Recent progress in EM GTS*

E.A. Startsev, W.W. Lee, W.X. Wang

Plasma Physics Laboratory, Princeton University
Princeton, NJ, USA

September 22, 2015

*Research supported by the U.S. Department of Energy.

Recent EM developments in GTS

- In the past year we have been working on implementation of recently developed perturbative particle simulation double-split-weight scheme for simulation of gyrokinetic finite- plasmas in the gyrokinetic tokamak code GTS.

- The scheme uses two delta-f weights carried by each particle to represent particles density and pressure.

- Use of separate weight to represent particle pressure allows to alleviate so called "cancelation" problem in finite- gyrokinetic simulations with fully kinetic electrons.

- We have successfully used this scheme for simulation of linear tearing and drift-tearing modes, in both collisionless and semi-collisional regimes in sheared slab and high-aspect ratio cylindrical cross-section tokamak geometries.

- In the last several month this scheme has been extended to includes key toroidal effects for the simulation of linear semi-collisional micro-tearing (MTM) and kinetic ballooning (KBM) modes in realistic aspect ratio cylindrical cross-section tokamak.

- The initial simulations tests of KBM modes using the modified turbulence code GTS has been successful.

Double-weight EM Perturbative Scheme

- Introducing particle weights $\delta f_e = w_e F_{0e}$, the Vlasov equation for electrons becomes ($\partial_y = (1/r)\partial_\theta$):

$$\frac{\partial w_e}{\partial t} + v_\parallel \partial_\parallel w_e + v_{dr}\partial_z w_e = -\partial_y(\phi - v_\parallel A)\kappa_e - v_\parallel E_\parallel + v_{dr}\partial_z\phi + C[w_e].$$

- Next, we introduce $\bar{w}_e = \partial_\parallel w_e - \kappa_e \partial_y A$ the equation for $\bar{w}$ becomes:

$$\frac{d\bar{w}_e}{dt} = \partial_y(\kappa_e E_\parallel - \nu\kappa_e^I A) - v_\parallel \partial_\parallel E_\parallel + (v_{dr}\partial_z)[\partial_\parallel \phi - \kappa_e \partial_y A] + C[\bar{w}_e].$$

- We also introduce second weight $\widehat{w}_e = w_e + \kappa_e \partial_y \int dt\phi$ which satisfies the equation:

$$\frac{d\widehat{w}_e}{dt} = -\partial_y \int dt(v_\parallel \kappa_e E_\parallel - \nu\kappa_e^I \phi) - v_\parallel E_\parallel + (v_{dr}\partial_z)[\phi + \kappa_e \partial_y \int dt\phi] + C[\widehat{w}_e].$$

where in the first term κ_e^I includes only current gradient part of κ_e:

$$\kappa_e \equiv -(\partial F_{0e}/\partial \mathrm{x})/F_{0e} = \kappa_n - \frac{3}{2}\kappa_{Te} + \frac{1}{2}\kappa_{Te}\frac{v_\parallel^2 + v_\perp^2}{v_{te}^2} + \frac{U'(r)}{\beta}\frac{v_\parallel}{v_{te}^2}$$

- Here the background current gradient is related to $q(r)$ profile by Ampere's law:

$$U'(r) = r\left(\frac{d}{rdr}\right)^2\left(\frac{r^2}{Rq}\right)$$

Field Equations

- With introduction of new weights $\bar{w}_e$ and $\widehat{w}_e$, the field equations become
$$\nabla_\perp^2 \phi = \widehat{S}_0 + (\kappa_n + \kappa_{Ti})(\widehat{\Gamma}_0 - 1)\partial_y \phi + \kappa_{Ti}\widehat{\Gamma}_1 \partial_y \phi, \tag{1}$$
and
$$[\nabla_\perp^2 - \beta(m+1)]E_\parallel = -[\nabla_\perp^2, \partial_\parallel]\phi - \bar{S}_0 + \beta(\bar{S}_2 + \widehat{S}_\nu) + \beta \partial_y \widehat{S}_{dr}$$
$$+\beta(\widehat{\Gamma}_0 - 1)E_\parallel + U'(r)\partial_y(\phi - \nu_0 \int dt\phi) - (\kappa_n + \kappa_{Ti})(\widehat{\Gamma}_0 - 1)\partial_y A - \kappa_{Ti}\widehat{\Gamma}_1 \partial_y A,$$

- Here $m = m_i/m_e$ and $\nu_0 = \int \nu(v)(v_\parallel/v_{the})^2 F_{0e}(v) d^3v$.

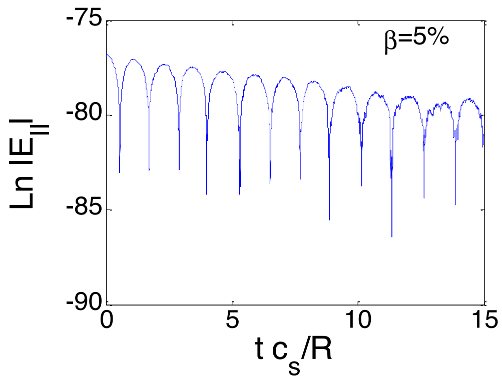
$$S_n = \int v_\parallel^n F_0(w_e - \langle w_i \rangle), \quad S_\nu^e = \int \nu(v) v_\parallel w_e F_{0e}, \quad S_{dr} = \int v_{dr} v_\parallel (w_e - \langle w_i \rangle) F_0$$

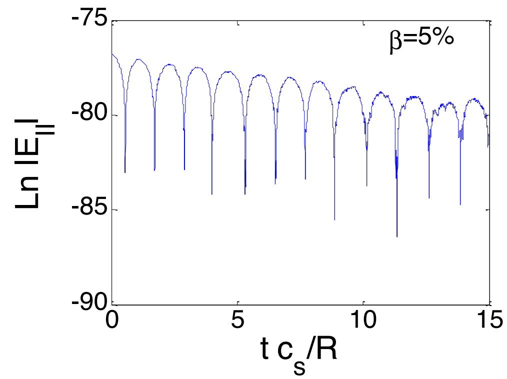
- where $\widehat{Q}$ quantities are now calculated by using new weight $\widehat{w}$ as for example $\widehat{S}_\nu^e = \int \nu(v) v_\parallel \widehat{w}_e F_{0e}$ and $\bar{Q}$ quantities are calculated by using the weight $\bar{w}$.

- To find A we integrate in time
$$A = -\int dt(\partial_\parallel \phi + E_\parallel).$$

- The first term on the right in equation for $E_\parallel$ involves commutator $[\nabla_\perp^2, \partial_\parallel]$ which requires only second derivative of potential and first derivative of q-profile and therefore can be neglected for modes with $k_\perp a \gg 1$.

Simulations of shear-Alfven wave





- Simulation of Shear Alfven wave with a minor radius similar to NSTX: $a/\rho_i = 165$.

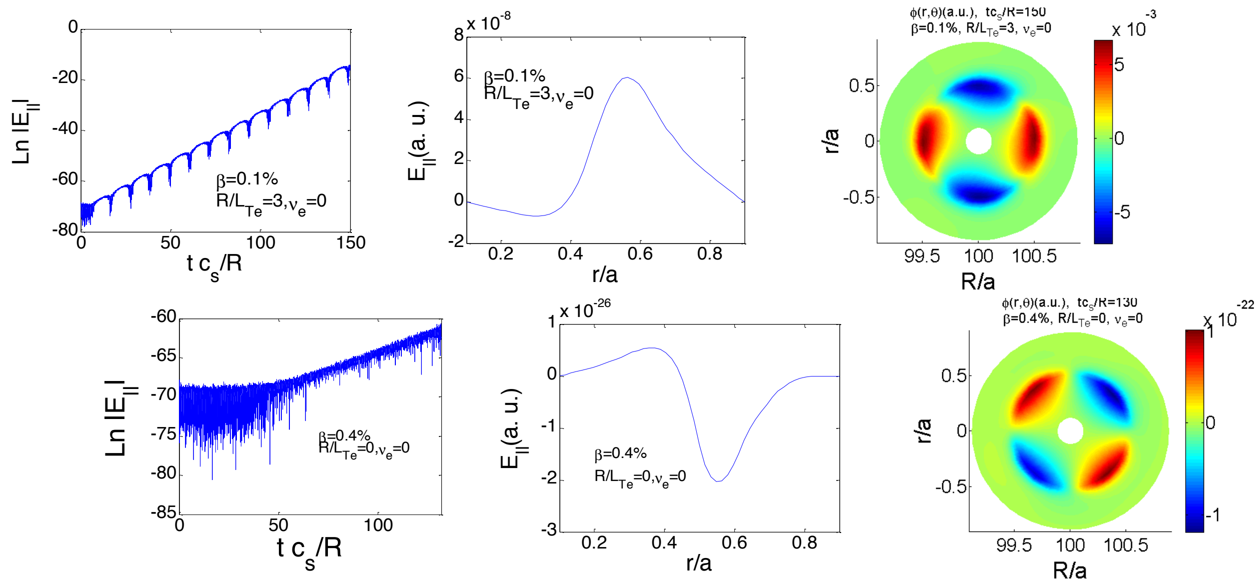
- $dl_\theta = 1.65\rho_i$, $dr = 1.65\rho_i$, $dt(c_s/R) = 0.06$ with 32 poloidal planes, $N_p = 20$.

- $(m, n) = (1, 1)$ component of $log|E_{||}|$ at r/a = 0.56 (corresponding to q = 2).

- The frequencies of the Alfven wave were $\omega = (2.7 + 0.15i)c_s/R$ for $\beta = 5\%$, and $\omega = (5.4 + 0.042i)c_s/R$ for $\beta = 1\%$.

- Approximate formula $\omega = (n - m/q)/\sqrt{\beta}(c_s/R)$ gives $\omega = 2.2c_s/R$ and $\omega = 5.0c_s/R$.

Simulations of global drift-tearing mode



- $dl_\theta = 0.25\rho_i$, $dr = 0.15\rho_i$,$dt(c_s/R) = 0.06$, $N_p = 20$.

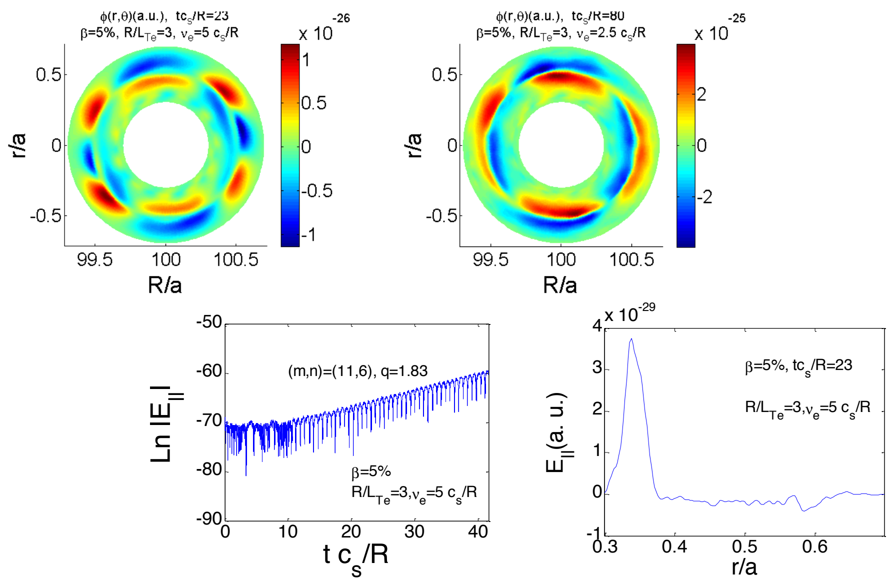
- $log|E_{\|}|$ at $r/a = 0.56$ (corresponding to q $= 2$); radial profile of $E_{\|}$ at $t(c_s/R) = 130$; $\phi(r,\theta)$ at $t(c_s/R) = 130$ for $a/\rho_i = 16$.

- $\gamma_{code} = 0.09(c_s/R)$, $\gamma_{eigen} = 0.1(c_s/R)$.

- $log|E_{\|}|$ at $r/a = 0.56$ (corresponding to q $= 2$); radial profile of $E_{\|}$ at $t(c_s/R) = 23$; $\phi(r,\theta)$ at $t(c_s/R) = 23$ for $a/\rho_i = 16$.

- $\omega_{code} = 0.28 + 0.38i(c_s/R)$, $\omega_{eigen} = 0.32 + 0.40i(c_s/R)$.

Simulations of micro-tearing mode with no curvature drifts

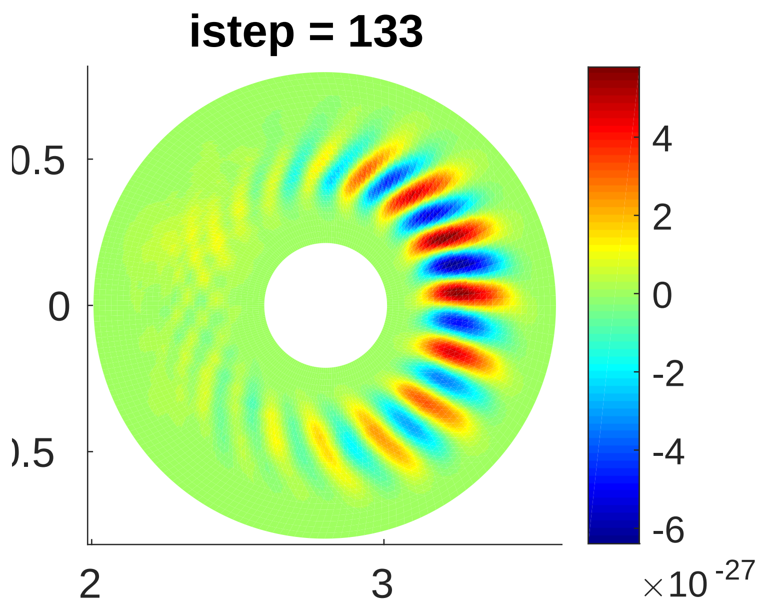


- $dl_\theta = 0.25\rho_i$, $dr = 0.055\rho_i$, $dt(c_s/R) = 0.06$, $N_p = 20$.

- $\phi(r, \theta)$ for runs with different values of electron collision frequency.

- $log|E_{||}|$ at $r/a = 0.35$ (corresponding to $q = 1.83 = 11/6$); radial profile of $E_{||}$ at $t(c_s/R) = 23$.

Simulations of Toroidal KBM mode for Cyclone-like parameters



- $\phi(r, \theta)$ in poloidal plane.
- $m_i/m_e = 3674$, $\beta_e = 3.3\%$, $R/L_{te} = R/L_{ti} = 6.92$, $R/L_n = 2.22$, $R/a = 2.8$, $a/\rho_i = 76$,
- $q(r) = 1.25 + 0.67(r/a)^2 + 2.38(r/a)^3 - 0.06(r/a)^4$.
- $dl_\theta = 0.7\rho_i$, $dr = 1.2\rho_i$, $dt(c_s/L_{ti}) = 0.01$, $N_p = 20$.

The EM algorithms generalized/reformulated for toroidal geometry; implementation into GTS is underway

Rigorously reformulate electron field-particle dynamics based on ideas developed/tested in simple cases, taking into account existing GTS framework

- Work on field quantity $E_\parallel = -\mathbf{b}\cdot\nabla\phi - (1/c)\partial A_\parallel/\partial t$ or $\int E_\parallel dt$ instead $A_\parallel$
  $A^h \equiv -c\int E_\parallel dt$; $A^s \equiv A_\parallel(t_0) - c\int \mathbf{b}\cdot\nabla\phi dt$

- Solve δf-equation for reformulated perturbed electron distribution δh_e
  $f_e \equiv \delta h_e + [1-(e/cT_e)A^h(v-u_\parallel)]f_{\rm sm}$

$$\frac{\partial \delta h_e}{\partial t} + \frac{1}{B^*}\nabla_{\mathbf{Z}}\cdot(\dot{\mathbf{Z}}B^*\delta h_e) = -\left[1-\frac{e(v-u_\parallel)}{cT_e}A^h\right](\dot{\mathbf{Z}}\cdot\nabla_{\mathbf{Z}} f_{\rm sm})$$

$$+\left\{\frac{e(v-u_\parallel)}{cT_e}\frac{\partial A^h}{\partial t} + \dot{\mathbf{Z}}\cdot\nabla_{\mathbf{Z}}[\frac{e(v-u_\parallel)}{cT_e}A^h]\right\} f_{\rm sm} + C^l$$

note: on RHS, $\partial A^h/\partial t$ term can be canceled by a term in $\dot{v_\parallel}\partial f_{\rm sm}/\partial v_\parallel$

Associated weight equation:

$$\dot{w} = \frac{1-w}{f_{\rm sm}}({\rm RHS}-C^l) + \frac{w-\langle w\rangle}{f_{\rm sm}}({\rm RHS}-C^l)$$

The EM algorithms generalized/reformulated for toroidal geometry; implementation into GTS is underway

- Field equations for A^h (generalized Amperes law), A^s and ϕ

$$(\nabla_\perp^2 - \frac{\omega_{pe}^2}{c^2})A^h = -\frac{4\pi e}{c}\int v_\parallel \delta h_e d^3v - \frac{4\pi e_i}{c} n_i \delta\bar{u}_{i,\parallel} - \nabla_\perp^2 A^s$$

$$\frac{\partial A^s}{\partial t} = -c\mathbf{b}\cdot\nabla\phi$$

$$-\nabla_\perp \cdot \frac{Z_i n_{i,0}}{B\Omega_i}\nabla_\perp\phi = \delta\bar{n_i} - \int \delta h_e d^3v$$

- Consistent Lagrangian equation of gyro-center motion using $E_\parallel$

$$\mathcal{L} = \frac{e_s}{c}[(\rho_\parallel + \frac{A_\parallel}{B})\mathbf{B} + \mathbf{A}]\cdot\mathbf{x} + \mu\dot{\xi} - \mathcal{H}$$

$$\mathcal{H} = \frac{1}{2}\frac{e_s^2}{m_s c^2}\rho_\parallel^2 B^2 + \mu B + e_s\phi$$

- We will focus on completion of implementing/testing/verifying generalized EM algorithms in GTS and initial applications, with a goal of bring this highly desirable capability to production for NSTX-U physics applications and other studies

Conclusions

- In the near future we will continue testing the double-split-weight EM scheme in GTS against available results for KBM and semi-collisional MTM eighenmodes in tokamaks with circular flux surfaces.
- Implementation of generalized, comprehensive EM algorithm into GTS is underway
- Apply GTS to current problems interest to NSTX-U.