Theory of Marangoni flow in Majesky-Kaita Li*quid* Li*thium* tray[1]

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Abstract

- *The Marangoni flow is generated by the reduced surface tension* $\boldsymbol{\sigma(T)}$ *at high temperatures,* $\boldsymbol{\sigma'(T) < 0}$*. Because of the fluid viscosity the flow penetrates into the bulk of the fluid.*
- *While so far, this effect has been essentially unknown and ignored in the MHD of liquid lithium, the presented theory shows that the fast heat removal from the heating zone is consistent with the recently discovered on (by R.Majesky and R.Kaita, PPPL) extraordinary heat propagation from the high power e-beam spot on the surface of the liquid Li tray on CDX-U machine.*
- *The theory of Marangoni flow in thin lithium layer MHD has been formulated. Because of the magnetic field in fusion devices, only thin layers are of practical interest. Effect of flow on heat removal from hot spots (or strike lines) has been assessed with and without magnetic field.*
- *The 3-D numerical code Cbebm was launched to simulate the heat transfer from the localized heating zone at the surface of liquid Li. At the moment, it includes a complete 3-D temperature evolution equation, while the 3-D distribution of flow velocity is calculated assuming its stationary viscous distribution. Extension of Cbebm to full viscous dynamics and MHD effects is envisioned in future.*
- *In contrast with conventional thermo-conduction case, when the heat zone remain localized, while the peak temperature is sensitive to the peak power deposition, in liquid lithium the peak temperature is not sensitive to the power deposition profile. Instead, the Marangoni flow expands the heat zone over entire area.*

Contents

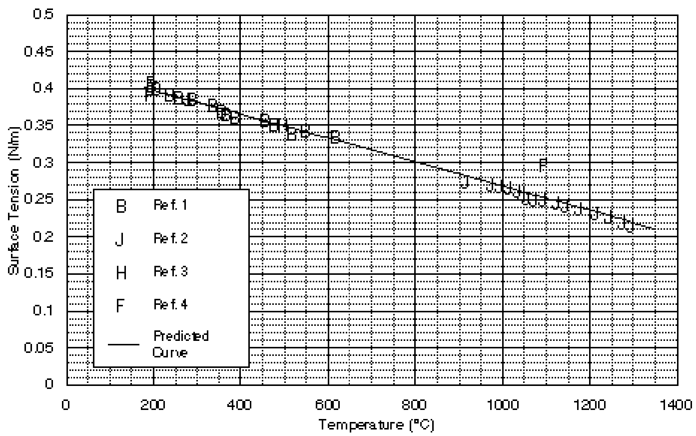
Li has negative derivative of the surface tension $\sigma(T)$ as a function of temperature T

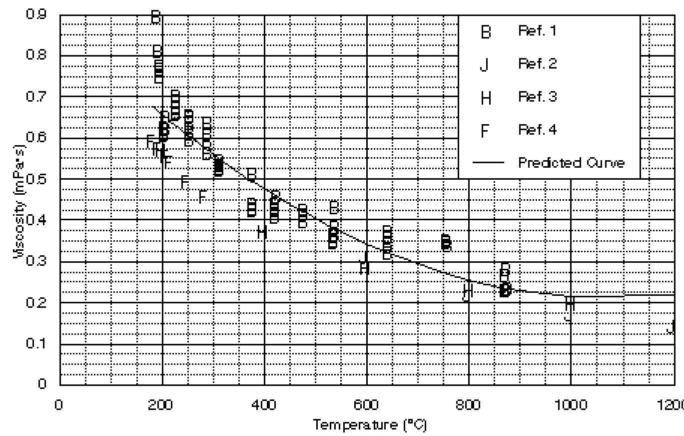
$$\text{Li dynamics:}\quad \rho\frac{D\vec{V}}{Dt} = -\nabla P + \mathrm{j}\times\mathrm{B} + \underbrace{\nu\Delta\vec{V}}_{viscosity}, \quad P = \underbrace{p}_{pressure} + \underbrace{\rho g z}_{gravity} \tag{1.1}$$

with the boundary condition

$$\nu\left.\frac{\partial\vec{V}_s}{\partial n}\right|_{surface} = -\nabla_s\underbrace{\sigma(T_s)}_{\substack{surface\\ tension}} = \underbrace{-\frac{d\sigma(T)}{dT}\nabla_s T_s}_{Marangoni\ flow\ drive}. \tag{1.2}$$

($\vec{n}$ is the normal vector to the pool, s is the surface projection).





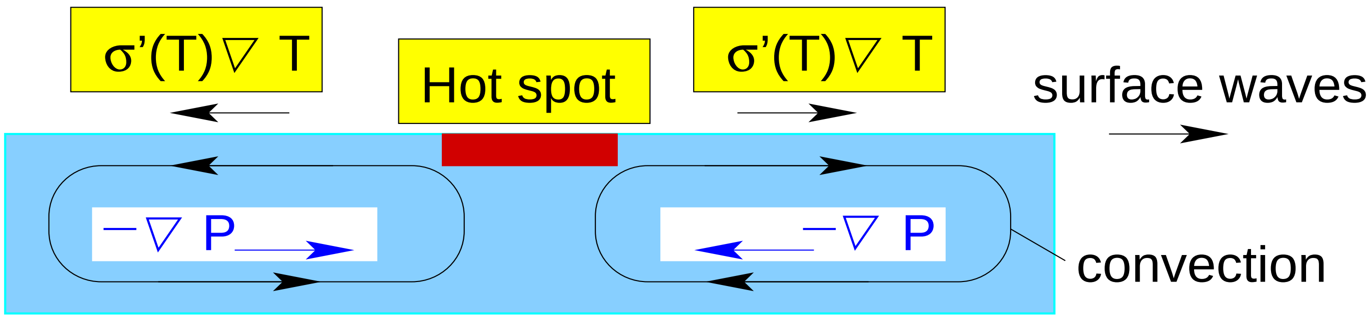
$\sigma(T) = 0.382 - 1.62\cdot 10^{-4}\bar{T}$ $\quad \boxed{\bar{T} \equiv T - 300^oC} \quad$ $\nu(T) = 5.6\cdot 10^{-4} - 9.0\cdot 10^{-7}\bar{T} + 5.8\cdot 10^{-10}\bar{T}^2$

Negative $\sigma'(T) < 0$ drives the fluid away from the hot spot.

2 Marangoni controls heat transport during e-beam spot heating

Surface tension gradient generates a viscous fbw inside liquid lithium

$$\frac{\partial \vec{V}}{\partial n} = \frac{\sigma'(T)}{\nu}\nabla_s T_s, \quad \vec{V} \simeq \frac{\sigma'(T)}{\nu}\nabla_s T_s d \simeq 0.12\nabla_s T_s d, \quad (d \text{ is the thickness of fbw}) \tag{2.1}$$



Marangoni fbw effects is dominant in physics of the e-beam spot heating.

1. Viscous fbw establishment across the pool (several secs) is determined by

$$d_{\nu-skin} = 1.8\sqrt{t}\cdot 10^{-3} < \frac{1}{2}d_{pool\ depth}, \quad \vec{V} = 2.2\cdot 10^{-4}\nabla T\sqrt{t} \tag{2.2}$$

Thermal conductivity based $\nabla T \simeq 10^5$ K^o/m would give $\vec{V} > 10$ m/sec in a fraction of sec.

Marangoni effect + gravity create convective cells inside liquid lithium

2. Surface tension elevates the fluid surface and establishes the pressure gradient along the pool: $p = p(x, y)z$.
3. Slowly evolving convective cells are established with dominant component

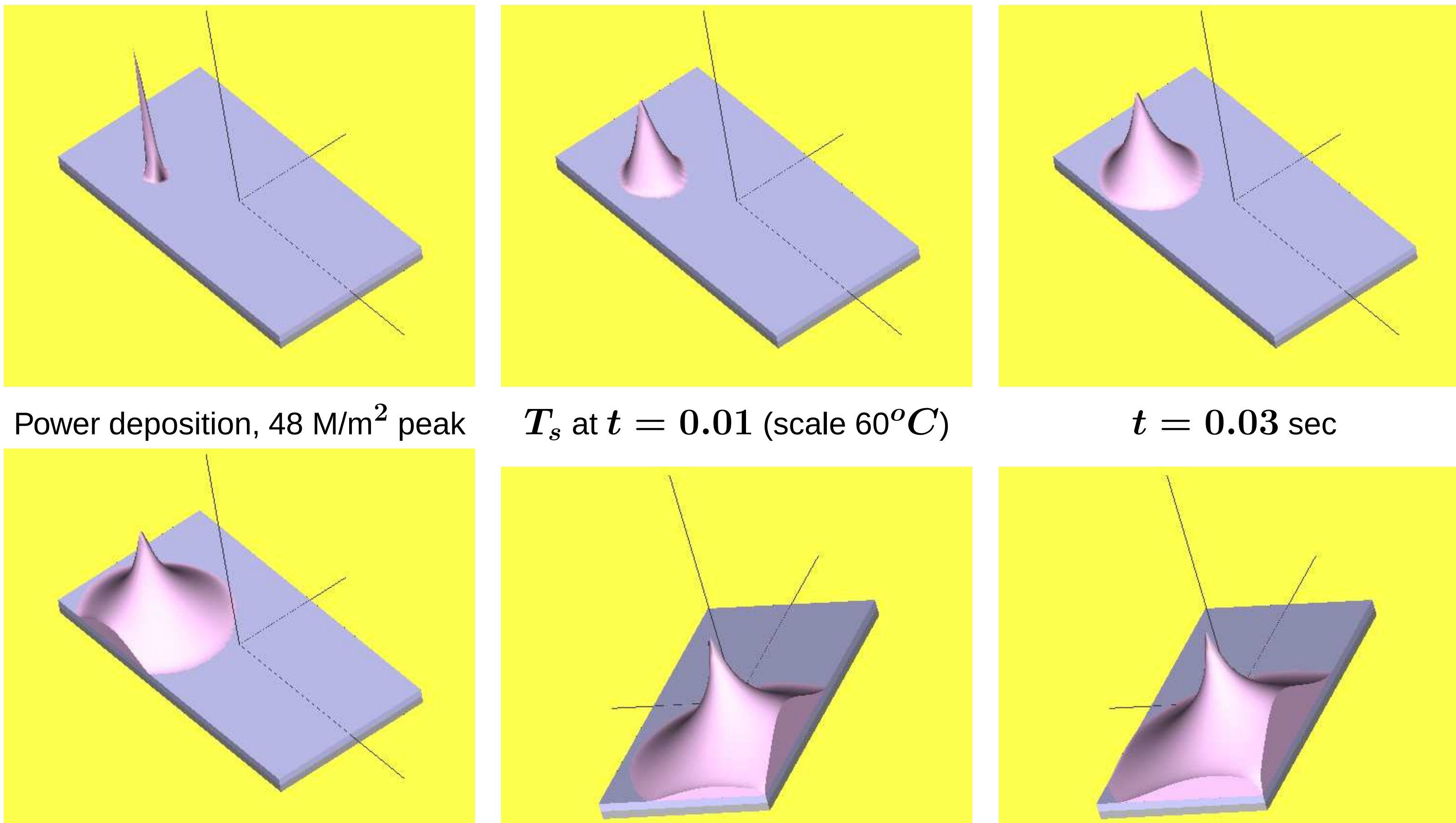
$$\begin{aligned} \vec{V} &= (z+d)\frac{-2(z+d)^2 + 6d(z+d) - 3d^2}{3d^2}\left(\frac{\sigma'(T)}{\nu}\nabla_s T\right)_{z=0}, \\ V_z &= (z+d)^2\frac{(z+d)^2 - 4d(z+d) + 3d^2}{6d^2}\nabla_s\left(\frac{\sigma'(T)}{\nu}\nabla_s T\right)_{z=0}. \end{aligned} \tag{3.1}$$

   They mix the heat inside the fluid and limite $\nabla_s T$.
4. Convective cells effciently transport heat from the hot spot to the cold fluid

$$\vec{\Gamma}_v = \rho c_p T\vec{V} = 2.1\cdot 10^6 T\vec{V} = 210\frac{T}{100}\vec{V}\left[\frac{\text{MW}}{\text{m}^2}\right]. \tag{3.2}$$

$\nabla_s T$ is self-consistently determined by balancing heating and convective transport, independent in peak power flux

Convective cell region expands toward the cold fluid (or yet unmelted Li)



Power deposition, 48 M/m^2 peak — T_s at $t = 0.01$ (scale 60^oC) — $t = 0.03$ sec

$t = 0.1$ sec — $t = 0.3$ sec — $\Delta T_s < 30^o, \quad t = 0.5$ sec

`Cbebm` *3-D simulations of flow expansion from e-beam spot in* $10 \times 20 \times 0.5$ *[cm^3] tray*

5. At the same time, elevates the surface of the fluid and generates surface waves.

Cbebm is a 3-D code utilizing a thin layer MHD approximation and explicit integration

Heat transport

$$\rho c_p \frac{DT}{Dt} = \rho c_p \underbrace{\left(\frac{\partial T}{\partial t} + (\vec{V}\cdot\nabla)T\right)}_{full\ time\ derivative} = -\kappa \Delta T, \quad (\nabla\cdot\vec{V}) = 0, \tag{4.1}$$

with fluid dynamics (not yet implemented)

$$\underbrace{\rho \frac{D\vec{V}}{Dt} = -\nabla p + \rho g z + \mathrm{j}\times\mathrm{B} + \nu\Delta\vec{V},}_{replaced\ by\ \vec{V}\ from\ Eq.[3.1]} \tag{4.2}$$

with the boundary condition at free surface

$$\nu \left.\frac{\partial \vec{V}_s}{\partial n}\right|_{surface} = -\nabla_s \underbrace{\sigma(T_s)}_{\substack{surface\\ tension}} = \underbrace{-\frac{d\sigma(T)}{dT}\nabla_s T_s}_{Marangoni\ flow\ drive} . \tag{4.3}$$

and Ohm's law (not yet implemented)

$$\vec{j} = \sigma_E\left(\nabla\varphi + \vec{V}\times\vec{B}\right), \quad \nabla\times\vec{j} = \sigma_E\left(\underbrace{\vec{B}_\perp\cdot\nabla_n)\vec{V}}_{\vec{B}_\perp\ drag} - \underbrace{(\vec{V}\cdot\nabla)\vec{B}}_{\nabla B_{tor}\ drag}\right) \tag{4.4}$$

Both $\vec{B}_\perp$ and gradient $\nabla_s|\vec{B}_s|$ of planar component lead to the drag force

The normal component $\vec{B}_\perp$ creates a pressure drop along the flow

$$\Delta\left(\rho\frac{V^2}{2}\right) = 2\Re_0\frac{B_\perp^2}{2\mu_0}, \quad \Re_0 \equiv \mu_0\sigma_E V L, \tag{5.1}$$

where L is the length of the flow. For Marangoni flow a new dimentionless drag parameter $Z_\perp$ determines the effect of $\vec{B}_\perp$ drag

$$Z_\perp \equiv \frac{2\mu_0\sigma_E V L\frac{B_\perp^2}{2\mu_0}}{\nu\frac{L}{h^2}V} = \frac{2\mu_0\sigma_E h^2}{\nu}\frac{B_\perp^2}{2\mu_0} < 1. \tag{5.2}$$

Numerically

$$\boxed{Z_\perp = \left(\frac{B_\perp}{0.011\ [\mathrm{T}]}\cdot\frac{h}{1\ [\mathrm{mm}]}\right)^2 < 1. \qquad (5.3)}$$

Both $\vec{B}_\perp$ and gradient $\nabla_s|\vec{B}_s|$ of planar component lead to the drag force

The gradient of the co-planar magnetic field B_{tor}/R along the flow creates the drag

$$\Delta\left(\rho\frac{V^2}{2}\right) = \Re_2\frac{B^2}{2\mu_0}, \quad \Re_2 \equiv \mu_0\sigma_E\frac{h^2}{R}V. \tag{5.4}$$

The dimentionless drag parameter $Z_{||}$ determines the effect of B_{tor}/R

$$Z_{||} \equiv \frac{\mu_0\sigma_E V h^2\frac{B_{tor}^2}{2\mu_0}}{\nu\frac{LR}{h^2}V} = \frac{\mu_0\sigma_E h^4}{RL\nu}\frac{B_{tor}^2}{2\mu_0} < 1. \tag{5.5}$$

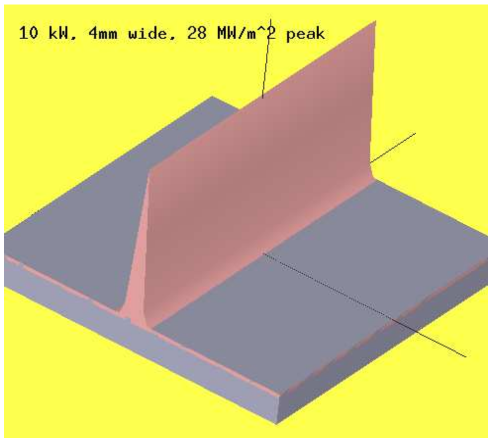
Numerically

$$\boxed{Z_{||} = \left(\frac{B_{tor}}{0.016\ [\mathrm{T}]}\cdot\frac{h}{\sqrt{RL}}\cdot\frac{h}{1\ [\mathrm{mm}]}\right)^2 < 1. \qquad (5.6)}$$

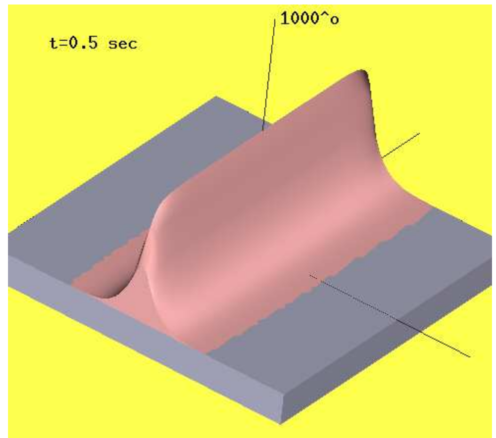
Liquid lithium MHD with Marangoni exhibits the effect of self-adjustment of flow thickness h to V independent conditions $Z_\perp < 1$ and $Z_{||} < 1$

This validates the thin layer approximation for liquid lithium.

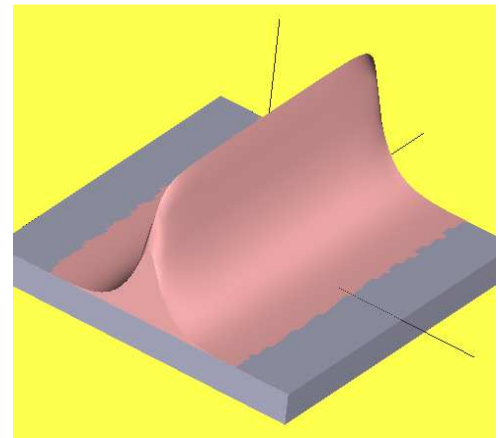
Liquid lithium tray is not sensitive to peaks in power deposition, while the W plate is.



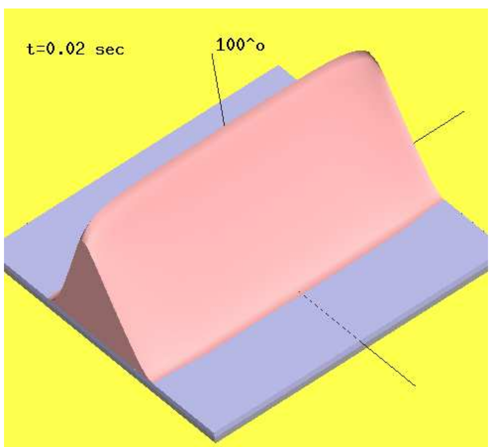
10 kW, 28 M/m^2 peak



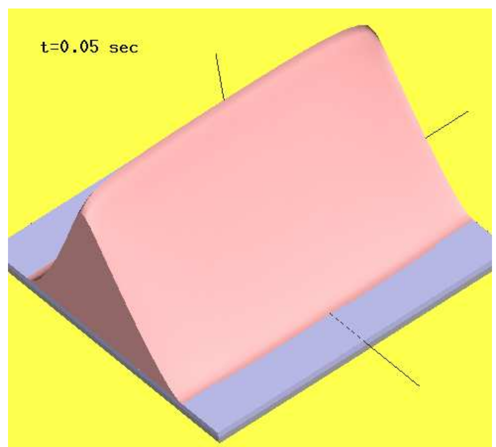
W plate 10x10x1 cm^3, T_s at $t = 0.5$



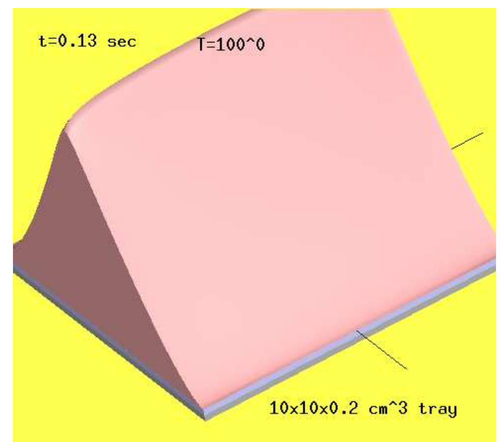
$t = 0.03$ sec



Li 2 mm tray, $t = 0.02$ sec



$t = 0.05$ sec



$\Delta T_s < 30^o, \quad t = 0.13$ sec

Majesky-Kaita Li-Li tray opens new opportunities for the spherical tokamaks, including IST