Neoclassical Theory Developments and Implementation

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Motivation

- The ion energy balance anomaly in NSTX has caused us to re-examine NBI driven energy fluxes as a possible explanation:
  - Work by Callen, et al (1974), Stacey, et al (1984-1993), and Hinton and Kim (1993) all have noted that an inward inward ion heat flux is driven by co-injection
- The expressions in the literature use limiting assumptions that are not readily applicable to NSTX:
  - Thermal ions deep in the banana regime (not true in the outer half of NSTX where the anomaly is strongest)
  - Large aspect ratio approximations (same reason)
  - Stacey, et al, and Hinton and Kim evaluated only the part driven by the NBI parallel momentum force (parallel heat force effect is likely the more dominant effect in NSTX, see next comment)
  - Expressions from Callen, et al, show the effect from NBI parallel heat force scales as Eb/Ti and can be dominant for NSTX (simple circular plasma approximation and NBI distribution)

➢ We are reworking the theory to remove the most questionable assumptions for NSTX, for incorporation into NCLASS using the fast ion distribution from the TRANSP NBI package

Neoclassical Parallel Force Balances with NBI

S.P. Hirshman and D.J. Sigmar, *Nucl. Fusion* **21** (1981) 1079
W.A. Houlberg, K.C. Shaing, S.P. Hirshman and M.C. Zarnstorff, *Phys. Plasmas* **4** (1997) 3230

- **In NCLASS we use the Hirshman-Sigmar formulation of neoclassical theory for a multiple species plasma.**
- **The flux surface averaged parallel force and heat force balance equations for thermal species j come from the odd velocity moments of the Boltzmann equation on a timescale $t \gg \tau_{jj}$:**

$$\langle \vec{B} \cdot \vec{\nabla} \cdot \vec{\Pi}_j \rangle = \langle \vec{F}_{1,j} \cdot \vec{B} \rangle + \langle \vec{F}^b_{1,j} \cdot \vec{B} \rangle + e_j n_j \langle \vec{E} \cdot \vec{B} \rangle \quad \text{Force}$$

$$\langle \vec{B} \cdot \vec{\nabla} \cdot \vec{\Theta}_j \rangle = \langle \vec{F}_{2,j} \cdot \vec{B} \rangle + \langle \vec{F}^b_{2,j} \cdot \vec{B} \rangle \quad \text{Heat force}$$

Viscous	Thermal Friction	Beam Friction	Ohmic

- **The beam friction terms provide a parallel force on the thermal species:**
  - **That is analogous to the thermal friction terms (between two thermal species)**
  - **That acts to drive fluxes just as the force from the parallel electric field drives fluxes (Ware pinch)**

The Thermal Friction and Viscous Forces are Functions of the Parallel and Poloidal Flows

- **Classical parallel friction forces between thermal species:**

$$\langle \vec{F}_{\alpha,j} \cdot \vec{B} \rangle = \sum_k \sum_\beta \ell_{\alpha\beta}^{jk} \hat{u}_{\parallel\beta,k}$$

$$\hat{u}_{\parallel 1,j} \equiv \langle \vec{u}_j \cdot \vec{B} \rangle$$

$$\hat{u}_{\parallel 2,j} \equiv \frac{2}{5} \frac{\langle \vec{q}_j \cdot \vec{B} \rangle}{p_j}$$

Thermal friction coefficients

Parallel flow

Parallel heat flow

Friction forces are a function of the parallel flows of all species

- **Neoclassical parallel viscous forces for thermal species:**

$$\langle \vec{B} \cdot \vec{\nabla} \cdot \vec{\Pi}_j \rangle = \langle B^2 \rangle \sum_\beta \hat{\mu}_{1\beta,j} \, \hat{u}_{\theta\beta,j}$$

$$\langle \vec{B} \cdot \vec{\nabla} \cdot \vec{\Theta}_j \rangle = \langle B^2 \rangle \sum_\beta \hat{\mu}_{2\beta,j} \, \hat{u}_{\theta\beta,j}$$

Thermal viscosity coefficients

$$\hat{u}_{\theta 1,j} \equiv \frac{\langle \vec{u}_j \cdot \vec{\nabla}\theta \rangle}{\langle \vec{B} \cdot \vec{\nabla} B \rangle}$$

$$\hat{u}_{\theta 2,j} \equiv \frac{2}{5} \frac{1}{p_j} \frac{\langle \vec{q}_j \cdot \vec{\nabla}\theta \rangle}{\langle \vec{B} \cdot \vec{\nabla} B \rangle}$$

Poloidal flow

Poloidal heat flow

Viscous forces are a function of the poloidal flows of own species

- **Kinetic theory provides the viscosity and friction coefficients**

The Radial Force Balances Relate the Poloidal and Toroidal Flows to T', p', and Φ'

- **The radial force balance equation for each odd velocity moment (α=1,2,3) relates the flows within a surface to gradients of flux surface quantities (driving forces):**

$$\begin{aligned}
\langle B^2 \rangle \hat{u}_{\theta\alpha,j} &= \hat{u}_{\|\alpha,j} + S_{\theta\alpha,j} \quad \alpha = 1,2,3 \\
S_{\theta 1,j} &= \frac{2\pi R B_t}{\Psi'} \left(\frac{p_j'}{e_j n_j} + \Phi' \right) \\
S_{\theta 2,j} &= \frac{2\pi R B_t}{\Psi'} \frac{k T_j'}{e_j}
\end{aligned}$$

- **This allows us to eliminate the toroidal flows and solve a matrix of equations for the poloidal flows for each charge state j**
- **The addition of NBI friction and heat friction terms leads to a modification to the poloidal flows (and the toroidal flows through the radial force balances), and consequently the other neoclassical transport properties**

Neoclassical Particle and Heat Fluxes with NBI

- **The banana plateau (BP) fluxes are related to the poloidal flows:**

$$\Gamma_j^{\mathrm{BP}} = -\frac{2\pi R B_t}{\Psi' e_j} \sum_\beta \hat{\mu}_{1\beta,j} \left(\hat{u}^{nc}_{\theta\beta,j} + \hat{u}^{b}_{\theta\beta,j} \right)$$

$$q_j^{\mathrm{BP}} = -\frac{2\pi R B_t k T_j}{\Psi' e_j} \sum_\beta \hat{\mu}_{2\beta,j} \left(\hat{u}^{nc}_{\theta\beta,j} + \hat{u}^{b}_{\theta\beta,j} \right)$$

Flows of thermal species driven by NBI modify BP

  - **Importance relative to standard neoclassical is governed by the ratio of induced poloidal flows**

- **The Pfirsch-Schlüter (PS) fluxes are related directly to the forces:**

$$\Gamma_j^{\mathrm{PS}} = \frac{2\pi R B_t}{\Psi' e_j} \left[1/\langle B^2 \rangle - \langle B^{-2} \rangle \right] \left[\langle \vec{F}_{1,j} \cdot \vec{B} \rangle + \langle \vec{F}^b_{1,j} \cdot \vec{B} \rangle \right]$$

$$q_j^{\mathrm{PS}} = \frac{2\pi R B_t k T_j}{\Psi' e_j} \left[1/\langle B^2 \rangle - \langle B^{-2} \rangle \right] \left[\langle \vec{F}_{2,j} \cdot \vec{B} \rangle + \langle \vec{F}^b_{2,j} \cdot \vec{B} \rangle \right]$$

NBI friction and heat friction modify PS

  - **Importance relative to standard neoclassical governed by the ratio of forces**

- **All of the NBI terms are inward with co-injection, outward with counter-injection (testable by co-/counter-NBI comparisons)**

The Radial Force Balances with NBI

- **Radial force balance equations in terms of parallel and poloidal flows:**

$$\langle B^2\rangle\left[\hat{u}^{nc}_{\theta 1,j}+\hat{u}^{b}_{\theta 1,j}\right] = \left[\hat{u}^{nc}_{\|1,j}+\hat{u}^{b}_{\|1,j}\right]+\frac{2\pi RB_t}{\Psi'}\left[\frac{p'_j}{e_j n_j}+\Phi'\right]$$

Force

$$\langle B^2\rangle\left[\hat{u}^{nc}_{\theta 2,j}+\hat{u}^{b}_{\theta 2,j}\right] = \left[\hat{u}^{nc}_{\|2,j}+\hat{u}^{b}_{\|2,j}\right]+\frac{2\pi RB_t}{\Psi'}\frac{kT'_j}{e_j}$$

Heat force

Poloidal flow induced by NBI (typically only use neoclassical value)

Parallel flow induced by NBI (typically use measured total toroidal flow)

- **NBI effects on poloidal rotation should be considered when:**
  - **Determining the radial electric field using the radial force balance with theoretical models for the poloidal rotation**
  - **Comparing measured poloidal rotation with theoretical models**

Viscous Heating with NBI

- **The neoclassical viscous heating is enhanced as the square of the poloidal flow velocities:**

$$
\begin{aligned}
P^{\mathrm{BP}}_{\mu,j} &= \hat{u}_{\theta 1,j}\langle \vec{B}\cdot\vec{\nabla}\cdot\vec{\Pi}_j\rangle \\
&= \left[\hat{u}^{nc}_{\theta 1,j} + \hat{u}^{b}_{\theta 1,j}\right]\langle B^2\rangle \sum_{\beta} \hat{\mu}_{1\beta,j} \left[\hat{u}^{nc}_{\theta\beta,j} + \hat{u}^{b}_{\theta\beta,j}\right]
\end{aligned}
$$

Poloidal flow from NBI modifies poloidal viscous heating

- **In addition to this are heating terms from classical (small) and toroidal viscosity (e.g., Stacey's gyroviscosity, turbulence induced viscosity, …)**
- **Viscous heating is likely important for:**
  - **ITBs with strong pressure gradients**
  - **Strong toroidal rotation (NSTX)**

NBI Friction and Heat Friction Terms

- The general expressions for the NBI friction terms are expressed in terms of Laguerre polynomial weighting of the collision integrals:

$$
\begin{aligned}
\vec{F}^{b}_{1,j} &= m_j \int d^3v\, \vec{v}\, L_0^{(3/2)}(x_j^2) C_{jb}[f_j, f_b] \\
\vec{F}^{b}_{2,j} &= -m_j \int d^3v\, \vec{v}\, L_1^{(3/2)}(x_j^2) C_{jb}[f_j, f_b]
\end{aligned}
$$

- After a lot of manipulation of the full collision operator expressed in terms of the Rosenbluth potentials, we have:

$$
\begin{aligned}
\vec{F}^{b}_{1,j} &= 3m_j \int d^3v\, \vec{J}_{jb} \\
\vec{F}^{b}_{2,j} &= 5m_j \int d^3v \left(\frac{3}{2} - x_j^2\right) \vec{J}_{jb}
\end{aligned}
$$

$$
\begin{aligned}
\vec{J}_{jb} = \; & \gamma_{jb}\left(3 + \frac{m_j}{m_b}\right) f_j(v) \int d^3v_b\, f_b(v_b) \frac{(\vec{v} - \vec{v}_b)}{|\vec{v} - \vec{v}_b|^3} \\
& -\frac{1}{2}\gamma_{jb} \vec{\nabla}_v f_j(v) \cdot \int d^3v_b\, f_b(v_b) \left[\frac{\overleftrightarrow{I}}{|\vec{v} - \vec{v}_b|} - \frac{(\vec{v} - \vec{v}_b)(\vec{v} - \vec{v}_b)}{|\vec{v} - \vec{v}_b|^3}\right]
\end{aligned}
$$

Electron-NBI Friction and Heat Friction Terms

- **For the electron thermal particle distribution we keep the flow and heat flow perturbations from the Maxwellian:**

$$
\begin{aligned}
f_e(v) &= \left[1 + \frac{2\vec{v}\cdot\vec{u}_e}{v_{te}^2} L_0^{(3/2)}(x_e^2) - \frac{2\vec{v}\cdot\vec{q}_e}{v_{te}^2 p_e}\frac{2}{5} L_1^{(3/2)}(x_e^2)\right] f_{0e} \\
f_{0e} &= \frac{n_e}{\pi^{3/2} v_{te}^3} e^{-x_e^2}
\end{aligned}
$$

- **The integrals are then evaluated using $m_e << m_b$ and $v_b << v_e$:**

$$
\begin{aligned}
\langle \vec{F}_{1,e}^b \cdot \vec{B} \rangle &= -\frac{m_e n_b}{\tau_{eb}} \left(\hat{u}_{||1,e} - \hat{u}_{||1,b} - \frac{3}{2}\hat{u}_{||2,e} \right) \\
\langle \vec{F}_{2,e}^b \cdot \vec{B} \rangle &= \frac{m_e n_b}{\tau_{eb}} \frac{3}{2} \left(\hat{u}_{||1,e} - \hat{u}_{||1,b} - \frac{13}{6}\hat{u}_{||2,e} \right)
\end{aligned}
$$

Ion-NBI Friction and Heat Friction Terms

- For the ion thermal particle distribution we keep only the Maxwellian (at least for now, but plan to examine the flow corrections later):

$$f_i(v) = f_{0i} = \frac{n_i}{\pi^{3/2} v_{ti}^3} e^{-x_i^2}$$

- We are still working on evaluating and checking the integrals using $m_i \sim m_b$ and $v_i << v_b$
  - This results in several velocity integrals over the NBI distribution
  - We hope these can be calculated accurately enough by post-processing binned fast ion distributions from the TRANSP NBI Monte Carlo package

Summary

- There is an extensive literature on the theory for NBI driven neoclassical fluxes and viscous heating
- Various of these analyses have shown the effects are comparable to the standard neoclassical effects
- That literature considers individual effects and approximations that need to be treated more comprehensively for NSTX
- These effects should be most visible in experiments with unbalanced NBI when turbulence induced transport is suppressed
  - NSTX cases where negative effective ion thermal conductivity is inferred
- Adding the terms to NCLASS should give us a good assessment of many NBI driven effects in low aspect ratio
- Schedule of remaining work:
  - Complete derivation and checking of NBI-ion integrals (APS)
  - Add terms to NCLASS and upgrade FORCEBAL/NCLASS (FORCEBAL has already been upgraded this year by R. Andre to work with NCLASS F90/95 version
  - Identify beam integrals appropriate for post-processing TRANSP NBI results and initial evaluation in FORCEBAL

Other Extensions to Neoclassical Theory for NSTX

- The low aspect ratio and toroidal field in NSTX limit the applicability of other approximations used in standard neoclassical theory:
  - Small banana width compared with system size
    - » Addressed by potato orbit models near the axis (which vary in their predictions of whether thermal ion transport is enhanced, decreased or unaffected)
  - Small gyroradius compared to banana width
    - » Classical and neoclassical effects become comparable in NSTX and theory/models might have to be revised
- Our approach is to watch for indications in NSTX of deviations from standard neoclassical theory, and extend the theory and models as necessary.
- The most vulnerable elements of standard neoclassical theory in NSTX are:
  - Ion thermal transport
  - Impurity transport
  - NBI current drive
  - Bootstrap current

A Selection of References on NBI Driven Effects

Introductory assessment including momentum and heat flux torques:

- **J.D. Callen, et al, "Neutral beam injection into tokamaks," 5th IAEA, Tokyo (1974), Vol 1, 645**

Momentum torque and viscous heating:

- **S.P. Hirshman, D.J. Sigmar, "Neoclassical transport of impurities in tokamak plasmas," *Nucl. Fusion* 21 (1981) 1079**
- **W.M. Stacey, Jr., "The effects of neutral beam injection on impurity transport in tokamaks," *Phys. Fluids* 27 (1984) 2076**
- **W.M. Stacey, Jr., "Rotation and Impurity Transport in a tokamak plasma with directed neutral-beam injection," *Nucl. Fusion* 25 (1985) 463**
- **W.M. Stacey, Jr., "Convective and viscous fluxes in strongly rotating tokamak plasmas," *Nucl. Fusion* 30 (1990) 2453**
- **W.M. Stacey, Jr., "Poloidal rotation and density asymmetries in a tokamak plasma with strong toroidal rotation," Phys. Fluids B 4 (1992) 3302**
- **F.L. Hinton, Y.-B. Kim, "Effects of neutral beam injection on poloidal rotation and energy transport in tokamaks," Phys. Fluids B 5 (1993) 3012**
- **W.M. Stacey, "Comments on 'Effects of neutral beam injection on poloidal rotation and energy transport in tokamaks,' " *Phys. Fluids* B 5 (1993) 4505**

Momentum and heat flux torques included in formal development of equations (no applications):

- **J.P. Wang, et al, "Momentum and heat friction forces between fast ions and thermal plasma species," *Nucl. Fusion* 34 (1994) 231**
- **W.A. Houlberg, et al, "Bootstrap current and neoclassical transport in tokamaks of arbitrary collisionality and aspect ratio," *Phys. Plasmas* 4 (1997) 3230**