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On scaling laws in scrape-off-layer plasmas

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Abstract. The scaling law in the scrape-off-layer (SOL) plasma is discussed, and the influence of the cross-field thermal conductivity, κ, is analysed. Analyses are done for the edge temperature, temperature fall-off length and the fluctuation level. The dependencies on the heating power, edge density and safety factor are investigated. The scaling law of the thickness of the heat channel, which is crucial in evaluating the performance of the future large devices, depends on the model of κ. It is found that the fluctuation level is most sensitive to the model of κ. The importance of simultaneous observation on the temperature, gradient and fluctuation level is shown.

1. Introduction

Among various issues of the plasma wall interactions, the problem of the high heat flux onto the diverter plate has been widely recognized. The prediction of the peak heat load in large devices, such as ITER, is one of the most important tasks for the physics of plasmas in the scrape-off-layer (SOL).

The area on the diverter plate, where the energy is deposited, is determined by the competition between the parallel plasma flow and the flow across the magnetic surface. There have been a lot of analytic studies which have analysed the SOL and diverter plasmas (for present status see, for example, [1]). The competition between the plasma flow along the field line and that across the field line is discussed. The task is composed of the two elements. One is the development of the method to have analytic insight for this two-dimensional plasma flow in the complex geometry. The other is the identification of the proper description of the transport coefficients in SOL.

In this paper, we discuss the influence of the choice of the model of thermal conductivity, κ, on the scaling law of SOL plasma parameters. The dependencies of the edge plasma temperature, T_b, thicknesses of the heat flow channel and temperature, and the fluctuation level, ϕ/T, are investigated. It is confirmed that thicknesses depend strongly on the model κ, so that the identification of κ in the present experiment is crucial for the prediction of the future devices and the prospective programme of fusion research. It is found that the dependence of ϕ/T on the model of κ is most prominent, so that the experimental study on the fluctuations in SOL would be very effective in identifying the proper scaling of the SOL and diverter plasmas. It is shown that the simultaneous study on the temperature, gradient and fluctuation level is important to obtain a correct form of the transport coefficient in the SOL plasma.

2. Analytic model of SOL transport

Analytic estimate for the SOL diverter plasma is based on the balance between the parallel heat transfer and perpendicular diffusion. Introducing the radial widths of the heat channel and the temperature, Δ_h and Δ_T respectively, the parallel and perpendicular energy balance is written as

$$-\kappa_{\|}\nabla_{\|}T = \frac{B_t}{B_p}\frac{P}{2\pi\Delta_h R} \tag{1}$$

$$\frac{\kappa_{\perp}T_b}{\Delta_T} = \frac{P}{4e\pi^2 aR} \tag{2}$$

where $\kappa_{\|}$ and $\kappa_{\perp}$ are the parallel and perpendicular thermal conductivity, B_t and B_p are the toroidal and poloidal magnetic field, a and R are the minor and major radii, respectively, subscript b indicates the plasma surface, $r = a$, and P stands for the total heat flow for one divertor leg. The numerical coefficient $1/e$ in (2) is introduced to denote that the average gradient $1/\Delta_T$ is evaluated at the position of the $1/e$ fall-off of the radial heat flux. The widths Δ_h and Δ_T are different in absolute values. In this article, however, we study the proportionality relation rather than the quantitative evaluation, so that we consider the ratio Δ_h/Δ_T is constant and analyse the dependence of Δ_T.

One example of the analytic theory on the SOL transport is given by Wagner and Lackner (which we call Wagner–Lackner (WL) scaling) [2]. With the assumptions that $\kappa_{\|}$ is classical, i.e., $\kappa_{\|} \propto T^{5/2}$ and $\kappa_{\perp}$ is Bohm-like in SOL, the dependence was given as

$$T_b^{WL} \propto P^{4/11}n_b^{-2/11} \qquad \Delta_T^{WL} \propto P^{-3/11}n_b^{7/11}. \tag{3}$$

More recently, 2D simulation has widely been performed [3]. Simulation studies seem to confirm the analytic theories. The 2D simulation has provided a scaling law [4]

$$T_b \propto P^{0.4}n_b^{-0.25} \qquad \Delta_T \propto P^{-0.11}n_b^{0.4} \tag{4}$$

by employing the same model for κ. This result is in a good agreement with (3), even though the realistic geometry and processes are taken into account in the simulation. This suggests that the analytic method to treat the two-dimensional transport in the complex geometry, which has yielded (3), gives the proper result for the first step of the analysis.

The comparison study of the models and experimental observations on the SOL plasma has also been made extensively [5–7]. There still remain discrepancies, and an improvement in the plasma modelling is necessary. One direction of the efforts is to study the roles of E_r and various drifts in SOL plasmas [8]. Contributions of various drifts, such as $E \times B$ drift and $\nabla T \times B$ heat flux, as well as the parallel current, were found to give quantitative differences. (The parallel current changes the heat transmission coefficient at the divertor plate.) However, the experimental observations still remain unexplained. The Bohm-like dependence of κ in SOL was supported in some experiments [6], but not necessarily in the other [7].

3. Dependence on the model of thermal conductivity

Equations (1) and (2) are solved for various models of the thermal conductivities. We employ the power law model for $\kappa_{\|}$ as

$$\kappa_{\|} = \kappa_{\|0}T^{\beta}. \tag{5}$$

When the mean free path (mfp) is much shorter than the connection length, L, the collisional diffusion prevails over the parallel transport and $\beta = 5/2$ holds and $\kappa_{\parallel 0}$ is independent of the density. (L is the connection length between the mid-plane and the divertor plate along the field line.) On the other hand, if the mfp becomes comparable to L, the heat flux is limited by the thermal speed and $\beta = 1/2$.

For the model of $\kappa_\perp$, the dependencies on the temperature as well as on the temperature gradient is taken into account, and we employ

$$\kappa_\perp = \kappa_{\perp 0} T^\alpha (\nabla T)^\gamma \tag{6a}$$

and

$$\kappa_{\perp 0} \propto n^\delta q^\mu R^\nu \tag{6b}$$

where q is the safety factor. The dependence of $\kappa_\perp$ on the gradient could be important, because the cross-field transport is often induced by the pressure-gradient-driven turbulence.

Substituting the expression (5) into (1), the analytic relation of T_b is given by requiring the limit $T_b \gg T_d$ (T_d being the temperature in front of the divertor plate) as

$$T_b^{\beta+1} = \frac{(\beta+1)^2 B_t L}{2 B_p \kappa_{\parallel 0} \pi R} \frac{P}{\Delta_T}. \tag{7}$$

(In obtaining (7) we use the fact that (5) gives the estimate $\Delta_T/\Delta_h \sim (\beta+1)$.) Equations (2) and (6) yield the relation

$$\frac{T_b^{\alpha+\gamma+1}}{\Delta_T^{\gamma+1}} = \frac{P}{4e\pi^2 a R \kappa_{\perp 0}}. \tag{8}$$

Equations (7) and (8) provide self-consistent solutions for the edge temperature and the heat channel thickness as

$$T_b = F_1^{d(\gamma+1)} F_2^d P^{d(\gamma+2)} \qquad d = \frac{1}{\alpha + (\beta+2)(\gamma+1)} \tag{9}$$

and

$$\Delta_T = F_1^{d(\alpha+\gamma+1)} F_2^{-d(\beta+1)} P^{d(\alpha-\beta+\gamma)} \tag{10}$$

where F_1 and F_2 are defined as

$$F_1 = \frac{(\beta+1)^2 B_t L}{2 B_p \kappa_{\parallel 0} \pi R} \tag{11a}$$

$$F_2 = \frac{1}{4e\pi^2 a R \kappa_{\perp 0}}. \tag{11b}$$

The fluctuation level, ϕ/T, is given by use of the strong turbulence estimate,

$$\kappa_\perp \simeq \frac{\phi}{B_t} n \tag{12}$$

(see, e.g., [9]). The strong turbulence limit is used because the fluctuation level is high in the edge and SOL plasmas. In calculating ϕ/T, T and $\nabla T/T$ are evaluated by T_b and Δ_T, respectively. The dependencies of T_b and Δ_T and (12) provide the dependence of ϕ/T as

$$\frac{\phi}{T} = \frac{\kappa_{\perp 0} B_t}{n} F_1^{d(\alpha-\gamma-1)} F_2^{d(\alpha+\beta\gamma+2\gamma-1)} P^{d(2\alpha+\beta\gamma+\gamma-2)}. \tag{13}$$

In terms of the density n_b, power P, safety factor q and the typical system size R, quantities T_b, Δ_T and ϕ/T are expressed as

$$T_b \propto n_b^{-\delta d} q^{(2\gamma+2-\mu)d} P^{(\gamma+2)d} R^{-(\nu+2)d} \tag{14}$$

$$\Delta_T \propto n_b^{(\delta\beta+\delta)d} q^{(2\alpha+2+2\gamma+\mu+\mu\beta)d} P^{(\alpha-\beta+\gamma)d} R^{(\nu+1)(\beta+1)d} \tag{15}$$

$$\phi/T \propto n_b^{(-\alpha-(\beta+2)(1+\gamma-\delta)+\delta)d} q^{(2\alpha-2-2\gamma+3\mu+\mu\beta)d} P^{(2\alpha-2+\beta\gamma+\gamma)d} R^{(-2\alpha+2+(\beta+2)(\nu-2\gamma)+\nu)d}. \tag{16}$$

These expressions are used to study the parameter dependencies.

As examples of the models for the cross-field transport coefficient, we study the cases of:

(1) pseudo-classical model ($\alpha = -3/2$, $\gamma = 1$, $\delta = 2$, $\mu = 2$)
(2) constant χ model ($\alpha = 0$, $\gamma = 0$, $\delta = 1$, $\mu = 0$)
(3) Bohm-like model ($\alpha = 1$, $\gamma = 0$, $\delta = 1$, $\mu = 0$)
(4) gyro-Bohm-like model ($\alpha = 3/2$, $\gamma = 0$, $\delta = 1$, $\mu = 0$)
(5) ∇T model ($\alpha = 0$, $\gamma = 1$, $\delta = 1$, $\mu = 1$)
(6) Rebut-like model (asymptotic limit of Rebut-type model [10]) ($\alpha = -1$, $\gamma = 1$, $\delta = 1$, $\mu = 1$)
(7) Itoh-like model [9] ($\alpha = 0$, $\gamma = 3/2$, $\delta = 1$, $\mu = 2$).

The thermal diffusivity χ has the relation $\chi = \kappa_\perp/n$. The models (4)–(7) have been used in analysing the core plasma confinement with some success [11]. The direct application of these models to SOL may violate the validity of the assumptions (such as collisionality and so on). The study here is not the explanation of the SOL plasma by these models, but the study of the influence of the transport model on the SOL plasma parameters.

Table 1. Scaling laws of the edge temperature, heat channel width and fluctuation level for various models of the cross-field transport. Thermal diffusivity $\chi_\perp$ is defined by $\kappa_\perp/n$.

$\chi_\perp$	T_b	Δ_T	ϕ/T
Pseudo-classical $\sim nq^2/\Delta_T\sqrt{T}$	$P^{2/5}n_b^{-4/15}q^{4/15}$	$P^{-2/5}n_b^{14/15}q^{16/15}$	$P^{-1/5}n_b^{7/15}q^{-22/15}$
Constant	$P^{4/9}n_b^{-2/9}q^{4/9}$	$P^{-5/9}n_b^{7/9}q^{4/9}$	$P^{-4/9}n_b^{2/9}q^{-4/9}$
Bohm-like $\sim T$	$P^{4/11}n_b^{-2/11}q^{4/11}$	$P^{-3/11}n_b^{7/11}q^{8/11}$	Const.
g Bohm-like $\sim T^{3/2}$	$P^{1/3}n_b^{-1/6}q^{1/3}$	$P^{-1/6}n_b^{7/12}q^{5/6}$	$P^{1/6}n_b^{-1/12}q^{1/6}$
∇T-type $\sim q\nabla T$	$P^{1/3}n_b^{-1/9}q^{1/3}$	$P^{-1/6}n_b^{7/18}q^{5/6}$	$P^{1/6}n_b^{-7/18}q^{1/6}$
R-L-type $\sim q\nabla T/T$	$P^{3/8}n_b^{-1/8}q^{3/8}$	$P^{-5/16}n_b^{7/16}q^{5/16}$	$P^{-1/16}n_b^{-5/16}q^{-1/16}$
Itoh model-type $\sim q^2(\nabla T)^{3/2}$	$P^{14/45}n_b^{-4/15}q^{4/15}$	$P^{-4/45}n_b^{14/45}q^{16/15}$	$P^{1/5}n_b^{-23/45}q^{8/15}$

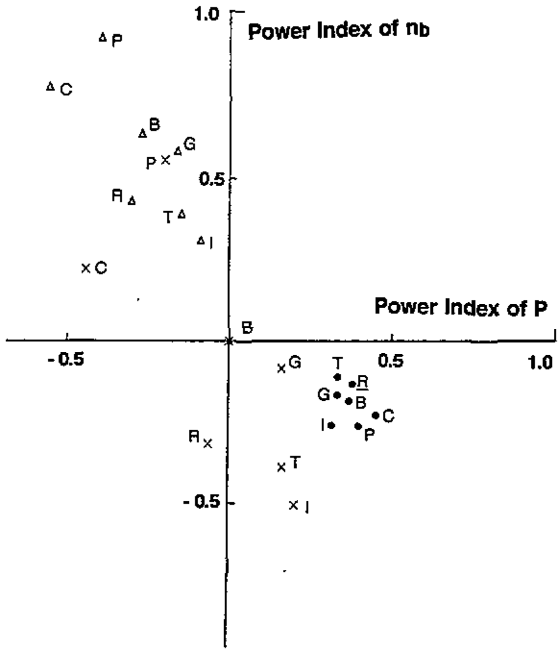

Figure 1. The power indices in the scaling laws of the edge temperature T_b (shown by ●) of the heat channel width (shown by Δ), and of the fluctuation level ϕ/T (shown by ×). Models of the cross-field transport are denoted by B (Bohm-like), C (constant), G (gyro-Bohm-like), I (Itoh-like model), P (pseudo-classical-like model), R (R-like model) and T (∇T model). Parameter β is chosen to be 5/2.

Table 1 summarizes the scaling laws of edge temperature, temperature fall-off length and fluctuation level for various types of the transport coefficient. Figure 1 illustrates the power indices of the edge density and loss power. The dependence of the temperature is not different among models. The fall-off length has wider variation. The models (4)–(7) give weaker dependence in comparison with the models (1)–(3). This confirms the knowledge that the uncertainty in the transport model causes the ambiguity in the estimate of the peak heat load onto the divertor plate. However, the sign of power index is still common for all models. Largest difference are seen in the dependence of the fluctuation level.

For the models of the pseudo-classical type and constant χ, there is a positive correlation between ϕ/T and Δ_T. When the fall-off length becomes shorter, i.e., the radial gradient in SOL is steeper, the fluctuation level decreases. In the Bohm model, ϕ/T is predicted to be independent. For other models (4)–(7) the anti-correlation holds. The steep gradient (short Δ_T) gives larger fluctuations. These dependencies of the fluctuation levels on the transport model suggest the importance of measuring the edge fluctuations, in order to identify the proper model of SOL transport.

The experiment on TEXTOR shows the positive correlation between $\tilde{n}/n$ and the density gradient [12]. It was found that when the density gradient in SOL becomes sharper, the fluctuation level is reduced. This is a clear contrast to the observation on the core plasma, where the increment of fluctuation level was found associated with the reduced gradient length [13].

4. Note on the collinearity

The result here suggests the importance of studying the dependencies of the temperature, fall-off length and fluctuation level simultaneously. If only some of them, say T_b, alone are analysed experimentally, one easily comes to a wrong conclusion about the nature of the thermal conductivity. This problem is known as collinearity in analysing the database (see, for example [14]). We illustrate this problem by choosing some examples from the models of the thermal conductivity.

Let us first take the case of the pseudo-classical transport. From table 1, we see that the scaling like

$$\chi_\perp \propto P^{1/5} n_b^{2/15} q^{4/5} \tag{17}$$

holds for the pseudo-classical transport. If one compares only the temperature T_b and thermal diffusity by changing the heating power and keeping other parameters fixed, one gets the relation $T_b \propto P^{2/5}$ and $\chi_\perp \propto P^{1/5}$. Eliminating P from these two relations, one obtains a *collinearity* as

$$\chi_\perp \propto T_b^{1/2}. \tag{18}$$

This dependence is completely different from the original dependence of the thermal transport coefficient in the case of pseudo-classical transport, i.e.,

$$\chi_\perp \propto T^{-1/2}. \tag{19}$$

As another example, we show the case of the ∇T model. By the same procedure, we find the relation $\chi_\perp \propto P^{1/2}$ and $T_b \propto P^{1/3}$. By eliminating P from these two relations, we arrive at the collinearity as

$$\chi_\perp \propto T_b^{3/2}. \tag{20}$$

This dependence is also different from the original form of $\chi_\perp$ for the ∇T model. The misinterpretation, which is shown in (18), is not the particular case of the pseudo-classical transport model, but happens more generally.

These examples clearly show that the comparison of a single parameter can easily lead to the misunderstanding of the transport property in the SOL plasma. It is also noted that both the cases, (18) and (20), yield the collinearity in which χ is an increasing function of T. This is one of the reasons that Bohm diffusion is often concluded from the study on the parameter dependence, although the radial shape of χ does not support this formula. It should be emphasized, again, that the study on the gradient scale length as well as on the fluctuation level is necessary, in order to avoid the misinterpretation by the collinearity.

5. Summary and discussion

In this article, we surveyed the relation between the various transport models and the SOL plasma parameters. By use of the point model of the SOL transport, the scaling law of the edge plasma temperature, that of the heat channel width and the one for the fluctuation level were obtained. It was found that the edge temperature has weak dependence on the transport model. The heat channel width is model dependent, stimulating the identification of the proper model of the cross-field transport. The model dependence was found most clearly for the level of fluctuations. This result suggests that the experimental study of the fluctuations will be more important in future for the study of the SOL and divertor plasmas. It is also shown that the collinearity in the data easily leads to the wrong conclusion: The study on the gradient scale length and fluctuation level are necessary.

The present database of the SOL fluctuations is very limited. The correlation seems to support modelling such as constant χ or pseudo-classical-type model. The pseudo-classical diffusion is caused by the nonlinear pressure gradient driven turbulence, and is relevant to the parameters of the SOL plasmas.

The analysis here is, however, based on the point model analysis. The two-dimensional simulation is required in order to confirm the dependence quantitatively. The influence of the radial electric field on the cross-field transport has also attracted attention recently [15]. Further research is required to identify the proper model of SOL transport.

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References

[1] Stangeby P C and McCracken G M 1990 *Nucl. Fusion* **30** 1035
[2] Wagner F and Lackner K 1984 *Physics of the Plasma Wall Interactions in Controlled Fusion (NATO ASI Series B 131)* ed D E Post and R Behrisch (New York: Plenum) p 931
[3] Ueda N *et al* 1988 *Nucl. Fusion* **28** 1183
Brooks J N 1987 *J. Nucl. Mater.* **145&147** 837
Neuhauser J *et al* 1989 *Plasma Phys. Control. Fusion* **31** 1551
Braams B *et al* 1990 *17th EPS Conf. on Controlled Fusion and Plasma Heating (Amsterdam)* part III, p 1417 (Amsterdam: European Physical Society)
[4] Itoh S-I *et al* 1990 *Plasma Phys. Control. Fusion* **32** 427
[5] Vlases G *et al* 1992 *J. Nucl. Mater.* **196&198** 392
Shimizu K *et al* 1992 *J. Nucl. Mater.* **196&198** 476
Porter G D *et al* 1994 *Contrib. Plasma Phys.* **34** 454
[6] McCormick K *et al* 1992 *J. Nucl. Mater.* **196&198** 264
[7] Watkins J G *et al* 1992 *J. Nucl. Mater.* **196&198** 829
[8] For a review see Ueda N *et al* 1994 *Contrib. Plasma Phys.* **34** 350 and papers cited therein; see also
Ueda N, Itoh K and Itoh S-I 1989 *J. Nucl. Mater.* **162&164** 607
Hinton F L and Staebler G M 1989 *Nucl. Fusion* **29** 405
Rognlien T D *et al* 1992 *Contrib. Plasma Phys.* **32** 485
[9] Itoh K *et al* 1994 *Plasma Phys. Control. Fusion* **36** 279
[10] Rebut P H, Lallia P P and Watkins M 1989 *Plasma Physics and Controlled Nuclear Fusion Research 1988* vol 2 (Vienna: IAEA) p 191
[11] Fukuyama A *et al* 1994 *Plasma Phys. Control. Fusion* **36**
[12] Komori A *et al* 1988 *Nucl. Fusion* **28** 1460
[13] Liewer P 1985 *Nucl. Fusion* **25** 543
[14] Christiansen J P *et al* 1992 *Nucl. Fusion* **32** 291
[15] Itoh S-I and Itoh K 1988 *Phys. Rev. Lett.* **32** 2276
Boileau A *et al* 1992 *Nucl. Fusion* **32** 995
Shoji T *et al* 1993 *Plasma Physics and Controlled Nuclear Fusion Research 1992* vol 1 (Vienna: IAEA) p 323