M3D Simulation Studies of NSTX

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Outline

- M3D code

  MHD, two-fluids, hybrid models.

- NSTX studies including flow effects

  2D steady states.

  Evolutions of IRE's.

  BAE modes.

M3D Project

W. Park et al., Phys. Plasmas **6,** 1796 (1999)
http://w3.pppl.gov/~wpark/pop_99.pdf

Multilevel 3D Project for Plasma Simulation studies
Various physics levels are needed to understand the physics.
The best method depends on the problem at hand.

Physics	Geometry	State
MHD 2 Fluids Gyrokin. Hot P./MHD Gyrokin.Ion/Fluid Elect. ….	MPP Serial Unstructured FE Structured FD	Equilibrium Linear Nonlinear

MHD model

- Solves MHD equations.

$$\rho\partial \mathbf{v}/\partial t + \rho\mathbf{v}\cdot\nabla\mathbf{v} = -\nabla p + \mathbf{J}\times\mathbf{B} + \mu\nabla^2\mathbf{v}$$

$$\partial\mathbf{B}/\partial t = -\nabla\times\mathbf{E}, \quad \mathbf{E} = (-\mathbf{v}\times\mathbf{B} + \eta\mathbf{J}), \quad \mathbf{J} = \nabla\times\mathbf{B}$$

$$\partial\rho/\partial t + \nabla\cdot(\rho\mathbf{v}) = 0$$

$$\partial p/\partial t + \mathbf{v}\cdot\nabla p = -\gamma p\nabla\cdot\mathbf{v} + \rho\nabla\cdot\kappa\nabla(p/\rho)$$

The fast parallel equilibration of T is modeled using wave equations;

$$\partial T/\partial t = s\,\mathbf{B}/\rho\cdot\nabla u$$

$$\partial u/\partial t = s\,\mathbf{B}\cdot\nabla T + \upsilon\nabla^2 u$$

s = wave speed/ v_A

Two-fluid MH3D-T

- Solves the two fluid equations with gyro-viscousity and neoclassical parallel viscousity terms in a torus.

- **Equations**

$$\mathbf{v} \equiv \mathbf{v}_i - \mathbf{v}_i^* = \mathbf{v}_e - \mathbf{v}_e^* + \mathbf{J}_\parallel/en,$$

$$\mathbf{v}_e^* \equiv -\mathbf{B}\times\nabla p_e/(enB^2), \quad \mathbf{v}_i^* \equiv \mathbf{v}_e^* + \mathbf{J}_\perp/en,$$

$$\rho\partial\mathbf{v}/\partial t + \rho\mathbf{v}\cdot\nabla\mathbf{v} + \rho(\mathbf{v}_i^*\cdot\nabla)\mathbf{v}_\perp = -\nabla p + \mathbf{J}\times\mathbf{B} - \mathbf{b}\cdot\nabla\cdot\Pi_i,$$

$$\partial\mathbf{B}/\partial t = -\nabla\times\mathbf{E}, \quad \mathbf{E} = (-\mathbf{v}\times\mathbf{B} + \eta\mathbf{J}) - \nabla_\parallel p_e/en - \mathbf{b}\cdot\nabla\cdot\Pi_e,$$

$$\mathbf{J} = \nabla\times\mathbf{B},$$

$$\partial\rho/\partial t + \nabla\cdot(\rho\mathbf{v}_i) = 0,$$

$$\partial p/\partial t + \mathbf{v}\cdot\nabla p = -\gamma p\nabla\cdot\mathbf{v} + \rho\nabla\cdot\kappa_\parallel\nabla_\parallel(p/\rho) - \mathbf{v}_i^*\cdot\nabla p + (1/en)\mathbf{J}\cdot\nabla p_e - \gamma p\nabla\cdot\mathbf{v}_i^* + \gamma p_e\mathbf{J}\cdot\nabla(1/en)$$

$$\partial p_e/\partial t + \mathbf{v}\cdot\nabla p_e = -\gamma p_e\nabla\cdot\mathbf{v} + \rho\nabla\cdot\kappa_\parallel\nabla_\parallel(p_e/\rho) + (1/en)\mathbf{J}_\parallel\cdot\nabla p_e - \gamma p_e\nabla\cdot(\mathbf{v}_e^* - \mathbf{J}_\parallel/en)$$

GK Hot Particle /MHD Hybrid MH3D-K

- **Fluid equations**

$\rho\partial \mathbf{v}/\partial t + \rho \mathbf{v}\cdot\nabla \mathbf{v} = -\nabla p - (\nabla\cdot \mathbf{P}_h)_\perp + \mathbf{J}\times\mathbf{B}$ (Pressure coupling)

or

$\rho\partial \mathbf{v}/\partial t + \rho \mathbf{v}\cdot\nabla \mathbf{v} = -\nabla p + (\nabla\times\mathbf{B} - \mathbf{J}_h)\times\mathbf{B} + q_h \mathbf{V}\times\mathbf{B}$

(Current coupling)

$\partial \mathbf{B}/\partial t = -\nabla\times\mathbf{E}, \quad \mathbf{E} = \mathbf{v}\times\mathbf{B} - \eta(\mathbf{J} - \mathbf{J}_h), \quad \mathbf{J} = \nabla\times\mathbf{B}$

$\partial\rho/\partial t + \nabla\cdot(\rho\mathbf{v}) = 0$

$\partial p/\partial t + \mathbf{v}\cdot\nabla p = -\gamma p \nabla\cdot\mathbf{v} + \rho\nabla\cdot\kappa\cdot\nabla\,(p/\rho)$

- **Gyrokinetic equations for energetic particles**

$d\mathbf{R}/dt = u[\,\mathbf{b} + (u/\Omega)\mathbf{b}\times(\mathbf{b}\cdot\nabla\mathbf{b})\,] + (1/\Omega)\mathbf{b}\times(\mu\nabla\mathbf{B} - q\mathbf{E}/m),$

$du/dt = -[\,\mathbf{b} + (u/\Omega)\mathbf{b}\times(\mathbf{b}\cdot\nabla\mathbf{b})\,]\cdot(\mu\nabla\mathbf{B} - q\mathbf{E}/m).$

GK Particle Ion / Fluid Electron Hybrid

- **Pressure coupling**

$$\rho\partial \mathbf{v}/\partial t + \rho \mathbf{v}\cdot\nabla \mathbf{v} = -\nabla\cdot\mathbf{P}\mathrm{i} - \nabla \mathrm{Pe} + \mathbf{J}\times\mathbf{B}$$

$$= -\nabla\cdot\mathbf{P}\mathrm{i}^{CGL} - \nabla\cdot\Pi\mathrm{i} - \nabla\mathrm{Pe} + \mathbf{J}\times\mathbf{B}$$

$\nabla\cdot\mathrm{Pi}^{CGL}$: from particles following GK eqns.

$\nabla\cdot\Pi\mathrm{i}$: fluid picture as 2 fluid eqns,

or from particles.

- **Fluid electrons**

$$\mathbf{E} = -\mathbf{V}_e\times\mathbf{B} + \eta\mathbf{J} + \nabla\cdot\mathbf{P}_e/ne$$

$$= -\mathbf{V}_e\times\mathbf{B} + \eta\mathbf{J} + \nabla P_e/ne + \mathbf{bb}\cdot\nabla\cdot\Pi_e/ne$$

$$\partial \mathbf{B}/\partial t = -\nabla\times\mathbf{E}, \quad \mathbf{J} = \nabla\times\mathbf{B}$$

P_e eqn currently, but $P_{\parallel}$ and $P_\perp$ eqns are planned.

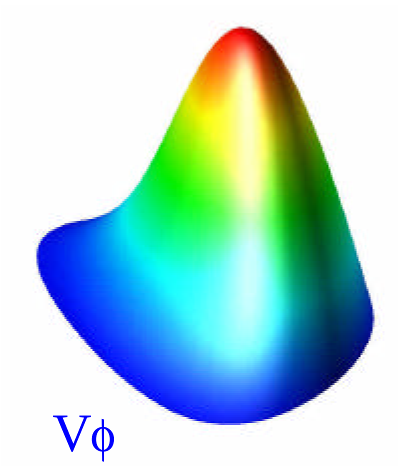
2D steady state with toroidal sheared flow

Quasi neutrality: $r\mathbf{V}\bullet\nabla\mathbf{V} + \nabla\bullet\overleftrightarrow{\mathbf{P}} - \mathbf{J}\times\mathbf{B} = 0$

$$\overleftrightarrow{\mathbf{P}} = \overleftrightarrow{\mathbf{P}}^{CGL} + \overleftrightarrow{\Pi}_g$$

$$= p\overleftrightarrow{I} + (P_{\parallel} - P_{\perp})\overleftrightarrow{\Pi}_{ii} + \overleftrightarrow{\Pi}_g$$

MHD Hot Particle/MHD 2-Fluids



MHD:

At the magnetic axis: $\mathbf{J}\times\mathbf{B} = 0$

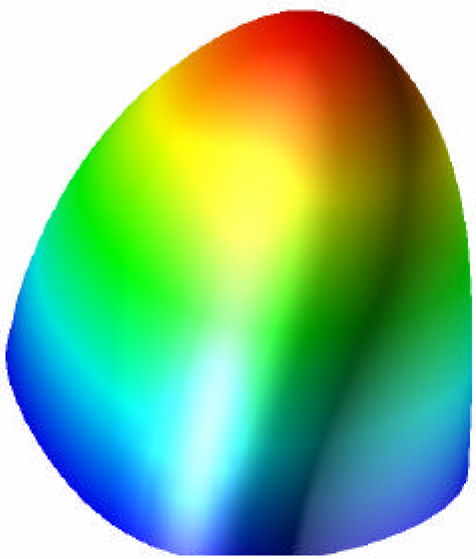
$$-\frac{r\mathrm{V}_f^{\,2}}{R} + \frac{T\partial r}{\partial R} = 0$$

Relative shift of $r \equiv \frac{R\partial r}{r\partial R} = \frac{\mathrm{V}_f^{\,2}}{T} = \frac{2M_A^{\,2}}{b}$

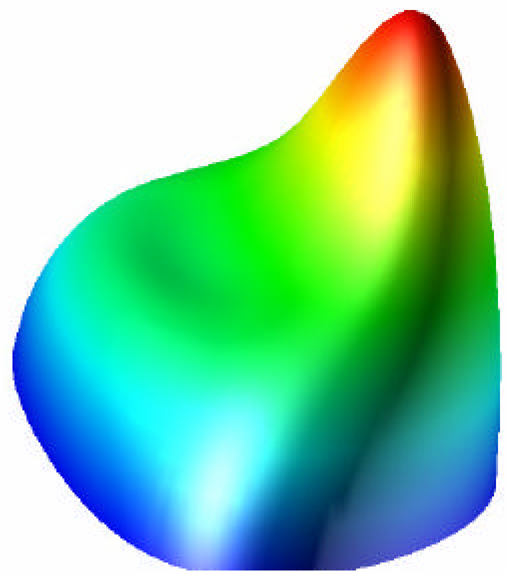
Density profile dependence on sheared Rotation

MHD

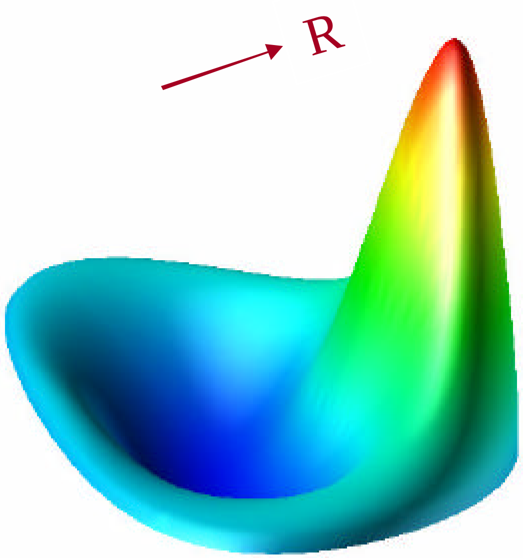
$\varepsilon=1.3$ $q_o=0.8$ $q_b=5$



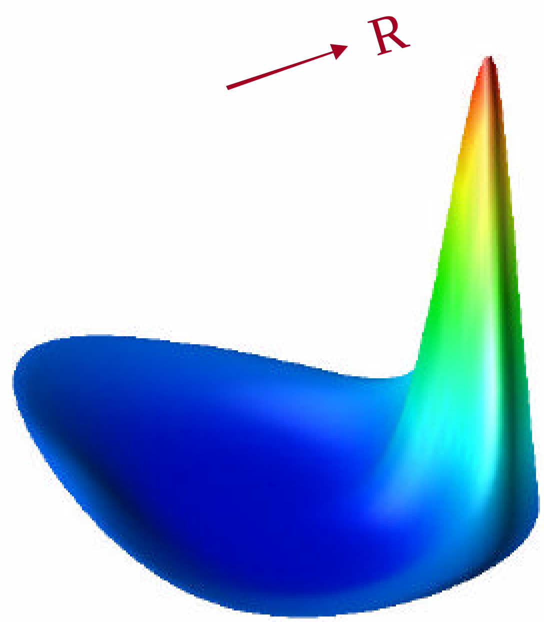
$M_A=0$
Sh=0
$\rho_{max}=1$
$\rho_{min}=0.5$



$M_A=0.2$
Sh=0.3
$\rho_{max}=1.1$
$\rho_{min}=0.5$



$M_A=0.5$
Sh=0.4-0.07=0.33
$\rho_{max}=1.9$
$\rho_{min}=0.2$

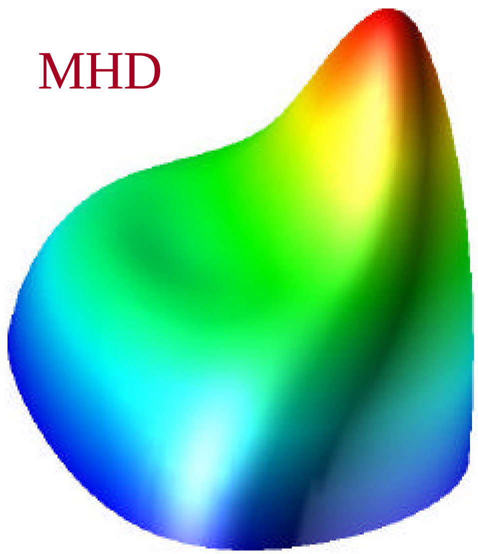


$M_A=0.8$
Sh=0.5-0.15=0.35
$\rho_{max}=5.2$
$\rho_{min}=0.005$

Density profile dependence on Physics model

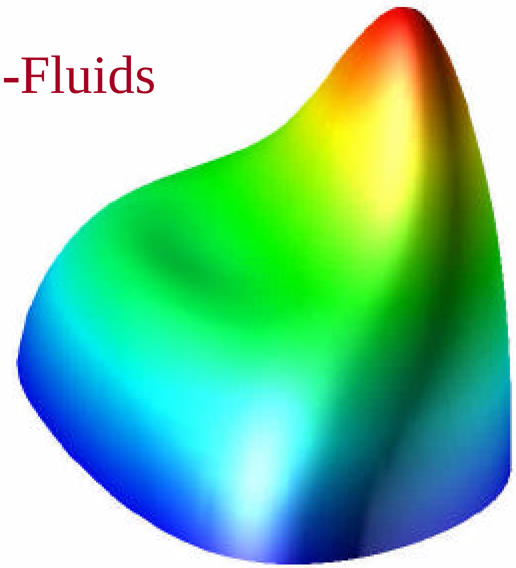
NSTX ε=1.3 q_o=0.8 q_b=5

MHD



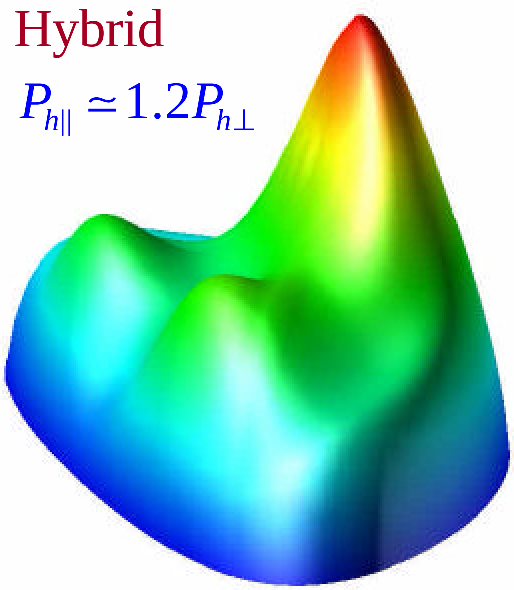
M_A=0.2
Sh=0.3
ρ_{max}=1.1
ρ_{min}=0.5
RelSh=1

Two-Fluids



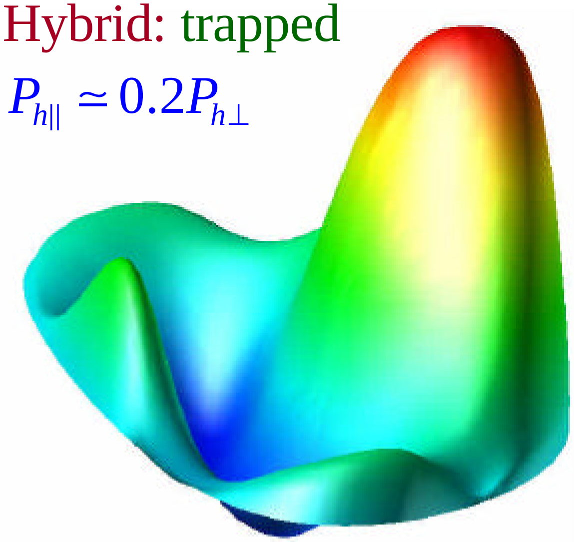
M_A=0.2
Sh=0.3
ρ_{max}=1.1
ρ_{min}=0.5
RelSh=1

Hybrid
$P_{h\|} \simeq 1.2 P_{h\perp}$



M_A=0.2
Sh=0.3
ρ_{max}=1.2
ρ_{min}=0.5
RelSh=0.8

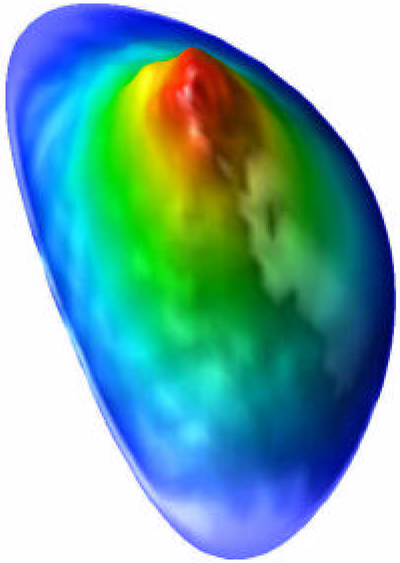
Hybrid: trapped
$P_{h\|} \simeq 0.2 P_{h\perp}$

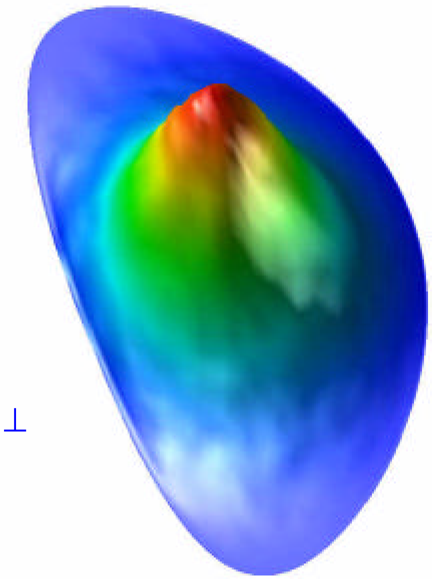


M_A=0.2
Sh=0.3
ρ_{max}=1.8
ρ_{min}=0.15
RelSh=1.9

Hot particle pressure $\mathbf{P}_h$ in the hybrid simulation

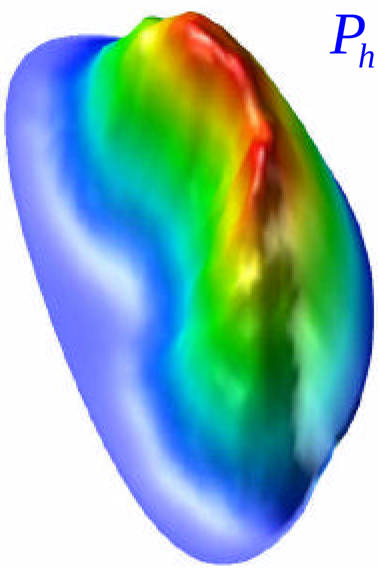
$P_{h\|}$ $\quad$ $P_{h\perp}$

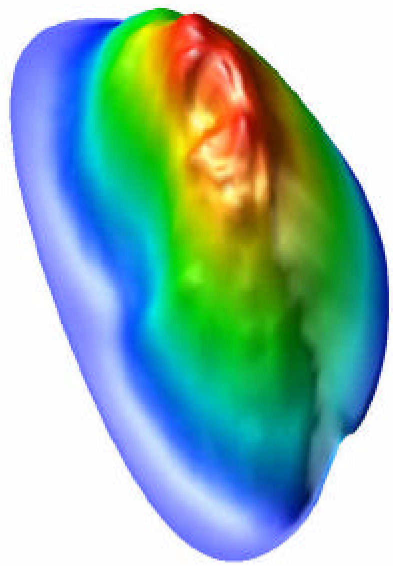




$P_{h\|} \simeq 1.2 P_{h\perp}$

Similar to
Experimental situation

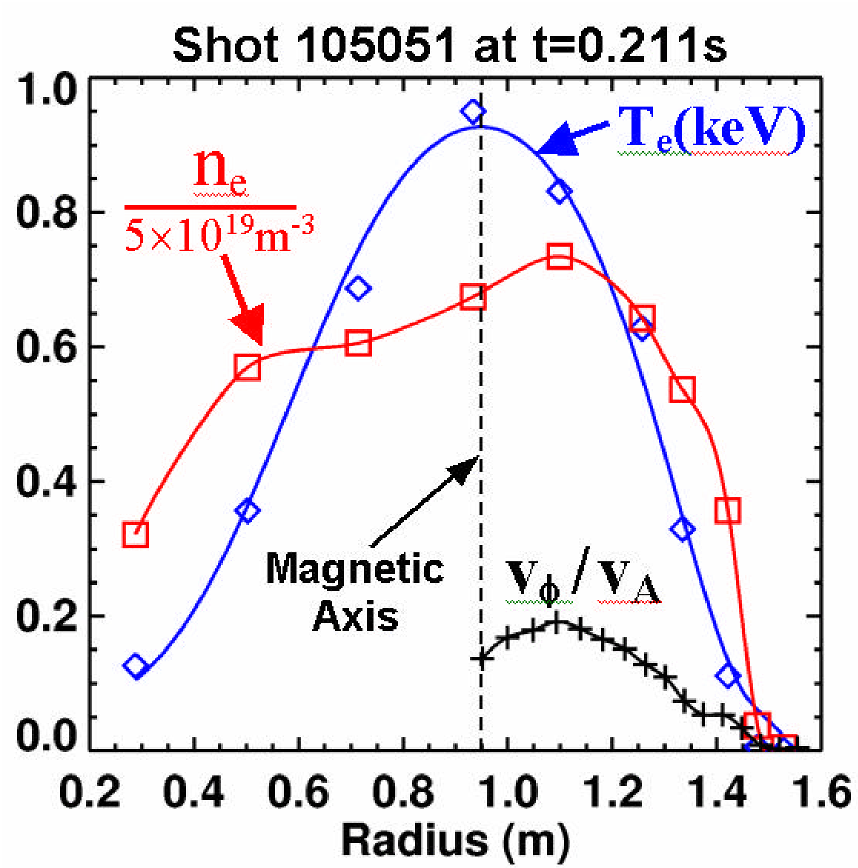




$P_{h\|} \simeq 0.2 P_{h\perp}$

Mostly trapped particles

NSTX experimental data

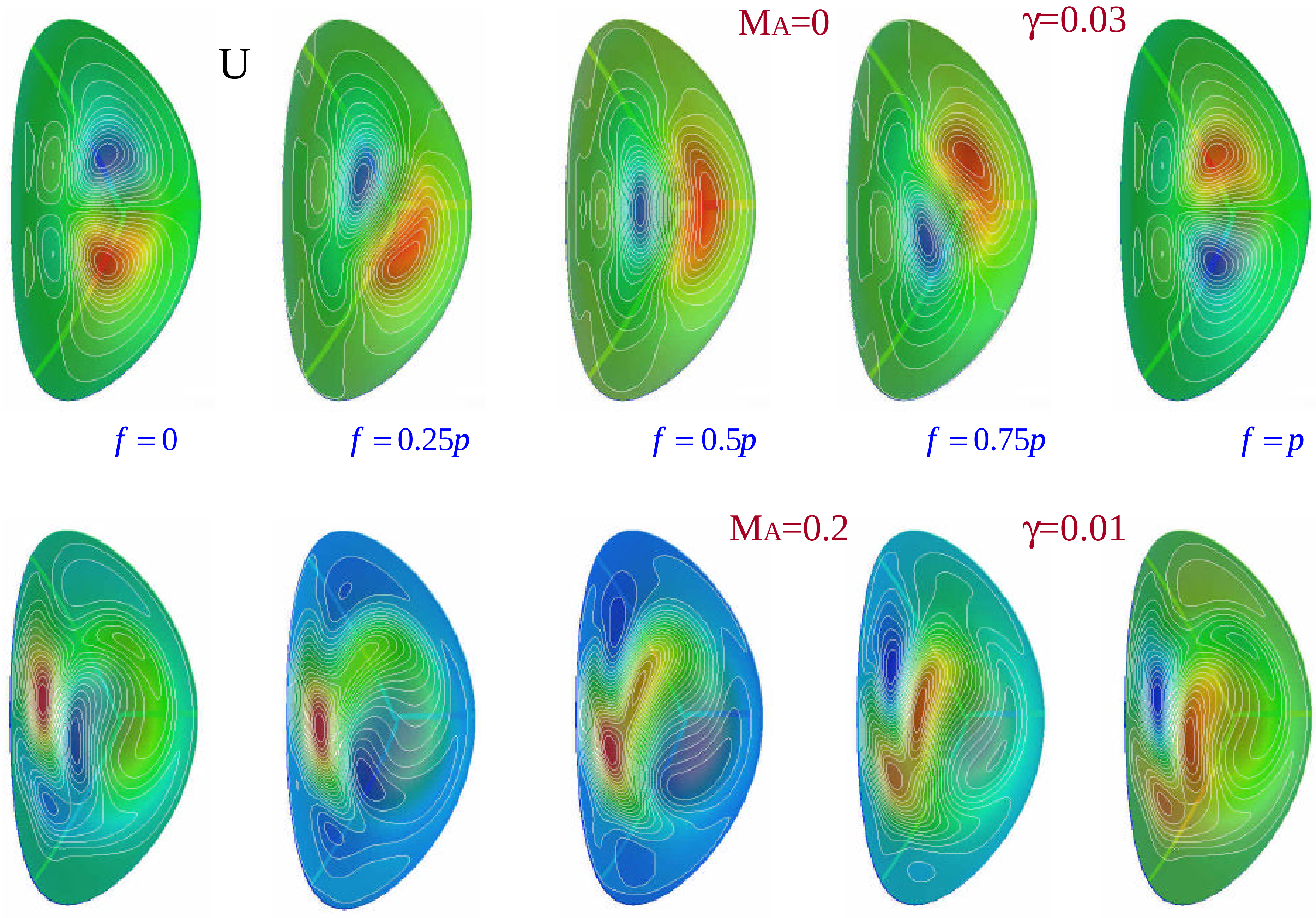


Relative shift of r

$$\frac{R\partial r}{r\partial R} = \frac{2M_A^2}{b}$$

Hot particle centrifugal force ~ Bulk plasma

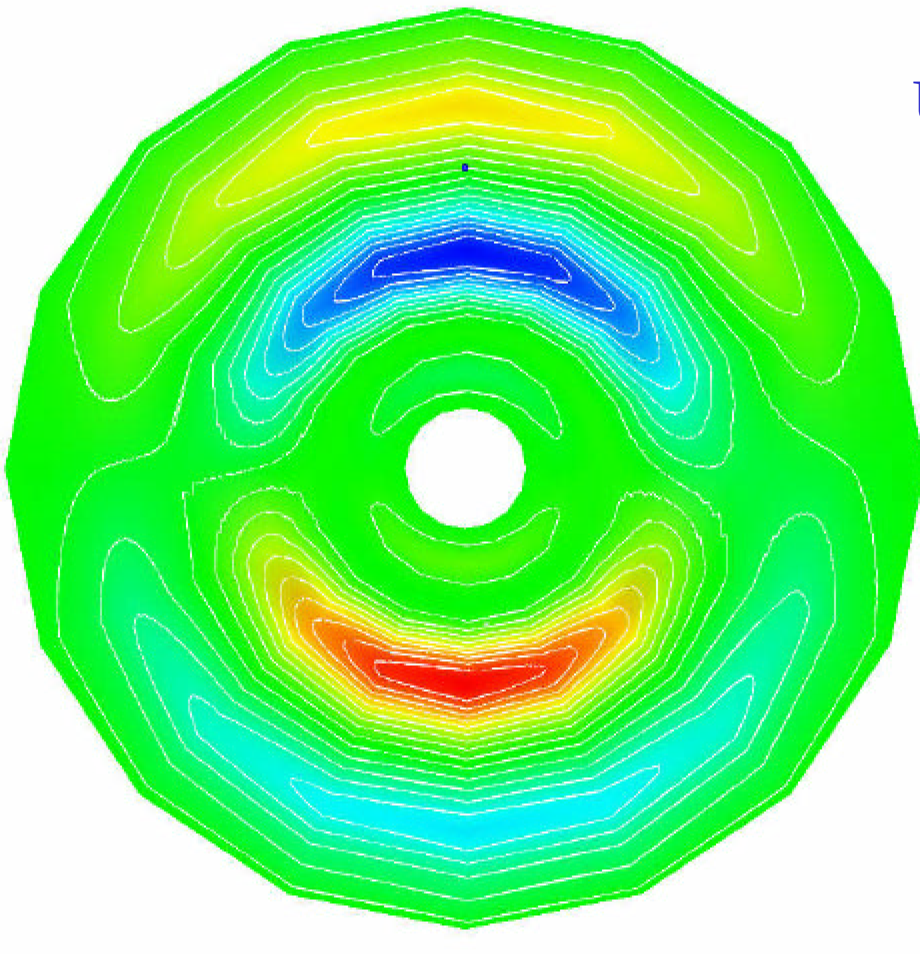
Linear Eigenmodes: shear flow reduces growth rate



Linear Eigenmodes

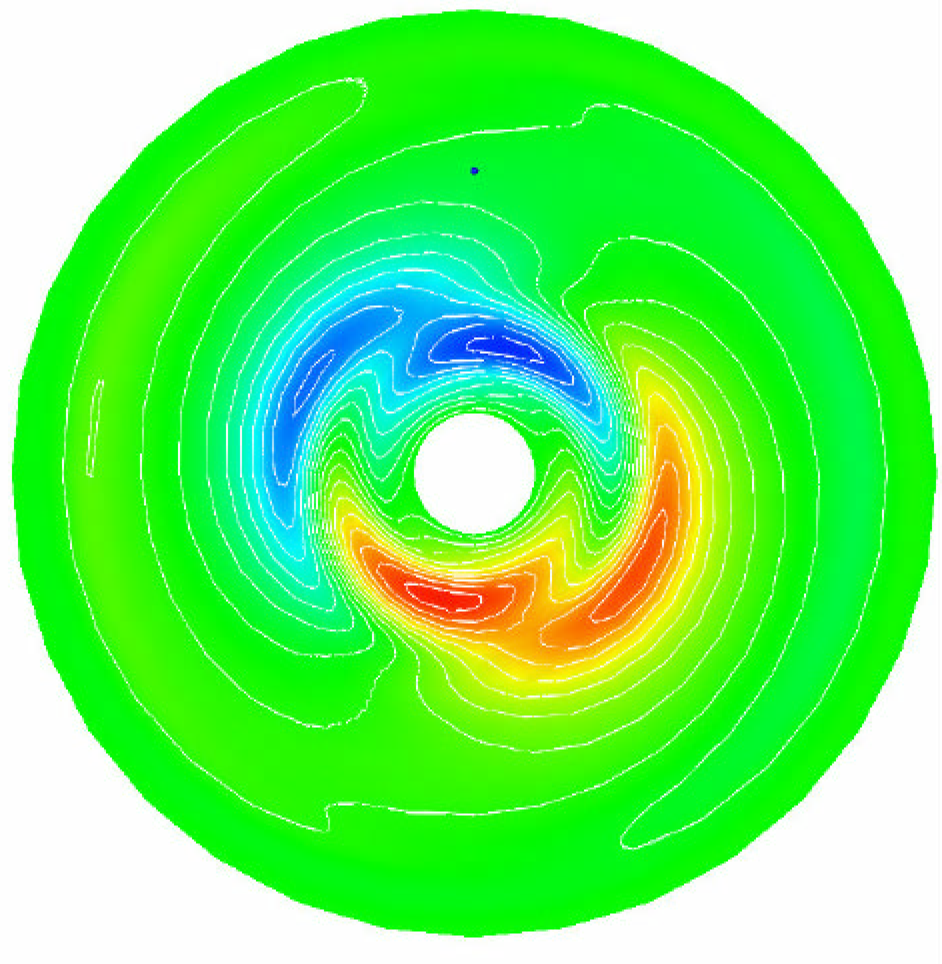
Top view on the mid-plane

$M_A=0$
Ωm=0

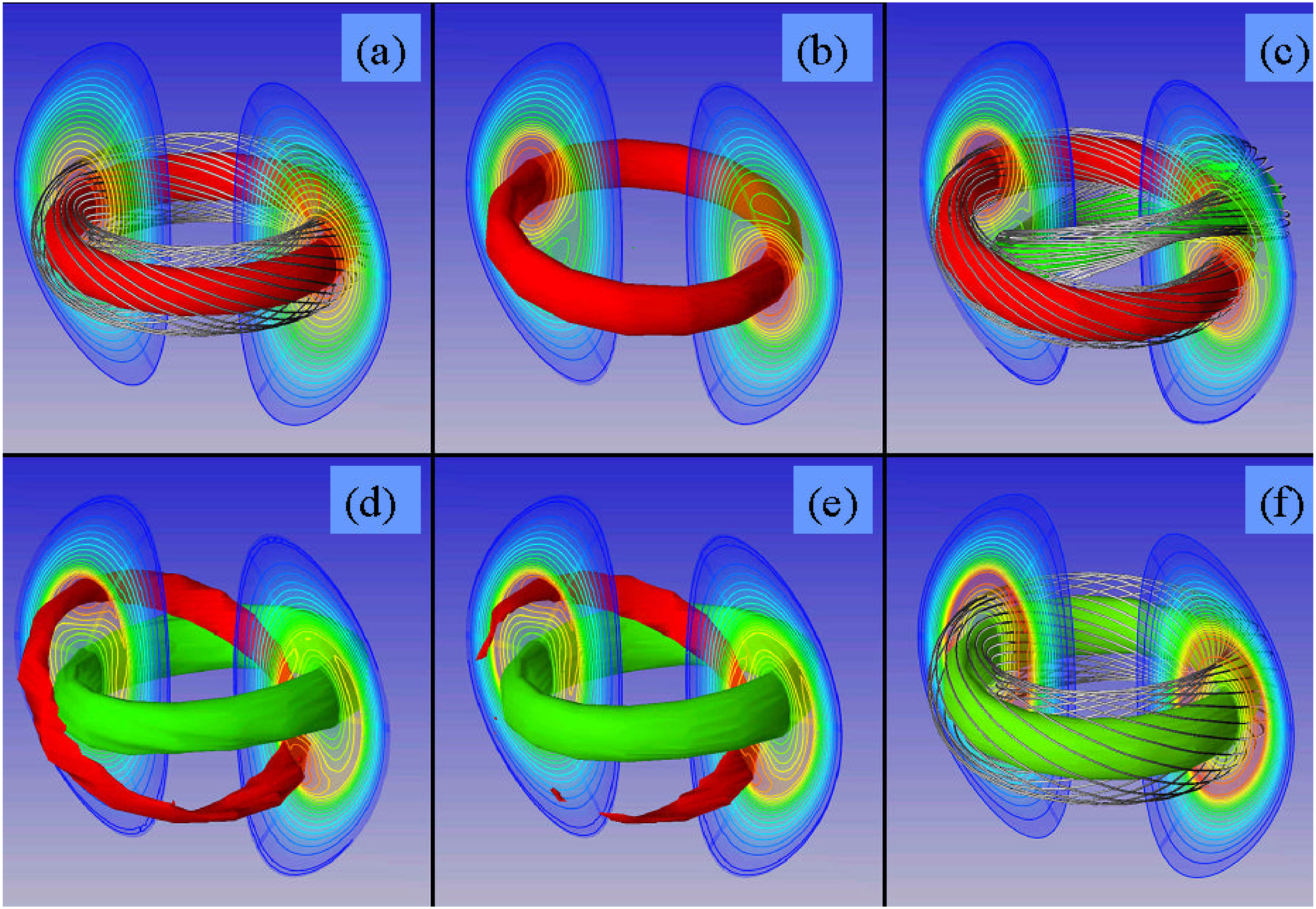


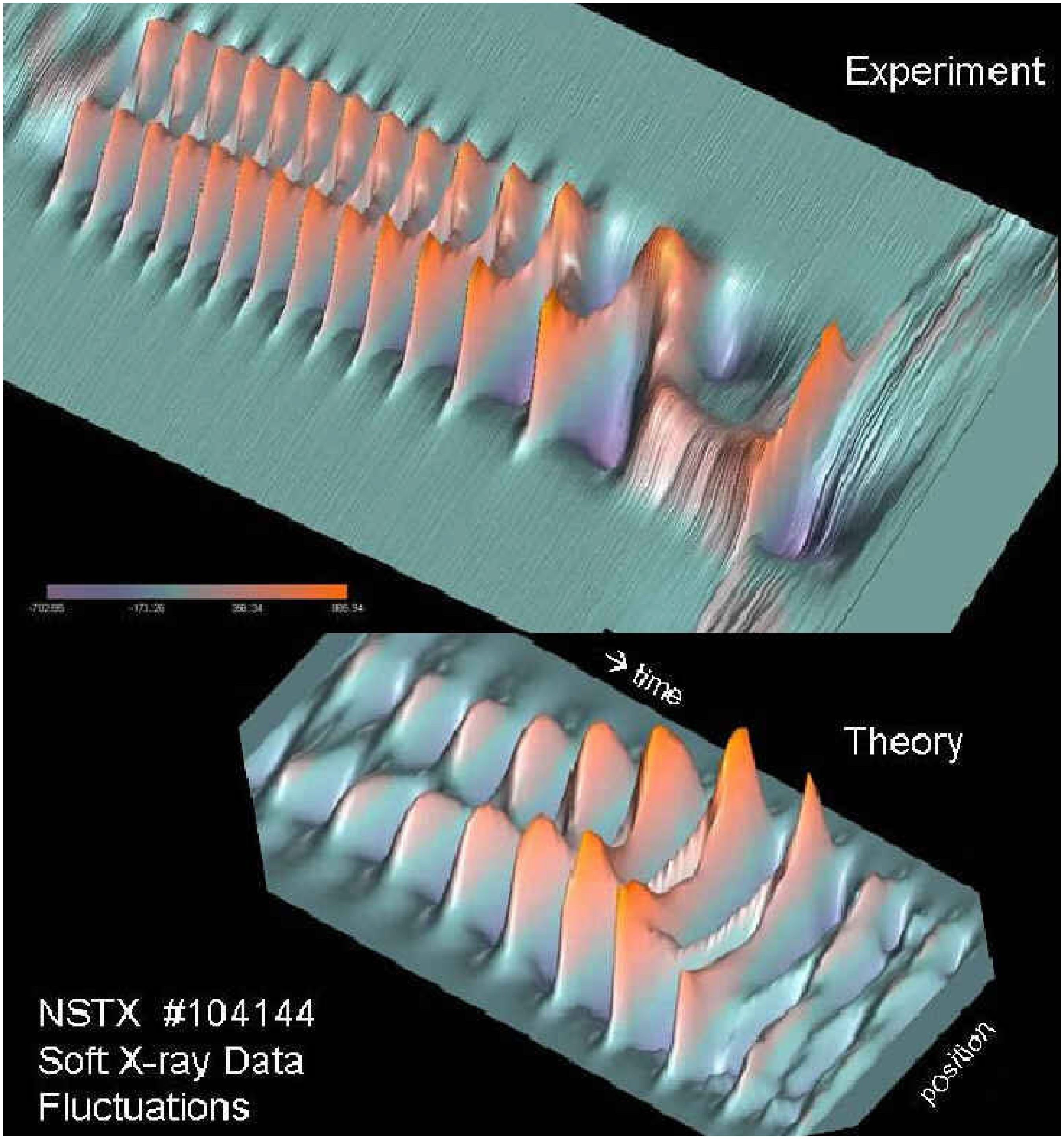
U

With shear flow: $M_A=0.2$
Rotating mode: Ωm=0.13



Nonlinear Evolution without strong flow: similar to a sawtooth crash

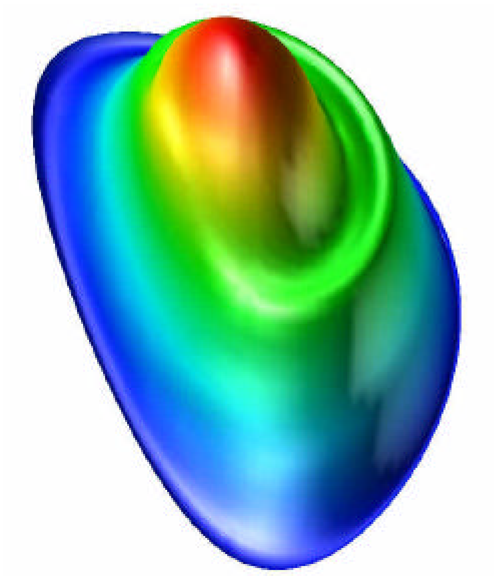




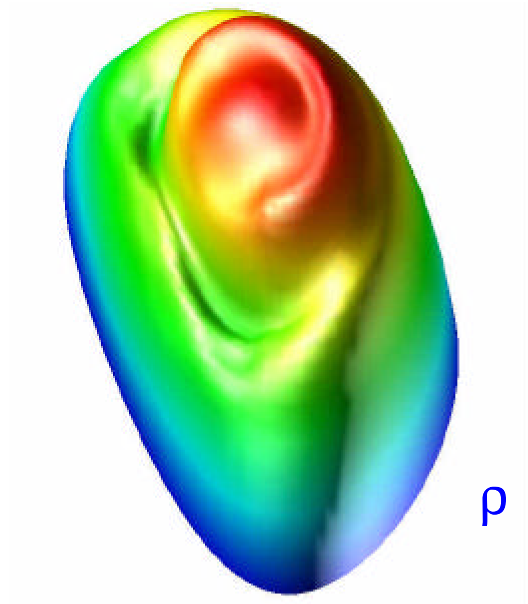
Soft X-ray signals compared:

Theory agrees with experiment on general characters, but does not have wall locking and a saturation phase.

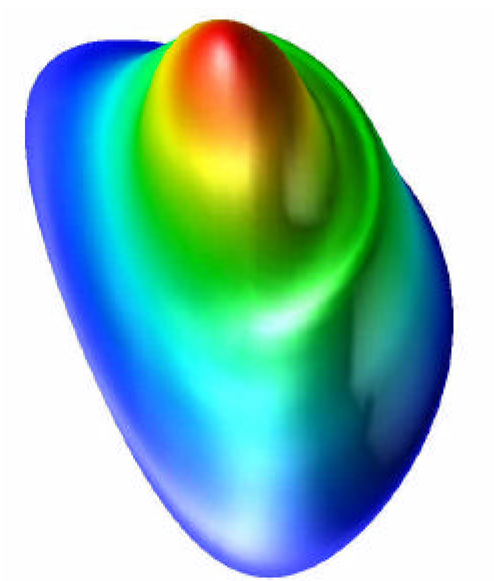
Nonlinear Evolution without strong flow



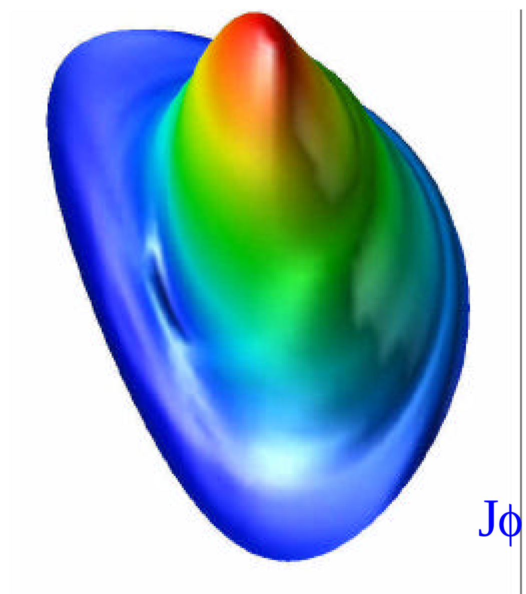
T



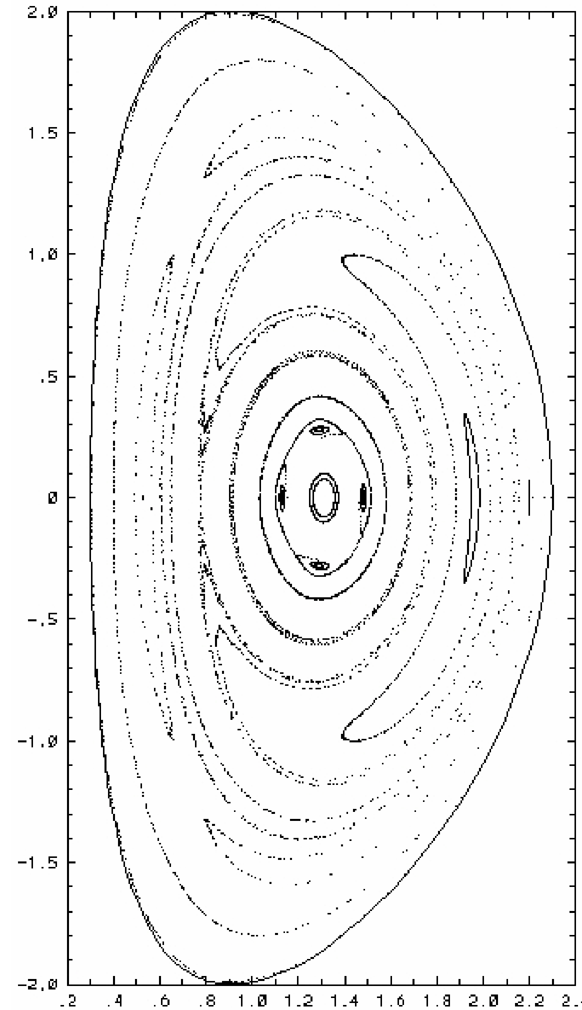
ρ

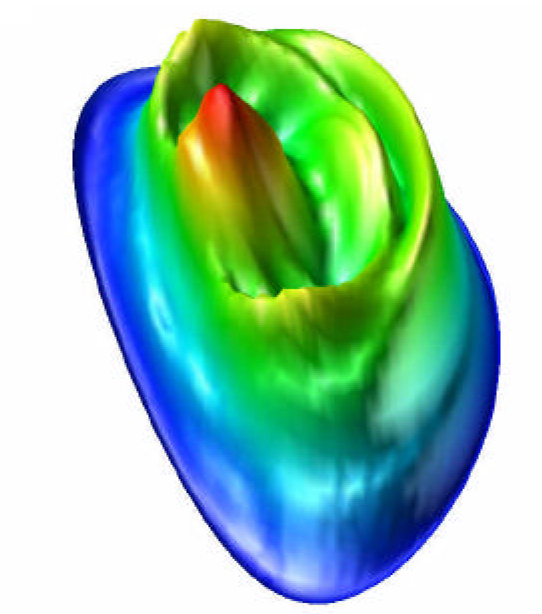


P

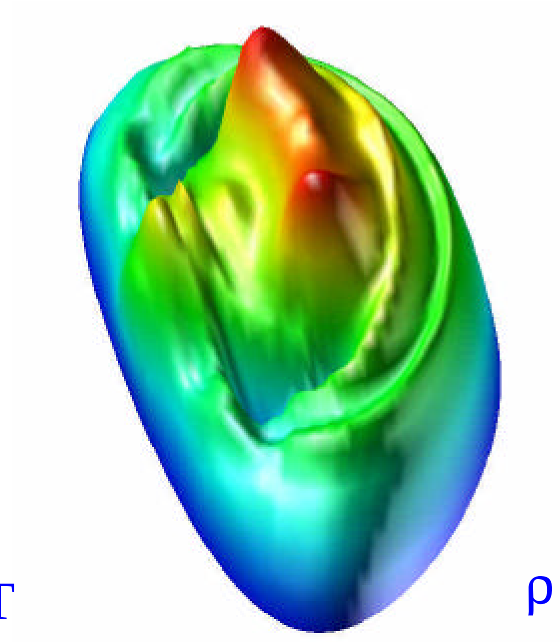


Jϕ

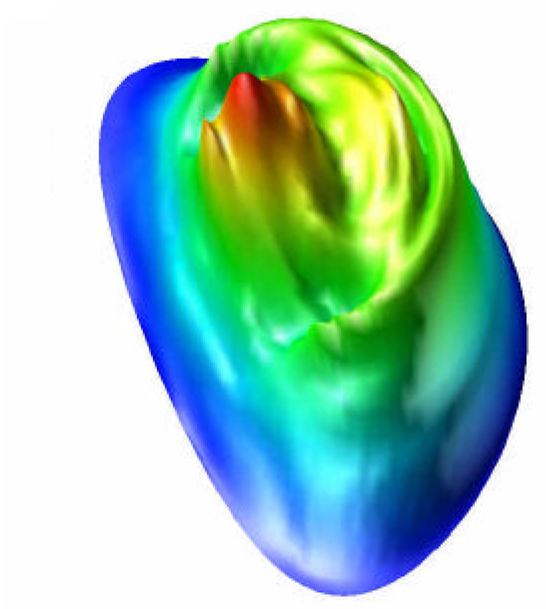




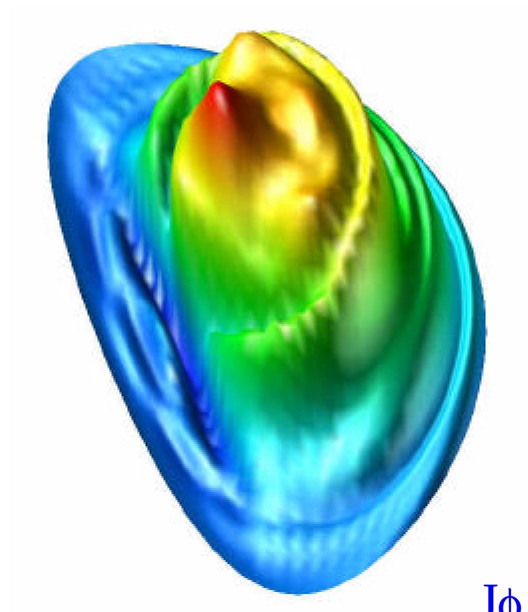
T



ρ

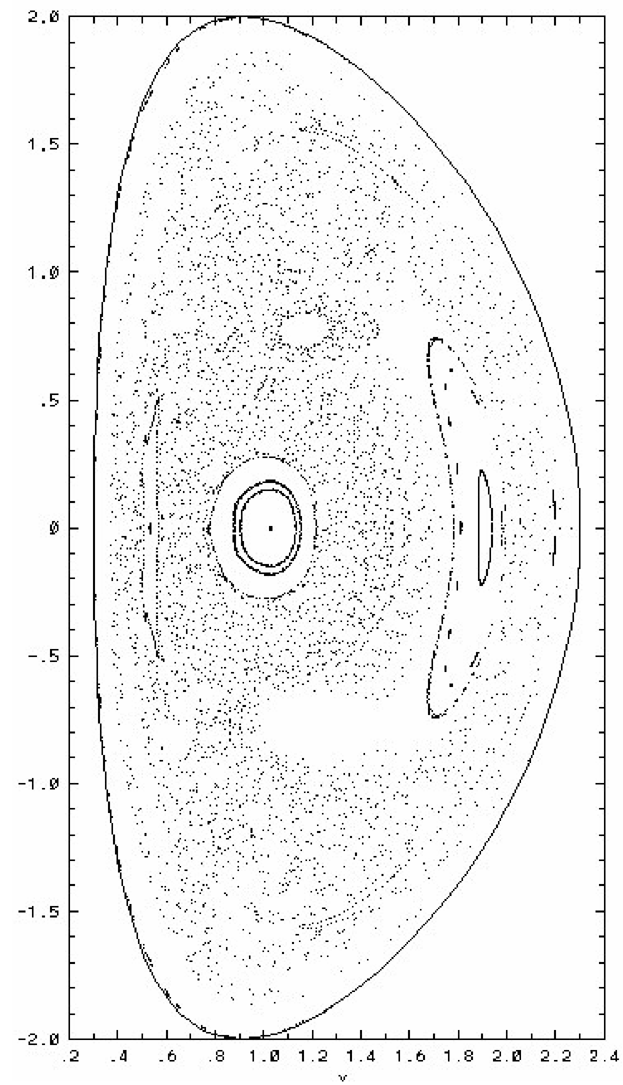


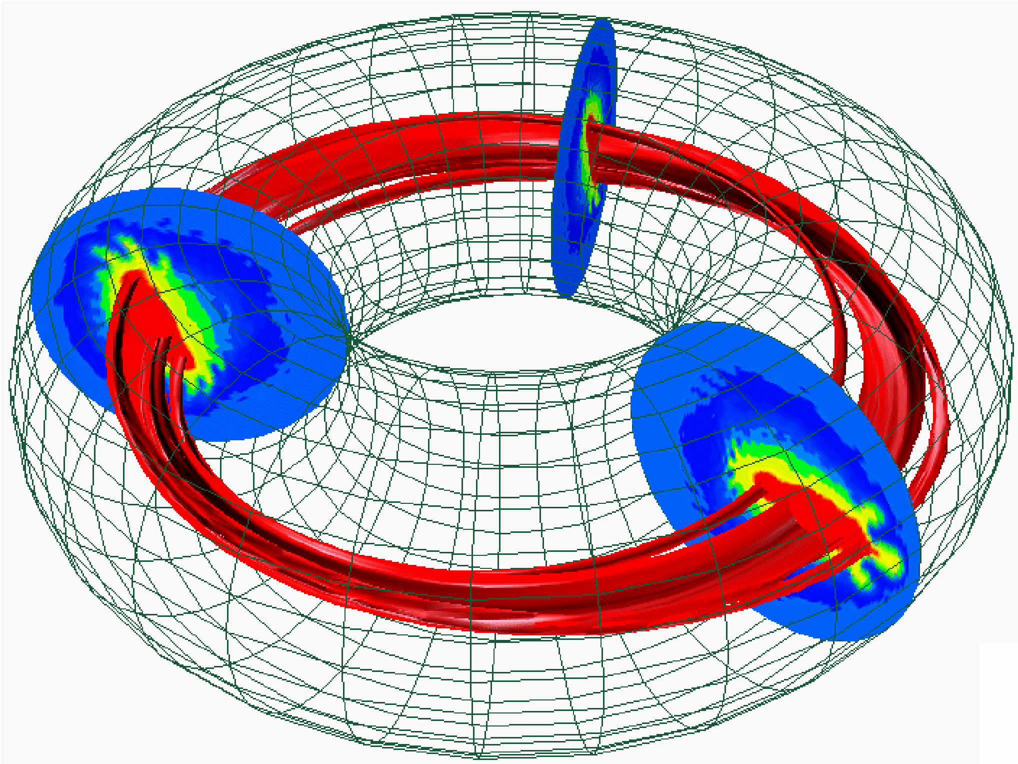
P



Jϕ

mostly stochastic

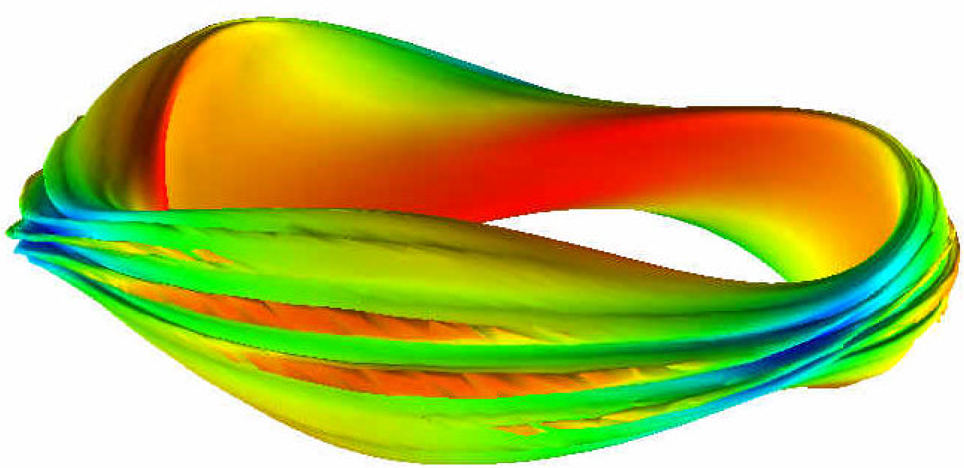


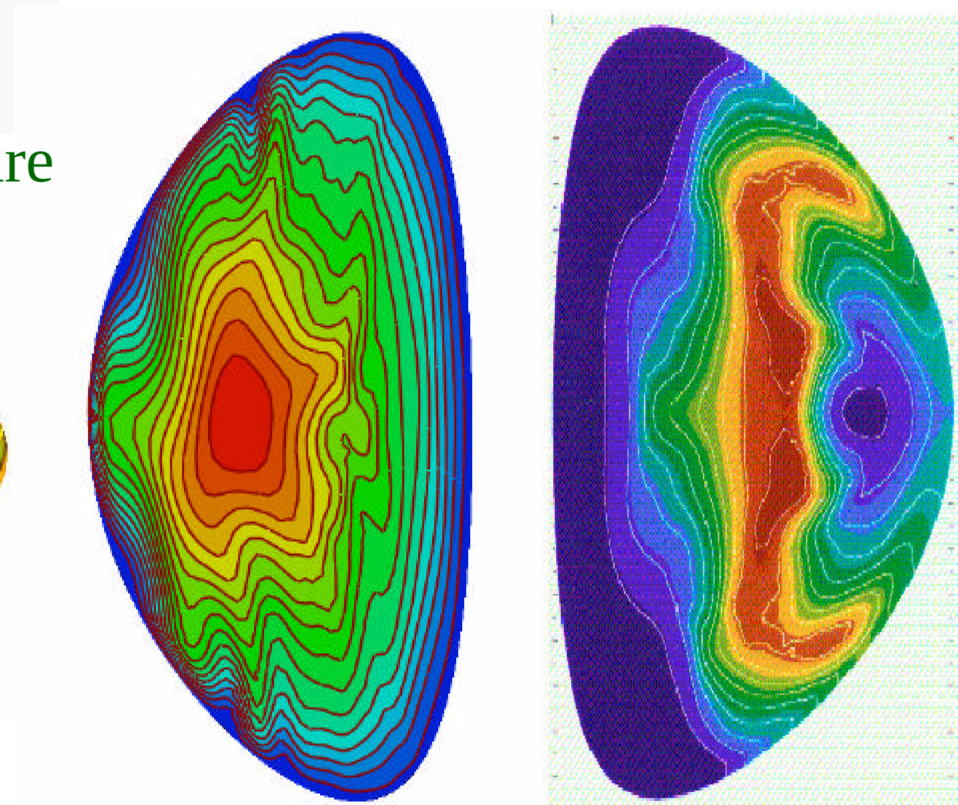


IRE : Disruption

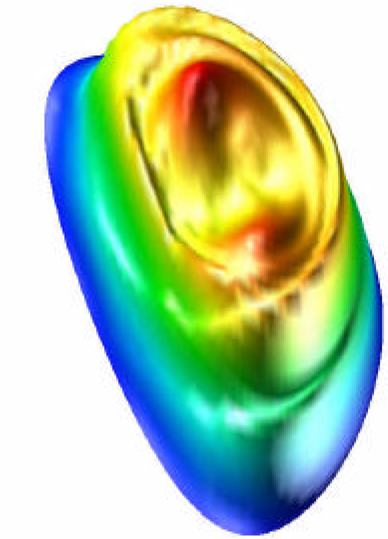
- Stochasticity as shown before.
- Localized steepening of pressure driven modes as shown here.

Pressure

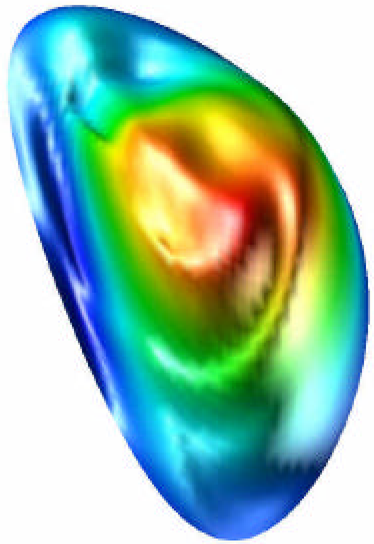




Nonlinear Evolution with peak rotation of M_A=0.2



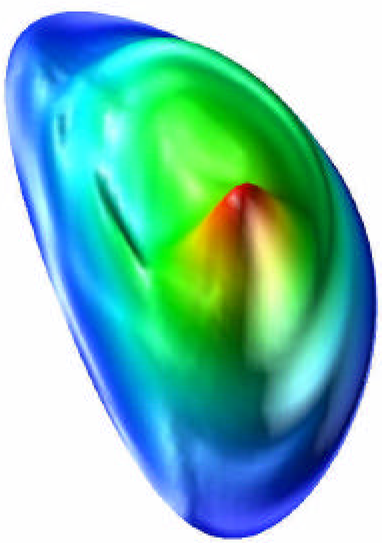
T



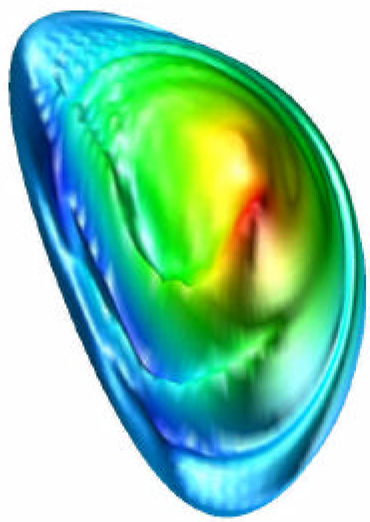
ρ

V_ϕ profile evolves
with reconnection

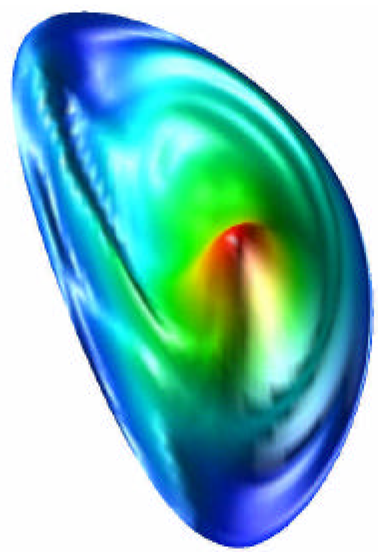
Momentum source rate
is an important factor



P

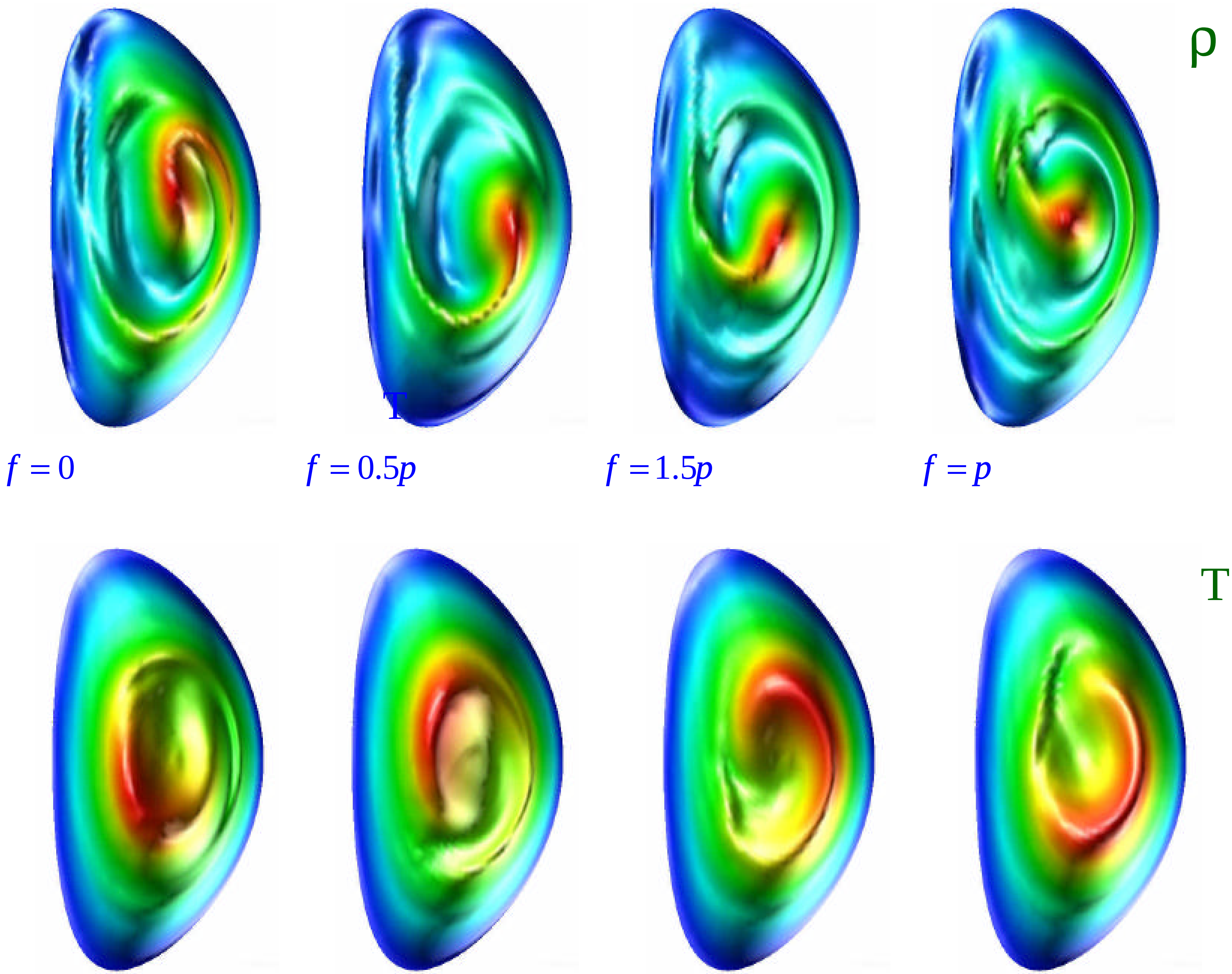


J_ϕ

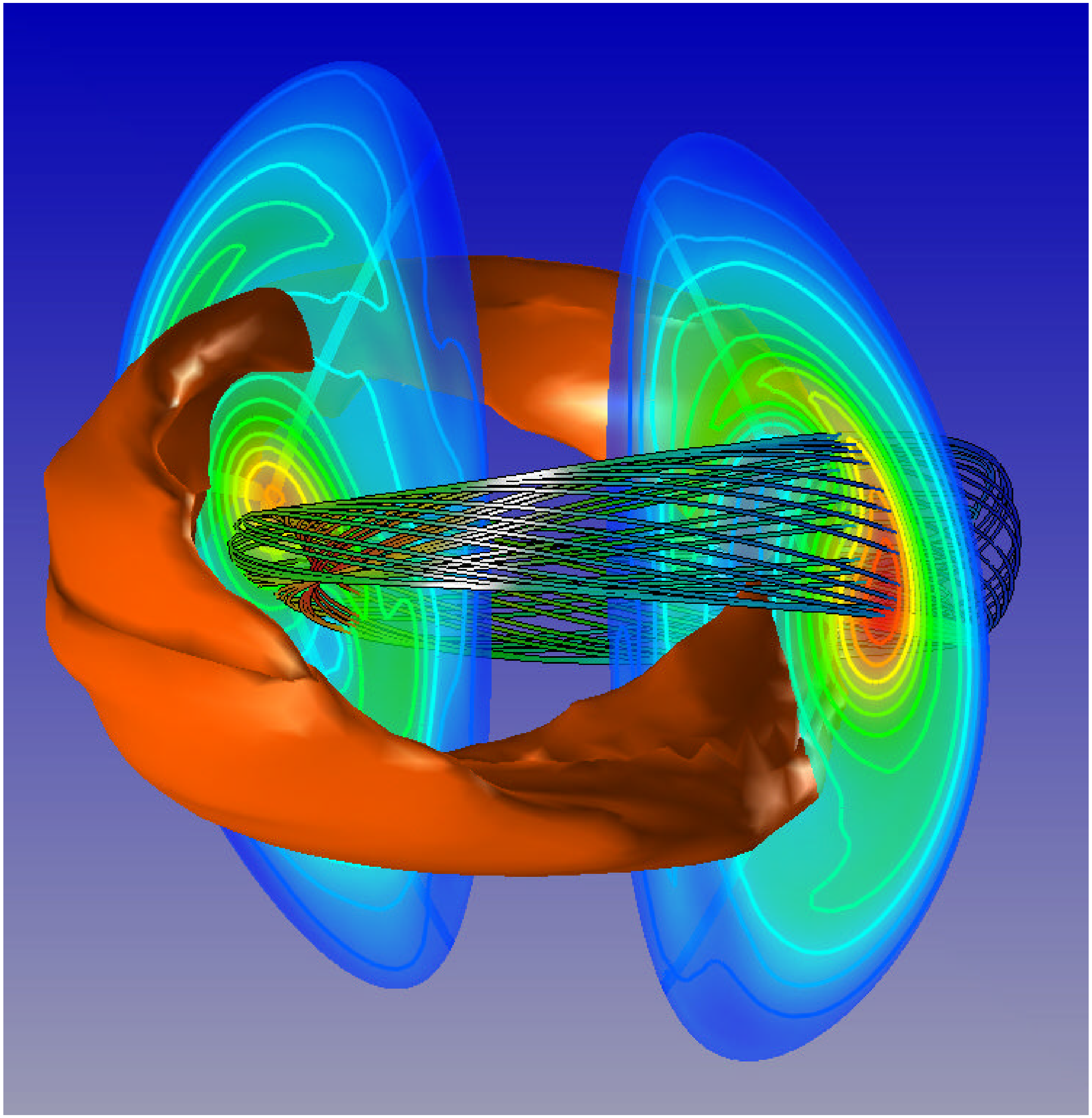


V_ϕ

ρ (P) and T out of phase in a saturated case

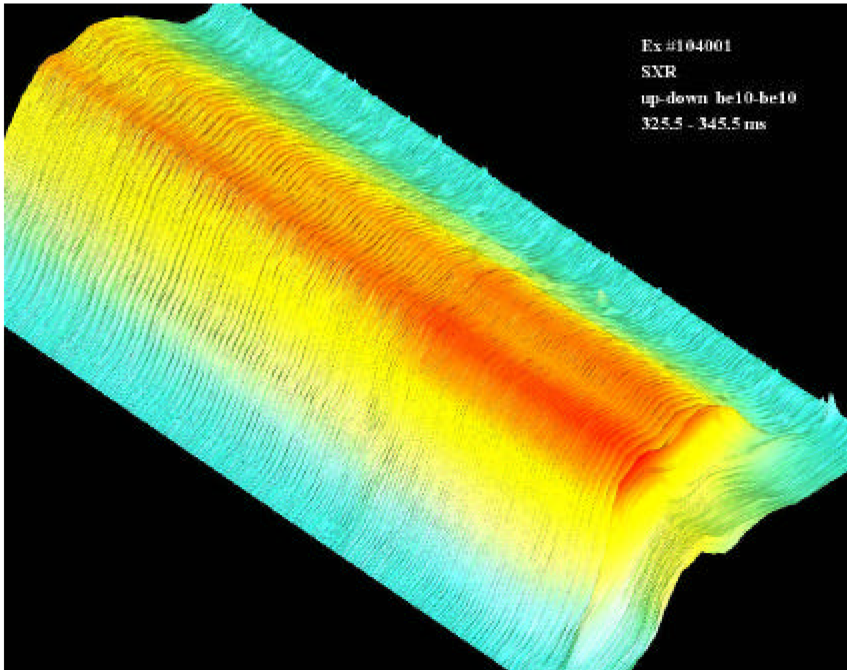


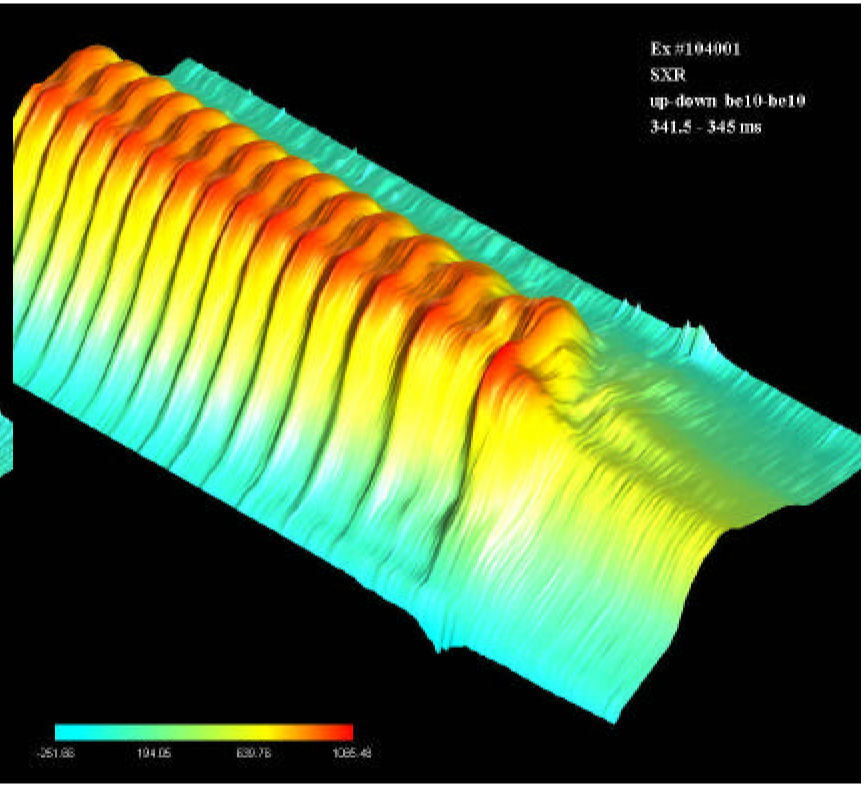
Saturated steady state with strong sheared flow

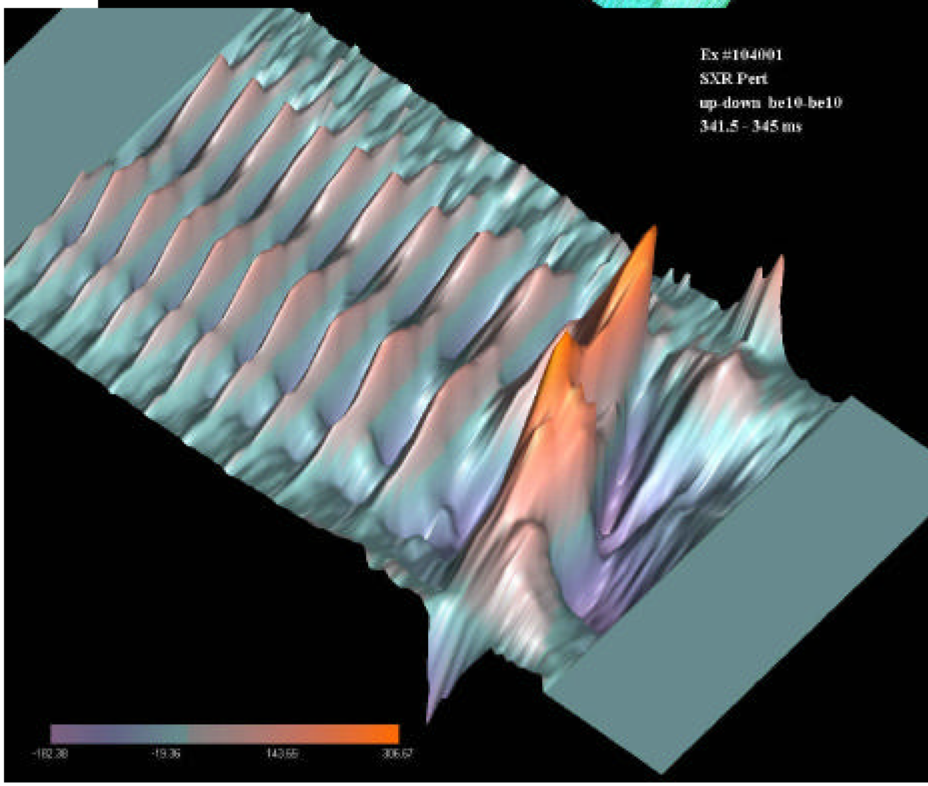


B Field line
in the island
Density (Pressure)
contours
Temperature
isosurface

Pressure peak inside the island together with shear flow causes the mode saturation.







Soft-Xray data often
shows long saturated period

EPM (BAE) is excited at high beta in hybrid simulations

More coupling to sound wave due to stronger curvature and high beta.

May explain experimental data.

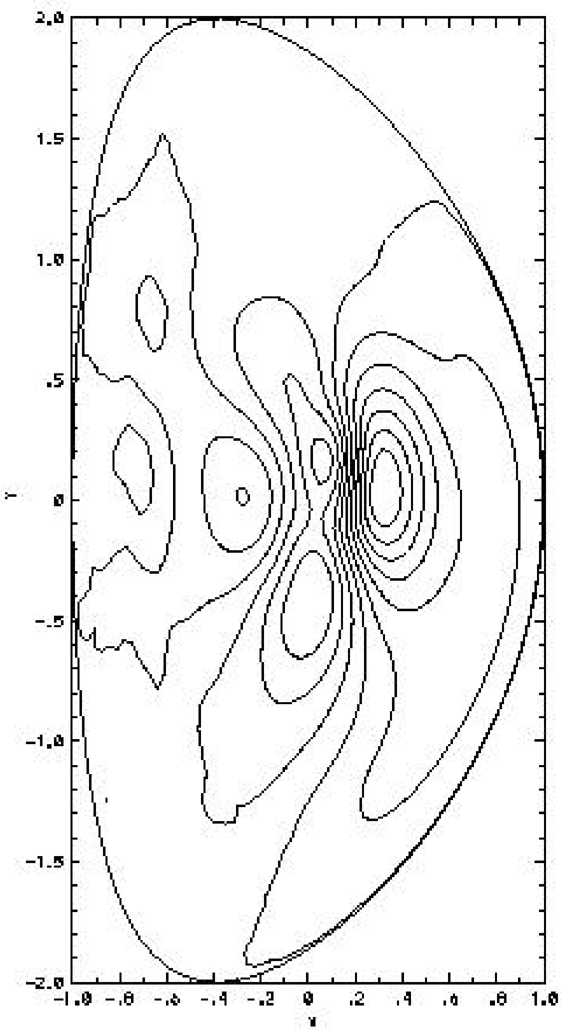
TAE

$b_{total} = 3.8\%$

$b_{h} = 3.8\%$

$w = 0.327$

$g = 0.013$



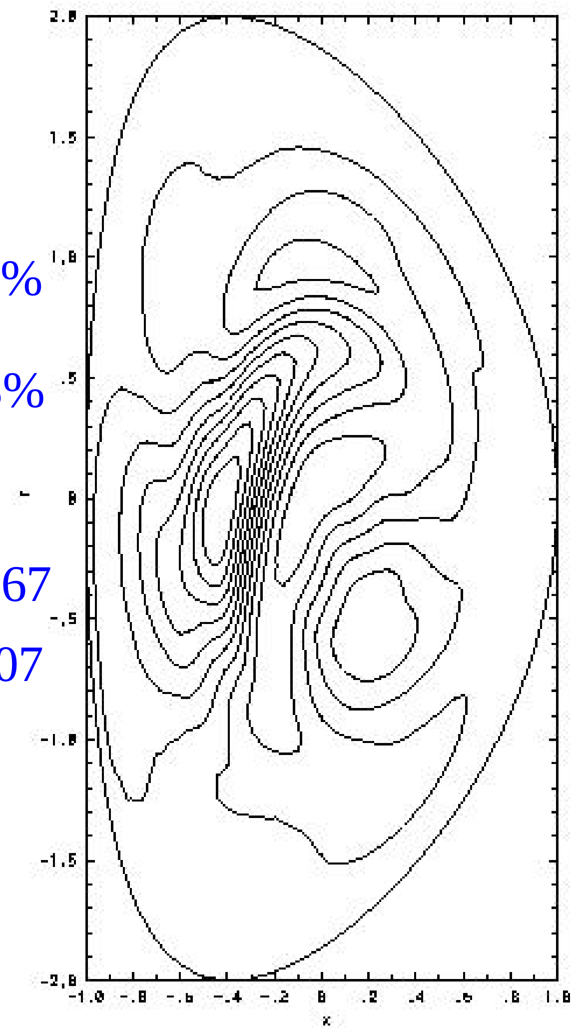
BAE

$b_{total} = 28\%$

$b_{h} = 3.8\%$

$w = 0.067$

$g = 0.007$



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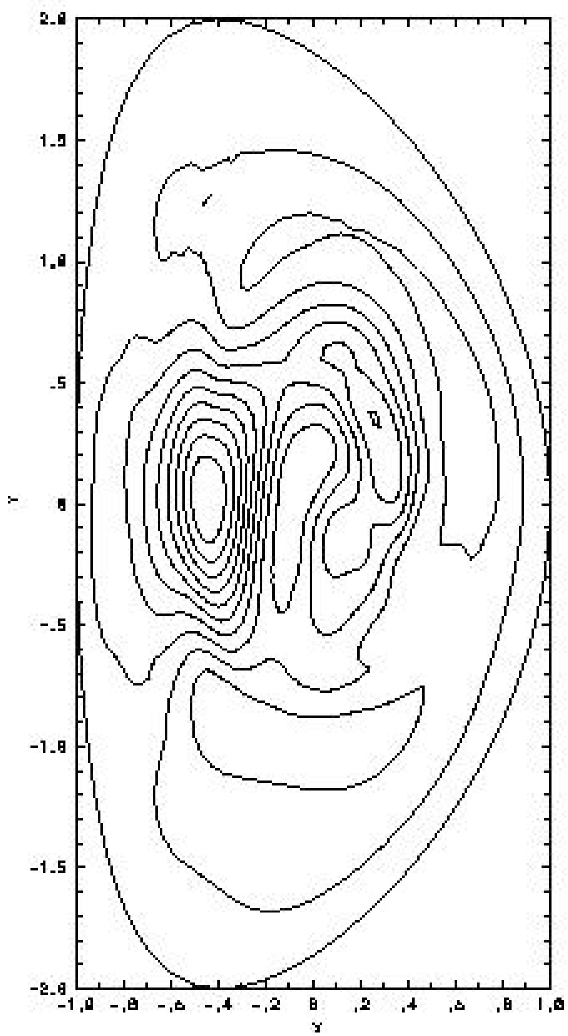
BAE changes to TAE when Γ is set to zero

BAE

$\Gamma=1.666$

$w = 0.05$

$g = 0.003$

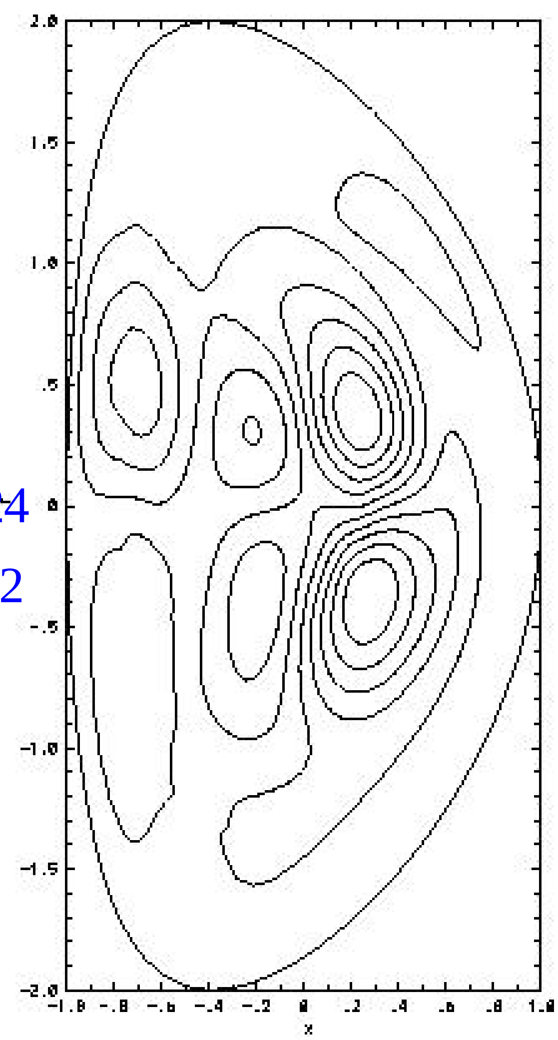


TAE

$\Gamma=0$

$w = 0.24$

$g = 0.02$



Summary

- M3D code with MHD, Two-fluids, and Particle/Fluid Hybrid levels is used to study NSTX.
- The relative density shift relation holds both in the simulation and experiment, with the centrifugal force of the hot component included.
- Toroidal sheared rotation reduces linear growth and can saturate internal kink.
- IRE:Disruption can occur in at least in two ways; due to stochasticity, and due to localized steepening of pressure driven modes.
- BAE mode is found which may explain experimental data.
- Resistive wall, vacuum region, and external coils are being added to M3D code to expand the regime of applicability.