Macroscopic Stability Research on NSTX - Update

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3rd Meeting of the ITPA MHD Stability Topical Group

October 6 - 9, 2009

Culham, UK

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v1.4

NSTX Macrostability Research in 2009 Addresses Topics of ITER Interest

- Goal
  - Understand plasma instabilities that limit steady operation of high performance plasmas and demonstrate/understand their control
- Status
  - 2009 experimental campaign completed (mid-Aug.)
  - 2009 results review collected data/initial analysis (mid-Sept.)
- Topics covered in this talk
  - Error fields (n = 1 and n = 3) (MDC-2)
  - Resistive wall modes (MDC-2)
  - Non-resonant magnetic braking physics (MDC-12)
  - NTM threshold physics; M3D-C^1 analysis (MDC-14)
  - Improving $\langle\beta_N\rangle_{pulse}$ with n = 1 and β_N feedback (MDC-2, MDC-17)

NSTX Macrostability Research – Topics Covered in Talk

- Error fields (n = 1 and n = 3) (MDC-2)
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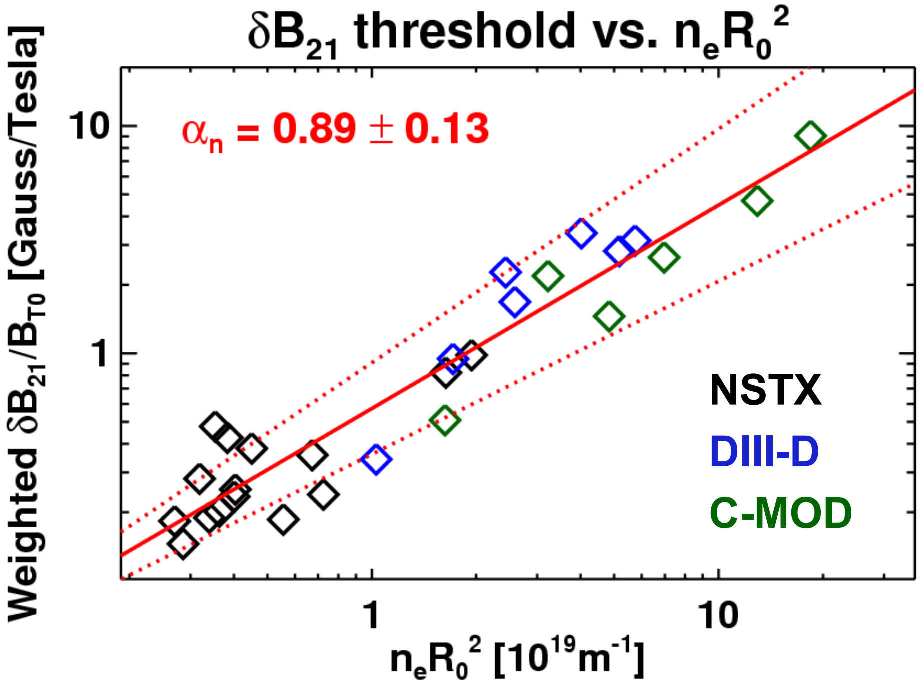
Mode locking due to n = 1 error fields investigated in plasmas with increased β

Low-β locked mode threshold scaling vs. density

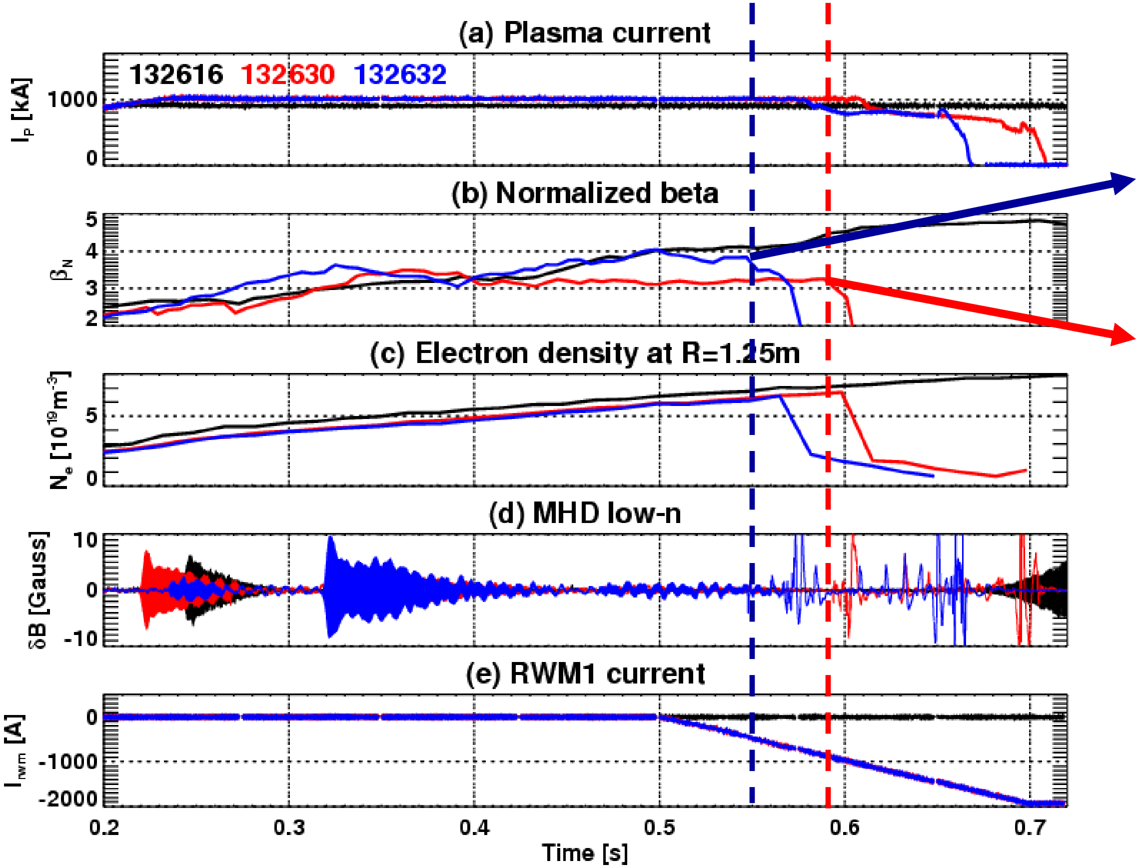


- ☐ Non-axisymmetric error field (n=1) can lock the plasma
- ☐ Well-known linear dependence of error field threshold for mode locking with plasma density in low-β plasmas

- ☐ XP903 Goals
  - ☐ investigate locking as plasma β is increased (but below n = 1 no-wall limit)
  - ☐ Examine the dependence on applied n = 1 error field as well as amplification due to the plasma

XP903: J.-K. Park

n = 1 Error field threshold for mode locking decreased as β_N increased



Higher-β locked earlier (w/ smaller RWM currents)

Lower-β locked later (w/ larger RWM currents)

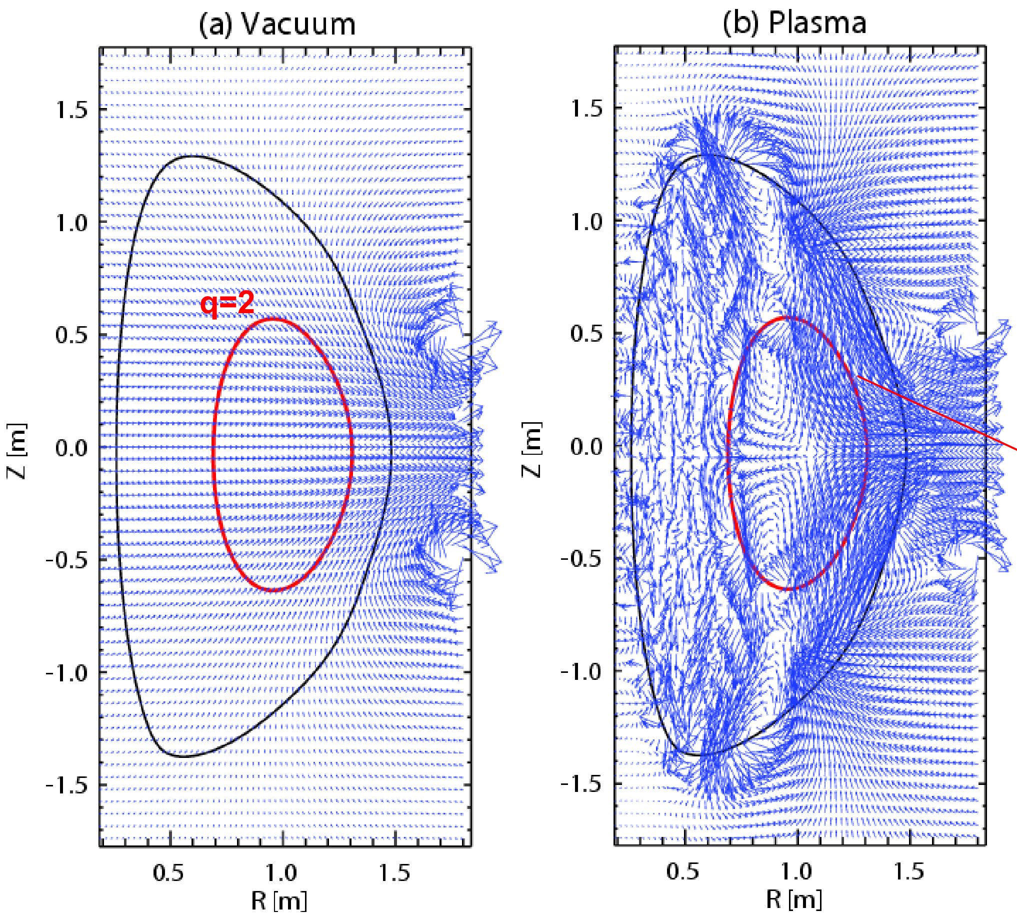
- ❑ 11 shots locked by n = 1 applied field (<1kA/turn in RWM control coils)
- ❑ n = 1 rotating modes observed in some cases, but study limited to static modes

XP903: J.-K. Park

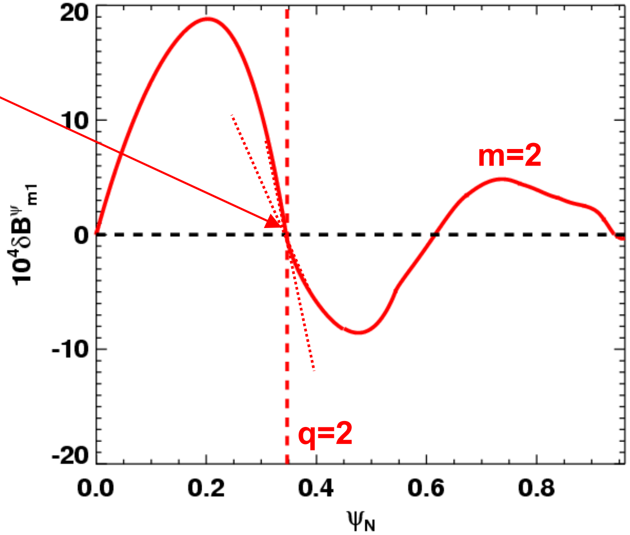
The IPEC code shows that resonant fields can be significantly amplified in higher-β plasmas

- ☐ Resonant field driving islands and its amplification increases with β_N
- ☐ Vacuum resonant fields underestimate threshold for mode locking at increased β

Perturbed field for #132633.00608



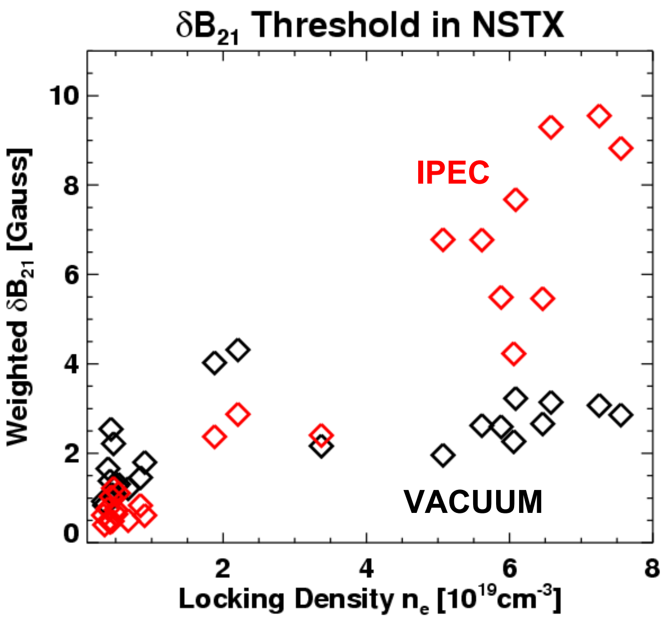
n=1 components of normal field



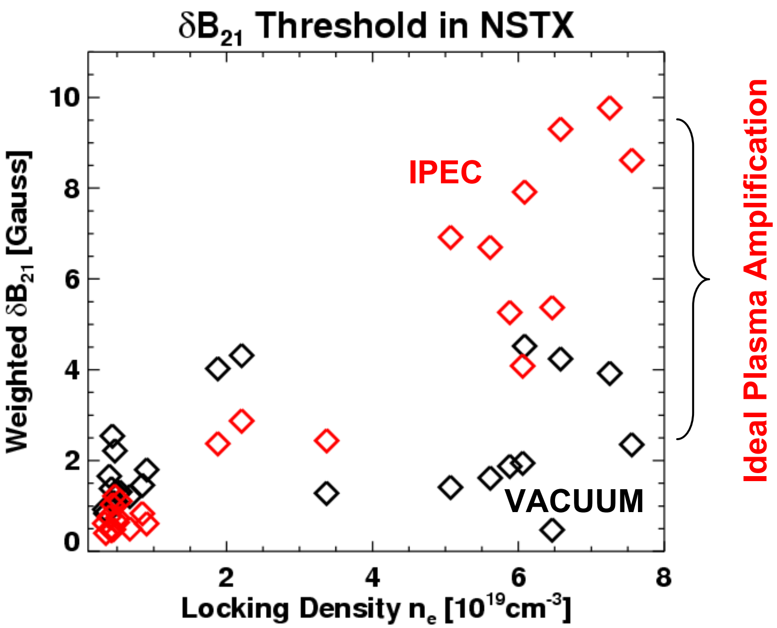
IPEC: J.-K. Park

Ideal plasma amplification of applied resonant field restores linear correlation of mode locking threshold with density

Without intrinsic error field for high-β



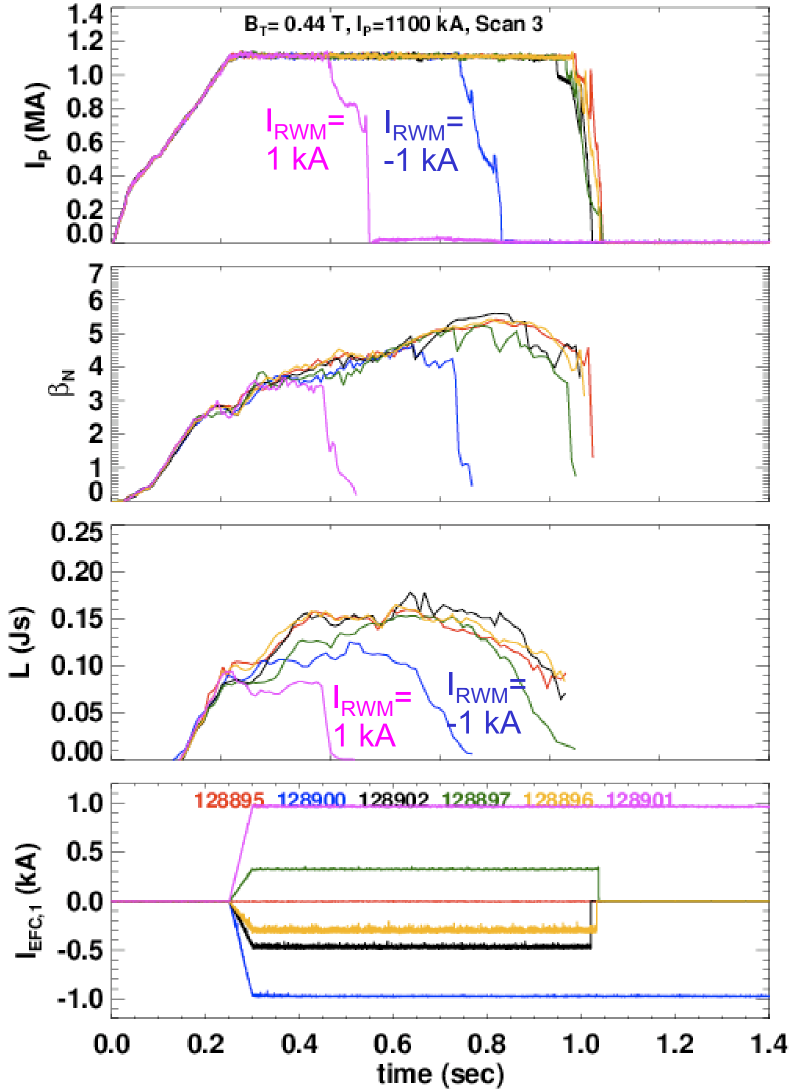
With intrinsic error field for high-β

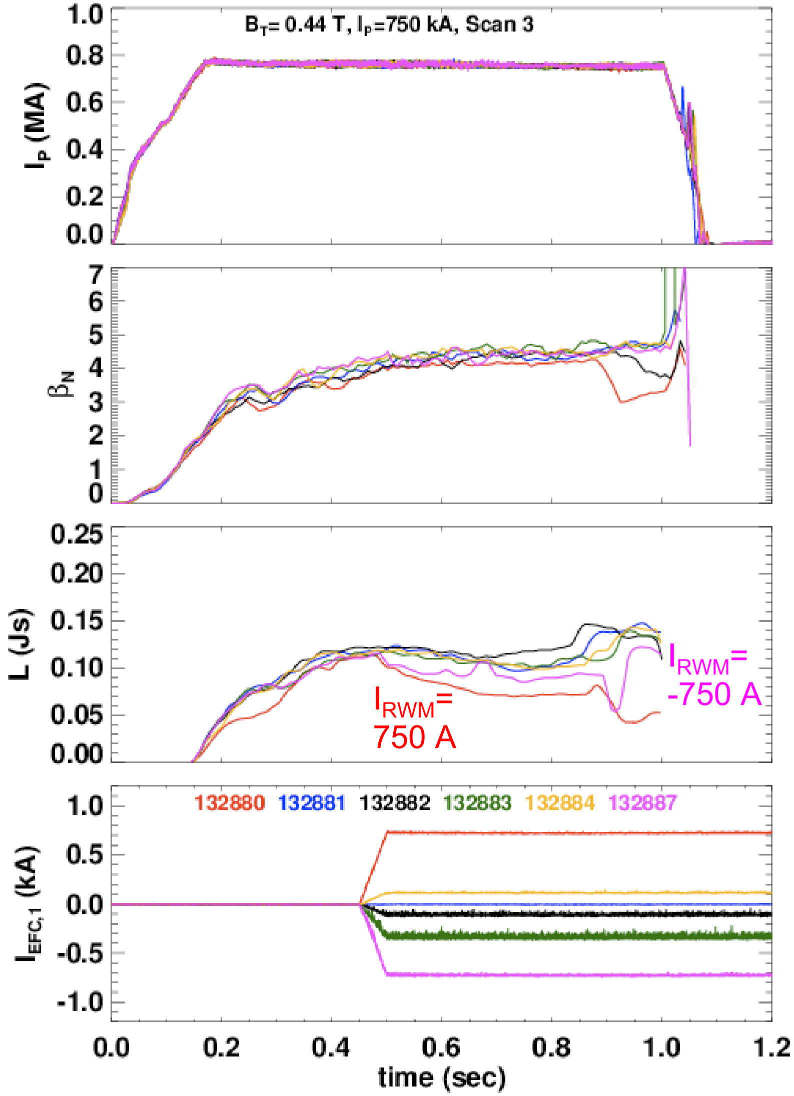


- ☐ IPEC resonant field joins linear density correlation from low-β to increased-β plasmas
- ☐ Intrinsic error field effects are weak due to large shielding of the unfavorable field spectrum

XP903: J.-K. Park

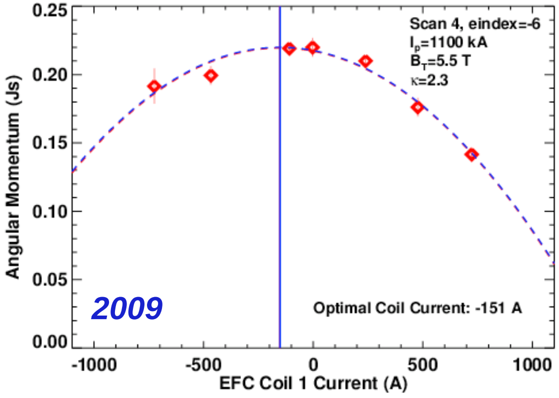
n=3 Error Field Inferred From Asymmetric Response of Plasma Rotation and Sustainment to n=3 Fields

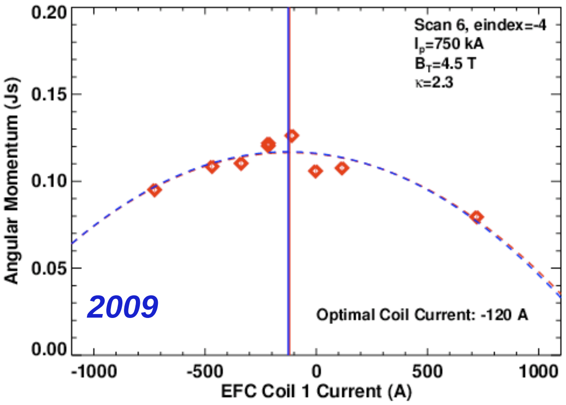


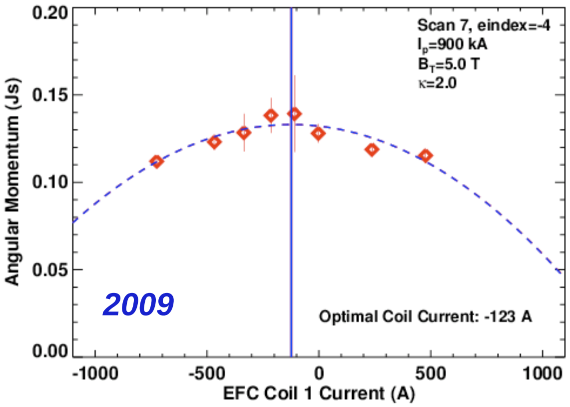


XP902: S. Gerhardt

Optimal n=3 Error Field Correction Determined vs. I_P, B_T







- "optimal" n = 3 error field correction attained by maximizing angular momentum, scanning I_p, B_t, elongation
- n = 3 error field consistent with known equilibrium field coil distortion
  - scales with equilibrium field coil current
  - field phase and amplitude of correction is consistent with that expected from coil distortion
- n = 3 error field correction routinely used to maximize plasma performance; used in conjunction with n = 1 RWM feedback control

XP902: S. Gerhardt

NSTX Macrostability Research – Topics Covered in Talk

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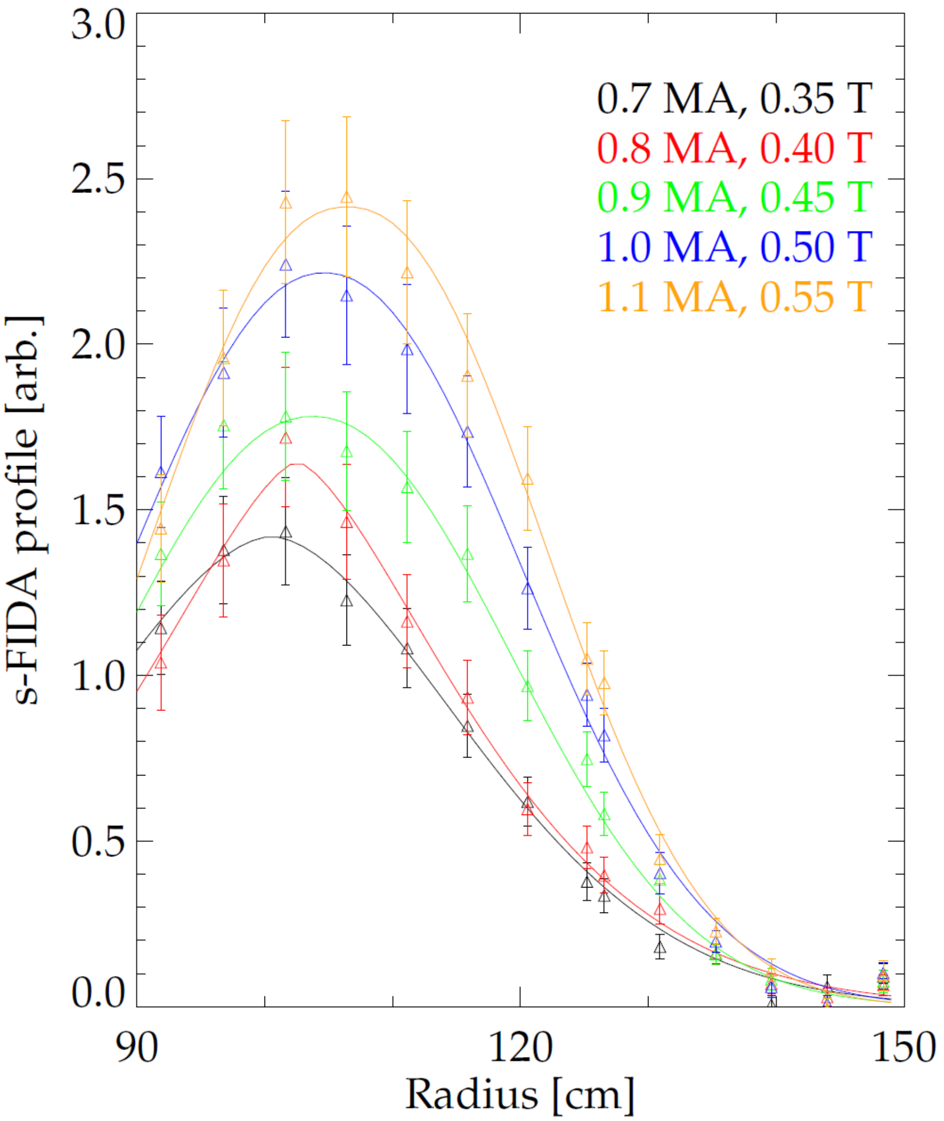
- ☐ Resistive wall modes (MDC-2)
- ☐ Non-resonant magnetic braking physics (MDC-12)
- ☐ NTM threshold physics; M3D-C^1 analysis MDC-14)
- ☐ Improving $<\beta_N>_{pulse}$ with n = 1 and β_N feedback (MDC-2, MDC-17)

Experiment performed on NSTX in 2009 to test energetic particle stabilization of RWMs

- RWMs destabilized by reduction in plasma rotation by non-resonant n = 3 magnetic braking
- Current and field changed at constant q, to change p_a/p_{tot}
- Fast-ion D_α measurements confirm successful scan of energetic particle fraction
- Stability analysis underway using the MISK code

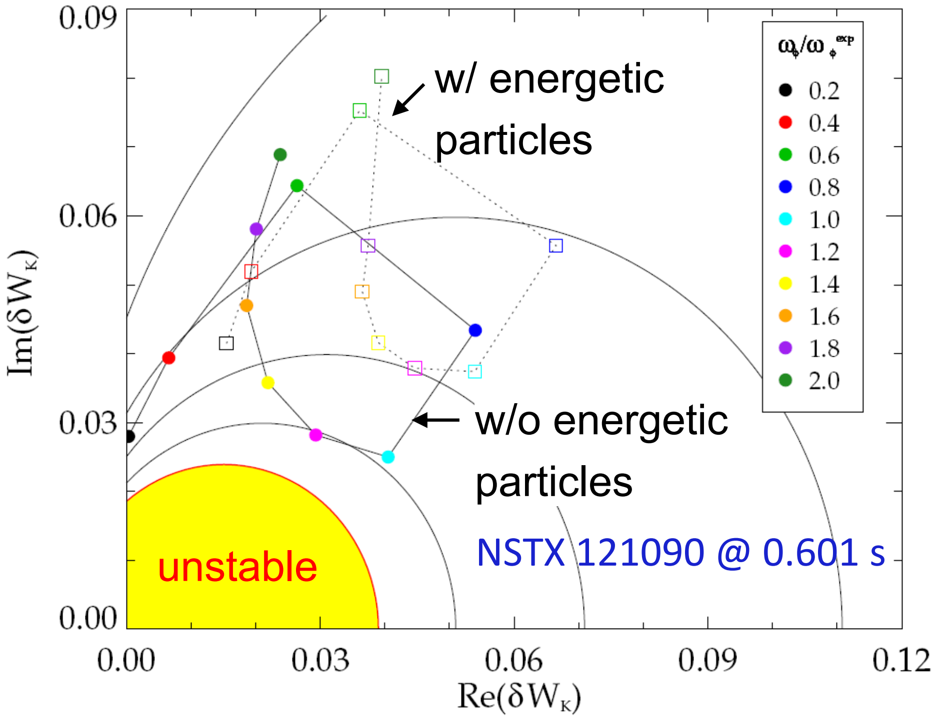


XP932: J.W. Berkery

MISK code used to evaluate RWM stability in NSTX including energetic particles from NBI

- NSTX RWM instability observed at intermediate plasma rotation
- Correlates with MISK code kinetic stability theory
  - Stabilization from precession drift resonance at low rotation and bounce resonance at high rotation
  - See backup slides for more detail and ITER calcs from last meeting
- Energetic particles are additionally stabilizing
  - Present model in MISK overestimates stability
  - Upgrading MISK to use realistic energetic particle distribution function for beam ions (vs. isotropic in pitch angle)

MISK Stability – plasma rotation scan

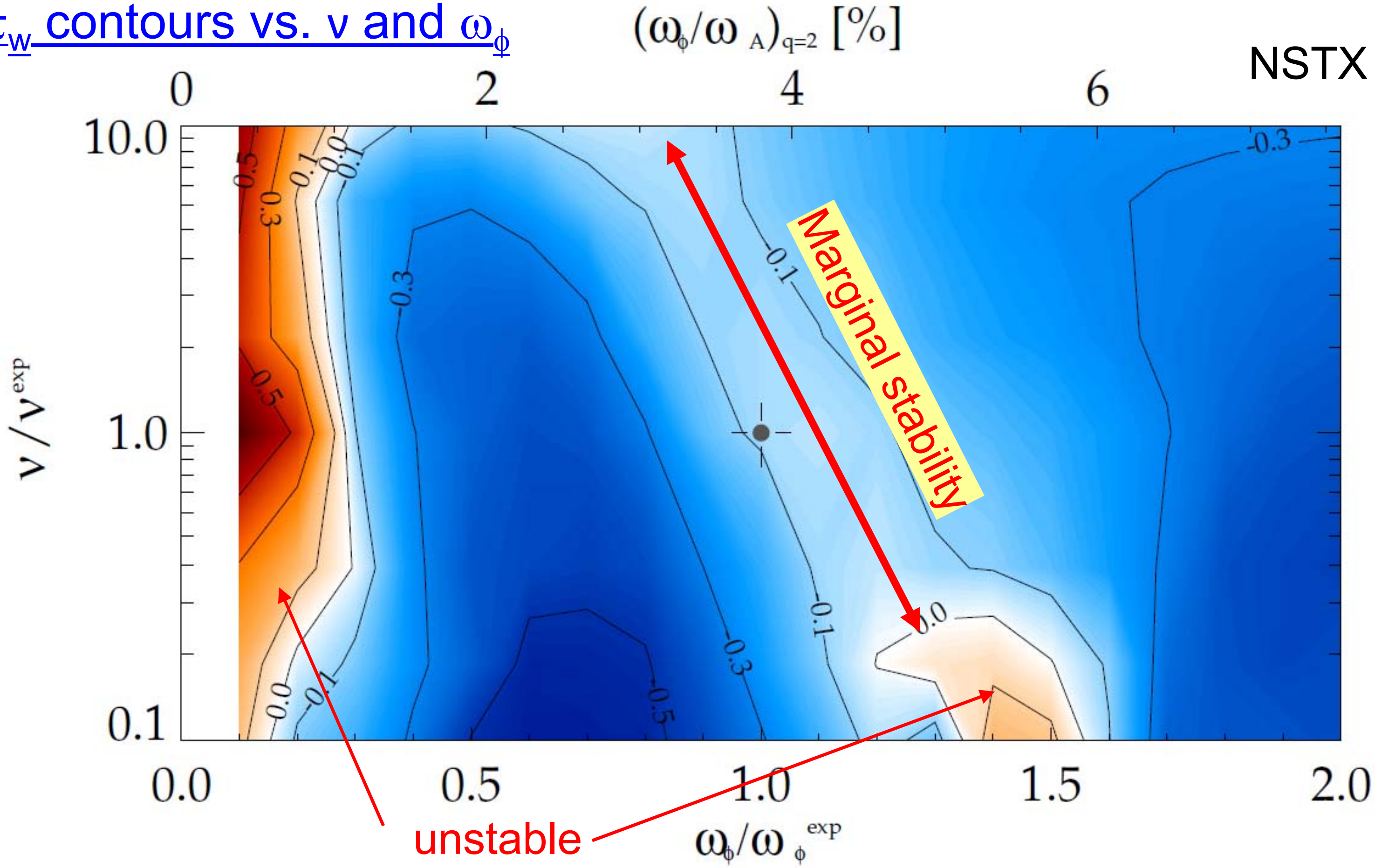


J.W. Berkery

Channel of "Weak Rotation" for RWM stabilization observed in MISK calculations for NSTX

$\gamma\tau_w$ contours vs. ν and ω_ϕ

NSTX 121083



Marginal stability

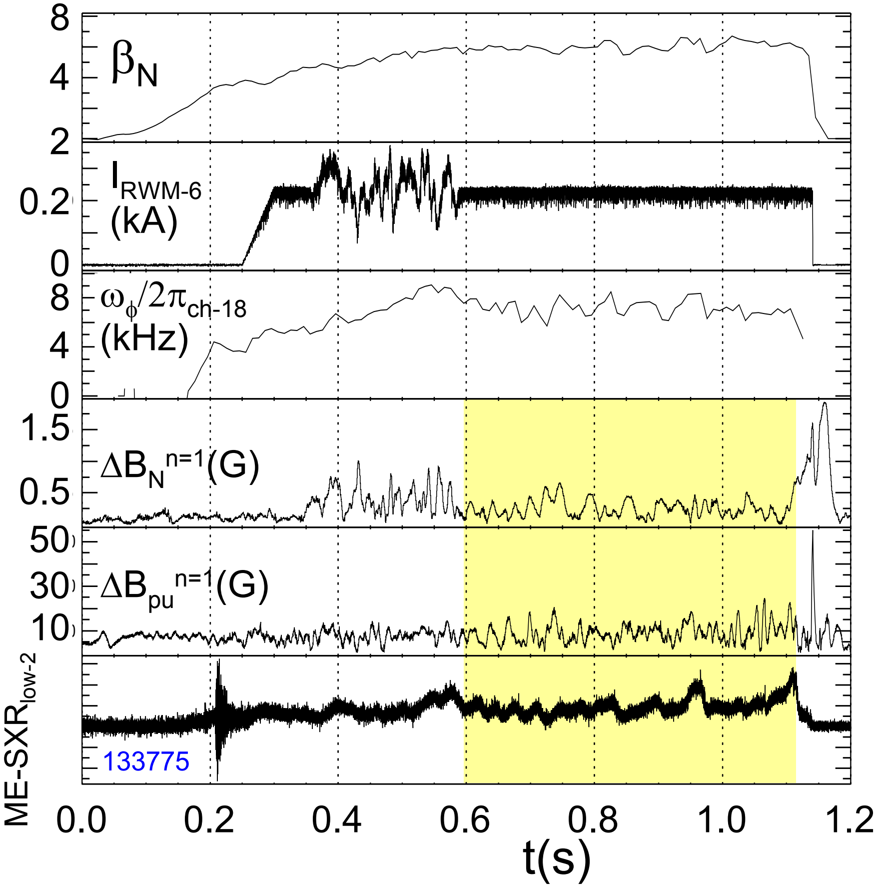
unstable

- Stabilization from precession drift resonance at low rotation and bounce resonance at high rotation
- Behavior of marginal channel dependent on profile details

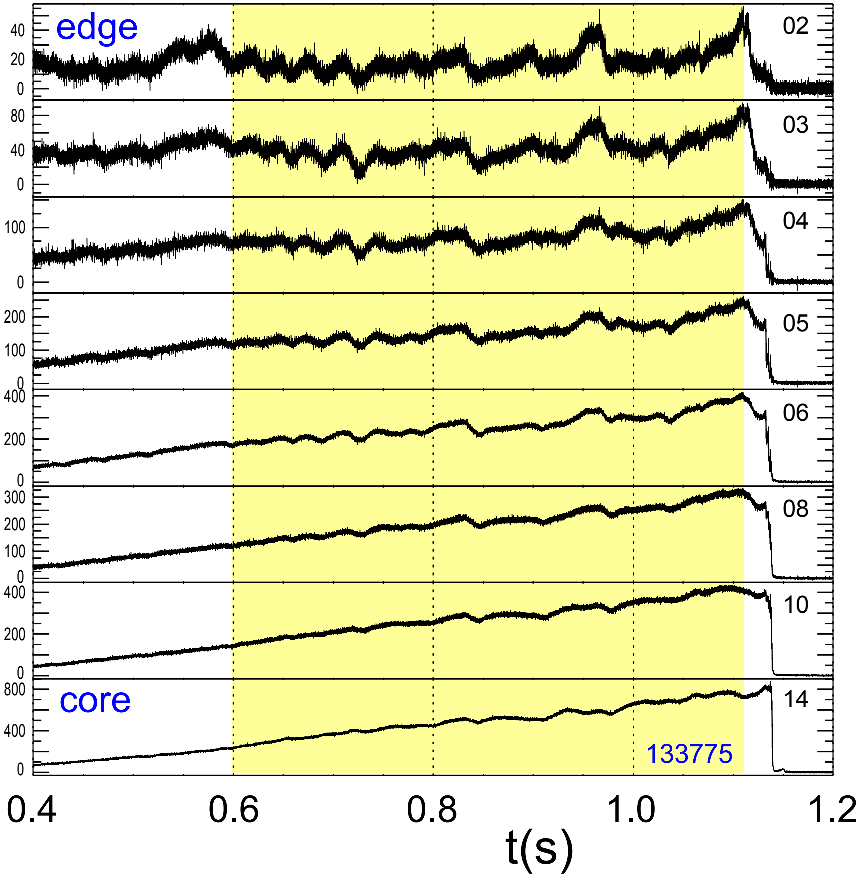
J.W. Berkery

High β_N shots exhibit low frequency activity in magnetic and kinetic diagnostics

I_p = 0.8 MA shot (programmed termination)



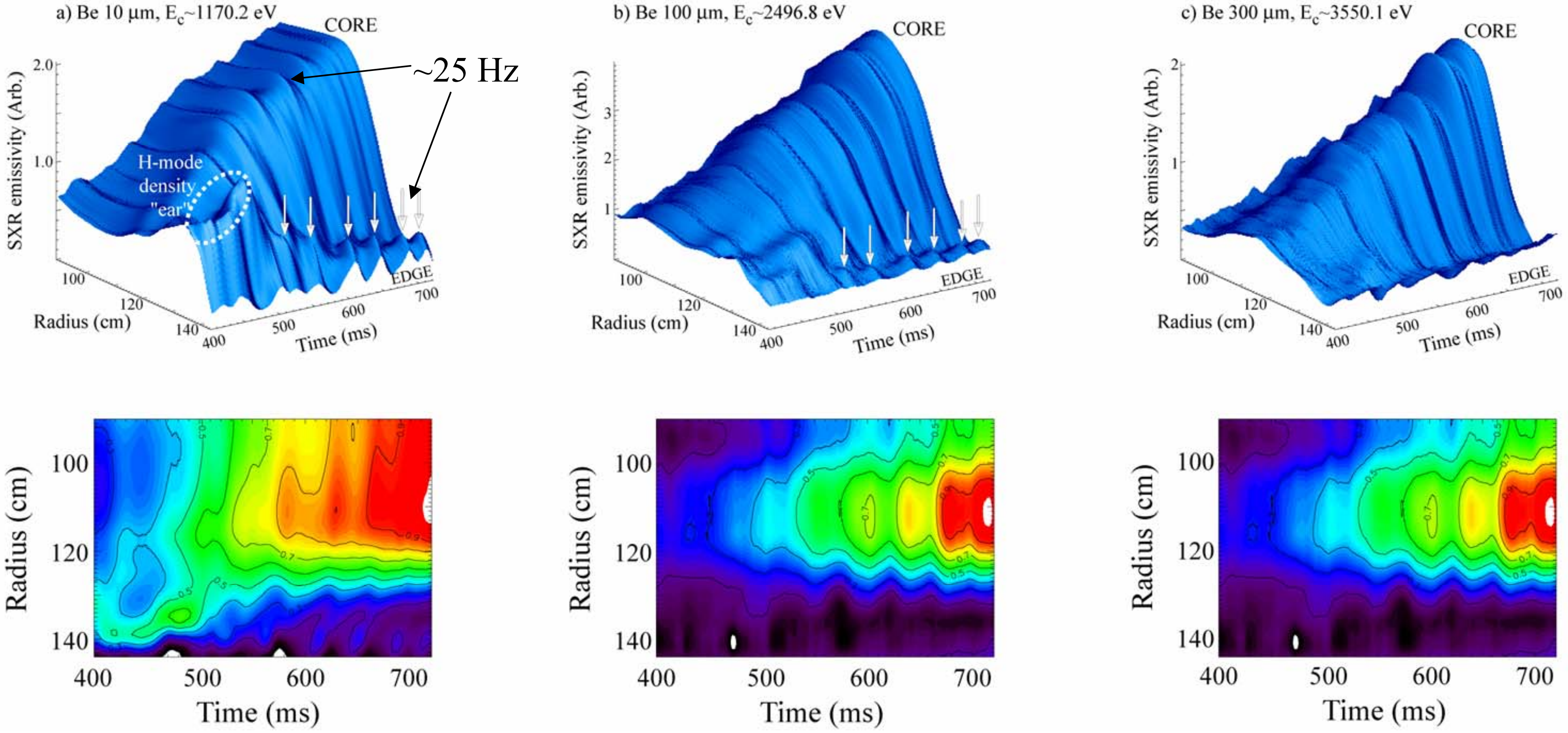
Multi-energy SXR data shows ~ 30 Hz mode activity



- Now examining characteristics of low frequency magnetic / kinetic fluctuations – explained by RWM theory?

XP935: S.A. Sabbagh

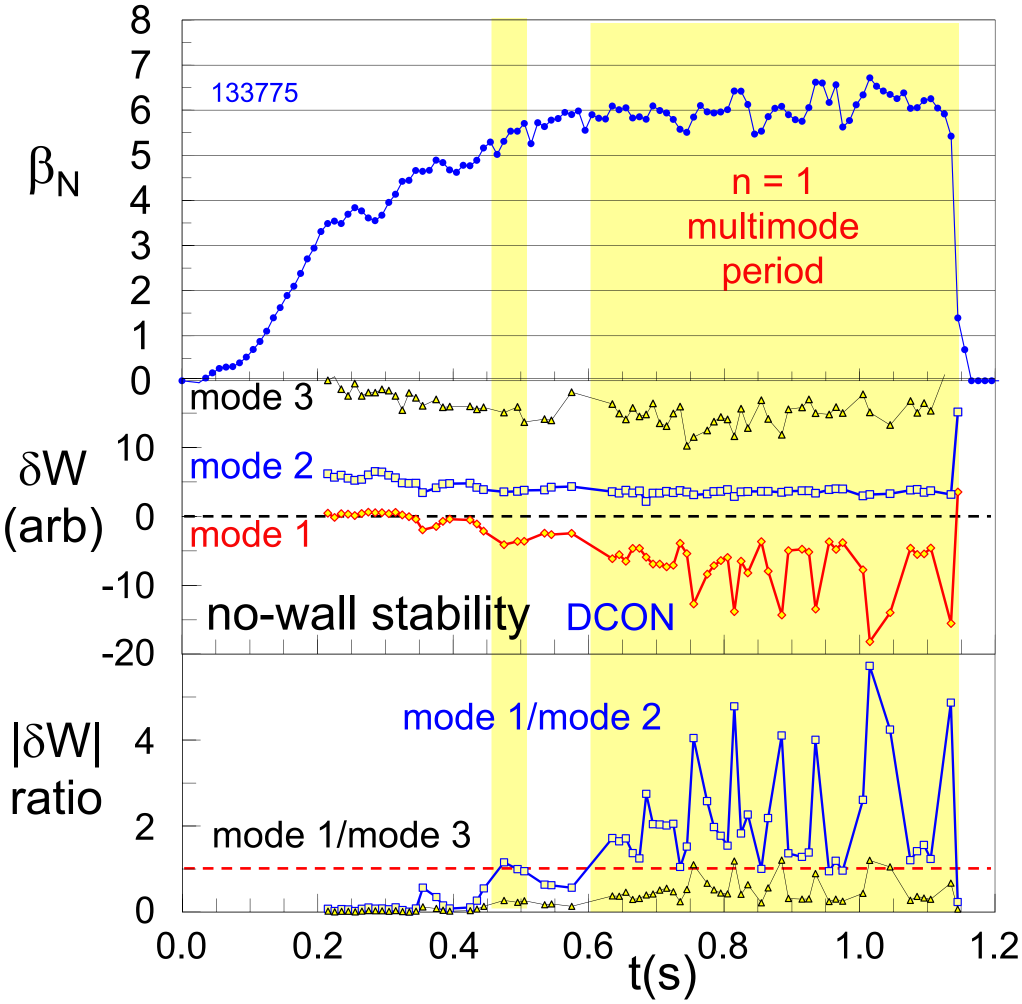
Multi-energy SXR reconstructions may indicate driven RWM

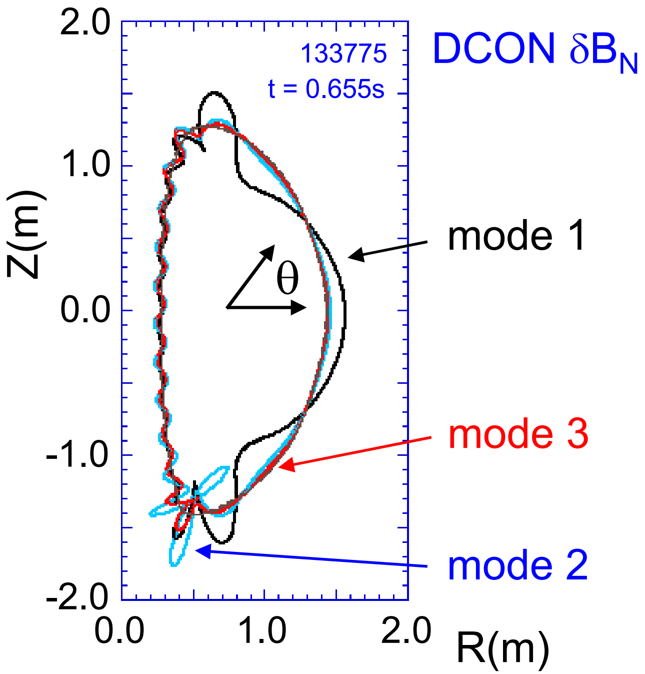


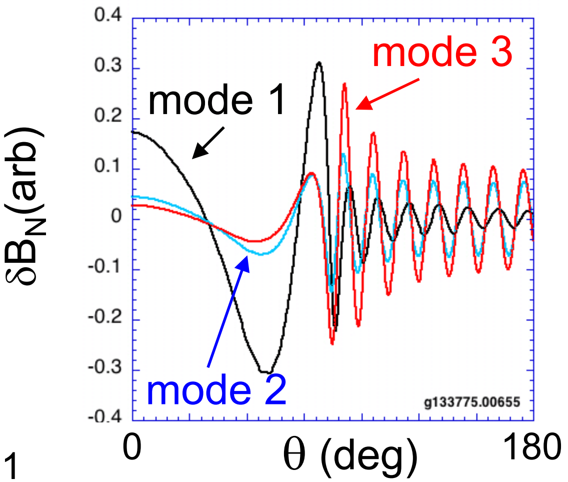
L. Delgado-Aparicio, PoP letter, to be submitted, (2009).

- *$n=3$* braking and *$n=1$* stabilizing fields modified kinetic profiles at early times.
- Increased edge n_Z blobs during stabilization; good correlation with drops in T_{e0} & S_n
- ***May have identified a driven RWM near the natural RFA resonance***.

Multimode response theoretically expected to be significant at high β_N



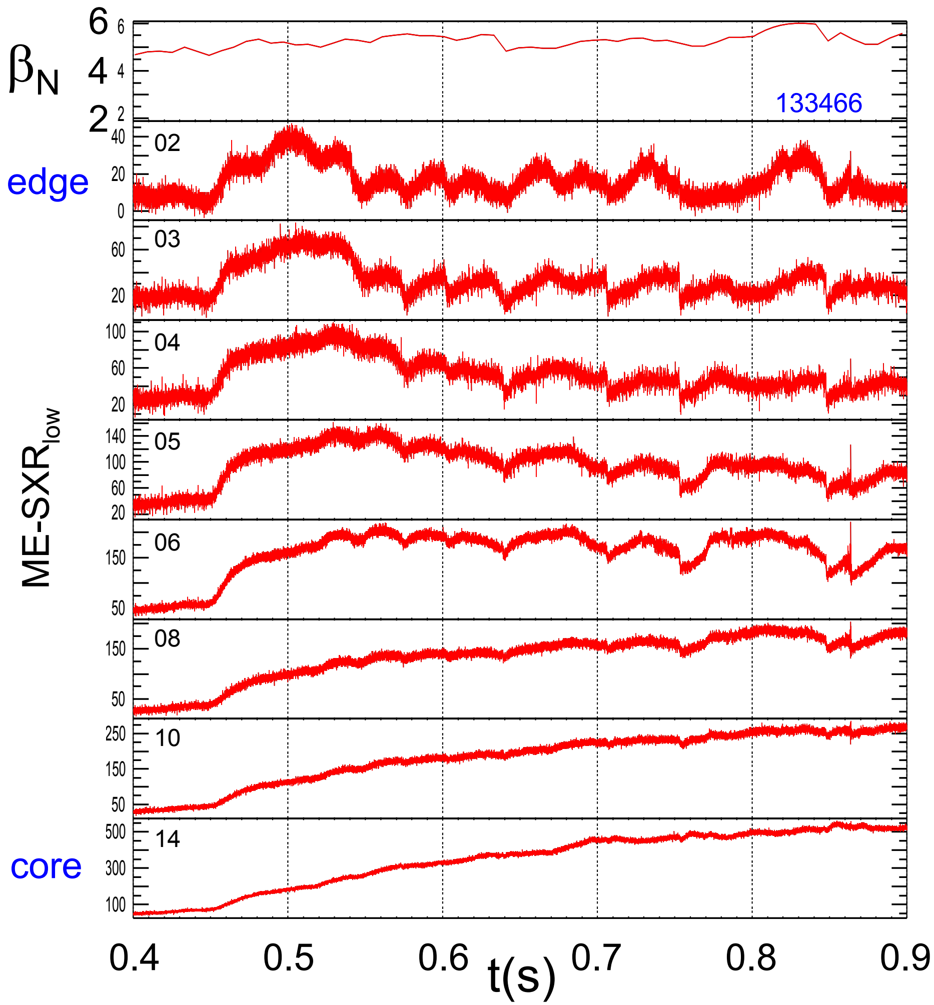




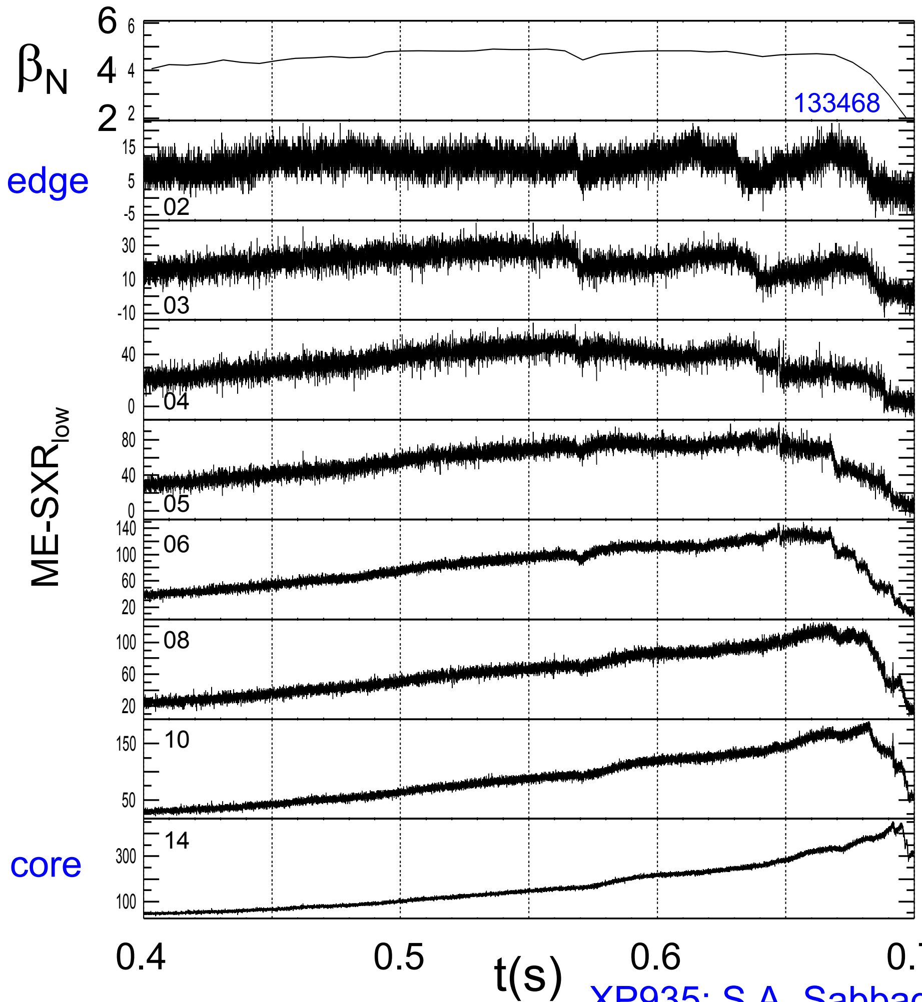
- Boozer multimode criterion for n = 1 met at high β_N (PoP 10 (2003) 1458.)
  - $|\delta W|$ smallest for 2nd n = 1 eigenfunction
  - Ratio of $|\delta W|$ for 3rd vs. 1st least stable mode sometimes also > 1

Mode observed in ME-SXR at ~30Hz covers greater radial extent as β_N increased

Heating with 3 NBI Sources – Higher β_N



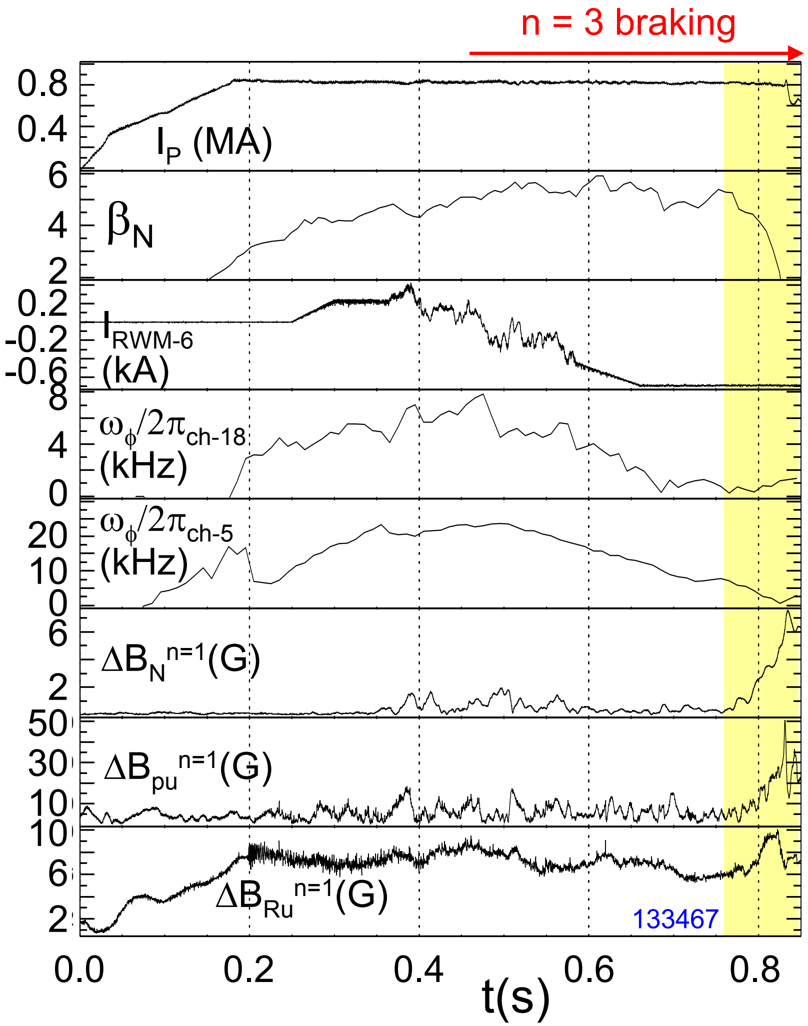
Heating with 2 NBI Sources – Lower β_N

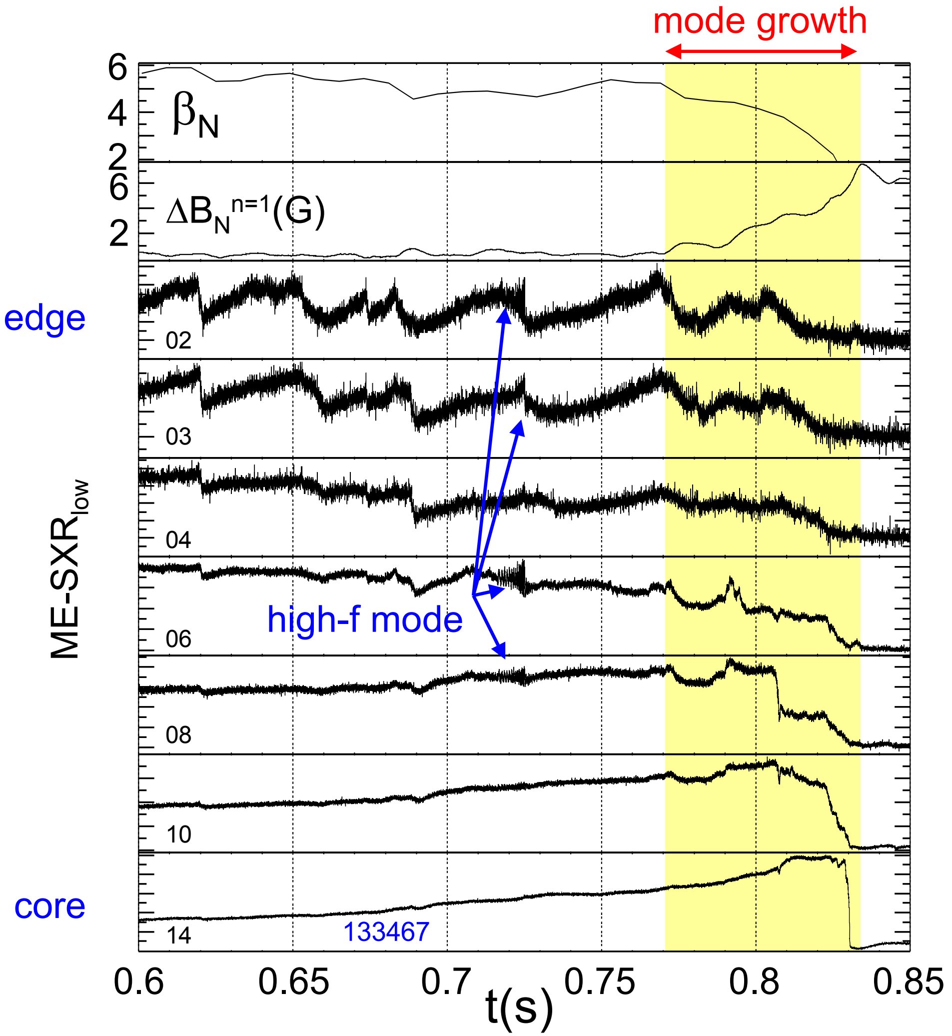


XP935: S.A. Sabbagh

- Note: proximity to marginal stability (e.g β_N plus ω_ϕ level) may be important

At reduced plasma rotation, observed growing RWM appears to be independent of low frequency ~30 Hz activity



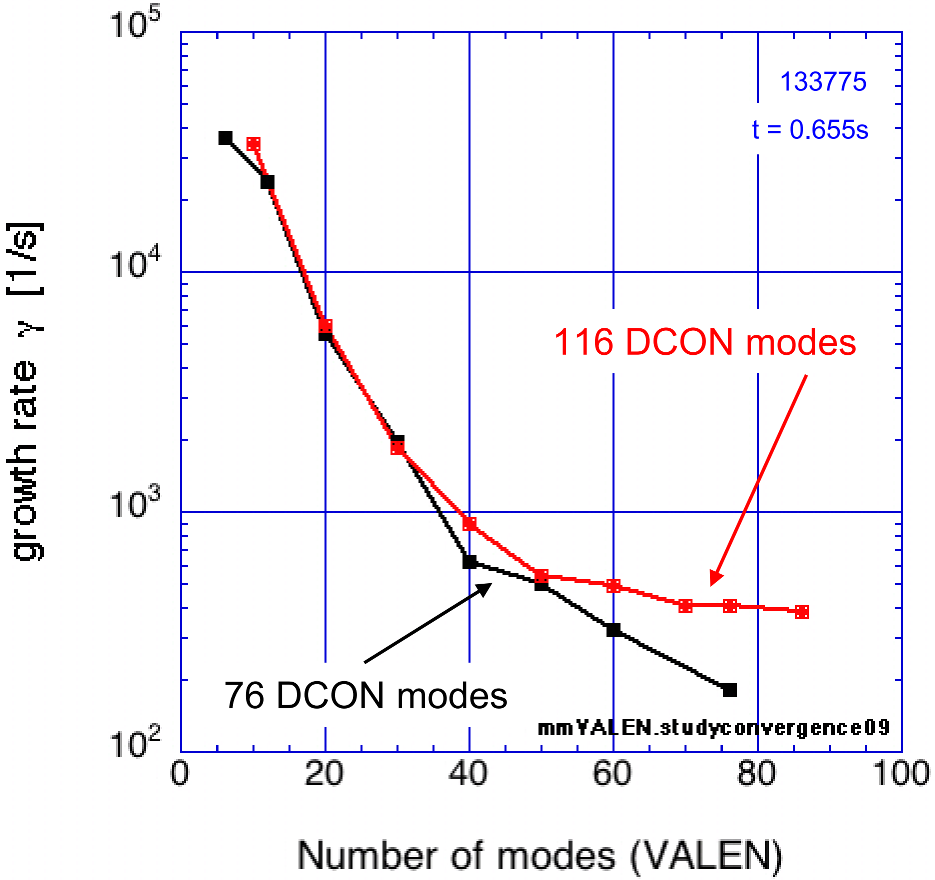


XP935: S.A. Sabbagh

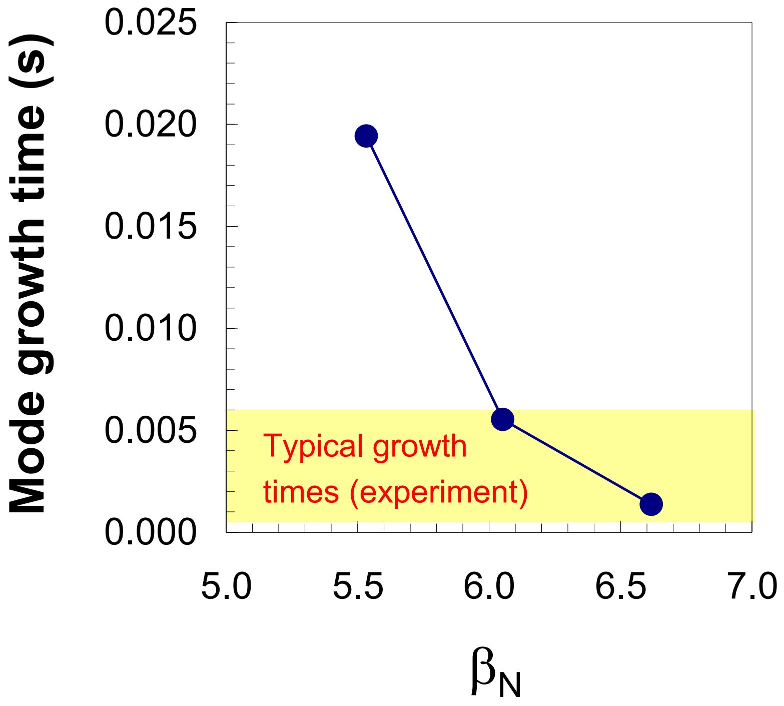
- ❑ Unstable mode is locked; ME-SXR mode apparently co-rotating
- ❑ Greater radial extent of ~ 30Hz during RWM growth

Multi-mode VALEN code (RWM control) testing successfully on high β_N NSTX plasmas

Growth rate vs. # of modes – mmVALEN



RWM growth time vs. betaN 133775 - mmVALEN

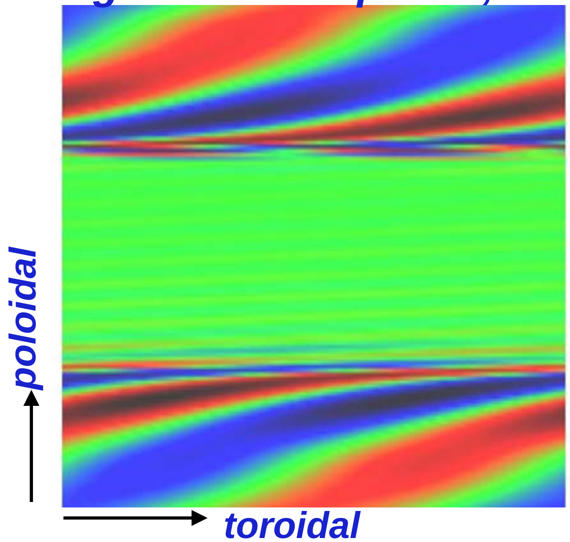


- See backup slides for mmVALEN equations
- mmVALEN to be used to examine response of 2nd mode to n = 1 feedback, error field and compare to experiment

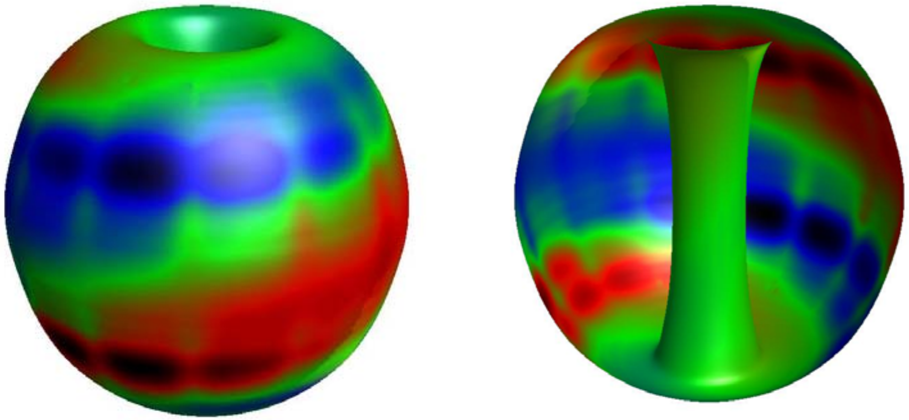
J. Bialek

Illustration of $B^n(\theta,\phi)$ on plasma surface from mmVALEN for NSTX shot 133775 (t=0.655s)

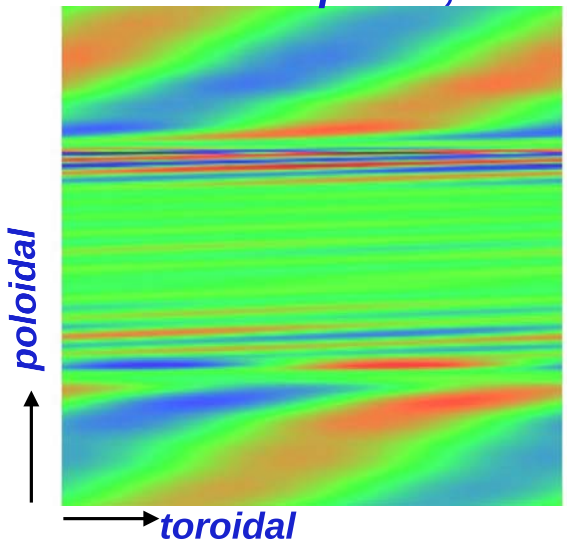
single mode response, total B^n



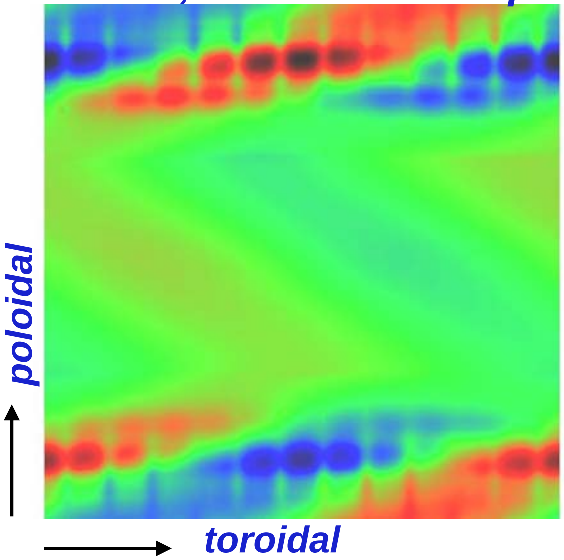
B^n from wall, multi mode response



multi mode response, total B^n

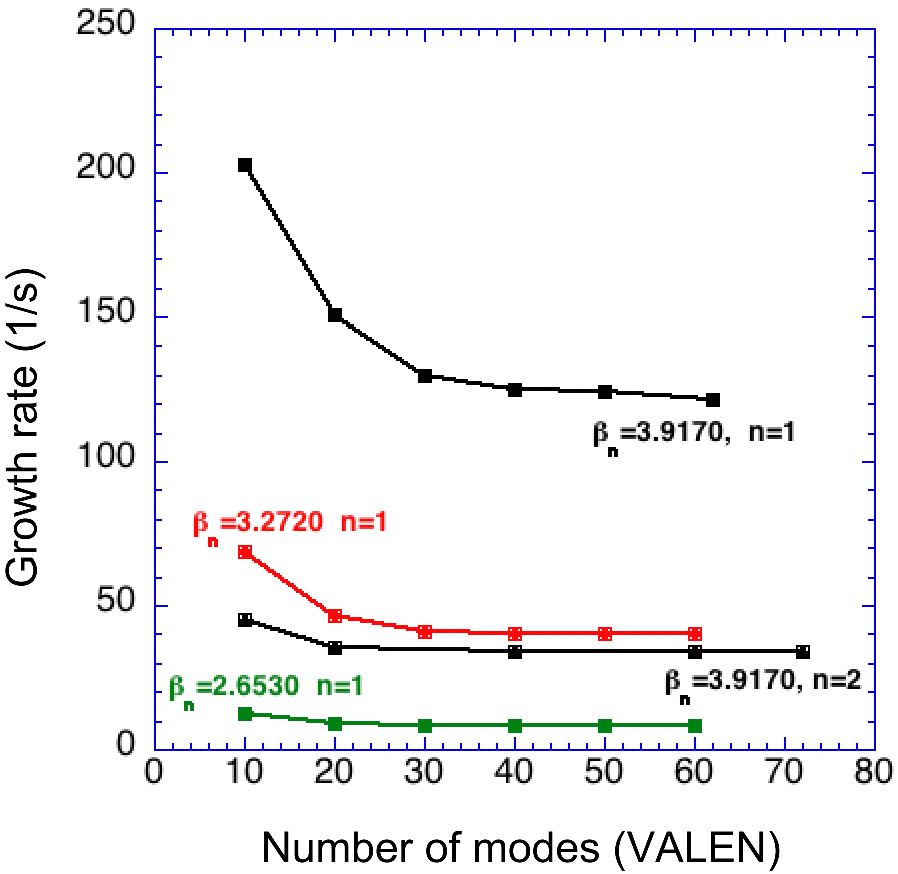


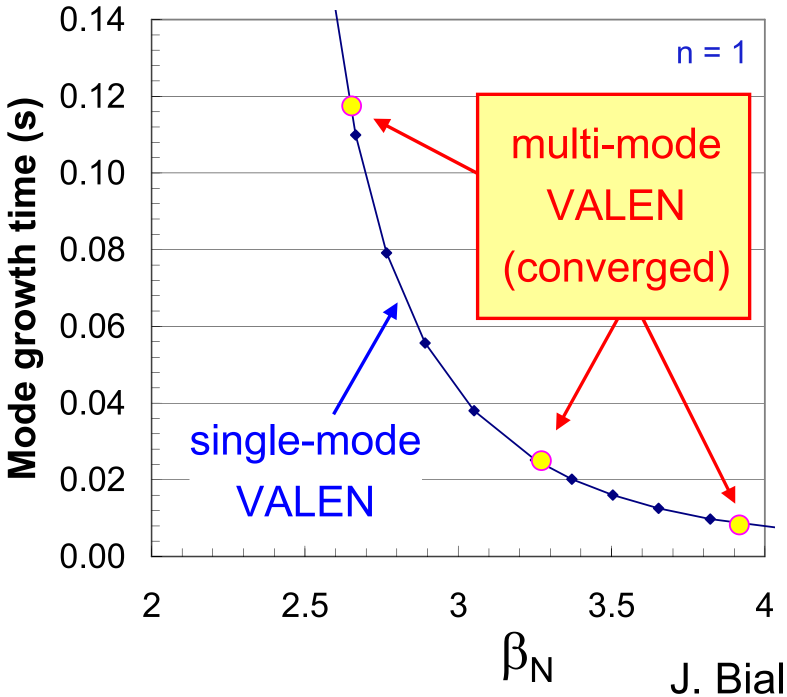
B^n from wall, multi mode response



J. Bialek

Multi-mode VALEN code (RWM control) testing successfully on ITER Scenario 4 cases


mmVALEN analysis of ITER
new scenario #4
convergence vs. # modes


Growth time vs. betaN - ITER Scen 4

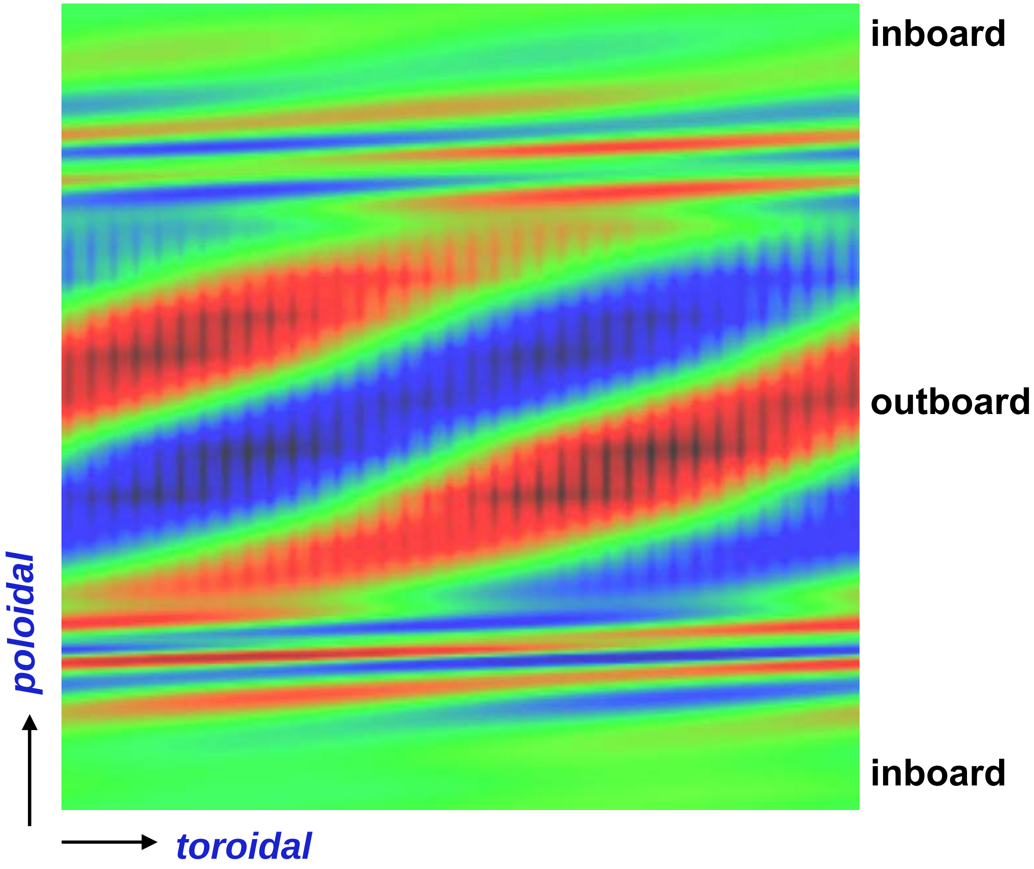
J. Bialek

- DCON δW shows several modes with high response
  - Three n = 1 modes at high β_N
  - Two n = 2 modes at high β_N

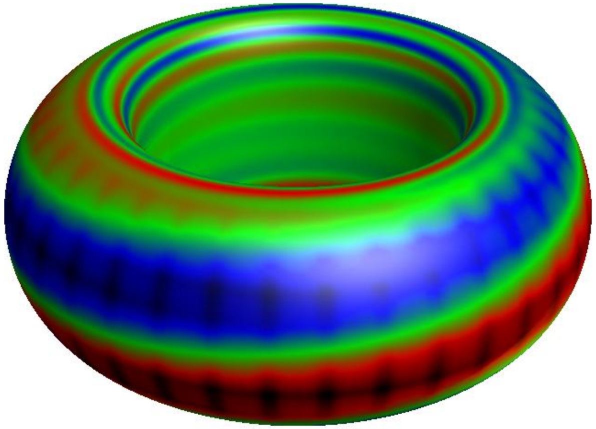
- At highest β_N, n =1 and 2 are unstable

Illustration of $B^n(\theta,\phi)$ on plasma surface from mmVALEN for ITER Scenario 4, $\beta_N = 3.92$

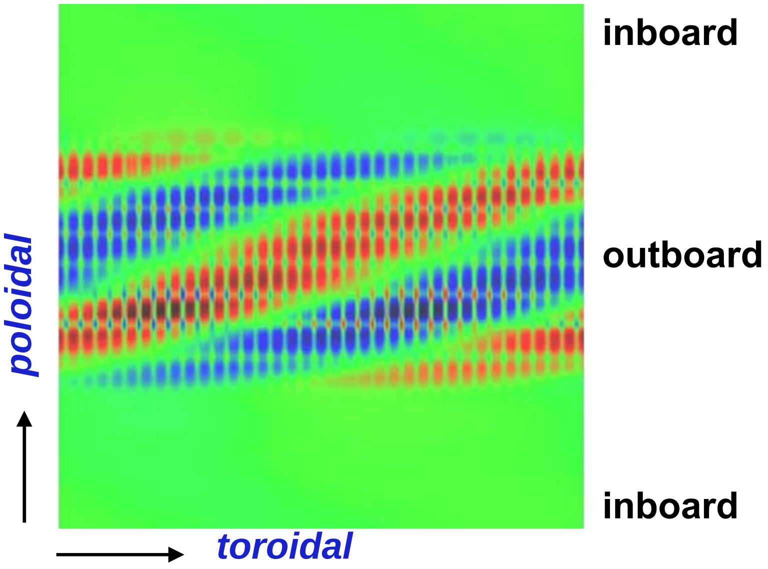
multi mode response (incl. wall), total B^n



B^n from wall, plasma



B^n from wall alone



- n = 1 eigenfunctions shown

J. Bialek

NSTX Macrostability Research – Topics Covered in Talk

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- ☐ NTM threshold physics; M3D-C^1 analysis MDC-14)
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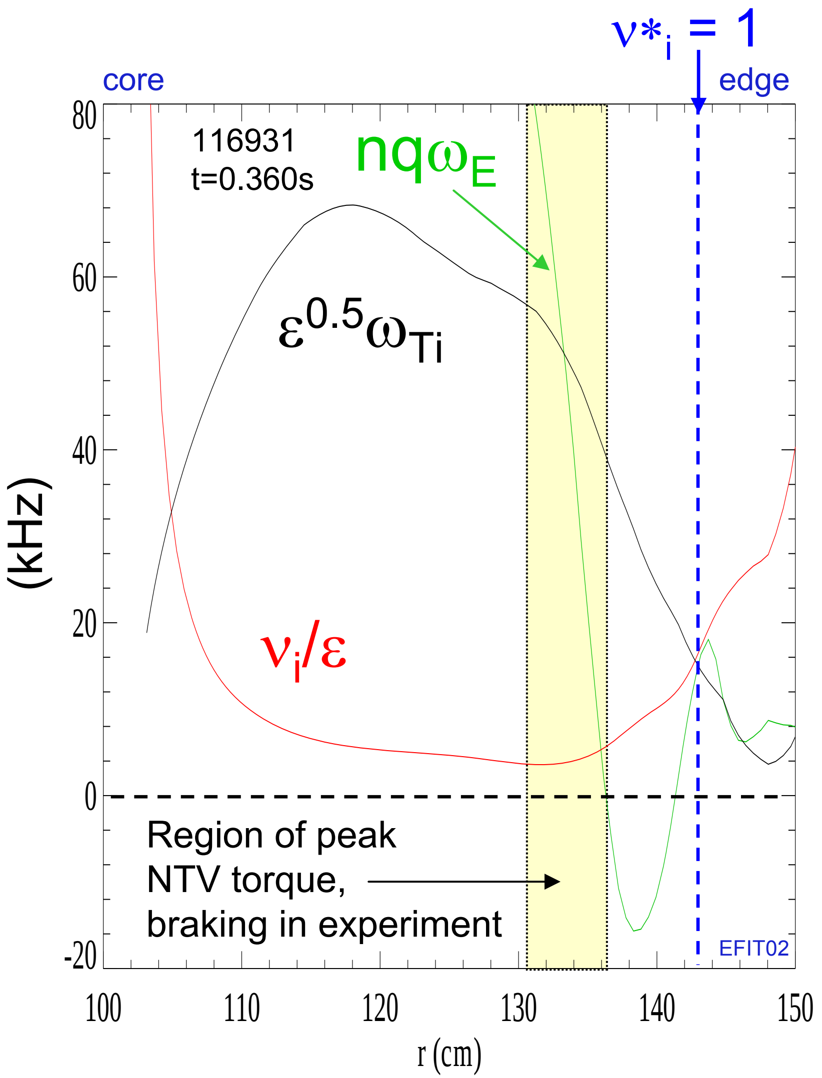
NSTX experiments examining applicability of NTV collisionless regime formulae / scaling

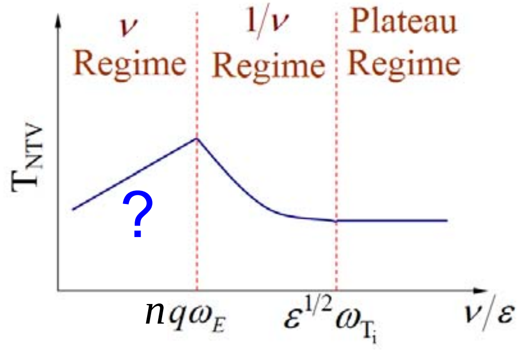


Region of peak NTV torque, braking in experiment

XP933: S.A. Sabbagh

- ❑ Past NSTX experiments
  - ❑ V_ϕ damping consistent with "1/nu regime" scaling ($T_i^{5/2}$) & magnitude

    Zhu PRL 2006; Sabbagh IAEA 2008 (sub. Nucl. Fusion)
- ❑ Frequency profiles
  - ❑ Collisionless NTV formulation region of validity: $nq\omega_E < \nu_i/\varepsilon < \varepsilon^{0.5}\omega_{Ti}$



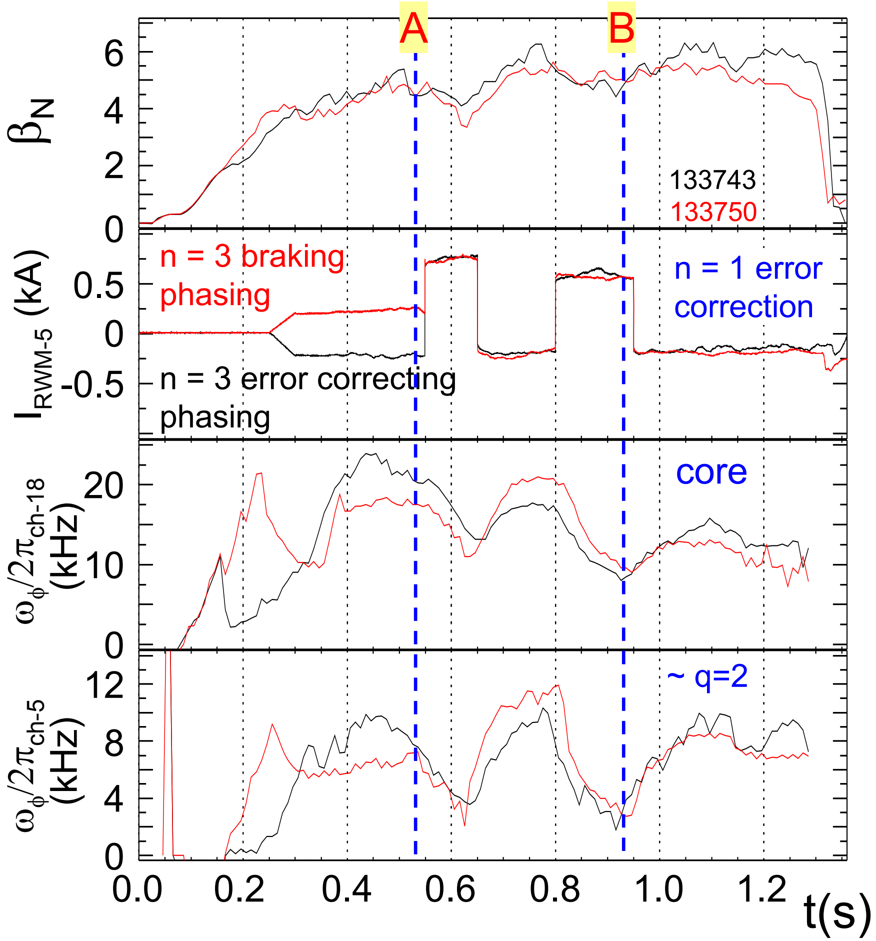
  - ❑ Is $nq\omega_E < \nu_i/\varepsilon$ (or $|nq\omega_E| < \nu_i/\varepsilon$) proper criterion for $1/\nu$ collisionless damping?
    - Work by Shaing, et al., J-K. Park, et al. postulate alternate low ν behavior
  - ❑ Important for low ν ITER plasma

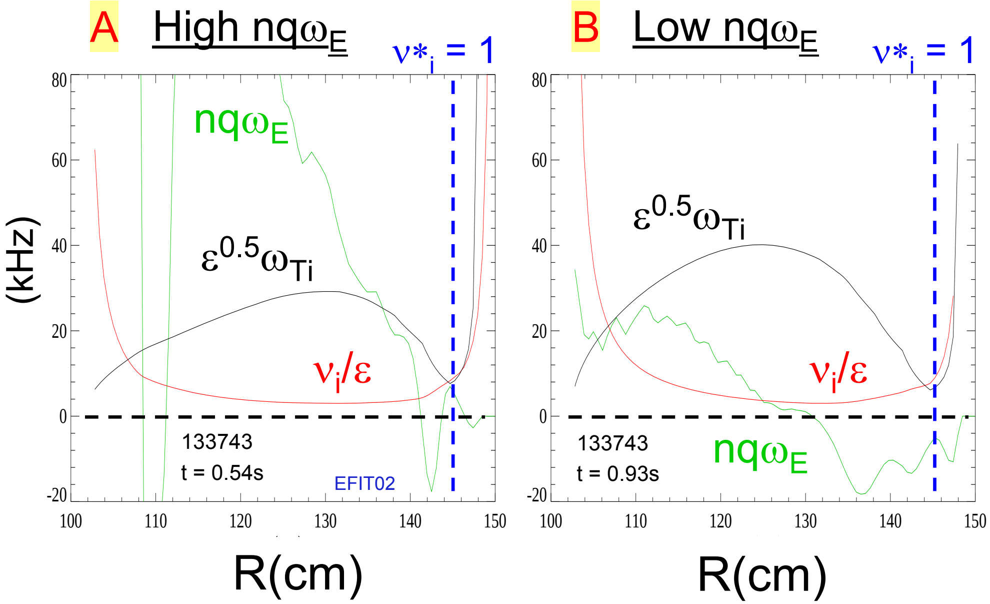
XP933: NTV physics at varied $\nu_i/nq\omega_E$ in NSTX – Brief Status

- Goals
  - Investigate damping over range of $\nu_i/nq\omega_E$ to determine if changes occur to NTV-induced magnetic braking
    - Key for both low and high rotation devices (ITER, ST-CTF)
    - Does ST data reveal new physics, or revise applicability criteria?
      - e.g. test recent criteria by K.C. Shaing, et al. (PPCF 51 (2009) 035004, also 035009)
- Status
  - NTV braking observed from all $\nu_i/nq\omega_E(R)$ variations made in experiment (n = 3 configuration)
    - Strong braking observed at increased T_i with lithium, even if $(\nu_i/\varepsilon)/nq\omega_E < 1$
    - Analysis has just begun (initial analysis of NTV relevant frequency profiles)
  - Braking of resonant surfaces appeared in plasmas at low ω_ϕ, but without locking (e.g. ω_ϕ would go to zero locally, then would increase)
    - Most likely occurred with Li wall preparation
  - Apparent lack of $1/\omega_\phi$ scaling of drag torque on resonant surfaces
    - Provocative result – either current layer / island width is decreasing at low ω_ϕ (why?), or perhaps drag due to "island NTV" ~ ω_ϕ (K.C. Shaing, PRL **87** (2001))

Significant variations made to $nq\omega_E(R)$ to examine effect on NTV braking

Example: variation of initial ω_ϕ





- Generally, rotation braking remains strong over large range of plasma rotation, $nq\omega_E$
- Analysis continues to examine rotation damping vs. $nq\omega_E$
  - Rotation damping rate
  - Damping profile broadness
  - Comparison to theory

XP933: S.A. Sabbagh

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Experiments Collected Good Data Sets on NTM Restabilization in 2009

Method: In both DIII-D and NSTX

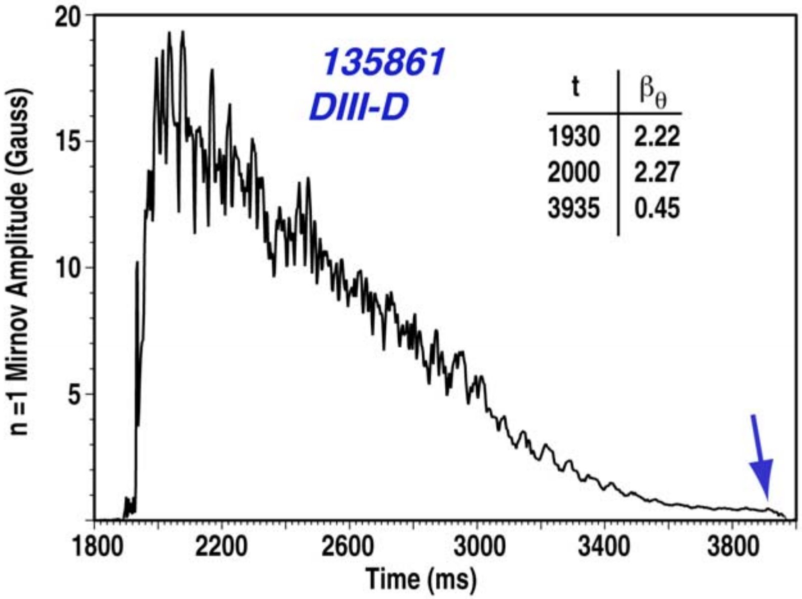
- Strike 2/1 mode in NBI heated ELMy H-mode discharge
- Ramp-down β_θ by reducing the beam power
- Determine the marginal island width (island width at the value of β_θ just sufficient to support an NTM)
- Marginal island width contains critical information about the small-island physics

Status: DIII-D

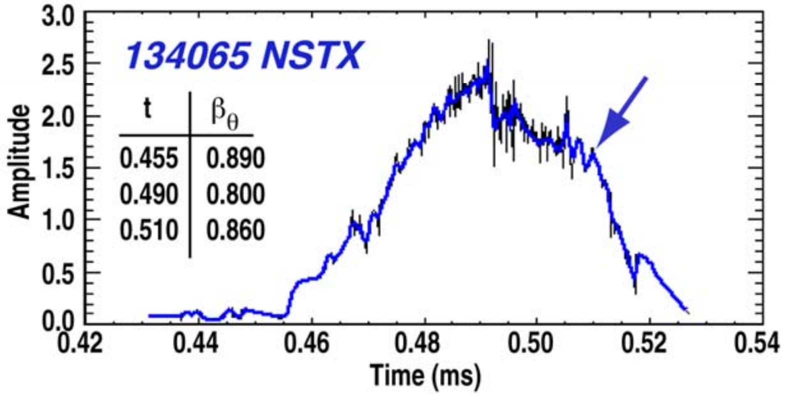
- Used gas puff to stay in H-mode
- 5 good 2/1 (and 2 good 3/1) cases
- Analysis complete

Status: NSTX

- Achieved a reproducible onset condition using modest Li evaporation
- Up to 9 good cases, 8 collected this year
- Analysis to be done



t	β_θ
1930	2.22
2000	2.27
3935	0.45

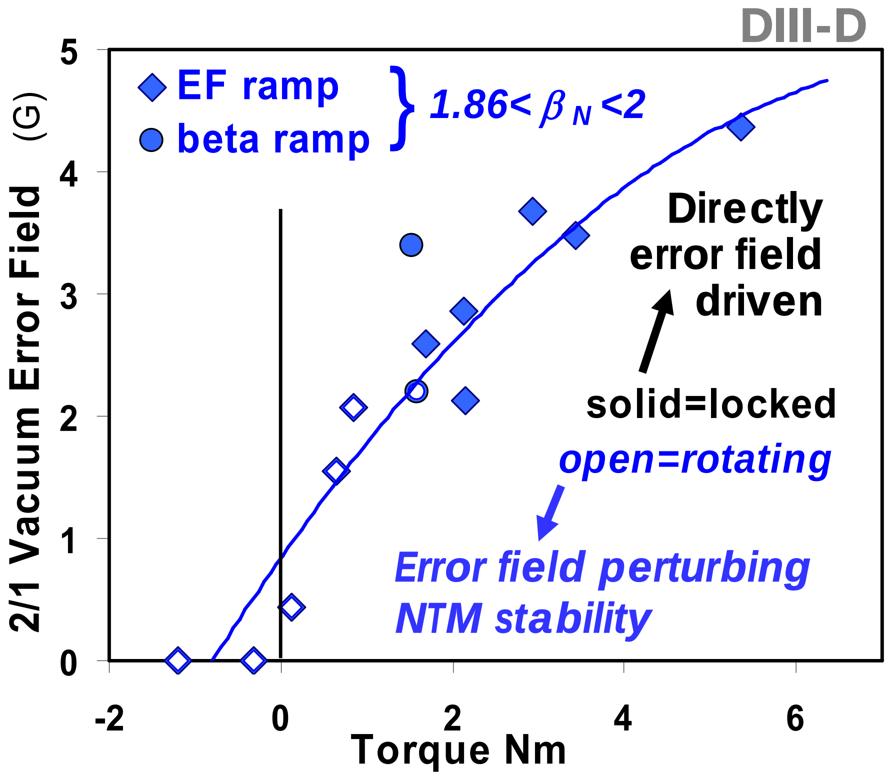


t	β_θ
0.455	0.890
0.490	0.800
0.510	0.860

NSTX XP914: R. LaHaye

XP915 Goals: Understand how error fields interact with plasma to change tearing stability

Error field can act through two mechanisms:



Directly error field driven

solid=locked

open=rotating

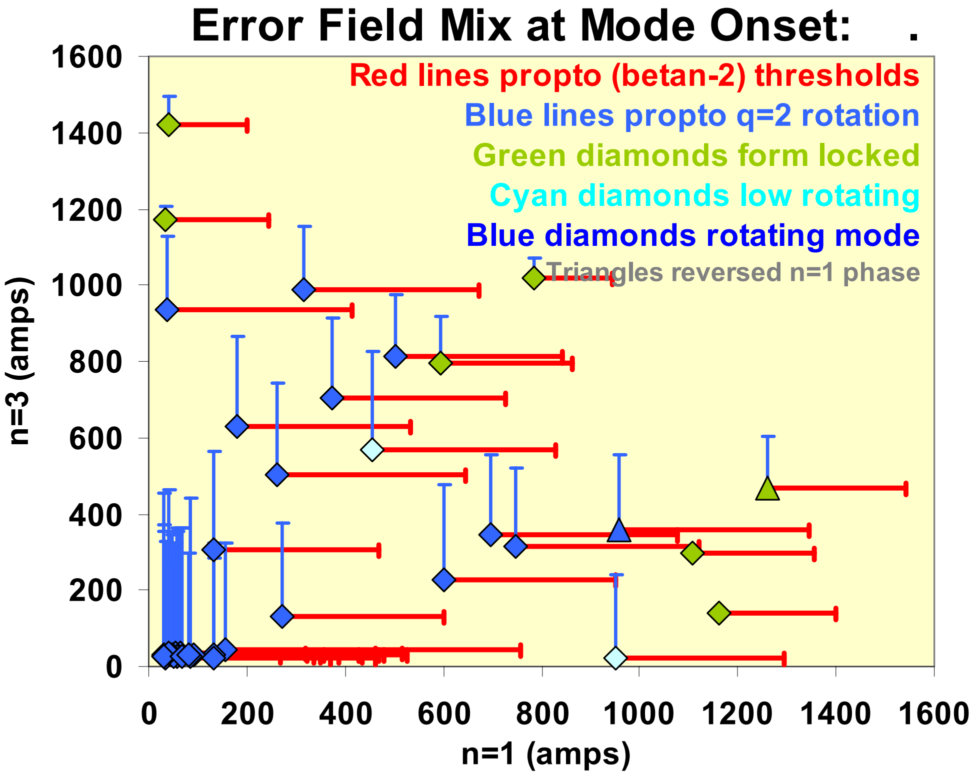
Error field perturbing NTM stability

How do β limits manifest in low torque plasmas?

- **Locked modes**
  - Error field amplification at high beta?
  - Proximity to
  - Role of rotation?
- **Rotating modes**
  - Perturbing classical / neoclassical stability?
  - Action through rotation or rotation shear?
    - Influences delta prime?

Successfully scanned n=1 and n=3 error fields up to their maximum effect on rotation and tearing mode β limits

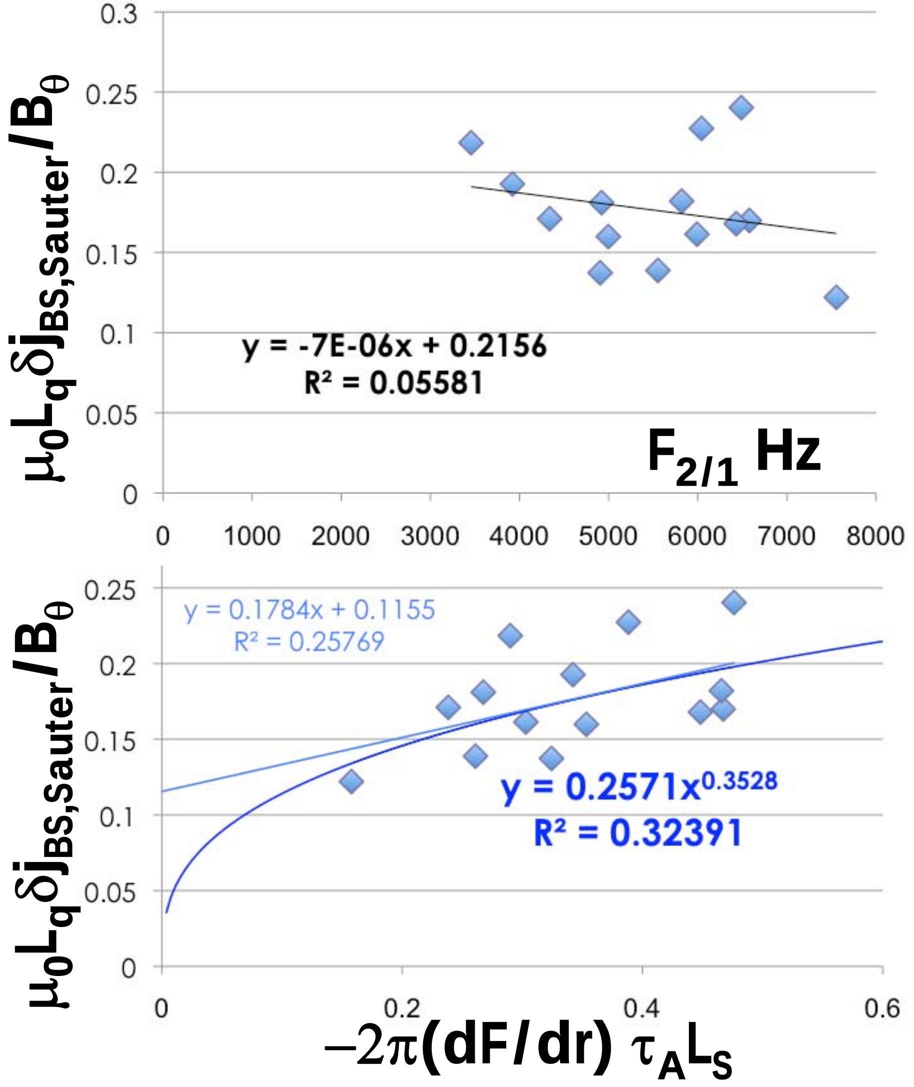
- n=1 and n=3 fields varied up to locked mode limits (◆)
  - See variation in mode onset β_N (red bars)
  - And underlying **rotation** (blue bars)
  - Very reproducible plasma thanks to H-mode tuning and lithium
- Some trends emerging…



XP915: R. Buttery

NTM bootstrap threshold favors rotation shear role

- No measureable trend vs rotation
- Weak positive correlation with normalised rotation shear
  - Lowest thresholds at low rotation shear
  - Highest thresholds at high rotation shear
  - Consistent with 2009 results (S.P. Gerhardt, et al.)
- No correlations if y-plot β_N
  - Also 2d fit vs rotation & rotation shear offers little improvement



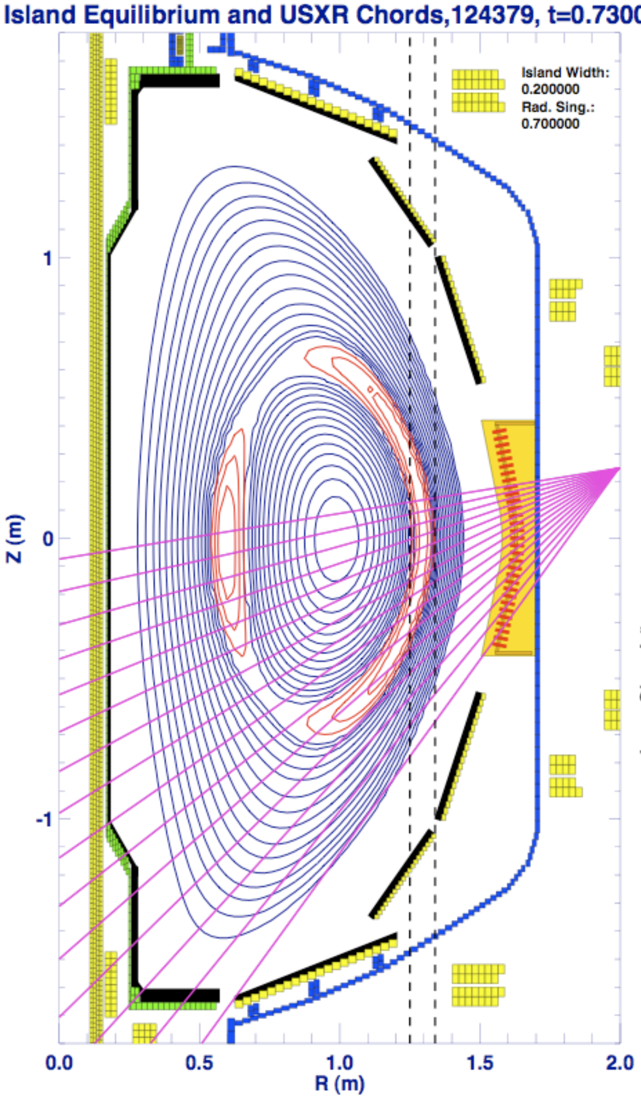
XP915: R. Buttery

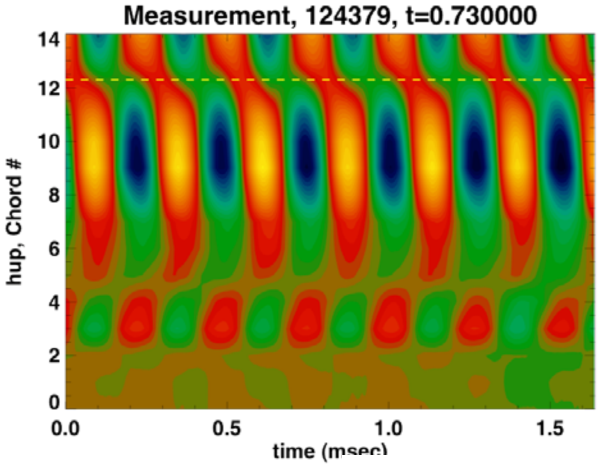
XP915: Error field influence on 2/1 NTM onset through rotation Conclusions

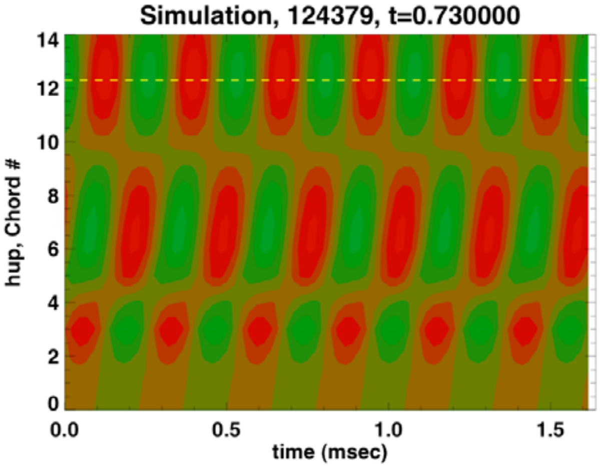
- Experiment successfully scanned n=1 and n=3 error fields up to their maximum effect on rotation and tearing mode β limits
  - Connected with high β_N locked mode data: shows $\omega/2$ rotation criteria for mode and lower β_N limits when rotation stopped
  - Accessed clear braking from n=1 and n=3 field and determined relative effects – both brake plasma significantly
  - Led to range of q=2 rotations and rotation shears, with loose correlation between the two
  - Although measurements of rotation shear are more noisy, **a distinct trend is observed for $\beta_{N\ 2/1NTM}$ with rotation shear**, *while rotation itself offer no significant correlations*
- This reinforces the idea that torque influences TM thresholds through rotation shear at the mode and therefore through modifications to underlying tearing stability

XP915: R. Buttery

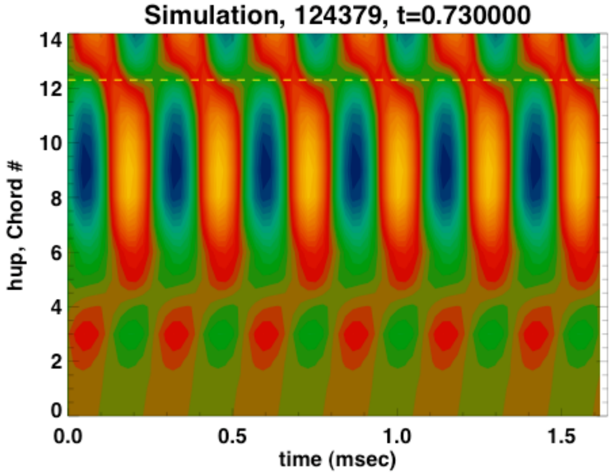
2/1 NTM is an important beta-limiting mode in NSTX







2,1 only

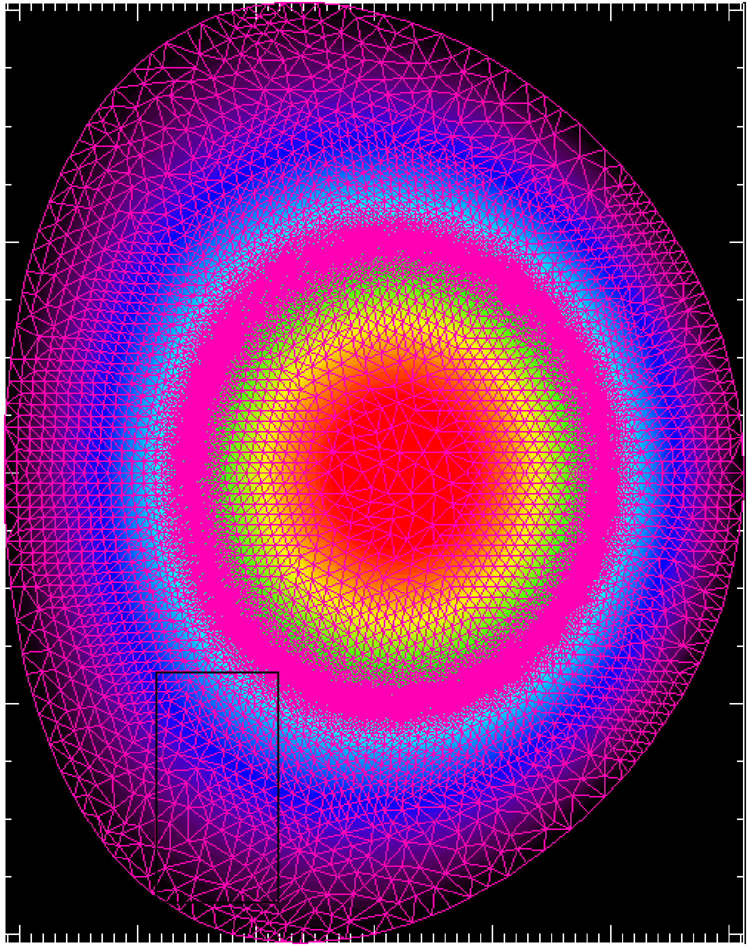


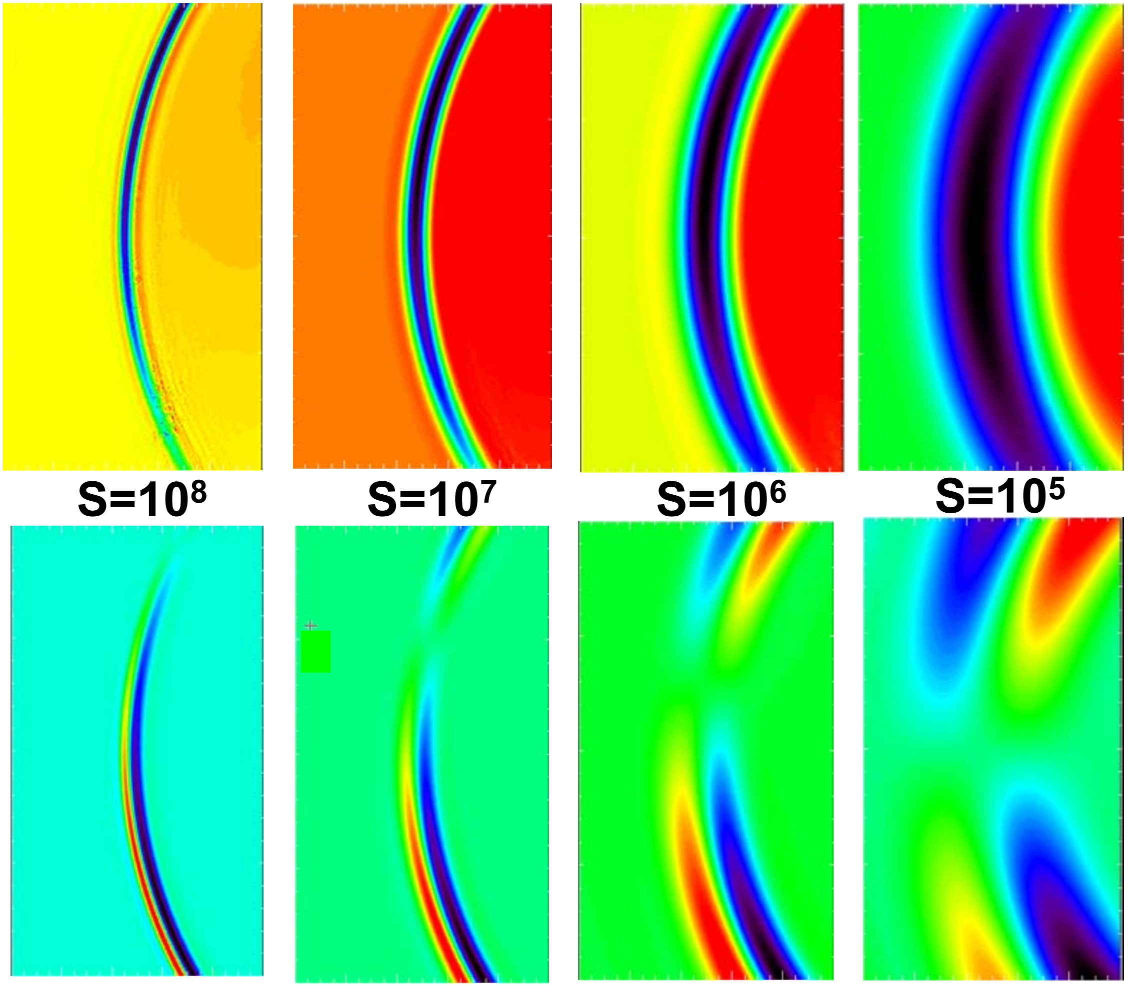
2,1 + 1,1 pert

- Eigenfunction analysis of multichord USXR data shows coupling of 2/1 mode NTM to 1/1 ideal kink (ubiquitous)

S.P. Gerhardt

M3D-C^1 Code allows MHD studies at high Lundquist number

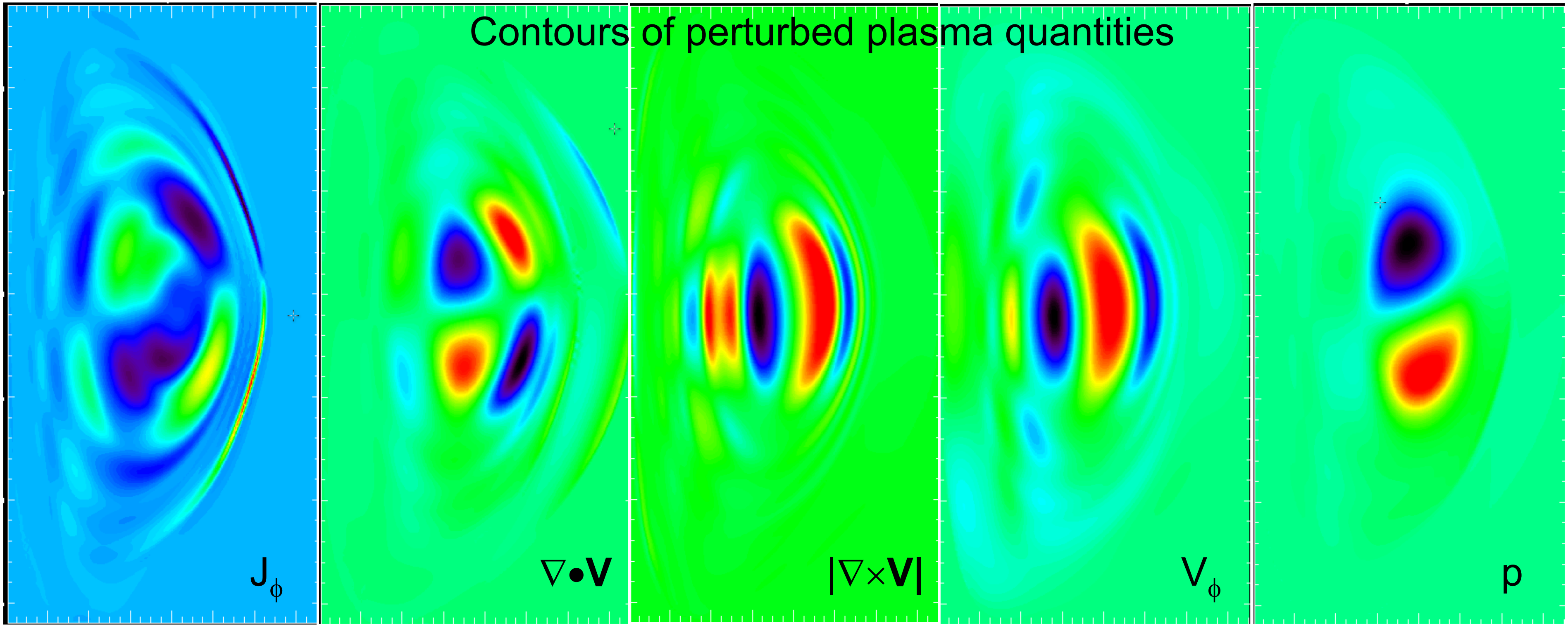




- M3D-C1 code has higher resolution, implicit time stepping
- Allow studies at higher S values
  - M3D code limited to S ~ 10^6

- Perturbed current density (top) and vorticity (bottom) for (1,1) tearing mode
- Structures, (e.g. current sheets) resolved at high S

M3D-C^1 code used to search for mode with experimental characteristics


Contours of perturbed plasma quantities

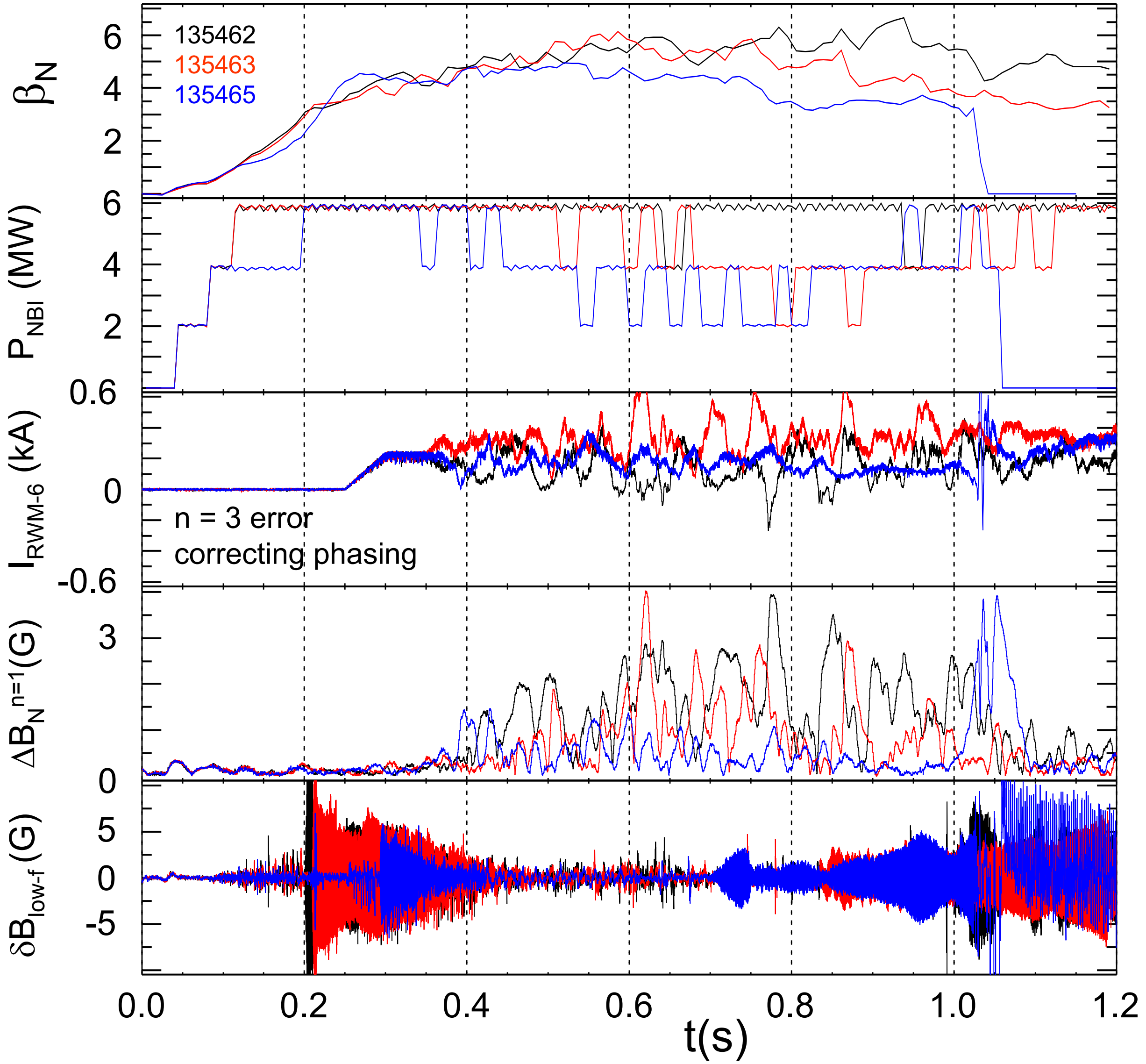
- Study conducted using experimental equilibrium
  - define η and thermal conductivity; impose random perturbation
  - solve time-dependent resistive MHD equations as initial-value problem

- An *n*=1 mode found with some characteristics of the experimental mode
  - Shows m=1 and m=2 components of plasma quantities
  - Internal ideal mode – no q=1 surface for resonance
  - Scaling study suggests experiment is close to marginal stability for this mode

NSTX Macrostability Research – Topics Covered in Talk

- ☐ Error fields (n = 1 and n = 3) (MDC-2)
- ☐ Resistive wall modes (MDC-2)
- ☐ Non-resonant magnetic braking physics (MDC-12)
- ☐ NTM threshold physics; M3D-C[1] analysis MDC-14)
- ☐ Improving $<\beta_N>_{pulse}$ with n = 1 and β_N feedback (MDC-2, MDC-17)

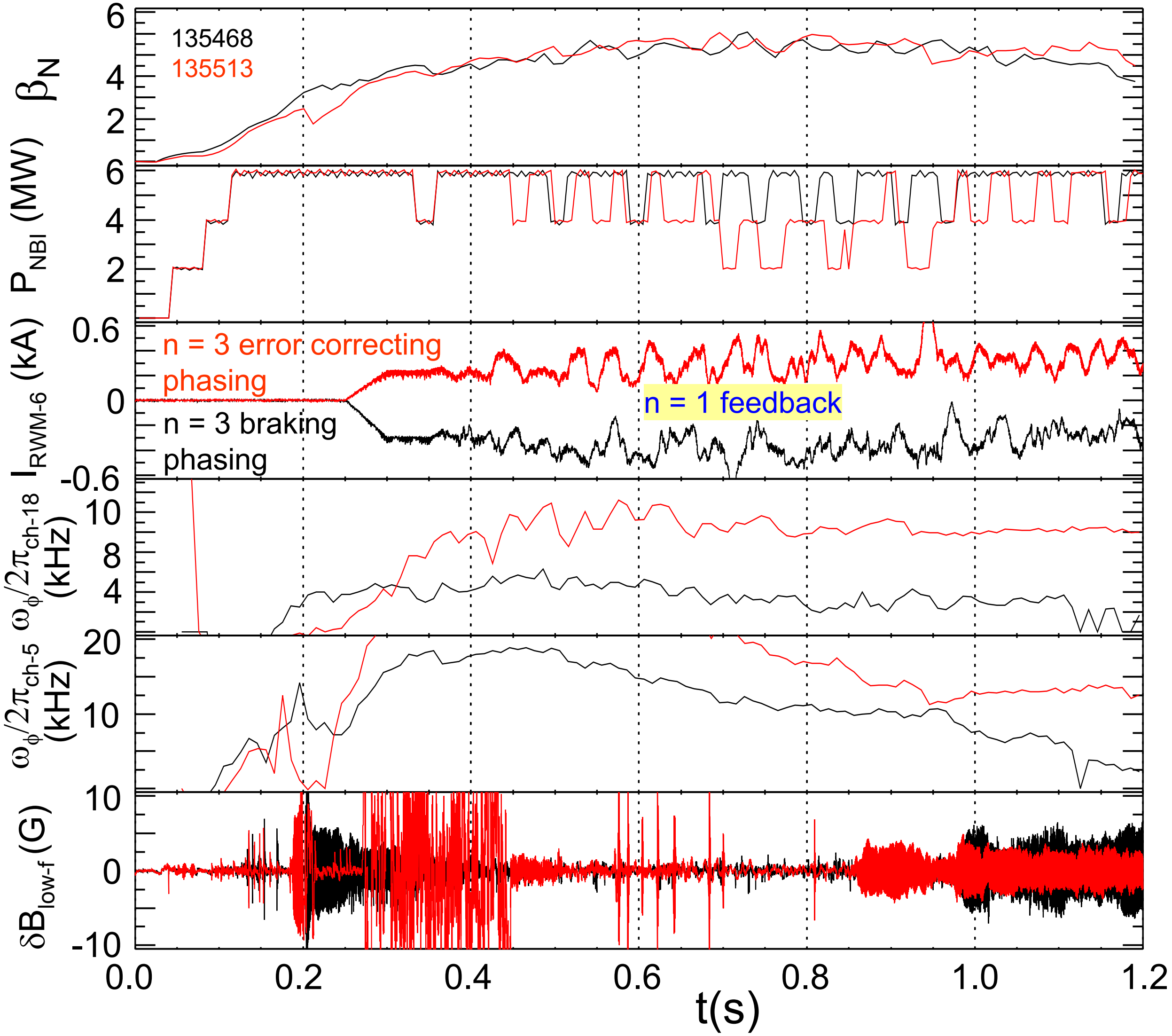
Successful NBI power limitation via β_N feedback in 2009 run



- Cases with n = 3 correcting field (highest ω_ϕ)
  - Nominal targets β_N = 4,5,6
  - NBI blocking shows FB
    - NBI power turned back on when n = 1 rotating mode appears
  - Higher activity in n = 1 LMD at highest β_N

XP934: S.A. Sabbagh

Successful β_N feedback at varied plasma rotation levels



- Prelude to ω_ϕ control
  - Reduced ω_ϕ by n = 3 braking does not defeat β_N FB
  - Increased P_{NBI} needed at lower ω_ϕ
- Steady β_N established over long pulse
  - independent of ω_ϕ over a significant range

XP934: S.A. Sabbagh

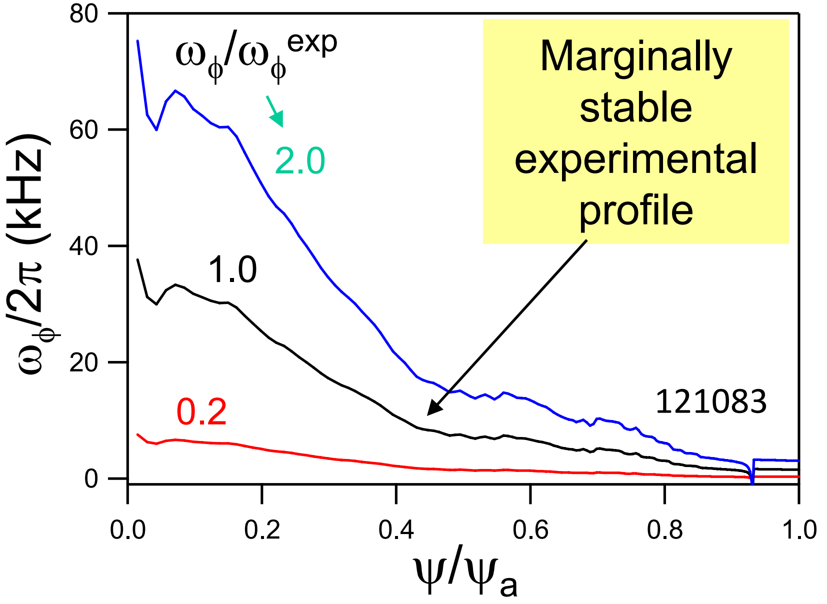
NSTX Macrostability Research in 2009 Addressing Topics of Furthering Steady Operation of High Performance Plasmas

- ❑ Ideal plasma amplification of applied n = 1 resonant field (IPEC) joins linear density scaling of mode locking threshold from low to increased-β
    - ❑ Optimal n=3 error field correction determined vs. I_P, B_T
- ❑ RWM instability, observed at intermediate plasma rotation, correlates with kinetic stability theory; role of energetic particles under study
- ❑ Low frequency ~ $O(1/\tau_{wall})$ mode activity at high β_N being investigated as potential driven RWM
- ❑ Theory shows multi-mode RWM response may be important at high β_N; multi-mode VALEN code now passing initial tests
- ❑ Strong non-resonant braking observed at high ω_E; applicability of NTV collisionless regime formulae / scaling under examination
- ❑ Expanded NTM experiments continue to find best correlation between NTM onset drive and flow shear
- ❑ Successful NBI power limitation via β_N feedback at varied plasma rotation levels

Backup Slides

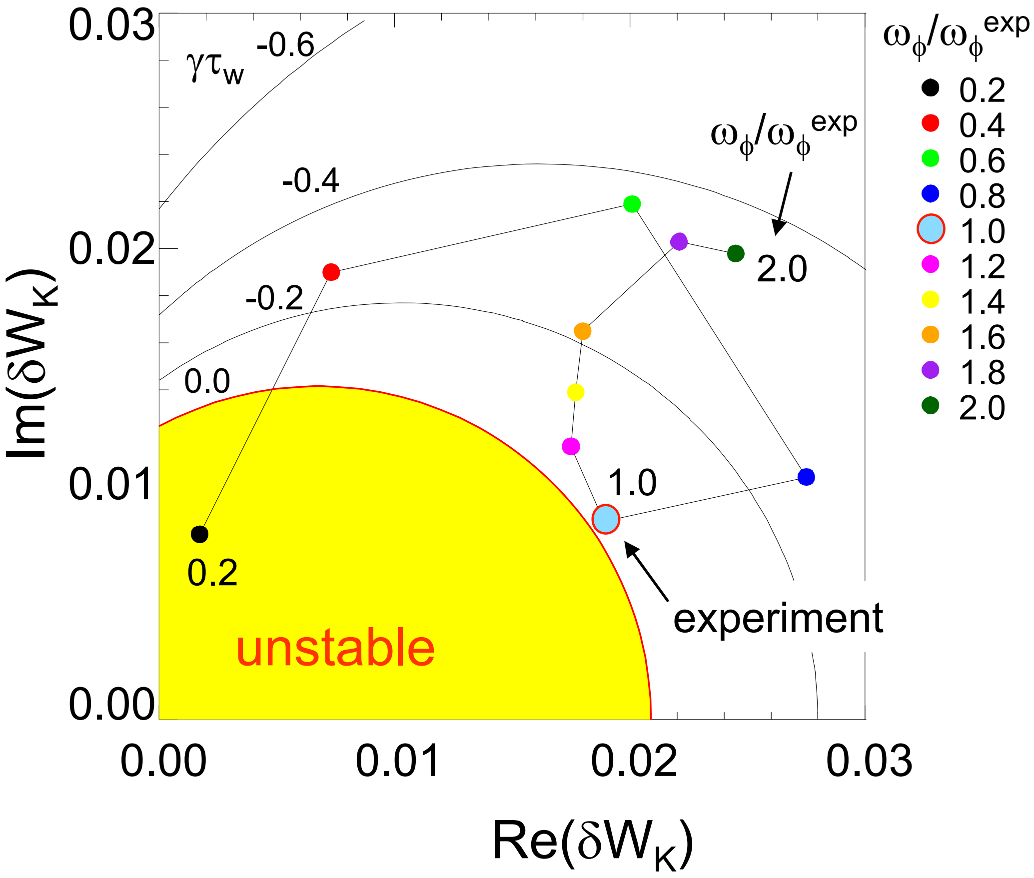
Kinetic modifications show decrease in RWM stability at relatively high V_ϕ – consistent with experiment

Theoretical variation of ω_ϕ



- ❑ Marginally stable experimental plasma reconstruction, rotation profile ω_ϕ^{exp}
- ❑ Variation of ω_ϕ away from marginal profile <u>increases</u> stability
- ❑ Unstable region at low ω_ϕ

RWM stability vs. V_ϕ (contours of $\gamma\tau_w$)



Modification of Ideal Stability by Kinetic theory (MISK code) investigated to explain experimental RWM stabilization

- Simple critical ω_ϕ threshold stability models or loss of torque balance do not describe experimental marginal stability

Sontag, et al., Nucl. Fusion **47** (2007) 1005.

- Kinetic modification to ideal MHD growth rate
  - Trapped and circulating ions, trapped electrons
  - Alfven dissipation at rational surfaces

$$\gamma\tau_w = -\frac{\delta W_\infty + \delta W_K}{\delta W_b + \delta W_K}$$

Hu and Betti, Phys. Rev. Lett **93** (2004) 105002.

- Stability depends on
  - Integrated ω_ϕ profile: resonances in δW_K (e.g. ion precession drift)
  - Particle collisionality

ω_ϕ profile (enters through ExB frequency)

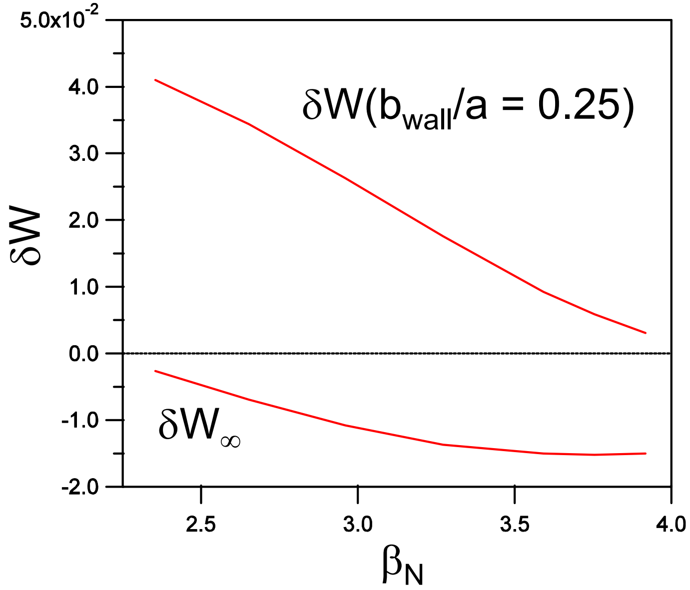
Trapped ion component of δW_K (plasma integral)

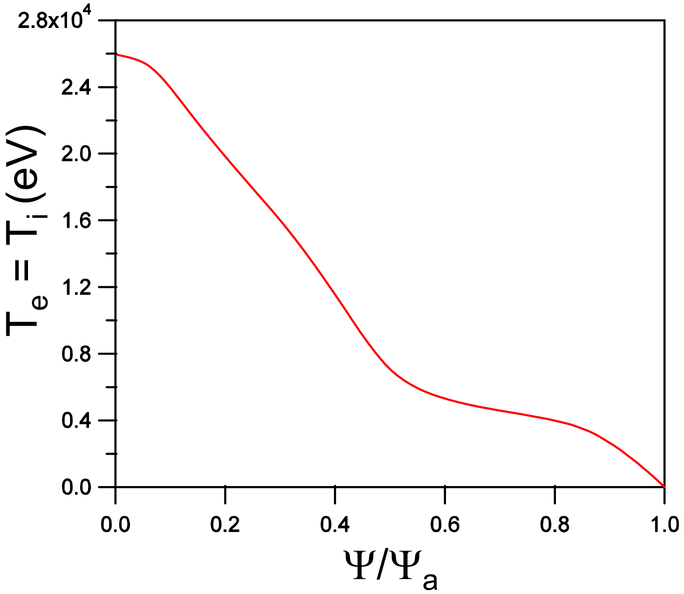
$$\delta W_K \propto \int \left[\frac{\omega_{*N} + \left(\hat{\varepsilon} - 3/2\right)\omega_{*T} + \omega_E - \omega - i\gamma}{\langle\omega_D\rangle + l\omega_b - i\nu_{eff} + \omega_E - \omega - i\gamma} \right] \hat{\varepsilon}^{\frac{5}{2}} e^{-\hat{\varepsilon}} d\hat{\varepsilon}$$

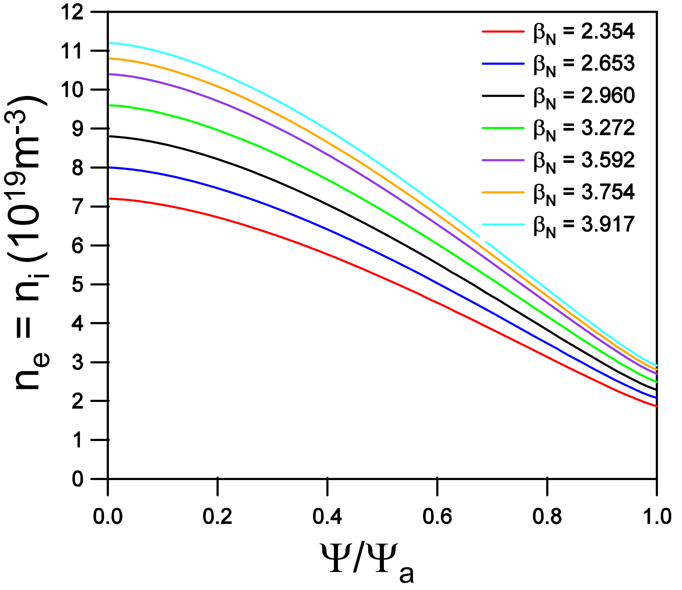
← Energy integral

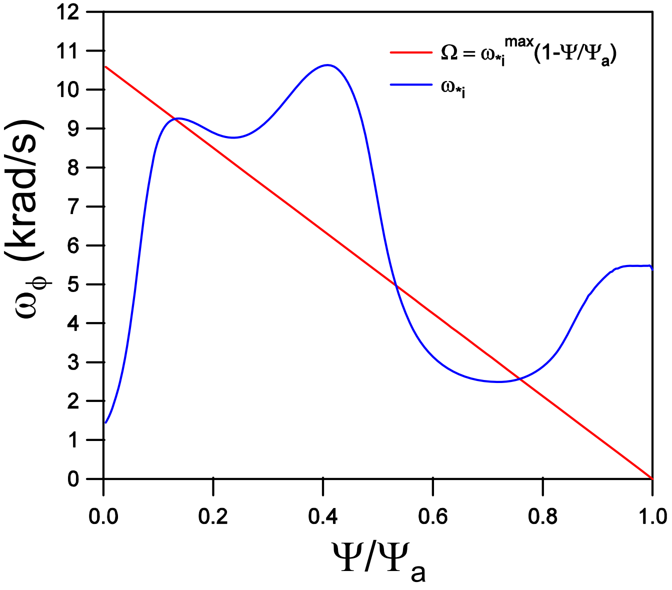
precession drift   bounce   collisionality

ITER Scenario 4 re-evaluated using MISK code



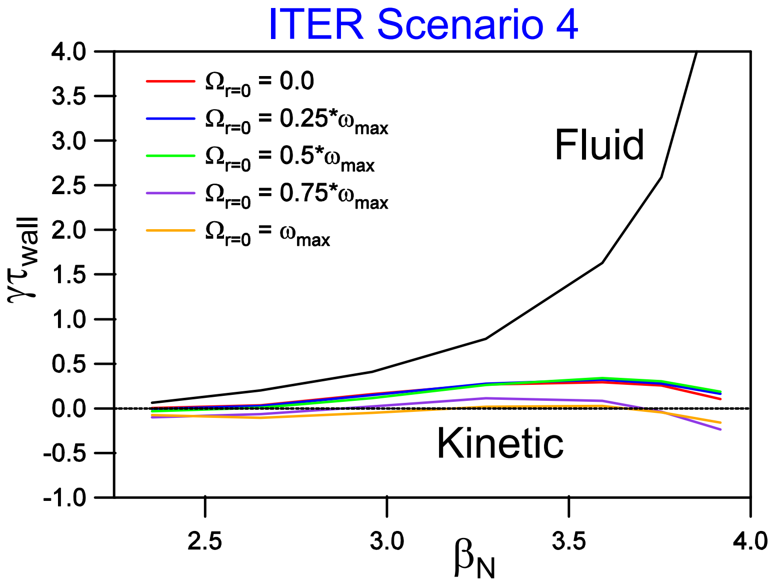




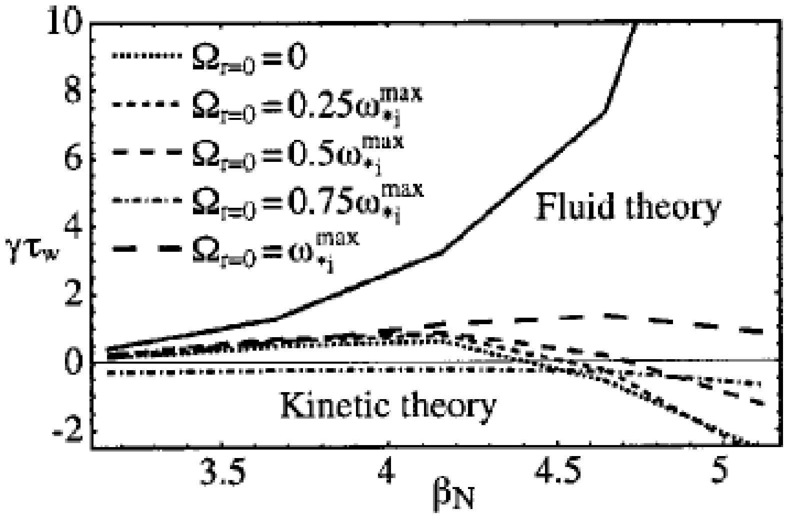


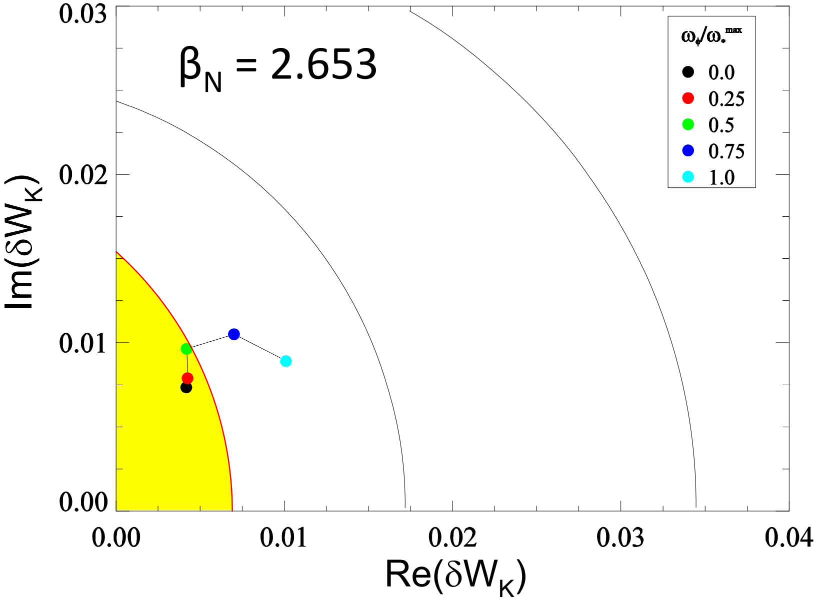
- ❑ Profile assumptions same as published by Hu/Betti
- ❑ Present calculations use Scenario 4 beta scan
- ❑ Simple plasma rotation profile, low peak values considered ($< \omega^*_i$)

Updated MISK code results using ITER Scenario 4 show similar behavior to published results



B. Hu, R. Betti, J. Manickam, Phys. Plasmas **12** (2005) 157301.





- Quantitative stability limits now appropriate for ITER Scenario 4
- The low levels of plasma rotation considered < ω^*_i indicates
  - Narrowing of unstable region
  - Simple behavior of marginal stability with plasma rotation
- Further study to include greater scoping of input profiles to make connection with present experimental results

VALEN L / R circuit formulation

$$\frac{d}{dt}\begin{Bmatrix} \{\Phi^{wall}\} \\ \{\Phi^{coil}\} \\ \{\Phi^{plasma}\} \end{Bmatrix} + \begin{bmatrix} [R_{ww}] & [0] & [0] \\ [0] & [R_{cc}] & [0] \\ [0] & [0] & [R_{pd}] \end{bmatrix} \begin{Bmatrix} \{I_w\} \\ \{I_c\} \\ \{I_d\} \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ \{V_c\} \\ \{0\} \end{Bmatrix} \begin{matrix} \text{1000's of equations} \\ \text{10's of equations} \\ \text{\# of plasma modes} \end{matrix}$$

where :

$\{\Phi^{\text{wall}}\}, \{\Phi^{\text{coil}}\}, \{\Phi^{\text{plasma}}\}$ are vectors of magnetic flux in the wall elements, coils, and plasma modes

$\{I_w\}, \{I_c\}, \{I_p\}, \{I_d\}$ are solution variables, i.e. vectors of mesh currents, coil currents, and plasma currents

The wall and coil equations are derived from standard circuit theory, the plasma circuit model is described in terms of plasma permeability i.e., P(δW_i $B^n{}_i(\theta,\phi)$) (no wall δW and B_normal dist):

ref:

A. Boozer, Physics of Plasmas, V5, No. 9, Sept. 1998, pg3350

A. Boozer, Physics of Plasmas, V10, No. 5, May 2003, pg1458

VALEN formulation (continued)

$$\left\{\Phi_w^{wall}\right\}=\left[L_{ww}\right]\left\{I_w\right\}+\left[L_{wc}\right]\left\{I_c\right\}+\left[L_{wp}\right]\left\{I_d\right\}+\left[L_{wp}\right]\left\{I_p\right\}$$

flux on wall elements

$$\left\{\Phi_c^{coil}\right\}=\left[L_{cw}\right]\left\{I_w\right\}+\left[L_{cc}\right]\left\{I_c\right\}+\left[L_{cp}\right]\left\{I_d\right\}+\left[L_{cp}\right]\left\{I_p\right\}$$

flux on coils

$$\left\{\Phi_p^{\substack{plasma\\total}}\right\}=\left[L_{pw}\right]\left\{I_w\right\}+\left[L_{pc}\right]\left\{I_c\right\}+\left[L_{pp}\right]\left\{I_d\right\}+\left[L_{pp}\right]\left\{I_p\right\}=\boxed{[P]}\left\{\Phi_p^{\substack{plasma\\ext}}\right\}$$

$$\left\{\Phi_p^{\substack{plasma\\ext}}\right\}=\left[L_{pw}\right]\left\{I_w\right\}+\left[L_{pc}\right]\left\{I_c\right\}+\left[L_{pp}\right]\left\{I_d\right\}$$

$$\left\{\Phi_s^{sensors}\right\}=\left[L_{sw}\right]\left\{I_w\right\}+\left[L_{sc}\right]\left\{I_c\right\}+\left[L_{sp}\right]\left\{I_d\right\}+\left[L_{sp}\right]\left\{I_p\right\}$$

flux in magnetic sensors

this gives:

$$\left\{I_p\right\}=\left[L_{pp}\right]^{-1}\left[P-1\right]\left\{\left[L_{pw}\right]\left\{I_w\right\}+\left[L_{pc}\right]\left\{I_c\right\}+\left[L_{pp}\right]\left\{I_d\right\}\right\}$$

$\left[P\left(B_i^n(\theta,\phi),\delta W\right)\right]$ ***is 'plasma permeability', the plasma response***

Plasma permeability [P] in single mode VALEN

in single mode VALEN we solve for the current potential $\kappa(\theta,\phi)$ of a single unstable mode B_normal distribution $B^n_u(\theta,\phi)$

$$\vec{j}(\theta,\phi) = \delta(r-a)\nabla\kappa(\theta,\phi)\times\nabla r$$ current density defined by current potential which produces $B^n_u(\theta,\phi)$

$$\boxed{P = \frac{1}{s}}\ \text{where:}\quad s = \frac{-\delta W}{L_B I_B^2/2},\quad s > 0 \text{ is unstable},\ s < 0 \text{ stable}$$

$$\Phi_B^2 = \oint_{\text{surface}} (B^n_u(\theta,\phi))^2 dA \oint_{\text{surface}} dA,\quad I_B = \frac{\oint_{\text{surface}} B^n_u(\theta,\phi)\kappa(\theta,\phi)dA}{\Phi_B},\quad L_B = \frac{\Phi_B}{I_B}$$

notice that plasma response has shape of $B^n_u(\theta,\phi)$

notice that plasma response is greatest for small δW (RFA !)

Plasma permeability [P] in multi mode VALEN

in multi mode VALEN we use collection of dcon $B^n_i(\theta,\phi)$ to define a basis set $\{f_i(\theta,\phi)\}$, these orthonormal functions define $\{\kappa_i(\theta,\phi)\}$, $[L_{pp}]$ & $[P_{pp}]$

$$\text{define inner product}: \quad \langle a,b\rangle = \oint_{\text{surface}} \frac{a(\theta,\phi)b(\theta,\phi)}{S}dA, \quad \text{where} \quad S = \oint_{\text{surface}} dA$$

$$\{f_i(\theta,\phi)\} = \sum_{\#\ \text{modes}} G_{i\alpha}\{B^n_\alpha(\theta,\phi)\} \quad \text{where}: \quad \oint_{\text{surface}} \frac{f_i(\theta,\phi)\, f_j(\theta,\phi)}{S} = \delta_{ij}, \quad \oint_{\text{surface}} f_i(\theta,\phi)dA = 0$$

$$\boxed{[P_{pp}] = [\Lambda_{pp}][L_{pp}]^{-1}} \quad \text{where}: \quad [\Lambda_{pp}]^{-1} = \frac{2[G][\varepsilon][G]^t}{S^2} \quad \text{where } [G] \text{ is gram schmidt xform}$$

$$\text{and}: \quad [\varepsilon] = \begin{bmatrix} \delta W_1 & 0 & \cdots & 0 \\ 0 & \delta W_2 & 0 & \vdots \\ \vdots & 0 & \ddots & 0 \\ 0 & \cdots & 0 & \delta W_{\substack{\text{lst}\\\text{mode}}} \end{bmatrix} \quad \text{or} \quad [\varepsilon] = \begin{bmatrix} \delta W_1 & \Gamma_1 & \cdots & 0 \\ -\Gamma_1 & \delta W_2 & 0 & \vdots \\ \vdots & 0 & \ddots & 0 \\ 0 & \cdots & -\Gamma_{\text{last}} & \delta W_{\substack{\text{lst}\\\text{mode}}} \end{bmatrix}$$

no rotation

with rotation

notice that we may combine $\{f_i\}$ from different 'n' values

notice that plasma response is a weighted sum of the $f_i(\theta,\phi)$

notice that plasma response is greatest for small δW (RFA)

Successful benchmark of mmVALEN NSTX test case – with-wall limit comparison

We compare estimates of the n=1 critical wall position, from DCON & mmVALEN using a complete ideal D-shaped wall. This is required because of the NSTX aspect ratio. (option ishape=5 in VACUUM).

- The DCON zero crossing in δW (black curve use left axis) compares well with vertical part of the mmVALEN dispersion curve (red & green curves, use right axis)
- Fair convergence with 76 modes, calculations continue
- Testing continues, reasonable initial comparison to growth rate in NSTX