NSTX

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Conceptual Design for β Control and Realtime Stability Limit Detection via RFA Measurements

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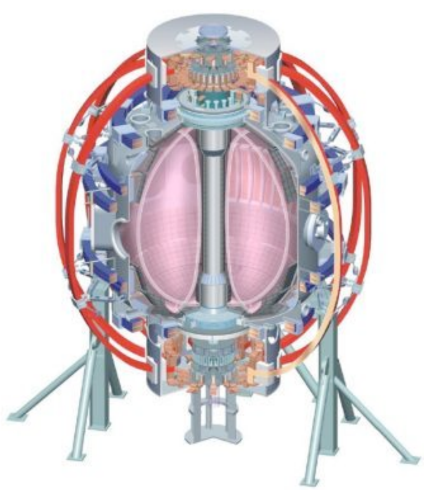

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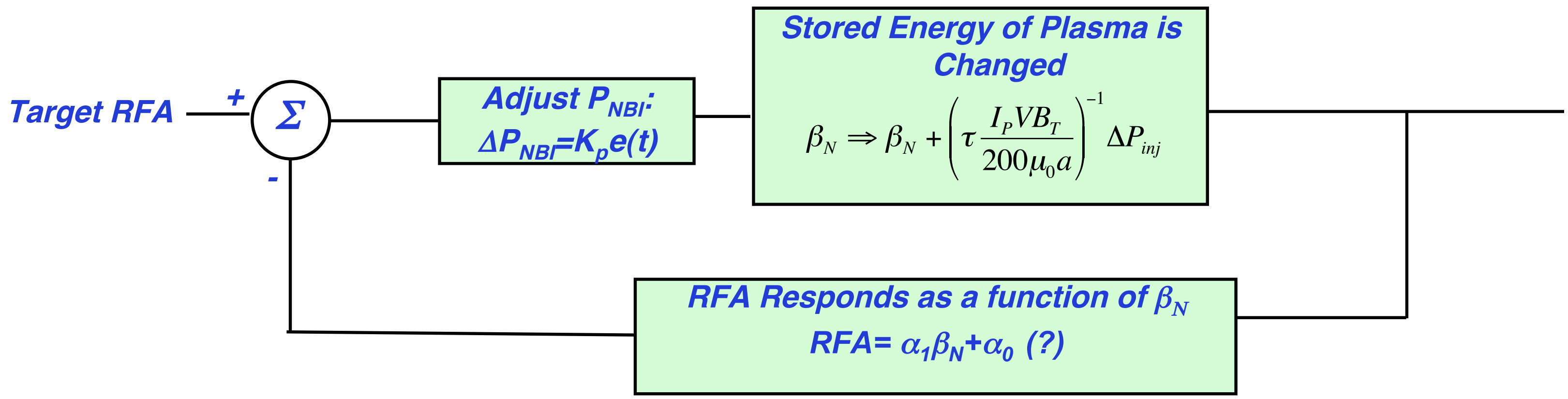
Ultimate Goal: Control NB Modulation in Order to Maintain β Just Below the "Real" Stability Limit



- RFA is the amplification of applied error fields by the plasma.
  - RFA is believed to increase rapidly near and above the no-wall β_N limit.
- By monitoring RFA in realtime, it may be possible to detect proximity to this stability limit.
- Using RFA to adjust the input power may be a component of non-disruptive operation near or just beyond the no-wall β_N limit.[1]
  - *Particularly attractive for a beam-driven device like CTF.*

1: H. Reimerdes, et al., NF 2005

Outline

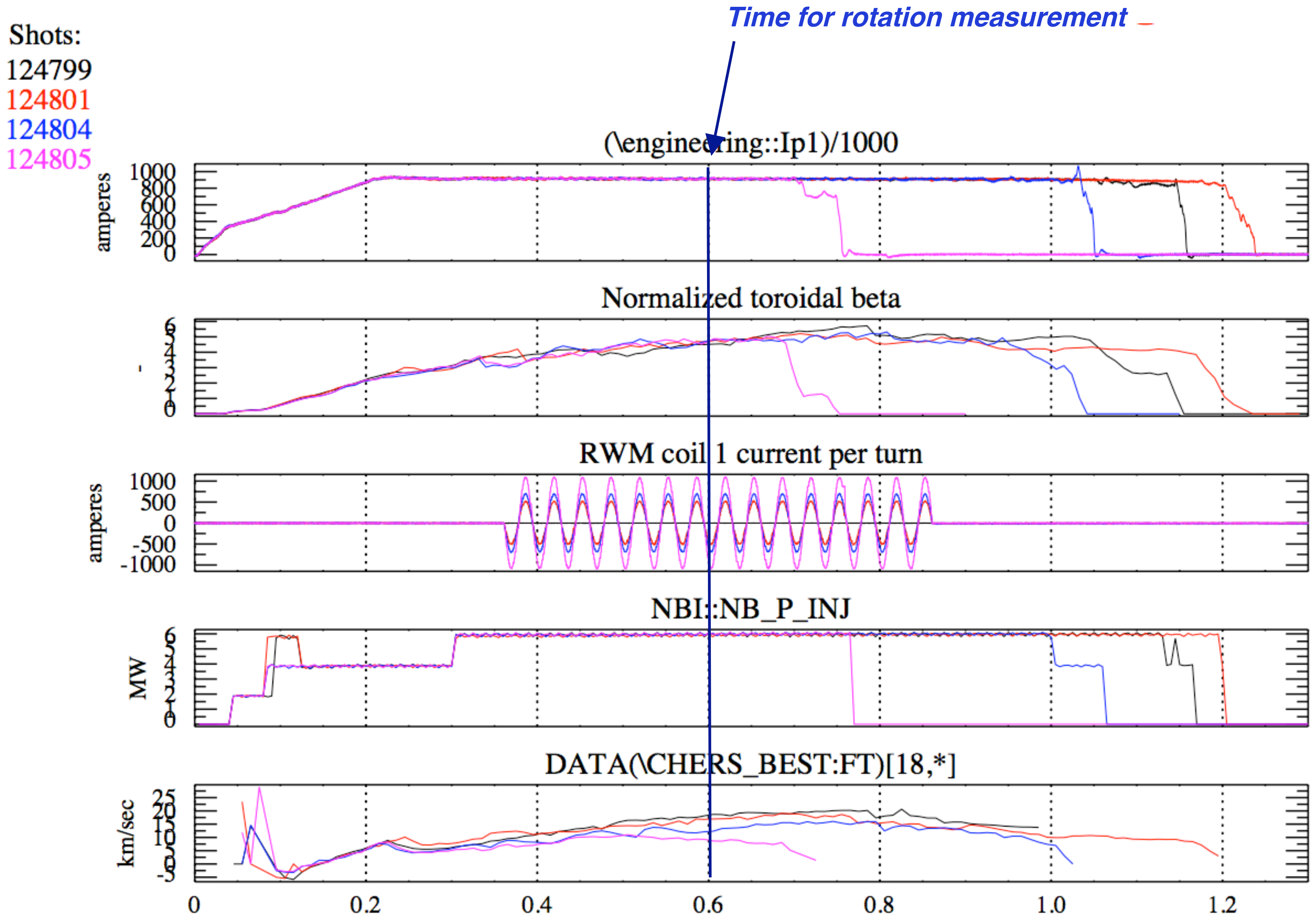
- Ultimate goal.
- Analysis of some 2007 data, XP-704
  - High-β, high-κ, high-δ, rapidly rotating targets similar to those we might want to use for RFA control.
  - RFA measurements in two ways:
    - Use of a single, highly filtered but minimally compensated anti-series pair of B_R sensors (scheme from J.K. Park).
    - Highly filtered, highly compensated n=1 mode decompositions.
  - Some results for RFA vs $\beta_{N,}$
- Simulation of NBI control based on RFA.
  - Example calculation based in single-sensor RFA detection.
- PCS Implementation.
- Proposed scoping XP to resolve issues raised above.

Always Pay Special Attention to :

Recursive Filters Used to Process Data

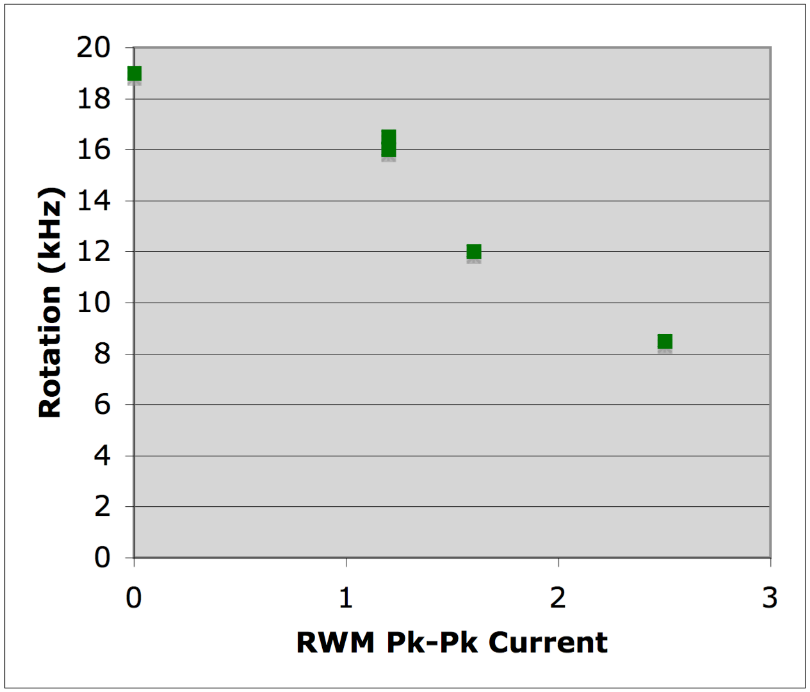
Computational Techniques Available in Realtime

High-Performance Shots Possible with 1kA Pk.-Pk. 30 Hz Traveling Waves



Larger Applied Fields Lead to Rotation Damping

CHERS Channel #18 rotation frequency, t=0.6 sec



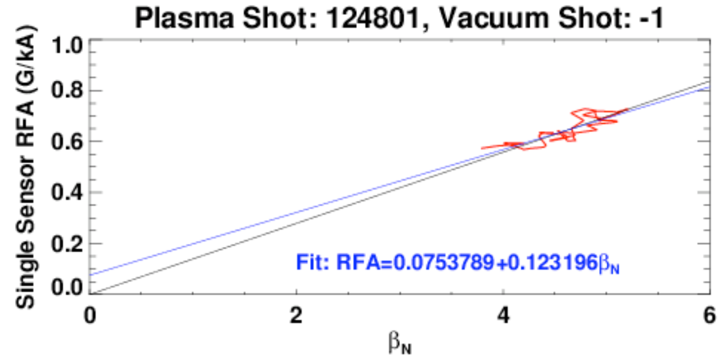
Severe rotation damping, and eventual disruption, for I_{RWM}>~1.4 kA

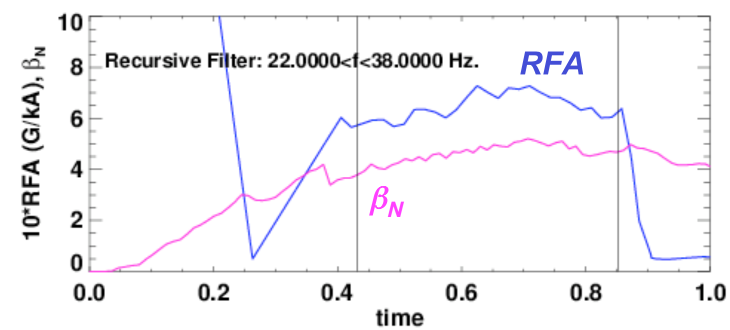
Example “Single Sensor” RFA Measurements: XP-704

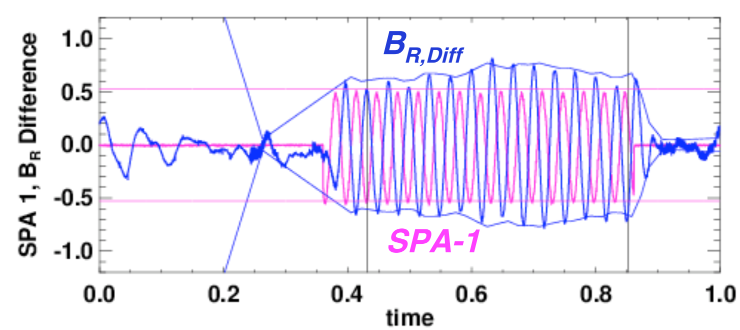
- Shot 124801: a “typical best shot”
  - 1.2 kA pk. to pk., 30 Hz, counter-rotating n=1 traveling wave (TW) perturbation.
  - 6MW input power, with β_N slowly evolving during TW application
- RFA Definition:

$$RFA = \frac{B_{R,Diff,Peak-to-Peak}}{I_{RWM,Peak-to-Peak}}$$

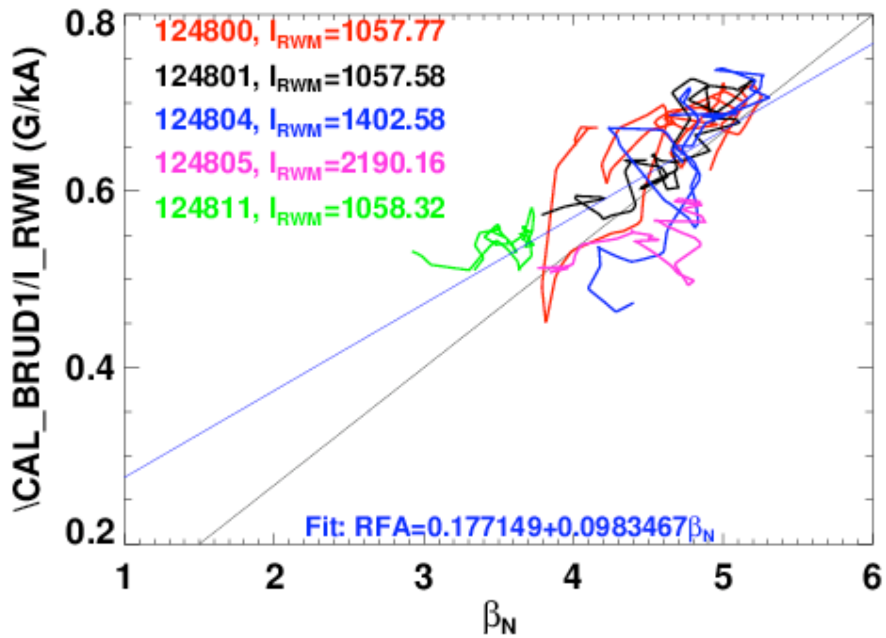
  - $B_{R\ Dif}$ is the difference field for a single coil pair, including all RWM coil pickup.
    - Equivalent to the ‘\Cal_’ signals in the tree.
    - May be good for realtime calculations, since the compensations are minimal.
  - $B_{R,Diff,Peak\text{-}to\text{-}Peak}$ determined from tracking zero-crossings.
    - Two amplitudes calculated for each cycle.
    - Relatively easy to code for realtime.
- Recursive filters used to isolate the correct frequency.
- Clear tracking of RFA with β_N during period of TW.







Look At All Co-Going Measurements For XP-704, Single Sensor RFA Definition



- Consider all shots with +30 Hz waves in XP-704
- All three source shots except 124811, which had 2 sources, and lower β.
- Scaling of RFA with β_N remains, though the scatter is large.
  - Can you actually do control based on these signals?
  - Including information from all sensors could improve the performance of the system.
    - Look at other definitions.

What about looking at the Full Plasma n=1 Response?

Basic RFA Model

H. Reimerdes, NF 2005

$$\tau_W \frac{dB_s}{dt} - \gamma_0 \tau_w B_a = M^*_{sc} I_C$$

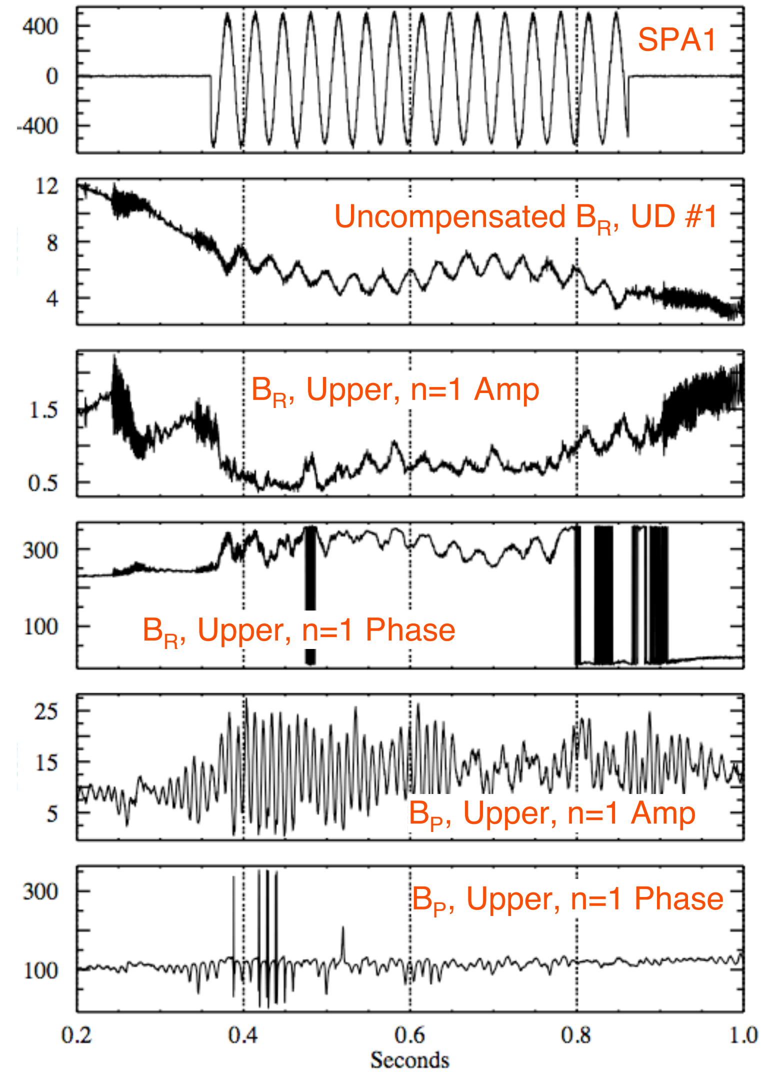
$$\gamma_0 = \gamma_{RWM} + i\omega_{RWM}$$

$$B_s^{ext} = \frac{M_{sc} I_C}{1 + i\omega_{ext}\tau_w}$$

$$A_{RFA,s} = \frac{B_s - B_s^{ext}}{B_s^{ext}} = c_s \frac{1+\gamma_0\tau_w}{i\omega_{ext}\tau_w - \gamma_0\tau_w}$$

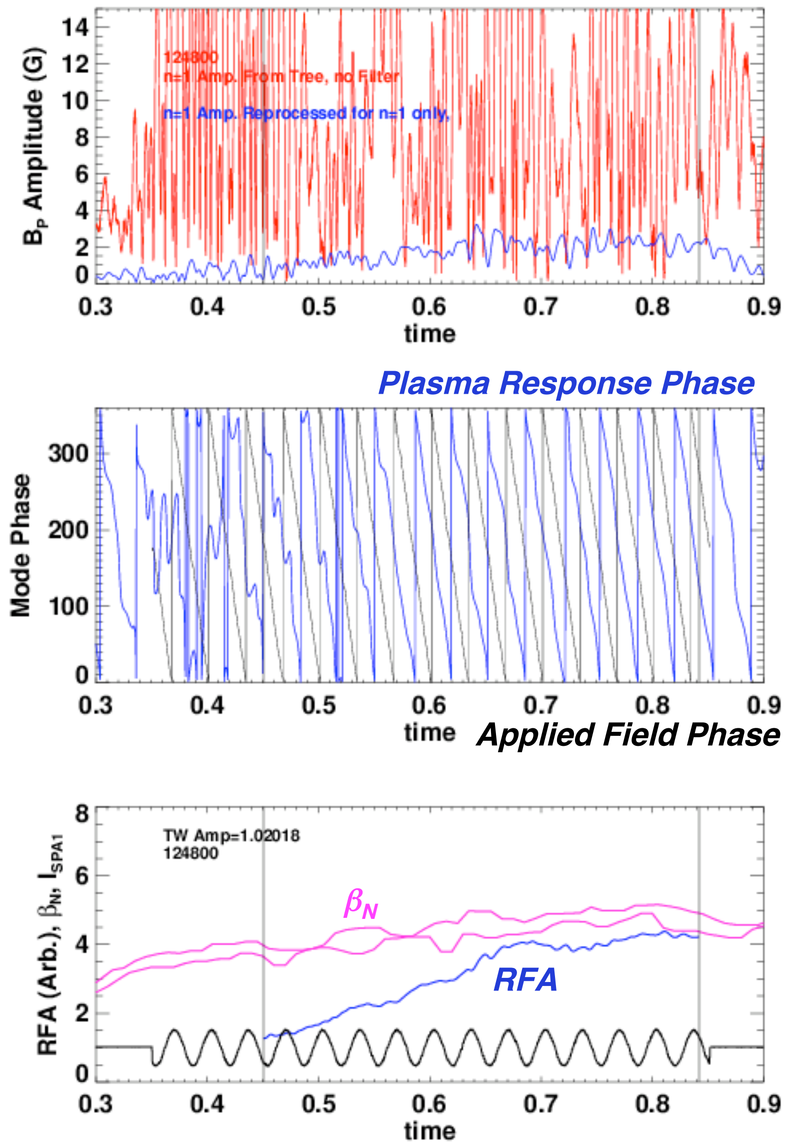
- Plasma response should be an n=1 perturbation, phase shifted with respect to the applied field.
- However, no rotating perturbation is seen in the archived n=1 decompositions.

Shot 124801

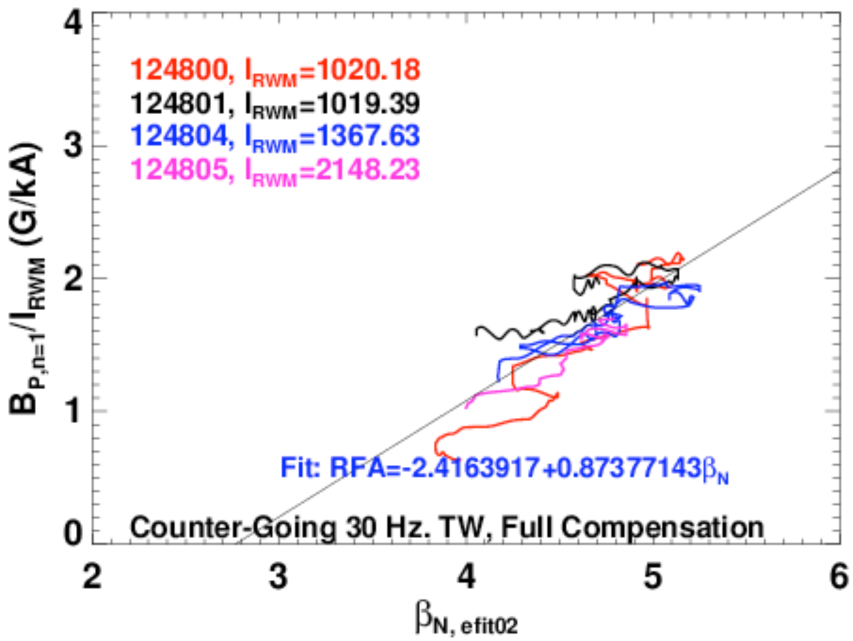


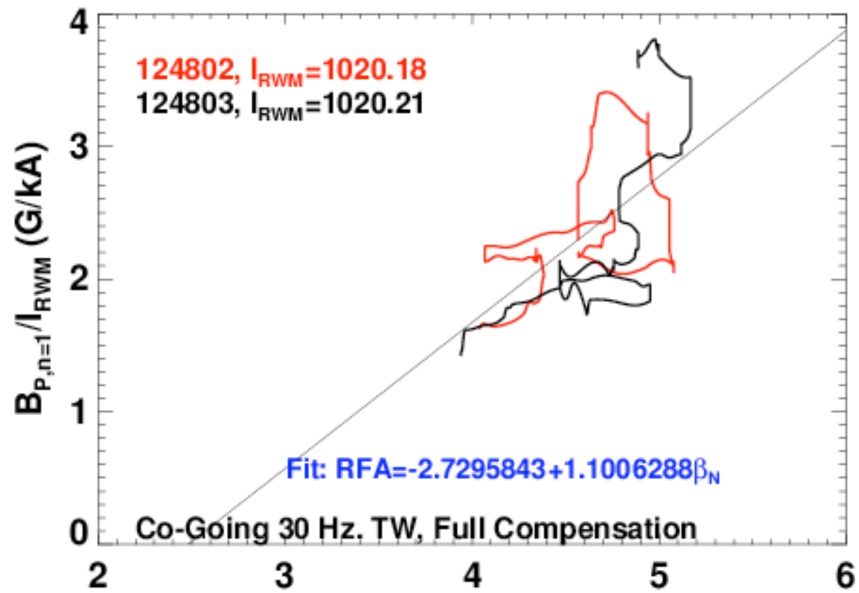
Extra Processing of the Sensor Data Resolves A Clear Rotating n=1 Perturbation

- Processing steps to observe rotating RFA:
  - Utilize fully compensated sensor data.
  - Compute mode amplitude and phase allowing n=1 (or 1+2) only.
  - Compute quadrature components of mode.
  - Apply high-order band-pass filter to quadrature components.
  - Recompute mode amplitude and phases.
- Clear rotating plasma response is observed.
- Plasma response scales with β_N.



RFA from n=1 Decomposition Shows Scaling with β_N





- $B_{P(upper)}$ sensors show best correlation with β_N.
  - Less noise than single sensor measurement, and more consistency that B_R data
  - These are the best RFA measurements SPG has yet found.
- B_p sensors may be less sensitive to plasma geometry than B_R (for fixed outer-gap)
  - Distance to B_R sensors is a stronger function or outer squareness and trianguarity.

Big Caveat

This Analysis Based on Fully Compensated Sensor Data, Including AC Compensation of EFC Coil Pickup

AC Compensations May Be Important Going Forward

$$C_i(t) = \sum_{j}^{NumRWMCoils} \sum_{k=0}^{k_{max}} p_{i,j,k} LPF\left(\frac{dI_{RWM,j}(t)}{dt};\tau_k\right)$$

$(\text{Compensated Signal})_i = (\text{Uncompensated Signal})_i - C_i$

Offline:

- ***For each sensor (i) and RWM coil (j), there are 5 time constants t, and 5 associated coefficients coefficients,***
- ***Total of 8×6×6×5×2=2880 (!) numbers***

Online Now:

- ***Include only k=0 term (static pickup)***
- ***8×6×6=288 numbers***

For RFA measurement, may only need to compensate in the vicinity of the applied frequency.

Can only static pickup and a single time-constant be useful for RFA feedback?

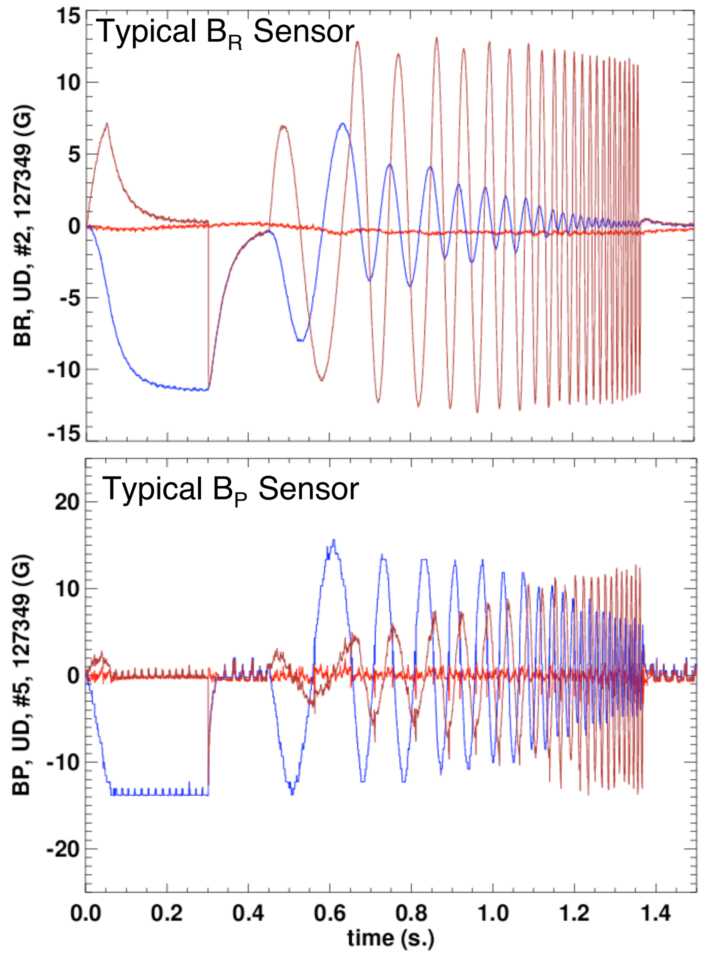
However, the overall feedback system might be improved with better AC compensation.

Example Compensations: Vacuum shot with a single RWM Coil Energized

Red: Fully Compensated

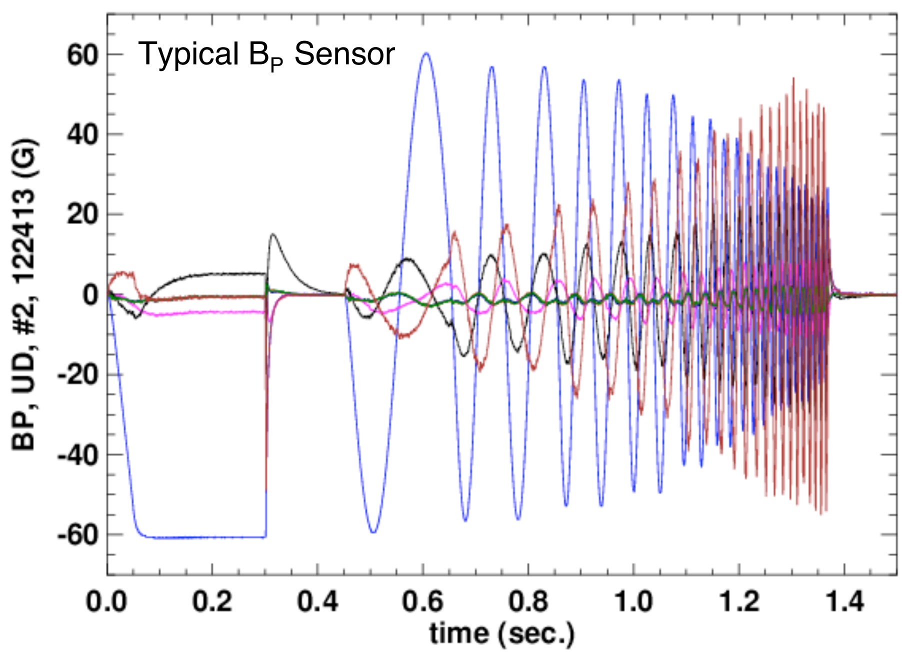
Blue: Full Pickup

Brown: Direct Pickup Only Subtracted (k=0 only, as in PCS)

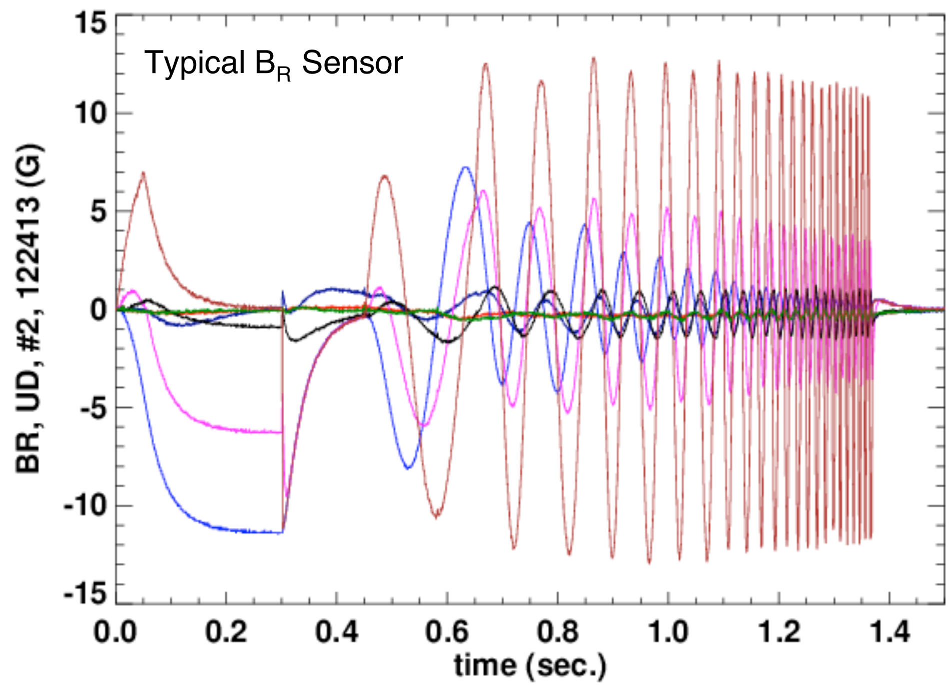


Two τ's (>0) May Be Sufficient For Compensation, while Three will Certainly Work

Blue: No Compensation
Brown: Static Compensation Only
Magenta: 2 term compensation, τ=0,3 (msec)
Black: 2 term compensation, τ=0,30 (msec)
Dark Blue: 3 term compensation, τ=0,4,16 (msec)
Green: 4 term compensation, τ=0,3,9,18 (msec)
Red: 5 term compensation, τ=0,2,4,8,16 (msec)



Blue: No Compensation
Brown: Static Compensation Only
Magenta: 2 term compensation, τ=0,3 (msec)
Black: 2 term compensation, τ=0,30 (msec)
Dark Blue: 3 term compensation, τ=0,24,96 (msec)
Green: 4 term compensation, τ=0,15,30,60 (msec)
Red: 5 term compensation, t=0,12,24,48,96 (msec)



Which Technique for RFA Detection is Best?

Editorial Color Code
Green is "good"
Red is "bad"

- Single Sensor Method
  - No additional compensations required.
  - Need zero-crossing/peak-finding algorithm.
  - The calibrated but uncompensated data is NOT presently available in mode-ID, so still need to bring in additional data from (I think) ACQ.
  - Time resolution is limited to essentially ~$1/f_{TW}$, based on zero-crossing/peak-peak analysis.
  - Requires one band-pass filter (the chosen difference signals).
- n=1 Decomposition Method
  - Uses (averages) all sensors to constrain the RFA response.
  - Provides instantaneous values for the RFA.
  - Requires AC compensations be applied to sensors.
    - These compensations can then be used for improved fast RWM feedback, as well as RFA control.
  - Requires 2 band-pass filters (the quadrature components).
- SPG Recommendation:
  - Implement AC compensations, with either 2 or 3 non-zero time constants, and use the filtered n=1 quadrature components for RFA measurements.

Modifications and/or New Algorithms Required in Mode-ID, RWM, and NBI Categories

Mode-ID Category
Processes RWM Sensor Data

(new) mar algorithm
- Compute AC compensated sensor data
- Calculate n=1 amplitude and phase for RWM feedback, as in present algorithm.
- Calculate band-pass filtered n=1 plasma response.
- Use requested $I_{SPA,TW}$ and frequency to generate an RFA measurement.

RWM Category
Controls The SPAs

(new) tmf algorithm
- Generate an n=1 traveling wave: $I_{SPAX,TW}$
- Allow a preprogrammed SPA request: $I_{SPAX,Pre\text{-}Prog}$
- Notch filter the quadrature components of the n=1 mode (B_P and B_R), to remove the pickup from the traveling wave.
  - Generate n=1 FB requests: $I_{SPAX,FB}$
- Error field correction directly proportional to the PF coil currents: $I_{SPAX,EFC}$
- Sum requests to generate SPA currents.

$$I_{SPAX}= I_{SPAX,TW}+ I_{SPAX,Pre\text{-}Prog}+ I_{SPAX,FB}+I_{SPAX,EFC}$$

NBI Category
Control NBI Pre-Programming and Modulation

(old & new) bnf algorithm
- Accept RFA measurements from mar algorithm.
- Adjust NB modulation to achieve desired RFA level.

Discuss each of these algorithms on the following slides

New "mar" Algorithm Will Produce Both B_P and B_R n=1 Mode Identification, and RFA Data As Well

- Add AC compensations.
  - Put AC compensation coefficients in model tree, read them into shared memory.
  - Add code to compute and subtract compensation terms
- For RMW control, retain features of "mid" algorithm.
  - Generate same amplitude and phase of n=1 modes for RWMF/DEFC, based on the decomposition matrix $Y_{mode\text{-}ID}$
  - Separate B_P, B_R, and "combined" n=1 mode amplitude and phases.
  - Send these numbers to "tmf" algorithm, or the older "imf" and "smf"" algorithm.
  - Same baseline rezeroing:
    - However, allow separate times for B_P and B_R sensors.
- RFA calculations
  - Use a different decomposition matrix, Y_{RFA}, to generate quadrature components of plasma response, and then bandpass filter the components.
    - Center frequency and BW of recursive filter are algorithm waveforms.
  - Calculate the amplitude of the plasma response $(C^2+S^2)^{1/2}$.
  - Normalize to SPA currents, to get RFA measurement in units of Gauss/kA.
    - SPA TW frequency and amplitude are waveforms in the "tmf" algorithm, and can be accessed here.
  - Send the RFA value to the "bnf" algorithm.

Simple Recursive Bandpass Filter Will Be Used to Isolate Plasma Response at f_{TW}

$$y_{out}[n] = a_0 y_{in}[n] + a_1 y_{in}[n-1] + a_2 y_{in}[n-2]$$

$$+b_1 y_{out}[n-1] + b_2 y_{out}[n-2]$$

$$a_0 = 1 - K, \quad a_1 = -2(K-R)\cos(2\pi f)$$

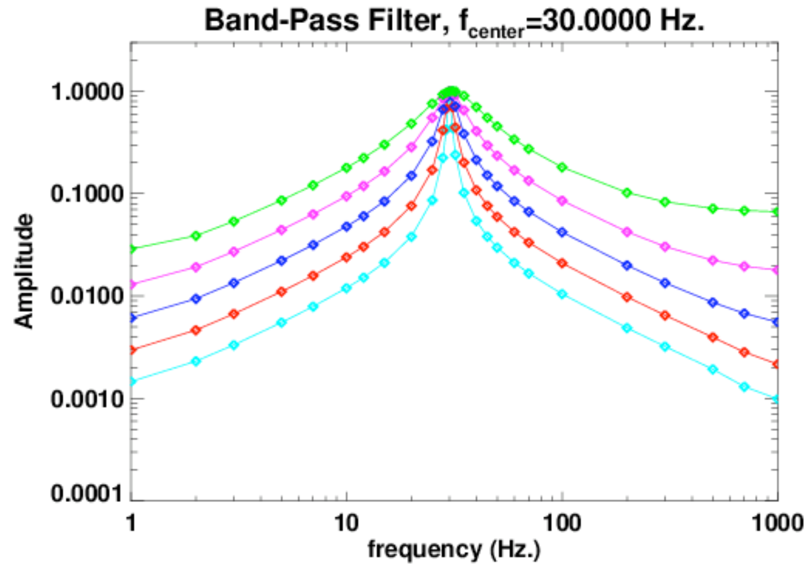
$$a_2 = R^2 - K$$

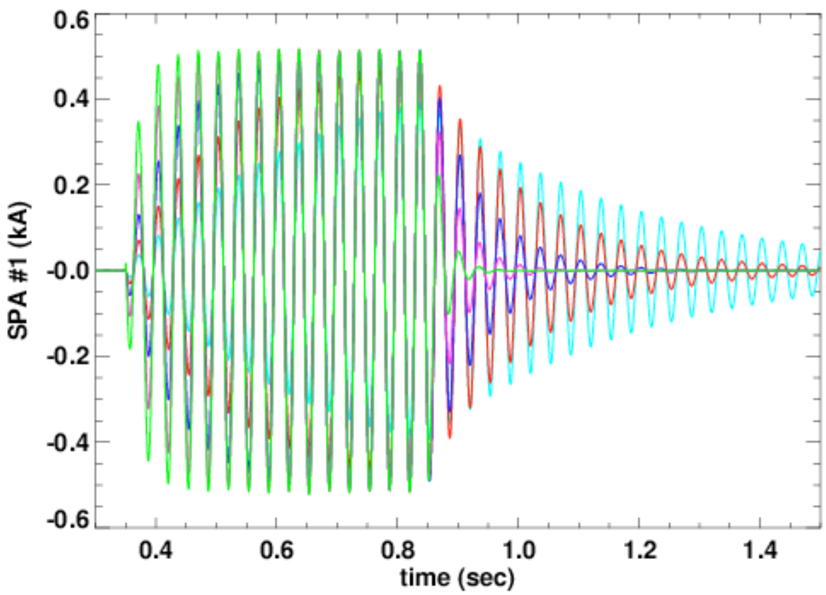
$$b_1 = 2R\cos(2\pi f) \quad b_2 = -R^2$$

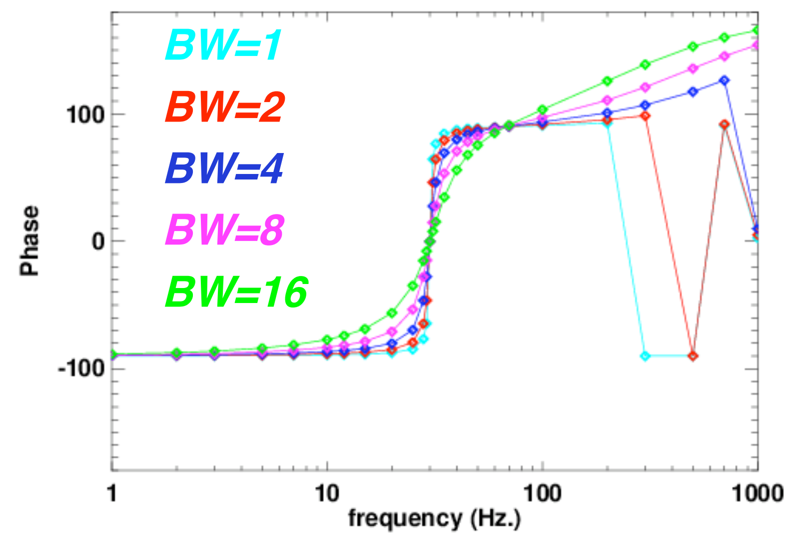
$$K = \frac{1 - 2R\cos(2\pi f) + R^2}{2 - 2\cos(2\pi f)}$$

$$R = 1 - 3 \times BW$$

Narrow passbands are possible, but maybe not desirable







New "tmf" Algorithm Designed to Allow RWMF in the Presence of the n=1 Spectroscopy Perturbation

- Feature #1: Generate n=1 traveling waves. $I_{SPA-1}^{TW}(t) = I^{TW}(t)\cos\left(\left(300° + \delta_{TW}(t)\right)\left(\frac{\pi}{180}\right) + 2\pi f_{TW}(t)t\right)$
  - Input wave amplitude, phase, frequency as waveforms.
- Feature #2: Include pre-programmed requests. $I_{SPA,X,\mathrm{Pre-Prog}}(t)$
- Feature #3: Include correction proportional to PF5, PF4, PF3U, PF3L, TF, OH

$$I_{SPA-i}^{FEC} = \sum_j G_{j,i}\left(I_j / 1000.\right), \quad j = \{PF5, PF4, PF3U, PF3L, TF, OH\}$$

- Feature #4: RWMF/DEFC, notch-filtered to remove applied TW (example equations for B_P sensors):
  - The "mar" algorithm provides the n=1 amplitude and phase, or alternatively the quadrature components: $B_{\mathrm{mode}}(\phi,t) = B_1(t)\cos(\phi - \theta_1(t)) = C_1(t)\cos(\phi) + S_1(t)\sin(\phi)$

$$C_{1,BP}(t) = B_{1,BP}(t)\cos(\theta_{1,BP}(t)) \quad S_{1,BP}(t) = B_{1,BP}(t)\sin(\theta_{1,BP}(t))$$

  - Calculate notch-filtered (NF) version of the quadrature components.

$$C_{1,FF,BP}(t) = LPF\left(C_{1,BP}(t); f_{center}, BW\right) \quad S_{1,NF,BP}(t) = LPF\left(S_{1,BP}(t); f_{center}, BW\right)$$

  - Reconstruct amplitude and phase of notch filtered data

$$B_{1,Bp,NF} = \sqrt{\left(C_{1,NF,BP}(t)\right)^2 + \left(S_{1,NF,BP}(t)\right)^2} \quad \theta_{1,Bp,NF} = a\tan\left(C_{1,NF,BP}(t) / S_{1,NF,BP}(t)\right)$$

  - Form feedback requests

$$I_{SPA,X,Bp-FB}(\phi,t) = P_{Bp}(t)B_{1,Bp,NF}(t)L_{eff}^{-1}\cos(\phi - \theta_{1,Bp,NF}(t) + \delta_{Bp}(t))$$

- Final SPA request is sum of all the above:

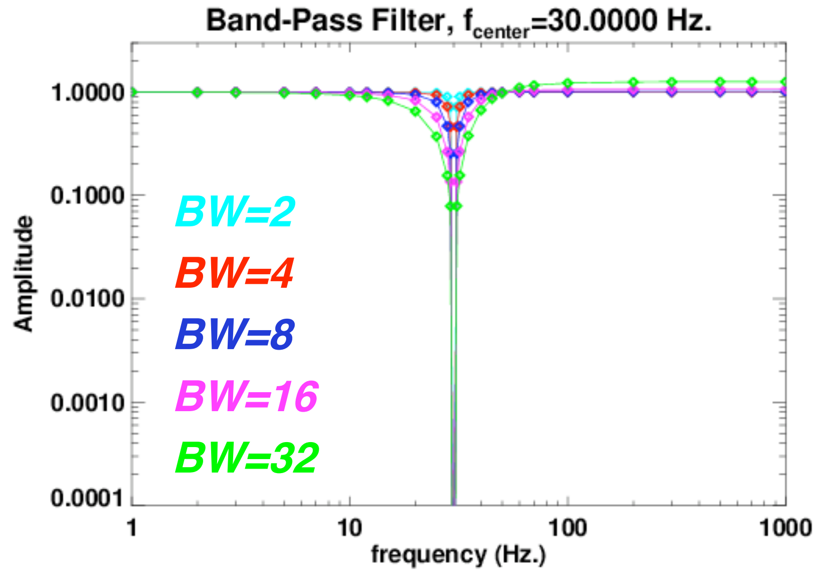
$$I_{SPA,X}(t) = I_{SPA,X,TW}(t) + I_{SPA,X,\mathrm{Pre-Prog}}(t) + I_{SPA,X,Bp-FB}(t) + I_{SPA,X,Br-FB}(t) + I_{SPA,X}^{EFC}(t)$$

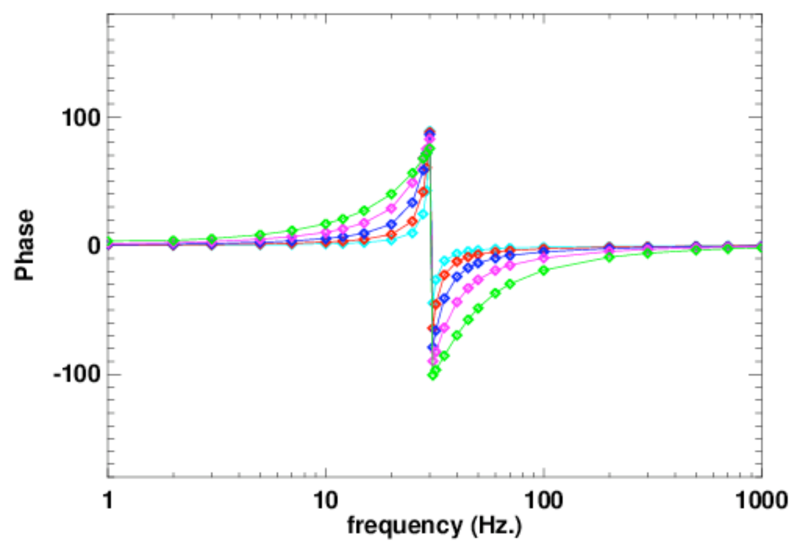
Need a Recursive Notch Filter In Order to do RWMF/DEFC

- Very narrow stop-band achievable with simple recursive filter.
- Minimal phase shift in the pass-band.

$$y_{out}[n] = a_0 y_{in}[n] + a_1 y_{in}[n-1] + a_2 y_{in}[n-2]$$
$$+ b_1 y_{out}[n-1] + b_2 y_{out}[n-2]$$
$$a_0 = K, \quad a_1 = -2K\cos(2\pi f)$$
$$a_2 = K$$
$$b_1 = 2R\cos(2\pi f) \quad b_2 = -R^2$$
$$K = \frac{1 - 2R\cos(2\pi f) + R^2}{2 - 2\cos(2\pi f)}$$
$$R = 1 - 3 \times BW$$

- BW and f are bandwidth and center frequency, normalized to sample frequency





Existing "bnf" Algorithm Can Likely be Utilized for the RFA Feedback

- NBI Category:
  - Existing "bnf" (= beta-normal feedback) algorithm already allows a wide variety of feedback targets:
    - W_{MHD}, β_N, β_T, for instance.
    - Waveforms exist for β_P control (P, I, D, deadband, LPF), but no internal code:
      - Simple to redefine those waveforms for RFA control, forget about β_P control.
    - Alternatively, add new waveforms for RFA-control
      - Need to think about the impact on reloading.
  - *Advantage is that we use the same (presumably optimized) methods for setting NBI modulation times, batting order, preprogramming*.
  - *Take credit for all the EPICS/PCS communication work presently being done.*

Plan is Designed For RFA Control, But Has Incremental Improvements For RWMF/DEFC Studies

- Mode-ID Improvements
  - Apply AC Compensations To Sensor Data
    - Accurate mode reconstruction during fast feedback
  - Allow B_R and B_P sensors to be re-zero'd at different times
    - Elimination of OH-TF pickup in the B_R sensors.
- RWM Coil Control Improvements
  - Introduce a notch filter, which can be used for any purpose
    - Elimination of 100 Hz noise?
  - Allow RWM coils to be directly tied to PF and TF coils.
    - Dynamic correction of the n=3 EF?

Issue To Be Addressed By An XP: How well does RFA Predict the Approach to Stability Limits?

Ideal stability limits depend on:

- Triangularity & Squareness
- q_0, q_{95}
- Pressure Peaking and l_i

Thesis: RFA measurement should be inherently sensitive to these dependencies (for fixed plasma-sensor distance)

Ferron et al, Phys. Plasmas **12** 056126
n=1 RFA Used to Study Stability Dependence on q_{min}

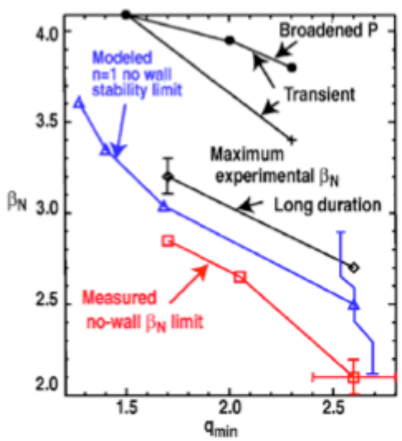


FIG. 1. (Color online). The measured and modeled dependence of the β_N limit on the minimum safety factor. Squares are the experimentally measured no-wall limit and triangles are the $n=1$ no-wall limit calculated for equilibria with profiles similar, but not identical to the profiles in the experimental discharges. Diamonds are the maximum β_N at which discharges have been operated for the duration of the machine pulse without significant instability, circles are the maximum β_N that has been obtained in steady-state scenario discharges. The cross represents the maximum β_N obtained without adding extra gas to broaden the pressure profile.

Menard et al, Phys. Plasmas **11** 639
Dependence of Stability on Triangularity

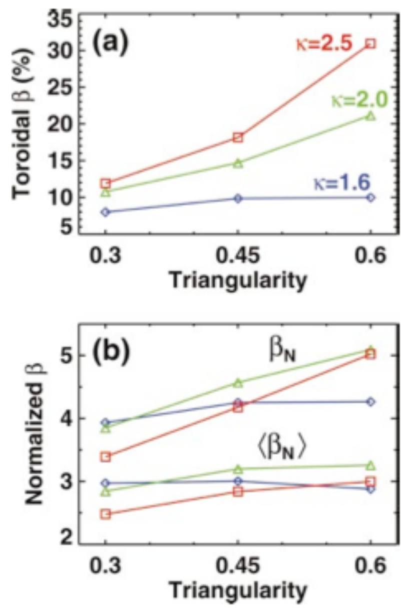


FIG. 2. (Color) (a) Marginally stable β_T(%) and (b) normalized beta values as a function of triangularity and elongation at 50% self-driven current fraction for aspect ratio $A=1.6$.

Strawman Shot Plan (which parts are most important?) (1)

Step 0: Establish 900 kA “fiducial” shape discharge with good β_N evolution (3 shots)

- κ~2.3-2.4, δ~0.6, dr_{sep}=-1cm, outer gap~10cm, Lithium conditioned
- Use n=3 correction to maintain optimal rotation.
- Start with 2 sources (3 or 4 MW?), add 3rd at ~500 msec (5 or 6MW) and *remove at 800 msec*, for β_N ramp-up and ramp down.
- Call this the reference configuration.
  - May need a second reference to get larger range of β_N evolution (?)
- Keep this β_N for all subsequent cases.

Step #1: Apply 30 Hz Co- Traveling Wave, 1kA pk-pk. (3 shots)

- 1kA and 30 Hz determined from previous XPs

Step #2: Frequency Scan (6 shots)

- Repeat at -30,10, 45, 60 Hz
- Use to assess:
  - Plasma perturbations with optical USXR as a function of frequency.
  - Low-frequency RFA measurement for comparison with IPEC.
  - Optimal frequency for RFA control

Shots: 18

Strawman Shot Plan (which parts are most important?) (2)

Step #3: Scan I_P to 700 kA, 1100kA (6 shots)

- Test robustness of these plasmas to 1kA traveling waves
- Test of RFA vs β_N at different q_{95}, l_i.

Step #4: At 900kA, scan triangularity at fixed elongation. (8 shots)

- Keep the outer-gap fixed.
- RFA should be inversely proportional to triangularity at fixed β_N (?)

Step #5: For reference shape, scan the outer gap. (3 shots)

- Increase to 15 cm, drop to 7 cm.
- Test geometry effects on RFA measurement.
  - Important for assessing the technique as a control tool: What is the tolerance for controlling the outer gap? -> $d(RFA)/d(gap_{out})$?

Step #6: Repeat with n=3 correction removed

Addendum: If/when PF4 is available, use RFA to understand the stability as a function of squareness.

Shots: 18+17=35

Diagnostics and Analysis

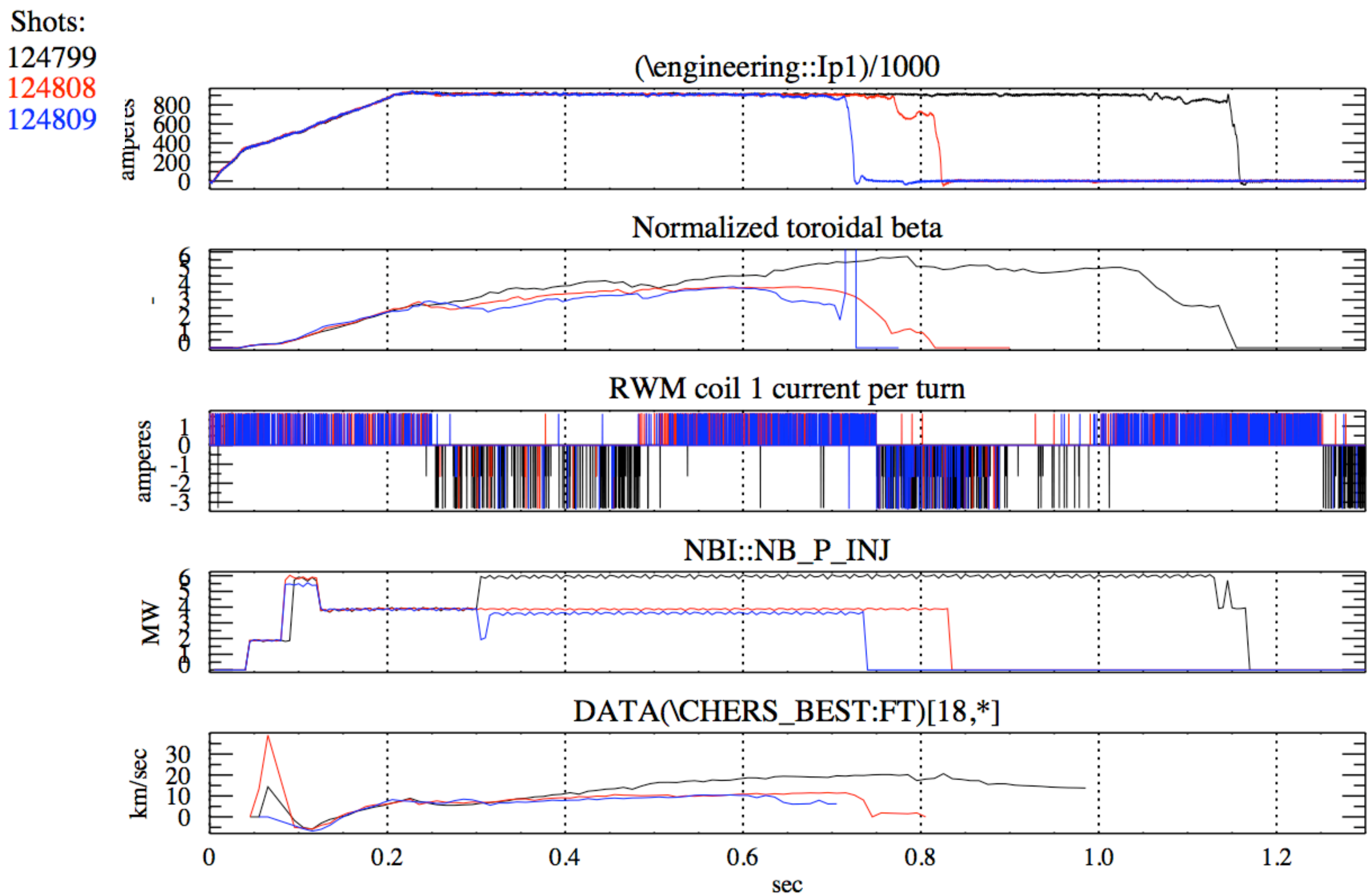
- Diagnostics
  - All profiles (T_i, T_e, Pitch Angle, Rotation, n_e,…)
  - RWM sensors (critical)
  - Optical SXR array
- Analysis
  - Equilibrium reconstruction with all constraints
  - DCON, for calculation of ideal stability limits.
  - IPEC

Most Aggressive Schedule Would Allow This to be Tried in 2010

- NBI Side
  - We have (essentially) never controlled beams from PCS.
  - Must develop/test PCS-EPICS communications links this year.
  - Rely on the NB modulation algorithm being tested (successfully) this year.
- “mar” and “tmf” algorithms
  - No new hardware to control, just PCS code (and a little ACQ).
  - 2-3 weeks to write the algorithm code
    - Assumes ~50 % of SPG’s time.
    - Need to test some algorithm code in idl.
    - Some Dana time to get AC-compensation coefficients from tree to algorithm.
  - Then Dana needs to compile all into PCS, check for memory conflicts…
  - 2-3 weeks for piggyback testing.
    - Background testing this run when the SPAs are not in use.
- Combined test: 2010 Run
- Real schedule may be slower, is certainly not faster.

Old and Discarded Stuff Follows

Three Reference Shots Taken For XP-704



All fiducial shape.

One reference with 6MW input, two with 4MW.

New "rid" Algorithm Will Produce Both B_P and B_R n=1 Mode Identification, and RFA Data As Well

- Retain features of "mid" algorithm.
  - Generate same amplitude and phase of n=1 modes
    - The data for this calculation uses all *static* coil compensations.
  - Separate B_P and B_R mode amplitude and phases.
  - Send these numbers to "tmf" algorithm, or the old "smf"" algorithm.
- Add calculation of zero-crossings for RFA measurement.
  - Pick a known good sensor pair (B_R, upper-difference #1)
  - Use "calibrated", but not "compensated" versions of signals.
    - Avoids need to apply AC compensations in realtime
  - Apply high-order causal bandpass filter
    - Essentially same code as in simulation
  - Check for zero crossings, identify TW amplitude.
    - Essentially same code as in simulation
  - Normalize to SPA currents, to get RFA measurement
    - SPA TW frequency and amplitude are waveforms in the "tmf" algorithm, and can be accessed here.
  - Send the RFA value to the "bnf" algorithm.

New "tmf" Algorithm Designed to Allow n=1 FB in the Presence of the n=1 Spectroscopy Perturbation

- Feature #1: Generate n=1 traveling waves.
  - Input wave amplitude, phase, and start time as waveforms:

$$I_{SPA,X,TW}(t) = \begin{cases} 0, \\ I_{TW}\cos(2\pi f - \theta_X), \end{cases}$$

- Feature #2: Generate pre-programmed requests. $I_{SPA,X,\mathrm{Pre-Prog}}(t)$
- Feature #3: Fast feedback and DEFC, notch-filtered to remove applied TW (example for B_P sensors):
  - The "rid" algorithm provides the n=1 amplitude and phase, or alternatively the quadrature components:

$$B_{\mathrm{mode}}(\phi,t) = B_1(t)\cos(\phi-\theta_1(t)) = C_1(t)\cos(\phi) + S_1(t)\sin(\phi)$$

$$C_{1,BP}(t) = B_{1,BP}(t)\cos(\theta_{1,BP}(t)) \quad S_{1,BP}(t) = B_{1,BP}(t)\sin(\theta_{1,BP}(t))$$

  - Calculate lowpass and highpass version of the quadrature components, using high-order filters (phase effects?).

$$C_{1,LPF,BP}(t) = LPF(C_{1,BP}(t)) \quad C_{1,HPF,BP}(t) = HPF(C_{1,BP}(t))$$

$$S_{1,LPF,BP}(t) = LPF(S_{1,BP}(t)) \quad S_{1,HPF,BP}(t) = HPF(S_{1,BP}(t))$$

  - Reconstruct amplitude and phase, for HP and LP filtered data

$$B_{1,Bp,LPF} = \sqrt{(C_{1,LPF,BP}(t))^2 + (S_{1,LPF,BP}(t))^2} \quad \theta_{1,Bp,LPF} = a\tan(C_{1,LPF,BP}(t)/S_{1,LPF,BP}(t))$$

$$B_{1,Bp,HPF} = \sqrt{(C_{1,HPF,BP}(t))^2 + (S_{1,LPF,BP}(t))^2} \quad \theta_{1,Bp,HPF} = a\tan(C_{1,HPF,BP}(t)/S_{1,HPF,BP}(t))$$

  - Form feedback requests, with different gains and phases for HP and LP values

$$I_{SPA,X,Bp-FB}(\phi,t) = P_{Bp,LP}(t)B_{1,Bp,LP}(t)L_{eff}^{-1}\cos(\phi-\theta_{1,Bp,LP}(t)+\delta(t)) + P_{Bp,HP}(t)B_{1,Bp,HP}(t)L_{eff}^{-1}\cos(\phi-\theta_{1,Bp,HP}(t)+\delta(t))$$

- Final SPA request is sum of all the above:

$$I_{SPA,X}(t) = I_{SPA,X,TW}(t) + I_{SPA,X,\mathrm{Pre-Prog}}(t) + I_{SPA,X,Bp-FB}(t) + I_{SPA,X,Br-FB}(t)$$

High-Order Causal Filter Realtime Implementation

HP and LP Filters

First use Nth order lowpass filter:

$$f_i^{(1)} = \overline{C}_{LP}\left[f_{i-1}^{(1)} + C\left(f_i^{(0)} + f_{i-1}^{(0)}\right)\right]$$

$$f_i^{(2)} = \overline{C}_{LP}\left[f_{i-1}^{(2)} + C\left(f_i^{(1)} + f_{i-1}^{(1)}\right)\right]$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$f_i^{(N)} = \overline{C}_{LP}\left[f_{i-1}^{(N)} + C\left(f_i^{(N-1)} + f_{i-1}^{(N-1)}\right)\right]$$

$$\overline{C}_{LP} = \left(1 + {dt}/{\tau}\right), \quad C = {dt}/{2\tau}$$

Then use Nth order lowpass filter:

$$f_i^{(1)} = \overline{C}_{HP}\left[f_{i-1}^{(1)} + \left(f_i^{(0)} - f_{i-1}^{(0)}\right)\right]$$

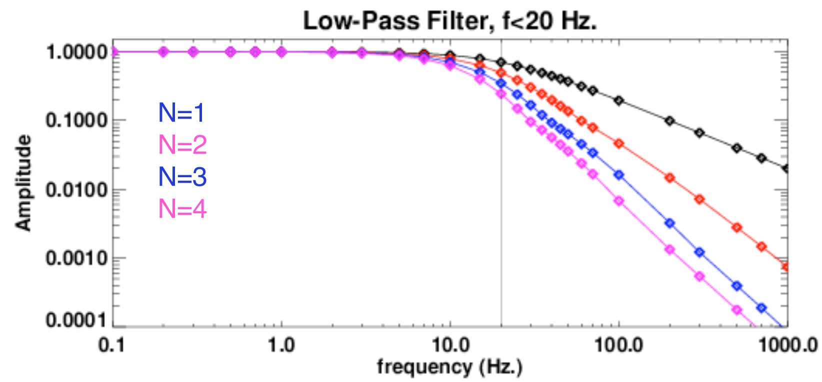
$$f_i^{(2)} = \overline{C}_{HP}\left[f_{i-1}^{(2)} + \left(f_i^{(1)} - f_{i-1}^{(1)}\right)\right]$$

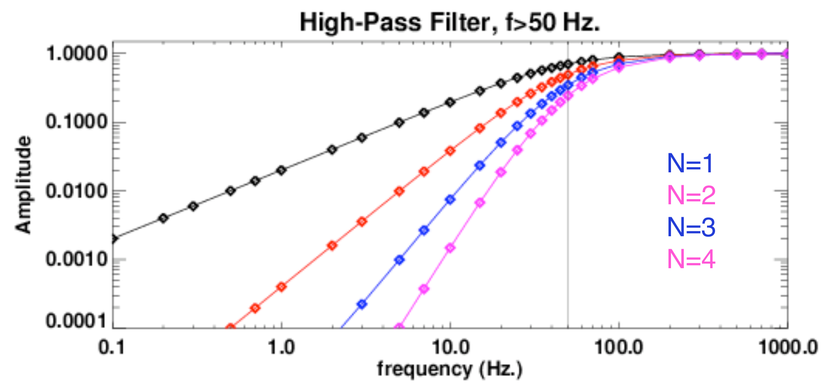
$$\vdots \qquad \vdots \qquad \vdots$$

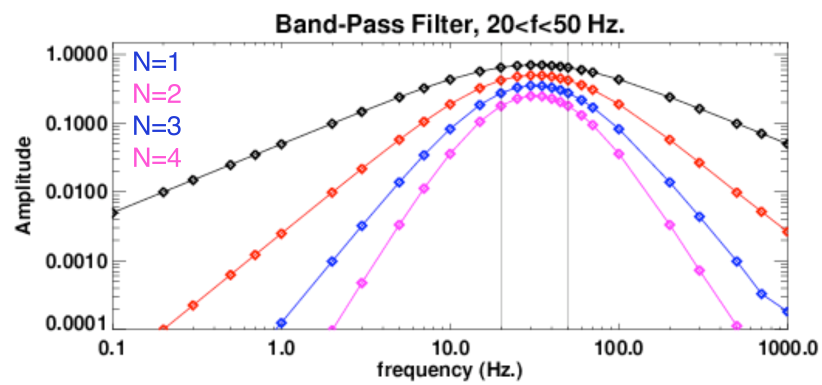
$$f_i^{(N)} = \overline{C}_{HP}\left[f_{i-1}^{(N)} + \left(f_i^{(N-1)} - f_{i-1}^{(N-1)}\right)\right]$$

$$\overline{C}_{HP} = \left(1 + {dt}/{\tau}\right)^{-1}$$

- For each signal undergoing an nth-order filter, need to keep an Nx2 array of previous values.
- Each filter requires a simple length-N FOR loop.
- Easy to implement.







Simulation of "Single Sensor" RFA Feedback Uses Simple Physics Model

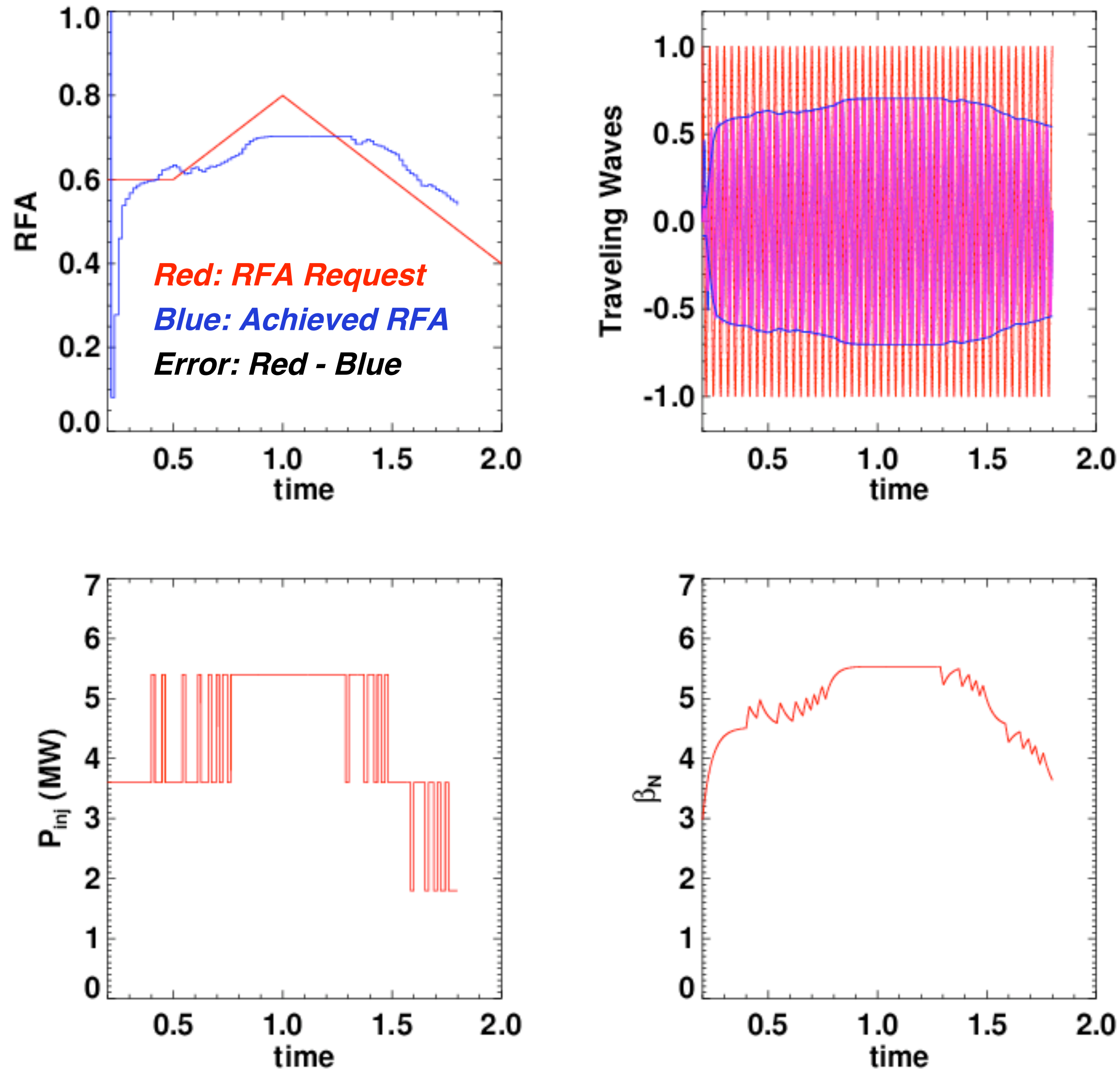
- Plasma Heating Model
  - Use 0-D model of plasma

$$\beta_N \Rightarrow \beta_N + \left(\tau \frac{I_P V B_T}{200\mu_0 a}\right)^{-1} \Delta P_{inj}$$

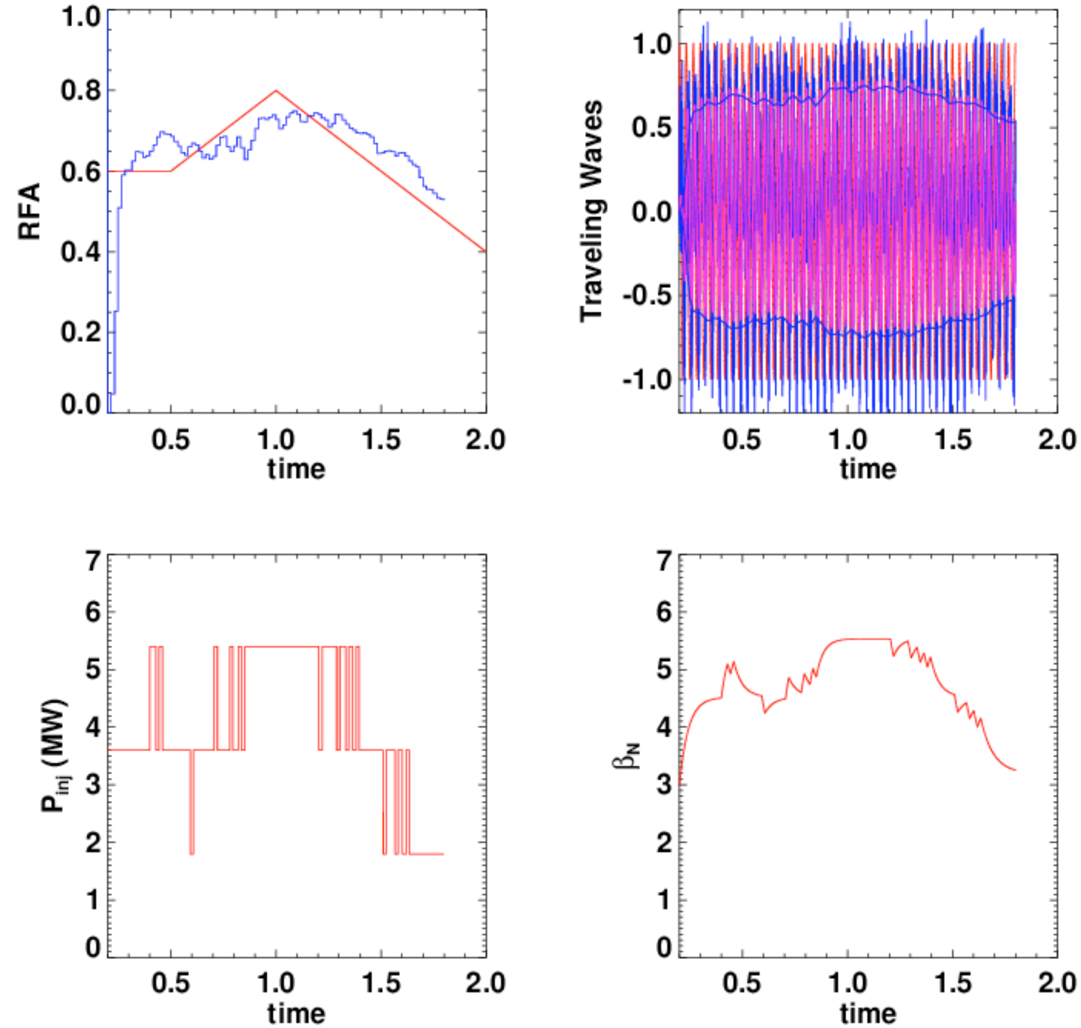
  - Neglect beam slowing down…power is instantly deposited
- RFA Model & Detection
  - Simple linear relationship between RFA and β_N: $M = a_{RFA}\beta_N + b_{RFA}$
    - a_{RFA} & b_{RFA} determined from previous measurements.
  - Detected wave is $M \times I_{RWM} \cos(wt)$
  - Add noise to the detected wave..
  - Apply a high-order causal filter to extract the eliminate noise
  - Detect the RFA from analysis of the traveling wave data.
    - Use zero-crossing identification to bracket maxima and minima.
- Feedback Scheme
  - Proportional gain on RFA error modifies the injected power request
- Neutral Beam
  - Injected power request leads to a duty cycle request for each source.
  - Use the same modulation methods as in the (untested) β_N control algorithm.

Simulation With No Noise Indicates RFA Control is Possible

- Proportional gain only, not optimized.
- No noise on detected wave
- Confinement time (40 msec) insufficient to reach requested RFA
- 3rd order causal HP and LP filters, passing 20 < f < 50 Hz.



Simulation With Considerable Noise Shows Success of High-Order Causal Filters



Red: SPA Waveform (kA)

Blue: Detected Difference Signal

Magenta: Difference Signal After Filtering