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Kinetic Effects on RWM Stabilization in NSTX: Initial Results

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Princeton Plasma Physics Laboratory

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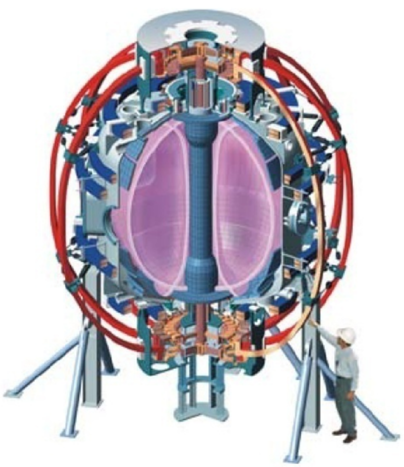

NSTX

RWM Energy Principle – Kinetic Effects

$$\gamma_F \tau_w = -\frac{\delta W_\infty}{\delta W_b}$$

(Haney and Freidberg, PoF-B, 1989)

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$$\gamma_K \tau_w = -\frac{\delta W_\infty + \delta W_K}{\delta W_b + \delta W_K}$$

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PEST

Hu/Betti code

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$$Re(\gamma_K \tau_w) = -\frac{\delta W_\infty \delta W_b + |\delta W_K|^2 + Re(\delta W_K)(\delta W_\infty + \delta W_b)}{\delta W_b \delta W_b + |\delta W_K|^2 + Re(\delta W_K)(\delta W_b + \delta W_b)}$$

Outline

- Introduction
  - The Hu/Betti code
  - Results: stability diagrams
  - Kinetic theory predicts near-marginal stability for experimental equilibria just before RWM instability.
- Collisionality
  - Kinetic theory predicts decrease in stability with increased collisionality.
- Rotation
  - Experimental rotation profiles are near marginal. Larger or smaller rotation is farther from marginal.
  - Unlike simpler "critical" rotation theories, kinetic theory allows for a more complex relationship between plasma rotation and RWM stability – one that may be able to explain experimental results.

The Hu/Betti code calculates δW_K

- Effects included:
  - Trapped Ions
  - Trapped Electrons
  - ~~Trapped Hot Particles~~
  - Circulating Ions
  - Alfven Layers

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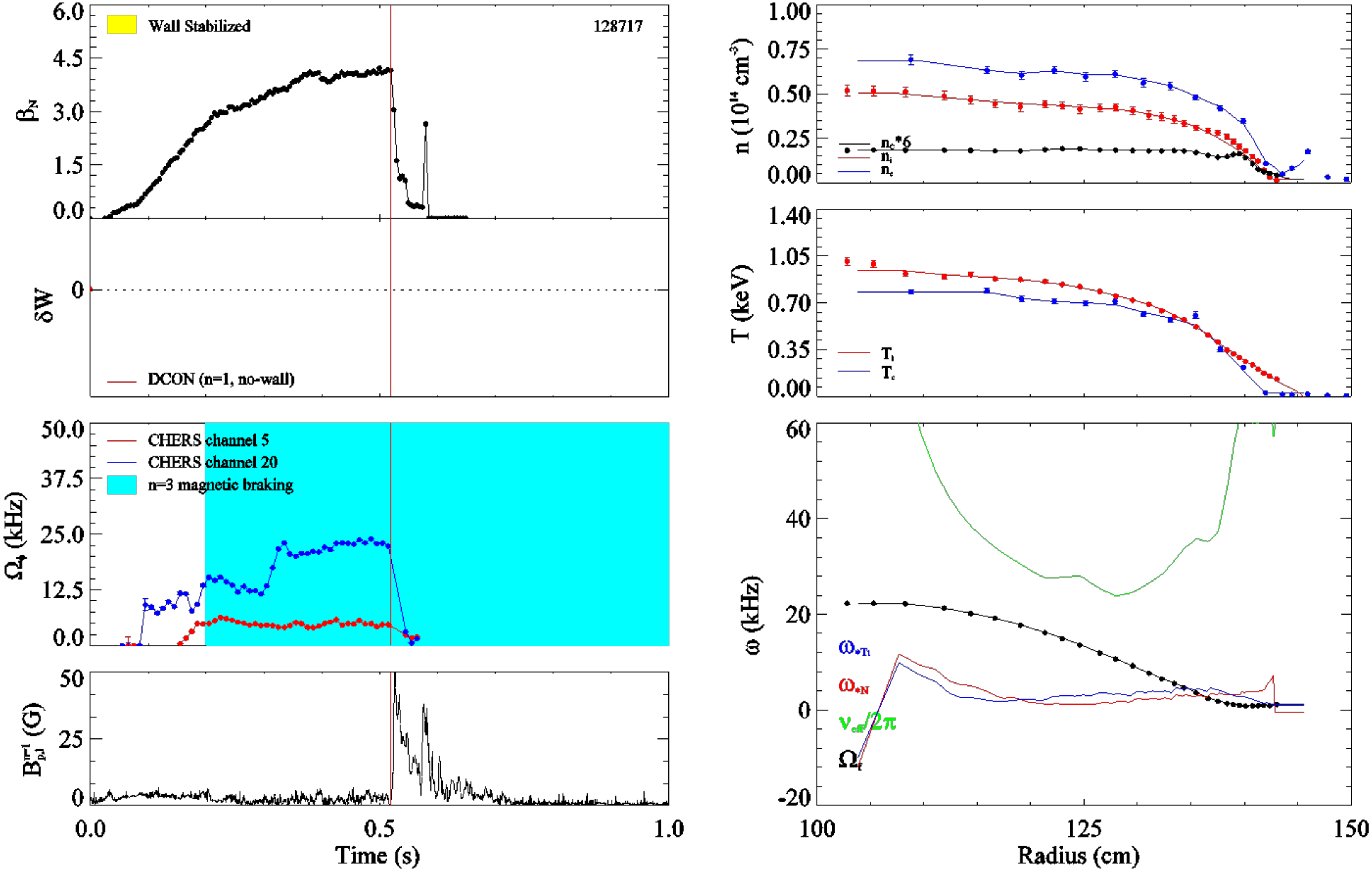
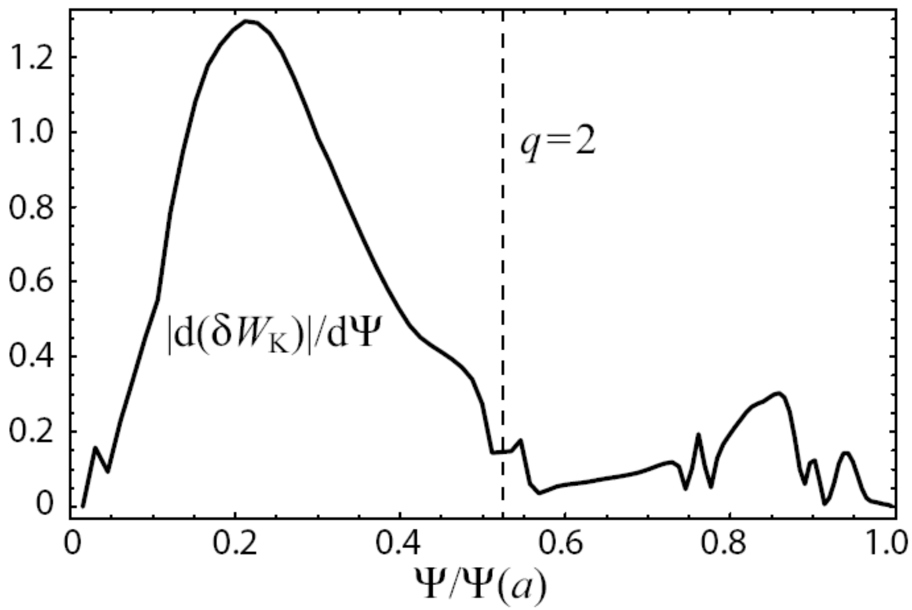
$$\delta W_K^{ti} = \int_0^{\Psi_a} d\Psi \left(\frac{p_s}{1+\frac{T_e}{T_i}} \right) \left(2\sqrt{\pi}\frac{r}{v} \right) \sum_{l=-\infty}^{\infty} \int_{B_0/B_{max}}^{B_0/B_{min}} d\Lambda \left(\frac{\hat{T}_b}{2} \right)$$
$$\times \int_0^{\infty} \left[\frac{\omega_{*N} + \left(\hat{\varepsilon} - \frac{3}{2}\right)\omega_{*T} + \omega_E - \omega}{\langle \omega_D \rangle + l\omega_b - i\nu_{\text{eff}} + \omega_E - \omega} \right] \hat{\varepsilon}^{5/2} e^{-\hat{\varepsilon}} d\hat{\varepsilon}$$
$$\times \left| \left\langle \left(2 - 3\frac{\Lambda}{B_0/B} \right) (\kappa \cdot \xi_\perp) - \left(\frac{\Lambda}{B_0/B} \right) (\nabla \cdot \xi_\perp) \right\rangle \right|^2$$

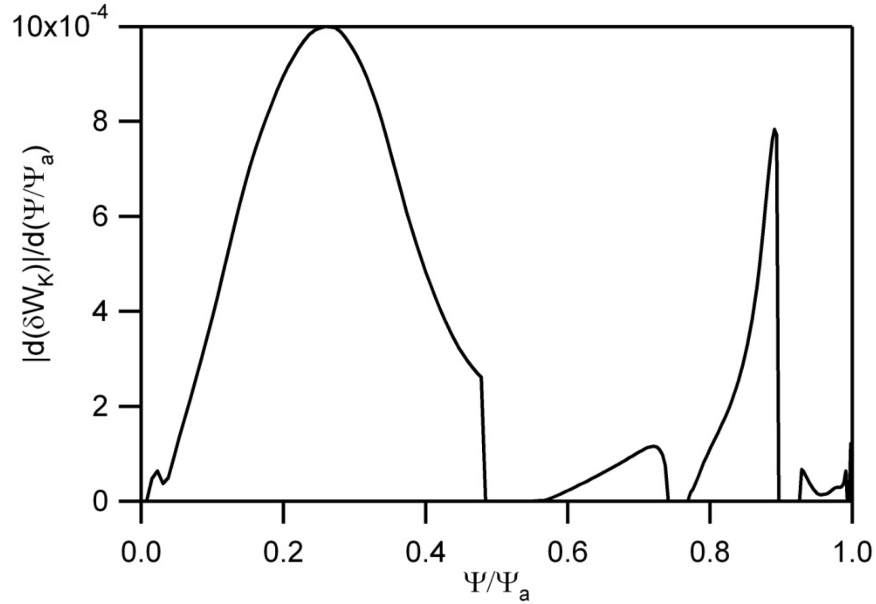
(Hu, Betti, and Manickam, PoP, 2006)

Experimental profiles used as inputs



Our implementation gives similar answers to Hu's





Hu

Berkery

(DIII-D shot 125701)

Stability diagrams: contours of constant Re($\gamma_K \tau_w$)

$$Re(\gamma_K \tau_w) = -\frac{\delta W_\infty \delta W_b + |\delta W_K|^2 + Re(\delta W_K)(\delta W_\infty + \delta W_b)}{\delta W_b \delta W_b + |\delta W_K|^2 + Re(\delta W_K)(\delta W_b + \delta W_b)}$$

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$$\gamma_K = 0: \; \left[Re(\delta W_K) + \tfrac{1}{2}(\delta W_b + \delta W_\infty)\right]^2 + \left[Im(\delta W_K)\right]^2 = \left[\tfrac{1}{2}(\delta W_b - \delta W_\infty)\right]^2$$

Stability diagrams: contours of constant $Re(\gamma_K \tau_w)$

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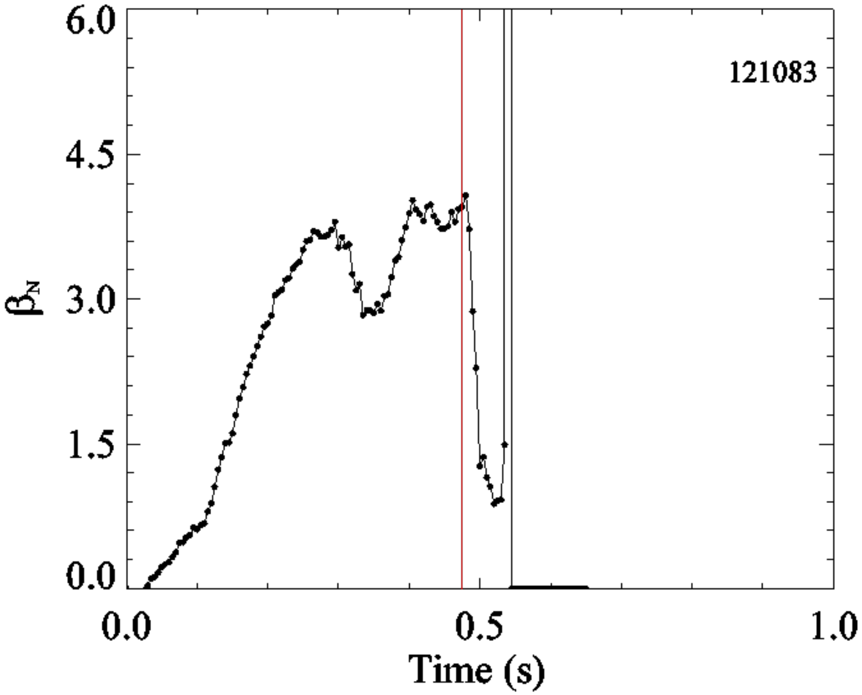

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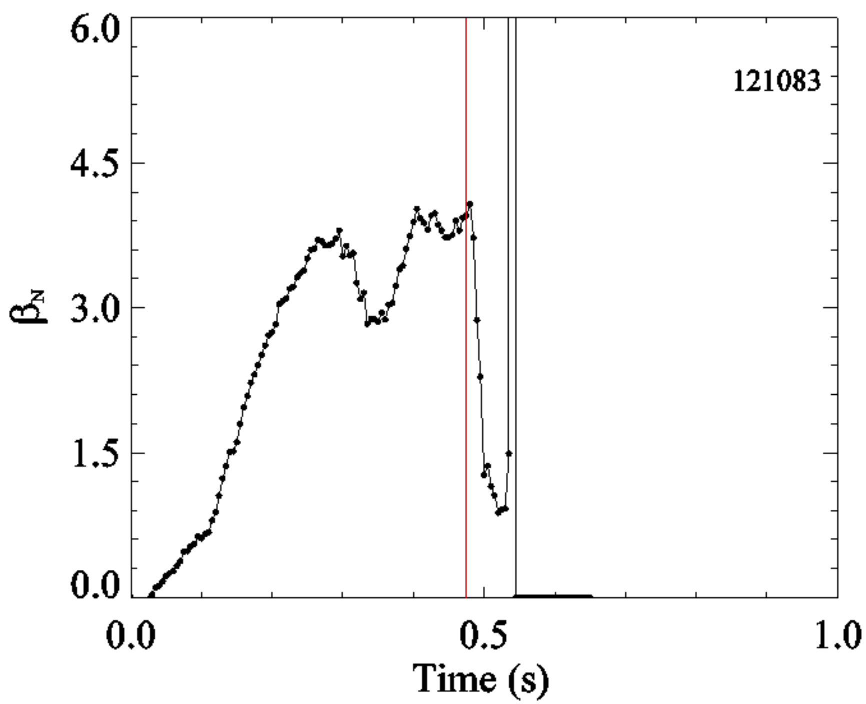

NSTX 121083 @ 0.475 s

$$\delta W_\infty = -2.09 \times 10^{-2}$$
$$\delta W_b = 7.42 \times 10^{-3}$$

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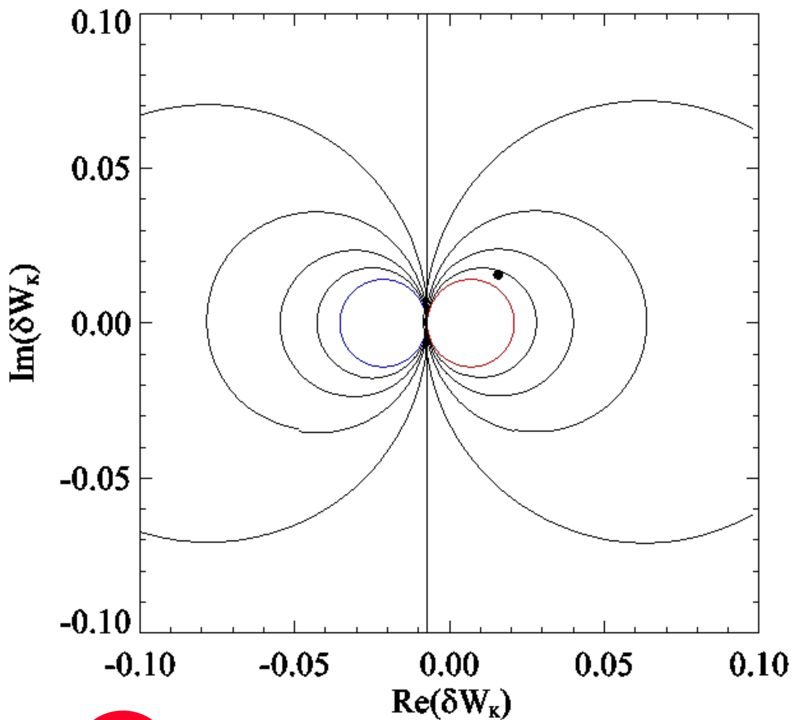

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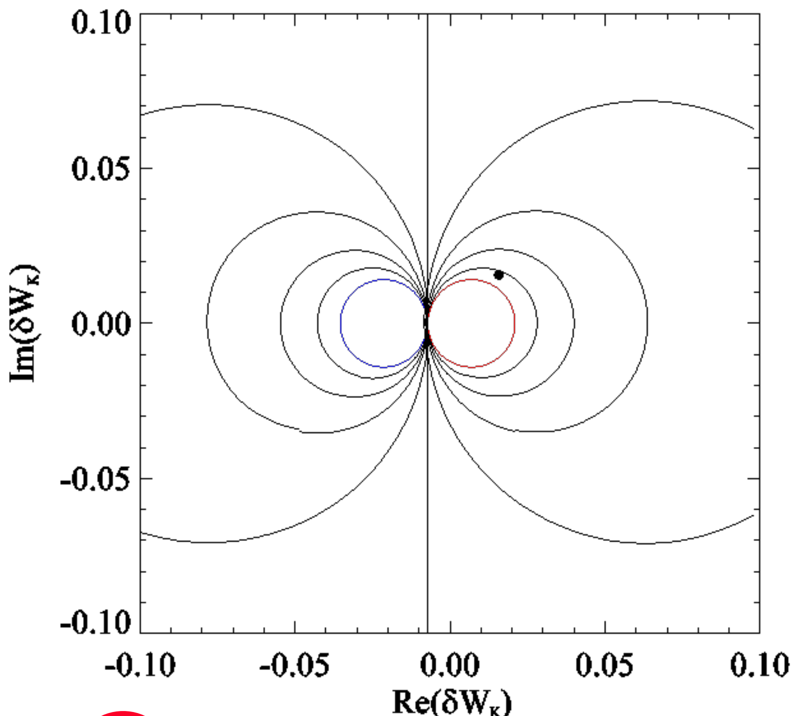


NSTX 121083 @ 0.475 s

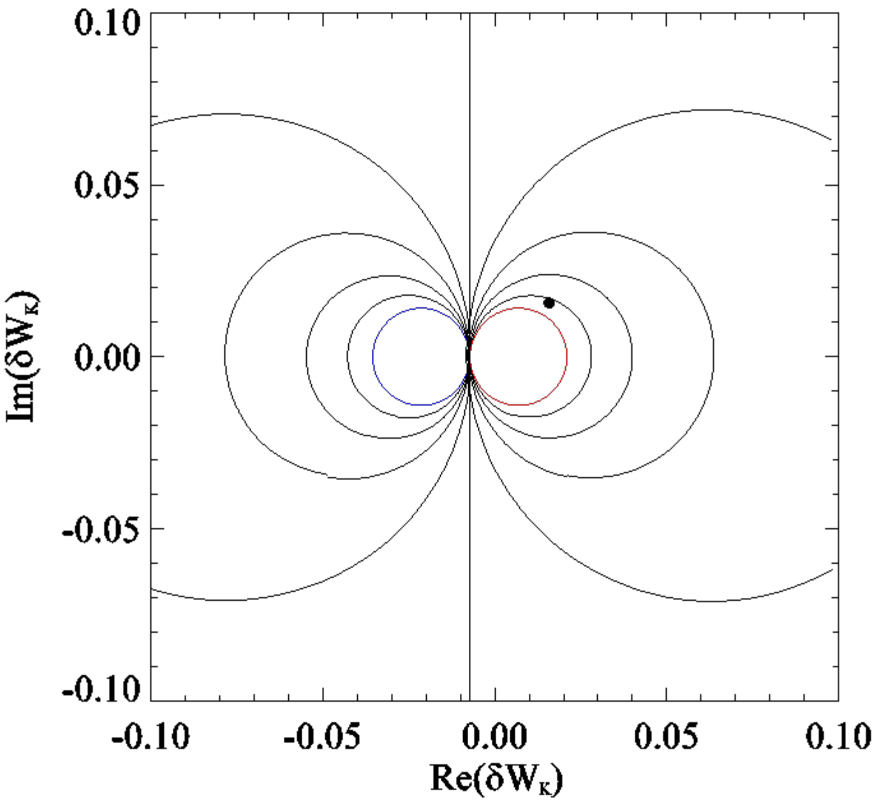
$\delta W_\infty = -2.09 \times 10^{-2}$

$\delta W_b = 7.42 \times 10^{-3}$

$Re(\delta W_K) = 1.58 \times 10^{-2}$

$Im(\delta W_K) = 1.57 \times 10^{-2}$

Stability results are all near marginal



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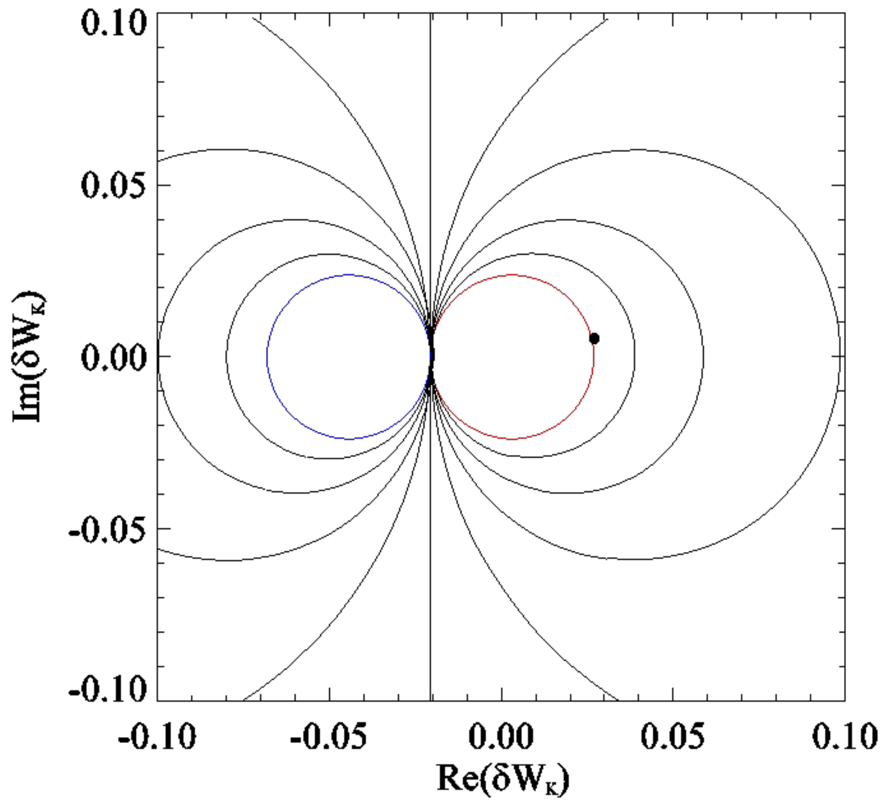

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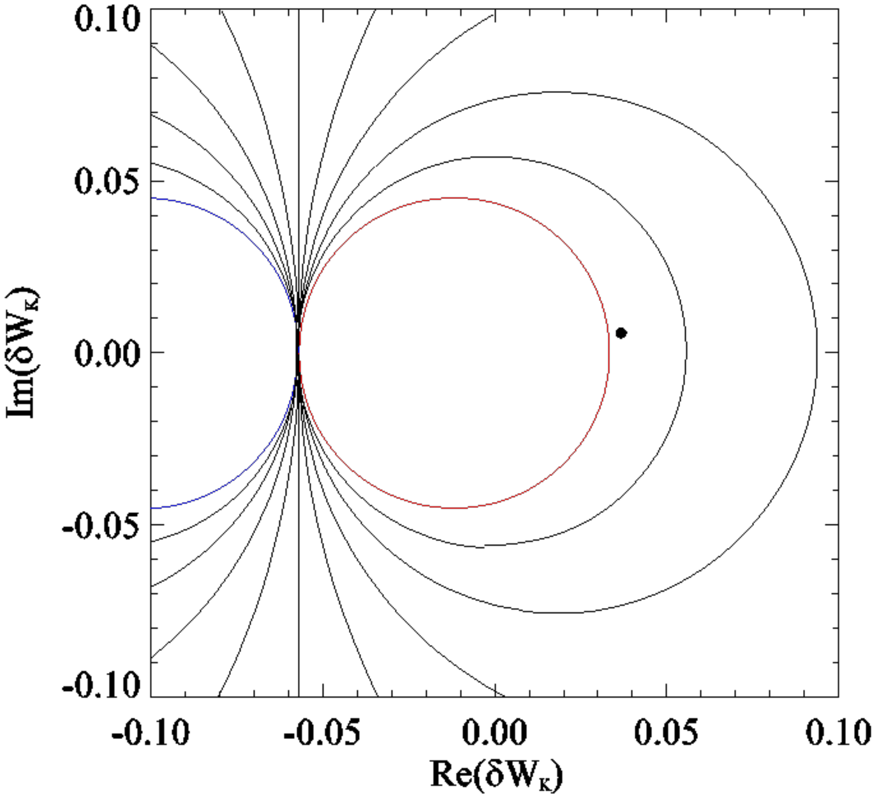
Stability results are all near marginal



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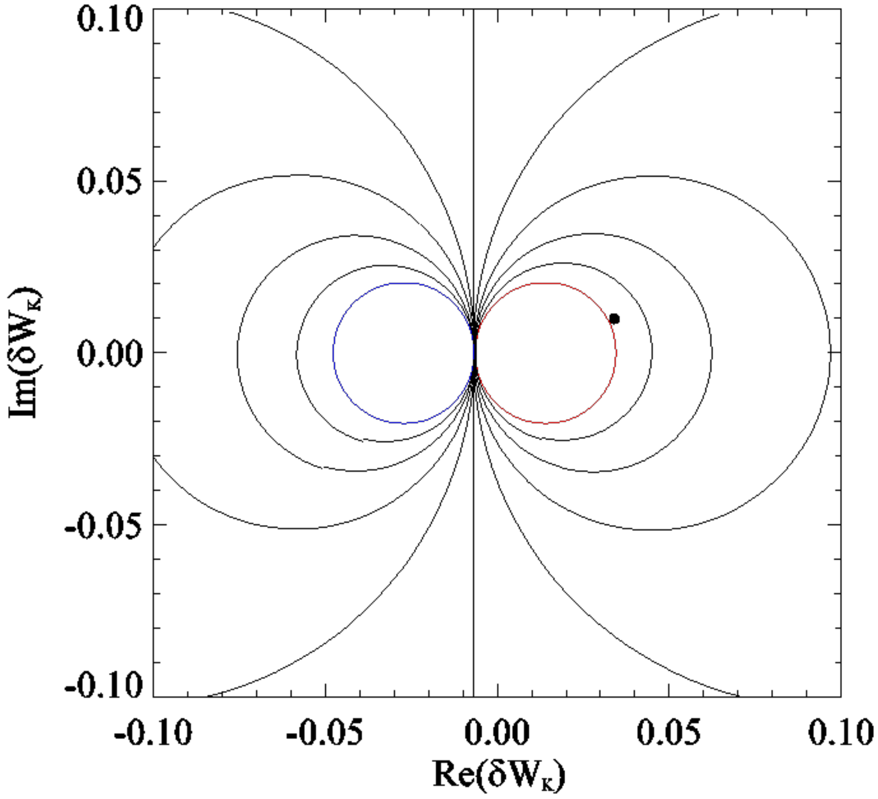

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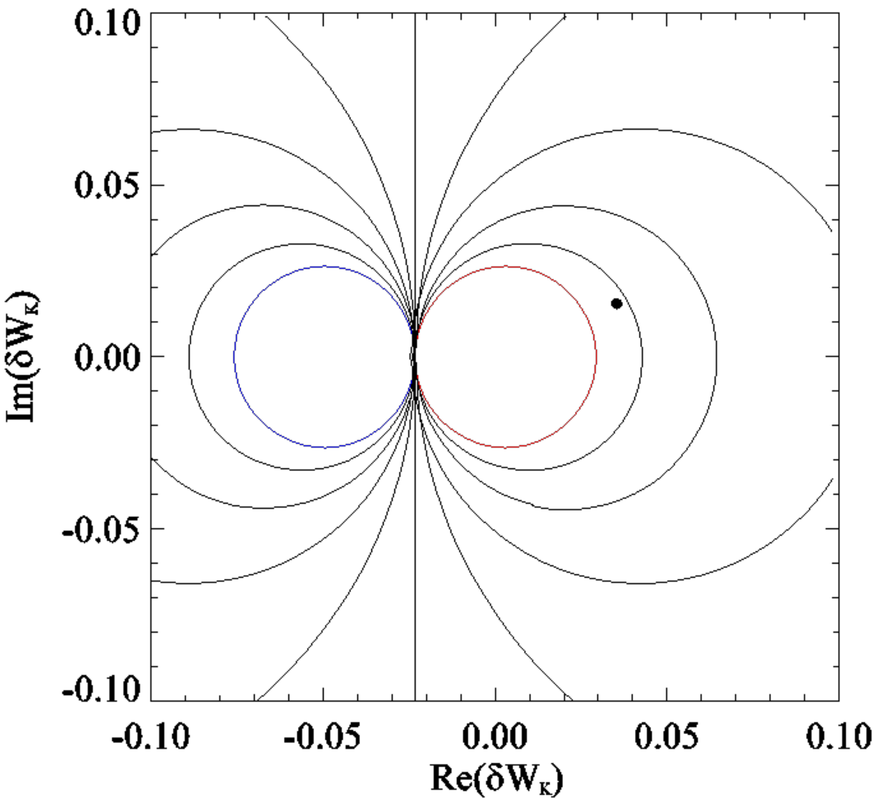
Stability results are all near marginal



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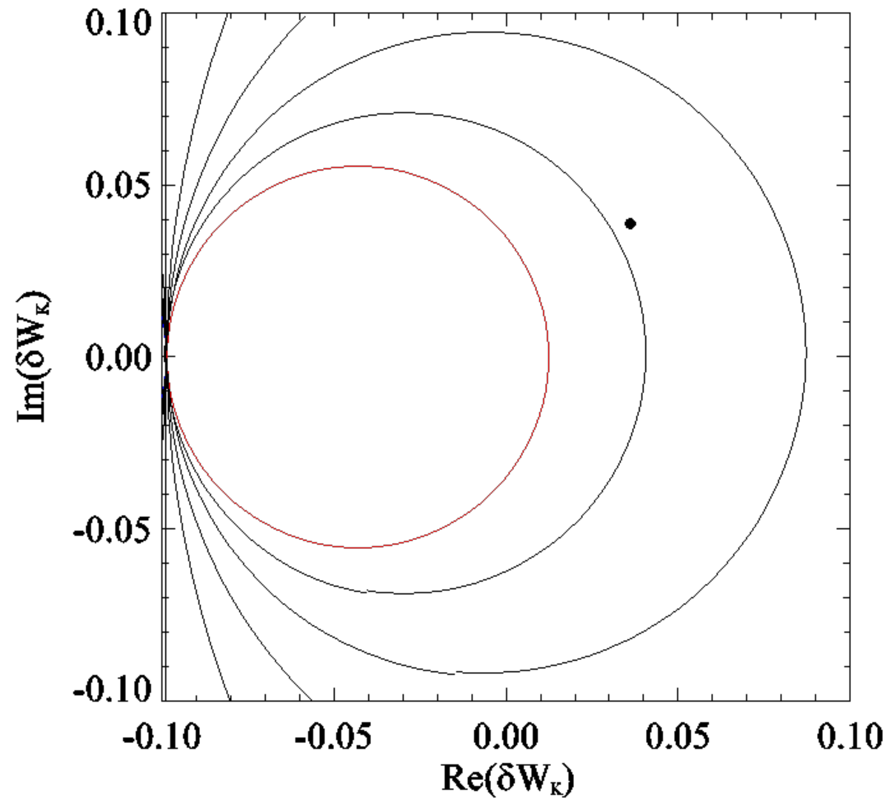


128863

Collisionality

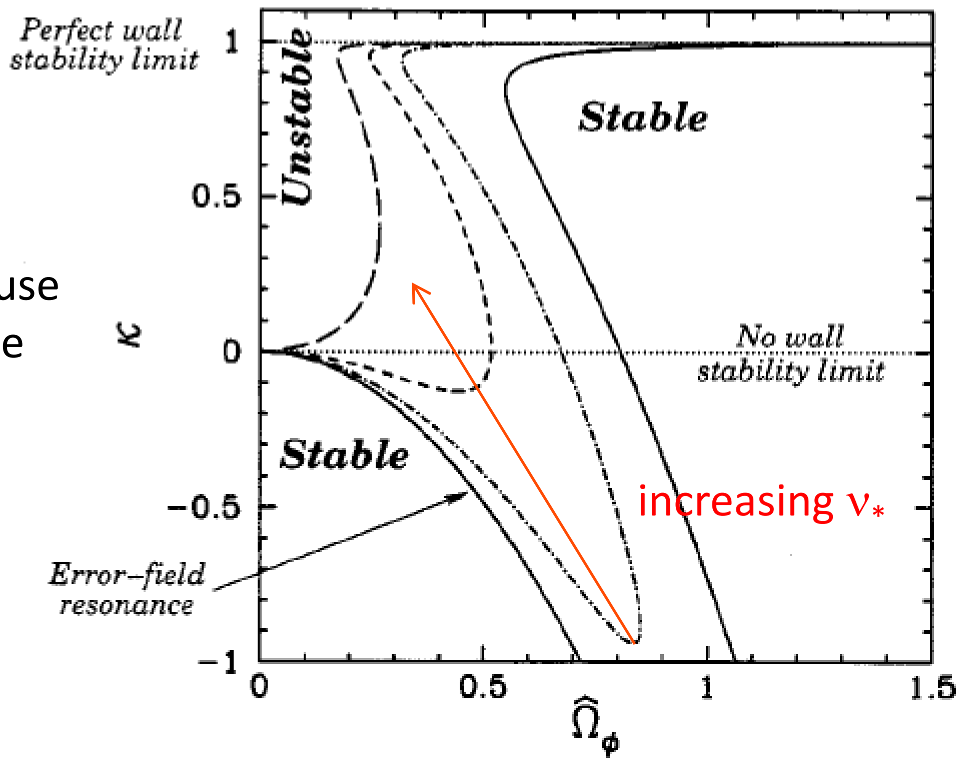
Simple model: collisions increase stability

$$[(\hat{\gamma}-i\hat{\Omega}_{\phi})^2+\nu_*(\hat{\gamma}-i\hat{\Omega}_{\phi})+(1-\kappa)(1-md)]$$
$$\times(\hat{\gamma}S_*+1+md)=1-(md)^2.$$

(Fitzpatrick, PoP, 2002)

"dissipation parameter"

- Fitzpatrick simple model
  - Collisions increase stability because they increase dissipation of mode energy.



Kinetic model: collisions decrease stability

$$\delta W_K \propto \left[\frac{\omega_{*N} + \left(\hat{\varepsilon} - \frac{3}{2}\right)\omega_{*T} + \omega_E - \omega}{\langle \omega_D \rangle + l\omega_b - i\nu_{\mathrm{eff}} + \omega_E - \omega} \right]$$

(Hu, Betti, and Manickam, PoP, 2006)

collision frequency (note: inclusion here is "ad hoc")

- Fitzpatrick simple model
  - Collisions <u>increase</u> stability because they increase dissipation of mode energy.
- Kinetic model
  - Collisions <u>decrease</u> stability because they reduce kinetic stabilization effects.

Collisionality should decrease δW_K: test with Z_{eff}

$$\delta W_K \propto \left[\frac{\omega_{*N} + \left(\hat{\varepsilon} - \frac{3}{2}\right)\omega_{*T} + \omega_E - \omega}{\langle \omega_D \rangle + l\omega_b - i\nu_{\text{eff}} + \omega_E - \omega} \right]$$

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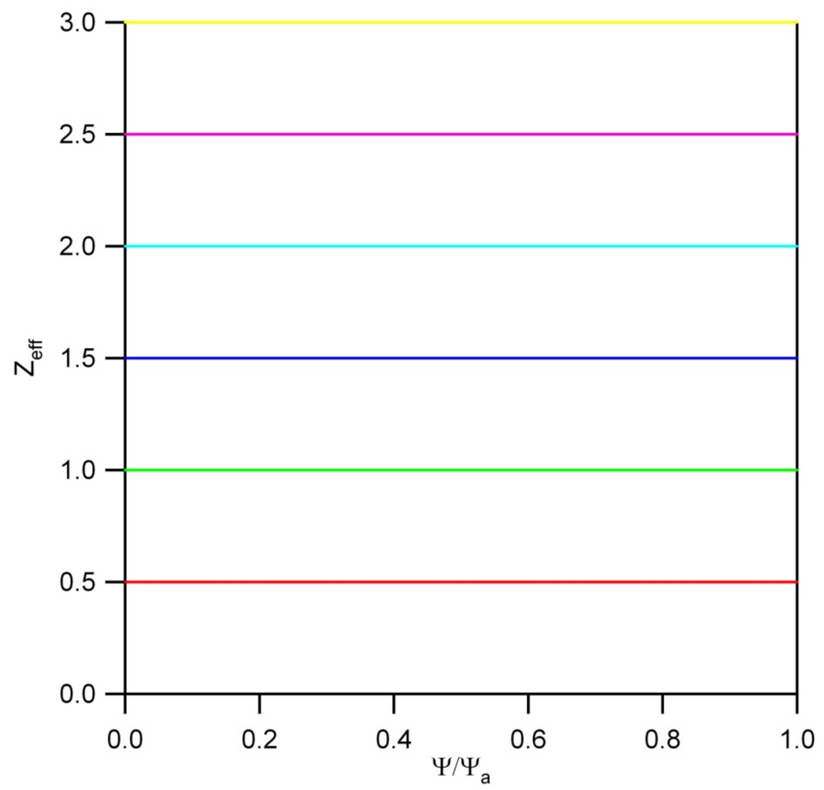
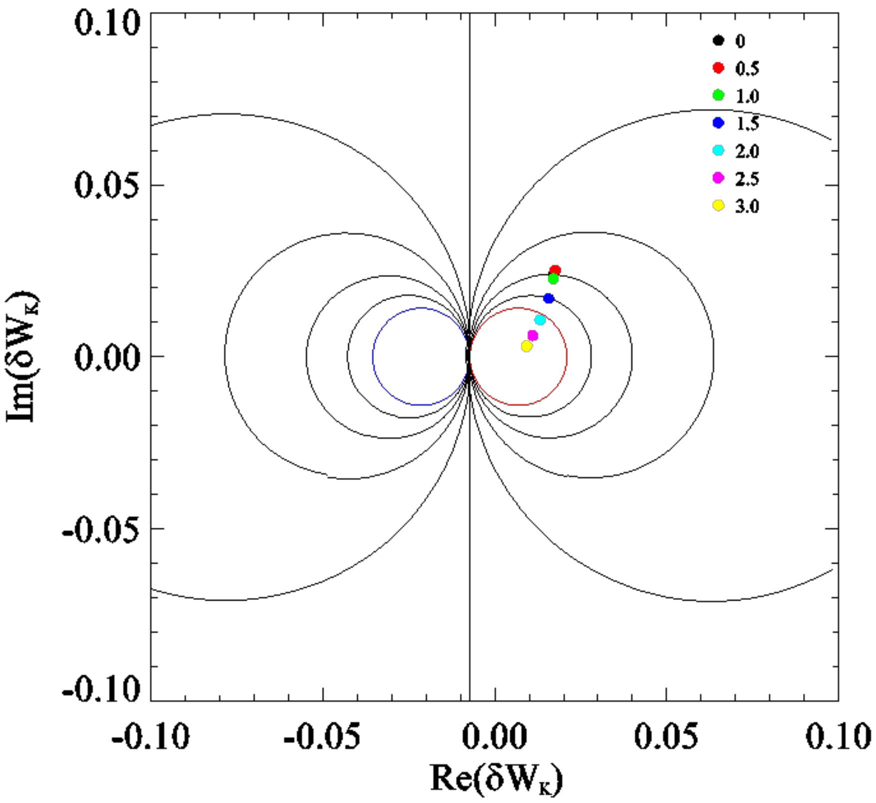
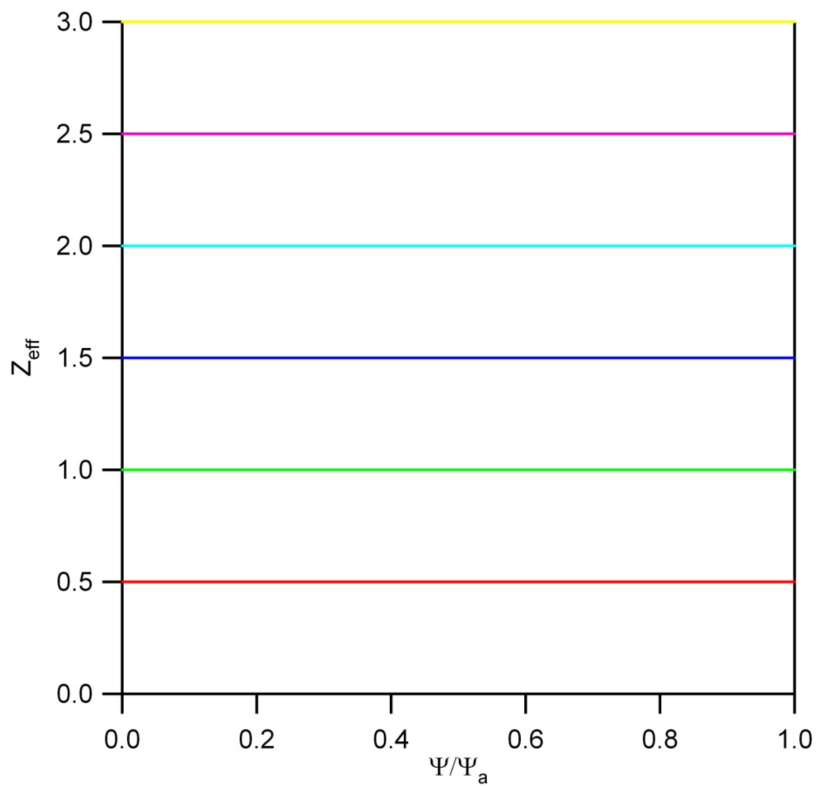


As expected, colisionality decreases stability





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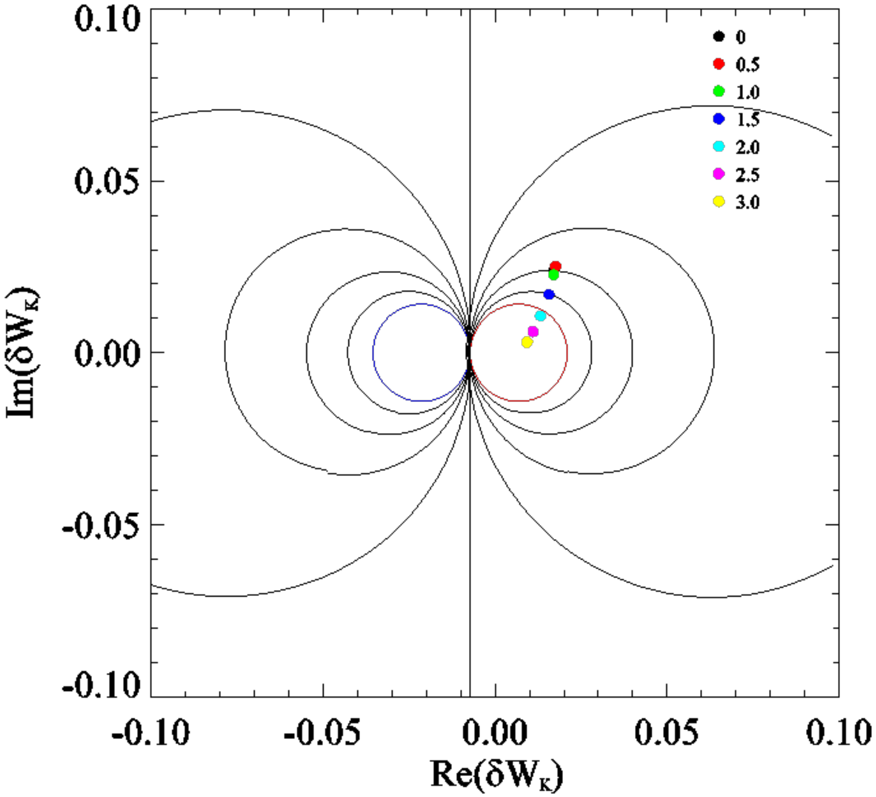

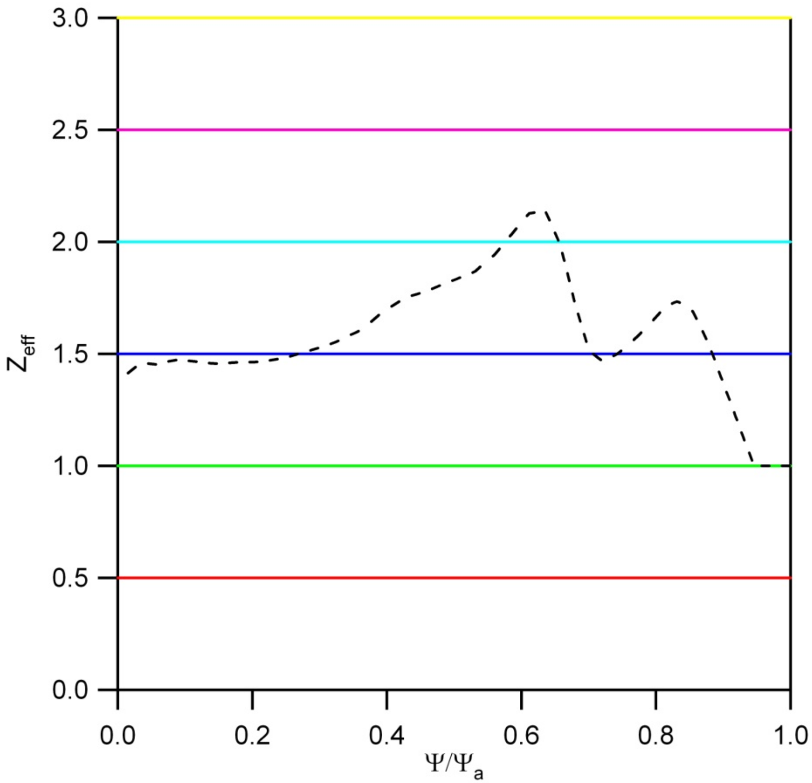

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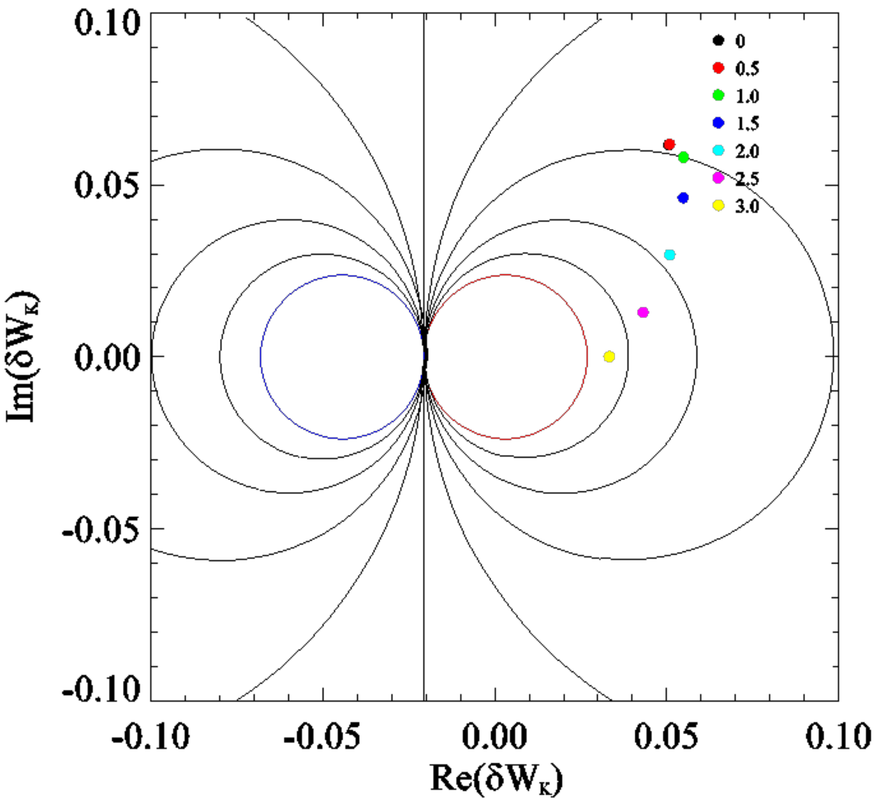

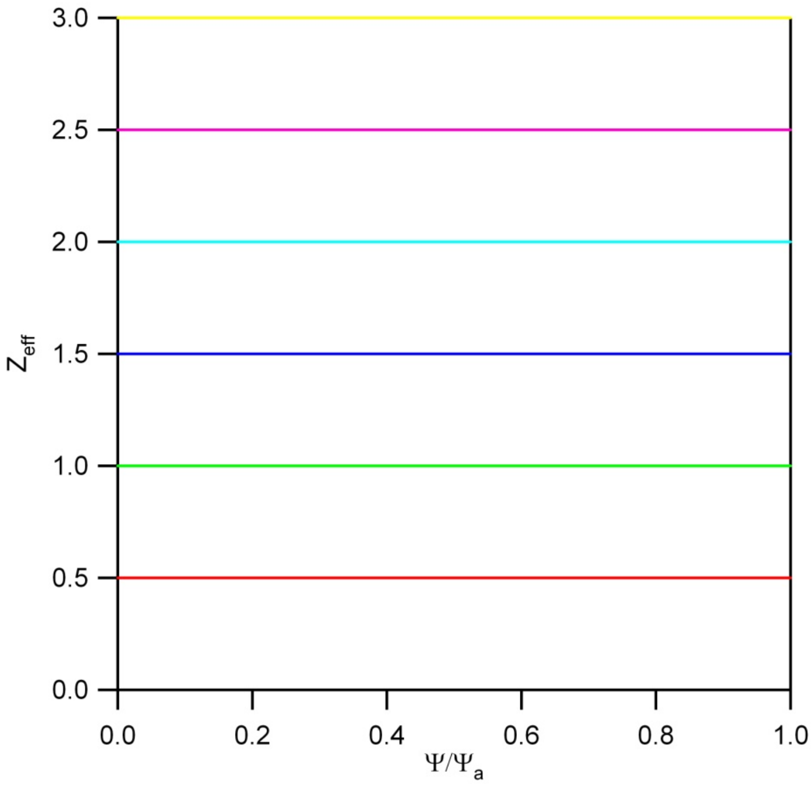

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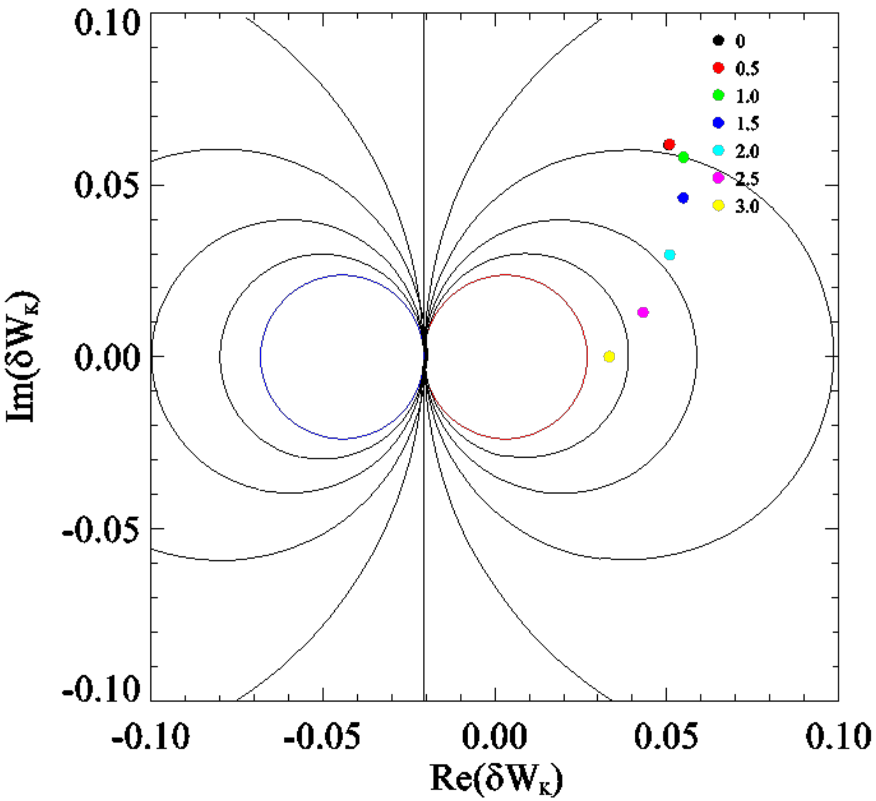

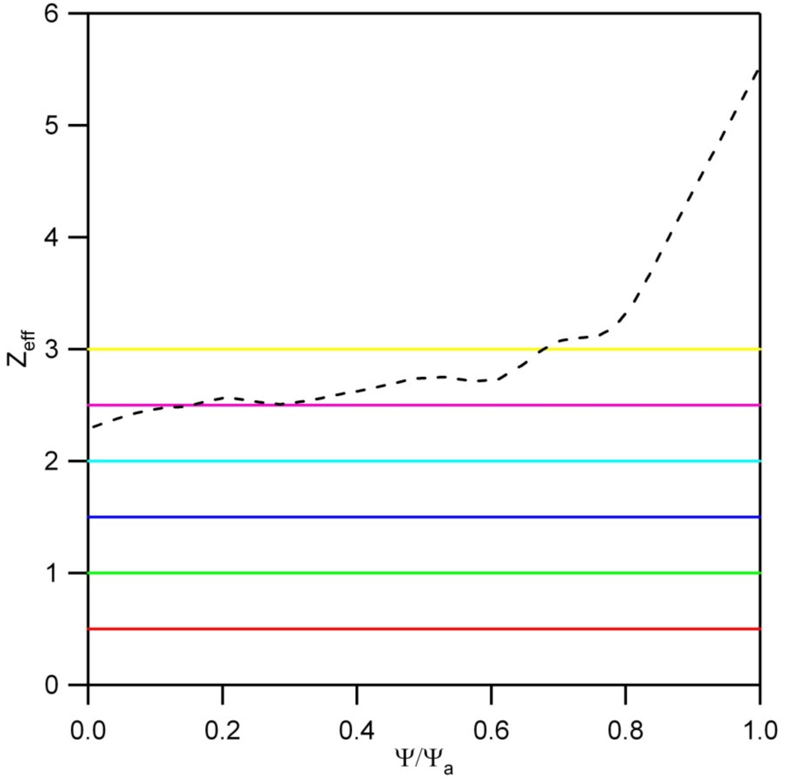

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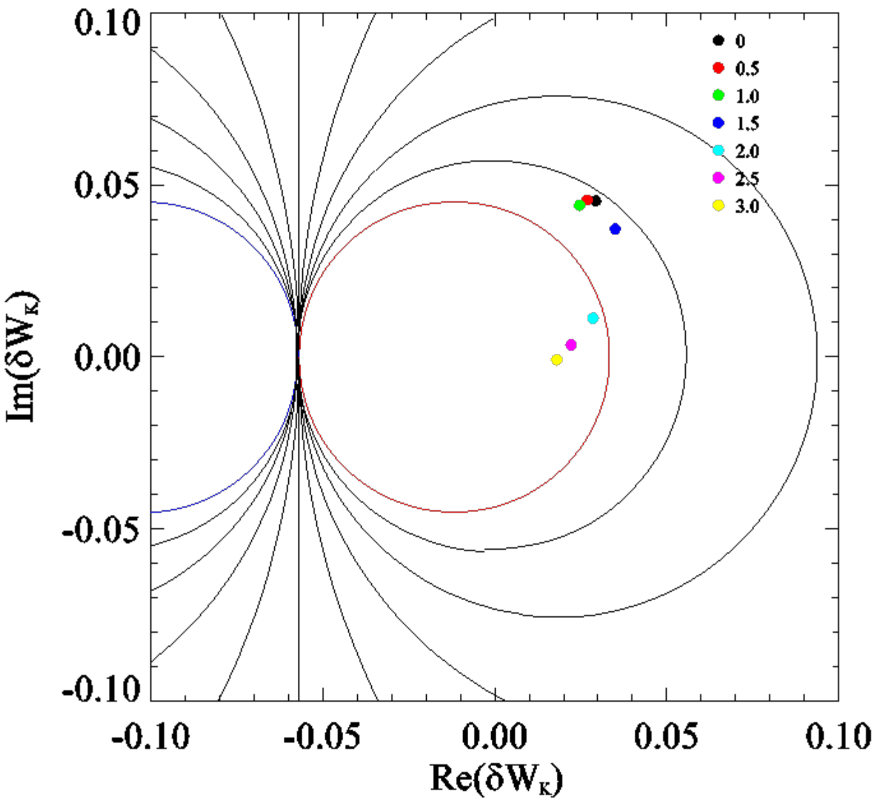

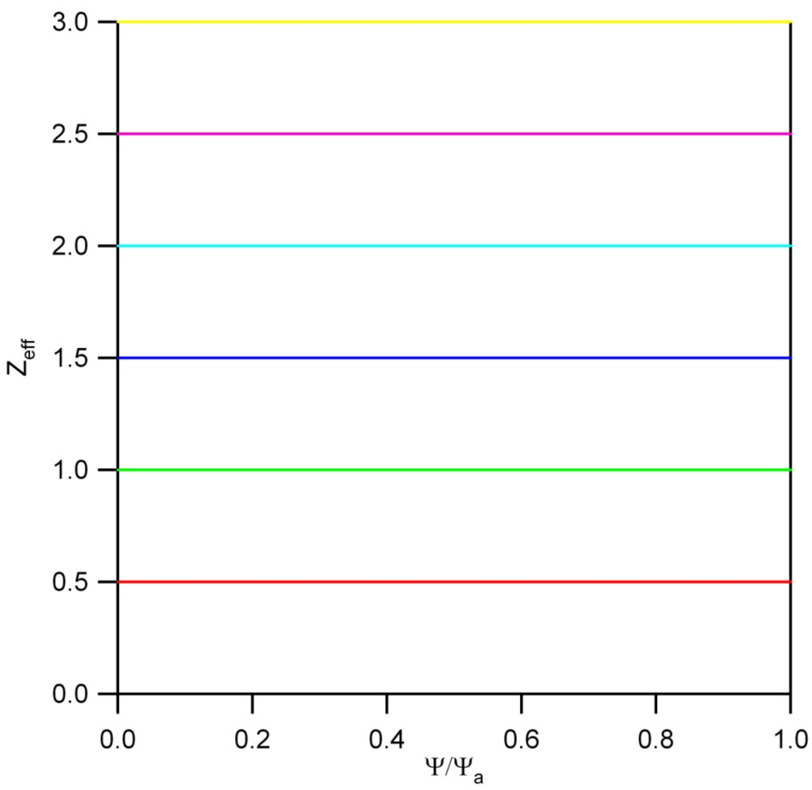

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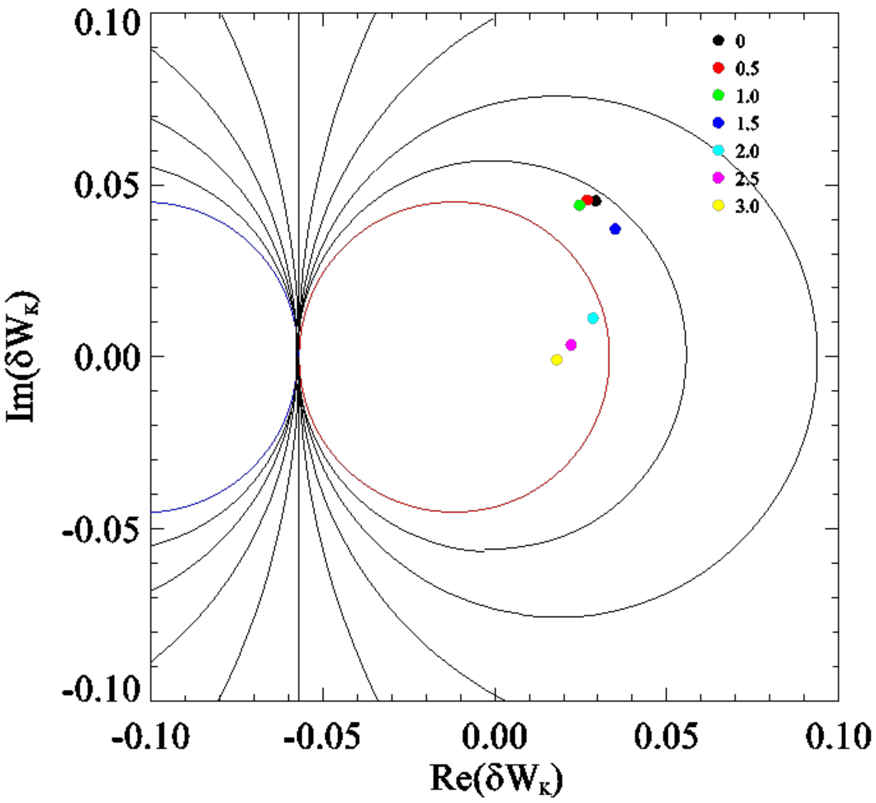

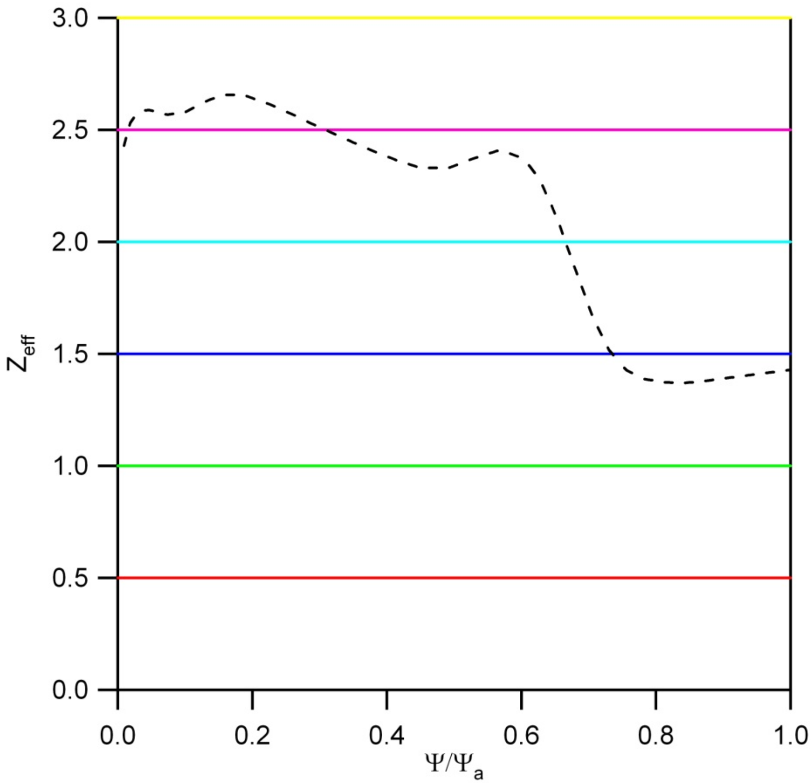

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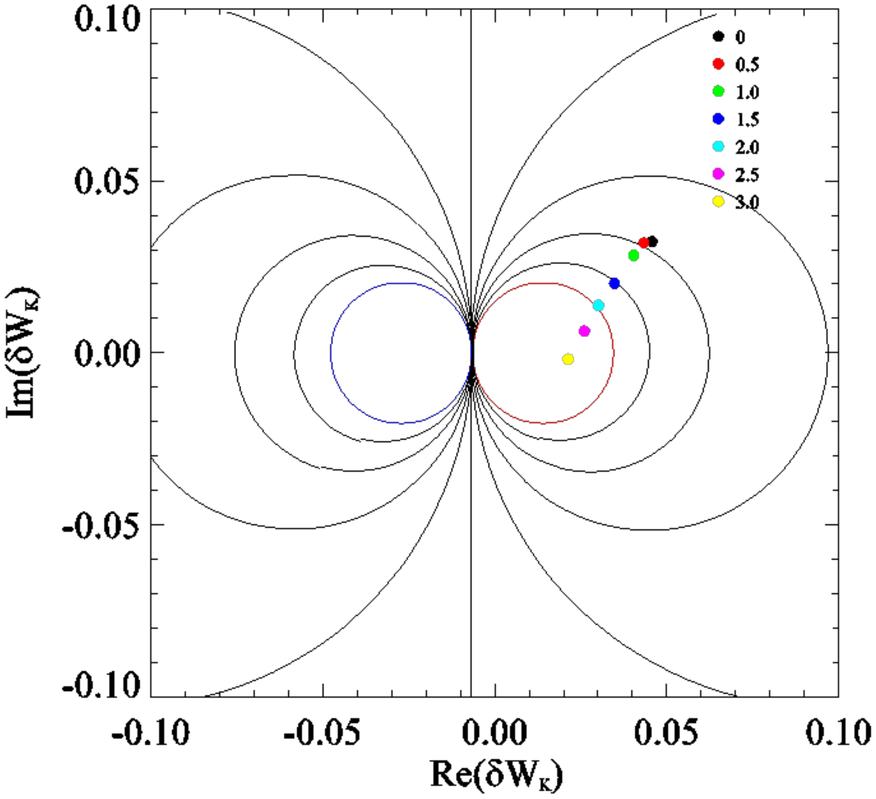

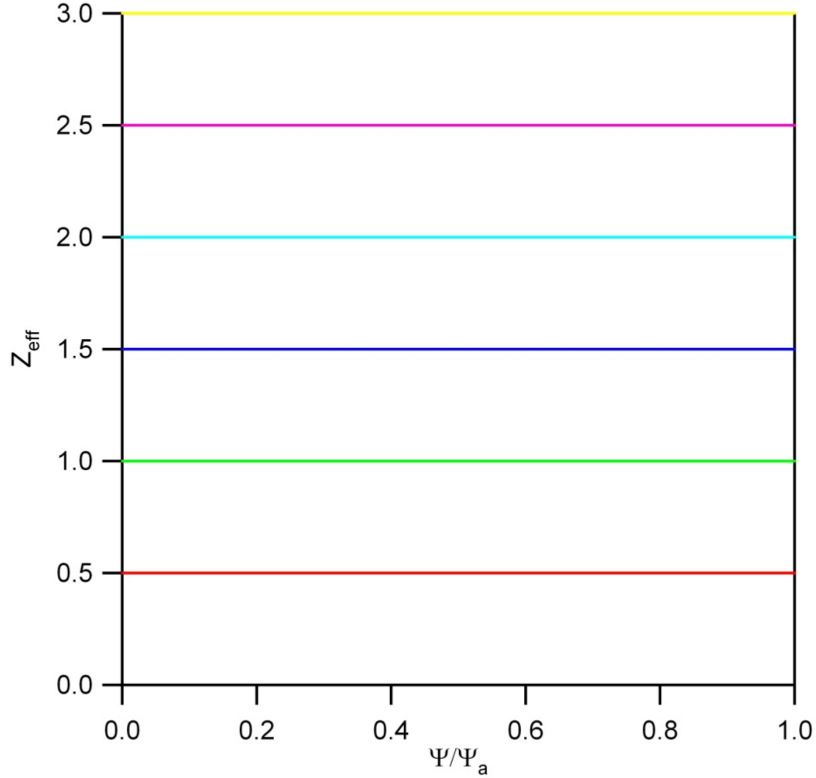

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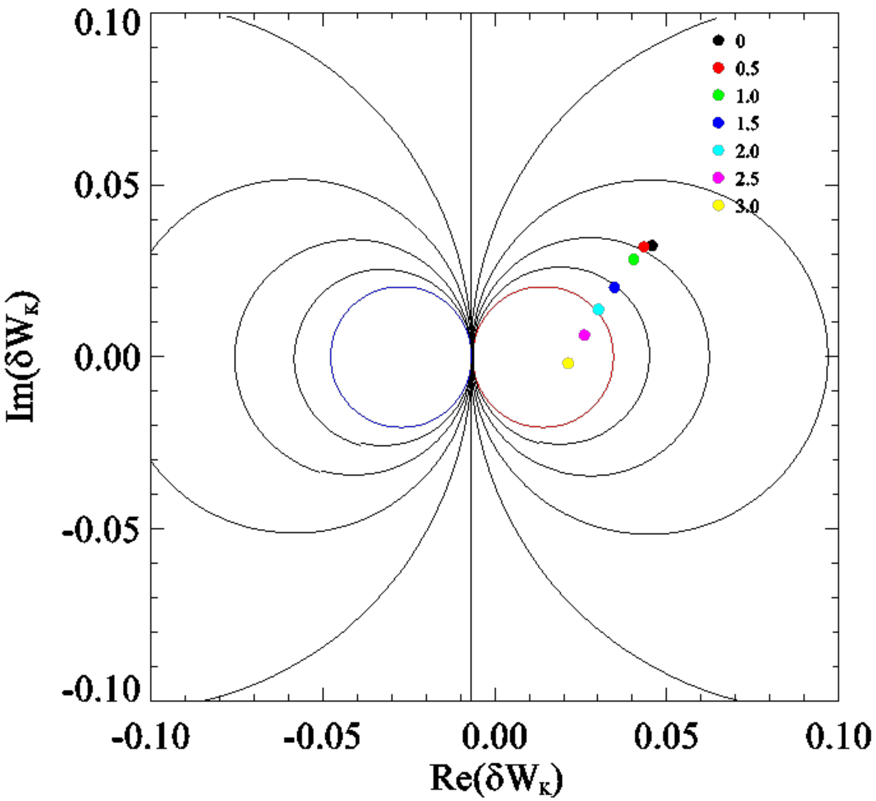

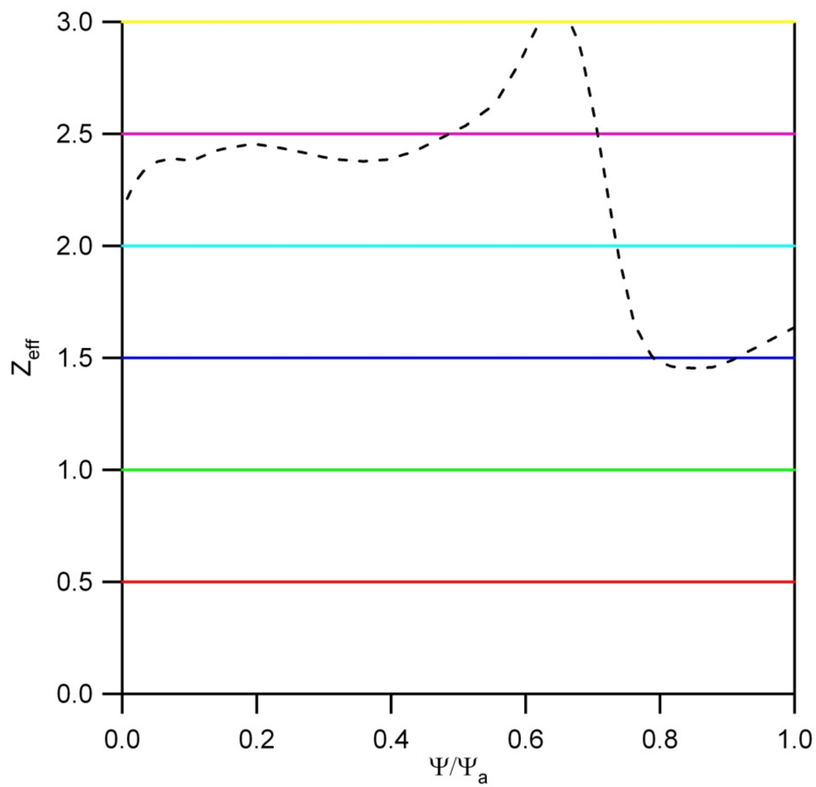

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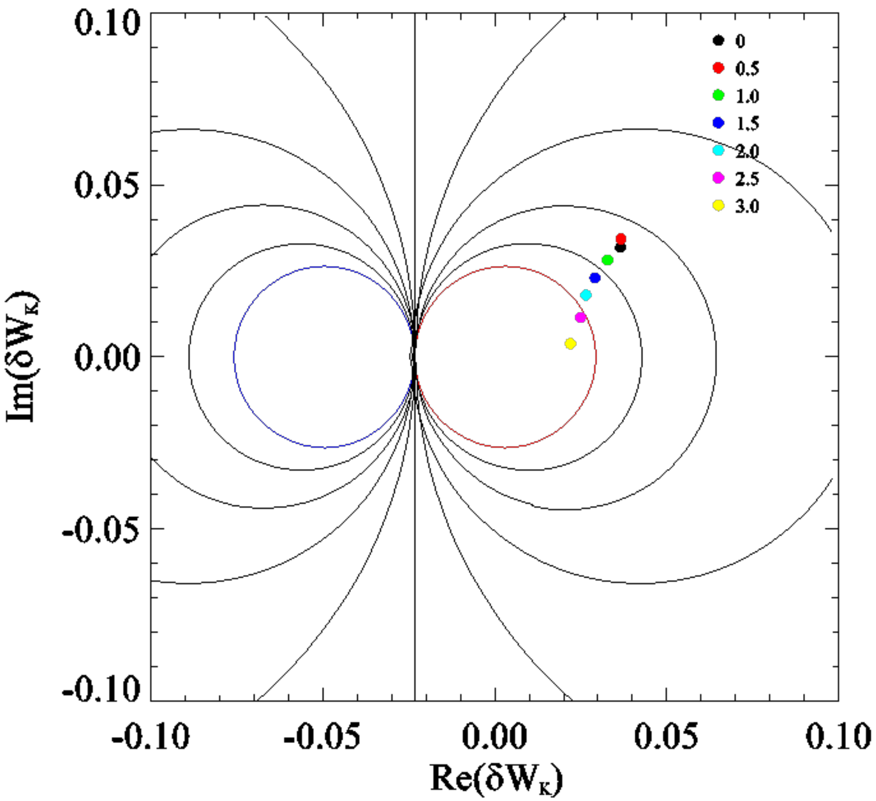

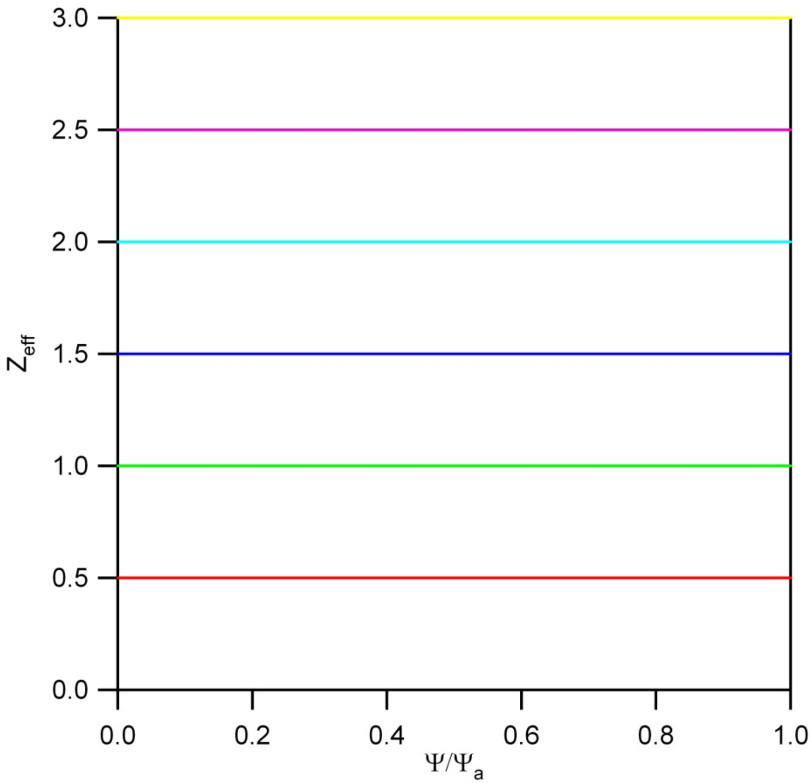

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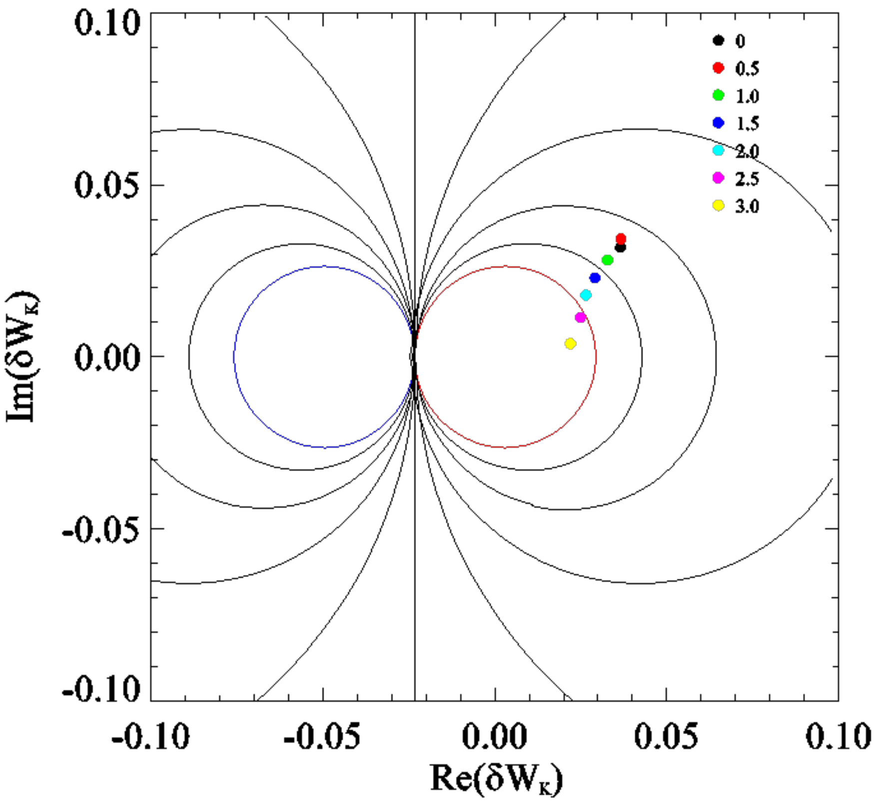

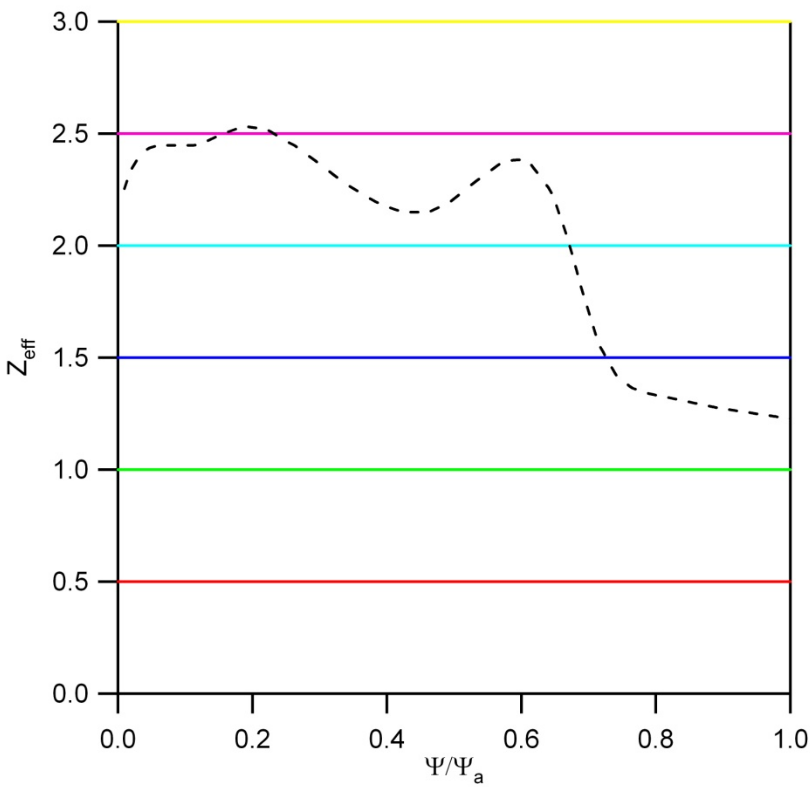

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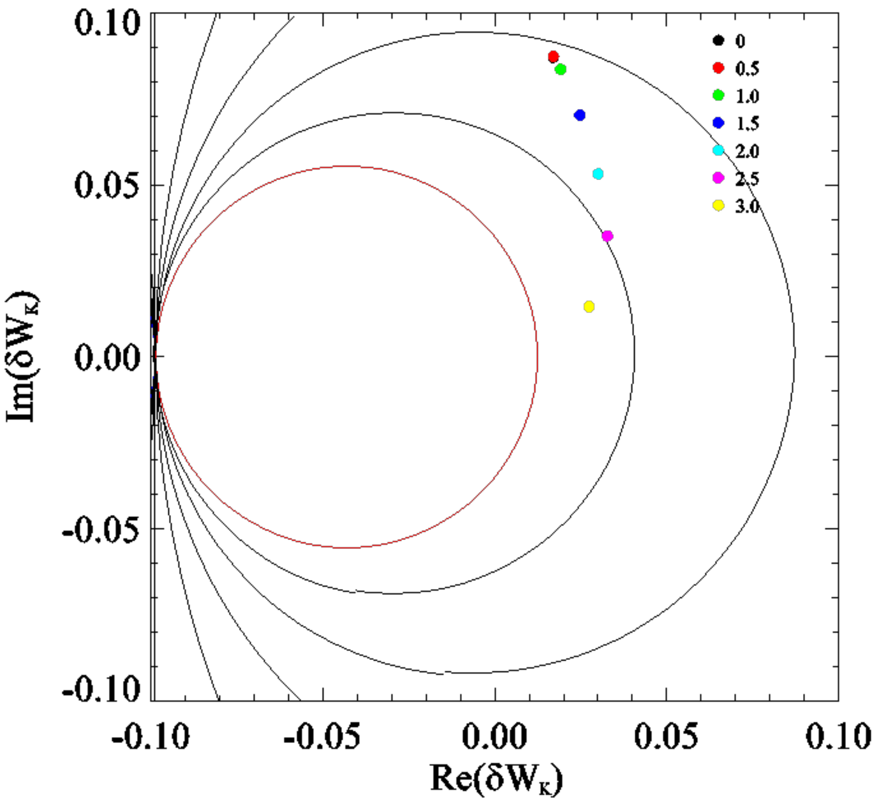

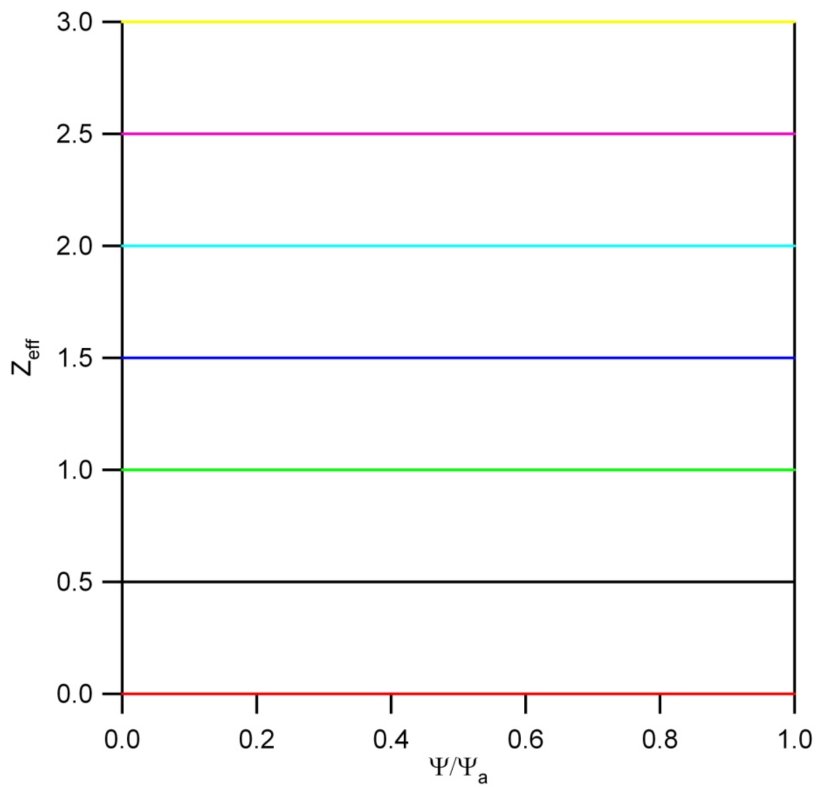

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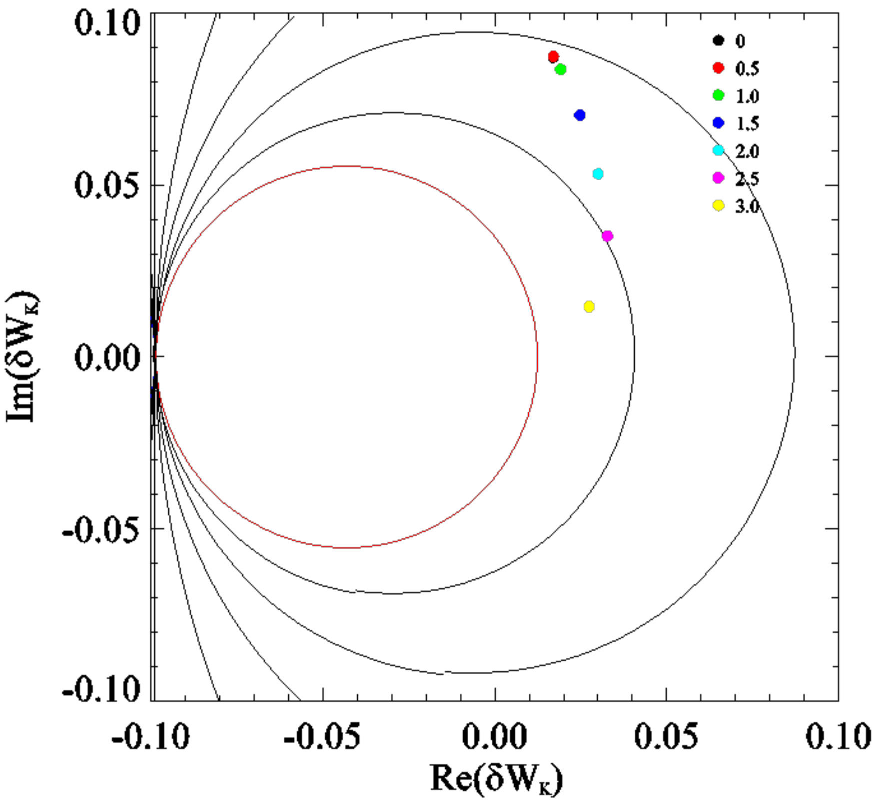

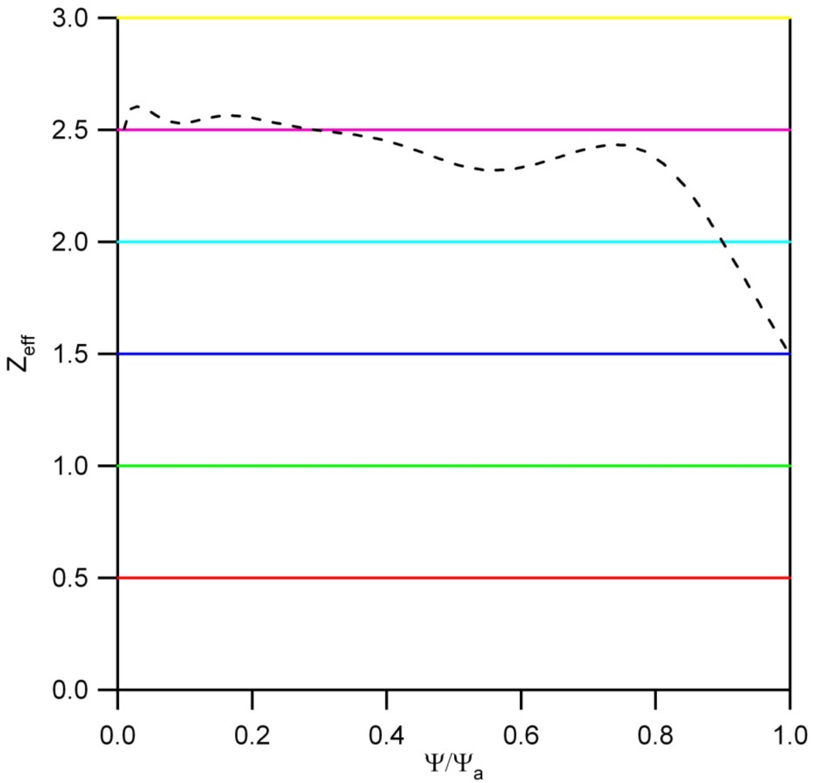

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Rotation

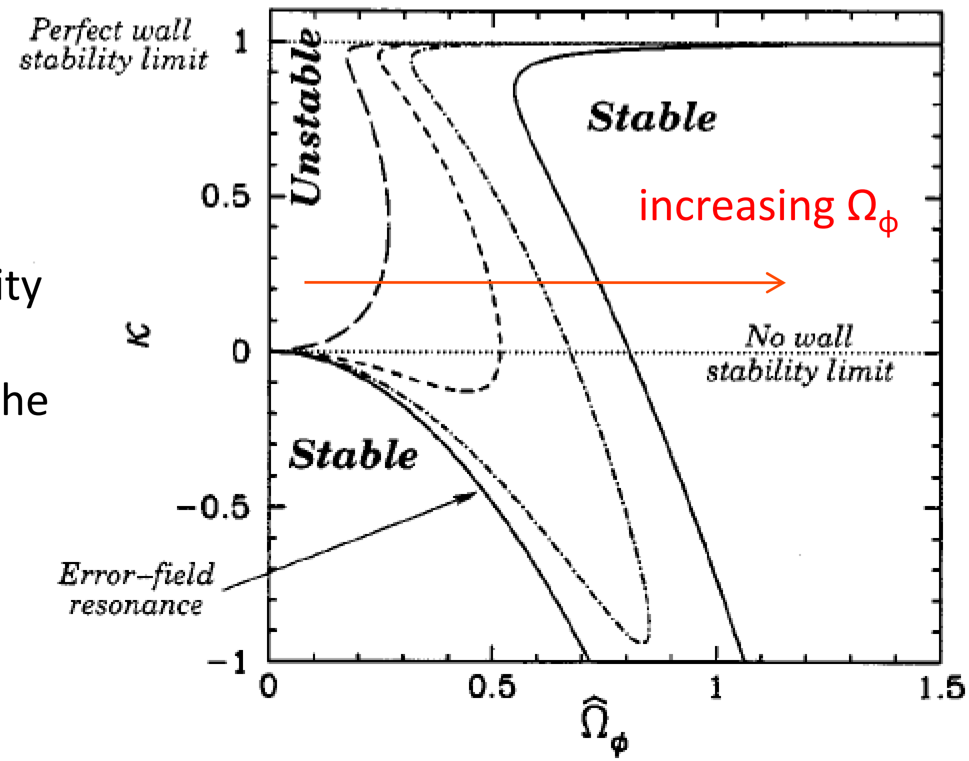
Simple model: rotation increases stability

$$[(\hat{\gamma}-i\hat{\Omega}_{\phi})^2+\nu_*(\hat{\gamma}-i\hat{\Omega}_{\phi})+(1-\kappa)(1-md)] \times(\hat{\gamma}S_*+1+md)=1-(md)^2.$$

(Fitzpatrick, PoP, 2002)

toroidal plasma rotation

- Fitzpatrick simple model
  - Plasma rotation increases stability and for a given β there is a "critical" rotation above which the plasma is stable.



Kinetic model: rotation/stability relationship is complex

$$\delta W_K \propto \left[\frac{\omega_{*N} + \left(\hat{\varepsilon} - \frac{3}{2}\right)\omega_{*T} + \omega_E - \omega}{\langle \omega_D \rangle + l\omega_b - i\nu_{\text{eff}} + \omega_E - \omega} \right]$$

(Hu, Betti, and Manickam, PoP, 2006)

E x B frequency

- Fitzpatrick simple model
  - Plasma rotation <u>increases</u> stability and for a given β there is a "critical" rotation above which the plasma is stable.
- Kinetic model
  - Plasma rotation <u>increases or decreases</u> stability and a "critical" rotation is not defined?

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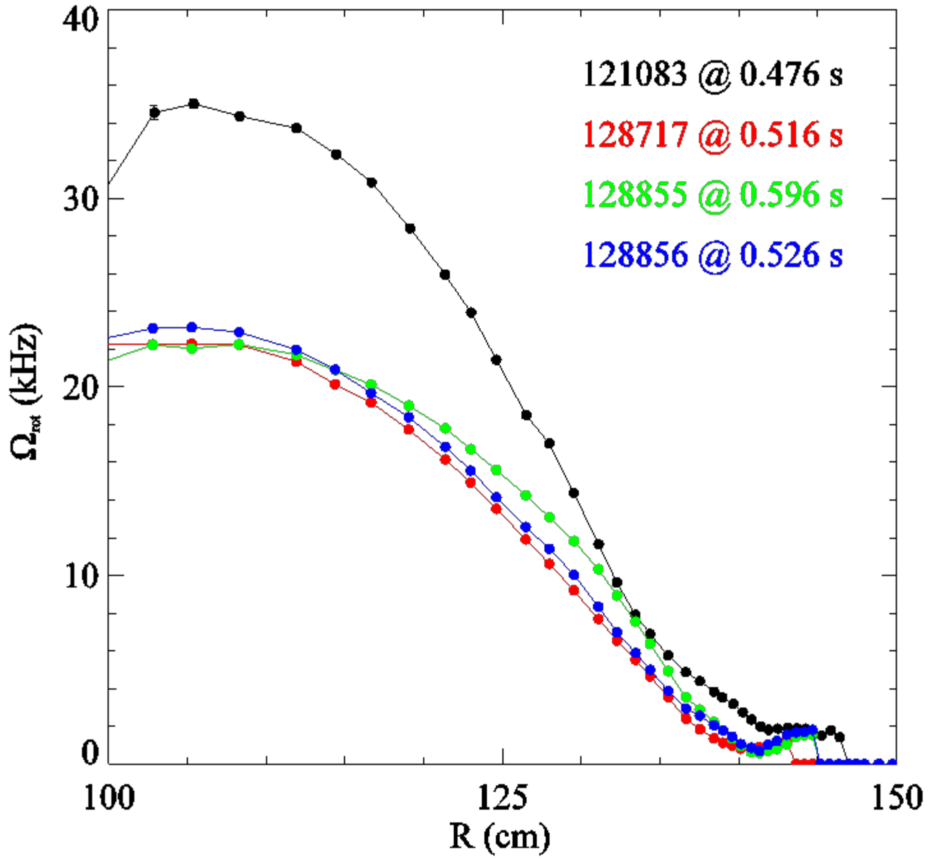
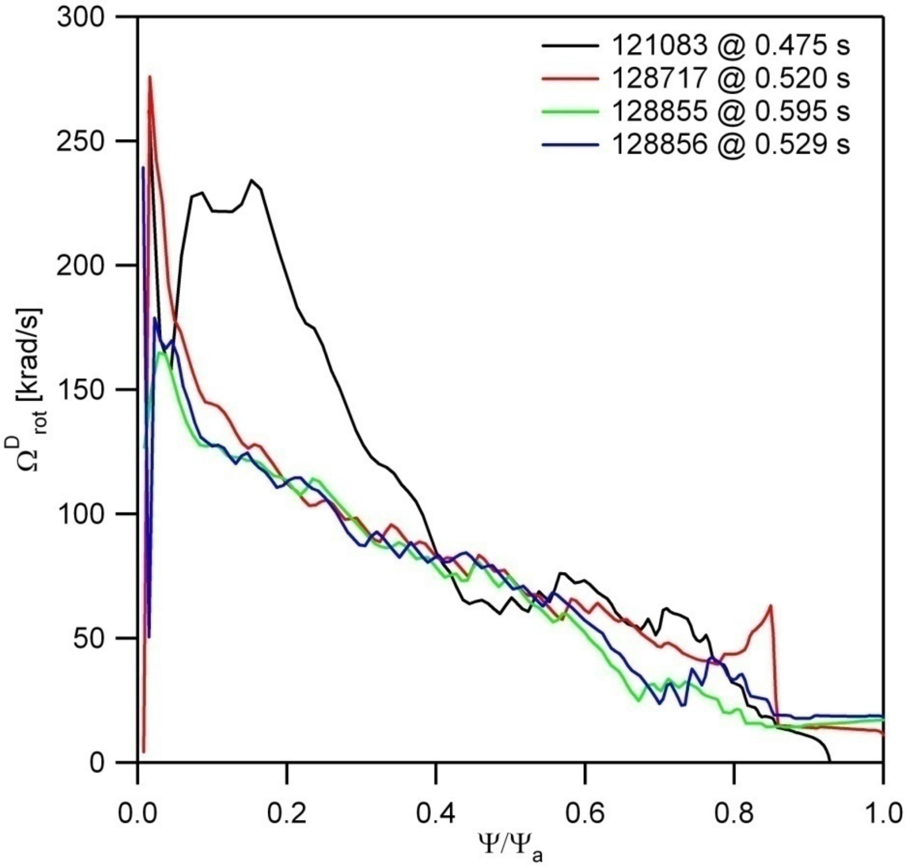
$$\omega_E = \Omega_\phi^D - \omega_{*i}^D - \frac{v_\theta^D}{2\pi R}\frac{B_\phi}{B_\theta}$$

$$\Omega_\phi^D = \Omega_\phi^C + \omega_{*i}^D - \omega_{*i}^C + \frac{(v_\theta^D - v_\theta^C)}{2\pi R}\frac{B_\phi}{B_\theta}$$

NSTX

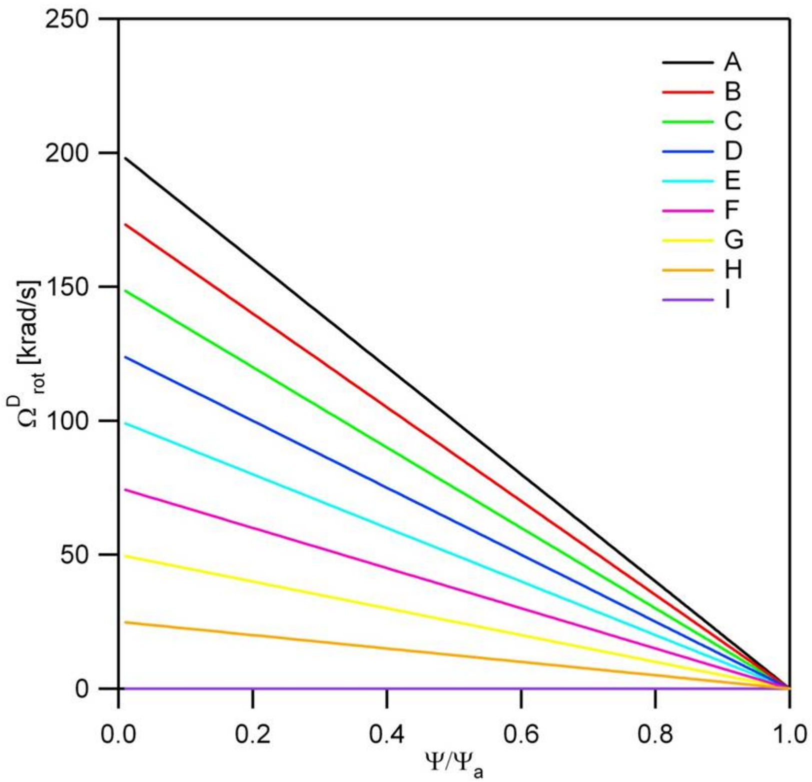
Rotation profiles just before instability



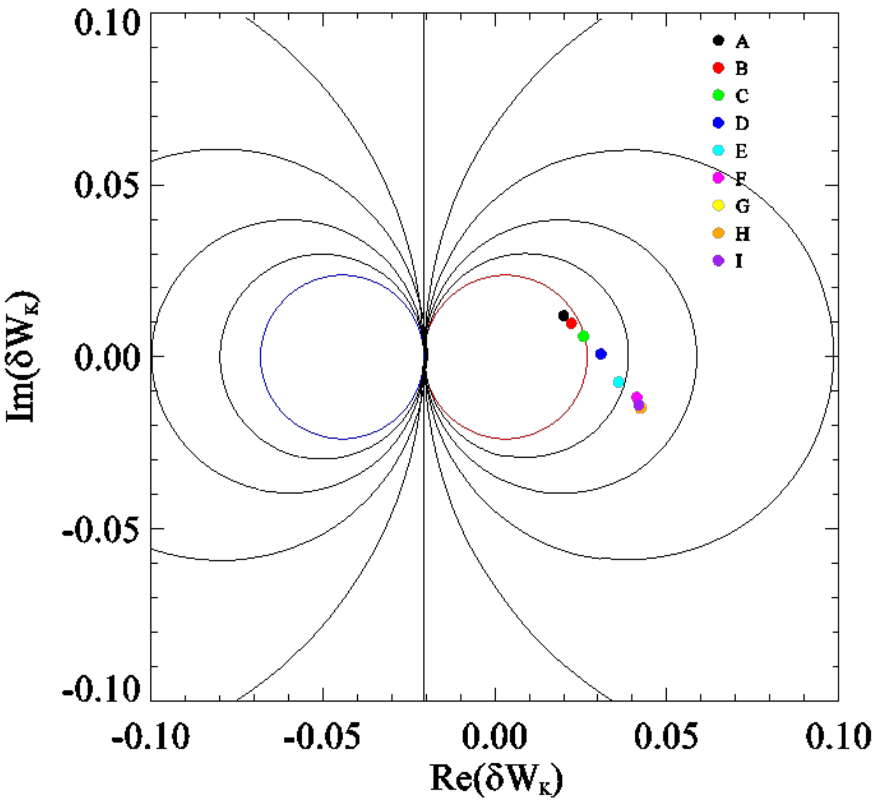


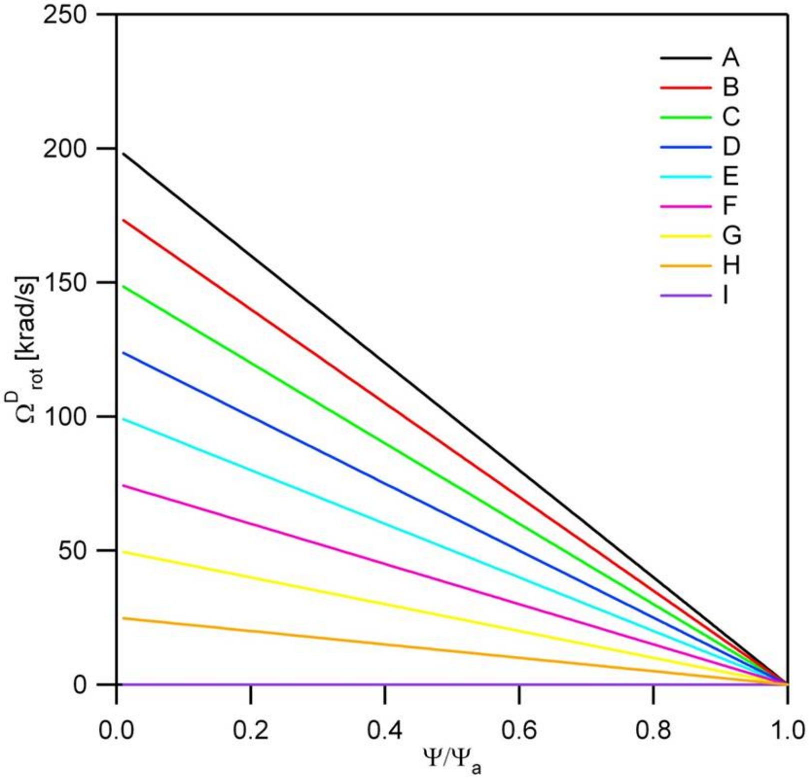
Convert to Hu/Betti code form

Using test rotation profiles shows the behavior



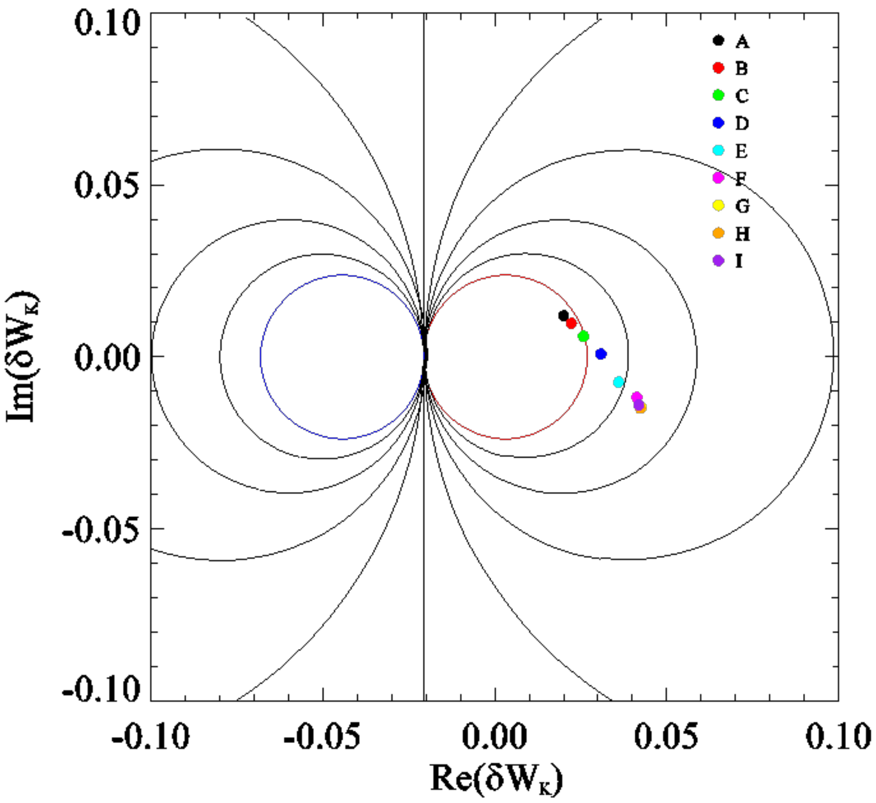
Using test rotation profiles shows the behavior

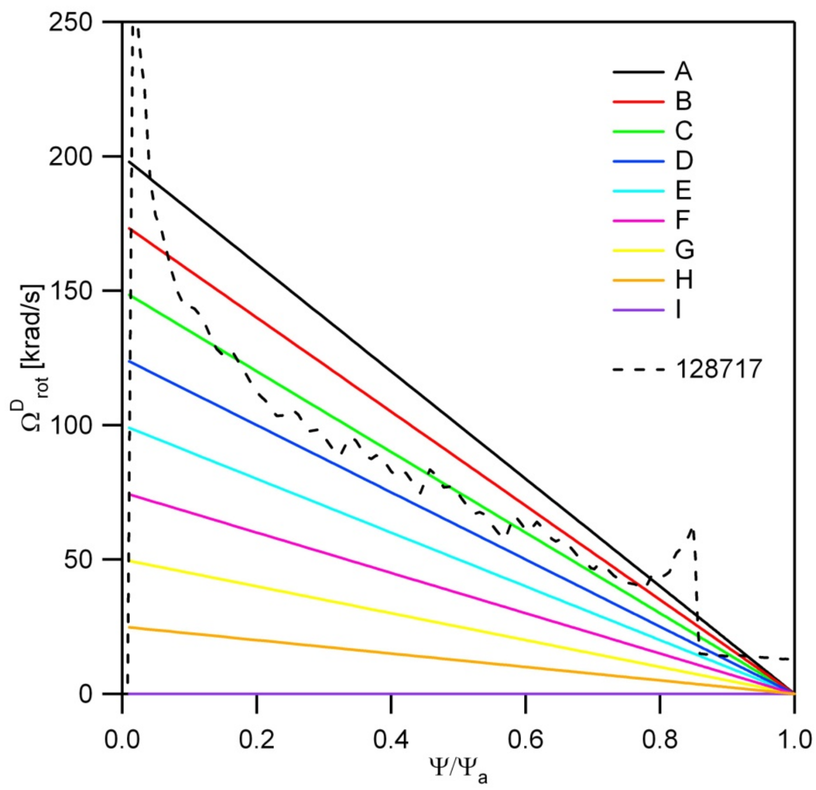




128717

Using test rotation profiles shows the behavior





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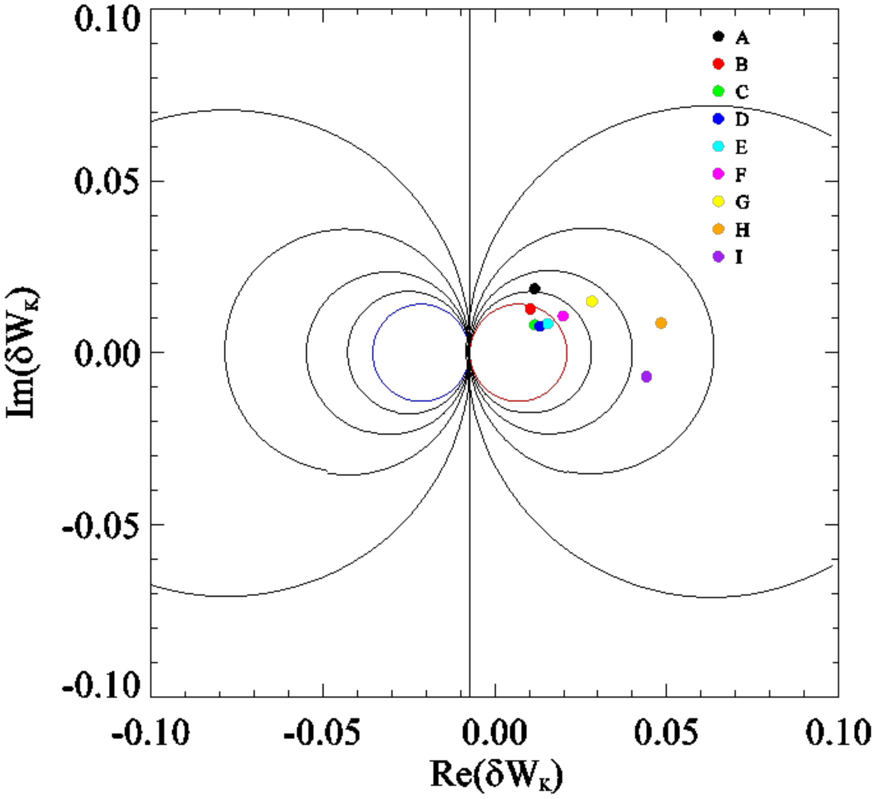
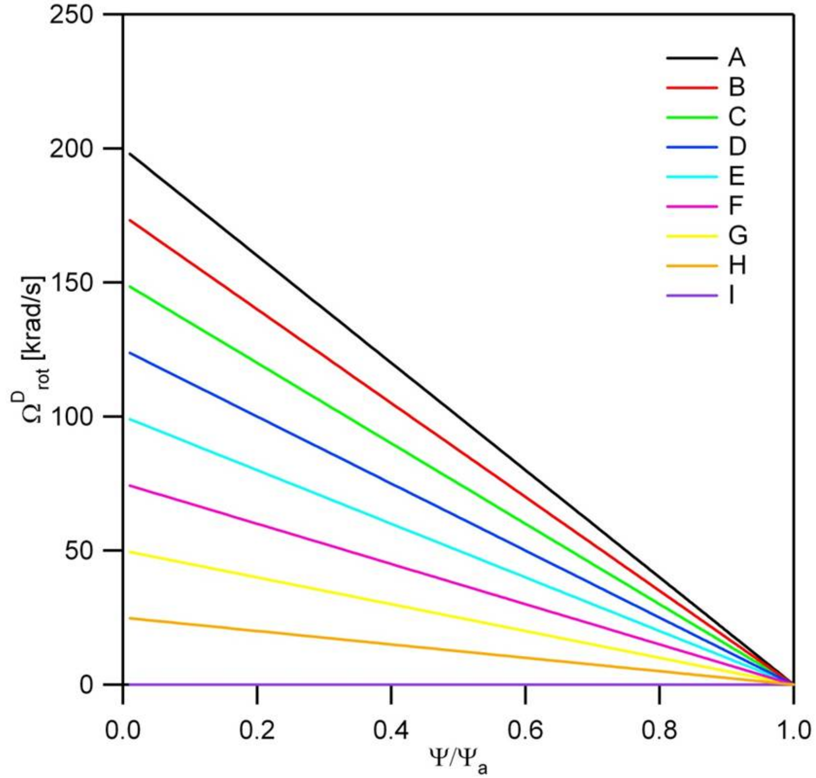
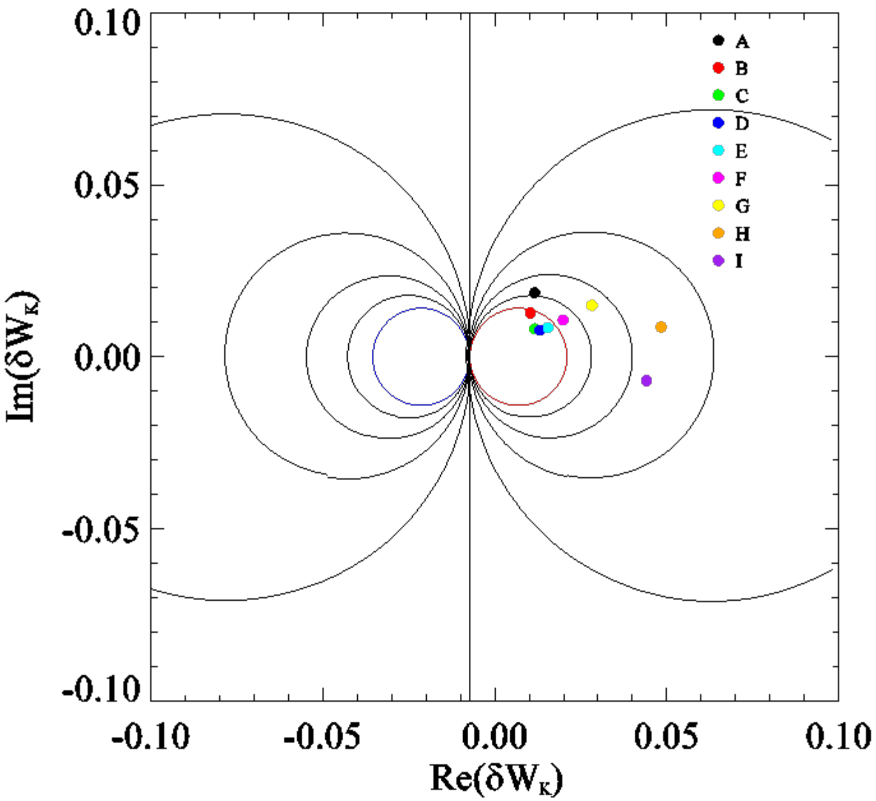
Using test rotation profiles shows the behavior

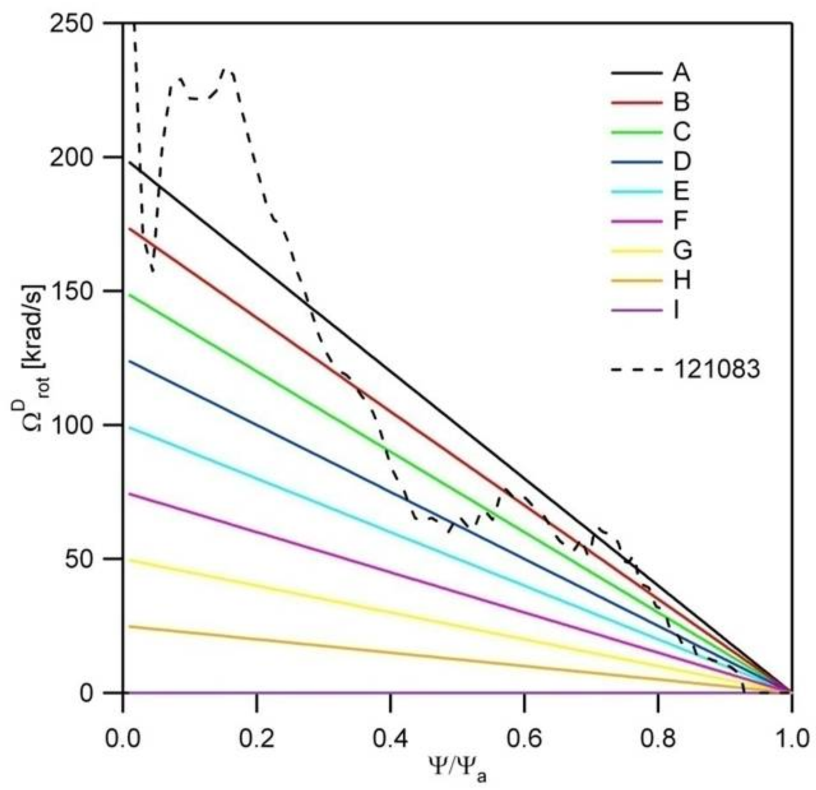




121083

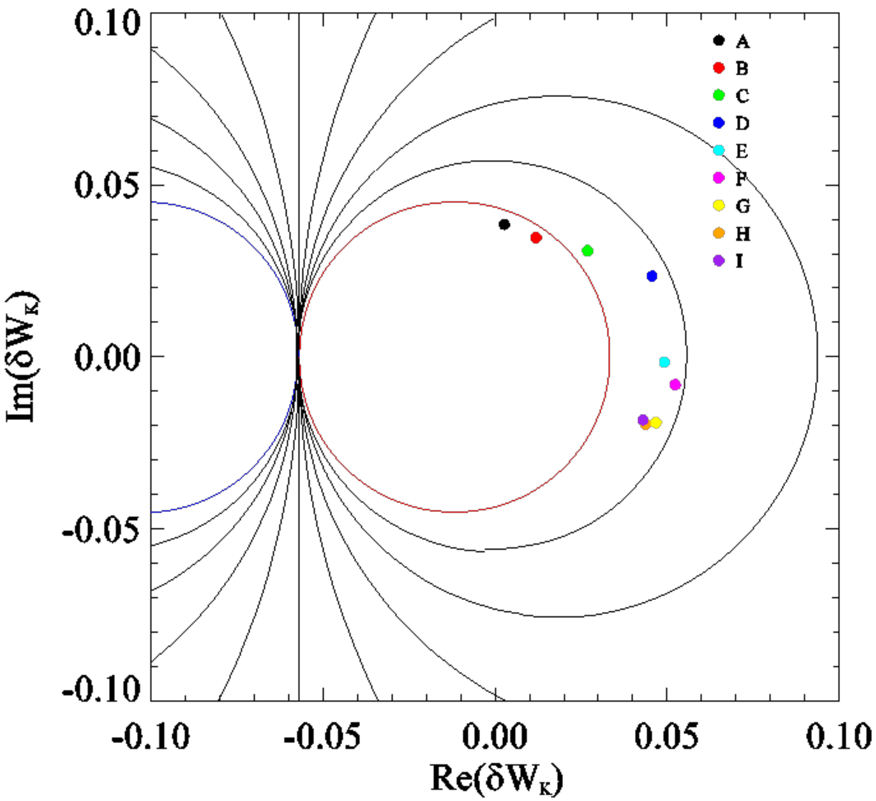
Using test rotation profiles shows the behavior

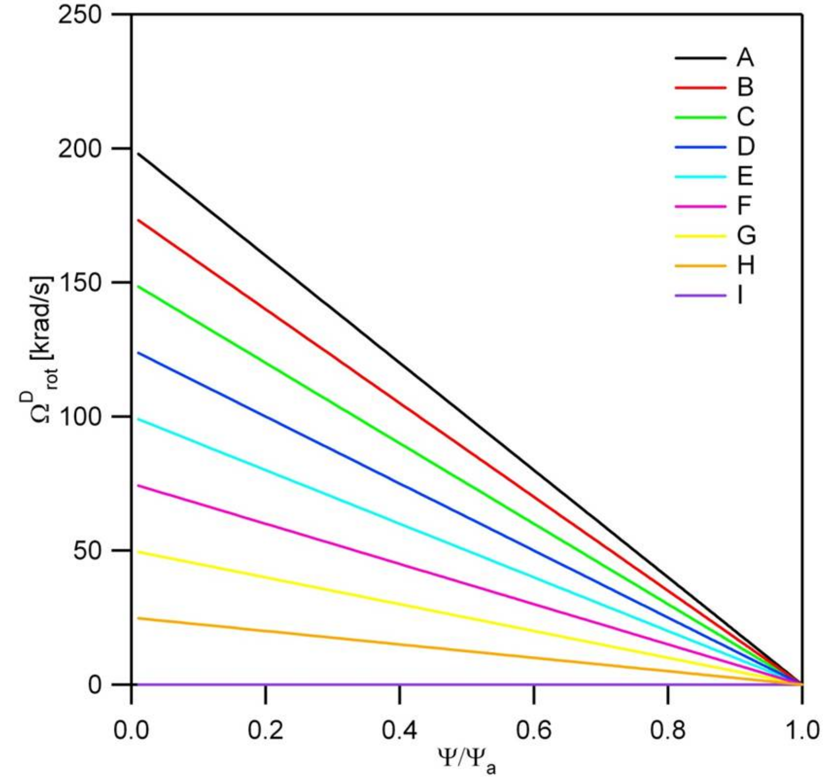




121083

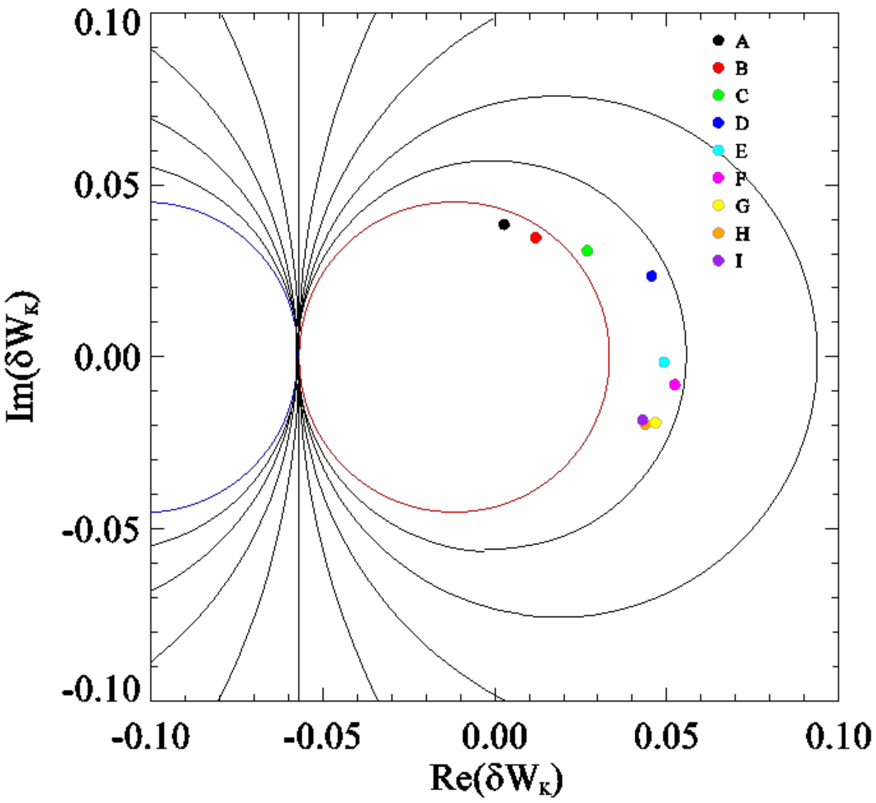
Using test rotation profiles shows the behavior

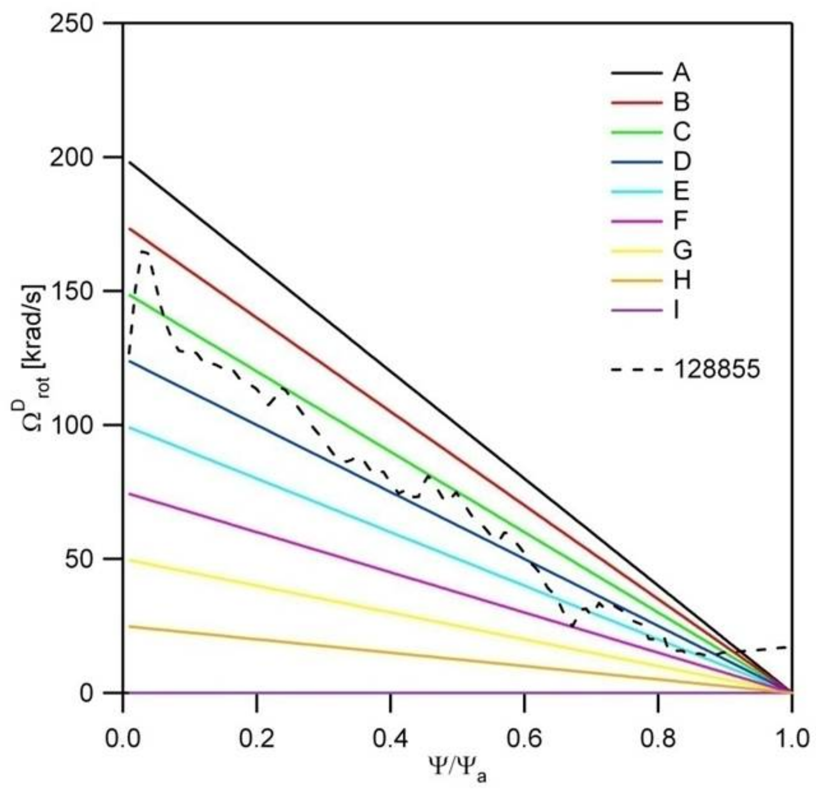




128855

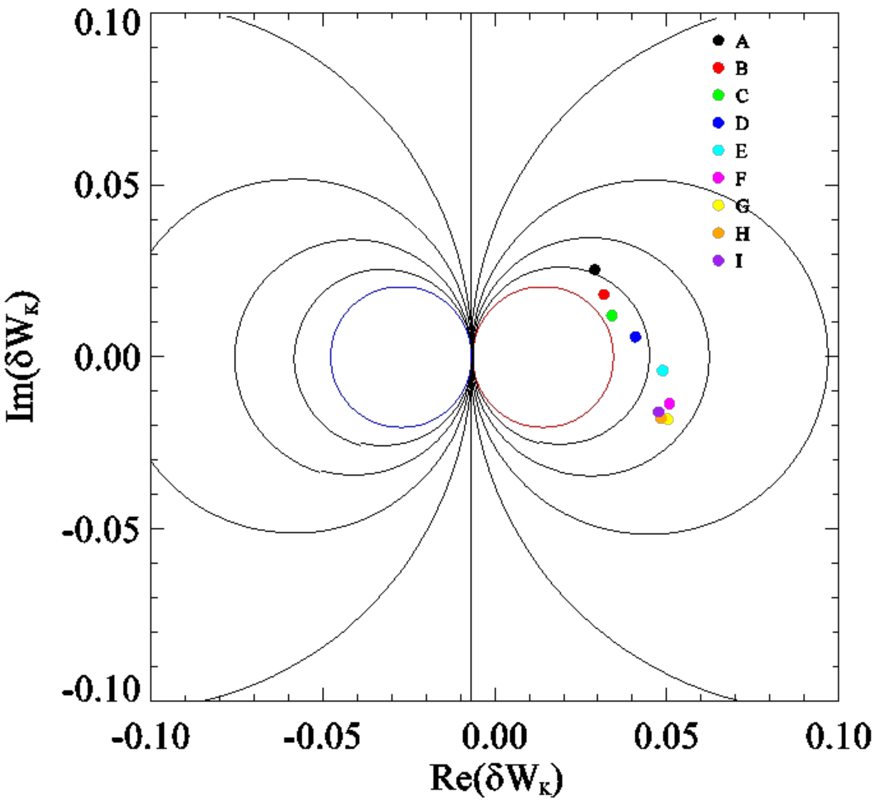
Using test rotation profiles shows the behavior

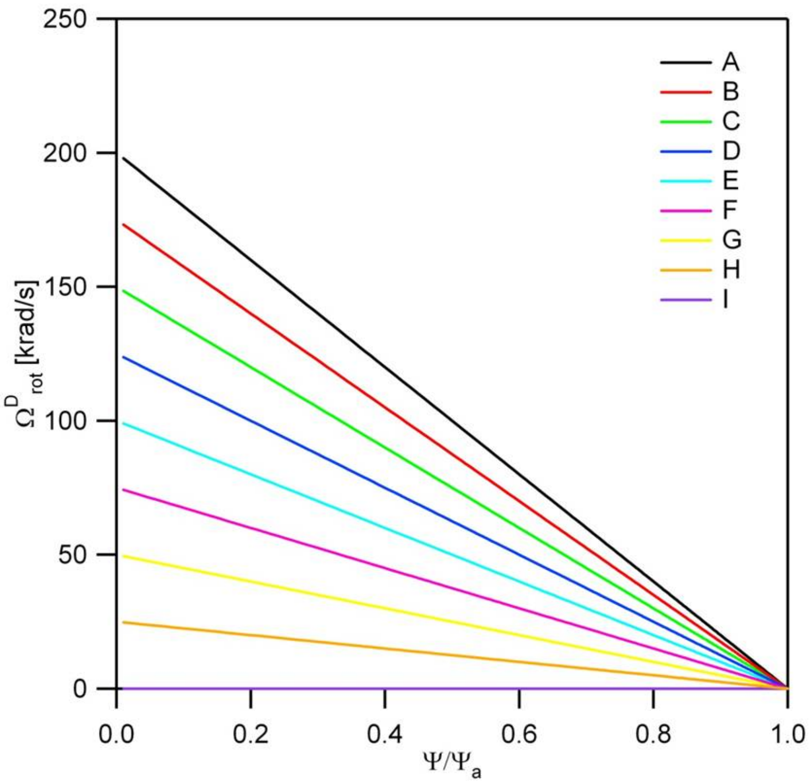




128855

Using test rotation profiles shows the behavior





128856

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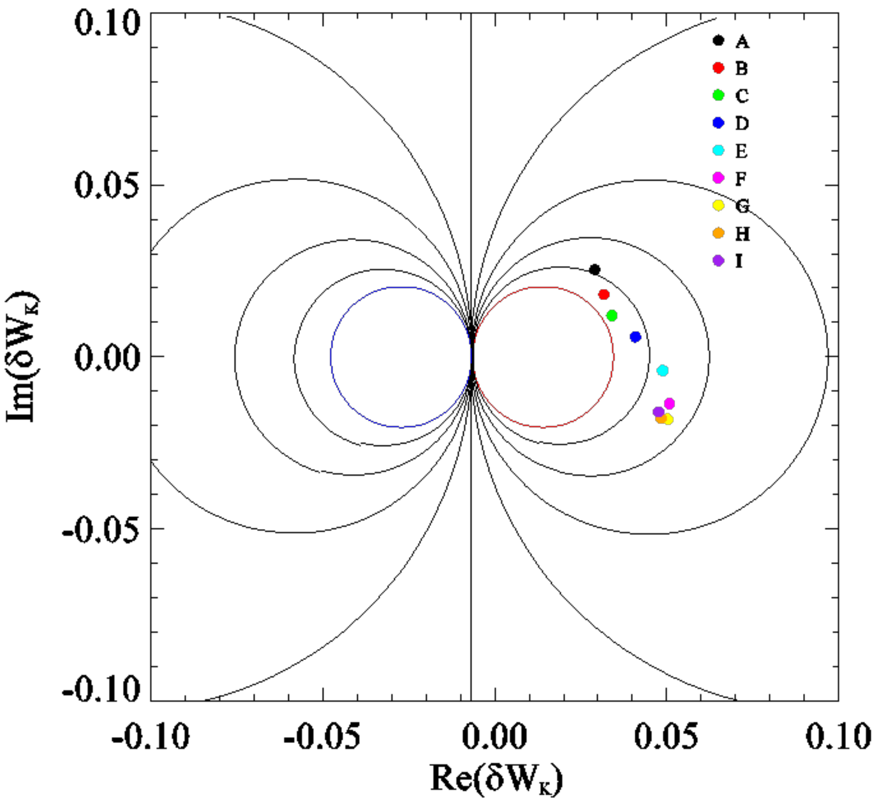

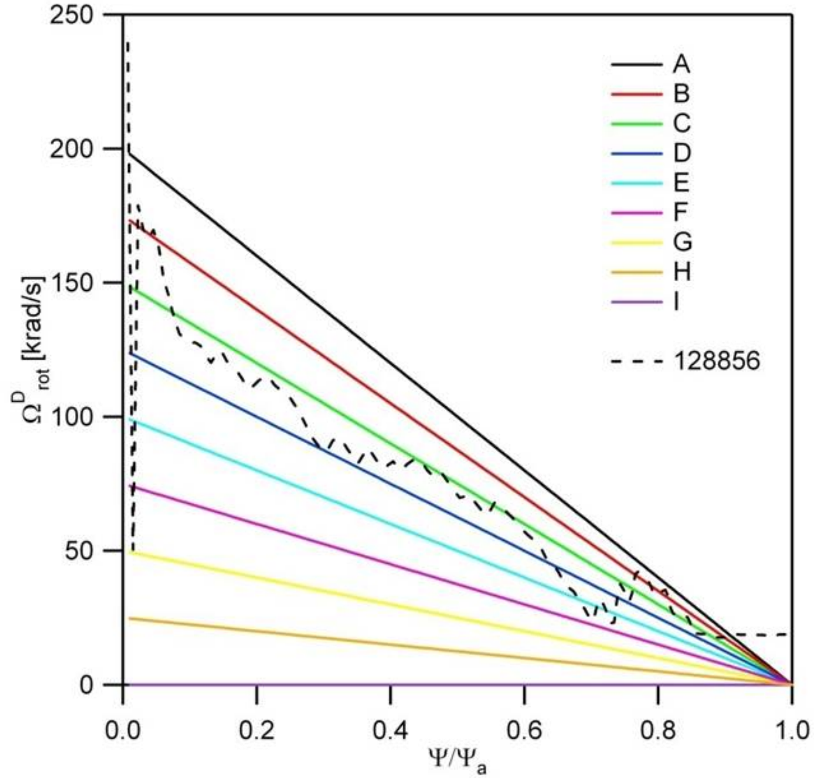

128856

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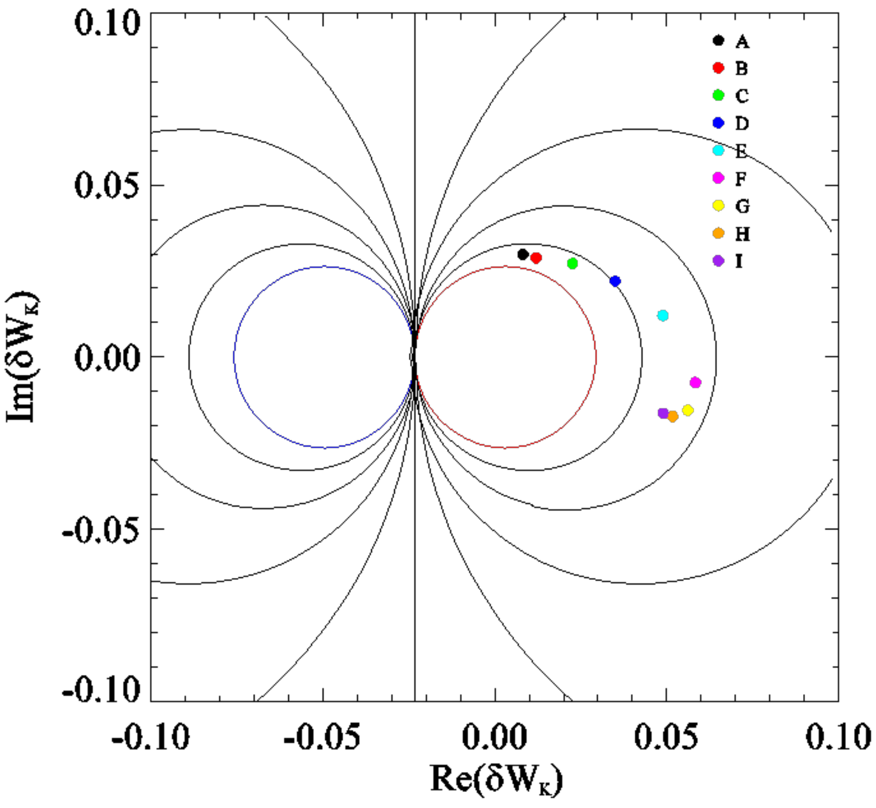
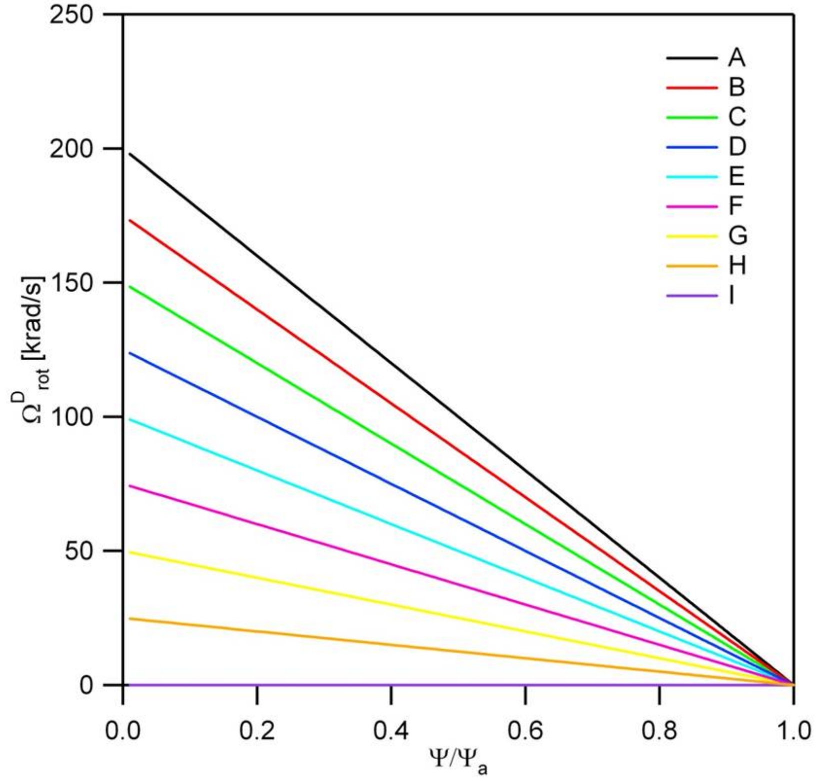
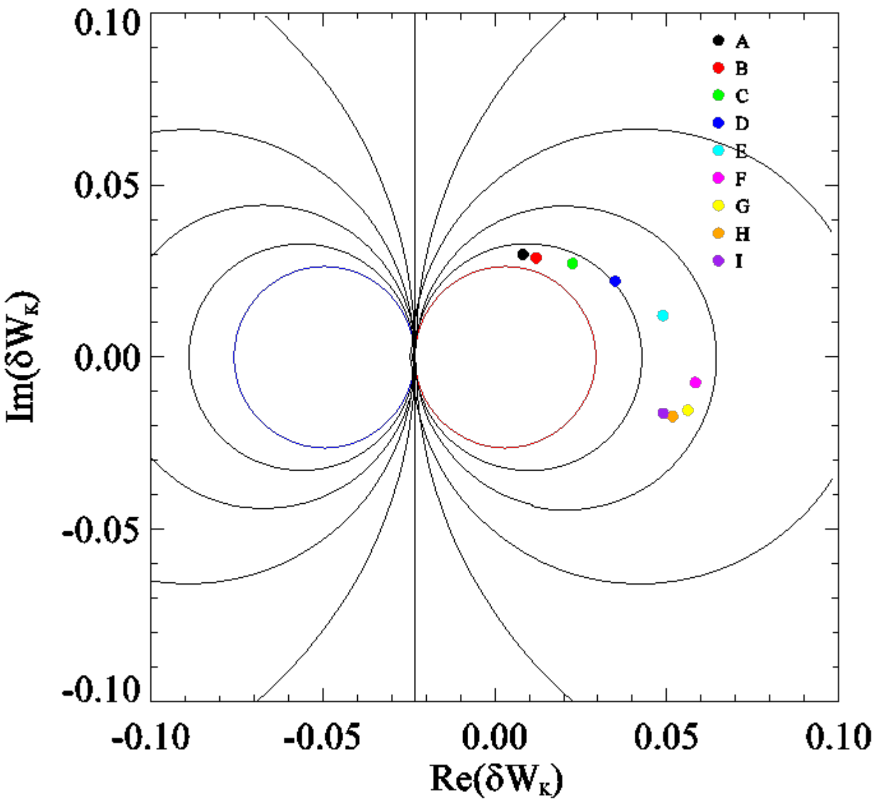
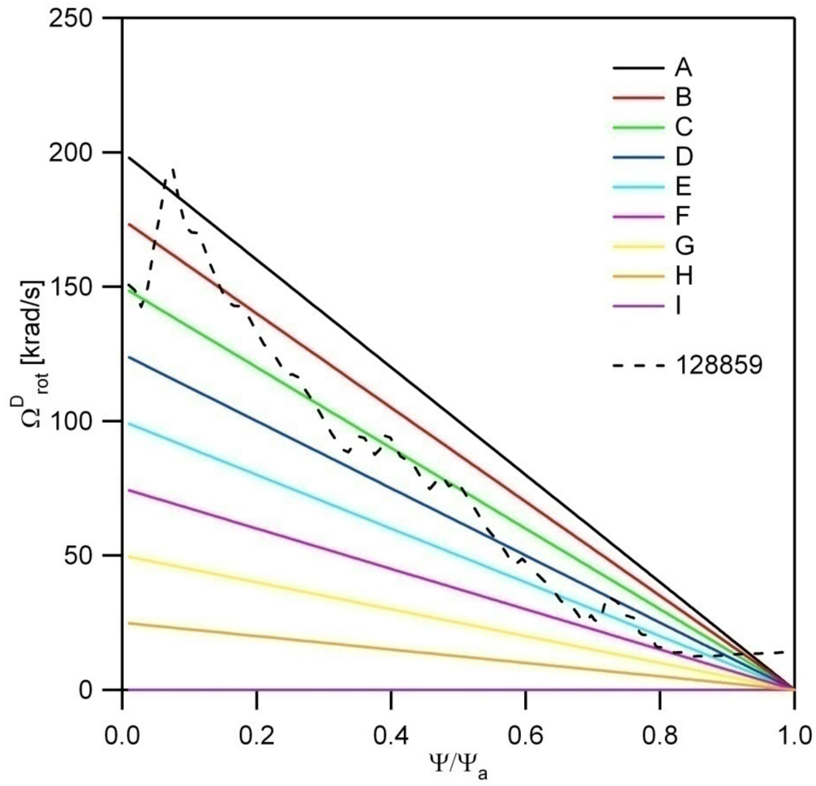




128859

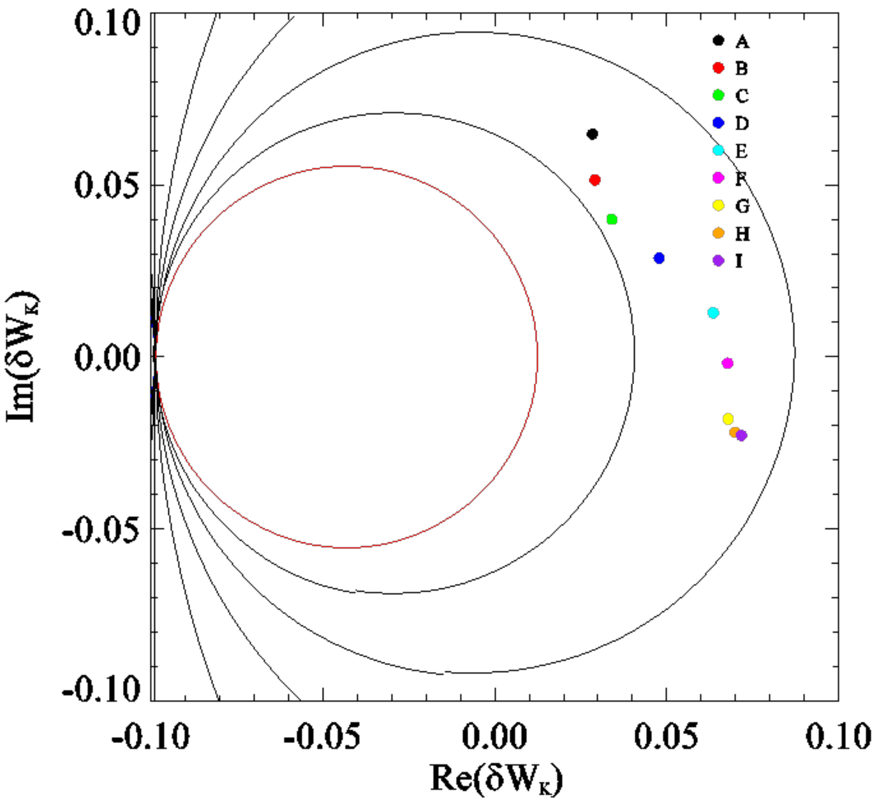
Using test rotation profiles shows the behavior

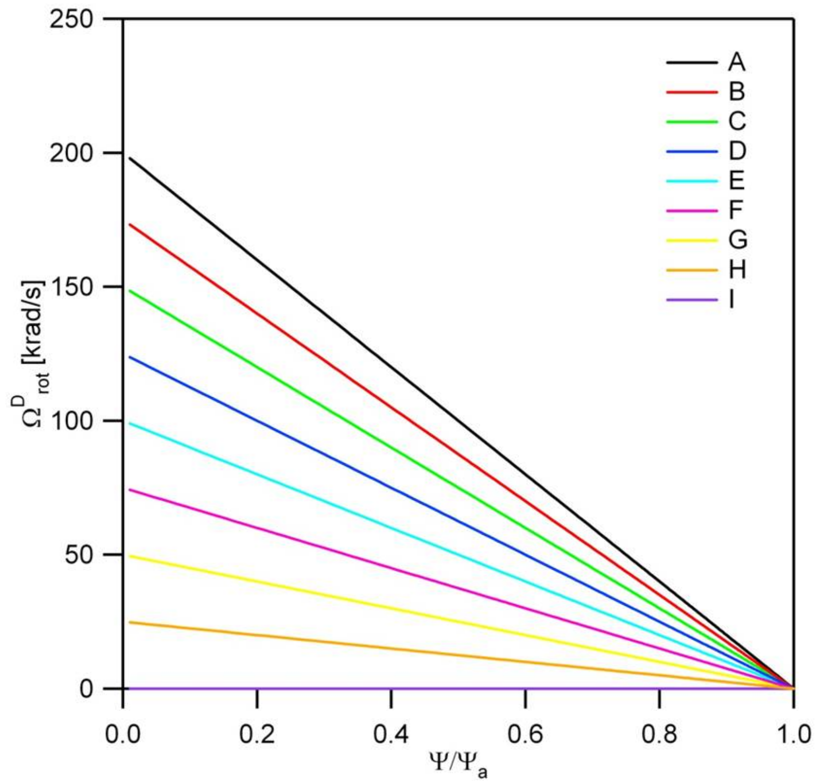




128859

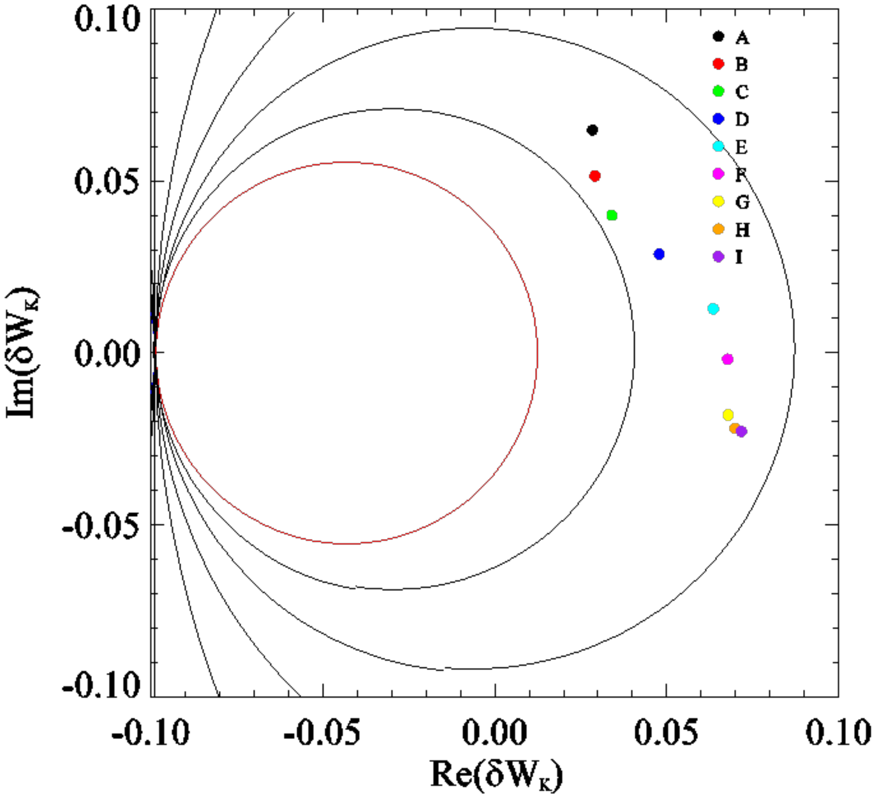
Using test rotation profiles shows the behavior

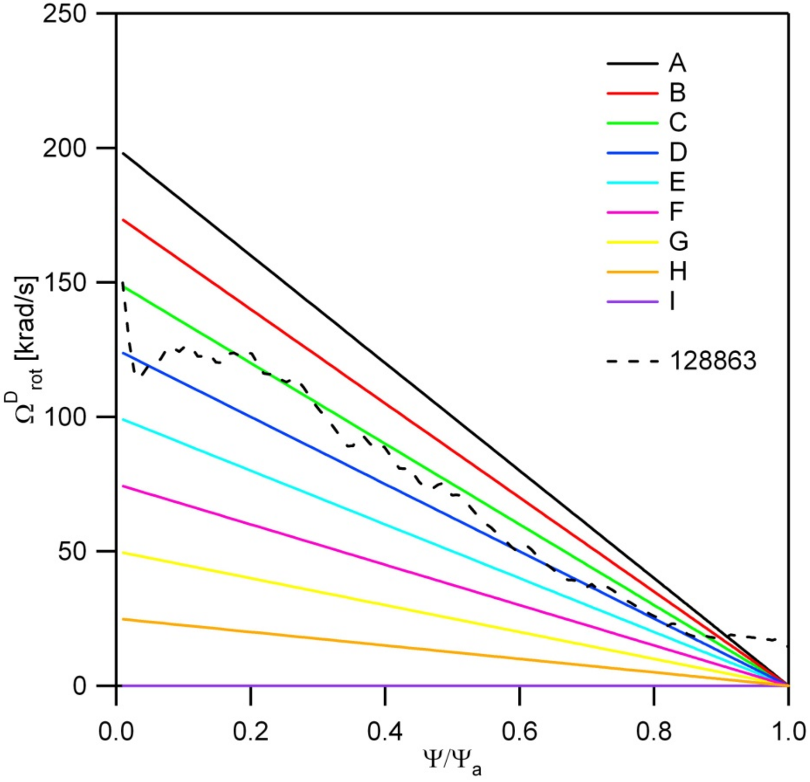




128863

Using test rotation profiles shows the behavior



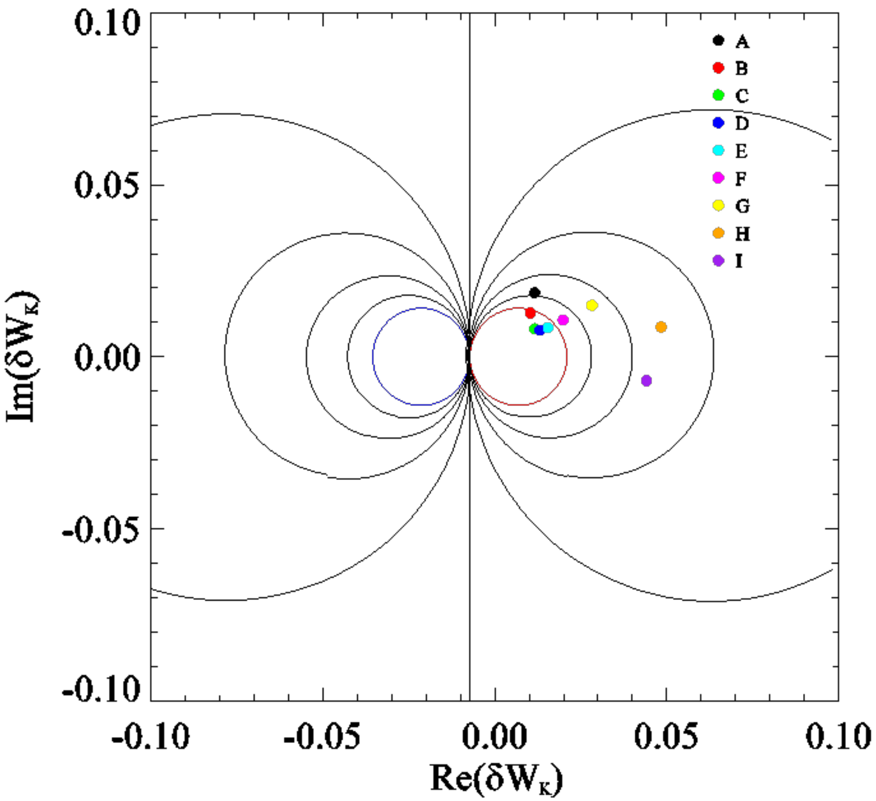


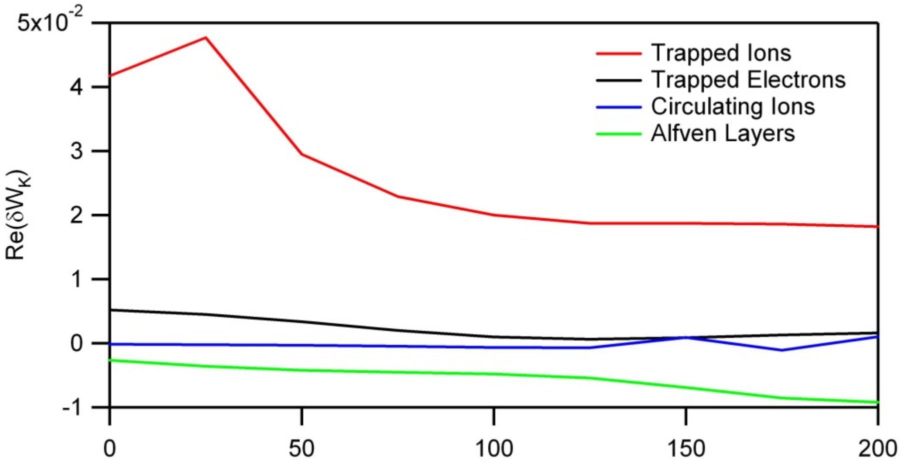
128863

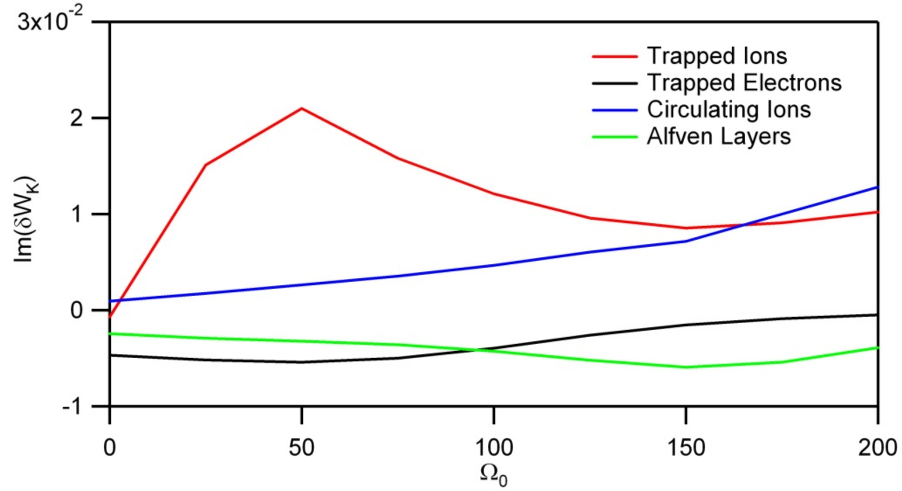
Conclusions

- Kinetic Stabilization
  - Analysis of multiple NSTX discharges from just before RWM instability is observed predicts near-marginal mode growth rates.
- Collisionality
  - The predicted effect of collisionality is as expected from the kinetic equation.
- Rotation
  - Increasing or decreasing the rotation in the calculation drives the prediction farther from the marginal point in either the stable or unstable direction.
  - Unlike simpler “critical” rotation theories, kinetic theory allows for a more complex relationship between plasma rotation and RWM stability.

Which components lead to stability/instability?

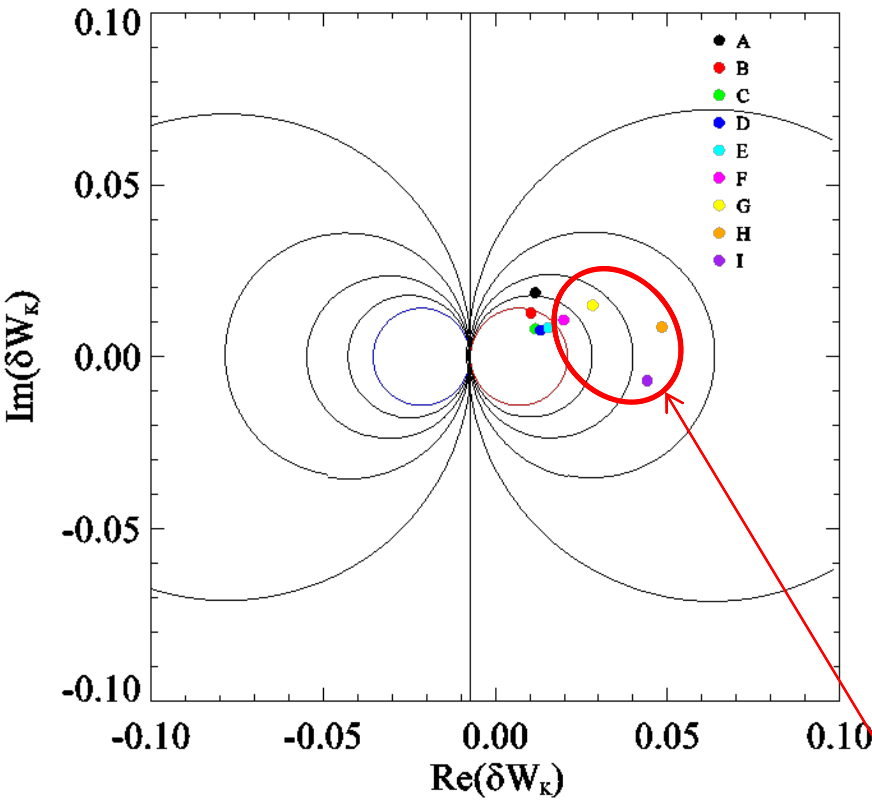


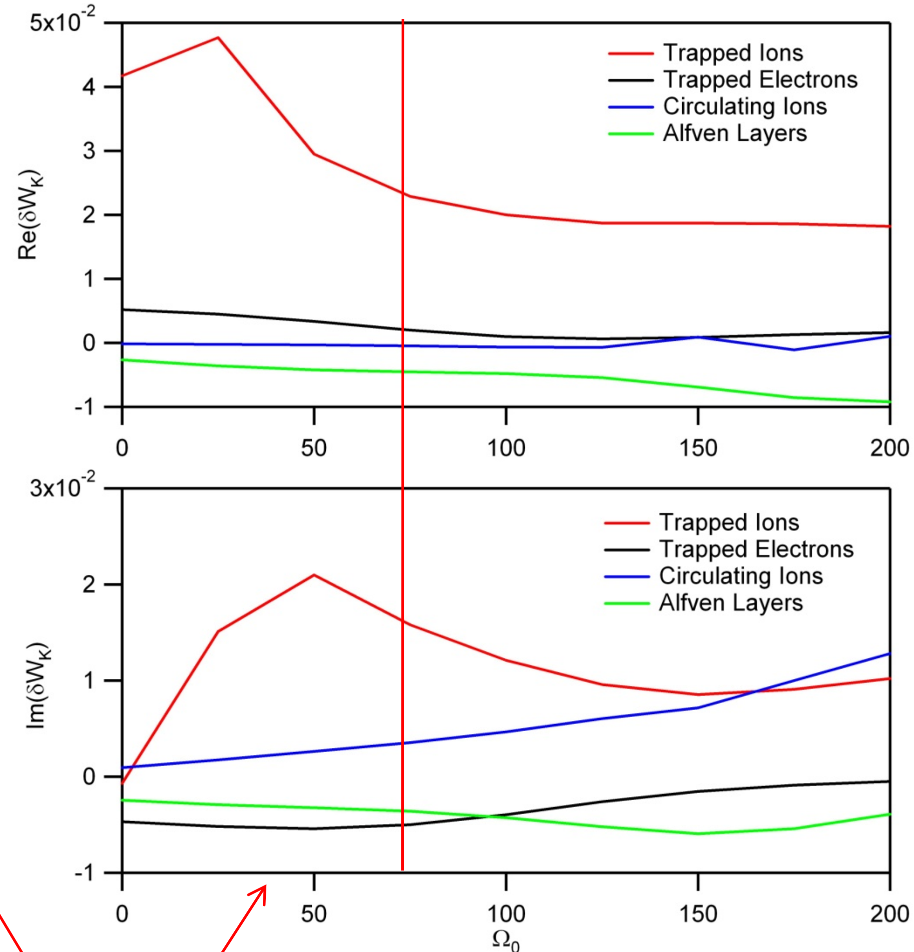




121083

Which components lead to stability/instability?

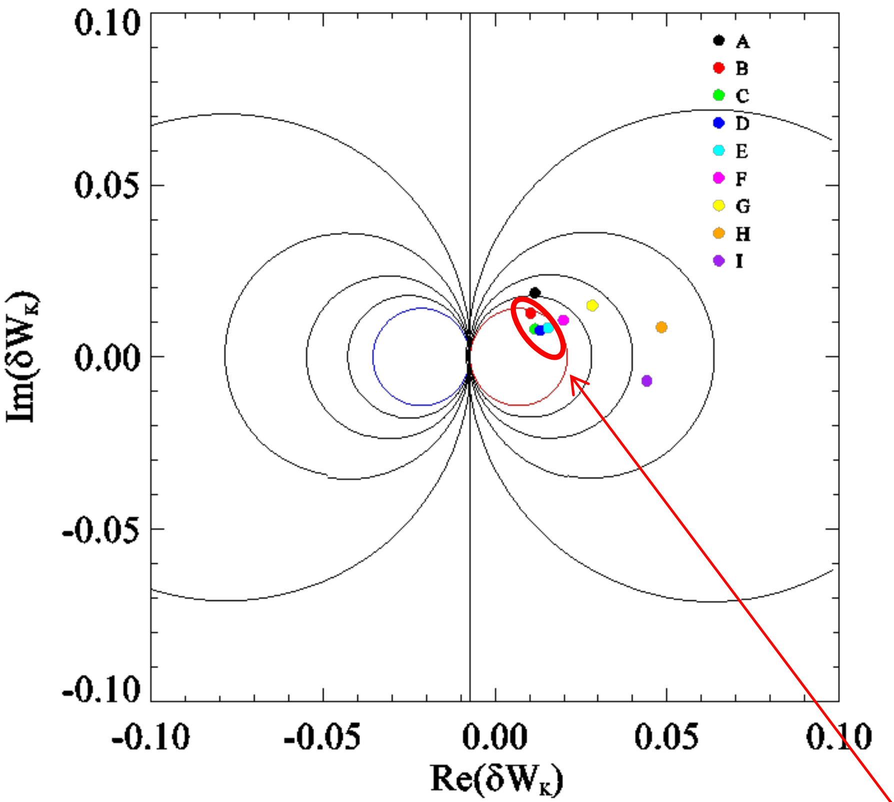


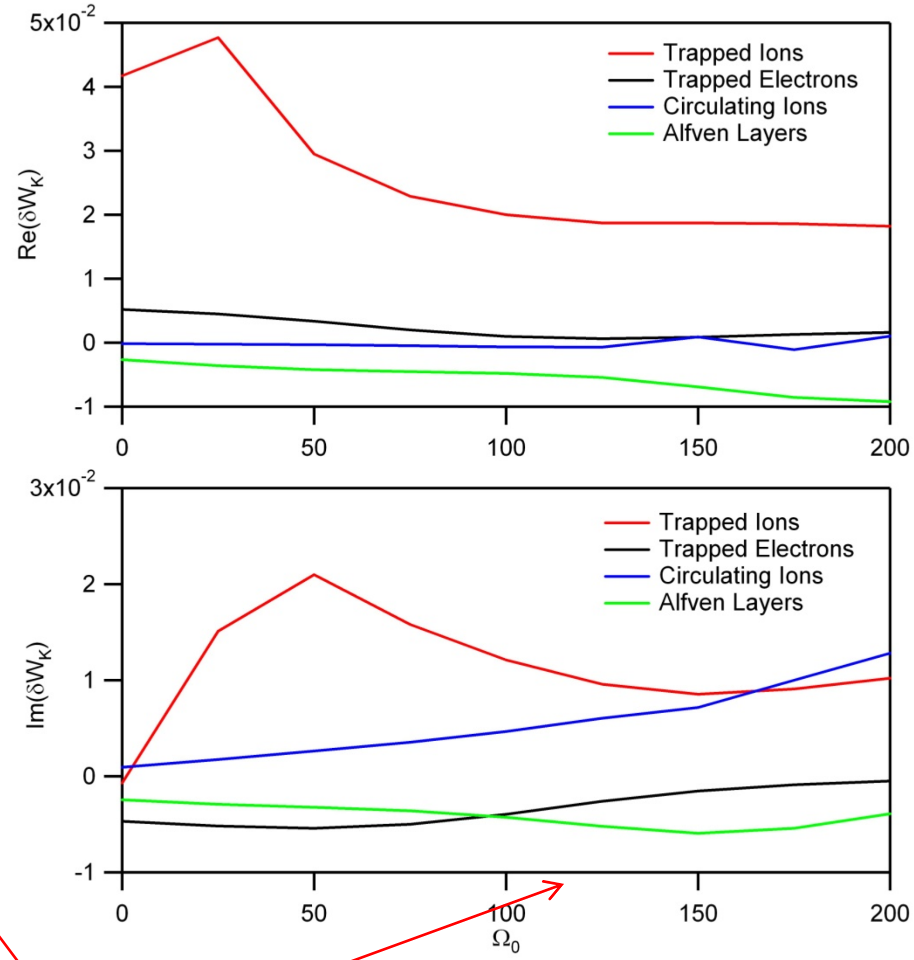


121083

stable – high trapped ion contribution

Which components lead to stability/instability?

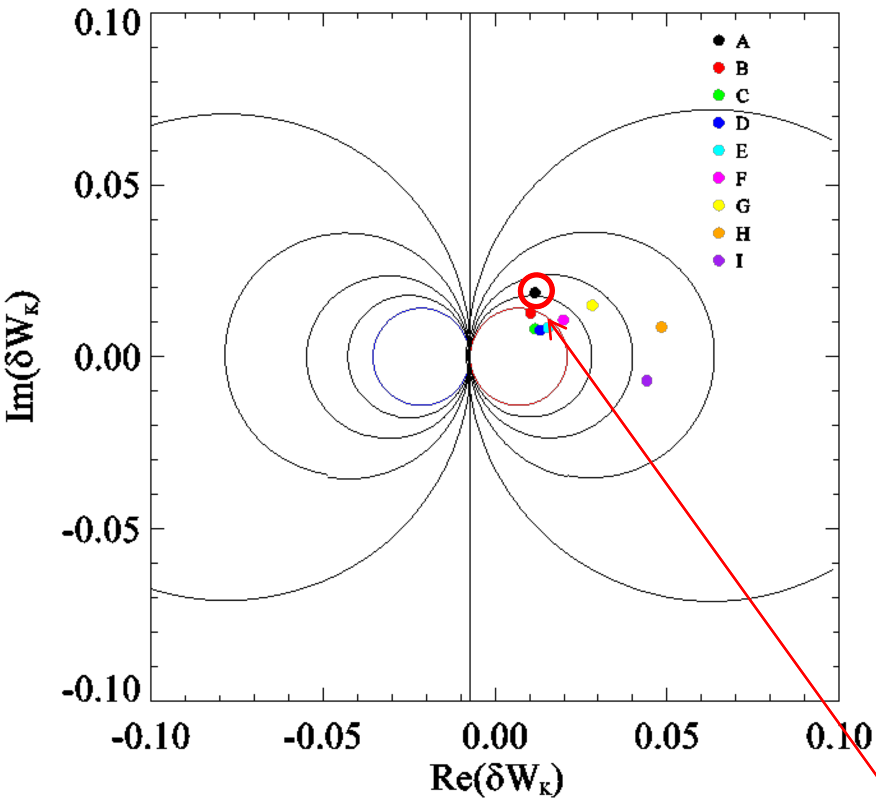


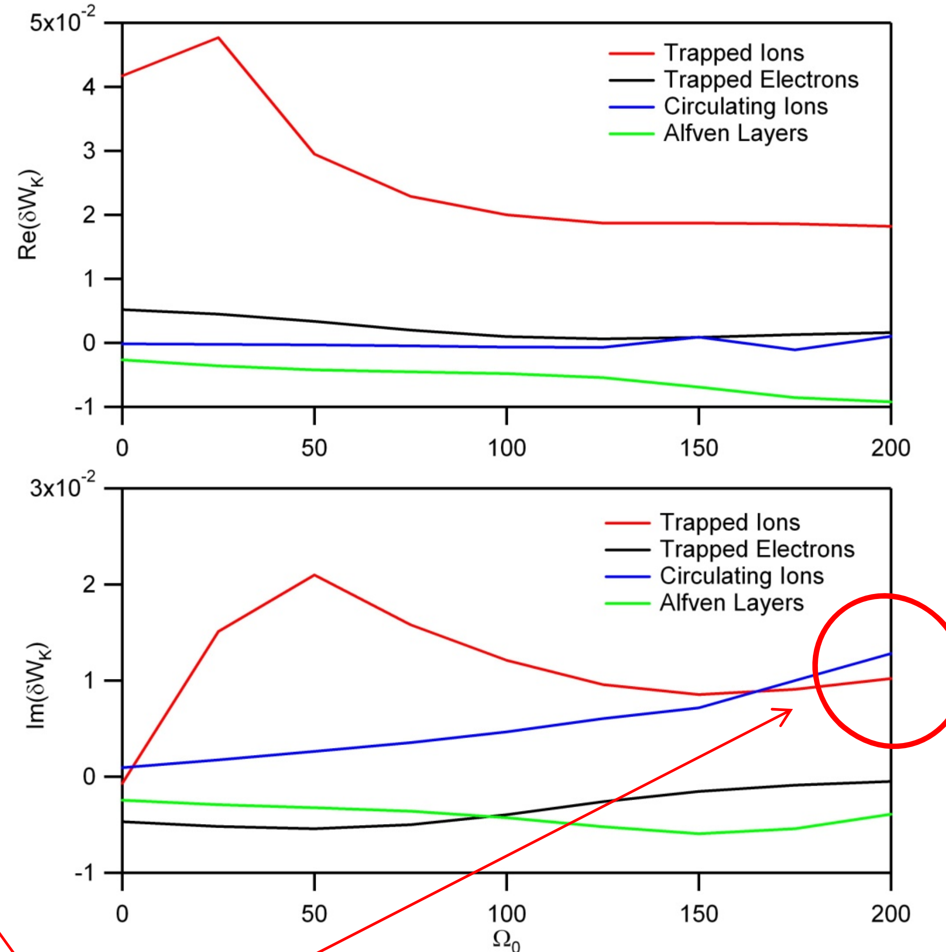


121083

unstable

Which components lead to stability/instability?

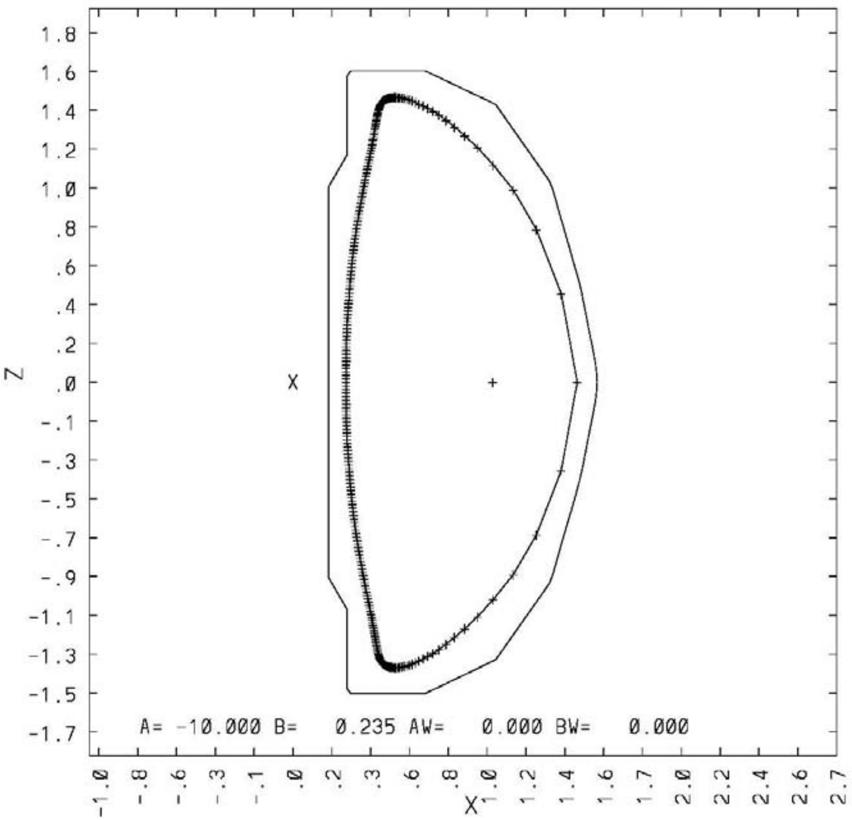




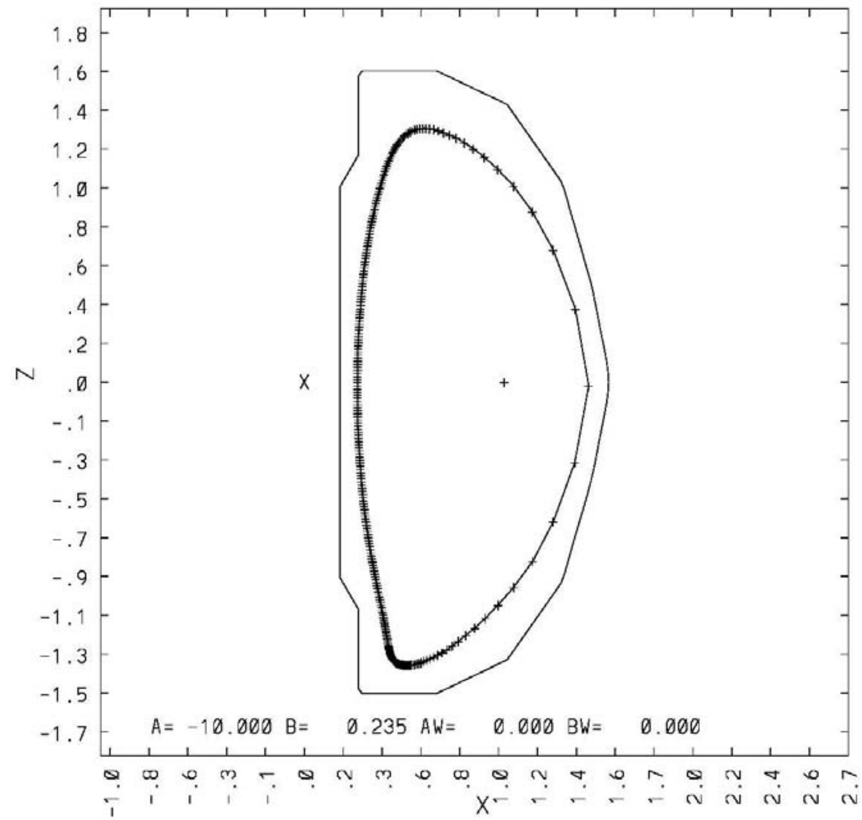
121083

stable – high circulating ion contribution

PEST symmetry setting makes a difference

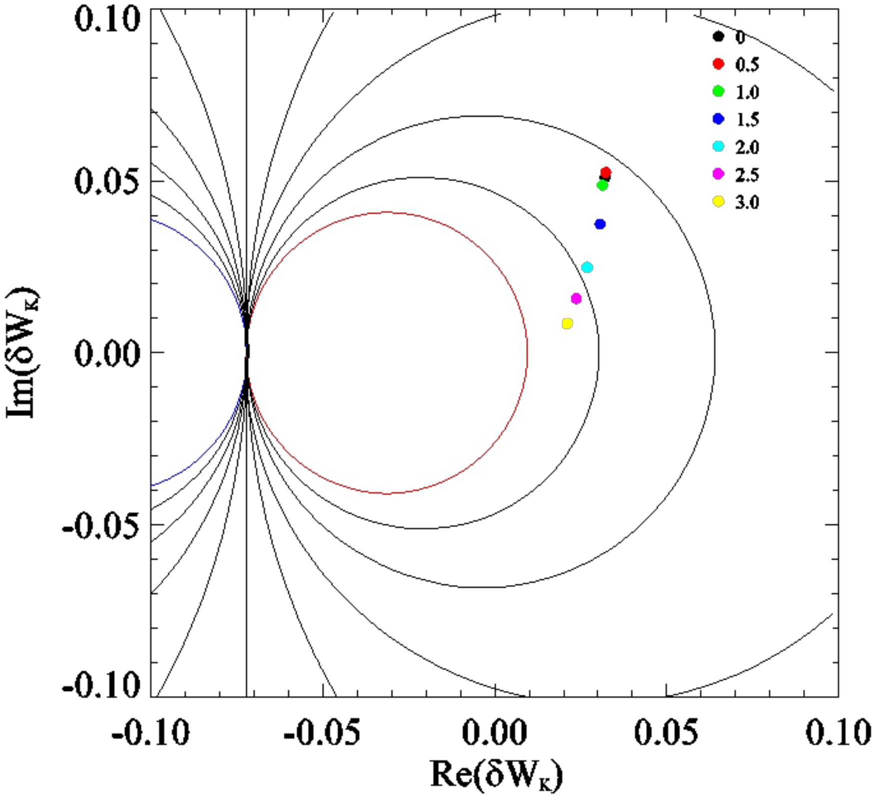


Symmetric

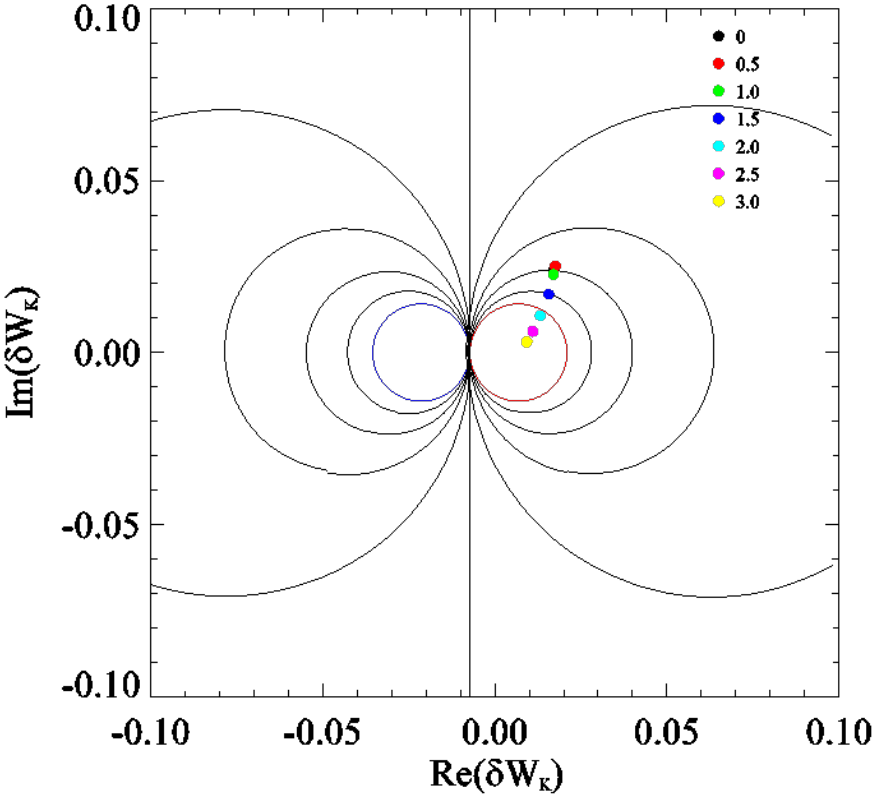


Asymmetric

PEST symmetry setting makes a difference

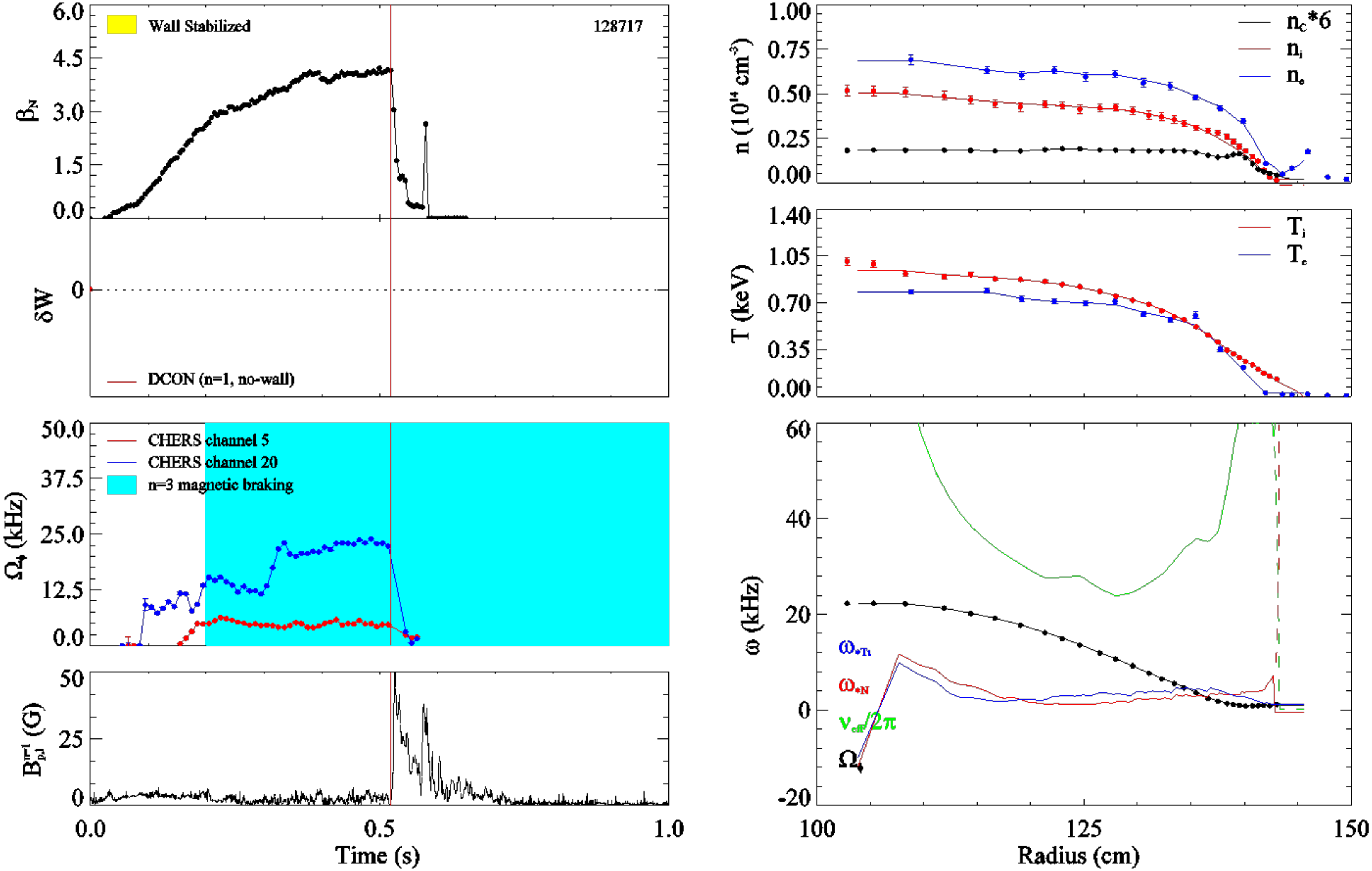


121083, Symmetric

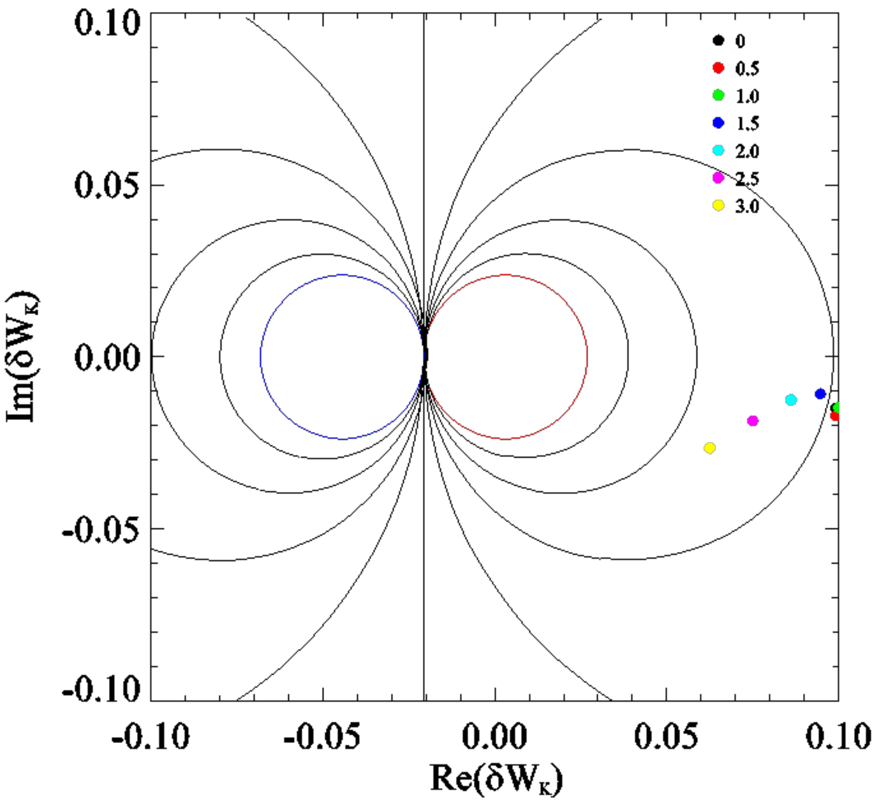


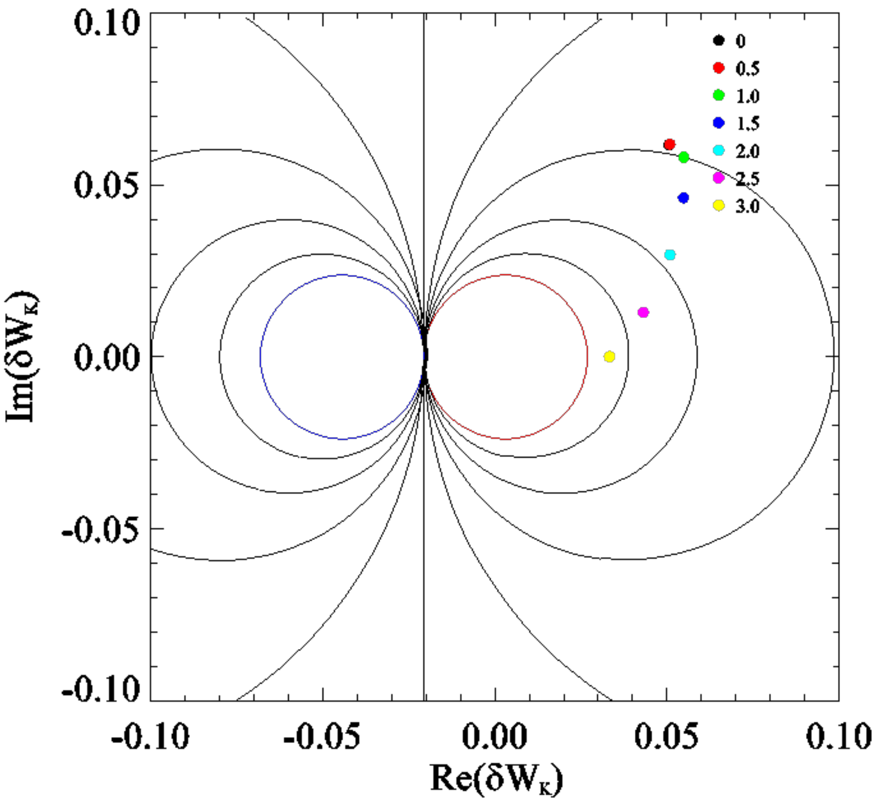
121083, Asymmetric

Experimental profile errors can make a big difference



Experimental profile errors can make a big difference



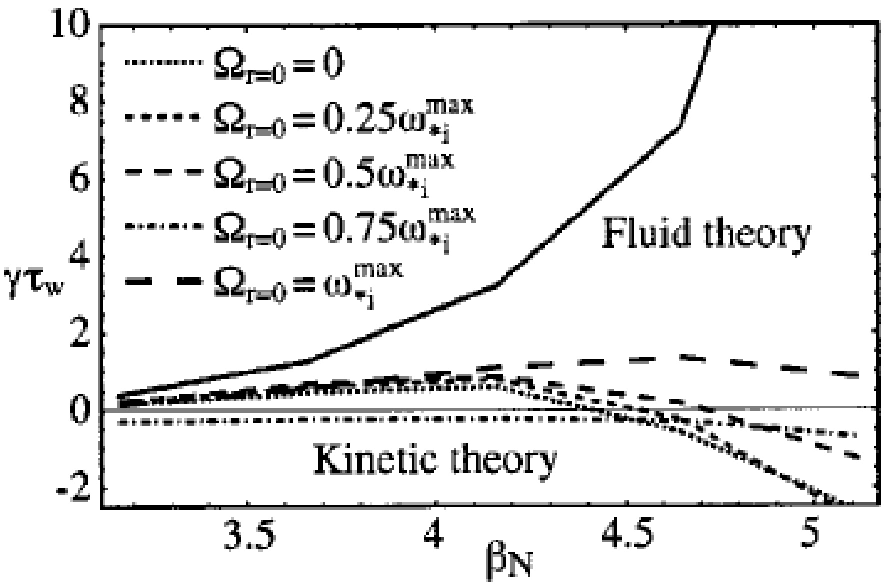


128717, With error

128717, Fixed

Previous ITER results also showed complex stabilization

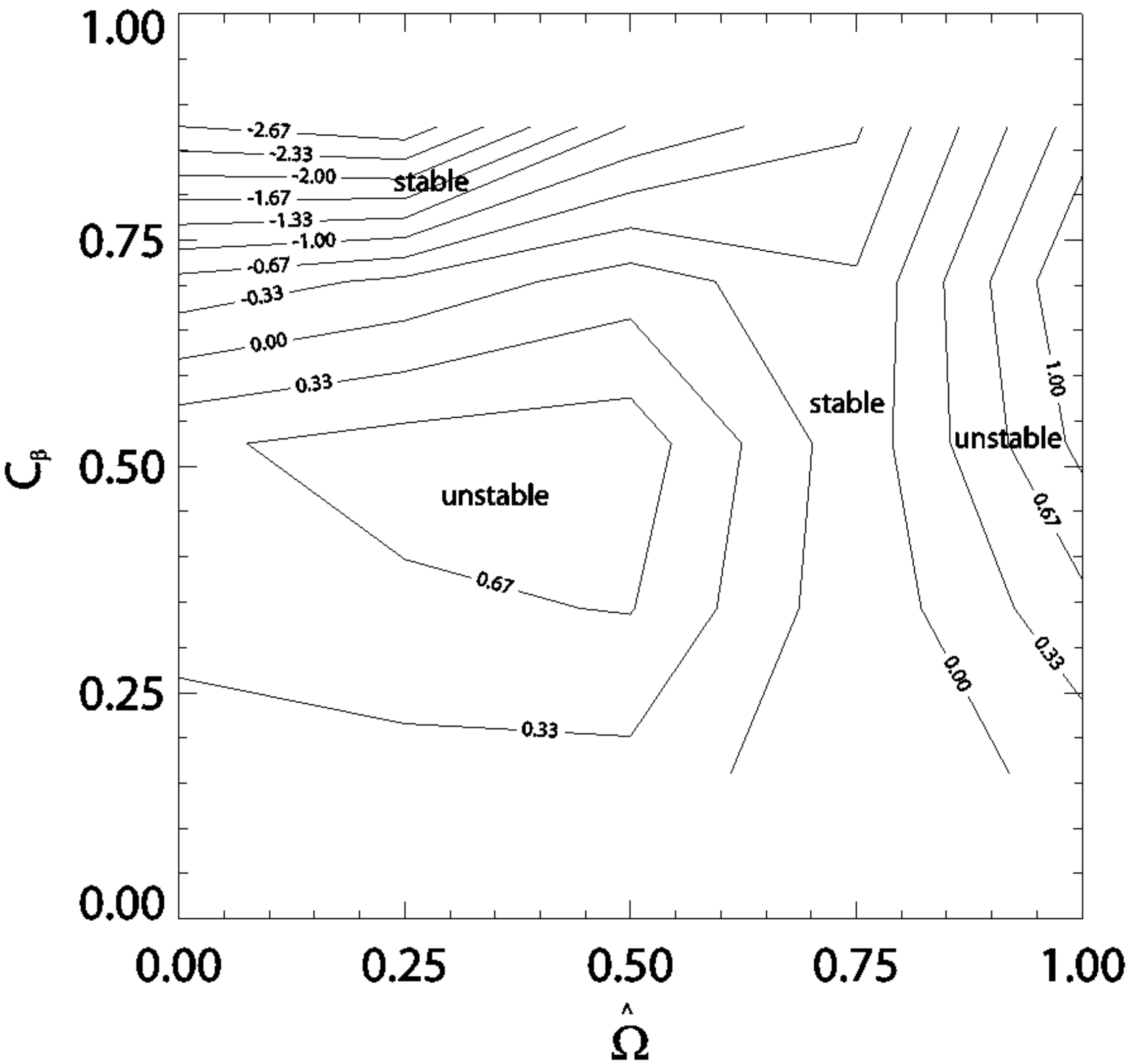
				δW_K for different $\Omega_{rot}(r=0)/\omega_{*i}^{max}$ values				
β_N	Δ^*/a	δW_{mhd}^{∞}	δW_{mhd}^{b}	0.0	0.25	0.50	0.75	1.0
3.16	0.500	−0.92	2.27	$0.52+0.20i$	$0.34+0.22i$	$0.26+0.39i$	$2.19+0.38i$	$0.51-0.25i$
3.66	0.360	−1.60	1.26	$0.61+0.36i$	$0.47+0.28i$	$0.35+0.38i$	$2.32+0.73i$	$0.54-0.16i$
4.16	0.297	−1.98	0.61	$0.51+0.74i$	$0.55+0.57i$	$0.60+0.47i$	$2.15+1.19i$	$0.62-0.09i$
4.65	0.272	−2.19	0.30	$-0.21+0.71i$	$0.04+1.01i$	$0.62+1.05i$	$1.77+1.73i$	$0.77-0.03i$
5.12	0.258	−2.31	0.11	$-0.20+0.35i$	$-0.34+0.49i$	$-0.23+0.98i$	$0.55+2.25i$	$1.01+0.46i$



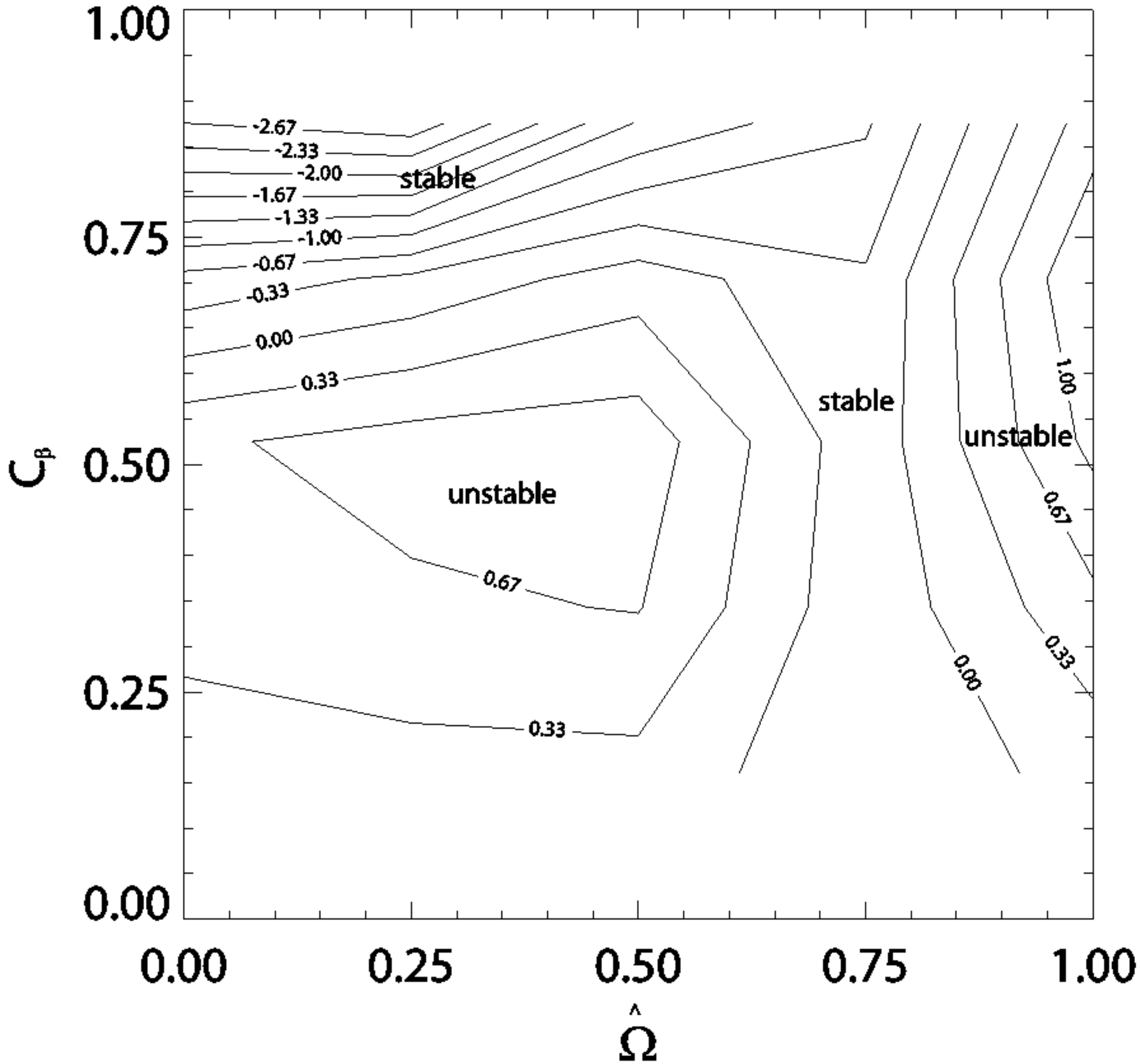
Caveat:
calculated with only trapped ion effect included.

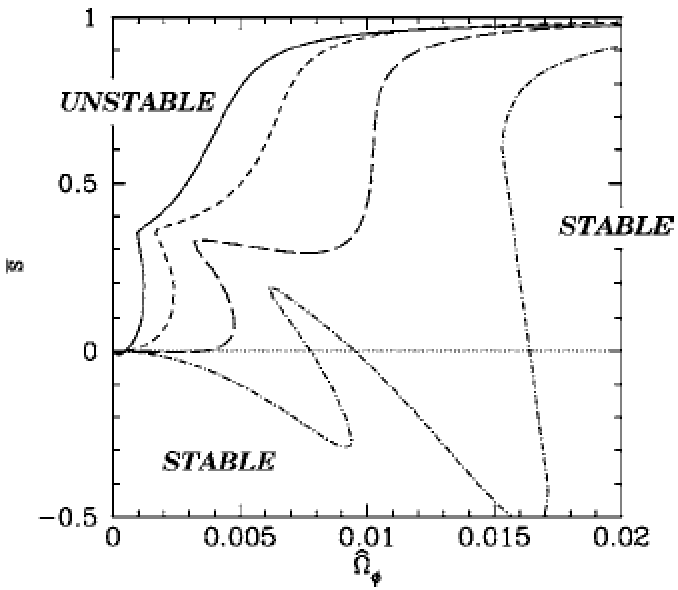
(Hu, Betti, and Manickam, PoP, 2005)

Ranges of stability – more complex than simple model



Ranges of stability – more complex than simple model





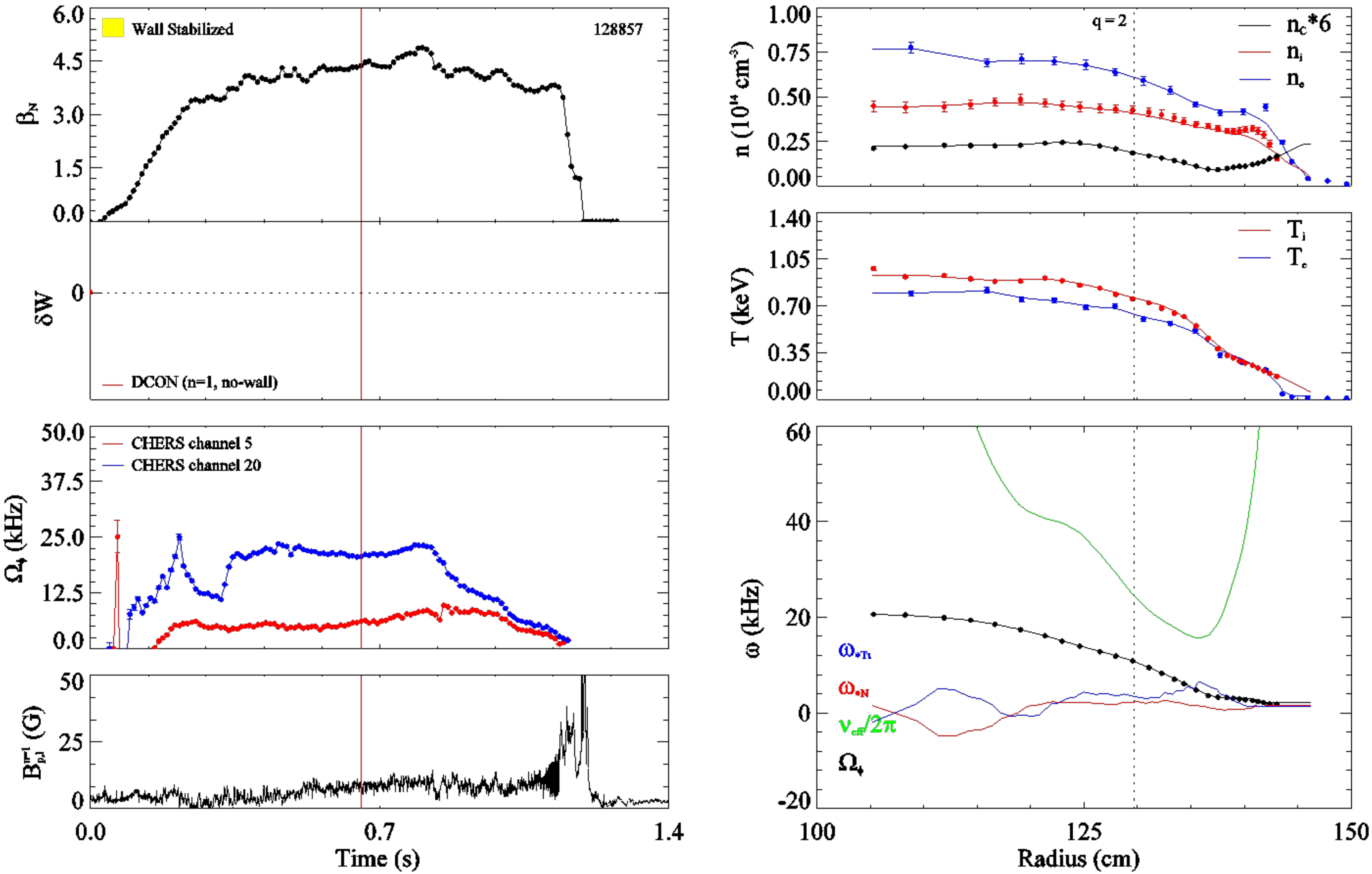
"simple model"

(Fitzpatrick and Bialek, PoP, 2006)

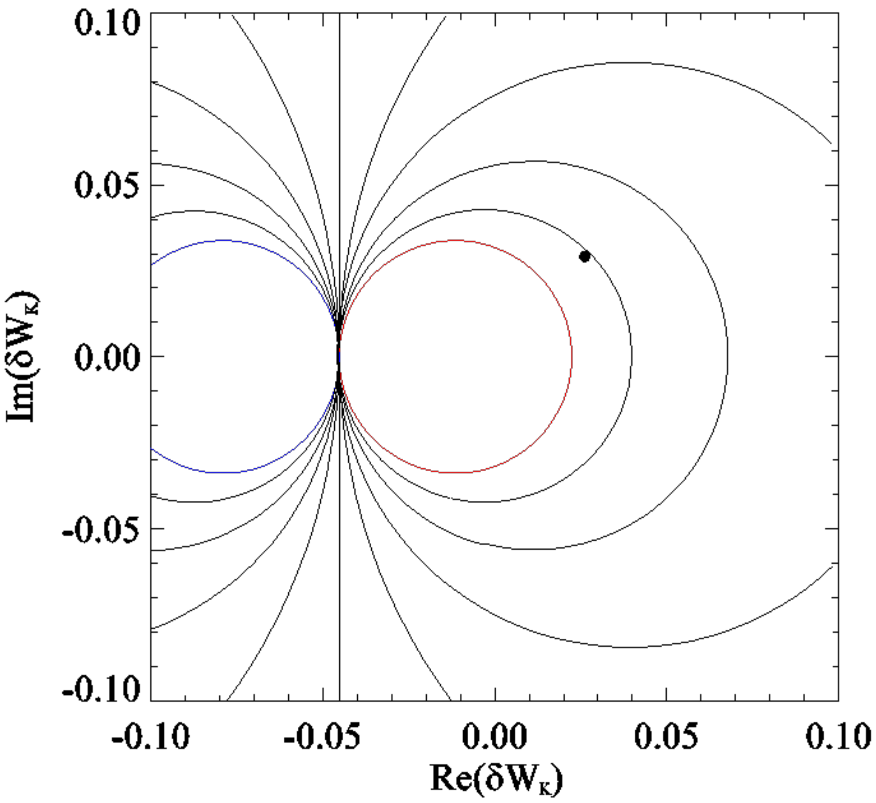
Changes to the code

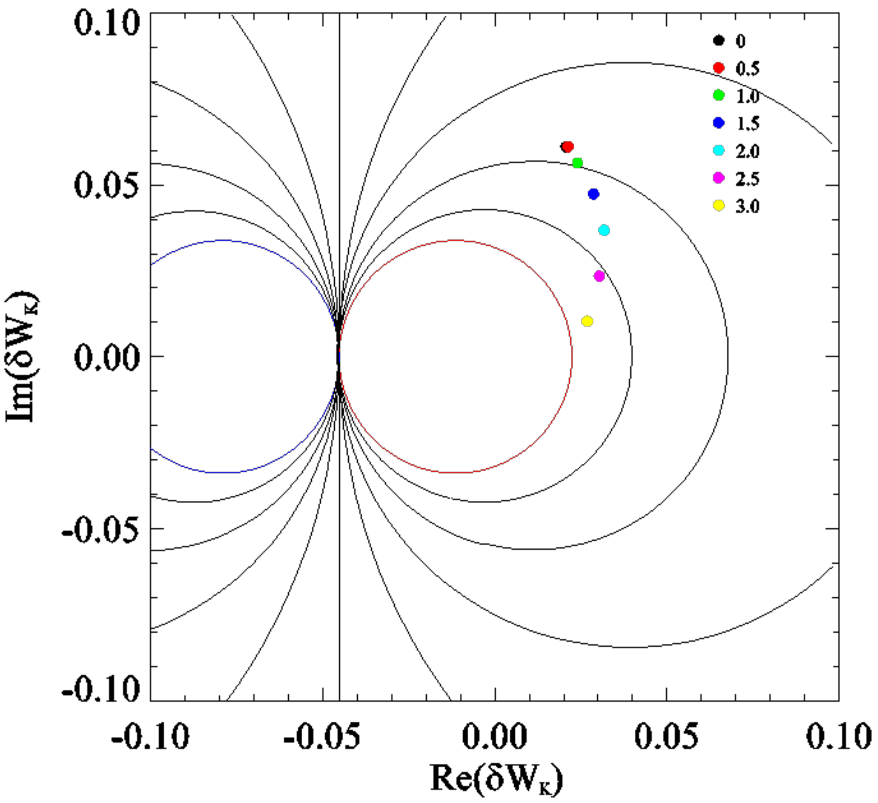
- Changes to coding
  - Changed from IMSL library integration to NAG library integration so it can run on PPPL computers.
  - Removed all hard-wired numbers - put in input files.
  - Automatic removal of rational surfaces and calculation of Alfven layer contributions.
  - Surface magnetic field doesn't have to be monotonic from Bmin to Bmax.
  - etc…
- Changes to physics
  - $\ln(\Lambda) \neq$ constant
  - $Z_{eff} \neq 1$
  - No small-aspect ratio assumption for $\kappa \cdot \xi$.
  - Automatically finds smallest mode frequency ω that doesn't give integration error.

What about a shot that doesn't go unstable?



What about a shot that doesn't go unstable?

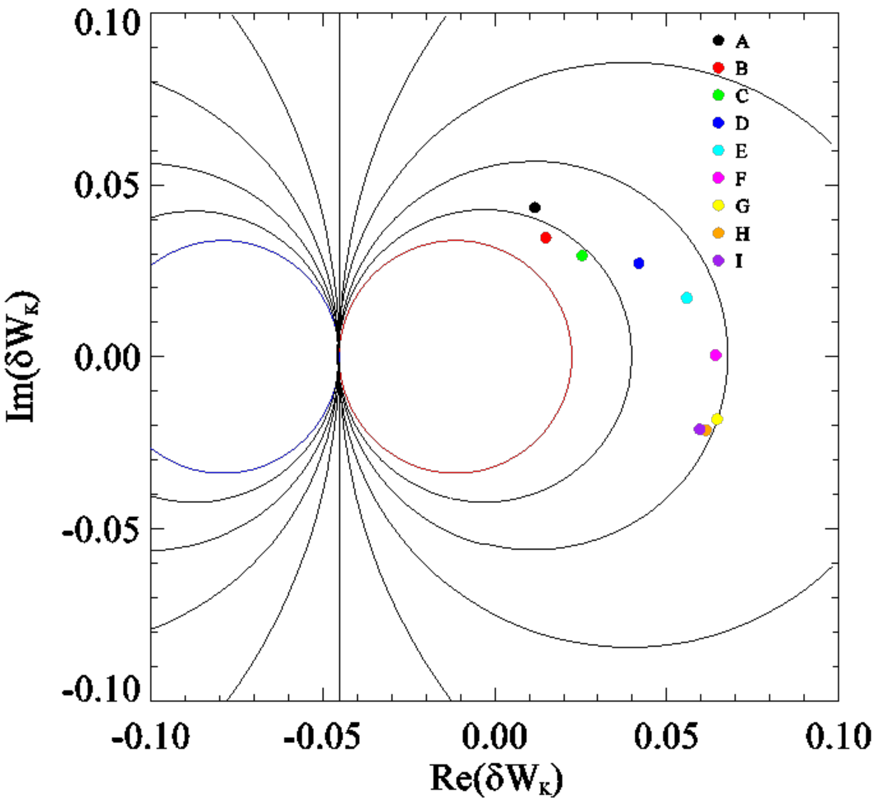




128857, Actual

128857, Collisionality

What about a shot that doesn't go unstable?



This actually comes closer to instability than the case of 128863, which was observed to go unstable, so certainly the code is not perfect.

128857, Rotation

Sontag's results

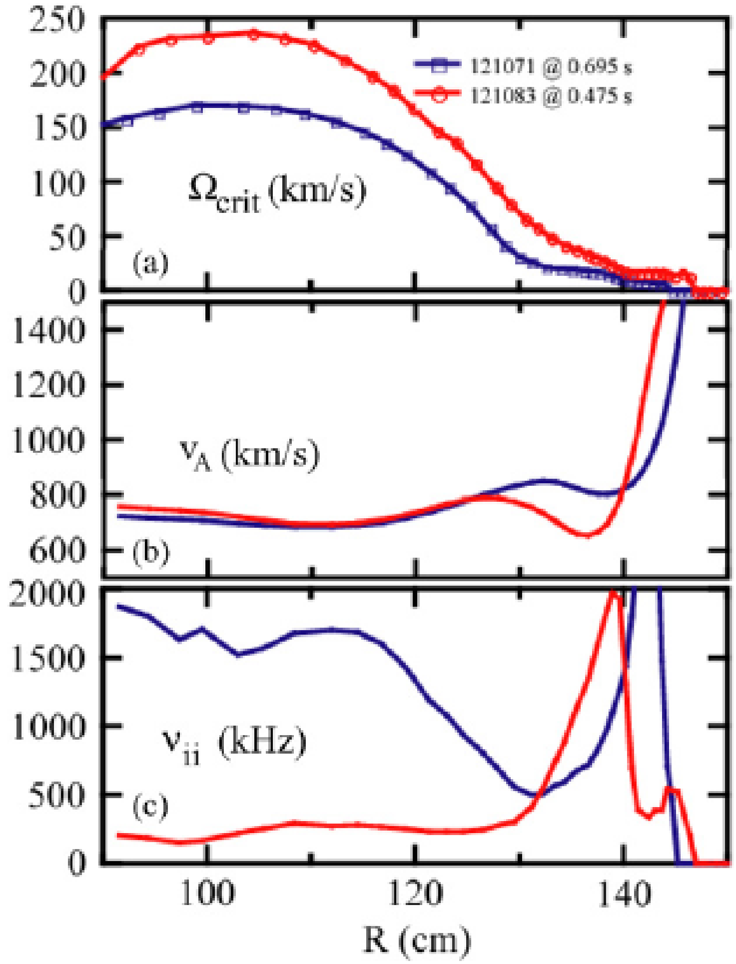


Figure 7. Ω_{crit} variation with ion collisionality for similar Alfven frequency profiles. Increased collisionality correlates with lower Ω_{crit}.

- High collisionality correlated with lower rotation at collapse.
  - Simple model interpretation: collisionality increases stability, leading to a lower "critical" rotation.
  - Possible kinetic model interpretation: collisionality decreases stability, meaning the lower rotation plasma, which would otherwise be stable, is now unstable.
  - More analysis is required.

(Sontag et al, NF, 2007)