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On geometric factors for neutral particle analyzers^{a)}

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Neutral particle analyzers (NPA) detect neutralized energetic particles that escape from plasmas. Geometric factors relate the counting rate of the detectors to the intensity of the particle source. Accurate geometric factors enable quick simulation of geometric effects without the need to resort to slower Monte Carlo methods. Previously derived expressions [G. R. Thomas and D. M. Willis, “Analytical derivation of the geometric factor of a particle detector having circular or rectangular geometry,” *J. Phys. E: Sci. Instrum.* **5**(3), 260 (1972); J. D. Sullivan, “Geometric factor and directional response of single and multi-element particle telescopes,” *Nucl. Instrum. Methods* **95**(1), 5–11 (1971)] for the geometric factor implicitly assume that the particle source is very far away from the detector (far-field); this excludes applications close to the detector (near-field). The far-field assumption does not hold in most fusion applications of NPA detectors. We derive, from probability theory, a generalized framework for deriving geometric factors that are valid for both near and far-field applications as well as for non-isotropic sources and nonlinear particle trajectories. © 2014 AIP Publishing LLC. [<http://dx.doi.org/10.1063/1.4885543>]

I. INTRODUCTION

Neutral particle analyzers (NPA) detect neutralized energetic particles (EP) that escape from plasmas. The neutralized particles provide information about the velocity distribution of the energetic particles which is important for understanding EP driven modes, instabilities, and transport phenomena.

Modeling NPA detector is often used in the analysis of the data and for planning future experiments. Geometric aspects of the NPA detectors such as the aperture size and detector depth affect model predictions. Typically, Monte Carlo methods are used to simulate the geometric effects of the detectors. Alternatively, a geometric factor relates the particle source intensity and the counting rate of the detector by a multiplicative scale factor that depends on the detector geometry. Accurate geometric factors enable quick simulation of the geometric aspects of NPA detector models without the need to resort to computationally costly Monte Carlo methods.

Early work in geometric factors was motivated by astrophysics experiments. Geometric factors for NPA detectors were adapted from these early results. In Sec. II we show that previously derived expressions^{1,2} for the geometric factor, while perfectly adequate for astrophysical systems, cannot be applied to NPA detectors due to near-field effects.

Previous expressions for geometric factors can only be applied to systems with isotropic particle sources that move in straight lines. In Sec. III we derive, from probability theory, a generalized framework for calculating geometric factors for systems that are non-isotropic and/or nonlinear.

II. THE FAR-FIELD ASSUMPTION

The geometric factor, f_g , of a detector is proportional to its solid angle. The geometric factor is then given by

$$f_g = \frac{1}{4\pi} \iint_S \frac{\mathbf{r} \cdot \hat{\mathbf{n}} dS}{r^3}, \quad (1)$$

where S is the viewable detector area. For most cases this equation cannot be solved analytically.

NPA detectors with circular geometries can be described by three parameters: the aperture radius (R_a), the detector radius (R_d), and the separation between the aperture and the detector (H). At some positions the aperture cuts off portions of the detector; reducing the detecting surface S and complicating the solid angle calculation. Thomas and Willis¹ calculated the detecting surface S by projecting the “shadow” created by the aperture onto the detector. This “aperture shadow” is parametrized by the angle of incident flux θ . For an isotropic distribution of incident particles, the total geometric factor of the detector is given by

$$f_g = 2\pi \int_0^{\theta_m} S(\theta) \sin \theta \cos \theta d\theta, \quad (2)$$

where θ_m is the maximum angle of incidence. This expression (and others like it) have been used in analyzing NPA detectors for decades.

When simulating NPA detectors, Eq. (2) is of limited use since it does not parametrize the aperture shadow S by position. We can parametrize the aperture shadow by position by circumscribing the aperture onto the detector plane. Defining the detector to be on the $z = 0$ plane with the normal vector co-linear with the z -axis, a source at point (x_p, y_p, z_p) projects a circle onto the detecting plane with radius and center given by

$$R_s = \frac{R_a z_p}{z_p - H} \quad (3)$$

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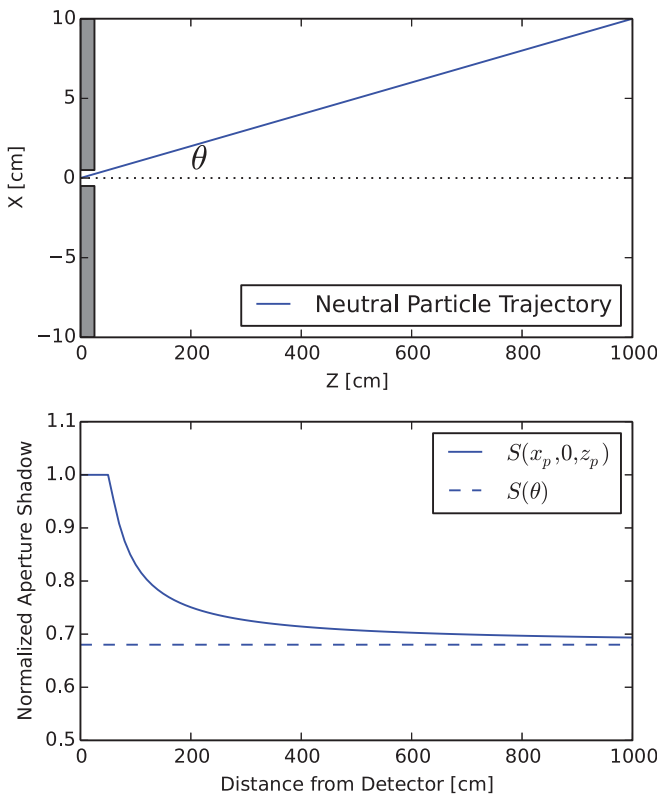


FIG. 1. Top: Trajectory of a neutral particle hitting a circular NPA detector with $R_a = R_d = 0.5$ cm, and $H = 25.4$ cm. Bottom: Normalized aperture shadow along the particle trajectory.

and

$$\vec{r}_{center} = \left(\frac{Hx_p}{H - z_p}, \frac{Hy_p}{H - z_p}, 0 \right), \quad (4)$$

where $z_p > H$. Integrating over the intersection of this circle and the detector circle gives an analytic expression for the aperture shadow $S(x_p, y_p, z_p)$.

The two expressions for the aperture shadow ($S(\theta)$ and $S(x_p, y_p, z_p)$) should be equivalent. However, comparing the two expressions reveals a discrepancy. Figure 1 shows that at a constant angle of incidence, the shadow area, $S(x_p, y_p, z_p)$ asymptotically approaches $S(\theta)$ as the distance from the detector increases. The discrepancy arises because the expression $S(\theta)$ implicitly assumes that the particle source is far away from the detector (far-field assumption) and can be parametrized by a single angle of incidence.

Consider an isotropically emitting source. Far away from the detector the choice of angle of incidence θ is trivial since they are approximately the same for every trajectory that strikes the detector. When the source is near the detector the choice of angle of incidence is not as clear since there are many possible particle trajectories each with a significantly different angle of incidence. As the particle source moves away from the detector the possible angles of incidence approach a single value.

In many fusion applications of NPA detectors the particle source is relatively close to the detector aperture; violating the requirements needed to use the θ parametrized geometric

factor. Instead, a brute force application of Eq. (1) over the aperture shadow should be used.

III. PROBABILISTIC FRAMEWORK FOR DERIVING GEOMETRIC FACTORS

The geometric factor can be thought of as the probability of a particle hitting the detector from a point $\vec{p}$ above the detector. The problem of calculating the geometric factor of the detector becomes the exercise of mapping the probability density function at the source onto the detector plane $z = 0$. In general, the mapping from $\mathbf{Y}$ to $\mathbf{X}$ space is done through the change in variable equation

$$prob(\mathbf{X}|I) = prob(\mathbf{Y}|I) \times \left| \frac{\partial \mathbf{Y}}{\partial \mathbf{X}} \right|, \quad (5)$$

where the term on the far right is the Jacobian of the transformation.³

In the case of NPA detectors, the transformation is from $\{\phi, \theta\}$ -space to $\{x, y\}$ -space. For linear trajectories the transformation is given by

$$\begin{bmatrix} x \\ y \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & \tan \theta \cos \phi \\ 0 & 1 & \tan \theta \sin \phi \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_p \\ y_p \\ z_p \end{bmatrix}, \quad (6)$$

where (x_p, y_p, z_p) is the position of the particle source. The Jacobian for this transformation is given by

$$\left| \frac{\partial(\phi, \theta)}{\partial(x, y)} \right| = \frac{z_p((x - x_p)^2 + (y - y_p)^2)^{-1/2}}{(x - x_p)^2 + (y - y_p)^2 + z_p^2}. \quad (7)$$

For an isotropic source the probability density function in $\{\phi, \theta\}$ -space is given by

$$prob(\phi, \theta|I) = \frac{1}{4\pi} \sin \theta. \quad (8)$$

Plugging Eqs. (7) and (8) into the change of variable equation and integrating over the aperture shadow $S(x_p, y_p, z_p)$ yields the geometric factor:

$$f_g = \frac{1}{4\pi} \iint_S \frac{z_p dS}{((x - x_p)^2 + (y - y_p)^2 + z_p^2)^{3/2}}. \quad (9)$$

This equation is equivalent to Eq. (1) which is expected since the solid angle calculation implicitly assumes an isotropic source with linear particle trajectories. The advantage of interpreting the geometric factor as a probability is that it can generalize to non-isotropic sources and to nonlinear particle trajectories.

For example, consider a non-isotropic neutral particle source with the following probability density function:

$$prob(\phi, \theta|I) = \frac{1}{\pi^2} \cos^2 \theta. \quad (10)$$

Following the same procedure that was used to derive Eq. (9) yields the following geometric factor:

$$f_g = \frac{1}{\pi^2} \iint_S \frac{z_p^3 ((x - x_p)^2 + (y - y_p)^2)^{-1/2} dS}{((x - x_p)^2 + (y - y_p)^2 + z_p^2)^2}. \quad (11)$$

In the case of charged particle detection, geometric factors for curved trajectories can be derived by defining transformation equations similar to Eq. (6). However, realistic equations are difficult to solve in practice.

IV. SUMMARY

It has been shown that the far-field assumption that is ubiquitously used in the calculation of NPA geometric factors is invalid for many fusion applications. It was also shown that interpreting the geometric factor as a probability leads

to a generalized framework for deriving geometric factors for non-isotropic sources and nonlinear particle trajectories.

¹G. R. Thomas and D. M. Willis, "Analytical derivation of the geometric factor of a particle detector having circular or rectangular geometry," *J. Phys. E: Sci. Instrum.* **5**(3), 260 (1972).

²J. D. Sullivan, "Geometric factor and directional response of single and multi-element particle telescopes," *Nucl. Instrum. Methods* **95**(1), 5–11 (1971).

³The transformation $X = H(Y)$ is subject to the constraint that H is bijective and differentiable. If H is not bijective Eq. (5) can be extended to a summation over all the Y values that correspond to a given X .