# Flux amplification in an Spherical Torus under CHI

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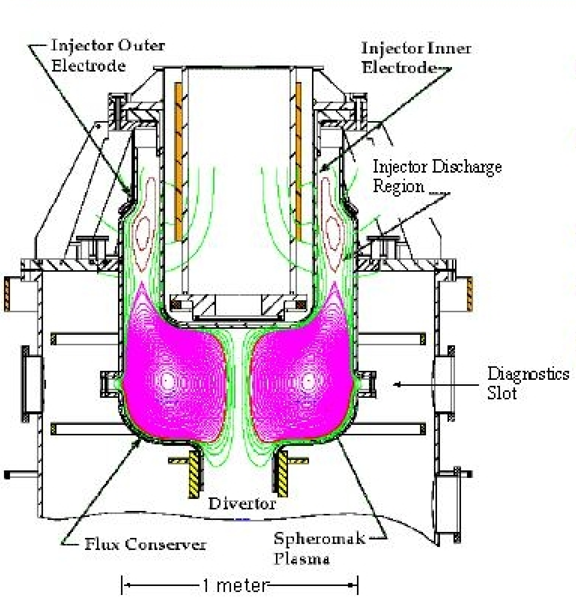
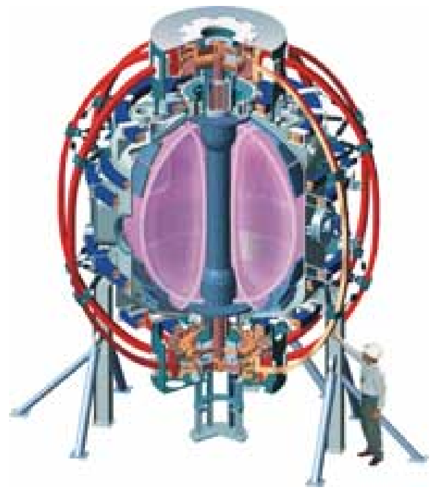
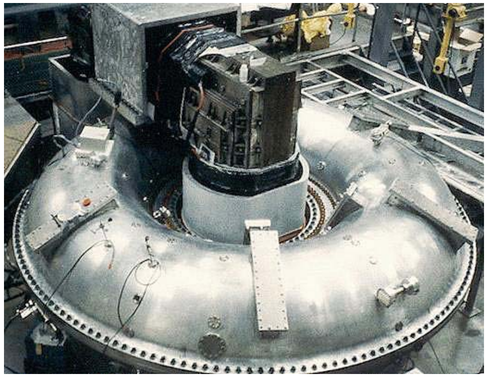
# Magnetic Relaxation



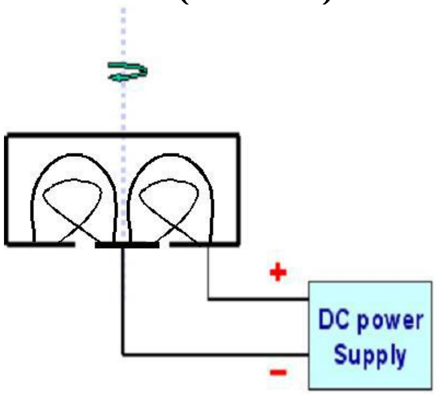

SSPX(LLNL)

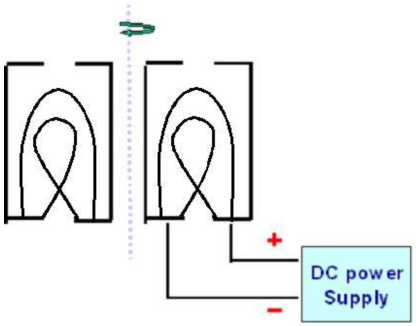


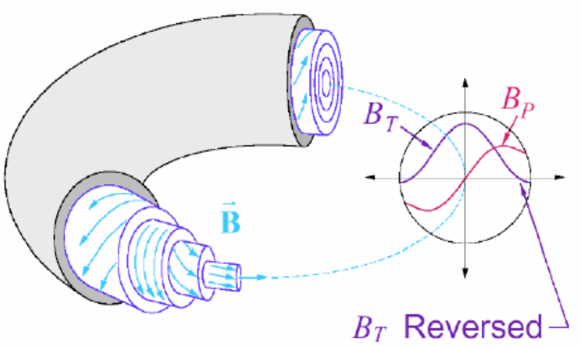
NSTX(PPPL, CHI)



MST (Wisconsin)



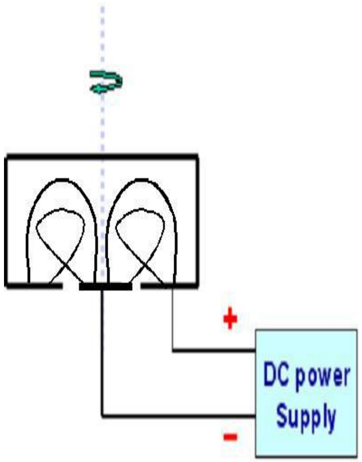

- Electro-static drive: power supply → magnetic energy.
- Helicity injection allows flux conversion: toroidal ⇔ poloidal.

# Resonances in magnetic relaxation

- Post-relaxation, modeled by Taylor state:

$$\nabla \times \vec{B} = k\vec{B}$$

- What is B made up of?
  - Vacuum field $B_x$ by external current. $\vec{B} \cdot \vec{n} \,|_{\partial\Omega} \neq 0$
  - Magnetic field $B_p$ by internal plasma current.
- Flux amplification: relative amplitudes of $B_x$ and $B_p$.
- "k" is the tunable parameter controlled by the external power source. $\vec{j} = k\vec{B}$
- Two classes of resonance:
  - without toroidal flux conserver: Jensen-Chu, 1984.
  - with toroidal flux conserver: Tang-Boozer, 2005.




[Tang and Boozer, Phys. Plasmas 12, 102102 (2005)]

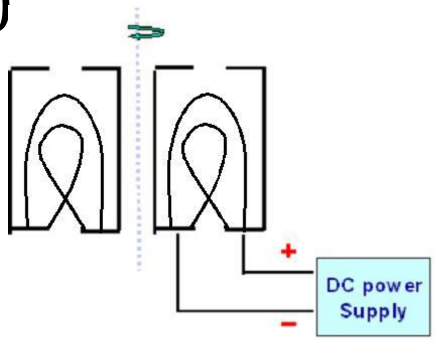
# Unconstrained resonance

- Axisymmetry: $\vec{B} = G(\chi)\nabla\varphi + \nabla\varphi \times \nabla\chi$
- G-S equation:

$$\Delta^*\chi + G\, {}^{dG}\!/_{d\chi} = \Delta^*\chi - k(G_0 - k)\chi = 0$$

- Solution: $\chi = \chi_v + \sum_i \alpha_i \chi_i$
- Vacuum field: $\Delta^*\chi_v = 0, \chi_v|_{\partial\Omega} = \chi|_{\partial\Omega}$
- CK modes: $\Delta^*\chi_i + k_i^2\chi_i = 0, \chi_i|_{\partial\Omega} = 0.$
- Jensen-Chu resonances (1984):

$$\alpha_i = \frac{k^2}{k_i^2 - k^2}\langle\chi_v\chi_i\rangle - \frac{k}{k_i^2 - k^2}G_0\langle\chi_i\rangle$$




[Tang and Boozer, PRL 94, 225004 (2005)]

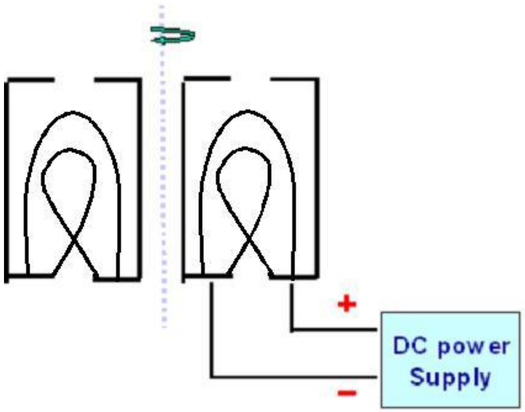
# Constrained resonance

[Tang and Boozer, PRL 95, 155002 (2005)]

- Finite net toroidal flux $\psi_t$ constraint.
- In DCD: $G(\chi) = G_0 - k\chi$

$$\Delta^* \chi - k(G_0 - k\chi) = 0$$

$$G_0 = \frac{\psi_t + \iint \frac{k\chi}{R} dS}{\iint \frac{1}{R} dS}$$




$$(k^2 - k_j^2)\alpha_j - \sum_i k^2 \overline{\chi}_i \langle \chi_j \rangle \alpha_i = k\overline{\psi}_t \langle \chi_j \rangle + k^2 \overline{\chi}_v \langle \chi_j \rangle - k^2 \langle \chi_v \chi_j \rangle$$

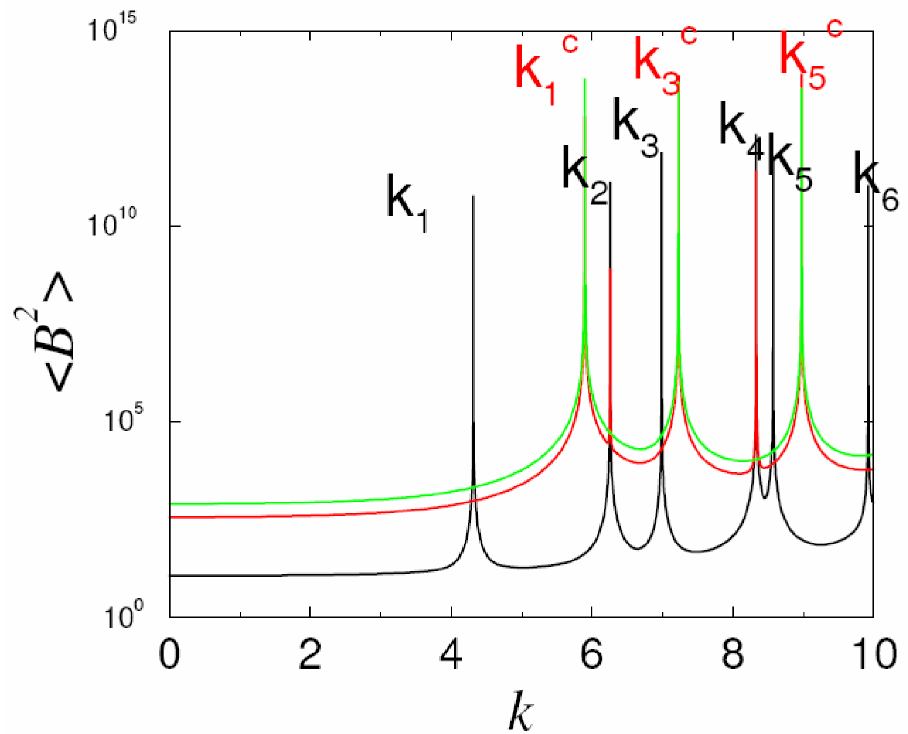
Complication arises because:

$$\langle \chi_j \rangle \neq 0 \qquad \overline{\chi}_j \neq 0$$

Mode has net toroidal flux

# Constrained Resonance

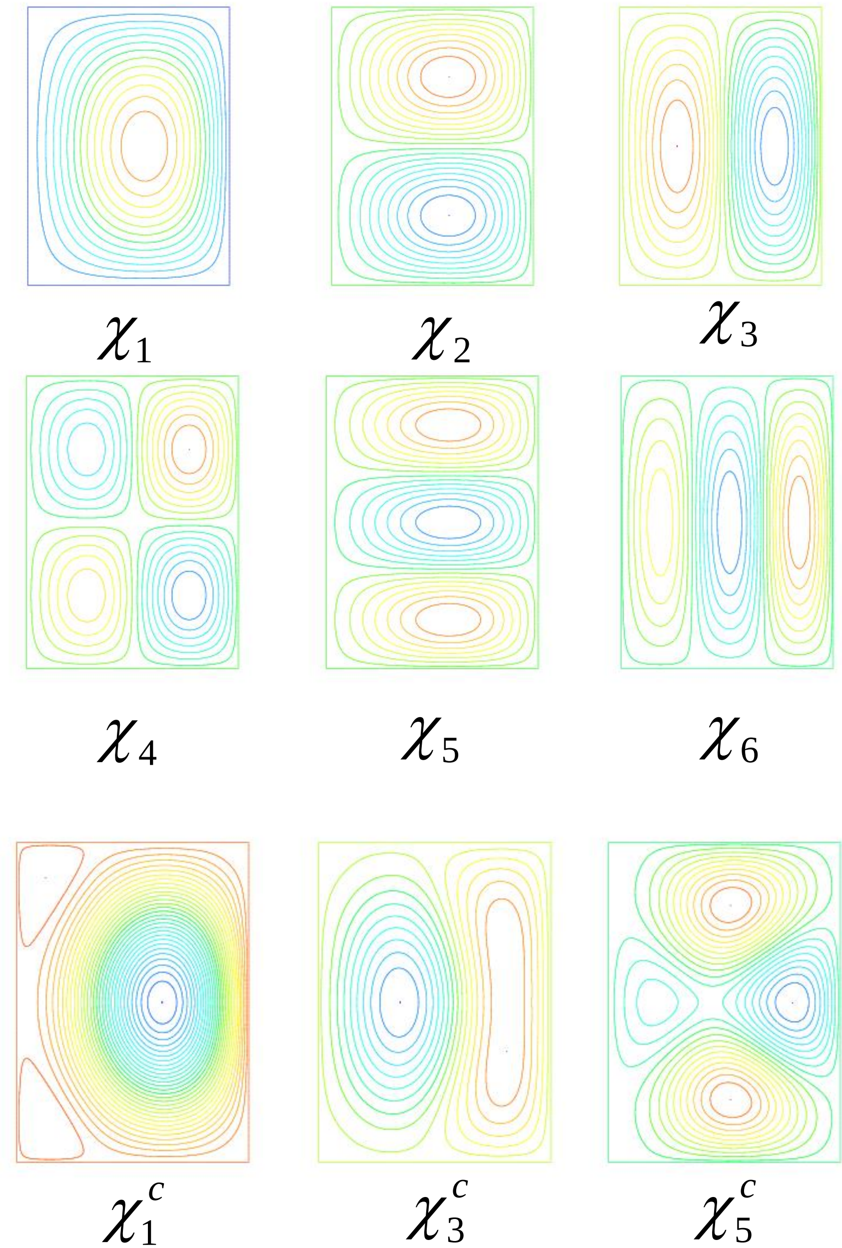
$$(k^2-k_j^2)\alpha_j-\sum_i k^2\overline{\chi}_i\langle\chi_j\rangle\alpha_i=k\overline{\psi}_t\langle\chi_j\rangle+k^2\overline{\chi}_v\langle\chi_j\rangle-k^2\langle\chi_v\chi_j\rangle$$




Black: $G_0=0$

Red (ST): $\psi_t = const.$

Green (RFP): $\psi_t = const.; \vec{B}\cdot\vec{n}=0$



# Flux amplif. in flux-conserving mode

[Tang and Boozer, Phys. Plasmas 12, 042113 (2005)]

- Characteristic parameter:

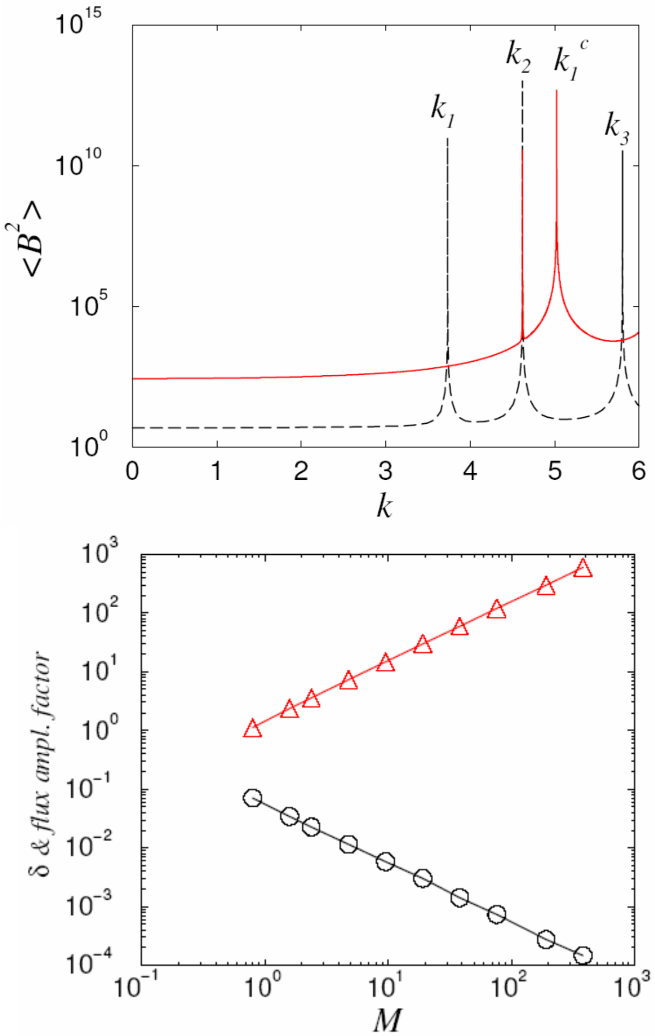
$$M = \psi_t / \chi_v^0$$

- Edge field reversal:

$$k_r = k_1 - \delta; \delta \approx k_1 / M$$

- Flux amplification factor at field reversal:

$$A_r = A(k = k_r) \approx M$$



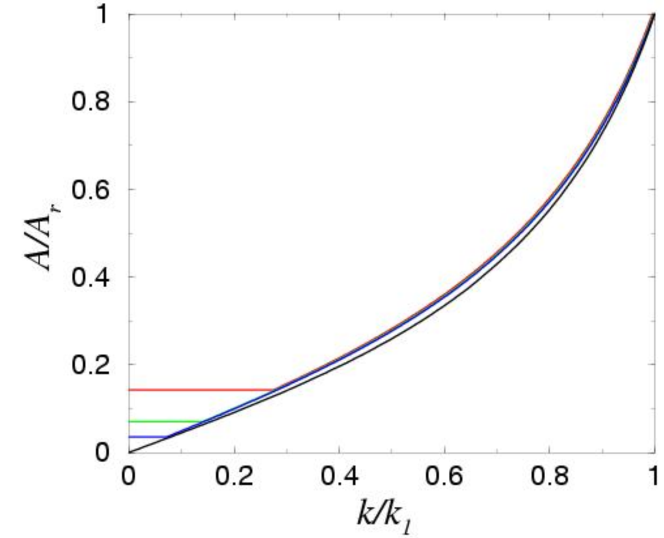
# Flux amplification versus k

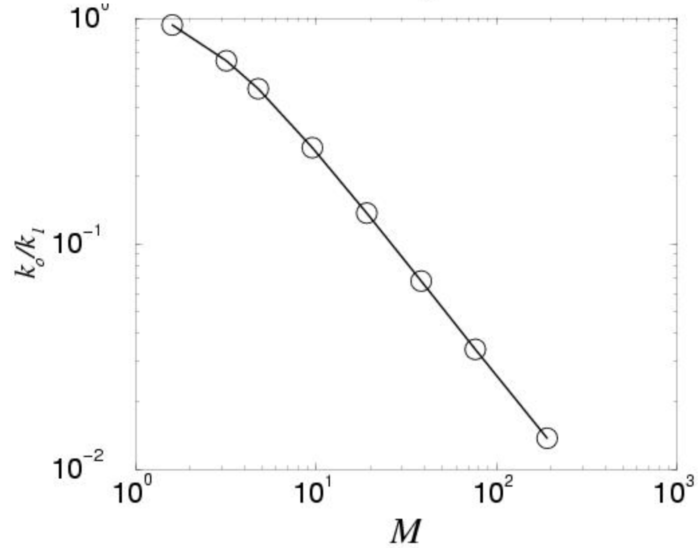
- k dependence of A:

$$\frac{A(k)}{A_r} = \frac{k_1^2 - k_1^{c^2}}{k^2 - k_1^{c^2}} \frac{k}{k_1}$$

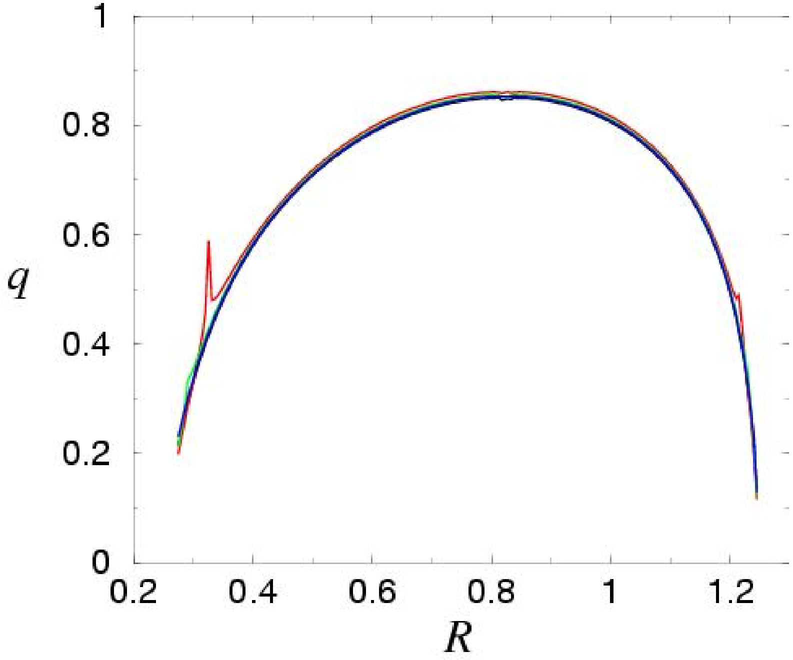
- Onset of flux amplif.

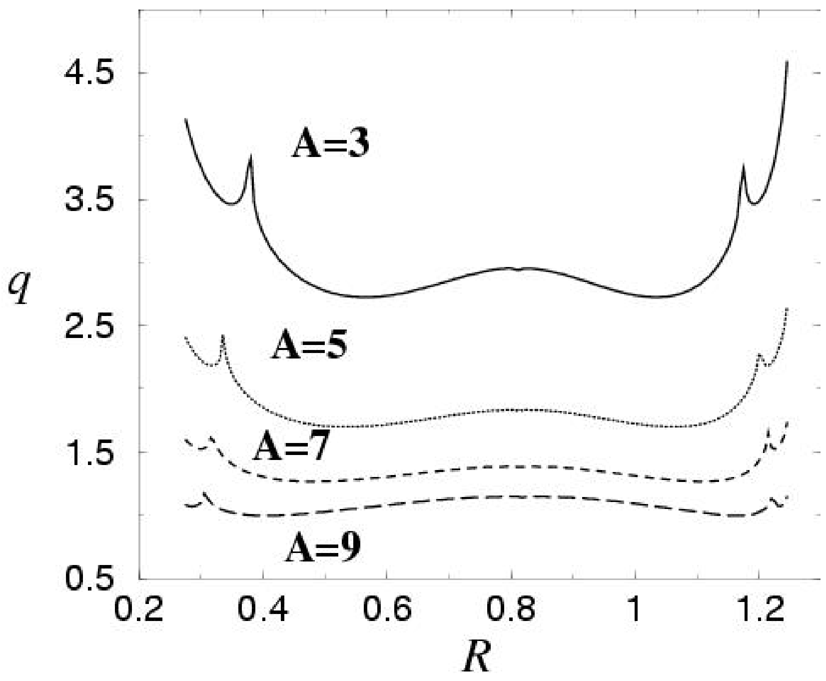
$$k_o \approx \frac{k_1}{M}$$



# q profile consideration




q profiles at $k_r$ are that of $\chi_1$

A < M for ST-consistency
(M=10, $A_r$=14)

# Summary

- ST-CHI under “spheromak” mode:
  - Must operate in the vicinity of the primary “spheromak” resonance for meaningful flux amplification.
  - Incomplete relaxation dominates the final answer.
    - Tang and Boozer, PRL 94, 225004 (2005).
- ST-CHI under flux-conserving mode:
  - Vacuum toroidal to injector poloidal flux ratio, M, defines the upper bound.
  - ST-relevant relaxed state can be obtained at $k \sim 0.7 k_1$, with Flux Amplification Factor about half of M.
  - This is obtained far from the actual up-shifted resonance, so more robust under incomplete relaxation.

# Backups

# Taylor state versus harmonic oscillators

- Taylor state: linear PDE

$$\nabla \times \vec{B} = k\vec{B}$$

- Homogeneous bdy condition
  - Eigenvalue problem.
    - No externally imposed vacuum magnetic field.
- Inhomogeneous bdy condition
  - Can be transformed into inhomogeneous linear PDE.
  - Driven problem.
    - With externally imposed vacuum magnetic field.

- Harmonic oscillator: linear ODE

$$d^2u/dt^2 + \omega_0^2 u = 0$$

- Driven harmonic oscillator:

$$d^2u/dt^2 + \omega_0^2 u = f \sin \omega t$$

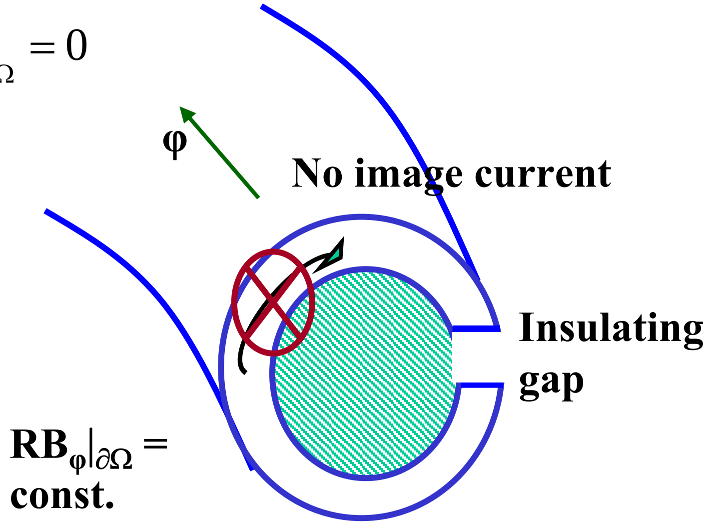
- Linear resonance

$$u = \frac{f}{\omega_0^2 - \omega^2} \sin \omega t$$

**Wavelength ⇔ Frequency, but what about f?**

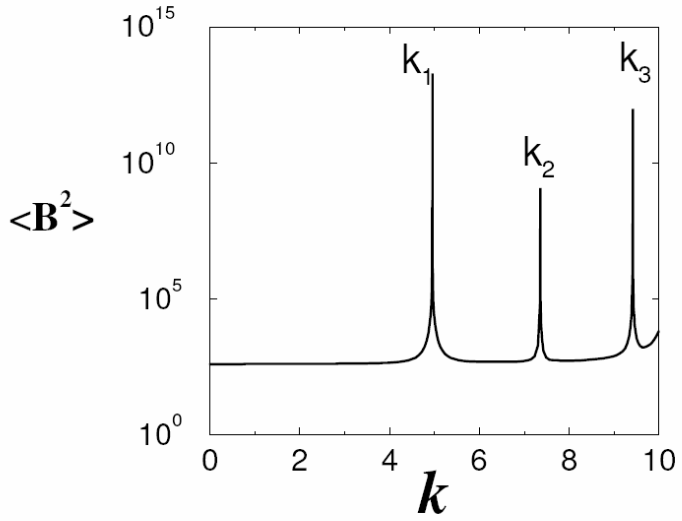
# Force-free modes: Chandrasekhar-Kendall

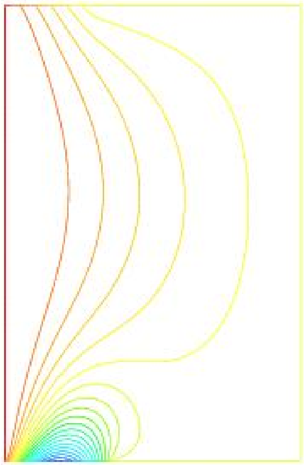
- Jensen-Chu-Taylor: $\left.\vec{A}\right|_{\partial\Omega} = 0$
  - Impose flux-free as boundary condition?
- Three classes of CK modes (no vacuum magnetic field contribution):
  - Axisymmetric CK modes with net toroidal flux.
  - Axisymmetric CK modes with no net toroidal flux.
    - Two conditions: $\left.\vec{B}\cdot\vec{n}\right|_{\partial\Omega} = 0 \qquad \left.B_\varphi\right|_{\partial\Omega} = 0$
  - Helical CK modes.
    - One is enough: $\left.\vec{B}\cdot\vec{n}\right|_{\partial\Omega} = 0$



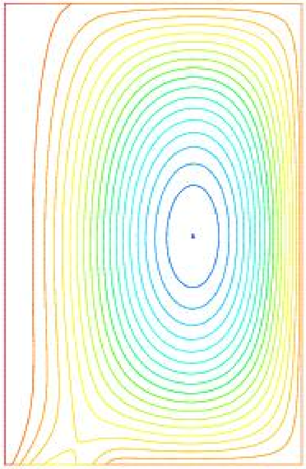

[Tang and Boozer, Phys. Plasmas 12, 102102, (2005)]

# Spheromak-mode flux amplification

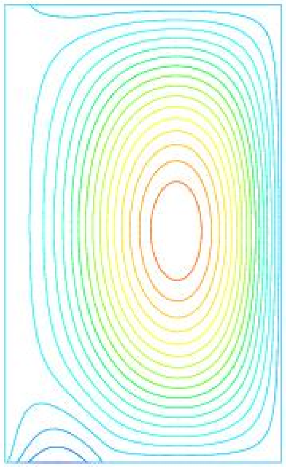


k=0



k=4.75: standard sp.



k=5.15: flipped

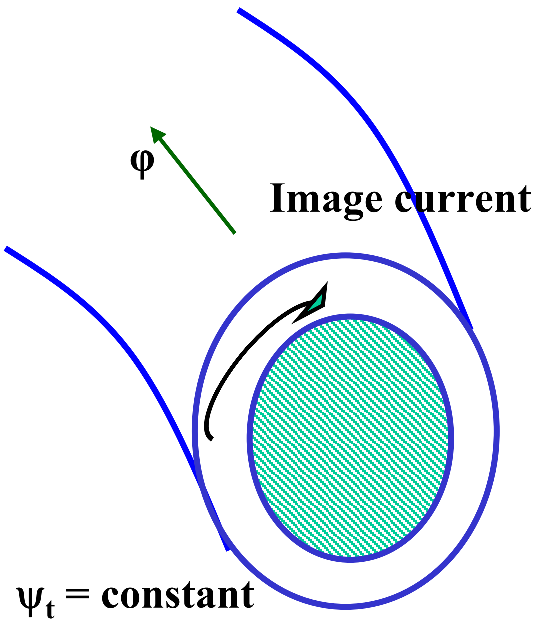
# Force-free modes: Yoshida-Giga

$$\nabla \times \vec{B} = k\vec{B}$$

$$\iint \vec{B} \cdot d\vec{a}_{\varphi} = 0$$

$$\vec{B} \cdot \vec{n}\Big|_{\partial\Omega} = 0$$

- With vanishing net flux (auxilliary integral constraint):
  - Two classes of flux-free CK modes.
  - Axisymmetric flux-free Yoshida-Giga modes
    - flux-free because vacuum field from the flux conserver exactly cancels the flux of flux-carrying CK modes.
    - Frequency upshifted from that of flux-carrying CK modes.
    - Yoshida-Giga and flux-carrying CK modes are not independent mode families.




[Tang and Boozer, Phys. Plasmas 12, 102102, (2005)]

# Partially relaxed Plasma

$$k(\chi) = k_c(1 - \varepsilon\chi^2)$$

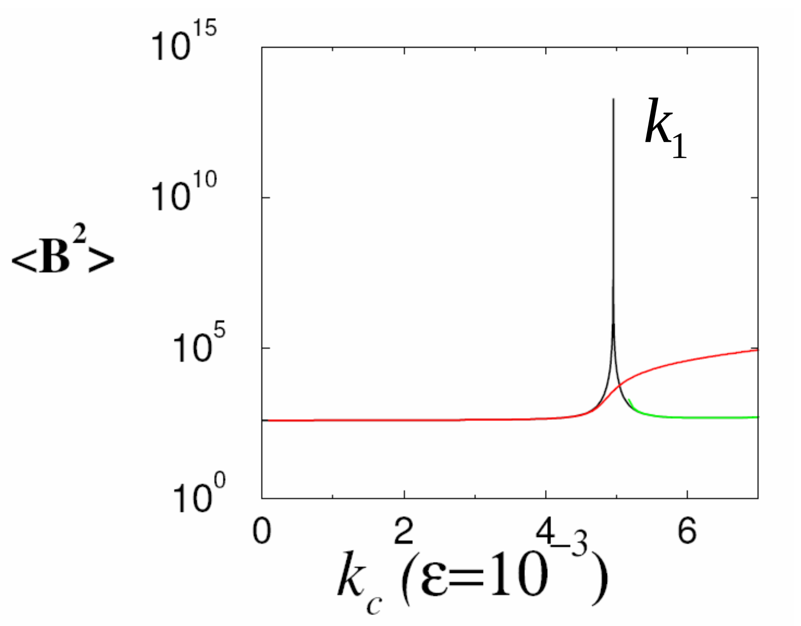
- around $k_1$

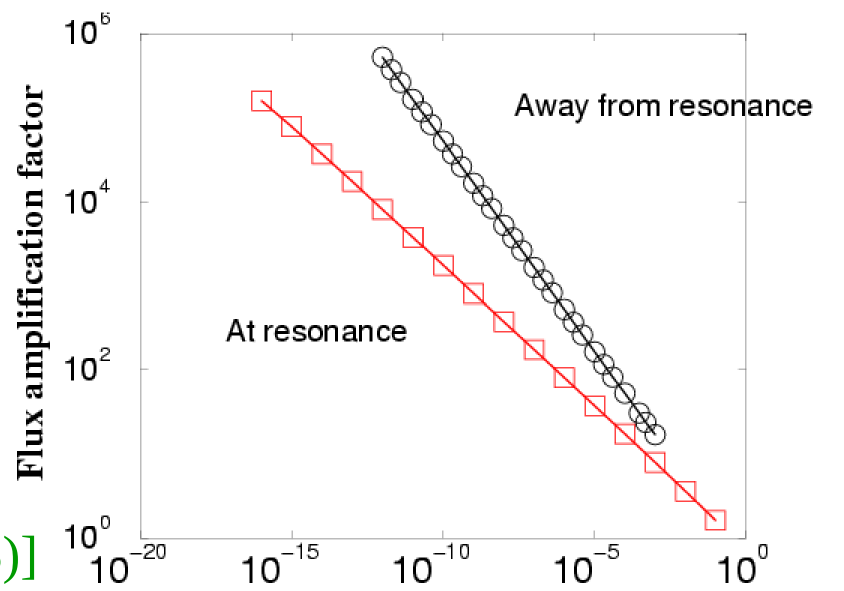
$$\alpha_1^r = \frac{\langle \chi_v \chi_1 \rangle^{1/3}}{\langle \chi_1^4 \rangle} \frac{1}{\varepsilon^{1/3}}$$

- away from $k_1$

$$\alpha_1^{(1)} = \sqrt{\frac{k_c^2 - k_1^2}{k_c^2 \langle \chi_1^4 \rangle} \frac{1}{\varepsilon}}$$

$$\alpha_1^{(2)} = -\alpha_1^{(1)}; \alpha_1^{(3)} = \alpha + o(\varepsilon)$$




[Tang and Boozer, PRL 94, 225004 (2005)]