TRANSP-Based Optimization Towards Non-inductive Ramp-up

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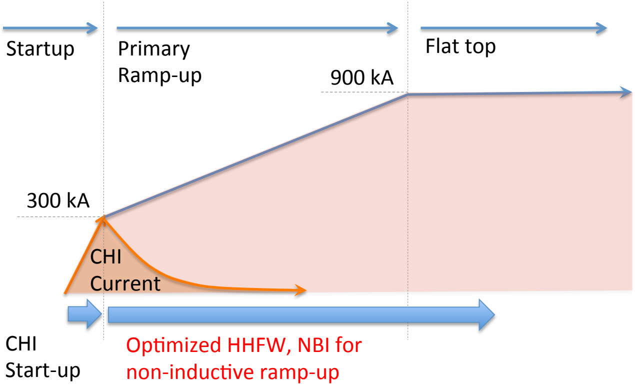
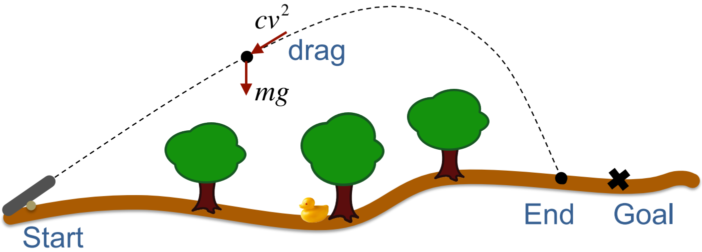
Objectives and Outline

- **Main Objective**
  - Combine Predictive-TRANSP with numerical optimization to find a strategy for non-inductive current ramp-up.



Example: Cannon Optimization



$$\begin{aligned} \underset{\mathbf{z}}{\text{minimize}} \quad & J(\mathbf{z}) \\ \text{subject to} \quad & h(\mathbf{z}) = 0 \\ & g(\mathbf{z}) \leq 0 \end{aligned}$$

- **IDEA:** Transform control problem (cannon aiming) into optimization problem
- Pass to numerical optimization solver

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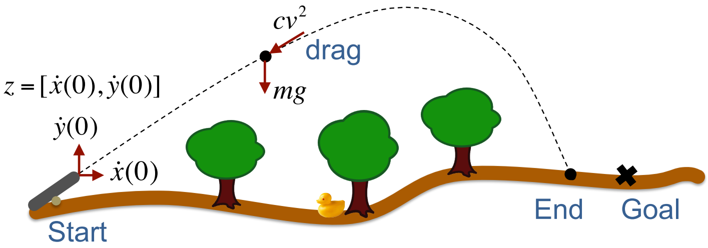

optimization variables: $\mathbf{z} = [\dot{x}(0), \dot{y}(0)]$

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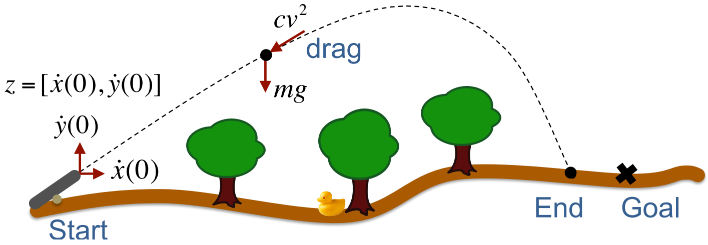

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$$\underset{\mathbf{z}}{\text{minimize}} \quad J(\mathbf{z})$$

$\leftarrow$ objective: i.e. powder consumed

$$J \propto v = \sqrt{\dot{x}(0)^2 + \dot{y}(0)^2}$$

$$\text{subject to} \quad h(\mathbf{z}) = 0$$
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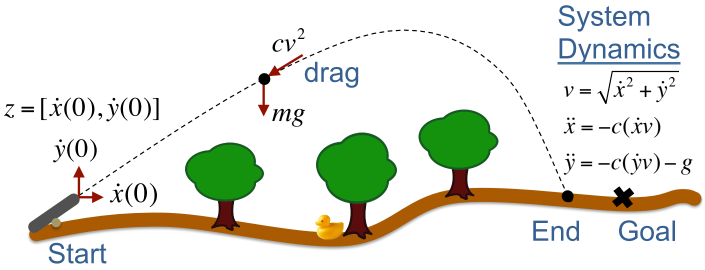

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minimize$_{\mathbf{z}}$ $\quad J(\mathbf{z}) \quad \leftarrow$ optimization objective: powder consumed

$$J \propto v = \sqrt{\dot{x}(0)^2 + \dot{y}(0)^2}$$

subject to $\quad h(\mathbf{z}) = 0 \quad \leftarrow$ equality constraints:

$$[x, y]_{\text{end}} = [x, y]_{\text{goal}}$$

$$v = \sqrt{\dot{x} + \dot{y}}, \quad \ddot{x} = -c(\dot{x}v), \quad \ddot{y} = -c(\dot{y}v) - g$$

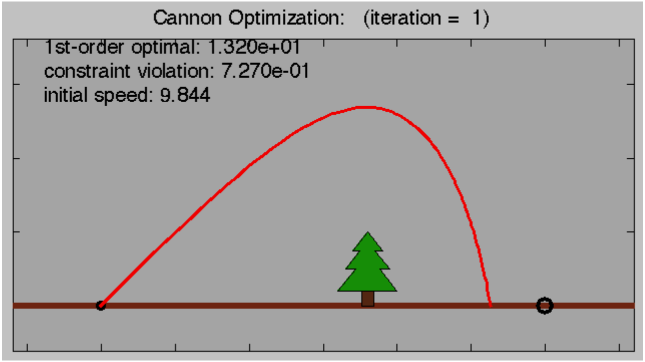
$g(\mathbf{z}) \leq 0 \quad \leftarrow$ inequality constraints:

$$y(t) \geq \text{Height Trees}$$

Cannon Targeting: Optimization-based Control

- Start with an approximate solution $\mathbf{z}_0$ (guess)
- Use gradient information of objective and constraints (and approximate hessian of the Lagrangian) to improve on the approximate solution

$$\mathbf{z}_{k+1} = \mathbf{z}_k + \Delta\mathbf{z}$$



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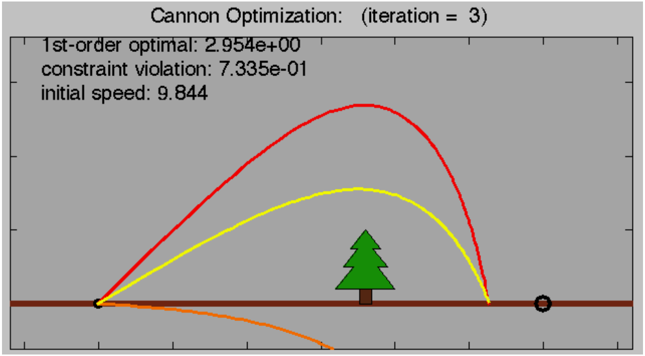

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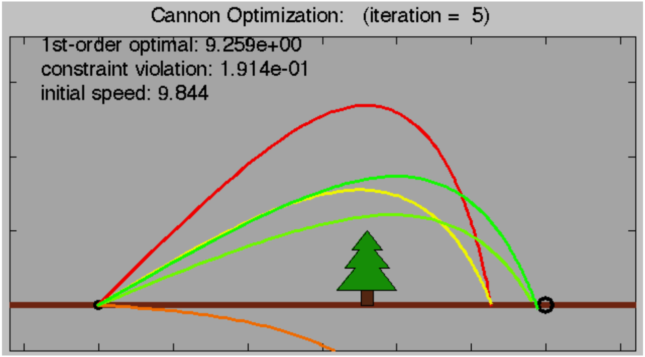

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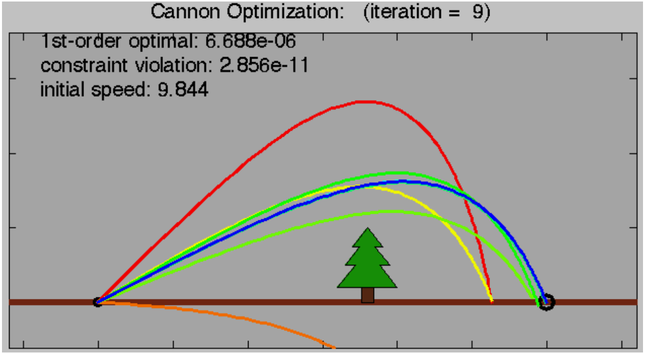


Formulation of Feedforward (FF) Control Optimization

- A feedforward (open-loop) control policy is obtained via nonlinear optimization - minimize a cost function subject to various constraints

$$\min_{u_{\mathrm{FF}}(t)} \quad J(I_{\mathrm{p}}, I_{\mathrm{p}}^{\mathrm{NI}}) \quad \Big\}\ \text{Cost Function, Optimization Objective}$$

s.t.:

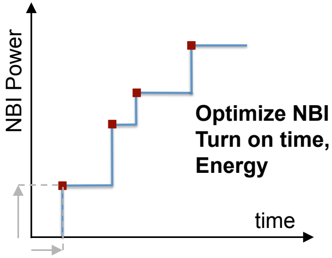
$$\dot{\psi} = f_{\psi}(\psi, u_{\mathrm{FF}}) \quad \Big\}\ \text{Model of poloidal magnetic flux evolution; } q \text{ profile / current profile are function of } \psi$$

$$u_{\mathrm{FF}}(t) \in \mathcal{U} \quad \Big\}\ \text{Physical limitation on actuators: Bounds / Rate Limit}$$

$$\beta_N(t) \leq \beta_{N_{\mathrm{max}}} \quad \Big\}\ \text{Nonlinear Constraint: MHD Stability Limit}$$

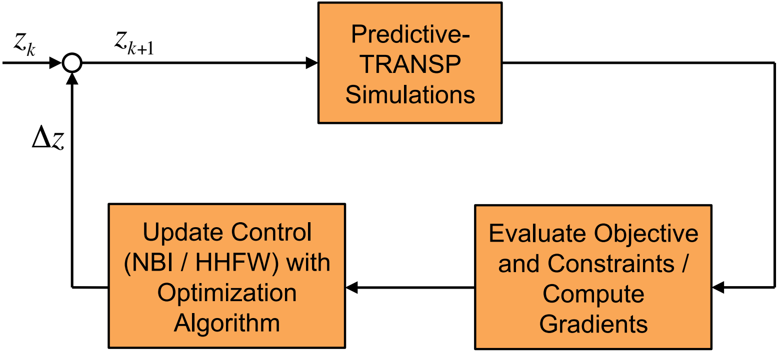
$$c(q) \leq 0 \quad \Big\}\ \text{Constraints on shape of } (q)\text{: MHD Stability Limit}$$

$$c(I_{\mathrm{p}}) \leq 0 \quad \Big\}\ \text{Constraint on current target}$$



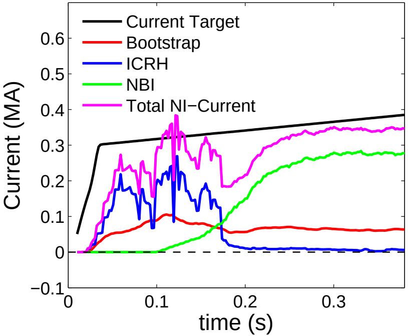
TRANSP-Based Optimization Code

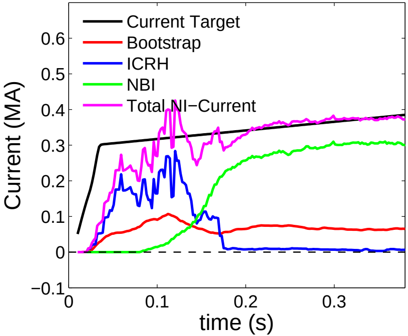
- Use predictive modeling capability of the TRANSP code
- Combine with numerical optimization (OMFIT) to do automated feedforward control optimization.



OMFIT

TRANSP-Based Optimization For Control Design





- **Left** Figure 9 - F. M. Poli et al. Simulations of non-inductive current ramp-up and sustainment in the National Spherical Torus Experiment Upgrade. Nucl. Fusion 55 (2015) 123011 (12pp)
- **Right** Result of an optimization of the ICRH and NBI powers to meet the current target with non-inductive sources.

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