Using the frequency domain for the real-time detection of the *n*=1 plasma response

By

H. Reimerdes

Columbia University, New York

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Potential of active MHD spectroscopy as a real-time stability measurement

From H. Reimerdes, et al.,
49th Annual Meeting of the Division of Plasma Physics
Orlando, 2007

Including corrected formulas

Fourier analysis of perturbation measurements yields plasma response at applied frequency

- **Fourier analysis in time-domain**
  - Signal of magnetic probe *s*

$$B_s(t) = \Re\left(\boldsymbol{B}_s^{\omega} \cdot e^{i\omega t}\right)$$

  - Current in coil *c*

$$I_c(t) = \Re\left(\boldsymbol{I}_c^{\omega} \cdot e^{i\omega t}\right)$$

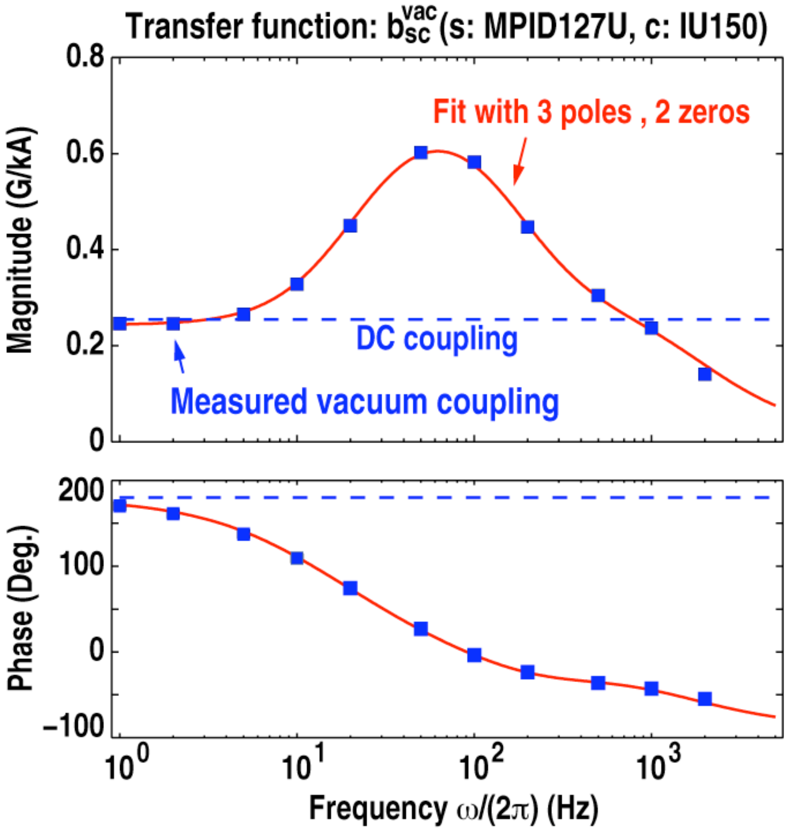
- **Fourier coefficients yield the transfer function**

$$\boldsymbol{b}_{sc}^{\omega} = \frac{\boldsymbol{B}_s^{\omega}}{\boldsymbol{I}_c^{\omega}}$$

- **Use vacuum coupling to extract plasma response from signals**

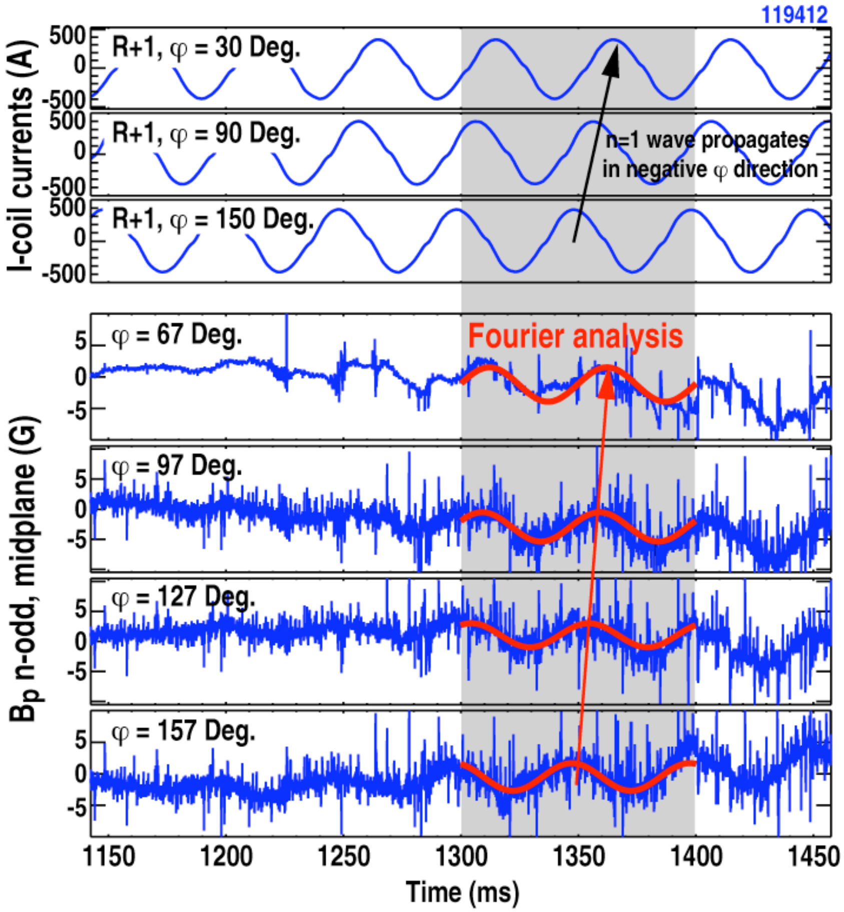
$$\boldsymbol{b}_{sc}^{\omega,plas} = \boldsymbol{b}_{sc}^{\omega} - \boldsymbol{b}_{sc}^{\omega,vac}$$

- **Frequency-dependent vacuum coupling is well documented**



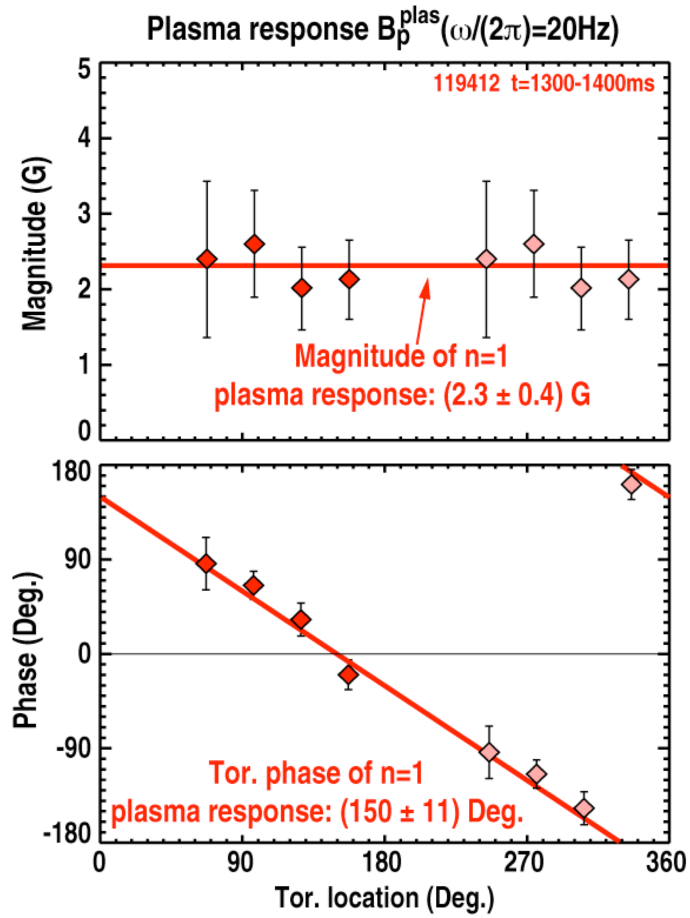
Fourier analysis and toroidal mode decomposition yields *n*=1 plasma response at applied frequency

- **Fourier analysis of sensor signals**



Subtract vacuum coupling →

- **Toroidal mode analysis of Fourier coefficients**



Calculation of *n*=1 plasma response from discretely sampled magnetic probe signals

Discrete time steps: $t_k = k\cdot\Delta$

Probe plasma with currents in coils c**:** $I_c(t_k) = I_C \mathbf{cos}(\omega_{ext}\, t_k + \phi_{c,0})$

Measure magnetic field with probes s**:** $B_s(t_k)$

Calculate Fourier series for each probe s **and time** t_K **over** L **preceding periods of applied field corresponding to** N = L· 2π/($\omega_{ext}\Delta$) **previous samples:**

$$a_{s,K} = \sum_{k'=K-N}^{K} B_s(t_{k'}) \cos(\omega_{ext} t_{k'}) \quad \text{and} \quad b_{s,K} = \sum_{k'=K-N}^{K} B_s(t_{k'}) \sin(\omega_{ext} t_{k'})$$

Subtract known vacuum coupling at ω_{ext} **using** $b_{sc}^{\omega_{ext},vac} = a_{sc}^{vac} + i b_{sc}^{vac}$ **:**

$$a_{s,K}^{plas} = a_{s,K} - I_C \sum_{c'=c}^{C} \left(a_{sc'}^{vac} \mathbf{cos}(\phi_{c',0}) - b_{sc'}^{vac} \mathbf{sin}(\phi_{c',0}) \right)$$

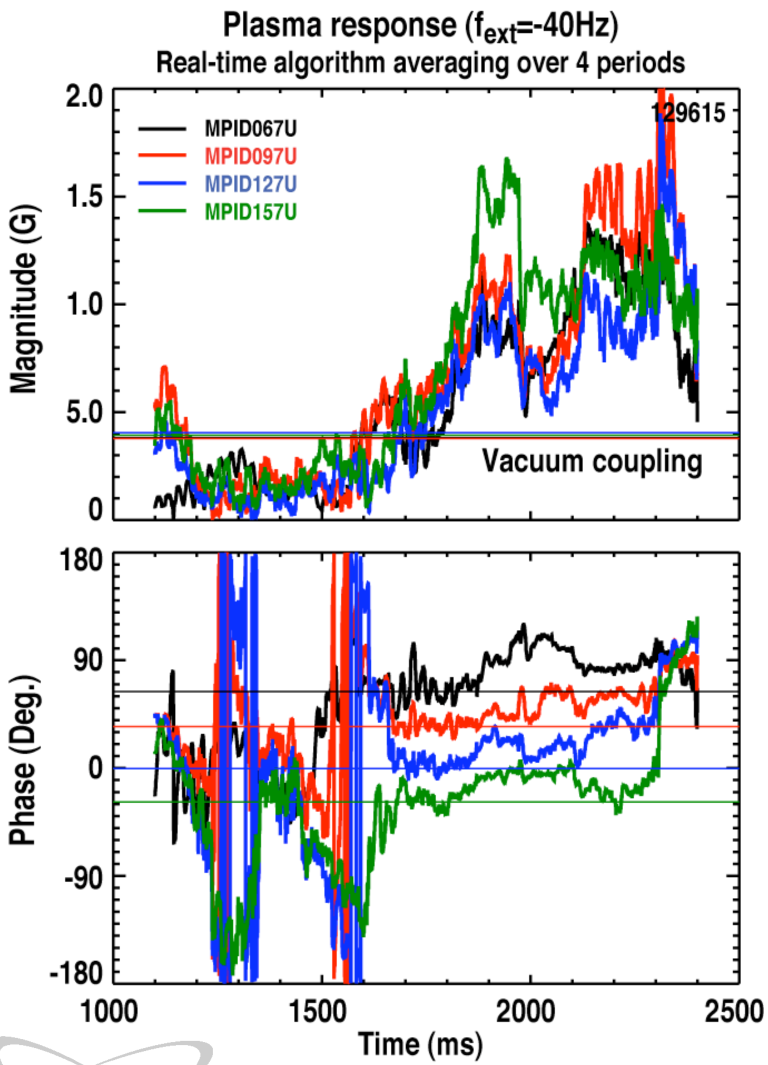
$$b_{s,K}^{plas} = b_{s,K} - I_C \sum_{c'=c}^{C} \left(a_{sc'}^{vac} \mathbf{sin}(\phi_{c',0}) + b_{sc'}^{vac} \mathbf{cos}(\phi_{c',0}) \right)$$

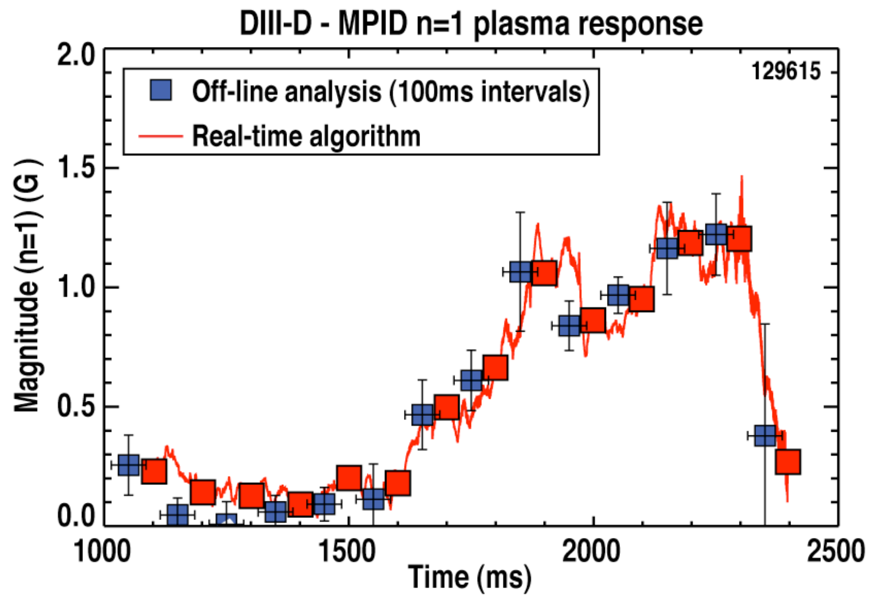
Extract ***n*****=1 plasma response:**

$$A_{S,K}^{plas} = 0.5 \sum_{s'=s} \left(c_{s'}^{r} a_{s',K}^{plas} - c_{s'}^{i} b_{s',K}^{plas} \right)$$

$$B_{S,K}^{plas} = 0.5 \sum_{s'=s} \left(c_{s'}^{r} b_{s',K}^{plas} + c_{s'}^{i} a_{s',K}^{plas} \right)$$

Real-time algorithm is equivalent to offline analysis on DIII-D





- Discrepancy possibly due to difference between I-coil current target (real-time algorithm) and actual current (offline analysis)

Additional slides

Calculation of toroidal mode component increases signal-to-noise ratio

- **General least square fit of toroidal mode components**
  - Solution by normal equations yields coefficients $\mathbf{C}_s^n$

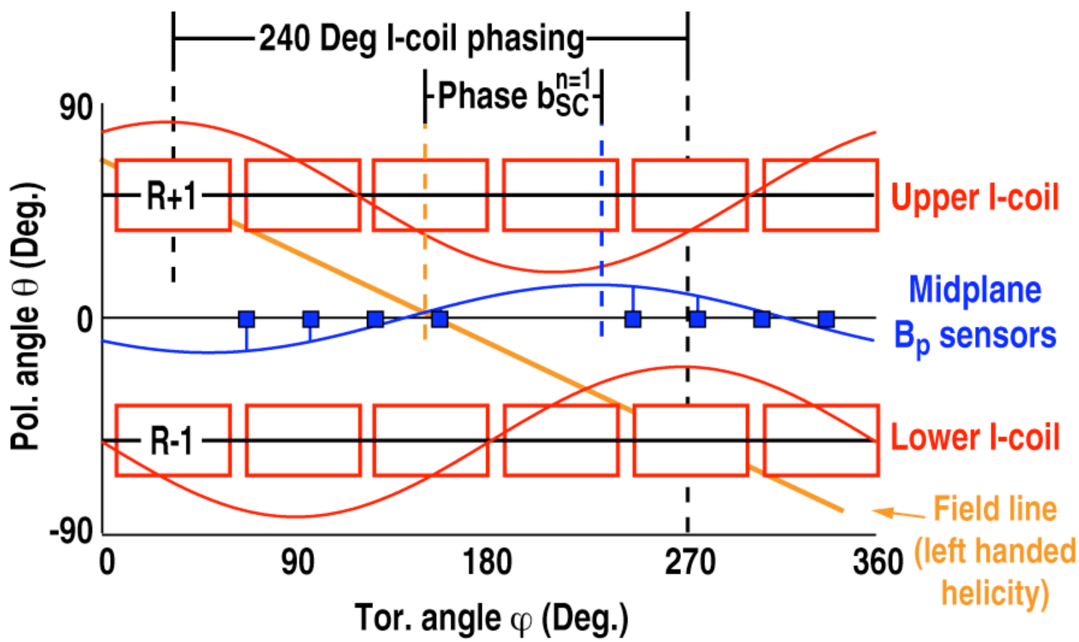
$$\boldsymbol{B}^n(t) = \sum_s \boldsymbol{C}_s^n \, B_s(t) \qquad \text{with} \qquad B_s(t) = \Re\left(\boldsymbol{B}^n(t) \cdot e^{-in\varphi_s}\right)$$

- **Generalize to obtain toroidal mode component at $\omega = \omega_{ext}$**

$$\boldsymbol{B}_S^{n,\omega} = 0.5 \sum_s \boldsymbol{C}_s^n \, \boldsymbol{B}_s^\omega$$

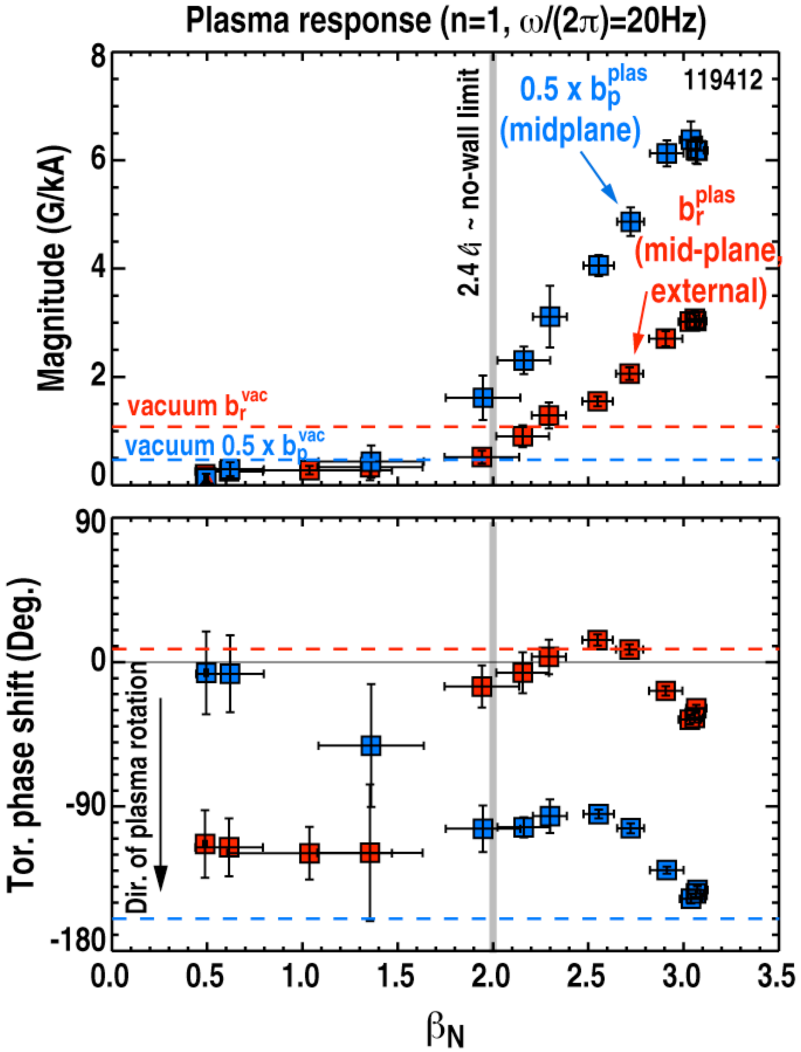
- **Coupling of coils to sensors for a mode number n at a frequency ω**

$$\boldsymbol{b}_{SC}^{n,\omega} = \frac{\boldsymbol{B}_S^{n,\omega}}{\boldsymbol{I}_C^{n,\omega}}$$



Plasma response increases at the (n=1) ideal MHD no-wall beta limit

- **No or very weak plasma response at low β_N (≤1.5)**
- **Significant amplification above a β-threshold**
- **Threshold is close to or coincides with the ideal MHD no-wall β-limit**
  - Limit set by n=1 kink mode
  - Typically (in this type of plasma)scales as $\beta_{N,nw}$~2.4 ℓ_i
- **In the wall stabilized regime the RFA continuously increases with β**



Frequency response well described by a single weakly damped mode

- **Frequency response of stable high β plasma is well described by a single mode model**

$$\tau_w \frac{dB_s}{dt} - \tau_w \gamma_0 B_s = M^*_{sc}\, I_c$$

with $\gamma_0 = \gamma_{RWM} + i\omega_{RWM}$

  – Fit of measured spectrum yields

Damping rate	$-\gamma_{RWM}$
Mode rotation frequency	ω_{RWM}
Effective mutual inductance	M_s^*

  – RFA spectrum has also been observed in NSTX [Sontag et al, *NF* (2007)] and JET [Gryaznevich et al, EPS (2007)]

- **Consistent with MARS-F calculations showing that a rotationally stabilized plasma is well described by a single pole** [Liu et al, *PoP* (2006)]