1. CHI EQUILIBRIUM MODELING
2. HELICITY

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NSTX Results Review
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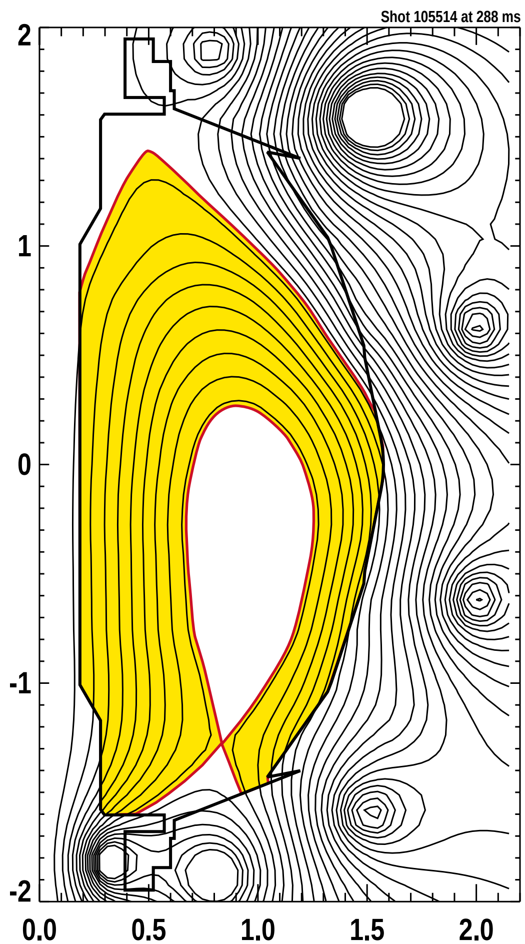
CHI Raises Basic Physics Questions

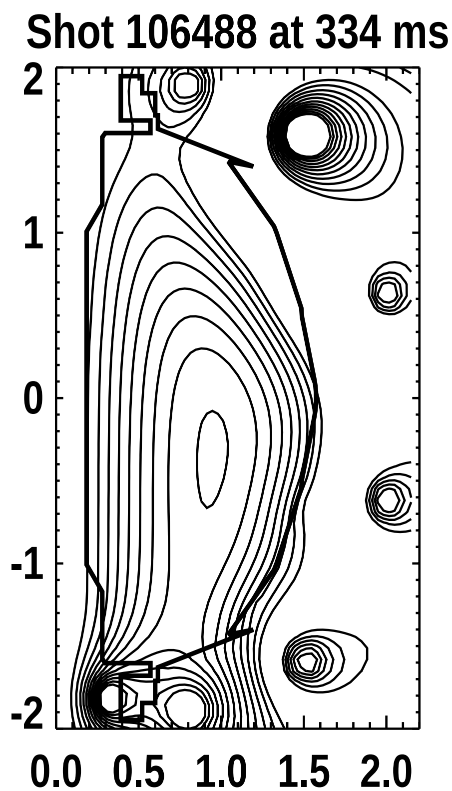
- Is there a sizeable central region of CLOSED magnetic surfaces by the end of a CHI non inductive startup pulse?
  - Needed to confine energetic HHFW electrons and/or NB injected ions during the handoff from CHI startup to another non inductive current driver.
- Helicity transport theory predicts that magnetic surfaces must be open (either intermittently or steadily) to sustain current.
  - Open surfaces reduce hot plasma confinement.
- How does CHI operate in STs to distribute plasma current on the closed surfaces?
  - Is this scalable to larger devices and stronger B?
  - Is this compatible with good confinement, or will CHI be limited to just a startup role?

EFIT Was Used to Analyze a High-Current CHI Shot for Indications of Flux Closure

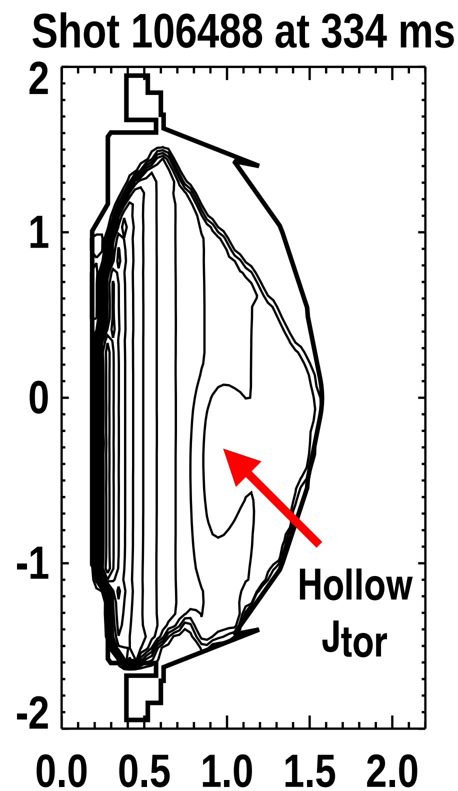


- A force–free current (yellow) was fit in the scrape–off layer (SOL).
- User specifies outer limit of SOL current; not automatic.
  - Here shown as near the 2nd X–point.
- EFIT requires at least a small closed flux region.
- Convergence is poor for NSTX CHI shots tried so far.
- Of course, EFIT <u>assumes</u> axisymmetry.

EFIT run with SOL Current Finds a Small Closed-Flux Region. It Has a Hollow J_{tor} Distribution.



Flux Surfaces



Current Contours

- EFIT finds a small closed-flux region, even though a hollow current profile makes less flux than a peaked one.
- Hollow J is qualitatively consistent with theoretical helicity transport concepts.
- EFIT cannot fit low-current CHI shots (no closed surfaces).

- SOL current out to 2nd X–pt.
- Fit indicators are quite poor in both E-and MFIT.

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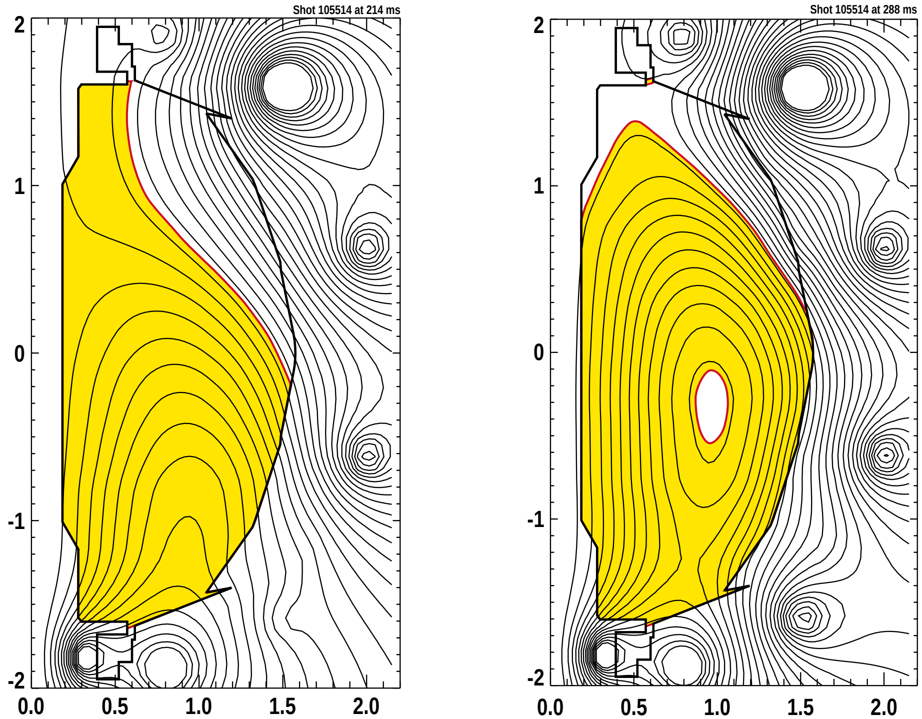
EFIT Can be Coded to Fit Current on the Correct Open Magnetic Lines



- Use the insulated gaps to define the minimum and maximum flux values that bound current-carrying flux (yellow).
  - This works for some common topologies and geometries.
  - It will not work once the closed flux is large.

Conclusion from Limited EFITting of CHI on NSTX

EFIT Magnetic Reconstruction
of a High-Current NSTX CHI Shot
is Consistent with Small Closed Flux Region

SOME CONSIDERATIONS ON MAGNETIC HELICITY

Magnetic Helicity and Its Transport Are Guiding Concepts for CHI

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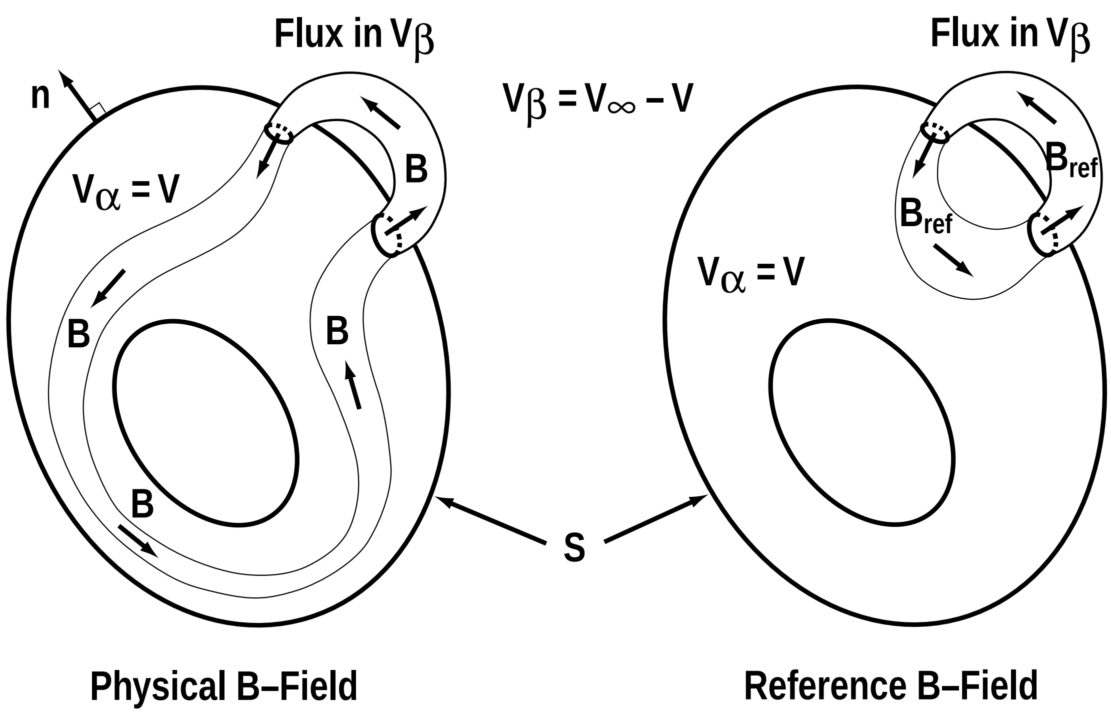
Berger-Field RELATIVE HELICITY is the <u>difference</u> between two simple A·B products over all space, V_∞,

Berger & Field, J. Fluid Mech. 147 (1984) 133.

$$K_{rel} = \int_{V_\infty} A \cdot B \, d^3x - \int_{V_\infty} A_{ref} \cdot B_{ref} \, d^3x$$



Outside of the volume V of interest, B_{ref} = B.

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Is the Moses-Gerwin-Schoenberg Helicity the Same as More Familiar Relative Helicities?

- Berger-Field relative helicity is gauge independent in an arbitrary V and independent of external fields and linkages, even if S is not a magnetic surface, under the generalized condition

$$A_{ref} \times n = A \times n \text{ on } S.$$

This includes the familiar $n \cdot B_{ref} = n \cdot B$ on S.

Best choice of a reference field can depend on the physical problem.

- In contrast, Moses et al. define a magnetic helicity in volume V as simply

$$K_{Moses} = \int_V A \cdot B \, d^3x \text{ with } A = n \times \nabla\Psi \text{ on S.}$$

This sets $\nabla \cdot A = 0$ and $A \cdot n = 0$ on S, which yields unique gauge and helicity.

They use no reference fields. Moses, Gerwin, Schoenberg, Phys. Plasmas 8 (2001) 4839.

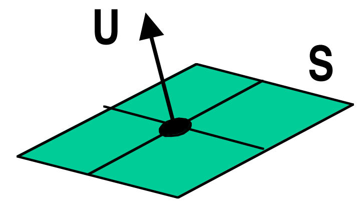
Time Derivative of Relative Helicity in General Requires Fields and Boundary Conditions on Moving Surface

Let U(x,t) be velocity of a coordinate point on S(x,t) and moving with it.

U(x,t) is measured in a fixed, non-deforming coordinate system.

U is perpendicular to S(x,t).

Let (´) denote quantities measured at rest on the moving S.



Then:

$$B' = B \qquad A' = A \qquad E' = E + U \times B \qquad \phi' = \phi - U \cdot A$$

$$\frac{\partial A'}{\partial t} = \frac{\partial A}{\partial t} - U \times B + \nabla(U \cdot A) = \frac{\partial A}{\partial t} + U \cdot \nabla A = \frac{dA}{dt}$$

$$E = -\frac{\partial A}{\partial t} - \nabla\phi, \qquad E' = -\frac{\partial A'}{\partial t} - \nabla\phi'$$

Boundary conditions: $\frac{\partial A'_{ref}}{\partial t} \times n = \frac{\partial A'}{\partial t} \times n \qquad \phi'_{ref} = \phi' \qquad E'_{ref} \times n = E' \times n$

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Time Derivative of Relative Helicity Can Take Various Forms

$$\frac{dK_{rel}}{dt} = -\int_{V(t)} 2\mathbf{E}\cdot\mathbf{B}\, d^3x + \int_{V(t)} 2\mathbf{E}_{ref}\cdot\mathbf{B}_{ref}\, d^3x$$

$$\frac{dK_{rel}}{dt} = -\int_{S(t)} (\mathbf{E}'\times\mathbf{A} + \phi'\mathbf{B})\cdot\mathbf{n}\, d^2x - 2\int_{V(t)} \mathbf{E}\cdot\mathbf{B}\, d^3x - \frac{d}{dt}\int_{V(t)} \mathbf{A}_{ref}\cdot\mathbf{B}_{ref}\, d^3x$$

$$= -\int_{S(t)} \left(\mathbf{A}\times\frac{\partial\mathbf{A}'}{\partial t} + 2\phi'\mathbf{B}\right)\cdot\mathbf{n}\, d^2x - 2\int_{V(t)} \mathbf{E}\cdot\mathbf{B}\, d^3x - \frac{d}{dt}\int_{V(t)} \mathbf{A}_{ref}\cdot\mathbf{B}_{ref}\, d^3x$$

Moses et al:

$$\frac{dK_{Moses}}{dt} = -\int_{S(t)} \left(\mathbf{A}\times\frac{\partial\mathbf{A}}{\partial t} + 2\phi\mathbf{B}\right)\cdot\mathbf{n}\, d^2x - 2\int_{V(t)} \mathbf{E}\cdot\mathbf{B}\, d^3x$$

- Much of the confusion over application of magnetic helicity arises from how to interpret the various terms.

Moses-Gerwin-Schoenberg Helicity is a Special Case of Berger-Field Relative Helicity

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- The Moses et al. choice of $A \cdot n = 0$ is consistent with Berger-Field, whose relative helicity does not constrain the choice of $\nabla \cdot A$ or $A \cdot n$.
- When $A \cdot n = 0$, then $U \cdot A = 0$, too, since $U \parallel n$.
  Then, it can then be shown that the Berger-Field boundary conditions

  $$\frac{\partial A'_{ref}}{\partial t} \times n = \frac{\partial A'}{\partial t} \times n \qquad \phi'_{ref} = \phi' \qquad E'_{ref} \times n = E' \times n$$

  can be written identically in terms of <u>either</u> moving-frame or fixed-frame variables.
- It appears to me at this time that Moses et al helicity is a special case of Berger-Field relative helicity, except for no reference helicity.
  - Moses et al is simpler than Berger-Field, but does Moses et al ever need to subtract reference fields?.
  - I want to derive Moses et al. explicitly for toroidal volumes and with close attention to conditions at moving surfaces.

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