Conceptual Design for β Control and Realtime Stability Limit Detection via RFA Measurements

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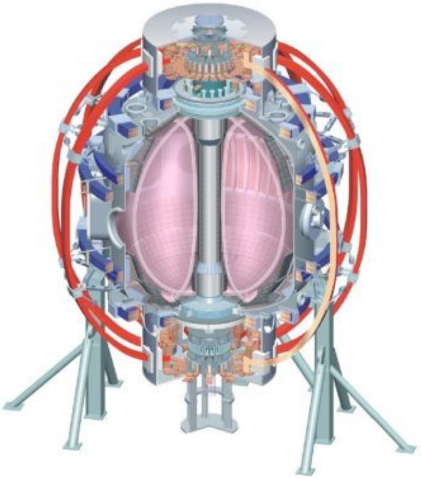

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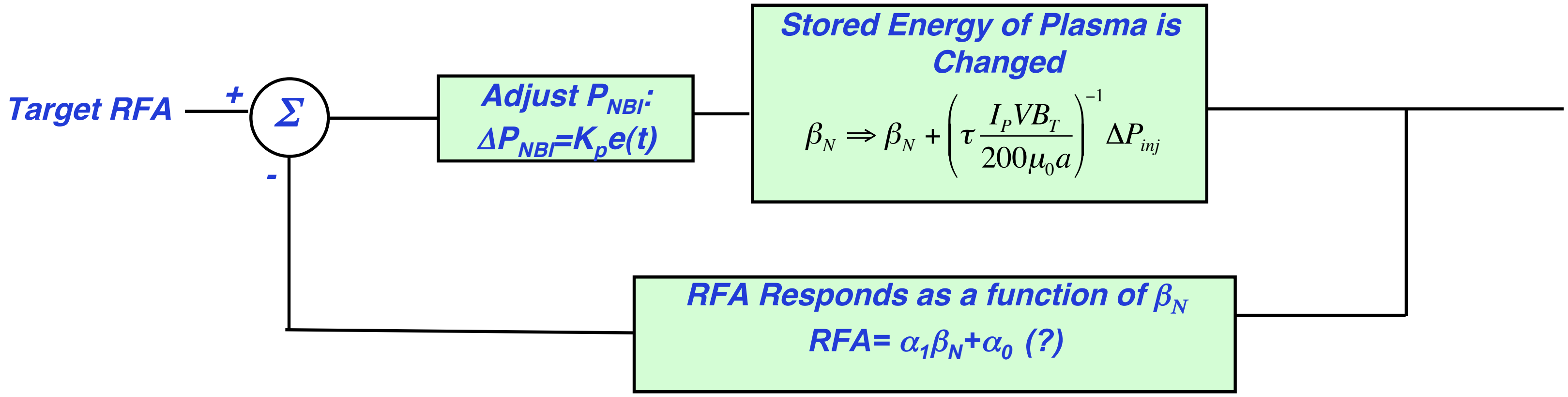
Ultimate Goal: Control NB Modulation in Order to Maintain β Just Below the “Real” Stability Limit



Target RFA + Σ -

Adjust P_{NBI}:
$\Delta P_{NBI}=K_p e(t)$

Stored Energy of Plasma is Changed

$$\beta_N \Rightarrow \beta_N + \left(\tau \frac{I_P V B_T}{200\mu_0 a}\right)^{-1} \Delta P_{inj}$$

RFA Responds as a function of β_N
RFA= $\alpha_1\beta_N+\alpha_0$ (?)

- RFA is the amplification of applied error fields by the plasma.
  - RFA is believed to increase rapidly near and above the no-wall β_N limit.
- By monitoring RFA in realtime, it may be possible to detect proximity to this stability limit.
  - May be better than rtEFIT+real-time stability codes (rtDCON?)
- Using RFA to adjust the input power may be a component of non-disruptive operation near or beyond the no-wall β_N limit.[1]
  - *Particularly attractive for a beam-driven device like CTF.*

1: H. Reimerdes, et al., NF 2005

Outline

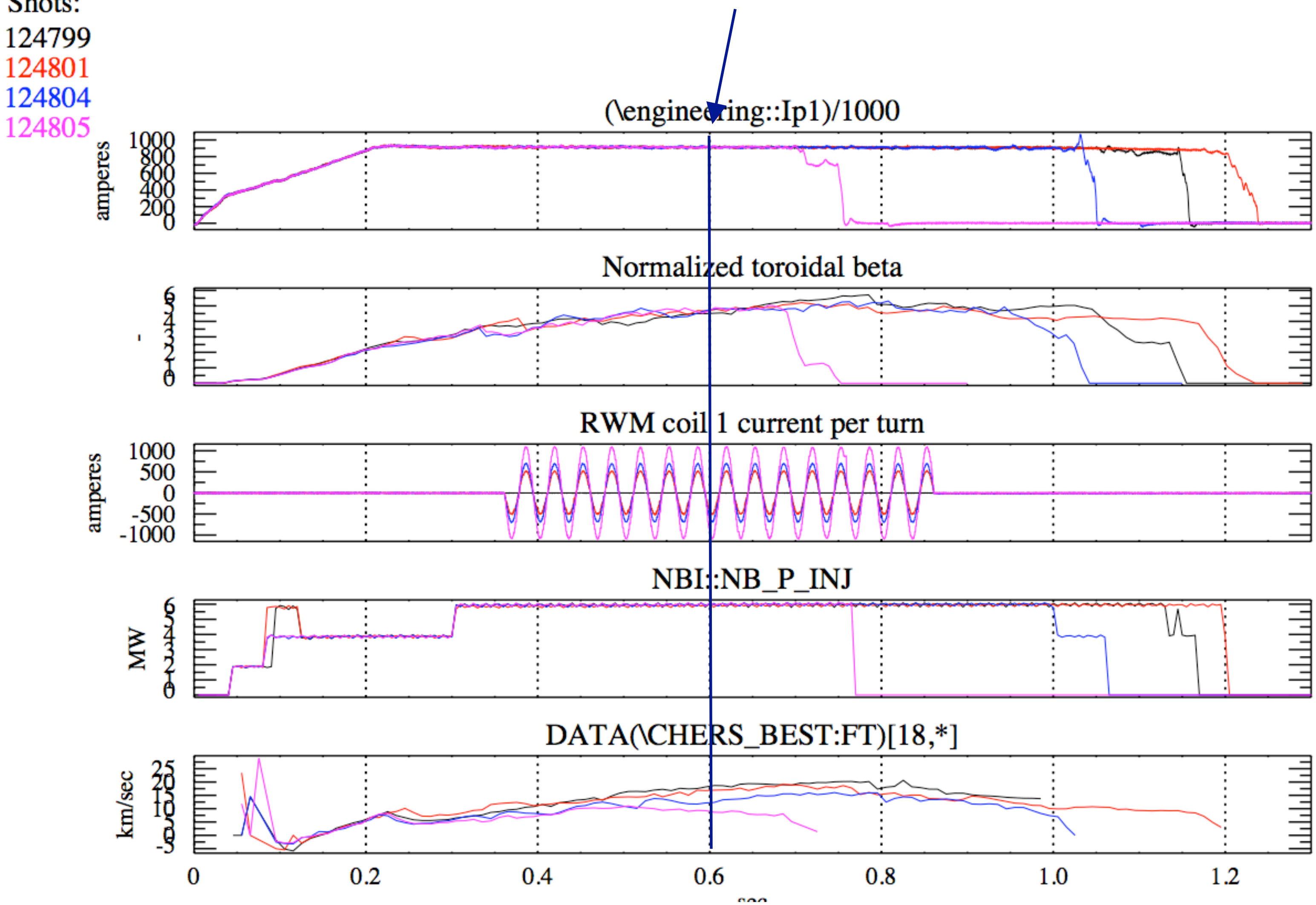
- Ultimate goal.
- Analysis of some 2007 data, XP-704
  - High-β, high-κ, high-δ, rapidly rotating targets similar to those we might want to use for RFA control.
  - RFA measurements in two ways:
    - Use of a single, highly filtered but minimally compensated anti-series pair of B_R sensors (scheme from J.K. Park).
    - Highly filtered, highly compensated n=1 mode decompositions.
  - Some results for RFA vs $\beta_{N,}$
- PCS Implementation.
- Proposed scoping XP to resolve issues raised above.

Pay Special Attention to :

Recursive Filters Used to Process Data

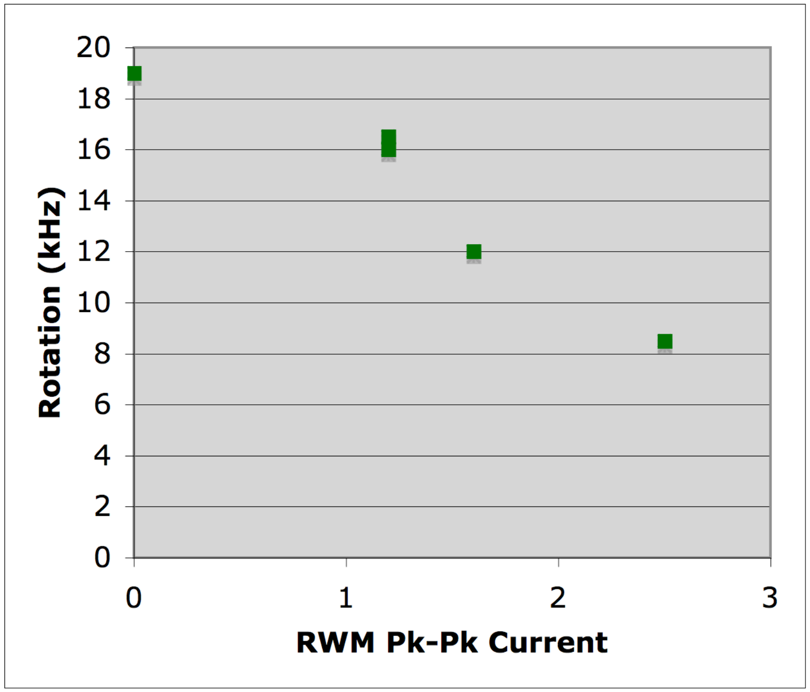
Computational Techniques Available in Realtime

High-Performance Shots Possible with 1kA Pk.-Pk. 30 Hz Traveling Waves


Time for rotation measurement

Larger Applied Fields Lead to Rotation Damping

CHERS Channel #18 rotation frequency, t=0.6 sec



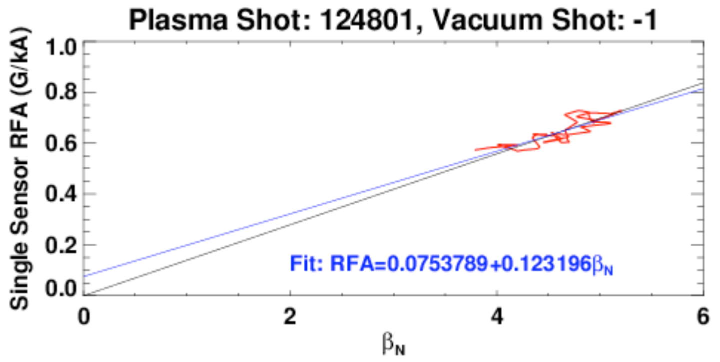
Severe rotation damping, and eventual disruption, for I_{RWM}>~1.4 kA

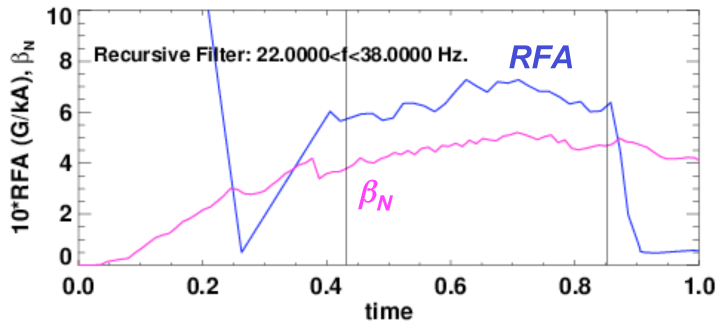
Example “Single Sensor” RFA Measurements: XP-704

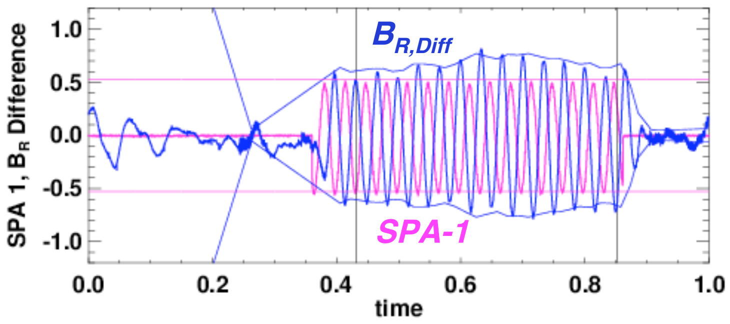
- Shot 124801: a “typical best shot”
  - 1.2 kA pk. to pk., 30 Hz, counter-rotating n=1 traveling wave (TW) perturbation.
  - 6MW input power, with β_N slowly evolving during TW application
- RFA Definition:

$$RFA = \frac{B_{R,Diff,Peak-to-Peak}}{I_{RWM,Peak-to-Peak}}$$

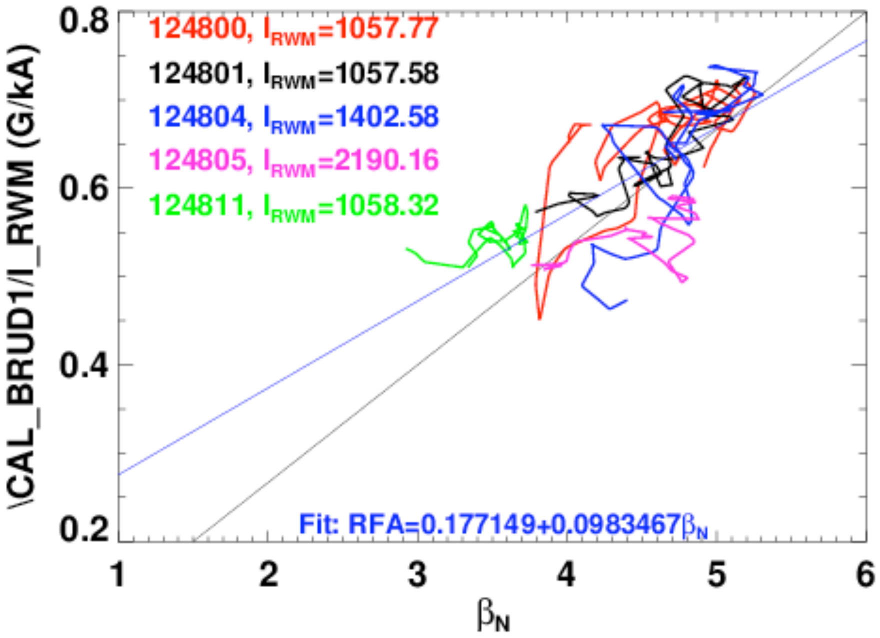
  - $B_{R\ Dif}$ is the difference field for a single coil pair, including all RWM coil pickup.
    - Equivalent to the ‘\Cal_’ signals in the tree.
    - May be good for realtime calculations, since the compensations are minimal.
  - $B_{R,Diff,Peak\text{-}to\text{-}Peak}$ determined from tracking zero-crossings.
    - Two amplitudes calculated for each cycle.
    - Relatively easy to code for realtime.
- Recursive filters used to isolate the correct frequency.
- Clear tracking of RFA with β_N during period of TW.







Look At All Co-Going Measurements For XP-704, Single Sensor RFA Definition



- Consider all shots with +30 Hz waves in XP-704
- All three source shots except 124811, which had 2 sources, and lower β.
- Scaling of RFA with β_N remains, though the scatter is large.
  - Can you actually do control based on these signals?
  - Including information from all sensors could improve the performance of the system.
    - Look at other definitions.

What about looking at the Full Plasma n=1 Response?

Basic RFA Model

H. Reimerdes, NF 2005

$$\tau_W \frac{dB_s}{dt} - \gamma_0 \tau_w B_a = M^*_{sc} I_C$$

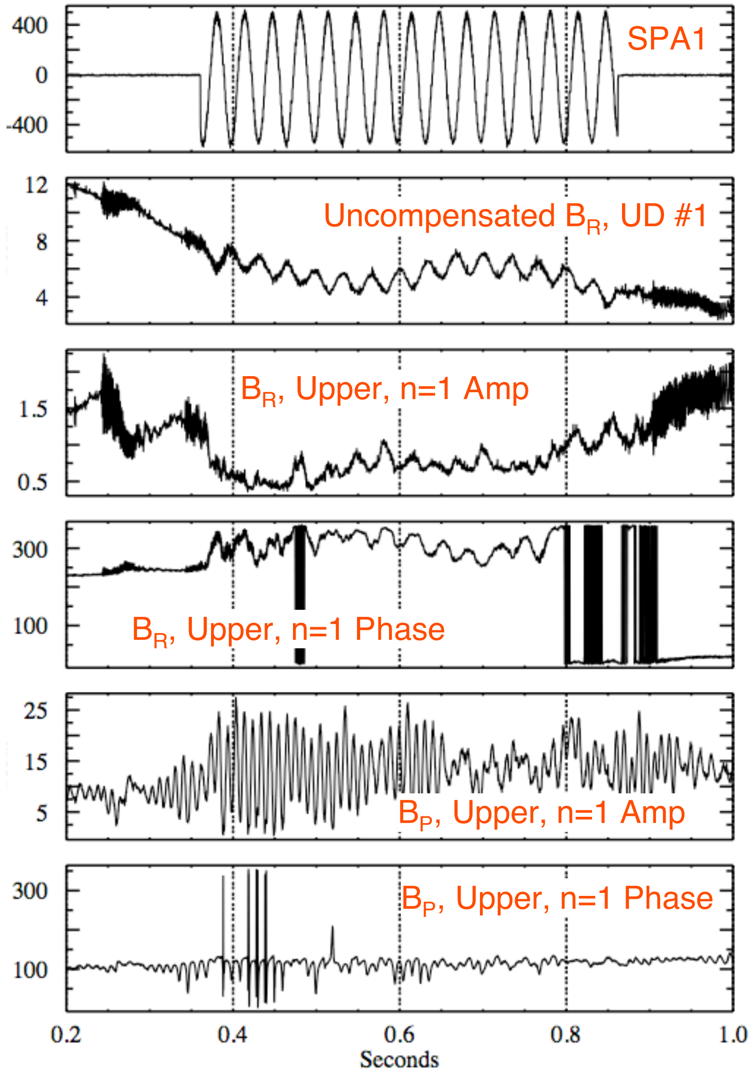
$$\gamma_0 = \gamma_{RWM} + i\omega_{RWM}$$

$$B_s^{ext} = \frac{M_{sc} I_C}{1 + i\omega_{ext}\tau_w}$$

$$A_{RFA,s} = \frac{B_s - B_s^{ext}}{B_s^{ext}} = c_s \frac{1 + \gamma_0 \tau_w}{i\omega_{ext}\tau_w - \gamma_0 \tau_w}$$

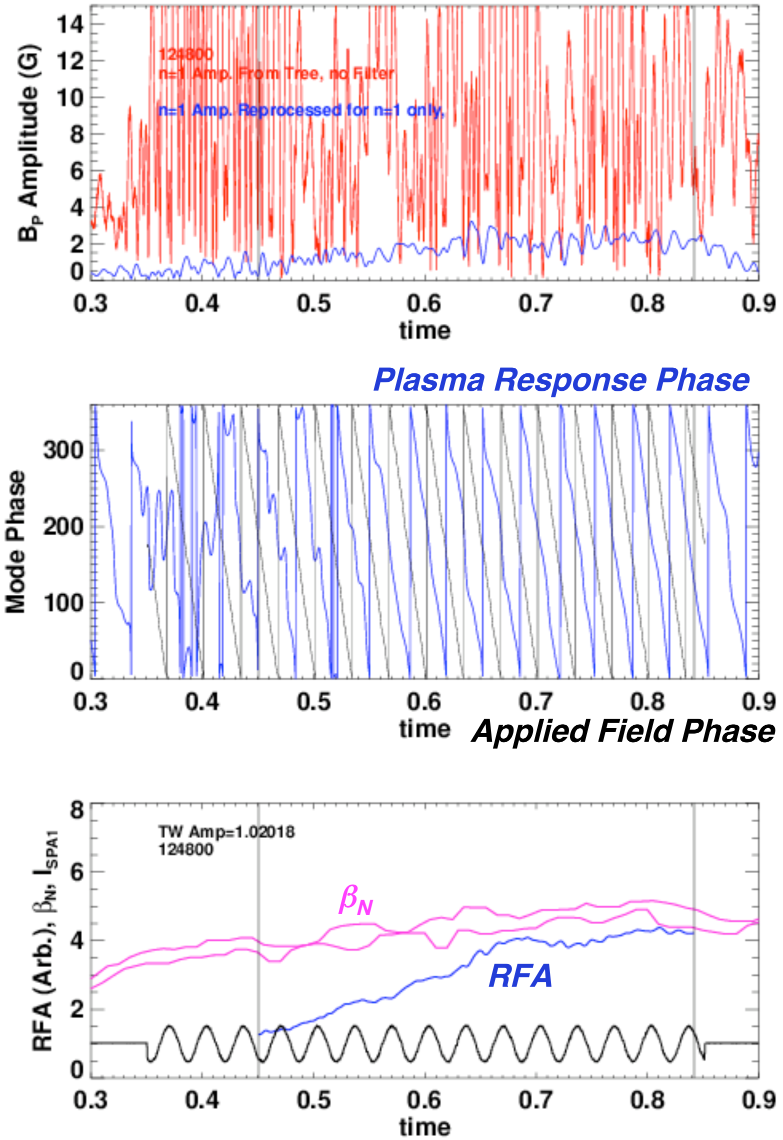
- Plasma response should be an n=1 perturbation, phase shifted with respect to the applied field.
- However, no rotating perturbation is seen in the archived n=1 decompositions.

Shot 124801

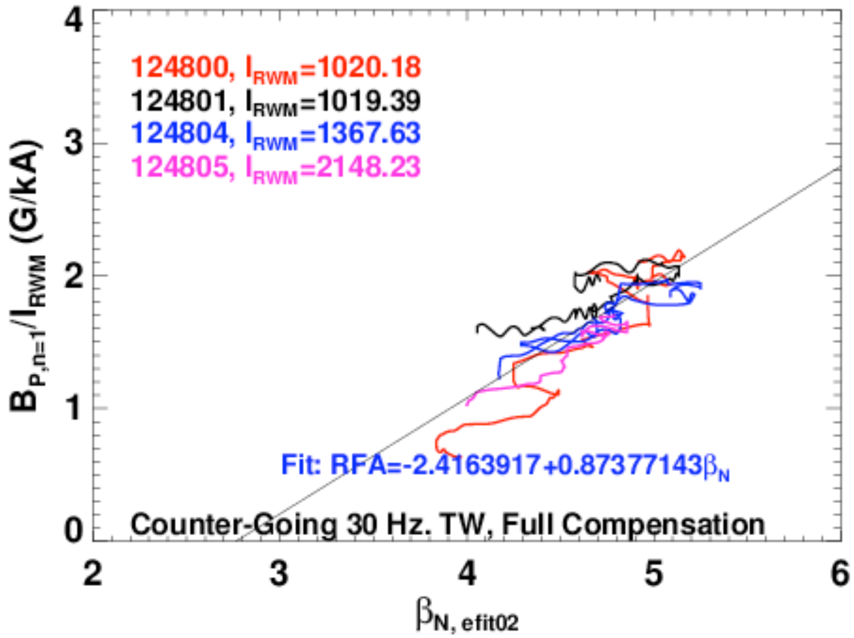


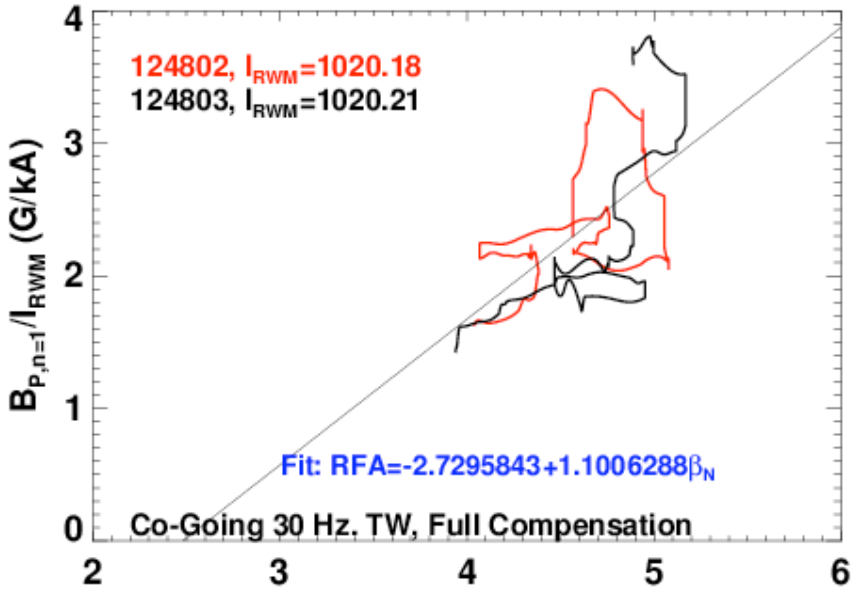
Extra Processing of the Sensor Data Resolves A Clear Rotating n=1 Perturbation

- Processing steps to observe rotating RFA:
  - Utilize fully compensated sensor data.
  - Compute mode amplitude and phase allowing n=1 (or 1+2) only.
  - Compute quadrature components of mode.
  - Apply high-order band-pass filter to quadrature components.
  - Recompute mode amplitude and phases.
- Clear rotating plasma response is observed.
- Plasma response scales with β_N.



RFA from n=1 Decomposition Shows Scaling with β_N



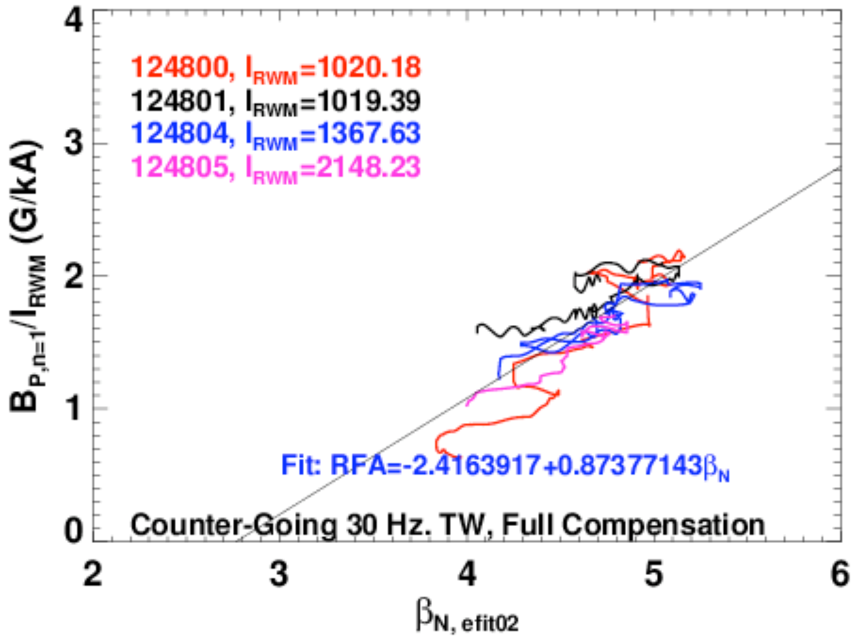


- $B_{P(upper)}$ sensors show best correlation with β_N.
  - Less noise than single sensor measurement, and more consistency that B_R data
  - These are the best RFA measurements SPG has yet found.
- B_p sensors may be less sensitive to plasma geometry than B_R (for fixed outer-gap)
  - Distance to B_R sensors is a stronger function or outer squareness and trianguarity.

Big Caveat

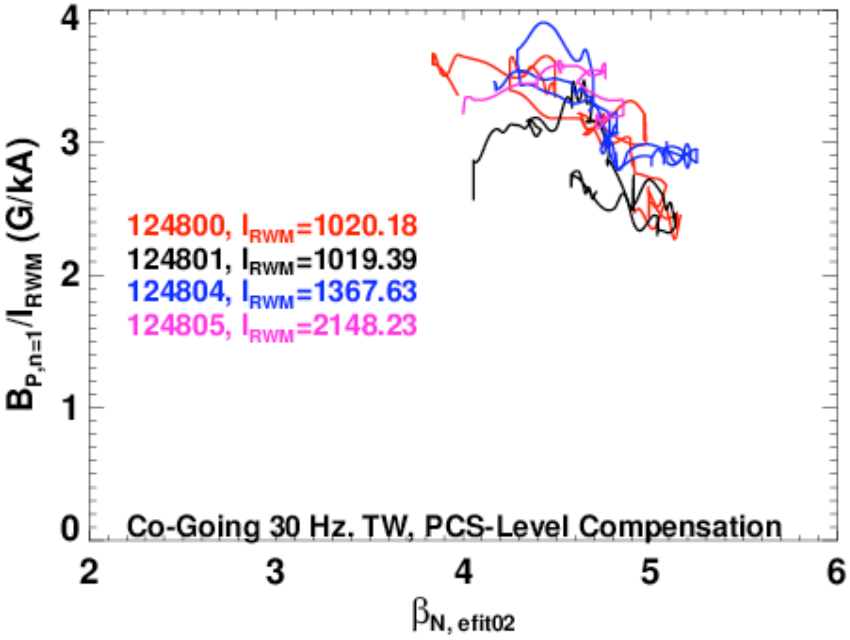
This Analysis Based on Fully Compensated Sensor Data, Including AC Compensation of EFC Coil Pickup

AC Compensations are Necessary



Counter-going TWs detected using full AC compensation

Clear RFA trend!



Counter-going TWs detected using only static compensation.

It is a mess.

Next Two Slides:

Details of the off time-domain AC compensation method…will skip unless people really want to hear it.

NSTX Uses Time-Domain AC Compensations

$$C_i(t) = \sum_{j}^{NumRWMCoils} \sum_{k=0}^{k_{\max}} p_{i,j,k} LPF\left(\frac{dI_{RWM,j}(t)}{dt};\tau_k\right)$$

(Compensated Signal)$_i$= (Uncompensated Signal)$_i$-C$_i$

Offline:

- ***For each sensor (i) and RWM coil (j), there are 5 time constants t, and 5 associated coefficients coefficients,***
- ***Total of 8×6×6×5×2=2880 (!) numbers***

Online Now:

- ***Include only k=0 term (static pickup)***
- ***8×6×6=288 numbers***

For RFA measurement, may only need to compensate in the vicinity of the applied frequency.

Can only static pickup and a single time-constant be useful for RFA feedback?

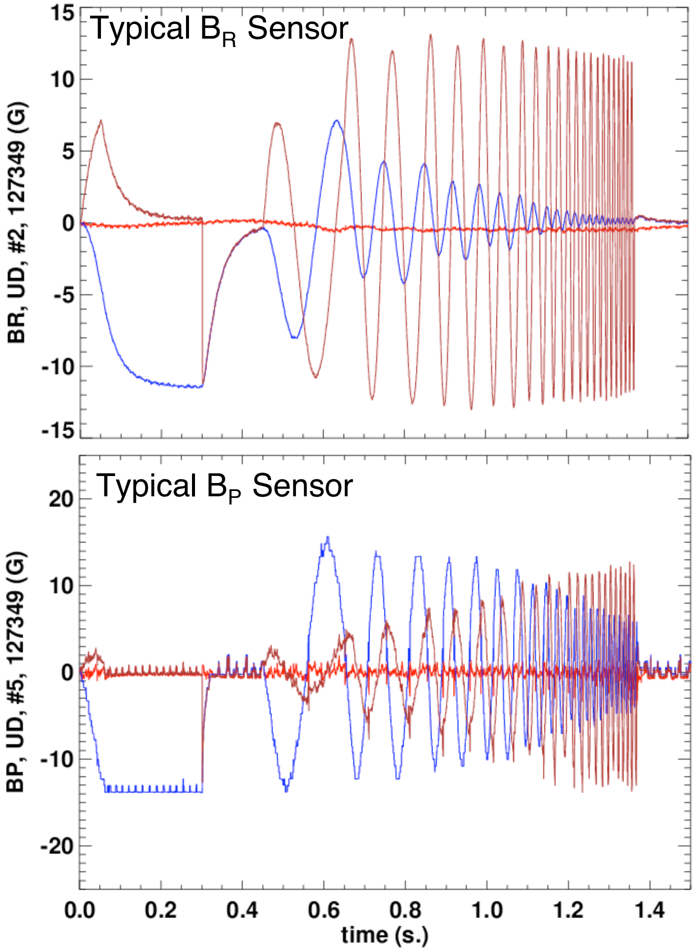
However, the overall feedback system might be improved with better AC compensation.

Example Compensations: Vacuum shot with a single RWM Coil Energized

Red: Fully Compensated

Blue: Full Pickup

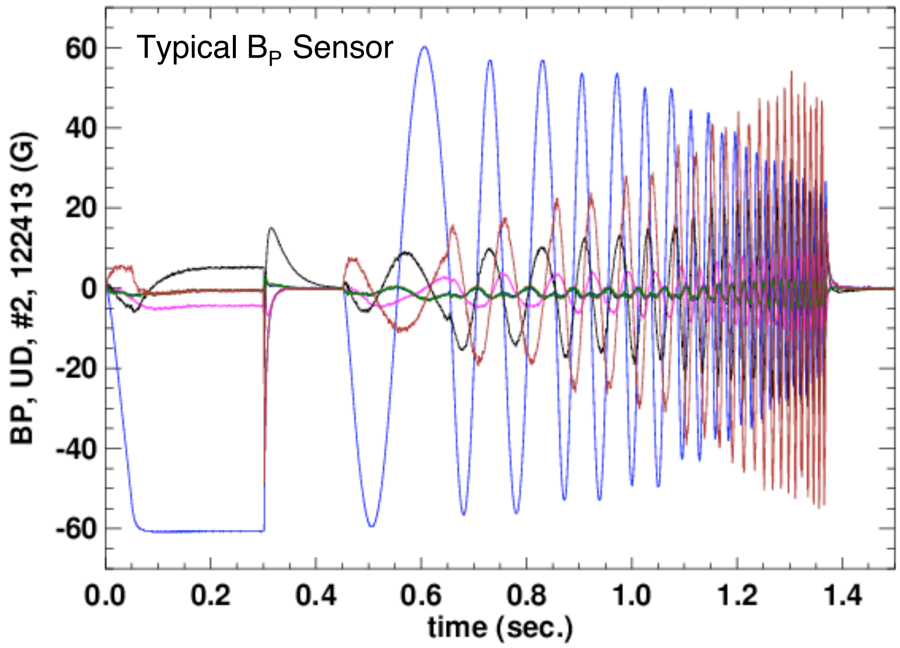
Brown: Direct Pickup Only Subtracted (k=0 only, as in PCS)

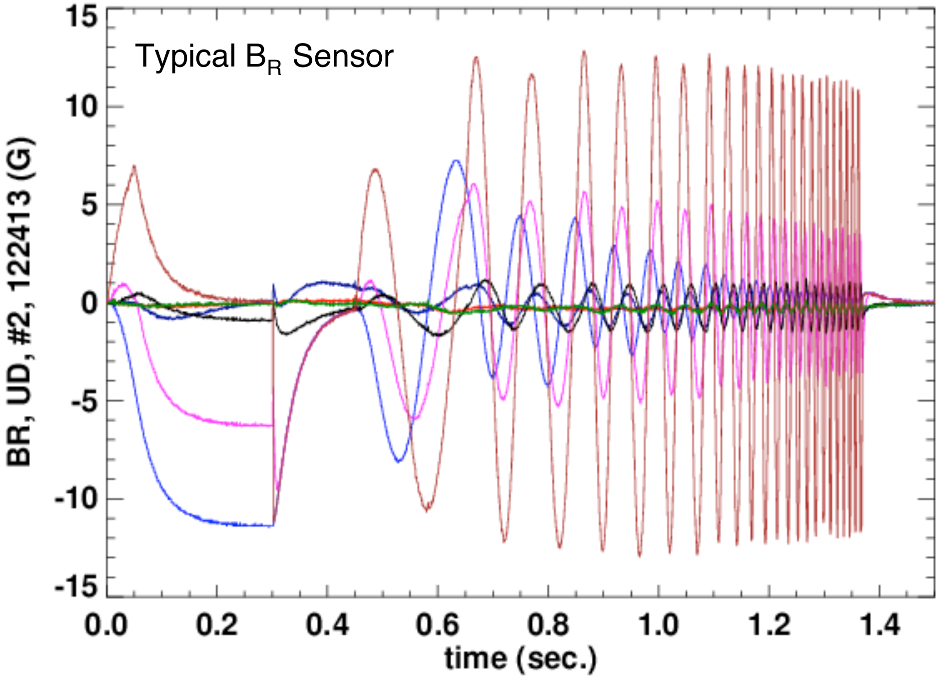


Two τ's (>0) May Be Sufficient For Compensation, while Three will Certainly Work

Blue: No Compensation
Brown: Static Compensation Only
Magenta: 2 term compensation, τ=0,3 (msec)
Black: 2 term compensation, τ=0,30 (msec)
Dark Blue: 3 term compensation, τ=0,4,16 (msec)
Green: 4 term compensation, τ=0,3,9,18 (msec)
Red: 5 term compensation, τ=0,2,4,8,16 (msec)

Blue: No Compensation
Brown: Static Compensation Only
Magenta: 2 term compensation, τ=0,3 (msec)
Black: 2 term compensation, τ=0,30 (msec)
Dark Blue: 3 term compensation, τ=0,24,96 (msec)
Green: 4 term compensation, τ=0,15,30,60 (msec)
Red: 5 term compensation, t=0,12,24,48,96 (msec)





Which Technique for RFA Detection is Best?

Editorial Color Code

Green is "good"

Red is "bad"

- Single Sensor Method
  - No additional compensations required.
  - Need zero-crossing/peak-finding algorithm.
  - The calibrated but uncompensated data is NOT presently available in mode-ID, so still need to bring in additional data from (I think) ACQ.
  - Time resolution is limited to essentially ~1/f_{TW}, based on zero-crossing/peak-peak analysis.
  - Requires one band-pass filter (the chosen difference signals).
- n=1 Decomposition Method
  - Uses (averages) all sensors to constrain the RFA response.
  - Provides instantaneous values for the RFA.
  - Requires AC compensations be applied to sensors.
    - These compensations can then be used for improved fast RWM feedback, as well as RFA control.
  - Requires 2 band-pass filters (the quadrature components).
- SPG Recommendation (pending discussion of frequency-domain method of HR):
  - Implement AC compensations, with either 2 or 3 non-zero time constants, and use the filtered n=1 quadrature components for RFA measurements.

Late Breaking…

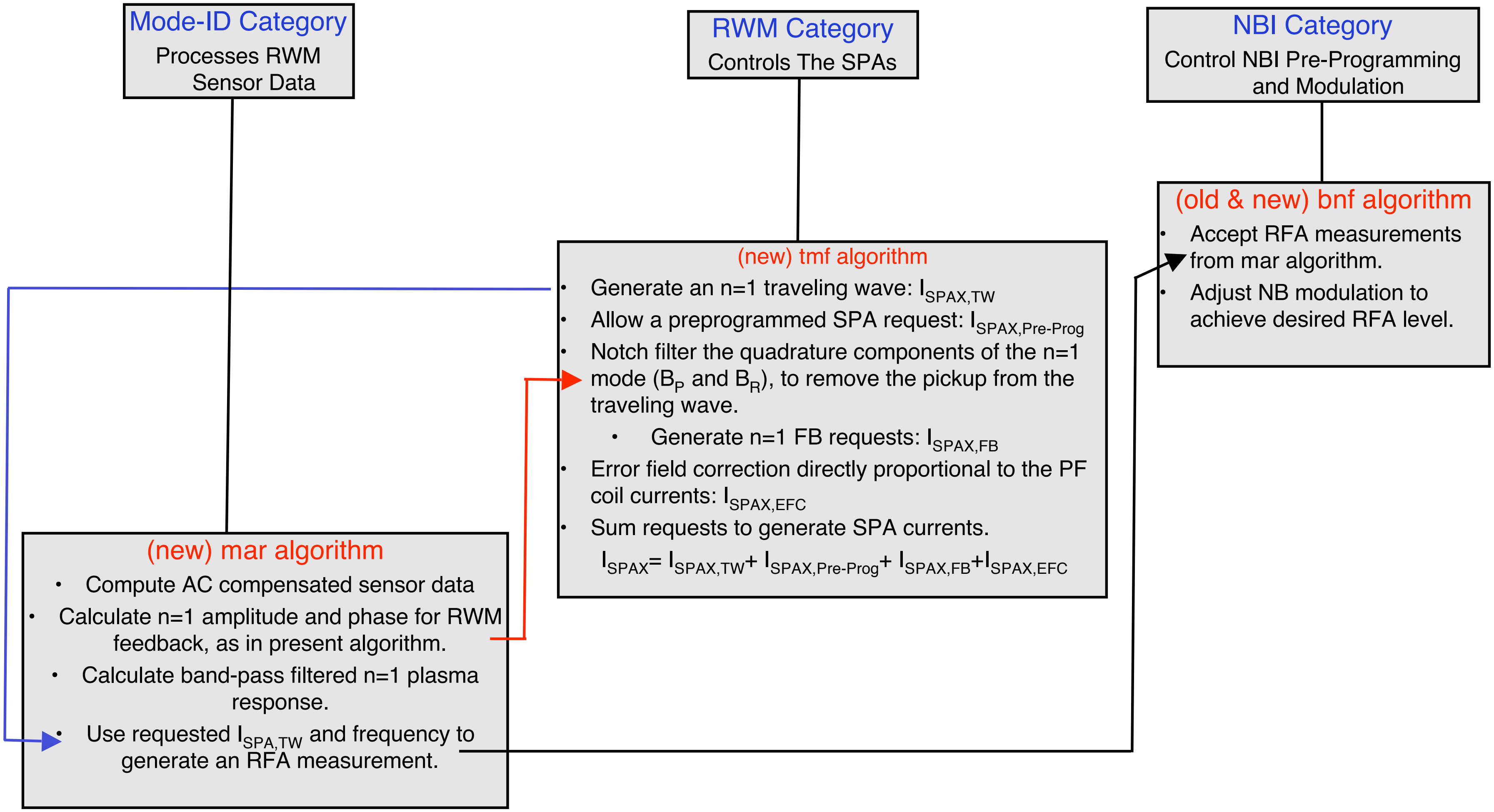
- Holger Reimerdes will present (next) a fast *frequency domain* method of isolating the RFA signal from an n=1 traveling wave.

- Implements the AC compensation at only the frequency of interest.
  - Fewer calculations, and possibly better signal to noise.

- Haven't yet tested it on NSTX data
  - data from 2005 should allow an initial comparison to time-domain methods

- All methods discussed in this presentation use time-domain analysis.

Modifications and/or New Algorithms Required in Mode-ID, RWM, and NBI Categories



Mode-ID Category
Processes RWM Sensor Data

RWM Category
Controls The SPAs

NBI Category
Control NBI Pre-Programming and Modulation

(old & new) bnf algorithm

- Accept RFA measurements from mar algorithm.
- Adjust NB modulation to achieve desired RFA level.

(new) tmf algorithm

- Generate an n=1 traveling wave: $I_{SPAX,TW}$
- Allow a preprogrammed SPA request: $I_{SPAX,Pre\text{-}Prog}$
- Notch filter the quadrature components of the n=1 mode (B_P and B_R), to remove the pickup from the traveling wave.
  - Generate n=1 FB requests: $I_{SPAX,FB}$
- Error field correction directly proportional to the PF coil currents: $I_{SPAX,EFC}$
- Sum requests to generate SPA currents.

$$I_{SPAX}= I_{SPAX,TW}+ I_{SPAX,Pre\text{-}Prog}+ I_{SPAX,FB}+I_{SPAX,EFC}$$

(new) mar algorithm

- Compute AC compensated sensor data
- Calculate n=1 amplitude and phase for RWM feedback, as in present algorithm.
- Calculate band-pass filtered n=1 plasma response.
- Use requested $I_{SPA,TW}$ and frequency to generate an RFA measurement.

Discuss each of these algorithms on the following slides

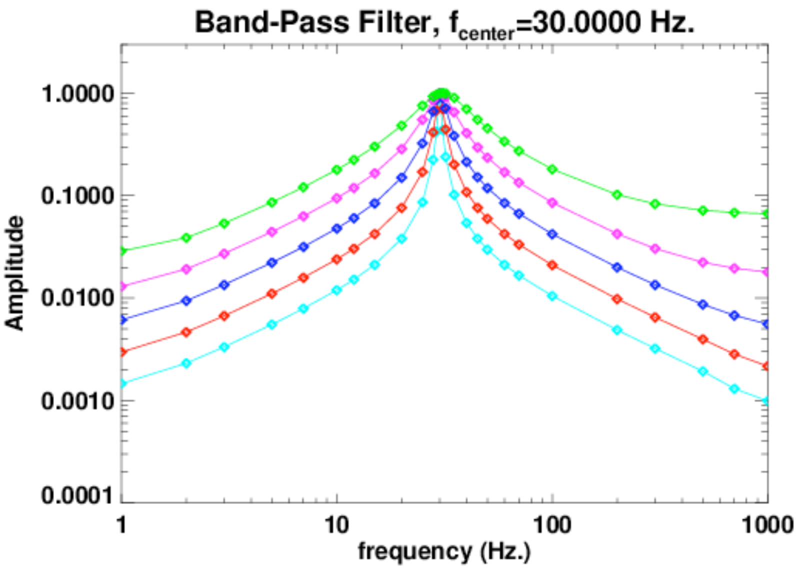
New “mar” Algorithm Will Produce Both B_P and B_R n=1 Mode Identification, and RFA Data As Well

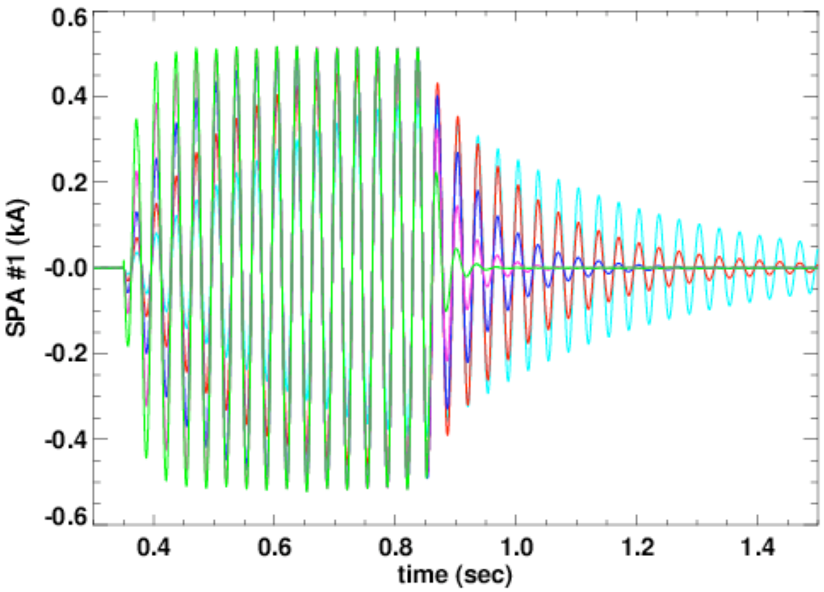
- Add AC compensations.
  - Put AC compensation coefficients in model tree, read them into shared memory.
  - Add code to compute and subtract compensation terms
- For RMW control, retain features of “mid” algorithm.
  - Generate same amplitude and phase of n=1 modes for RWMF/DEFC, based on the decomposition matrix $Y_{\text{mode-ID}}$
  - Separate B_P, B_R, and “combined” n=1 mode amplitude and phases.
  - Send these numbers to “tmf” algorithm, or the older “imf” and “smf”” algorithm.
  - Same baseline rezeroing:
    - However, allow separate times for B_P and B_R sensors.
- RFA calculations
  - Use a different decomposition matrix, Y_{RFA}, to generate quadrature components of plasma response, and then bandpass filter the components.
    - Center frequency and BW of recursive filter are algorithm waveforms.
  - Calculate the amplitude of the plasma response $(C^2+S^2)^{1/2}$.
  - Normalize to SPA currents, to get RFA measurement in units of Gauss/kA.
    - SPA TW frequency and amplitude are waveforms in the “tmf” algorithm, and can be accessed here.
  - Send the RFA value to the “bnf” algorithm.
  - This section is the part that changes under Holger’s frequency-domain method.

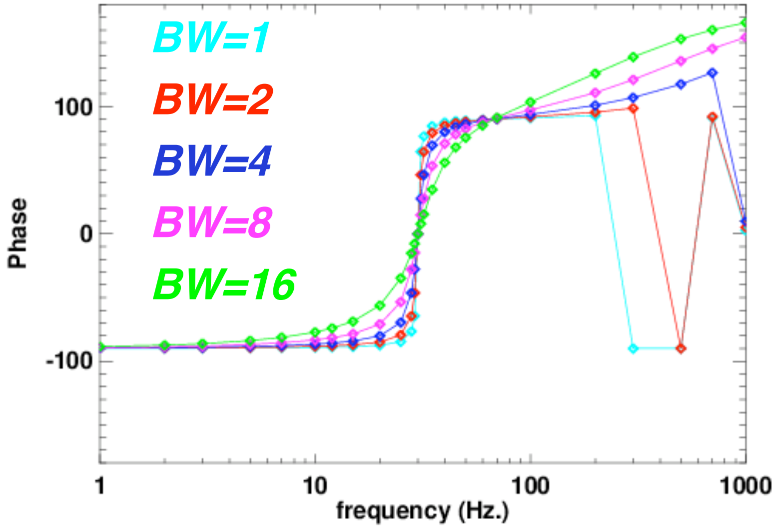
Simple Recursive Bandpass Filter Will Be Used to Isolate Plasma Response at f_{TW}

$$y_{out}[n] = a_0 y_{in}[n] + a_1 y_{in}[n-1] + a_2 y_{in}[n-2]$$
$$+ b_1 y_{out}[n-1] + b_2 y_{out}[n-2]$$
$$a_0 = 1 - K, \quad a_1 = -2(K-R)\cos(2\pi f)$$
$$a_2 = R^2 - K$$
$$b_1 = 2R\cos(2\pi f) \quad b_2 = -R^2$$
$$K = \frac{1 - 2R\cos(2\pi f) + R^2}{2 - 2\cos(2\pi f)}$$
$$R = 1 - 3 \times BW$$

Narrow passbands are possible, but maybe not desirable







New “tmf” Algorithm Designed to Allow RWMF in the Presence of the n=1 Spectroscopy Perturbation

- Feature #1: Generate n=1 traveling waves. $I_{SPA-1}^{TW}(t) = I^{TW}(t)\cos\left(\left(300° + \delta_{TW}(t)\right)\left(\frac{\pi}{180}\right) + 2\pi f_{TW}(t)t\right)$
  - Input wave amplitude, phase, frequency as waveforms.
- Feature #2: Include pre-programmed requests. $I_{SPA,X,\mathrm{Pre-Prog}}(t)$
- Feature #3: Include correction proportional to PF5, PF4, PF3U, PF3L, TF, OH

$$I_{SPA-i}^{FEC} = \sum_j G_{j,i}\left({}^{I_j}\!/_{1000.}\right), \quad j = \{PF5, PF4, PF3U, PF3L, TF, OH\}$$

- Feature #4: RWMF/DEFC, notch-filtered to remove applied TW (example equations for B_P sensors):
  - The “mar” algorithm provides the n=1 amplitude and phase, or alternatively the quadrature components:

$$B_{\mathrm{mode}}(\phi,t) = B_1(t)\cos(\phi - \theta_1(t)) = C_1(t)\cos(\phi) + S_1(t)\sin(\phi)$$

$$C_{1,BP}(t) = B_{1,BP}(t)\cos(\theta_{1,BP}(t)) \quad S_{1,BP}(t) = B_{1,BP}(t)\sin(\theta_{1,BP}(t))$$

  - Calculate notch-filtered (NF) version of the quadrature components.

$$C_{1,FF,BP}(t) = LPF\left(C_{1,BP}(t); f_{center}, BW\right) \quad S_{1,NF,BP}(t) = LPF\left(S_{1,BP}(t); f_{center}, BW\right)$$

  - Reconstruct amplitude and phase of notch filtered data

$$B_{1,Bp,NF} = \sqrt{\left(C_{1,NF,BP}(t)\right)^2 + \left(S_{1,NF,BP}(t)\right)^2} \quad \theta_{1,Bp,NF} = a\tan\left(C_{1,NF,BP}(t)/S_{1,NF,BP}(t)\right)$$

  - Form feedback requests

$$I_{SPA,X,Bp-FB}(\phi,t) = P_{Bp}(t)B_{1,Bp,NF}(t)L_{eff}^{-1}\cos(\phi - \theta_{1,Bp,NF}(t) + \delta_{Bp}(t))$$

- Final SPA request is sum of all the above:

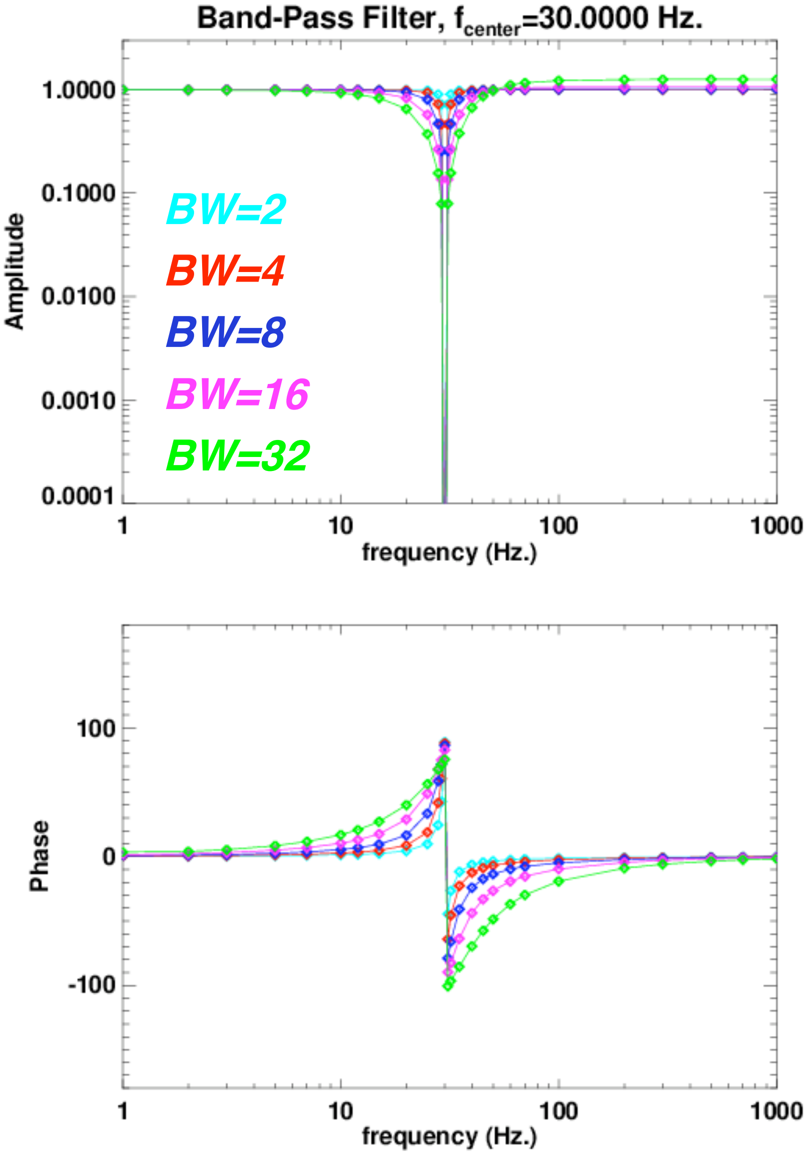
$$I_{SPA,X}(t) = I_{SPA,X,TW}(t) + I_{SPA,X,\mathrm{Pre-Prog}}(t) + I_{SPA,X,Bp-FB}(t) + I_{SPA,X,Br-FB}(t) + I_{SPA,X}^{EFC}(t)$$

Need a Recursive Notch Filter In Order to do RWMF/DEFC

- Very narrow stop-band achievable with simple recursive filter.
- Minimal phase shift in the pass-band.

$$y_{out}[n] = a_0 y_{in}[n] + a_1 y_{in}[n-1] + a_2 y_{in}[n-2]$$
$$+b_1 y_{out}[n-1] + b_2 y_{out}[n-2]$$
$$a_0 = K, \quad a_1 = -2K\cos(2\pi f)$$
$$a_2 = K$$
$$b_1 = 2R\cos(2\pi f) \quad b_2 = -R^2$$
$$K = \frac{1 - 2R\cos(2\pi f) + R^2}{2 - 2\cos(2\pi f)}$$
$$R = 1 - 3 \times BW$$

- BW and f are bandwidth and center frequency, normalized to sample frequency



Existing "bnf" Algorithm Can Likely be Utilized for the RFA Feedback

- NBI Category:
  - Existing "bnf" (= beta-normal feedback) algorithm already allows a wide variety of feedback targets:
    - W_{MHD}, β_N, β_T, for instance.
    - Waveforms exist for β_P control (P, I, D, deadband, LPF), but no internal code:
      - Simple to redefine those waveforms for RFA control, forget about β_P control.
    - Alternatively, add new waveforms for RFA-control
      - Need to think about the impact on reloading.
  - *Advantage is that we use the same (presumably optimized) methods for setting NBI modulation times, batting order, preprogramming.*
  - *Take credit for all the EPICS/PCS communication work presently being done.*

Plan is Designed For RFA Control, But Has Incremental Improvements For RWMF/DEFC Studies

- Mode-ID Improvements
  - Apply AC Compensations To Sensor Data
    - Accurate mode reconstruction during fast feedback.
    - Doesn't happen with HR's frequency-domain method and single frequency AC compensation method.
    - There are interesting combinations of the time & frequency domain methods.
  - Allow B_R and B_P sensors to be re-zero'd at different times
    - Elimination of OH-TF pickup in the B_R sensors?

- RWM Coil Control Improvements
  - Introduce a notch filter, which can be used for any purpose
    - Elimination of 100 Hz noise?
  - Allow RWM coils to be directly tied to PF and TF coils.
    - Dynamic correction of the n=3 EF?

Issue To Be Addressed By An XP: How well does RFA Predict the Approach to Stability Limits?

<u>Ideal stability limits depend on:</u>

- Triangularity & Squareness
- q_{0}, q_{95}
- Pressure Peaking and l_i

Thesis: RFA measurement should be inherently sensitive to these dependencies

Proposal: Test these in an XP

Ferron et al, Phys. Plasmas **12** 056126
n=1 RFA Used to Study Stability Dependence on q_{min}

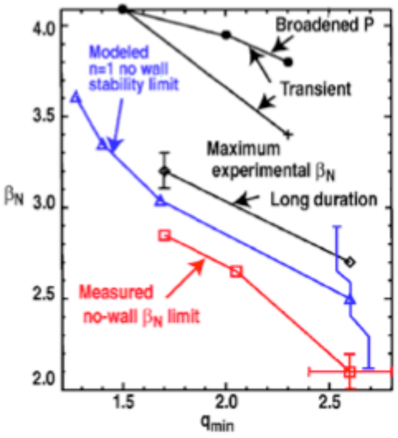


FIG. 1. (Color online). The measured and modeled dependence of the β_N limit on the minimum safety factor. Squares are the experimentally measured no-wall limit and triangles are the $n=1$ no-wall limit calculated for equilibria with profiles similar, but not identical to the profiles in the experimental discharges. Diamonds are the maximum β_N at which discharges have been operated for the duration of the machine pulse without significant instability, circles are the maximum β_N that has been obtained in steady-state scenario discharges. The cross represents the maximum β_N obtained without adding extra gas to broaden the pressure profile.

Menard et al, Phys. Plasmas **11** 639
Dependence of Stability on Triangularity

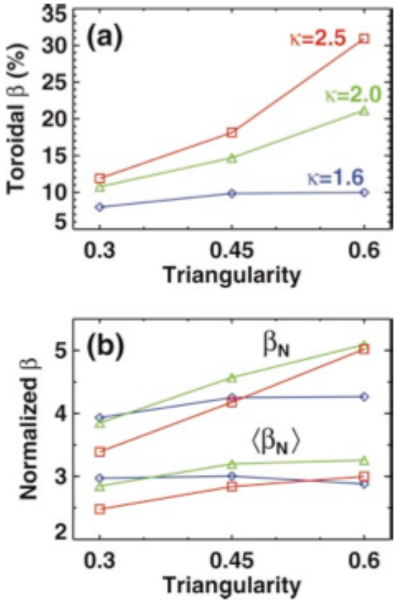


FIG. 2. (Color) (a) Marginally stable $\beta_T(\%)$ and (b) normalized beta values as a function of triangularity and elongation at 50% self-driven current fraction for aspect ratio $A = 1.6$.

Strawman Shot Plan (which parts are most important?) (1)

Step 0: Establish 900 kA "fiducial" shape discharge with good β_N evolution (3 shots)

- κ~2.3-2.4, δ~0.6, dr_{sep}=-1cm, outer gap~10cm, Lithium conditioned
- Use n=3 correction to maintain optimal rotation.
- Start with 2 sources (3 or 4 MW?), add 3rd at ~500 msec (5 or 6MW) and *remove at 800 msec*, for β_N ramp-up and ramp down.
- Call this the reference configuration.
  - May need a second reference to get larger range of β_N evolution (?)
- Keep this β_N for all subsequent cases.

Step #1: Apply 30 Hz Co- Traveling Wave, 1kA pk-pk. (3 shots)

- 1kA and 30 Hz determined from previous XPs

Step #2: Frequency Scan (6 shots)

- Repeat at -30,10, 45, 60 Hz
- Do we need vacuum shots?
- May be able to extract this data from XP-501? Should at least provide guidance.
- Use to assess:
  - Plasma perturbations with optical USXR as a function of frequency.
  - Low-frequency RFA measurement for comparison with IPEC.
  - Optimal frequency for RFA control

Shots: 18

Strawman Shot Plan (which parts are most important?) (2)

Step #3: Scan I_P to 700 kA, 1100kA (6 shots)

- Test robustness of these plasmas to 1kA traveling waves
- Test of RFA vs β_N at different q_{95}, l_i.

Step #4: Modify pre-heating in 900 kA case (4 shots)

- Later pre-heating should allow a higher-li plasma, and change in stability limits.
- Delay the 2nd and/or third sources.

Step #5: At 900kA, scan triangularity at fixed elongation. (8 shots)

- Keep the outer-gap fixed.
- RFA should be inversely proportional to triangularity at fixed β_N (?)

Step #6: For reference shape, scan the outer gap. (3 shots)

- Increase to 15 cm, drop to 7 cm.
- Test geometry effects on RFA measurement.
    - Important for assessing the technique as a control tool: What is the tolerance for controlling the outer gap? -> $d(RFA)/d(gap_{out})$?

Step #7: Repeat (some steps) with n=3 correction removed (3 shots)

Addendum: If/when PF4 is available, use RFA to understand the stability as a function of squareness.

Shots: 18+24=42

Diagnostics and Analysis

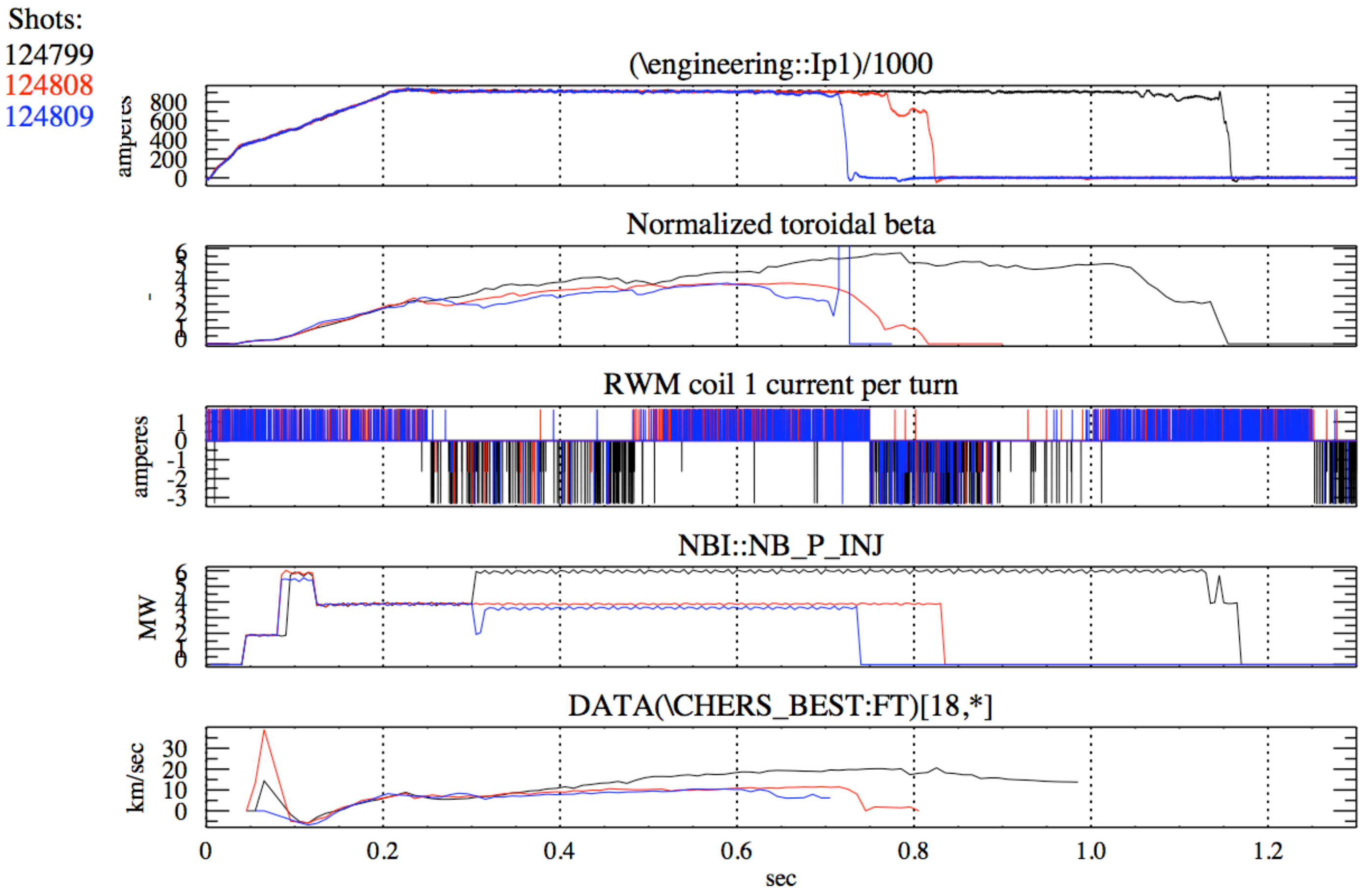
- Diagnostics
  - All profiles (T_i, T_e, Pitch Angle, Rotation, n_e,…)
  - RWM sensors (critical)
  - Optical SXR array
    - Look SXR perturbation due to the stable RWM.
- Analysis
  - Equilibrium reconstruction with all constraints
  - DCON, for calculation of ideal stability limits.
  - IPEC (?)

Most Aggressive Schedule Would Allow This to be Tried in 2010

- NBI Side
  - We have (essentially) never controlled beams from PCS.
  - Must develop/test PCS-EPICS communications links this year.
  - Rely on the NB modulation algorithm being tested (successfully) this year.
  - This is the biggest uncertainty in the schedule.
- “mar” and “tmf” algorithms
  - No new hardware to control, just new PCS code (and a little new ACQ code).
  - 1-1.5 months to write the algorithm code, assuming 50-75% of SPG’s time.
    - Need to code-up Holger’s frequency domain method and compare to time-domain analysis.
      - May modify the “mar” algorithm specification.
    - Need to test some algorithm code in idl.
    - Need to write the PCS code.
    - Some Dana time to get AC-compensation coefficients from tree to algorithm.
  - Then Dana needs to compile all into PCS, check for memory conflicts…
  - 2-3 weeks for piggyback testing.
    - Background testing this run when the SPAs are not in use.
- First combined test of RFA detection+NB control: mid-2010 Run
- Real schedule may be slower, is certainly not faster.

Old and Discarded Stuff Follows

Three Reference Shots Taken For XP-704



All fiducial shape.

One reference with 6MW input, two with 4MW.

New “rid” Algorithm Will Produce Both B_P and B_R n=1 Mode Identification, and RFA Data As Well

- Retain features of “mid” algorithm.
  - Generate same amplitude and phase of n=1 modes
    - The data for this calculation uses all *static* coil compensations.
  - Separate B_P and B_R mode amplitude and phases.
  - Send these numbers to “tmf” algorithm, or the old “smf”” algorithm.
- Add calculation of zero-crossings for RFA measurement.
  - Pick a known good sensor pair (B_R, upper-difference #1)
  - Use “calibrated”, but not “compensated” versions of signals.
    - Avoids need to apply AC compensations in realtime
  - Apply high-order causal bandpass filter
    - Essentially same code as in simulation
  - Check for zero crossings, identify TW amplitude.
    - Essentially same code as in simulation
  - Normalize to SPA currents, to get RFA measurement
    - SPA TW frequency and amplitude are waveforms in the “tmf” algorithm, and can be accessed here.
  - Send the RFA value to the “bnf” algorithm.

New “tmf” Algorithm Designed to Allow n=1 FB in the Presence of the n=1 Spectroscopy Perturbation

- Feature #1: Generate n=1 traveling waves.
  - Input wave amplitude, phase, and start time as waveforms:

$$I_{SPA,X,TW}(t) = \left\{ \begin{array}{c} 0, \\ I_{TW}\cos(2\pi f - \theta_X), \end{array} \right.$$

- Feature #2: Generate pre-programmed requests. $I_{SPA,X,\mathrm{Pr}e-\mathrm{Pr}og}(t)$
- Feature #3: Fast feedback and DEFC, notch-filtered to remove applied TW (example for B_P sensors):
  - The “rid” algorithm provides the n=1 amplitude and phase, or alternatively the quadrature components:

$$B_{\mathrm{mod}\,e}(\phi,t) = B_1(t)\cos(\phi-\theta_1(t)) = C_1(t)\cos(\phi) + S_1(t)\sin(\phi)$$

$$C_{1,BP}(t) = B_{1,BP}(t)\cos(\theta_{1,BP}(t)) \quad S_{1,BP}(t) = B_{1,BP}(t)\sin(\theta_{1,BP}(t))$$

  - Calculate lowpass and highpass version of the quadrature components, using high-order filters (phase effects?).

$$C_{1,LPF,BP}(t) = LPF\left(C_{1,BP}(t)\right) \quad C_{1,HPF,BP}(t) = HPF\left(C_{1,BP}(t)\right)$$

$$S_{1,LPF,BP}(t) = LPF\left(S_{1,BP}(t)\right) \quad S_{1,HPF,BP}(t) = HPF\left(S_{1,BP}(t)\right)$$

  - Reconstruct amplitude and phase, for HP and LP filtered data

$$B_{1,Bp,LPF} = \sqrt{\left(C_{1,LPF,BP}(t)\right)^2 + \left(S_{1,LPF,BP}(t)\right)^2} \quad \theta_{1,Bp,LPF} = a\tan\left(C_{1,LPF,BP}(t)/S_{1,LPF,BP}(t)\right)$$

$$B_{1,Bp,HPF} = \sqrt{\left(C_{1,HPF,BP}(t)\right)^2 + \left(S_{1,LPF,BP}(t)\right)^2} \quad \theta_{1,Bp,HPF} = a\tan\left(C_{1,HPF,BP}(t)/S_{1,HPF,BP}(t)\right)$$

  - Form feedback requests, with different gains and phases for HP and LP values

$$I_{SPA,X,Bp-FB}(\phi,t) = P_{Bp,LP}(t)B_{1,Bp,LP}(t)L_{eff}^{-1}\cos(\phi-\theta_{1,Bp,LP}(t)+\delta(t)) + P_{Bp,HP}(t)B_{1,Bp,HP}(t)L_{eff}^{-1}\cos(\phi-\theta_{1,Bp,HP}(t)+\delta(t))$$

- Final SPA request is sum of all the above:

$$I_{SPA,X}(t) = I_{SPA,X,TW}(t) + I_{SPA,X,\mathrm{Pr}e-\mathrm{Pr}og}(t) + I_{SPA,X,Bp-FB}(t) + I_{SPA,X,Br-FB}(t)$$

High-Order Causal Filter Realtime Implementation

HP and LP Filters

First use Nth order lowpass filter:

$$f_i^{(1)} = \overline{C}_{LP}\left[f_{i-1}^{(1)} + C\left(f_i^{(0)} + f_{i-1}^{(0)}\right)\right]$$

$$f_i^{(2)} = \overline{C}_{LP}\left[f_{i-1}^{(2)} + C\left(f_i^{(1)} + f_{i-1}^{(1)}\right)\right]$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$f_i^{(N)} = \overline{C}_{LP}\left[f_{i-1}^{(N)} + C\left(f_i^{(N-1)} + f_{i-1}^{(N-1)}\right)\right]$$

$$\overline{C}_{LP} = \left(1 + {dt}/{\tau}\right), \quad C = {dt}/{2\tau}$$

Then use Nth order lowpass filter:

$$f_i^{(1)} = \overline{C}_{HP}\left[f_{i-1}^{(1)} + \left(f_i^{(0)} - f_{i-1}^{(0)}\right)\right]$$

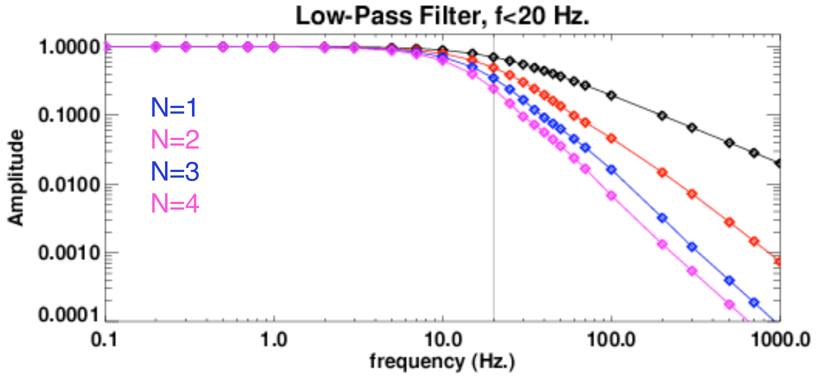
$$f_i^{(2)} = \overline{C}_{HP}\left[f_{i-1}^{(2)} + \left(f_i^{(1)} - f_{i-1}^{(1)}\right)\right]$$

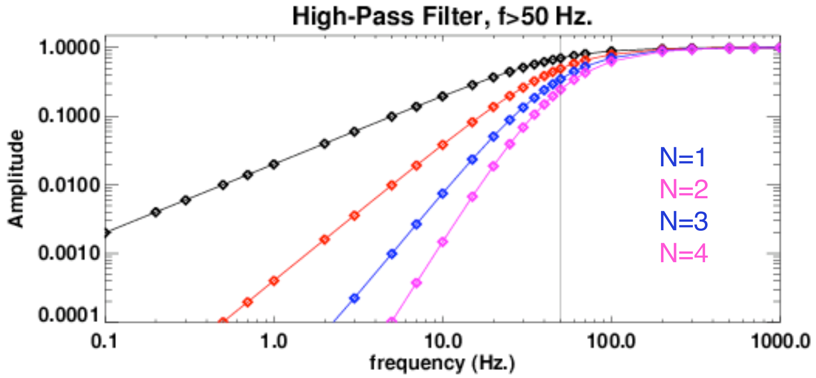
$$\vdots \qquad \vdots \qquad \vdots$$

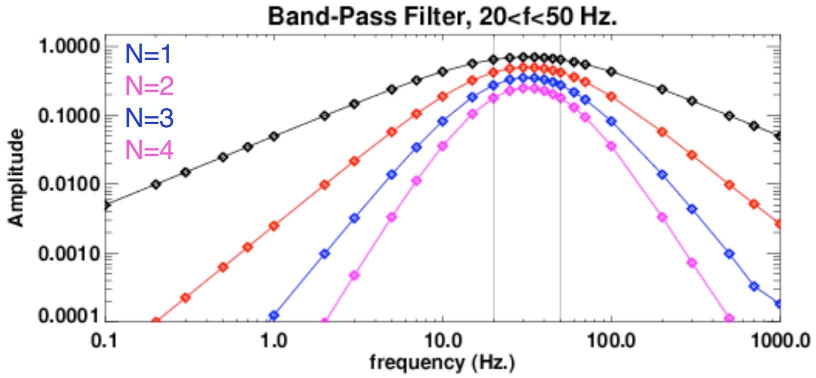
$$f_i^{(N)} = \overline{C}_{HP}\left[f_{i-1}^{(N)} + \left(f_i^{(N-1)} - f_{i-1}^{(N-1)}\right)\right]$$

$$\overline{C}_{HP} = \left(1 + {dt}/{\tau}\right)^{-1}$$

- For each signal undergoing an nth-order filter, need to keep an Nx2 array of previous values.
- Each filter requires a simple length-N FOR loop.
- Easy to implement.





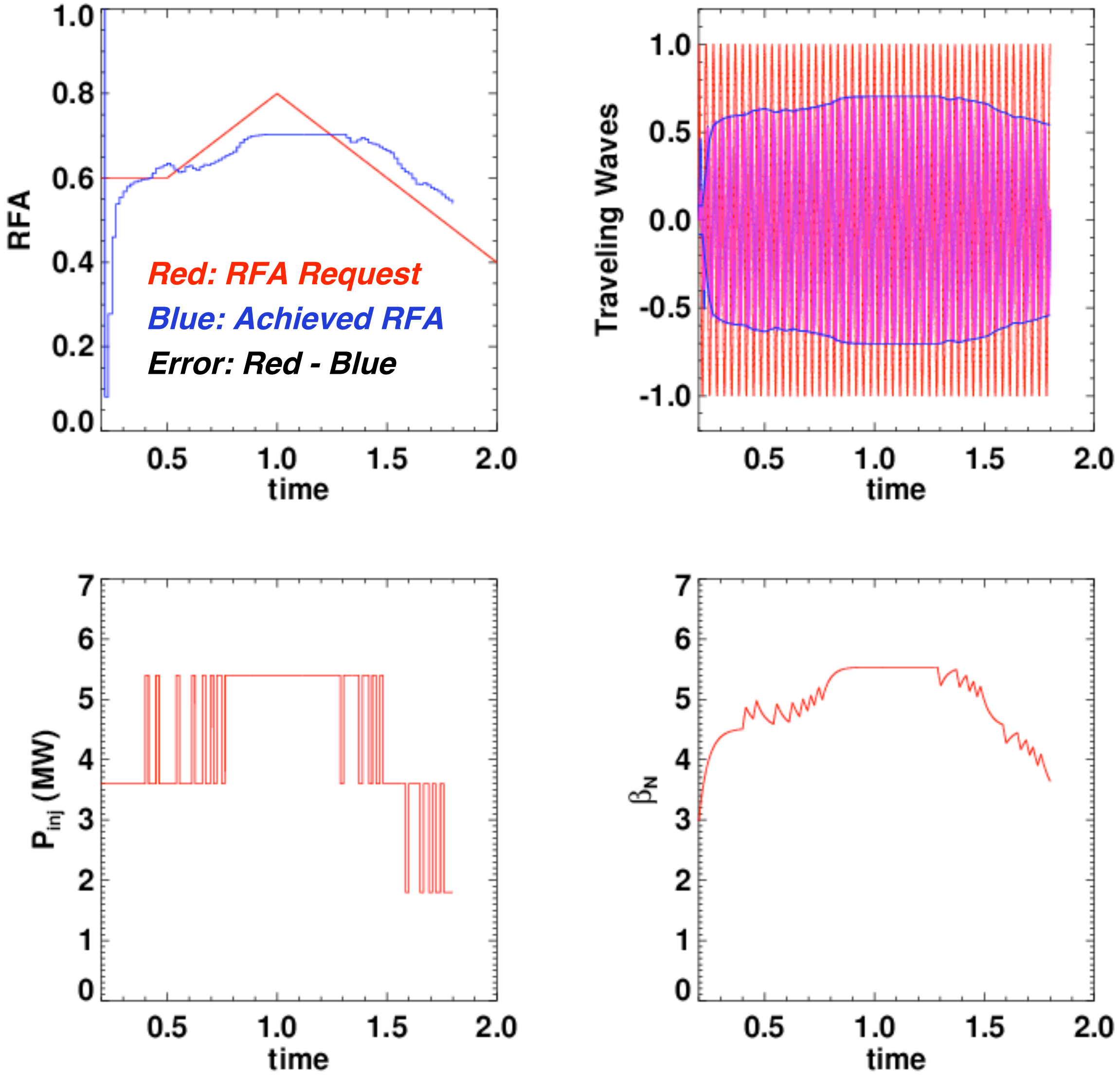


Simulation of “Single Sensor” RFA Feedback Uses Simple Physics Model

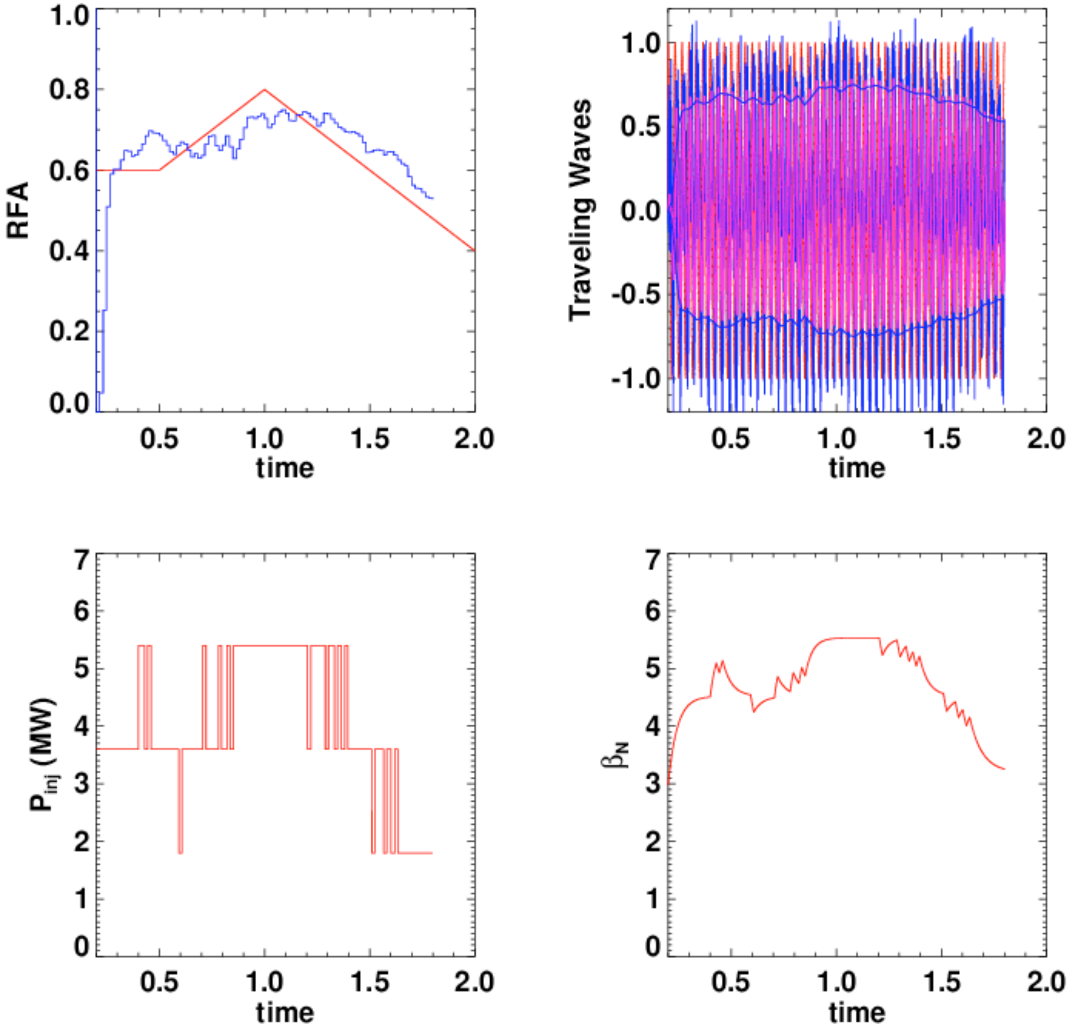
- Plasma Heating Model
  - Use 0-D model of plasma
    $$\beta_N \Rightarrow \beta_N + \left(\tau \frac{I_P V B_T}{200\mu_0 a}\right)^{-1} \Delta P_{inj}$$
  - Neglect beam slowing down…power is instantly deposited
- RFA Model & Detection
  - Simple linear relationship between RFA and β_N: $M = a_{RFA}\beta_N + b_{RFA}$
    - a_{RFA} & b_{RFA} determined from previous measurements.
  - Detected wave is $M \times I_{RWM}\cos(wt)$
  - Add noise to the detected wave..
  - Apply a high-order causal filter to extract the eliminate noise
  - Detect the RFA from analysis of the traveling wave data.
    - Use zero-crossing identification to bracket maxima and minima.
- Feedback Scheme
  - Proportional gain on RFA error modifies the injected power request
- Neutral Beam
  - Injected power request leads to a duty cycle request for each source.
  - Use the same modulation methods as in the (untested) β_N control algorithm.

Simulation With No Noise Indicates RFA Control is Possible

- Proportional gain only, not optimized.
- No noise on detected wave
- Confinement time (40 msec) insufficient to reach requested RFA
- 3rd order causal HP and LP filters, passing 20 < f < 50 Hz.



Simulation With Considerable Noise Shows Success of High-Order Causal Filters



Red: SPA Waveform (kA)

Blue: Detected Difference Signal

Magenta: Difference Signal After Filtering