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Statistical Analysis of diffusion coefficients in NSTX

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Motivation

- Understanding of plasma transport in tokamak plasmas is one of the most important issues.
- In Spherical Torus(ST), ion transport is thought to be the neoclassical level. However, electron transport is more complicated, which is thought to be governed by multiple microinstabilities and fast particle instabilities.
- Because of this non-linearity and complexity, statistical approaches have been taken to understand heat transport using experimental database of NSTX.
- Multiple Linear Regression(MLR) and Neural Network(NN) are used in this research.



Setup for statistical analysis with MLR and NN

Target Machine	NSTX
Total datasets	~680 sets from TRANSP output of ~680 shots (50% for training, 30% for validation to improve training, 20% for TEST)
Input parameters	$R/L_{n_e}, R/L_{n_i}, R/L_{T_e}, R/L_{T_i}, R/L_{P_{fast}}, n_i/n_e, T_i/T_e, P_{fast}/P, q, \hat{s},$ $R\nabla V_\phi/V_{th,i}, V_\phi/V_{th,i}, V_{E \times B}/c_s, \kappa, \delta_{lower}, \delta_{upper}, Z_{eff}, \rho_*, v_{ei}/(c_s/a),$ β_t, α_{MHD} (21 parameters believed to determine transport, instabilities)
Input data mining	3 radial points ($\rho = 0.35, 0.50, 0.65$) at a single time point averaged over radius (± 0.05) & time (± 15 ms)
Target parameters	$Q_e/Q_{GB}, Q_i/Q_{GB} (Q_{GB} = \rho_*^2 c_s n_e T_e)$

• Statistical Method

- Multiple Linear regression : linear regression using stepwise method
- Neural Network: training by Levenberg-Marquardt algorithm
scanning 1-3 layers/5-15 neurons for optimization



R^2 with MLR and NN at $\rho = 0.35, 0.50, 0.65$

- **Multiple Linear Regression**

ρ	$Q_{e,N}$	$Q_{i,N}$
0.35	0.643	0.488
0.50	0.617	0.488
0.65	0.714	0.517

- **Neural Network**

ρ	$Q_{e,N}$	$Q_{i,N}$
0.35	0.873	0.787
0.50	0.895	0.796
0.65	0.889	0.735

* R^2 : determination coefficient \rightarrow shows regression's accuracy



Dominant Input Parameters Determining Transport Fluxes – MLR

• Multiple Linear Regression

- Dominant input parameters

ρ	0.35		0.50		0.65	
	Q_e	Q_i	Q_e	Q_i	Q_e	Q_i
dominant parameters	$v_{ei}/(c_s/a), \alpha_{MHD}, q$	$n_i/n_e, Z_{eff}$	$v_{ei}/(c_s/a), n_i/n_e, Z_{eff}$	T_i/T_e	$v_{ei}/(c_s/a), q, \alpha_{MHD}$	T_i/T_e

$$Q_{e,N}(\rho = 0.35) = -0.006 - 0.253 * \rho^* + 0.005 * T_i/T_e + 0.008 * v_{ei}/(c_s/a) + 0.001 * P_{fast}/P + \varepsilon * \hat{s} + 0.004 * n_i/n_e$$

$$R^2 = 0.643 \quad -0.011 * \beta_t + \varepsilon * R/L_{T_e} + \varepsilon * R/L_{P_{fast}} - 0.001 * V_{E \times B}/c_s - 8.544 * 10^{-6} * \alpha_{MHD} + \varepsilon * R/L_{T_i} + 0.001 * \kappa + \varepsilon * R \nabla V_\phi / V_{th,i}$$

$$Q_{i,N}(\rho = 0.35) = 0.028 - 0.124 * \rho^* - 0.002 * T_i/T_e - 0.004 * Z_{eff} + \varepsilon * R/L_{P_{fast}} - 6.107 * 10^{-6} * \alpha_{MHD} + \varepsilon * R/L_{T_e}$$

$$R^2 = 0.488 \quad +0.002 * \delta_{upper} + 0.002 * v_{ei}/(c_s/a) - 0.021 * n_i/n_e + \varepsilon * q$$

$$Q_{e,N}(\rho = 0.50) = 0.244 - 1.065 * \rho^* + 0.001 * R/L_{T_i} + 0.017 * v_{ei}/(c_s/a) + 0.003 * R/L_{n_e} - 0.003 * R/L_{T_e} - 0.038 * Z_{eff}$$

$$R^2 = 0.617 \quad -5.376 * 10^{-5} * \alpha_{MHD} + 0.016 * T_i/T_e + 0.002 * \hat{s} - 0.016 * V_\phi/V_{th,i} + 0.003 * q - 0.209 * n_i/n_e$$

$$Q_{i,N}(\rho = 0.50) = 0.015 - 0.022 * \rho^* - 0.009 * T_i/T_e + 0.006 * n_i/n_e - 0.305 * \beta_t - 0.001 * R/L_{T_i} + \varepsilon * R/L_{P_{fast}}$$

$$R^2 = 0.384 \quad +0.002 * v_{ei}/(c_s/a) + \varepsilon * R/L_{n_e} + 0.001 * R \nabla V_\phi / V_{th,i} - 0.002 * \kappa + 0.001 * \hat{s}$$

$$Q_{e,N}(\rho = 0.65) = -0.067 + 0.031 * v_{ei}/(c_s/a) + 0.003 * R/L_{T_i} + 0.026 * P_{fast}/P + 0.002 * R/L_{P_{fast}} + 0.009 * R/L_{n_e}$$

$$R^2 = 0.613 \quad -0.022 * T_i/T_e + 0.009 * R \nabla V_\phi / V_{th,i} - 0.007 * Z_{eff} - 0.055 * V_\phi/V_{th,i} + \varepsilon * Z_{eff} + 0.019 * q + 0.012 * V_{E \times B}/c_s$$

$$Q_{i,N}(\rho = 0.65) = 0.052 - 0.052 * T_i/T_e + 0.003 * R/L_{T_e} + 0.016 * n_i/n_e - 2.690 * \rho^* + 0.001 * R/L_{P_{fast}} + 0.001 * R/L_{n_e}$$

$$R^2 = 0.403 \quad +0.005 * V_{E \times B}/c_s$$

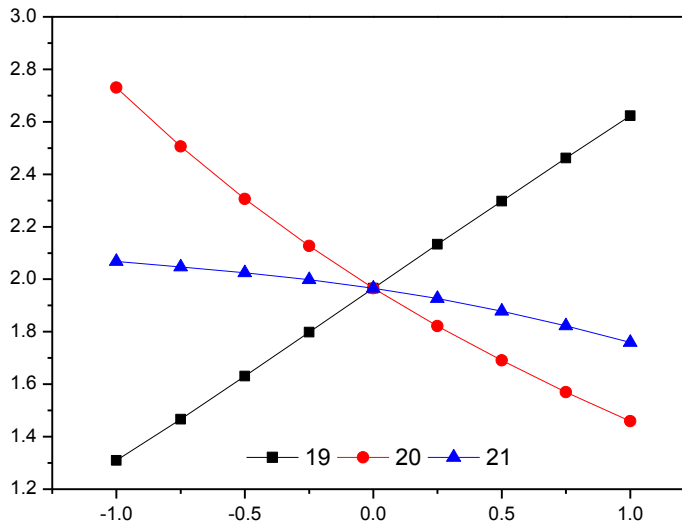
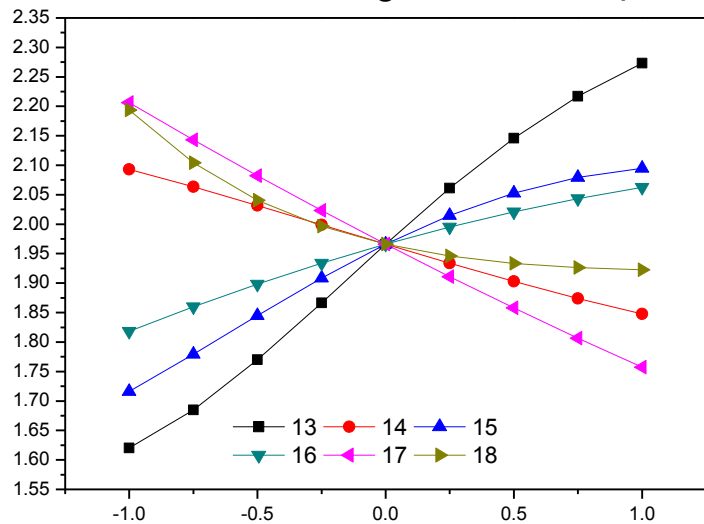
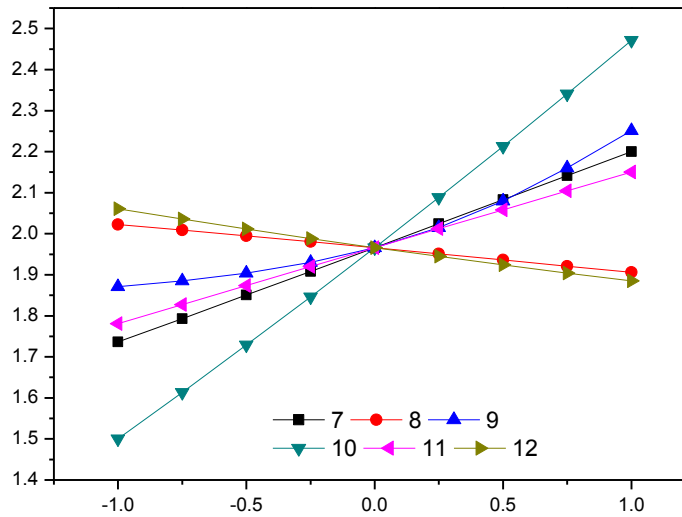
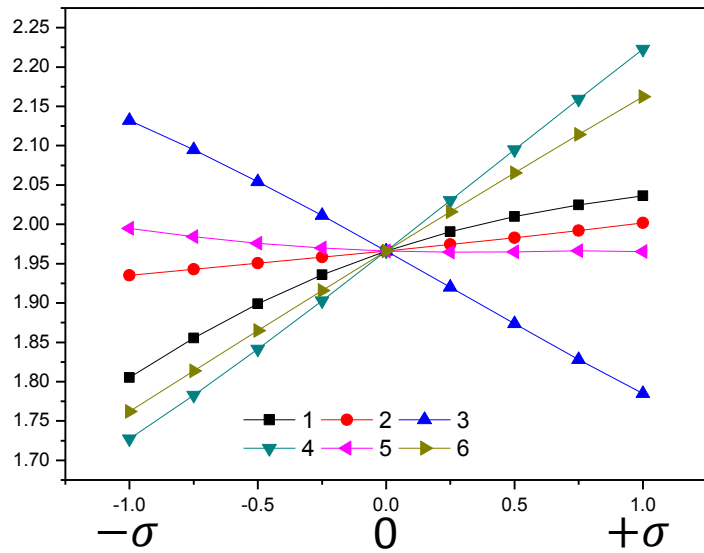


Dominant Input Parameters Determining Transport Fluxes – NN

- Neural Network is a ‘black-box’ by itself, thus it is difficult to identify dominant input parameters.
- There are several ways to interpret Neural Network results,
 - 1) Visualization of Neural Network
 - connect lines between neurons with different color or thickness based on weighting factor
 - 2) Sensitivity analysis
 - Find out how much does the input parameter affect on the output by scanning each input parameter from $+\sigma$ to $-\sigma$ with fixing other input parameters at average value in the database
 - 3) Randomization test
 - Randomize weighting factor between input-hidden-output layer
- Here, we will use **sensitivity analysis**.



Example, Sensitivity scan for Q_e/Q_{GB} at $\rho = 0.65$



#	Input	#	Input
1	R/L_{n_e}	12	$V_\phi/V_{th,i}$
2	R/L_{n_i}	13	$V_{E \times B}/c_s$
3	R/L_{T_e}	14	r/a
4	R/L_{T_i}	15	κ
5	$R/L_{P_{fast}}$	16	δ_{lower}
6	n_i/n_e	17	δ_{upper}
7	T_i/T_e	18	Z_{eff}
8	P_{fast}/P	19	ρ_*
9	q	20	$v_{ei}/(c_s/a)$
10	\hat{s}	21	β_t
11	$R\bar{V}V_\phi/V_{th,i}$		



Dominant Input Parameters Determining Transport Fluxes – NN

- Neural Network**

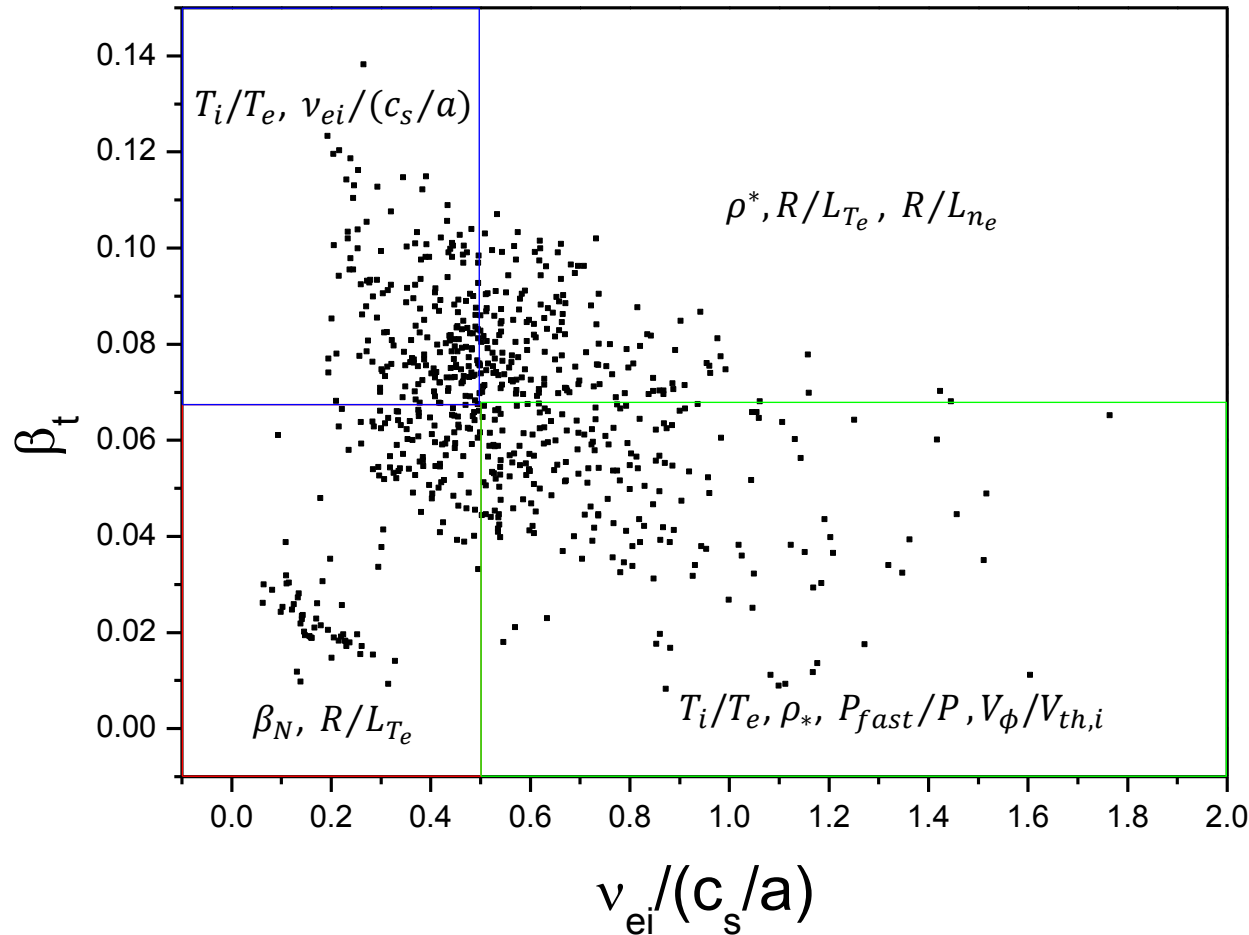
- Dominant input parameters

ρ	0.35		0.50		0.65	
	Q_e	Q_i	Q_e	Q_i	Q_e	Q_i
dominant parameters	$P_{fast}/P, Z_{eff}$	$P_{fast}/P, Z_{eff}$	$v_{ei}/(c_s/a), \beta_t, R/L_{ne}$	$P_{fast}/P, Z_{eff}$	$v_{ei}/(c_s/a), \beta_t, \hat{s}$	$R/L_{Te}, R/L_{P_{fast}}, T_i/T_e, q, \kappa$



Dominant Input Parameters in Various Plasma Regimes

- Database is separated into four different regimes with respect to toroidal beta and collisionality and NN is applied to extract dominant input parameters for each regime.
- Preliminary results are shown below (dominant parameters are shown in each regime).



Conclusion and Future Work

- As statistical methods, MLR and NN are applied to derive scaling laws of transport fluxes using NSTX experimental database.
- NN, more suitable for non-linear systems, shows higher accuracy over MLR.
- Dominant parameters determining the transport fluxes are extracted from MLR and NN sensitivity scan. Difference of the dominant parameters between MLR and NN is mainly due to difference in R^2 .

- Dominant parameters will be identified in various plasma regimes in terms of toroidal beta and collisionality for physics studies.

- The MLR and NN scaling laws for transport fluxes will be implemented into transport codes for transport modelling. Predictive modelling will be done for NSTX shots to evaluate their performance first, then applied to NSTX-U for further study.

- Comparison with other ST devices such as MAST and conventional tokamaks is planned.

