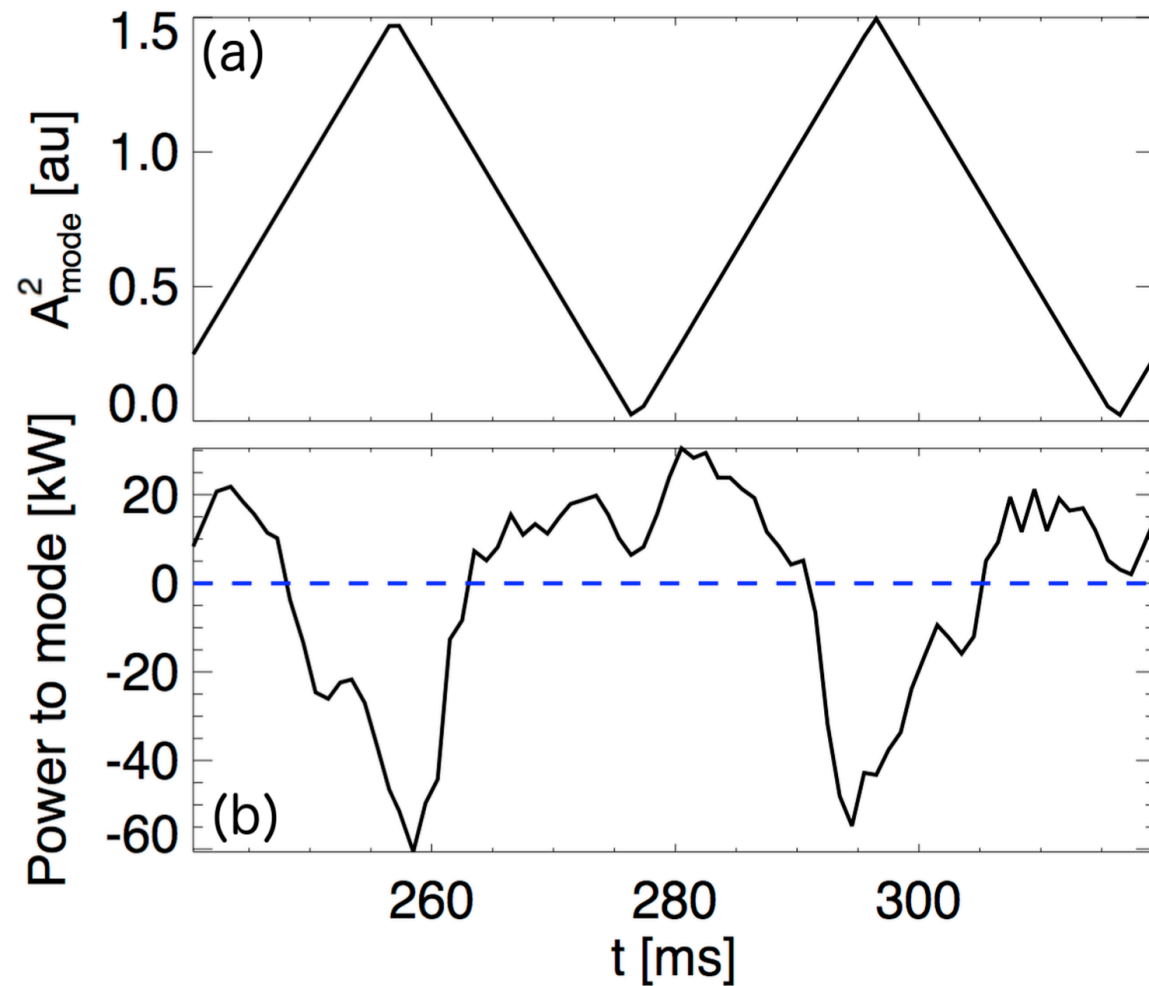
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# Linear growth rate and saturation amplitude can be inferred from $P_{EP}(E_w)$ characteristic

TRANSP run: sweep mode amplitude  $\sim \sqrt{E_w}$ , record power from EPs to the mode

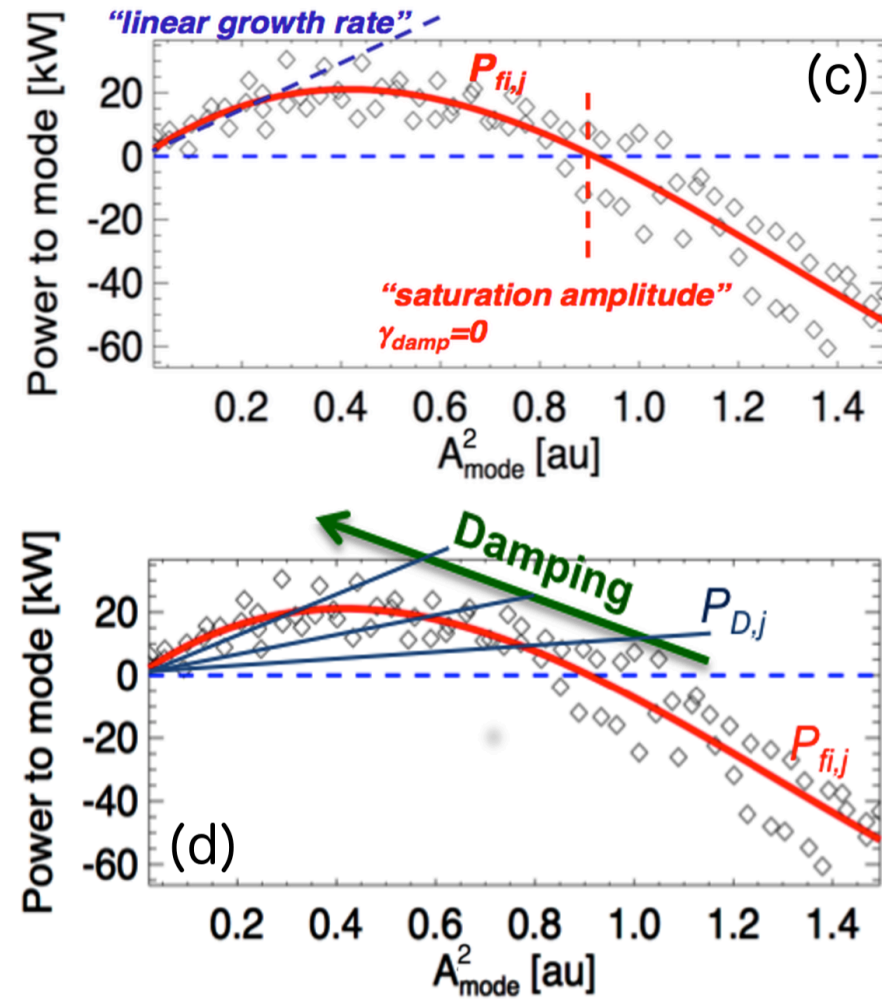
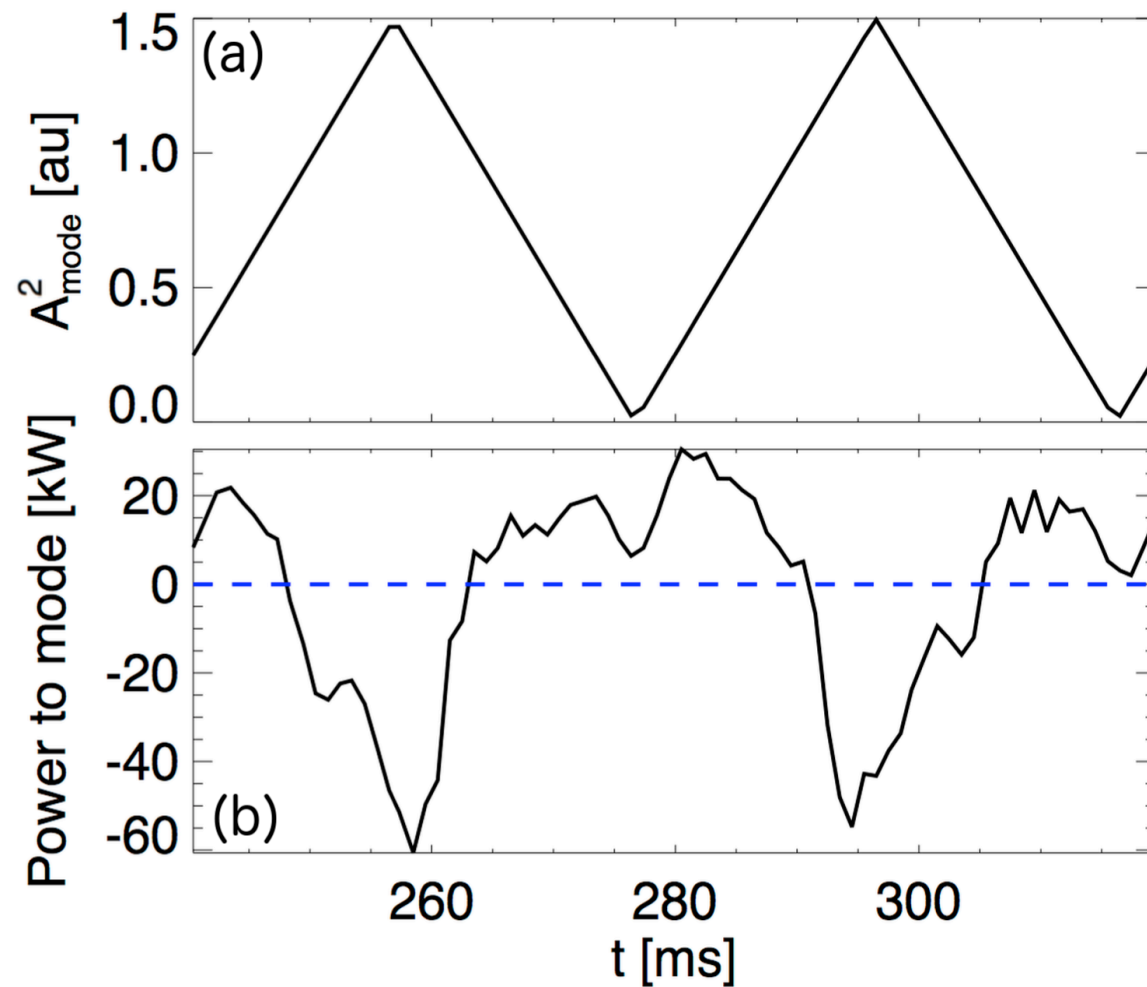
Build  $P_{EP}(E_w)$  curve, infer  $\gamma_{lin}$  and saturation amplitude



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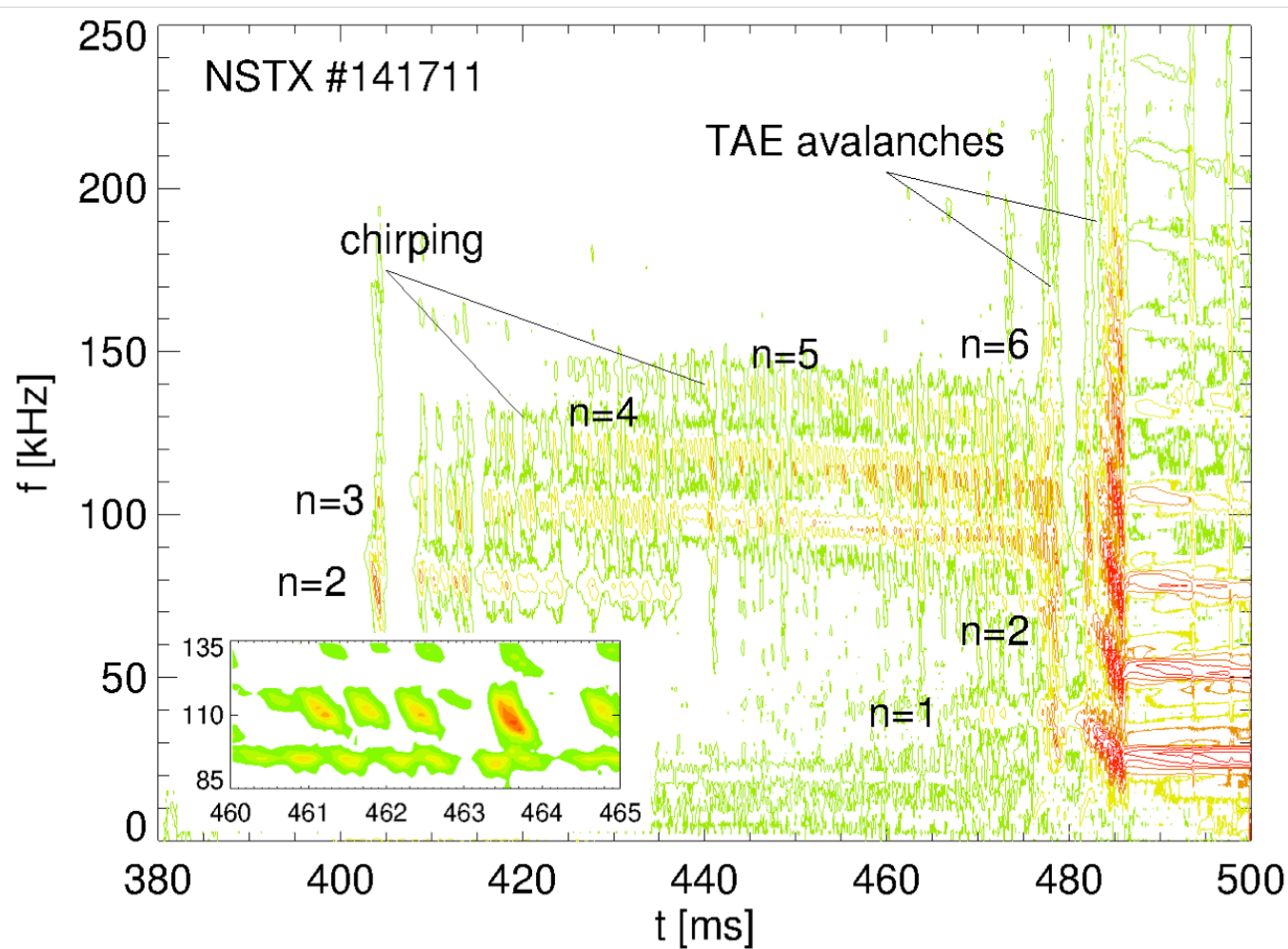
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# Target scenario: NSTX #141711

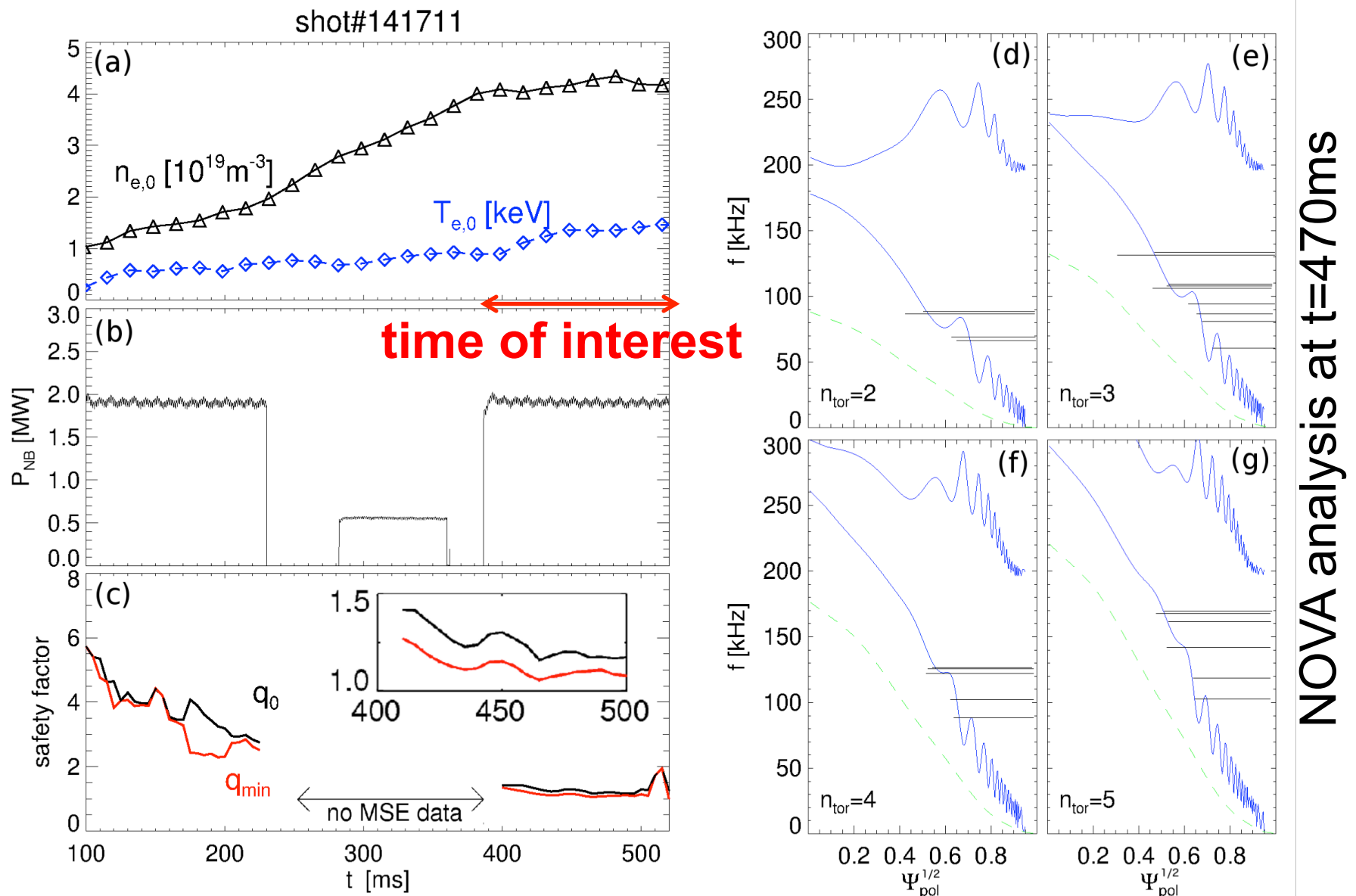
NB-heated L-mode featuring robust TAE activity

> *Neglect TAE avalanches in this work*



# Target scenario: NSTX #141711

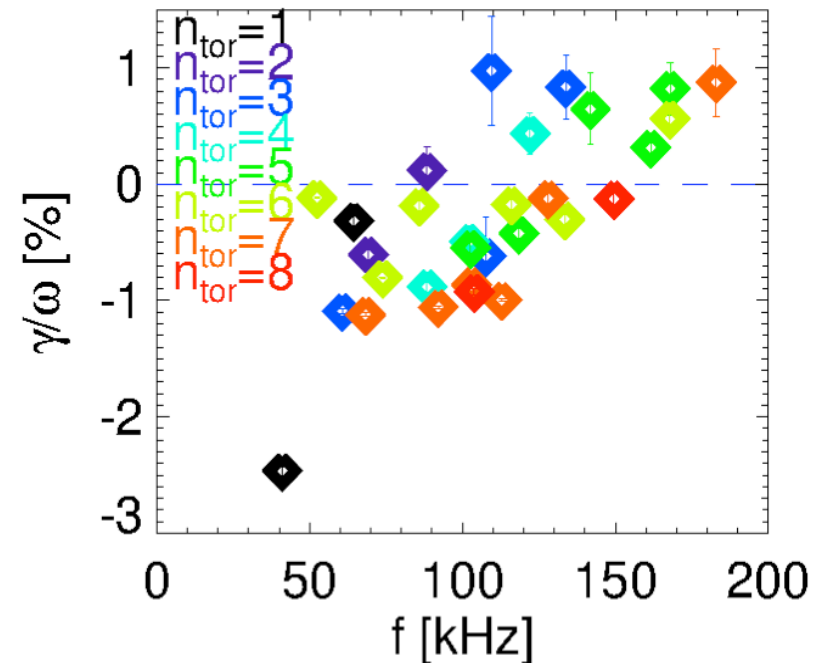
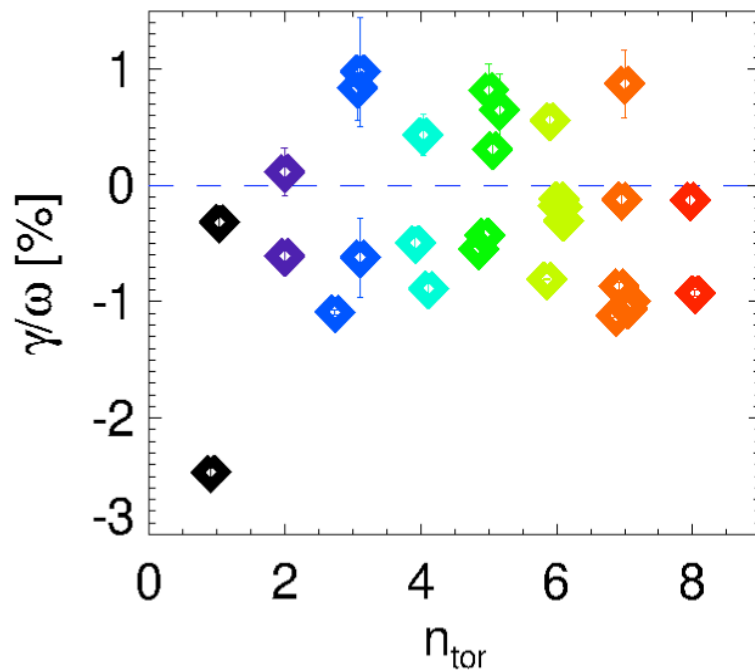
## NB-heated L-mode featuring robust TAE activity



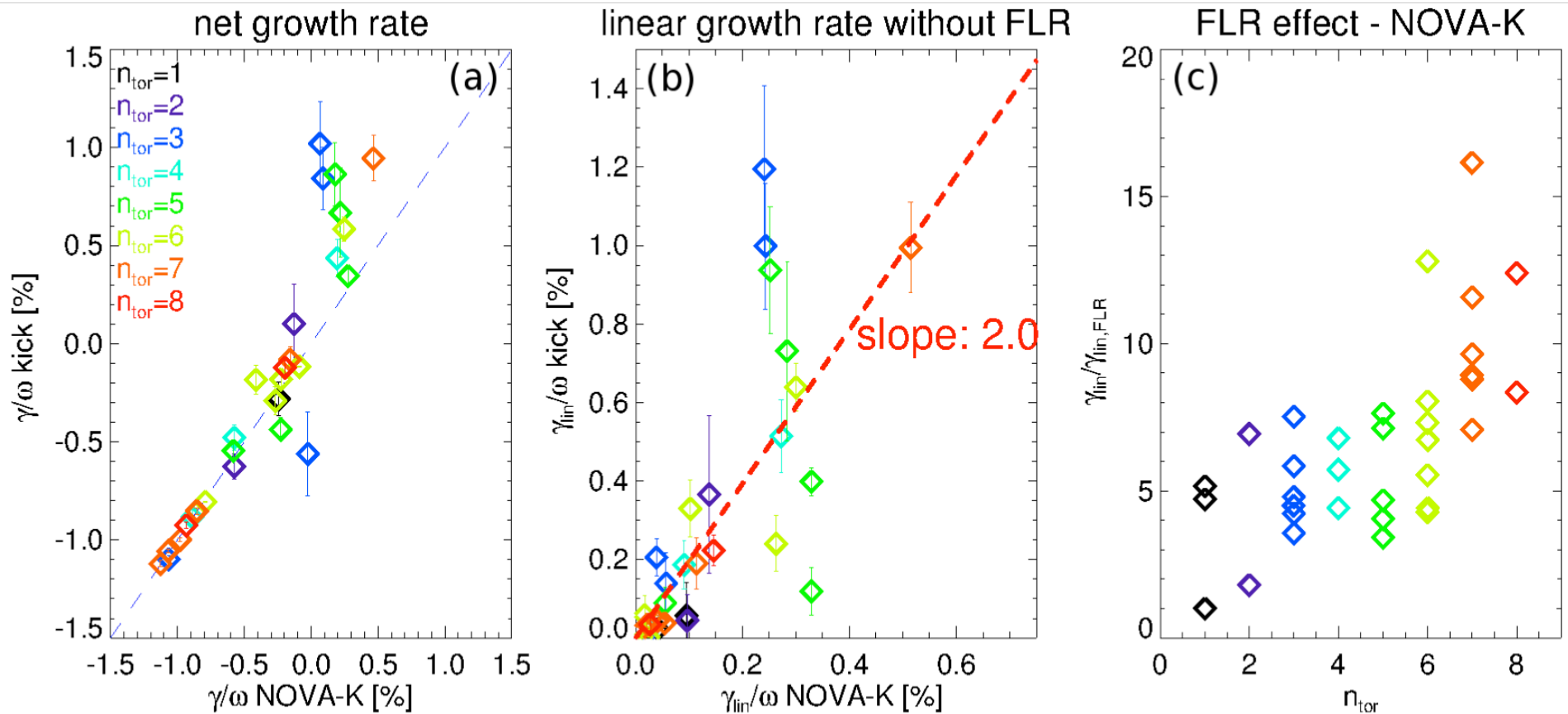
# NOVA analysis provides $\sim 50$ candidate modes with toroidal mode numbers $n=1-8$

- NOVA finds *all* eigenmodes for given toroidal mode number/frequency range - regardless of their stability
- ‘Linear’ kick model analysis identifies 9 unstable modes with  $n=2-7$

*net growth rate including damping from NOVA*



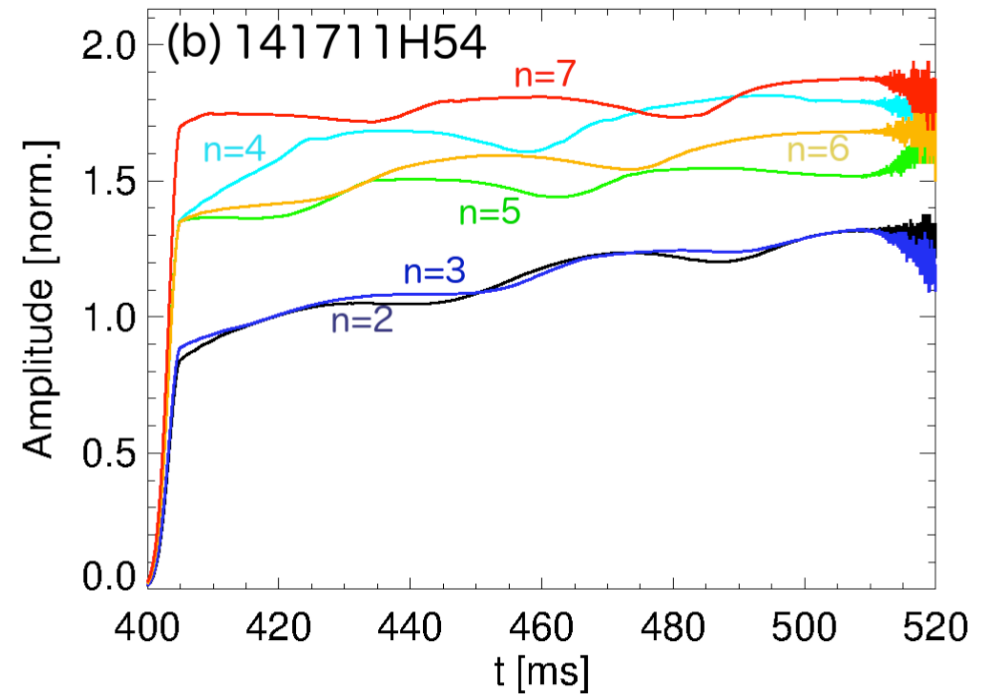
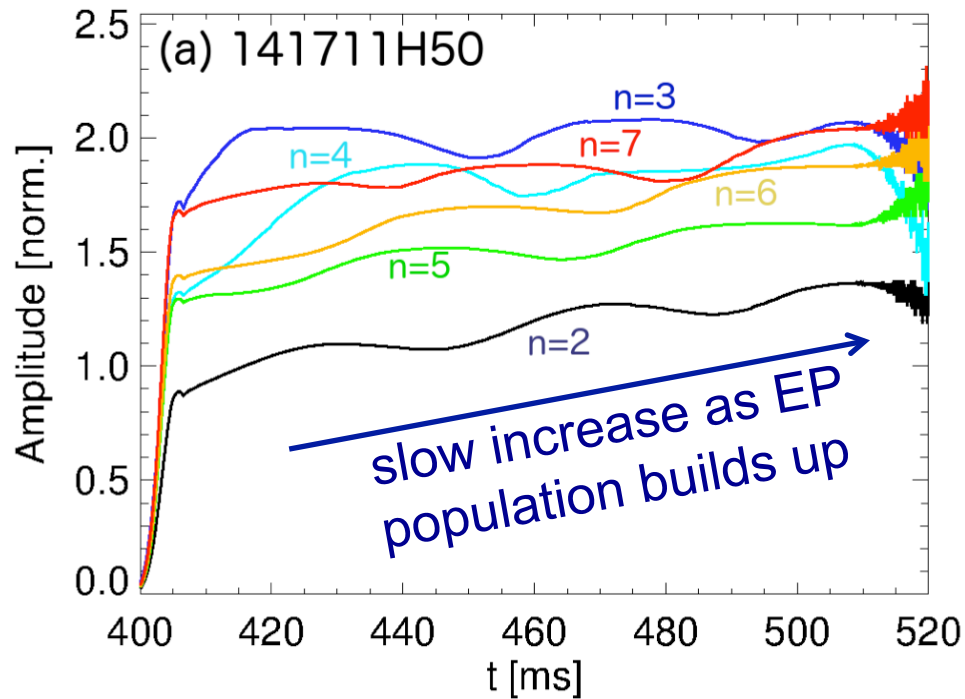
# On average, kick model computes 2x larger $\gamma_{lin}$ than NOVA-K



- Difference likely due to different fast ion distribution, lack of sources/sinks in NOVA-K
- FLR effects *not* included in ORBIT (-> kick model)
  - Inferred growth rates, saturation amplitudes are upper limit

# Saturation amplitudes are $\delta B_r/B \sim 10^{-4} - 10^{-3}$ for weakly bursting/chirping phase

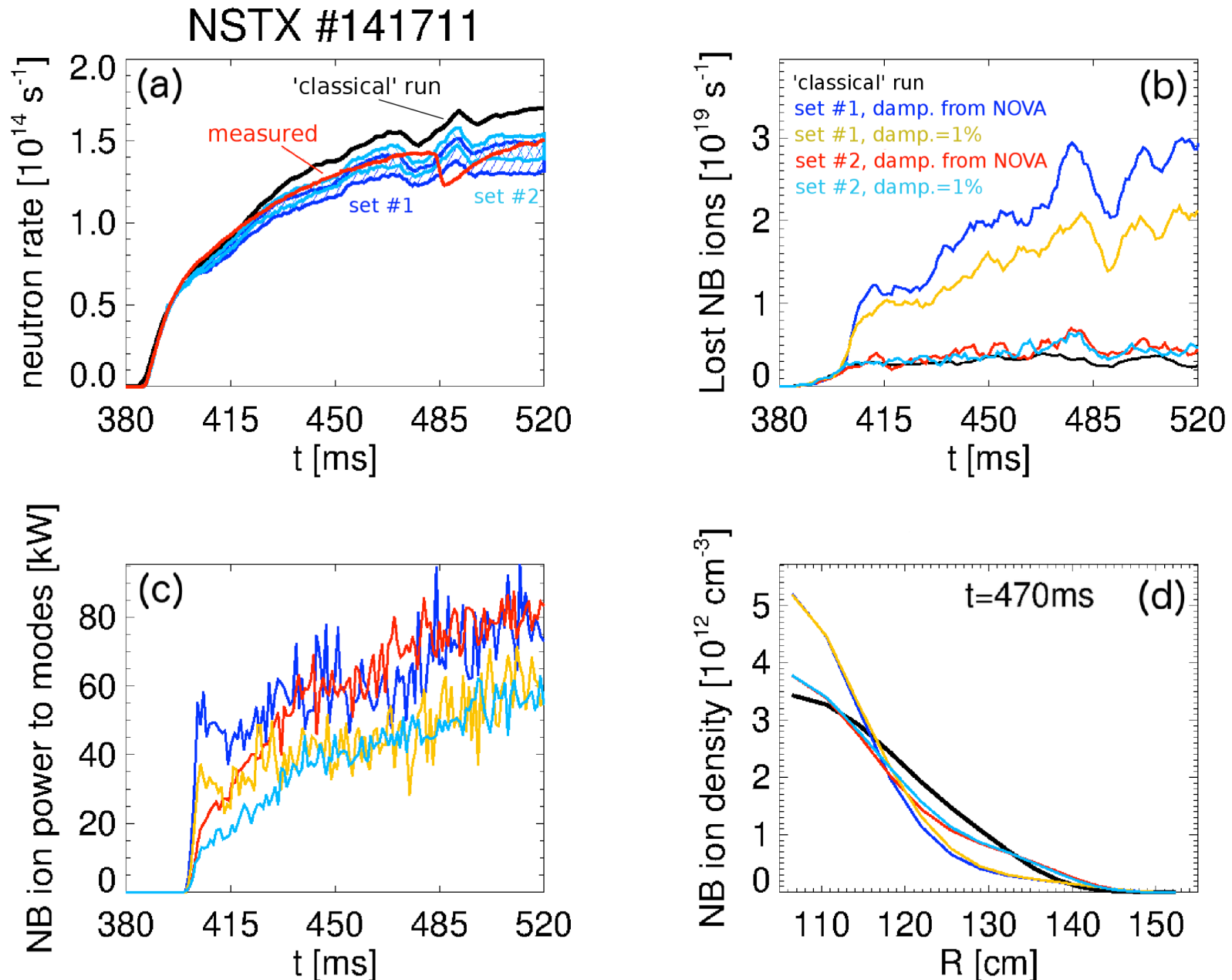
- Divide 9 unstable modes into two sets,  $n=2-7$



- Amplitude =1 corresponds to  $\delta B_r/B = 5 \times 10^{-4}$
- Inferred saturation amplitudes appear reasonable, based on previous analysis
  - $\delta n/n$  from UCLA reflectometer
- *Results can vary considerably if different damping rates are used*



# TRANSP computes similar neutron deficit for the two sets of 'unstable' modes



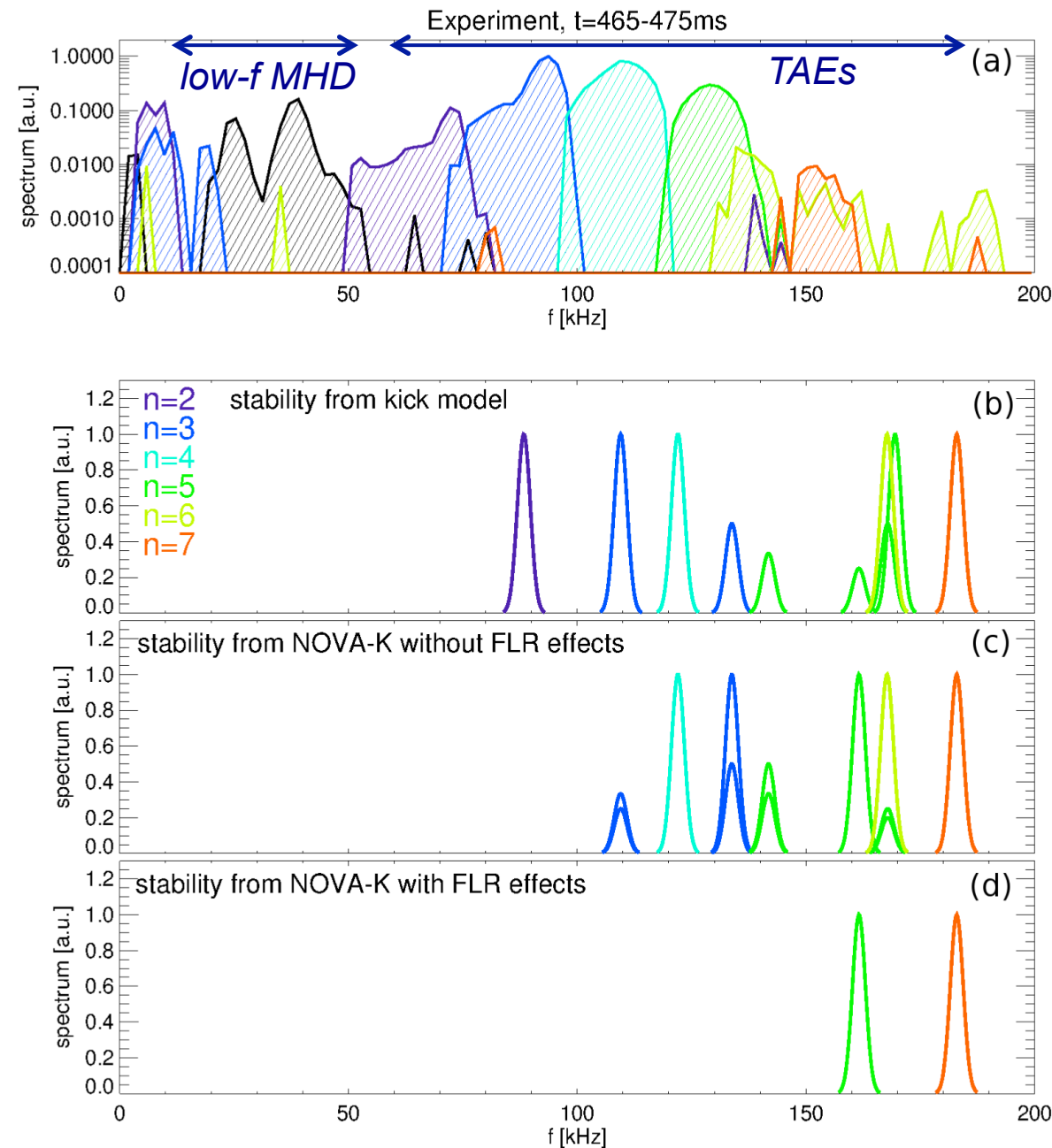
# Summary of *semi-predictive* kick model analysis

- Kick model *can* be used for semi-predictive analysis
  - Requires thermal profiles
  - Requires candidate modes (, damping rates) from NOVA/NOVA-K
- NUBEAM/TRANSP implementation has several advantages over single-time-slice analysis
  - Provides well-diagnosed time-dependent simulations
  - (Classical) sources, sinks are accurately simulated
  - Integrates non-classical EP effects into whole discharge simulation
- Main limitations:
  - Relies on other codes to infer mode structure, damping rates
  - Best suited for slowly varying background profiles ( $I_p$ ,  $q(r)$ )
  - Requires additional analysis to “predict” bursting/chirping modes (cf. recent work by Duarte, Berk, Gorelenkov)

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# Predicted unstable spectrum is in reasonable agreement with measurements

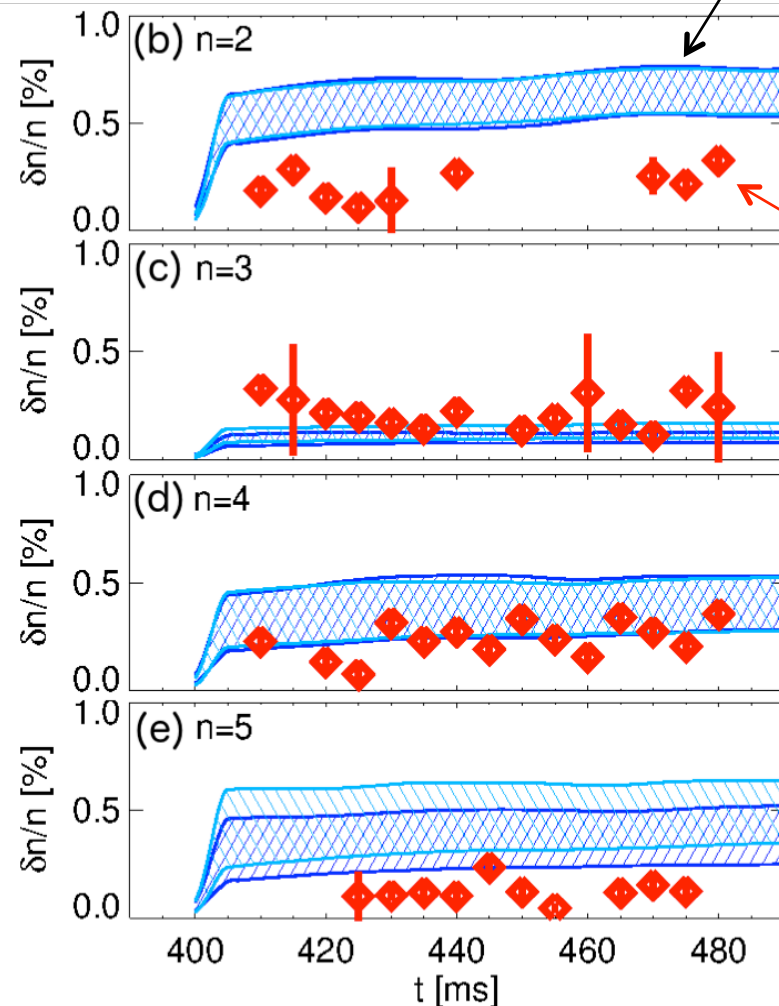
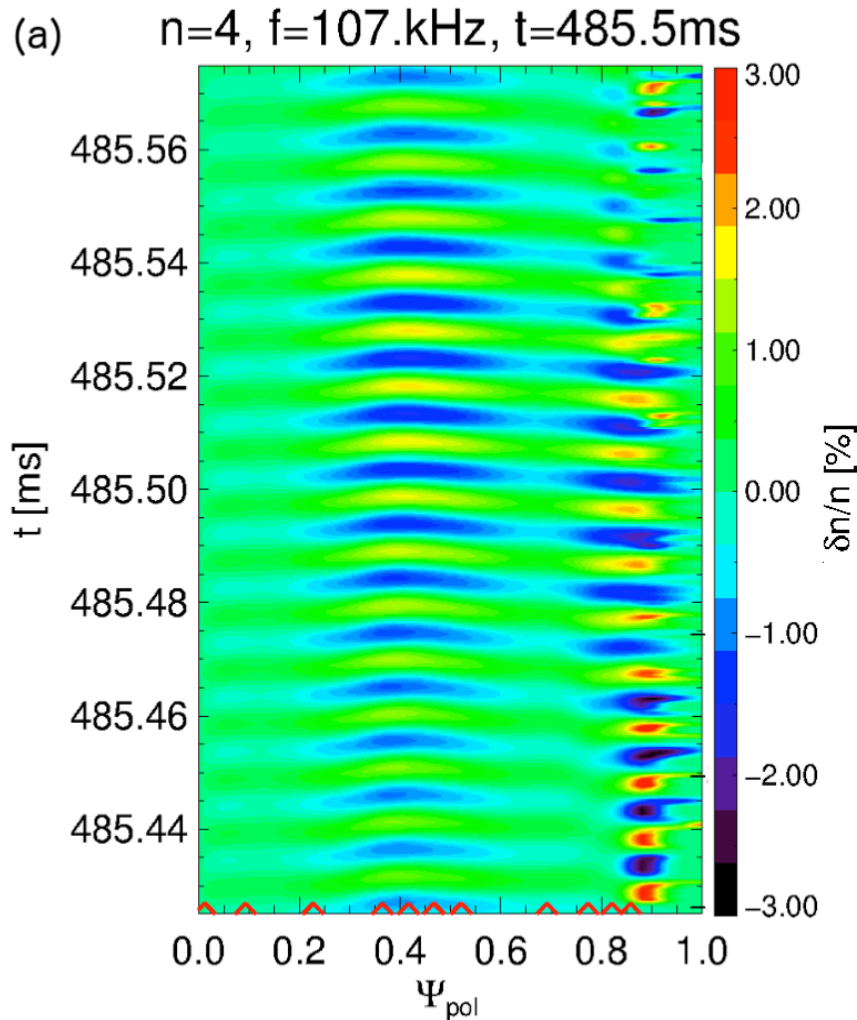


- Experiment shows a rich spectrum:
  - TAEs, low-f MHD (, EHO?)
- Predicted spectrum looks ~OK
- Frequency (NOVA) shifted up by ~10-20kHz, probably due to choice of  $\gamma_{\text{adiab}}$
- NOVA-K finds similar unstable spectrum *if FLR effects are neglected*
- NOVA-K seems to underestimate  $\gamma_{\text{lin}}$  when FLR effects are included

# (Average) predicted saturation amplitude is within x2-3 from measurements

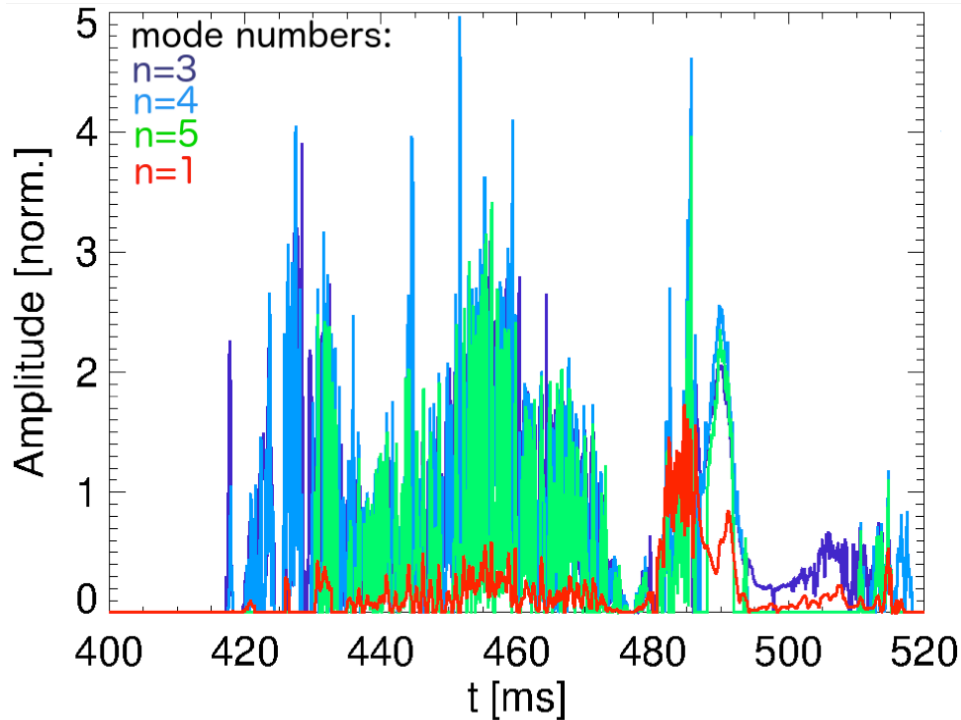
- $\delta n/n$  from UCLA reflectometer
  - Average over TAE bursts every  $\sim 5$ ms

Colors: two sets of modes  
Range: damping from NOVA/  
NOVA-K vs  $\gamma_{\text{damp}} = 1\%$

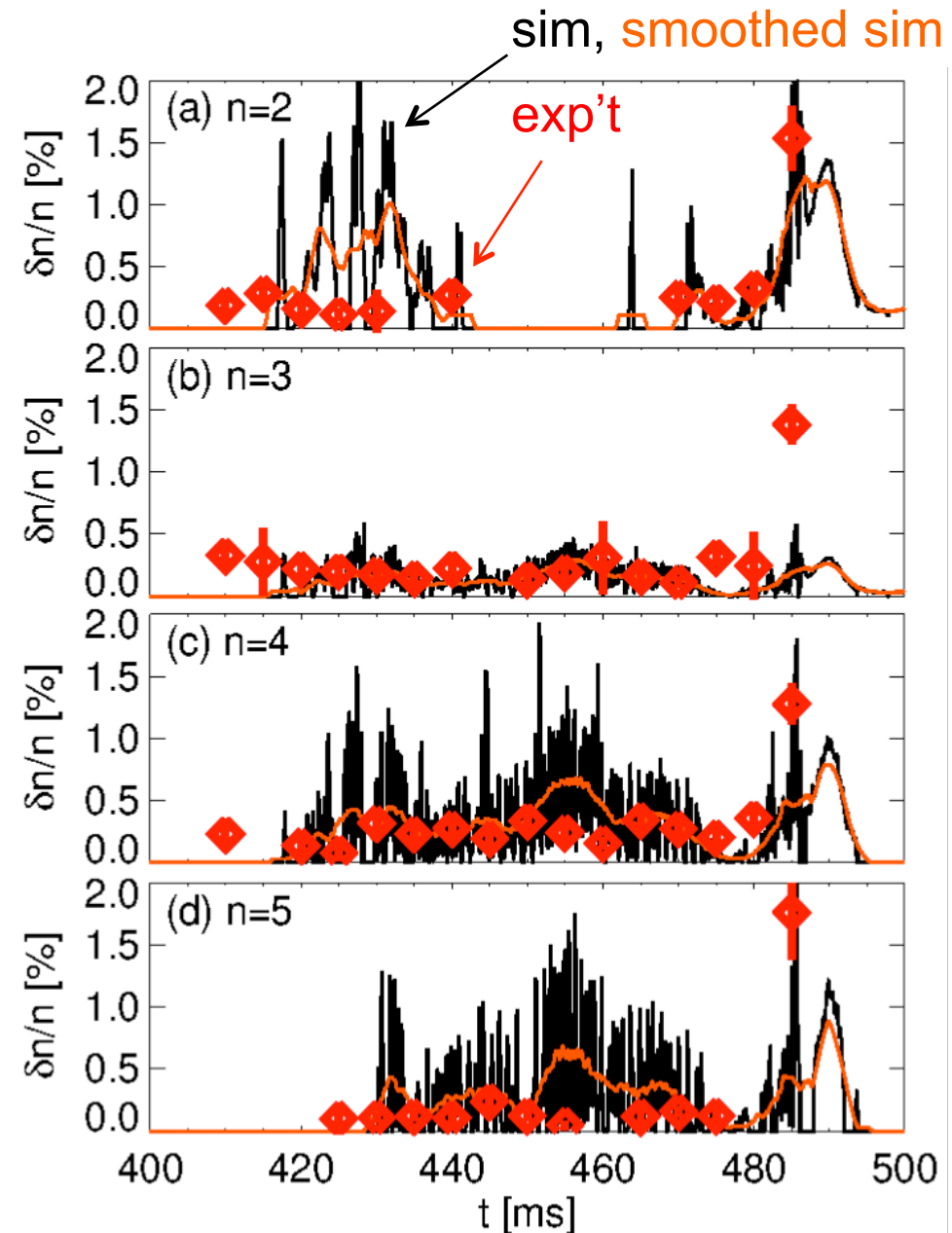


from  
experiment

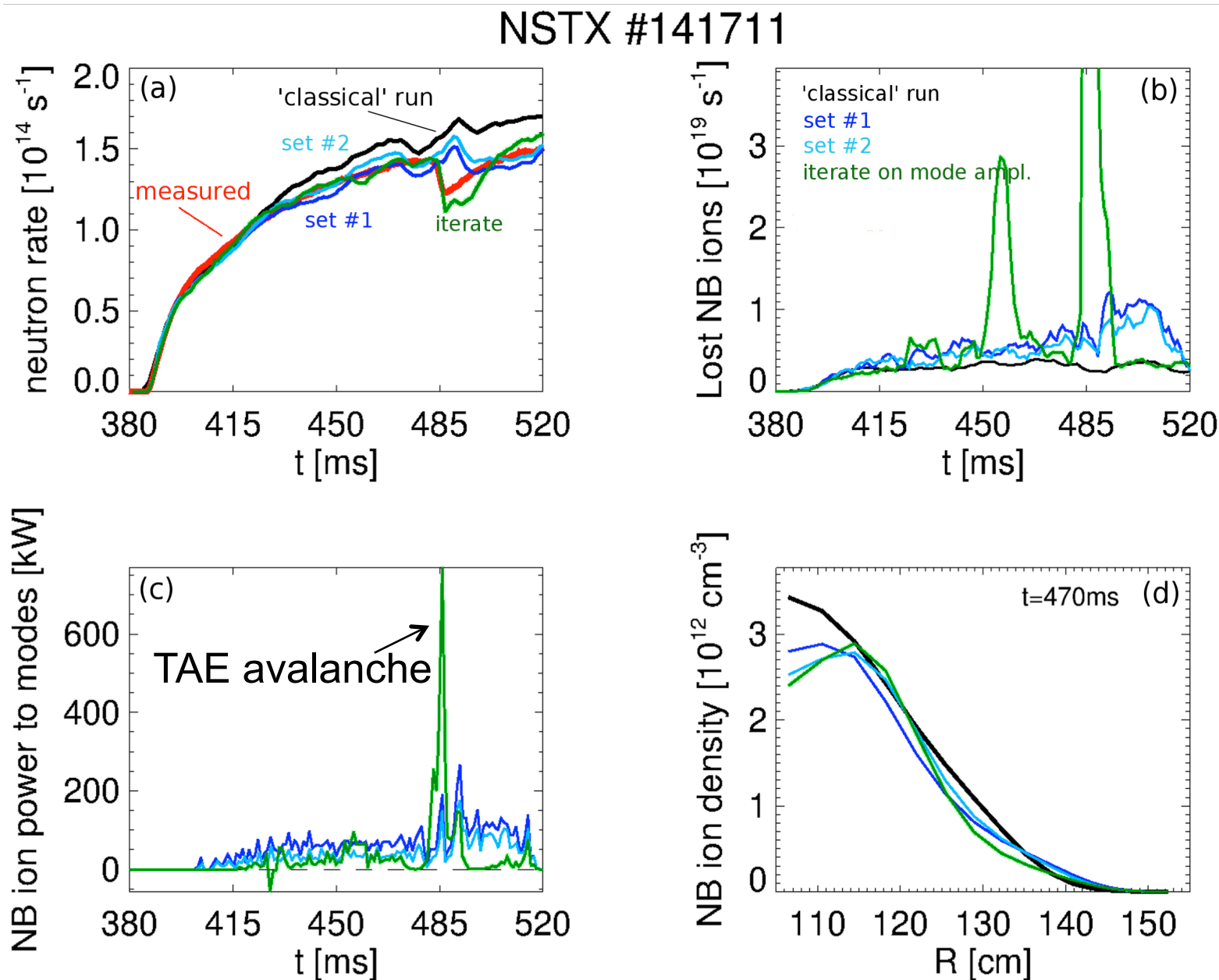
# Including bursting behavior & iterating to match neutron rate brings $\delta n/n$ closer to measurements



- Infer bursts from Mirnov coils
- Iterate TRANSP runs to match measured neutrons
- *Also include  $n=1$  from mode-mode coupling (exp't evidence)*



# Interpretive TRANSP runs lead to closer match with experiment



# Summary of (semi-)interpretive kick model analysis

- Additional information from actual experiment greatly improves kick model results
- In most cases, actual analysis is a mix of prediction/interpretation
  - E.g. mode amplitude(s) known at limited times, then ‘extrapolate’ based on match with neutrons
- Overall, semi-interpretive analysis works well for NSTX/NSTX-U, DIII-D scenarios
  - Promising results so far, including ‘counter-TAE’ case with off-axis NB2A (NSTX-U #203609)
  - Initial comparison with extended set of EP diagnostics (neutrons, FIDA, NPA, mode amplitudes) looks very promising (Heidbrink, PoP 2016 - submitted)



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# Outlook

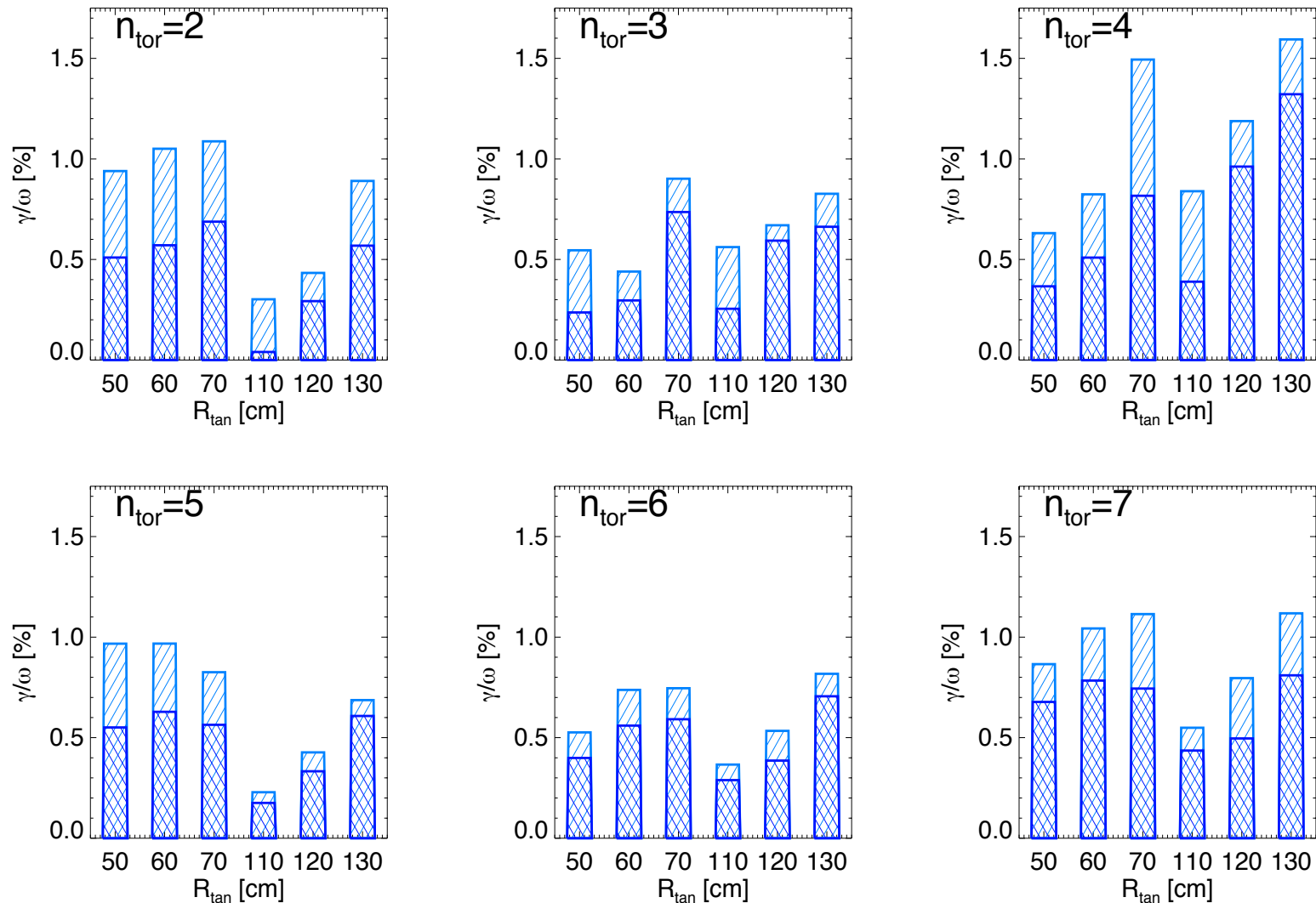
- Analysis procedure has been established - moving on to more extensive validation
  - Target NSTX/NSTX-U and DIII-D scenarios to cover broad range of experimental conditions
  - Include comparison with FIDA, NPA, possibly sFLIP/FILD
- Mid-term: exploit kick model infrastructure in NUBEAM to implement CGM/RBQ model in TRANSP (Gorelenkov)
- Longer term developments:
  - Extend kick model to 3D-fields effects
    - May require some modifications to the code (NUBEAM)
    - Would enable time-dependent analysis with sources/sinks
  - Include gyro-averaging effects as 'default' (ORBIT)
  - Remove constraint on  $\mu=\text{const}$ : extend to high-f CAEs/GAEs
  - Extend computation of kick probabilities to full-orbit codes (SPIRAL, others)

# Backup

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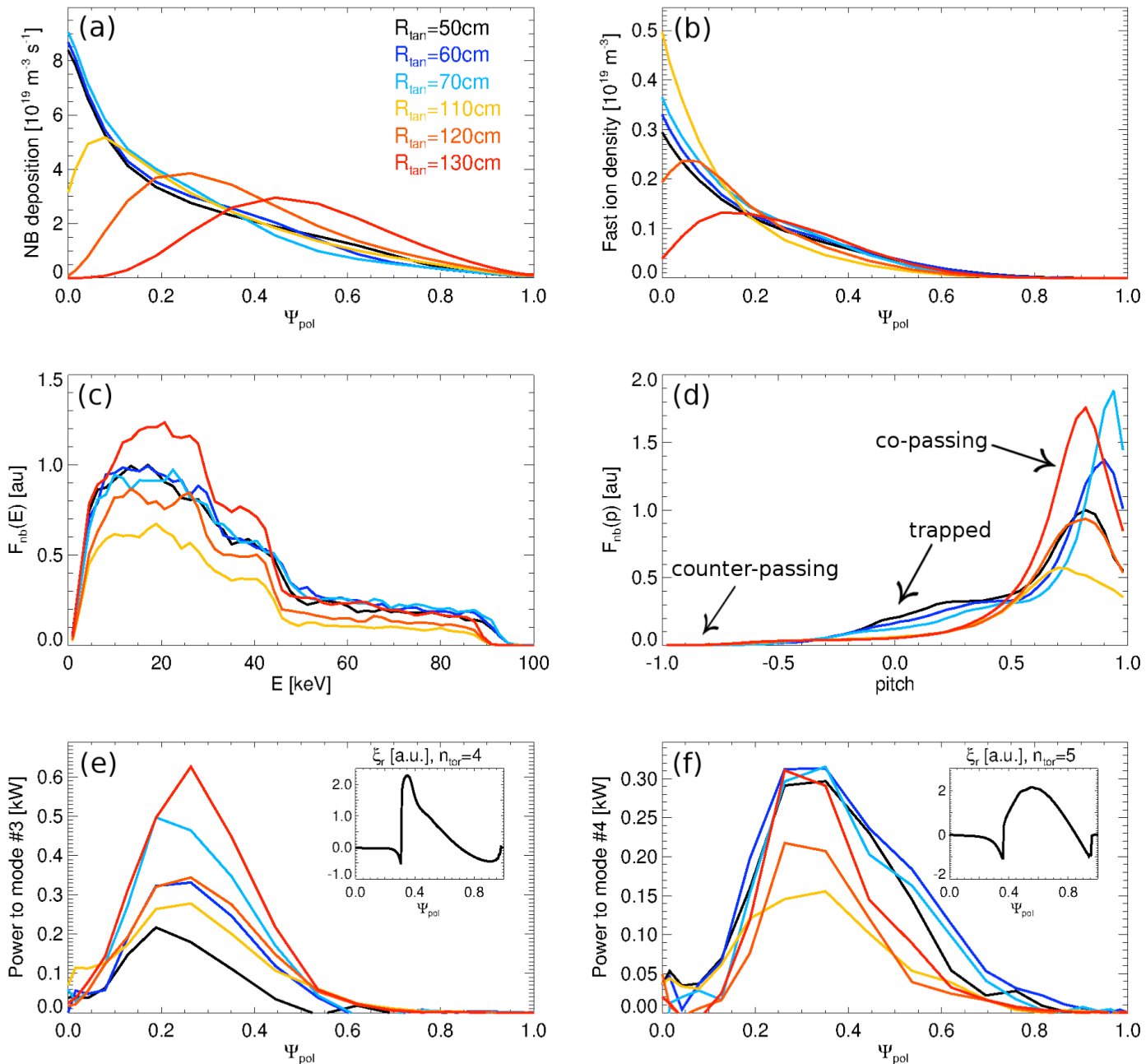
# Kick model enables *numerical experiments* for scenario development, e.g. to optimize NB mix

- Computed growth rate varies with NB tangency radius

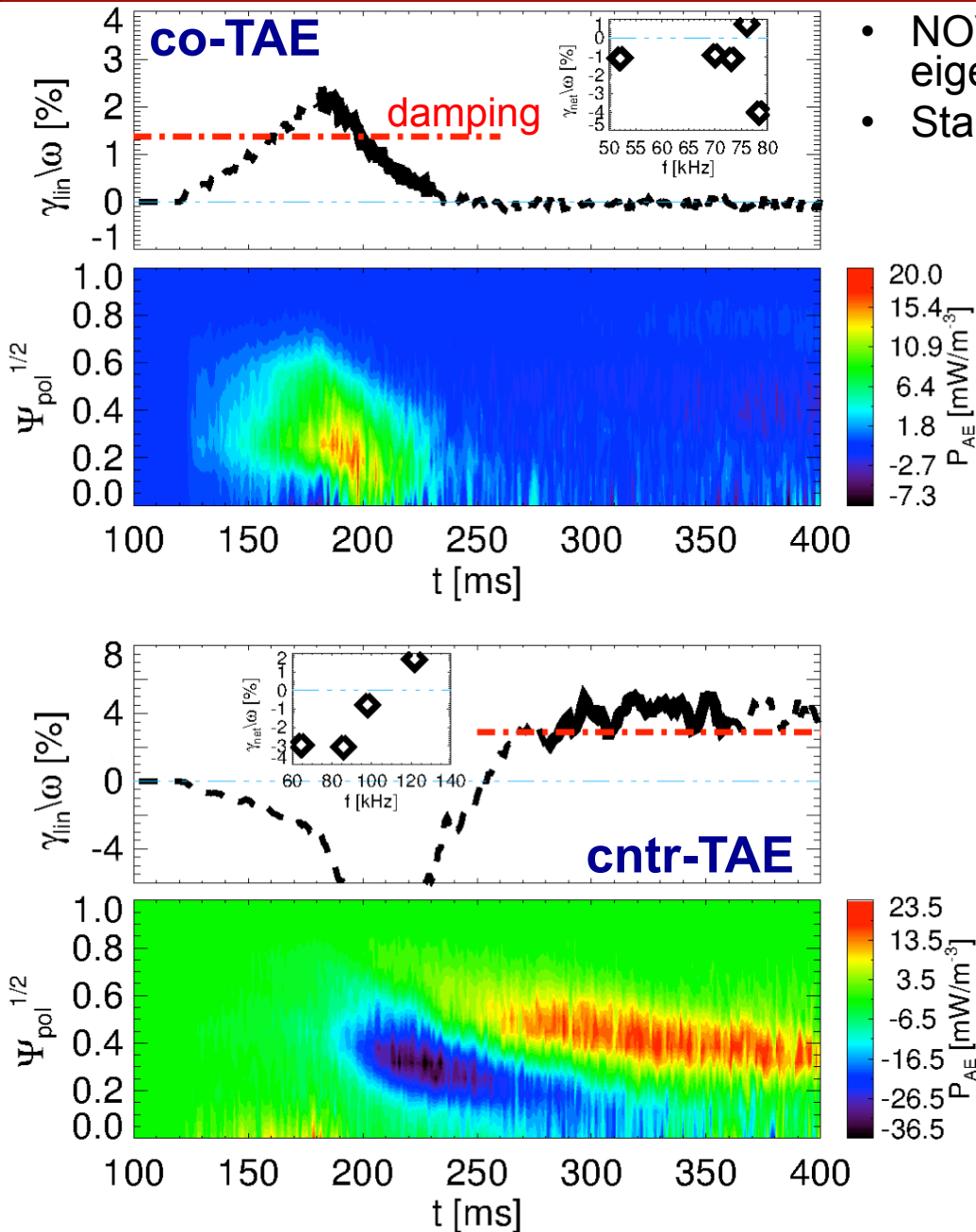


- No obvious correlation found...

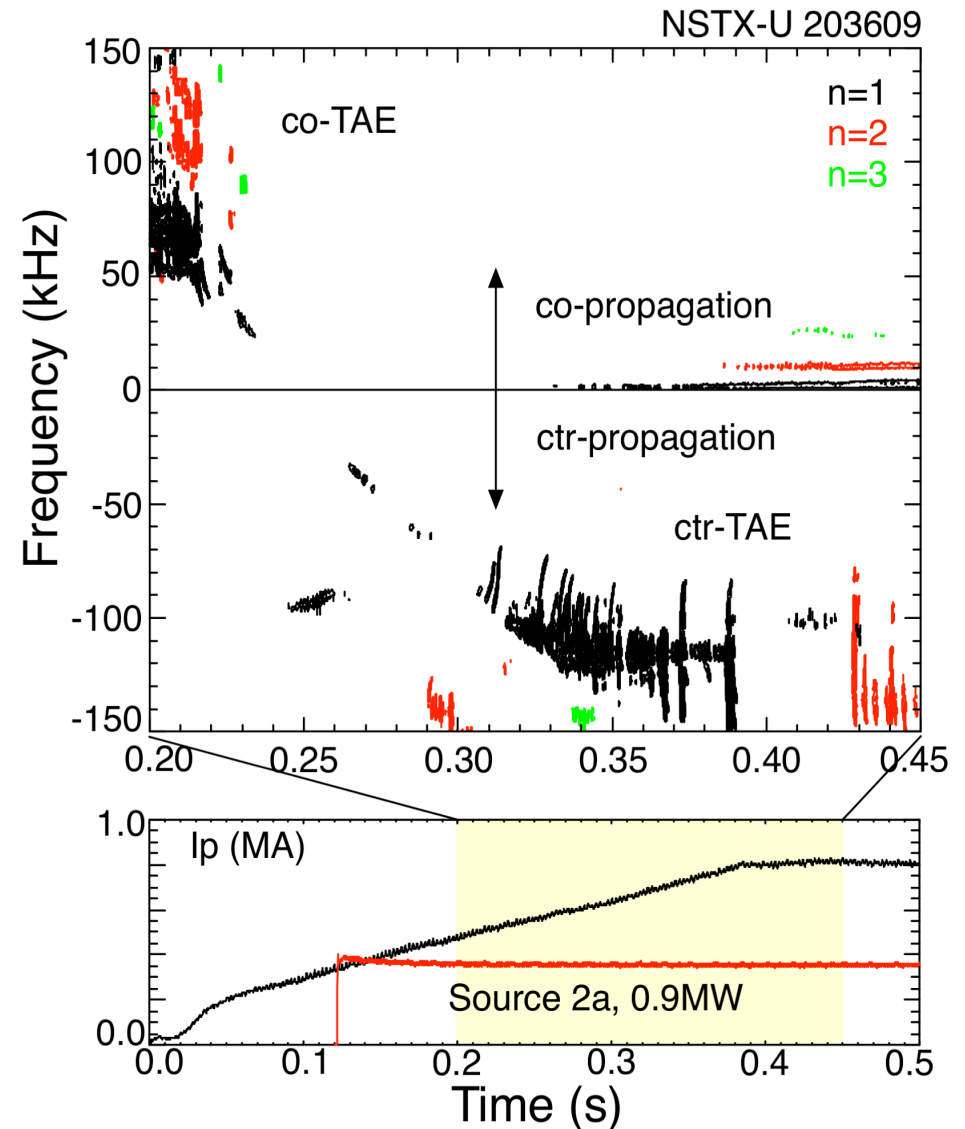
# Diagnostics in TRANSP are crucial to unfold complex dependence NBI $\leftrightarrow$ TAE stability



# Kick model analysis recovers transition from co- to cntr-TAEs during off-axis NBI (*all preliminary!*)



- NOVA modeling at  $t=200$ ms and  $t=320$ ms provides  $n=1$  eigenmodes
- Stability analysis used to select co- and cntr  $n=1$  modes



# Comparison of some reduced models used for EP transport

	<i>ad-hoc</i> $D_{fi}$	<i>CGM model</i> (*)	' <i>kick</i> ' model
<b>physics-based</b>	no	yes	yes
<b>required input</b>	$D_{fi}(\rho, t)$	growth/damping rates	probability, mode amplitude
<b>applicability</b>			
	indirectly	multiple AEs	AEs, kinks, NTMs. Fishbones/ EPMs?
<i>multi-mode</i>			
<i>steady-state</i>	yes	yes	yes
<i>transients</i>	yes	OK for $\tau > \tau_{relax}$	yes
<b>phase-space selectivity</b>	modest	no(t yet)	yes
<b>predictive runs</b>	requires guess $D_{fi}$	requires mode spectrum: growth/damping	requires mode spectrum, damping
<b>improvements</b>	none planned	extend to 2D in velocity space	remove $\mu$ conservation

(\*) CGM – Critical Gradient Model, see IAEA-FEC 2014: Gorelenkov TH/P1-2, Heidbrink EX/10-1

# Kick model vs Critical Gradient model

Start from the evolution of mode energy:  $\frac{\partial E_w}{\partial t} = P_{EP} - 2\gamma_{damp} E_w$

Introduce 'growth rate':  $\gamma_{gr}(E_w) \doteq \frac{P_{EP}(E_w)}{2E_w} \Rightarrow \frac{\partial E_w}{\partial t} = 2[\gamma_{gr} - \gamma_{damp}] E_w$

## CGM

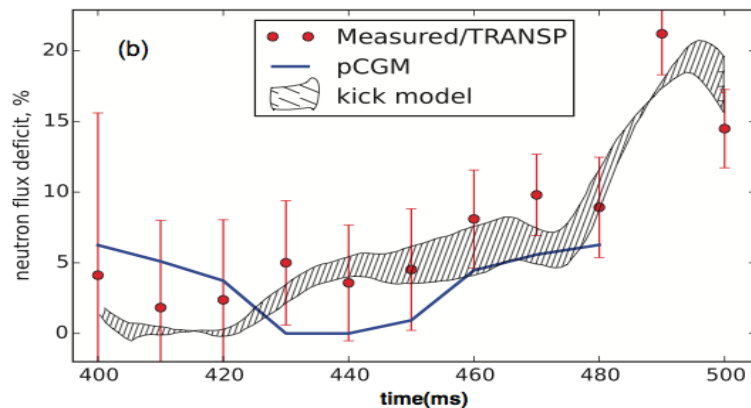
- Assume  $\gamma_{damp}$  is known (NOVA)
- Assume  $\gamma_{gr} \sim d\beta(r, E_w)/dr$
- Compute local  $\beta(r, E_w)$  from 'saturation':  

$$\gamma_{gr}(E_w) - \gamma_{damp} \rightarrow 0 \Rightarrow E_w \rightarrow E_w^{sat}$$
- Infer EP density profile from  $\beta(r, E_w)$  at saturation

## Kick model

- Assume  $\gamma_{damp}$  is known (NOVA)
- Compute  $P_{EP}(E_w)$  for each mode
- Set  $E_w^{sat}$  from saturation:  

$$\gamma_{gr}(E_w) - \gamma_{damp} \rightarrow 0 \Rightarrow E_w \rightarrow E_w^{sat}$$
- Infer EP density profile from TRANSP run
- *OR:* compute  $E_w$  by matching neutrons (, FIDA, NPA, ...), verify  $\gamma_{damp}$  values
- *OR:* get  $E_w$  from experiment, verify saturation condition,  $\gamma_{damp}$  values





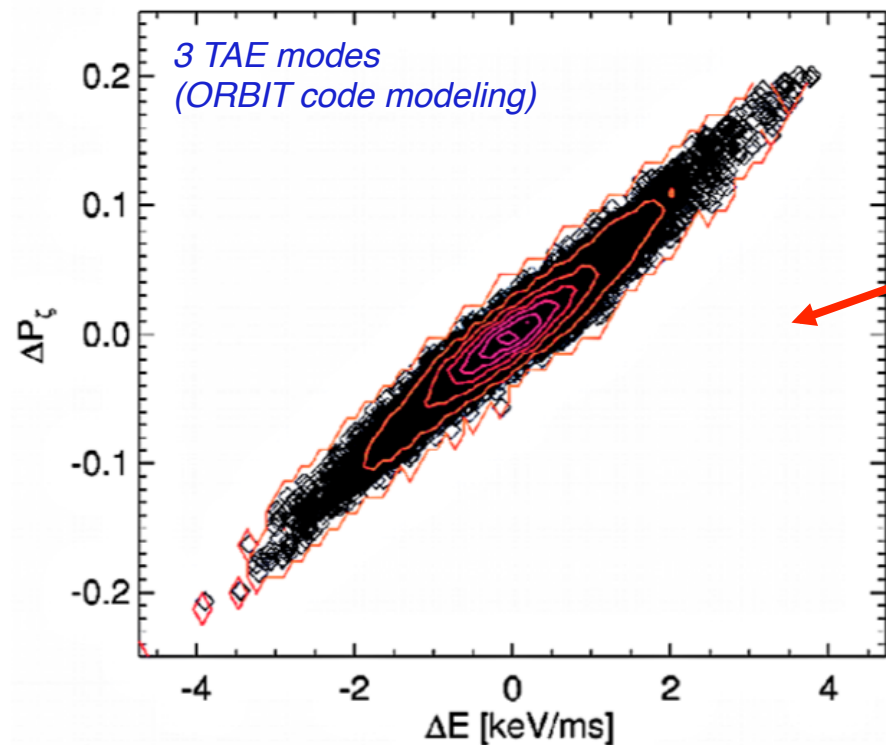
# New 'kick model' uses a *probability distribution function* to describe particle transport in $(E, P_\zeta, \mu)$ space

Kicks  $\Delta E$ ,  $\Delta P_\zeta$  are described by  

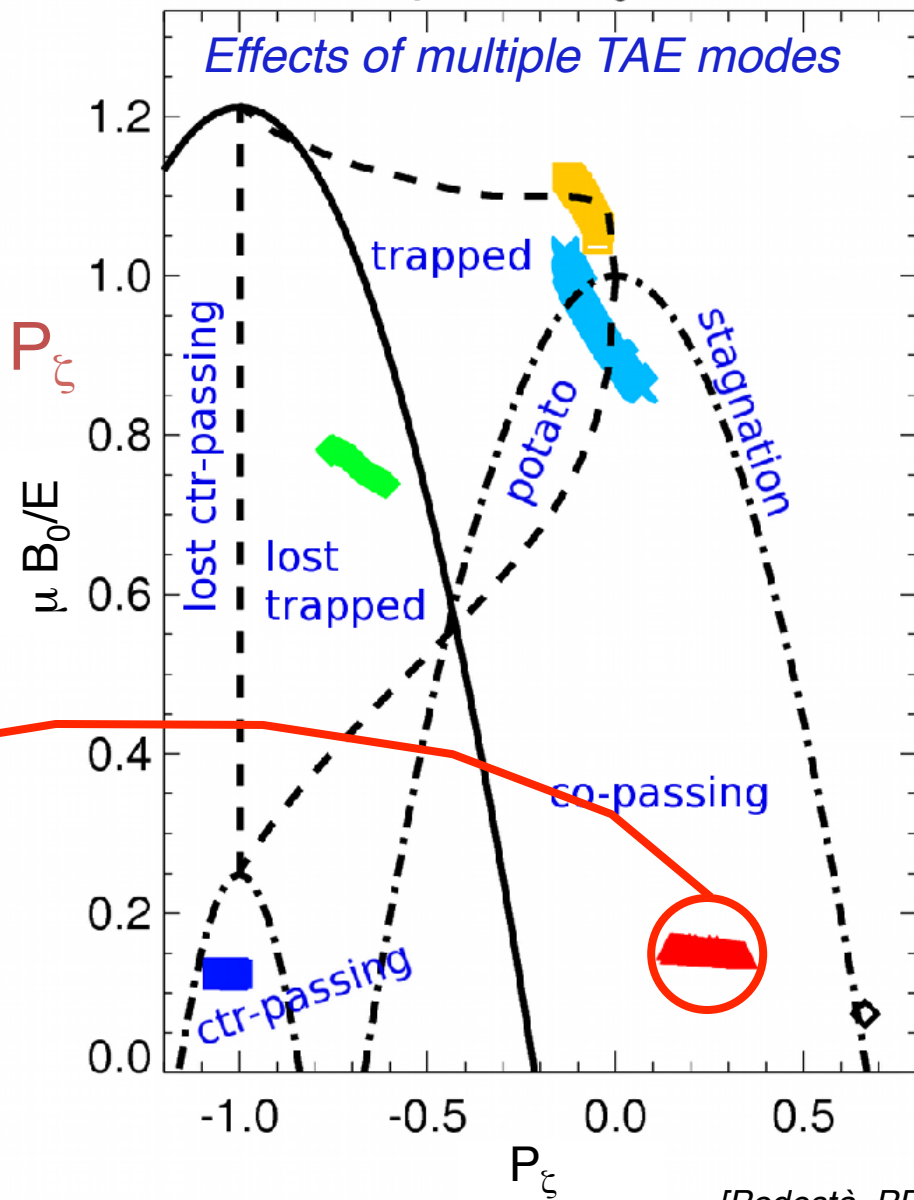
$$p(\Delta E, \Delta P_\zeta | P_\zeta, E, \mu, A)$$

which includes the effects of multiple modes, resonances.

→ correlated random walk in  $E, P_\zeta$



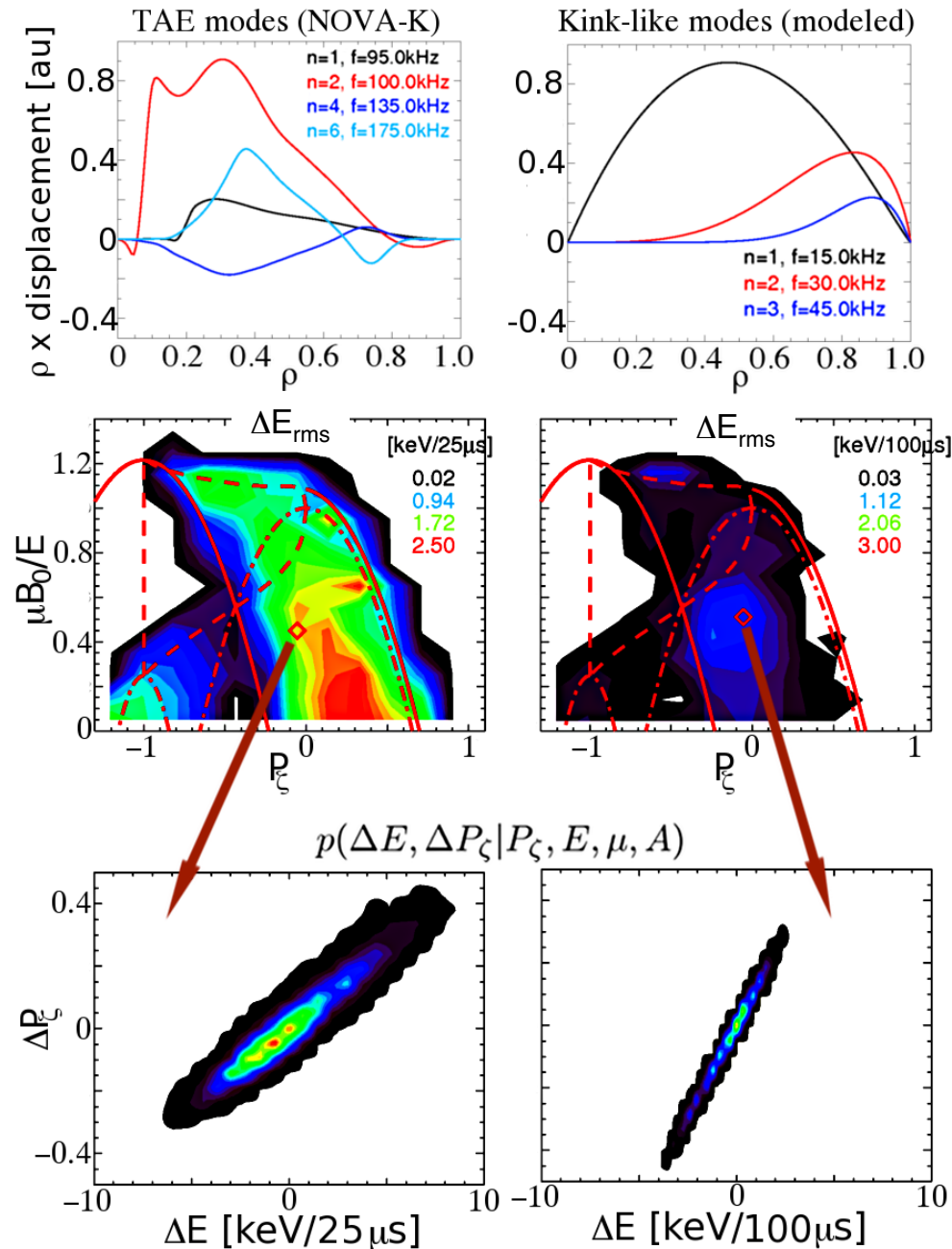
Phase space,  $E_0=80.0\text{keV}$



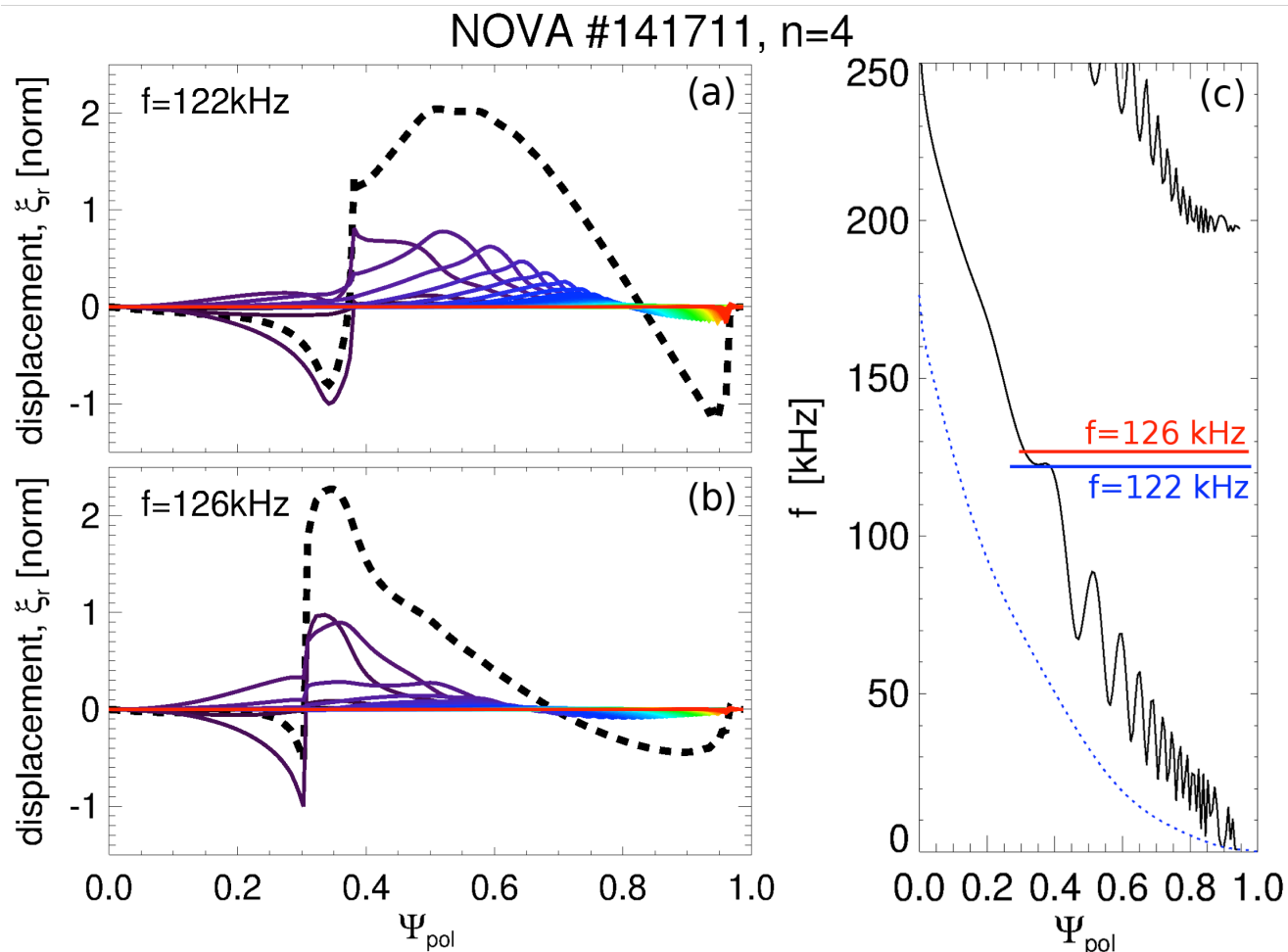
[Podestà, PPCF 2014]

# $\rho(\Delta E, \Delta P_\zeta | P_\zeta, E, \mu)$ and a time-dependent 'mode amplitude scaling factor' enable multi-mode simulations

- Example: toroidal AEs (TAEs) and low-frequency kink
- $\rho(\Delta E, \Delta P_\zeta | P_\zeta, E, \mu)$  from particle-following code ORBIT
- Each type of mode has separate  $\rho(\Delta E, \Delta P_\zeta)$ ,  $A_{\text{mode}}(t)$
- TAEs and kinks act on different portions of phase space
- Amplitude vs. time can differ, too
- Effects on EPs differ
  - > TAEs: large  $\Delta E$ ,  $\Delta P_\zeta$
  - > kinks: small  $\Delta E$ , large  $\Delta P_\zeta$



# Accurate computation of mode structure is critical for reliable kick model analysis



- Main source of uncertainty in kick model runs comes from selection of candidate modes
- Example: ideal MHD doesn't resolve intersection with AE continuum
- Discontinuity in mode structure propagates to kick probability, e.m. energy associated with the mode

# Scheme to advance fast ion variables according to transport probability in NUBEAM module of TRANSP

