Predicting frequency chirping of Alfvénic modes

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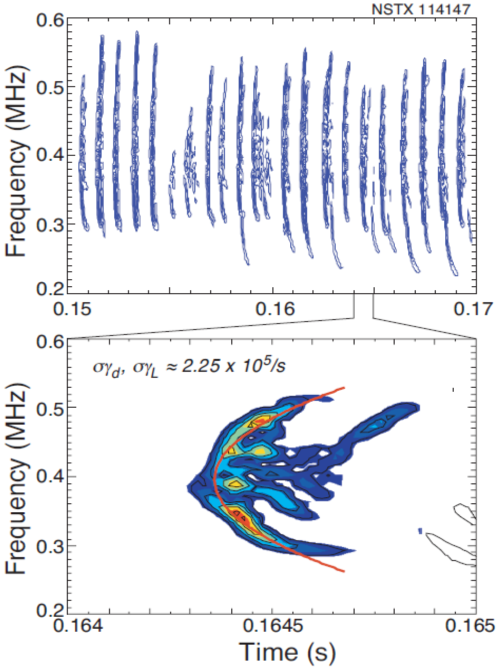
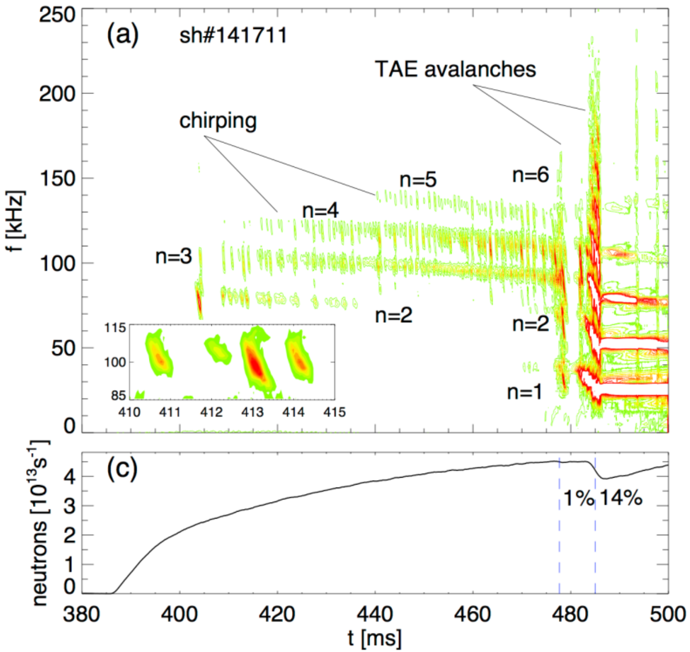
Transport induced by chirping modes can seriously degrade the confinement of energetic particles

GAE/CAE
Fredrickson et al, PoP 2006



TAE   Podestà et al, NF 2012



Up to 40% of injected beam is observed to be lost in DIII-D and NSTX

Chirping behavior is observed to be a precursor to avalanches in NSTX

What is the dominant fast ion transport mechanism (convective or diffusive)? When is quasilinear theory applicable? Gorelenkov's talk

Why chirping is ubiquitous in NSTX but rare in DIII-D?

What makes wave chirping likely to happen?

Starting point: **kinetic equation** plus **wave power balance** close to marginal stability

Berk, Breizman and Pekker, PRL 1996 Lilley, Breizman and Sharapov, PRL 2009

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Cubic equation: lowest-order nonlinear correction to the evolution of mode amplitude C

$$\frac{dC(t)}{dt} - C(t) = -\sum_j \int d\Gamma \mathcal{H} \int_0^{t/2} d\tau \tau^2 C(t-\tau) \times$$
$$\times \int_0^{t-2\tau} d\tau_1 e^{-\hat{\nu}_{stoch}^3 \tau^2 (2\tau/3+\tau_1) + i\hat{\nu}_{drag}^2 \tau(\tau+\tau_1)} \times$$
$$\times C(t-\tau-\tau_1) C^*(t-2\tau-\tau_1)$$

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- If nonlinearity is weak: linear stability, solution saturates at a low level and *f* merely flattens (system not allowed to further evolve nonlinearly).
- If *C* blows up: system enters a strong nonlinear phase with large mode amplitude and can be driven unstable (**precursor of chirping modes**).

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stabilizing

destabilizing (makes integral sign flip)

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A general criterion for Alfvén wave chirping

(strongly dependent on competition between fast ion scattering and drag)

$$Crt = \frac{1}{N}\sum_{j,\sigma_{\parallel}}\int dP_{\varphi}\int d\mu \frac{|V_j|^4}{\omega_{\theta}\nu_{drag}^4}\left|\frac{\partial \Omega_j}{\partial I}\right|\frac{\partial f}{\partial I} Int$$

>0: fixed-frequency likely

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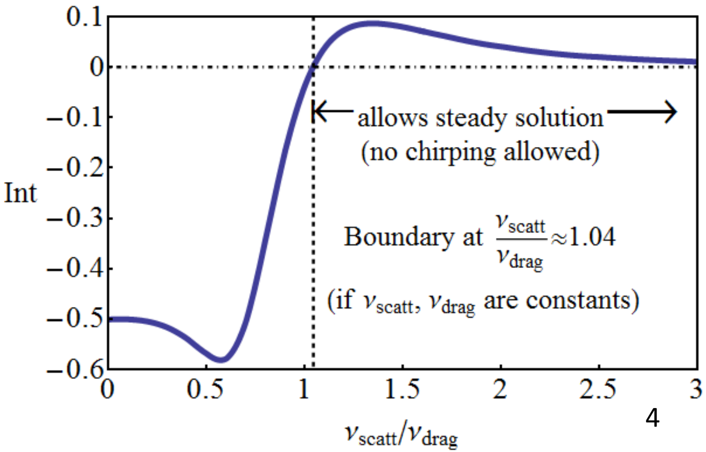
$$\boxed{Int} \equiv Re\int_0^\infty dz \frac{z}{\frac{\nu_{stoch}^3}{\nu_{drag}^3}z - i} exp\left[-\frac{2}{3}\frac{\nu_{stoch}^3}{\nu_{drag}^3}z^3 + iz^2\right]$$

Crt accounts for collisional coefficients varying along resonances and particle orbits



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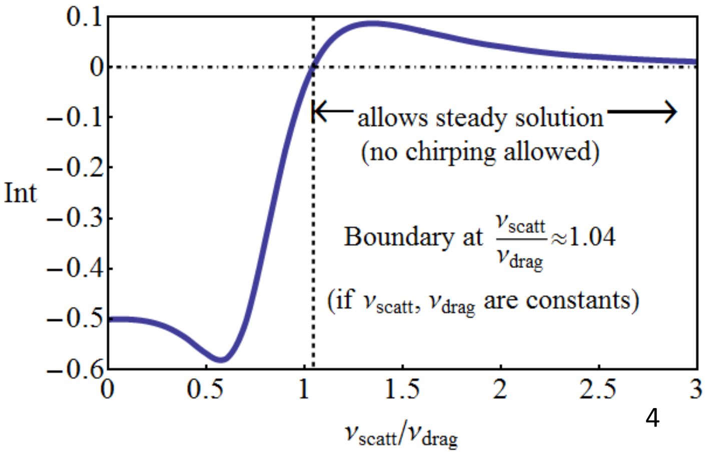

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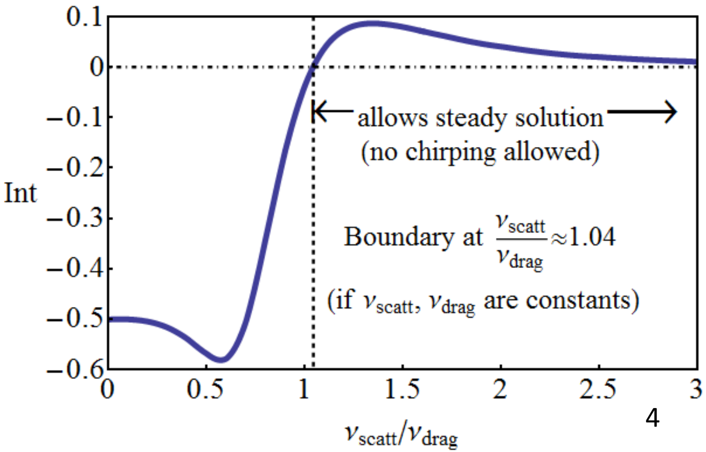
Eigenstructure information:

$$q\int dt \mathbf{v}_{dr} \cdot \delta \mathbf{E} e^{i\omega t}$$



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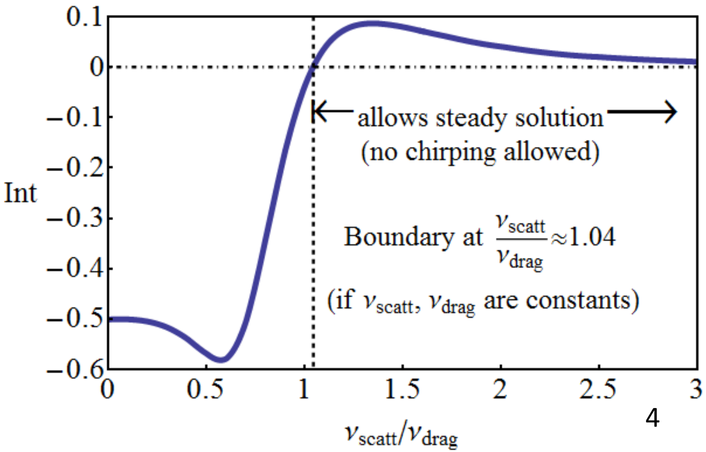
$$\Omega_l\left(\mathcal{E}' + \omega P_\varphi/n, P_\varphi, \mu\right) \equiv$$
$$\equiv n\frac{d\varphi}{dt} - l\frac{d\theta}{dt} - \omega_0$$



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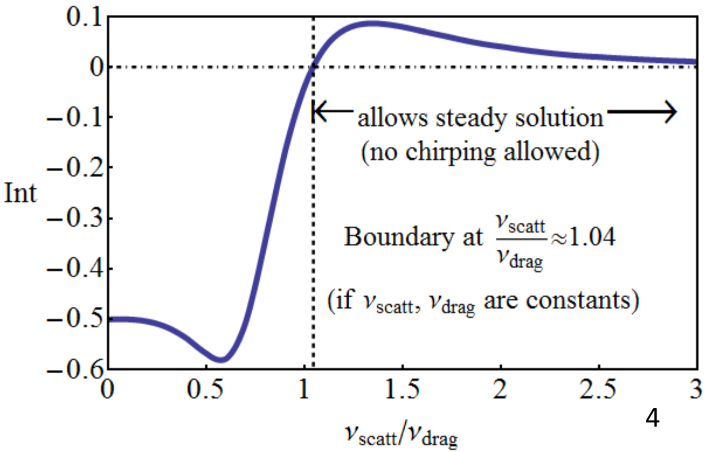

Criterion was incorporated into NOVA-K: nonlinear prediction from linear physics elements

Turbulence scattering explains why chirping is common in NSTX but rare in DIII-D

Proposed criterion for *Alfvén wave* chirping onset:

Duarte, Berk, Gorelenkov *et al*, PRL (submitted)

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Inclusion of fast ion micro-turbulence

From GTC gyrokinetic simulations for passing particles (Zhang, Lin and Chen, PRL 2008):

$$D_{EP}\left(E_{EP}\right) \approx D_{th,i} \frac{5T_e}{E_{EP}}$$

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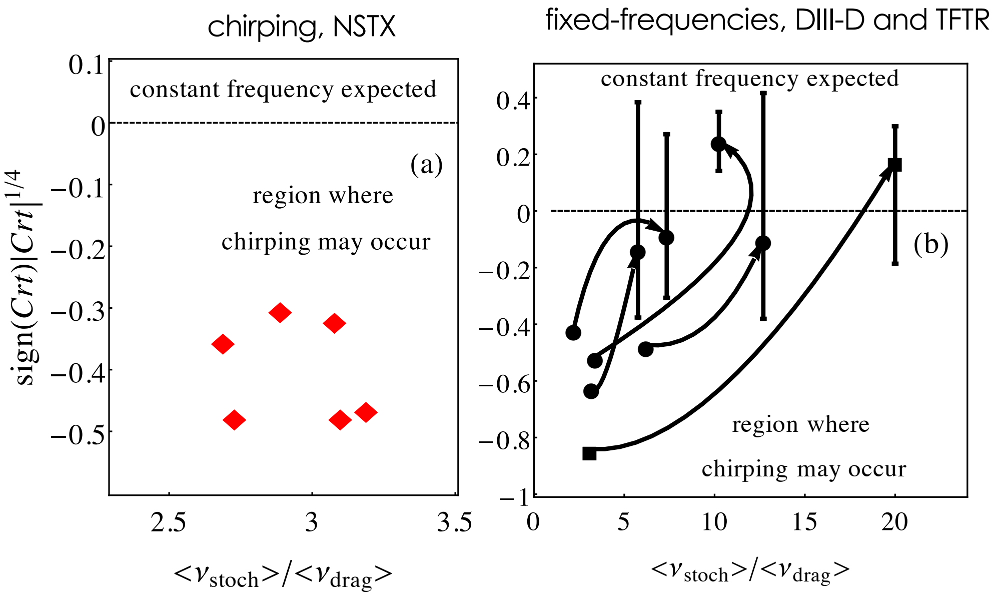
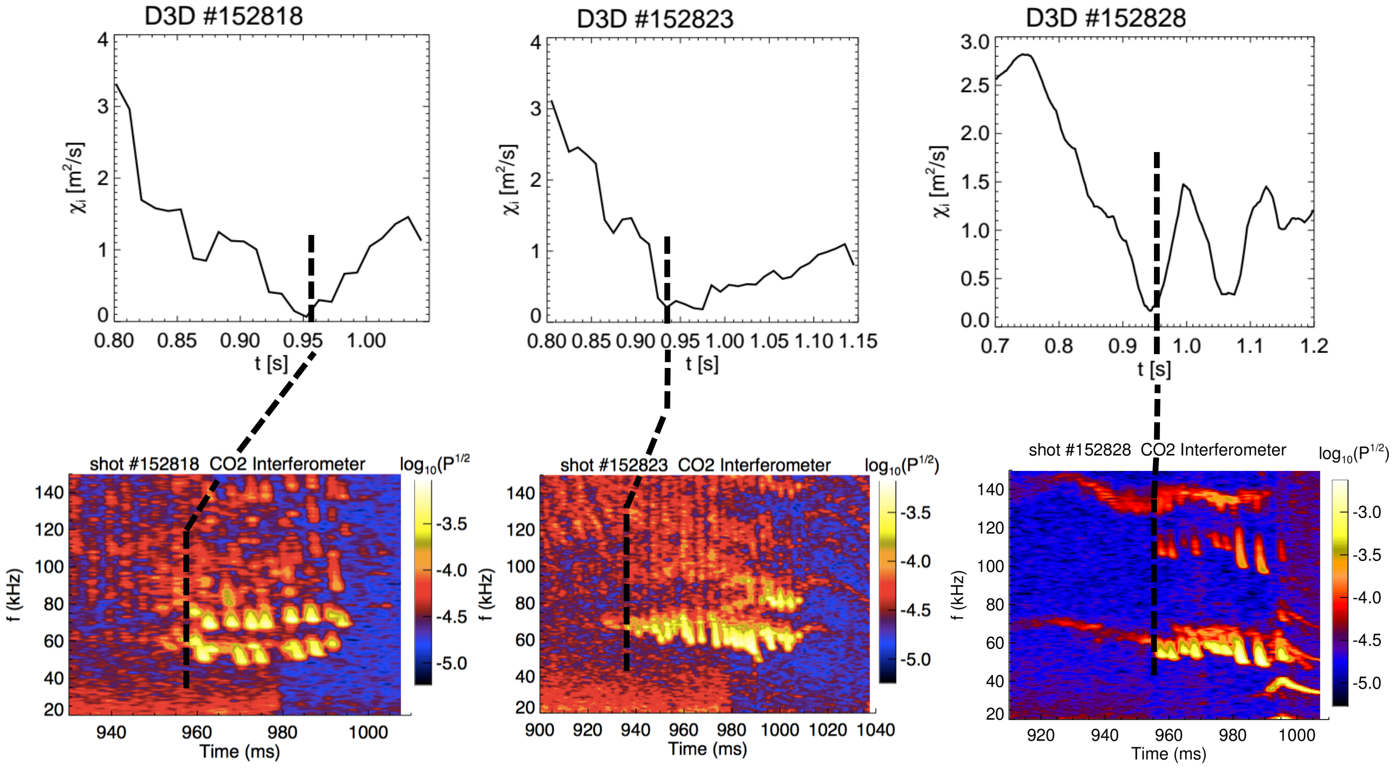


Alfvén wave chirping quantitatively agrees with the criterion

Arrows represent the turbulent diffusion that adds up to pitch-angle scattering

Correlation between the emergence of chirping and a substantial decrease of ion micro-turbulence in DIII-D:



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Conclusions

- Theory and experiments have indicated that wave chirping response is linked with low turbulent activity;
- Although micro-turbulence-induced fast ion transport is low compared with Alfvén wave-induced transport, it competes with collisional transport (e.g., during the early non-linear evolution);
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Future possibilities

- Dedicated experiments with negative triangularity on DIII-D will explore the consequences of this chirping study;
- NSTX-U: possibility of use of HHFW and 3D fields to increase fast ion stochasticity;
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The ultimate goal of this dedicated study is to identify the applicability of reduced models for fast ion transport

Thank you