NSTX

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Anisotropy in MISK – part 2

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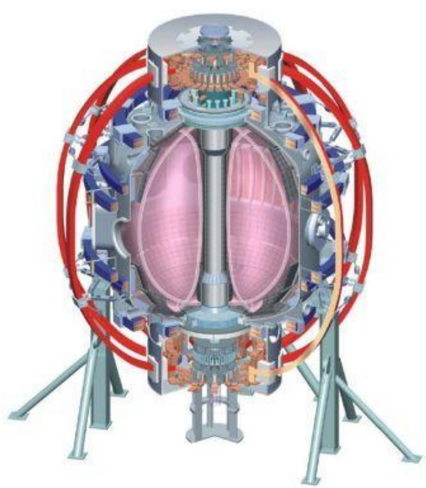
December 20, 2010



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Reminder of MISK formulae, previous status, and what's new

$$\delta W_H = \sum_j \sum_{l=-\infty}^{\infty} 2\sqrt{2}\pi^2 \int\int\int \left[|\langle HT_j \rangle|^2 \frac{(\omega - n\omega_E)\frac{\partial f_j}{\partial \varepsilon} - \frac{n}{Z_j e}\frac{\partial f_j}{\partial \Psi}}{n\langle \omega_D^j \rangle + l\omega_b^j - i\nu_{\text{eff}}^j + n\omega_E - \omega} \right] \frac{\hat{\tau}}{m_j^{\frac{3}{2}} B} |\chi| \varepsilon^{\frac{1}{2}} d\varepsilon d\chi d\mathbf{\Psi},$$

$$\delta W_{\tilde{B}_\parallel} = \sum_j 2\sqrt{2}\pi^2 \int\int\int \left[\langle HT_j \rangle^* \frac{\tilde{\mathbf{B}}_\parallel}{B} \frac{\chi^2 - 1}{2|\chi|} \frac{\partial f_j}{\partial \chi} \right] \frac{\hat{\tau}}{m_j^{\frac{3}{2}} B} |\chi| \varepsilon^{\frac{1}{2}} d\varepsilon d\chi d\mathbf{\Psi},$$

- Simple Gaussian for EPs
  - $\partial f/\partial \chi$ expression was wrong in $\delta W_{B_\parallel}$! Now fixed.
  - Circulating EPs are now included in $\delta W_{B_\parallel}$
  - Only Re($\delta W_{B_\parallel}$) used
  - Many more cases were run

A general expression for an anisotropic beam ion distribution function has been worked out*

$$f_j^b(\varepsilon,\Psi,\chi) = \sum_s \sum_k \sum_p f_{s,k,p}(\varepsilon,\Psi,\chi) \quad (242)$$

s = # of sources (NSTX: 3), k = #of energy components (3),
p = # of deposition surfaces (2)

$$= \sum_s \sum_k \sum_p n_{s,k,p}(\Psi) A_{s,k,p}(\Psi) \left(\frac{m_j}{\varepsilon_{s,k}}\right)^{\frac{3}{2}} \frac{1}{\hat{\varepsilon}_{s,k}^{\frac{3}{2}} + \hat{\varepsilon}_c^{\frac{3}{2}}(\Psi)} \frac{1}{\delta\chi_{s,k,p}(\hat{\varepsilon}_{s,k},\Psi)}$$

$$\times \left[e^{-(\chi-\chi_{0\ s,p}(\Psi))^2/\delta\chi^2_{s,k,p}(\hat{\varepsilon}_{s,k},\Psi)} + e^{-(\chi+2+\chi_{0\ s,p}(\Psi))^2/\delta\chi^2_{s,k,p}(\hat{\varepsilon}_{s,k},\Psi)} + e^{-(\chi-2+\chi_{0\ s,p}(\Psi))^2/\delta\chi^2_{s,k,p}(\hat{\varepsilon}_{s,k},\Psi)} \right],$$

$$0 \le \hat{\varepsilon}_{s,k} \le 1, -1 \le \chi \le 1. \quad (243)$$

$$f_j^b(\varepsilon,\Psi,\chi) = n_j A_b \left(\frac{m_j}{\varepsilon_b}\right)^{\frac{3}{2}} \frac{1}{\hat{\varepsilon}^{\frac{3}{2}} + \hat{\varepsilon}_c^{\frac{3}{2}}} \frac{1}{\delta\chi} \left(\exp\left[\frac{-(\chi-\chi_0)^2}{\delta\chi^2}\right] + \exp\left[\frac{-(\chi+2+\chi_0)^2}{\delta\chi^2}\right] + \exp\left[\frac{-(\chi-2+\chi_0)^2}{\delta\chi^2}\right] \right)$$

$$\frac{\partial f_j}{\partial\chi} = f_j^b \left(-\frac{2(\chi-\chi_0)}{\delta\chi_0^2} - \frac{2(\chi+2+\chi_0)}{\delta\chi_0^2} - \frac{2(\chi-2+\chi_0)}{\delta\chi_0^2} \right)$$

$$= f_j^b \left(-\frac{2(3\chi+\chi_0)}{\delta\chi_0^2} \right).$$

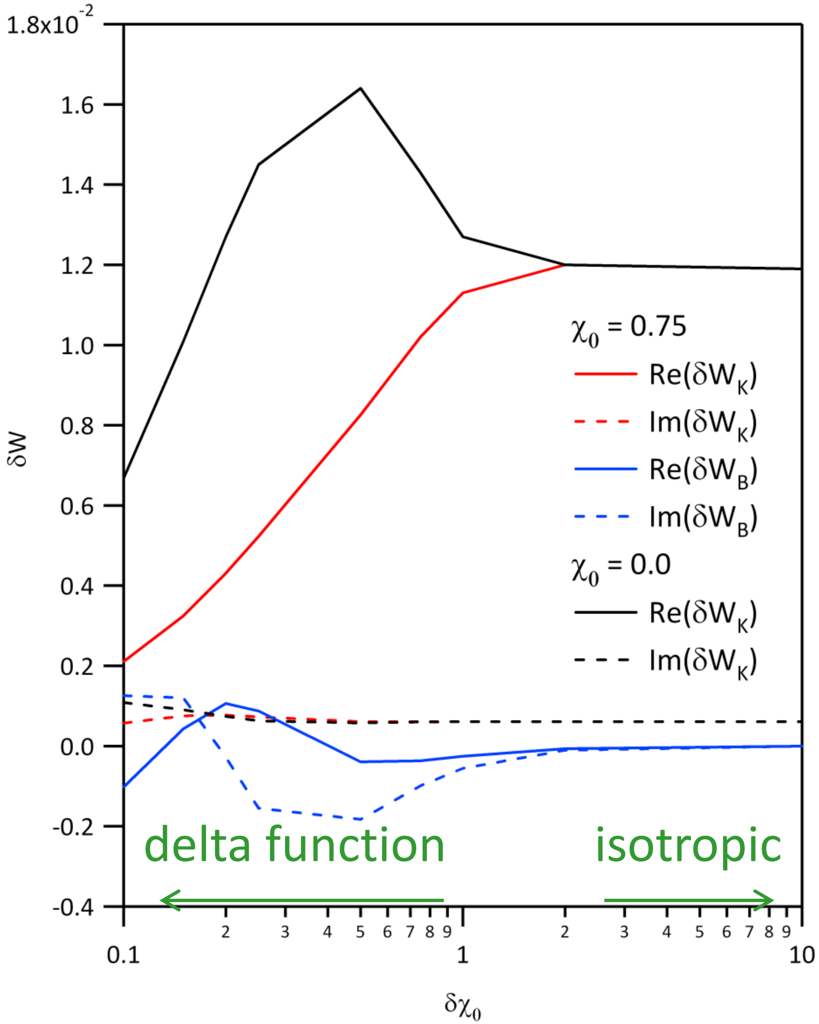
* special thanks to Nikolai Gorelenkov, Mario Podesta, and Brian Grierson

If χ_0 depends on Ψ and δχ depends on Ψ and ε, d*f*/dΨ and d*f*/dε are very complicated. Two simplifications:

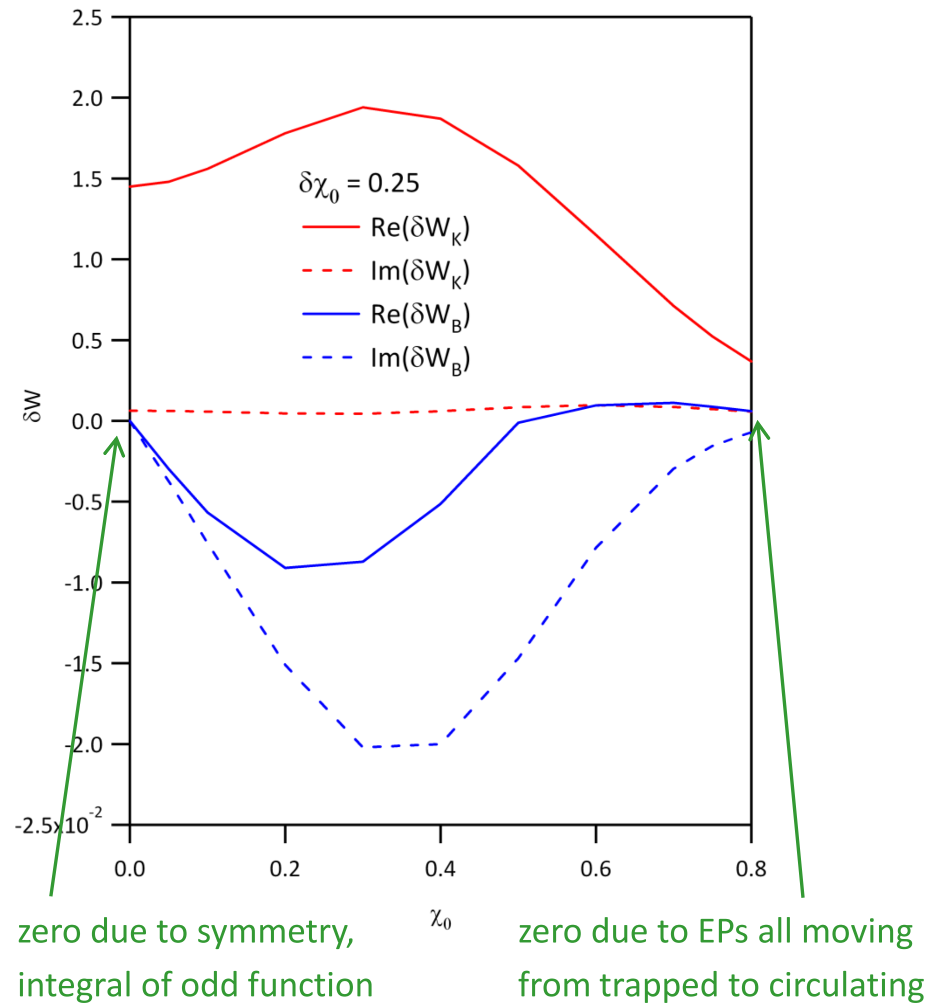
1. χ_0 = constant and δχ = constant → "Simple Gaussian"
2. δχ → ∞ → recover Isotropic

Previous: Anisotropy of EPs makes a difference; δW_B for simple Gaussian EPs is not negligible

Keeping χ_0 constant, varying $\delta\chi$

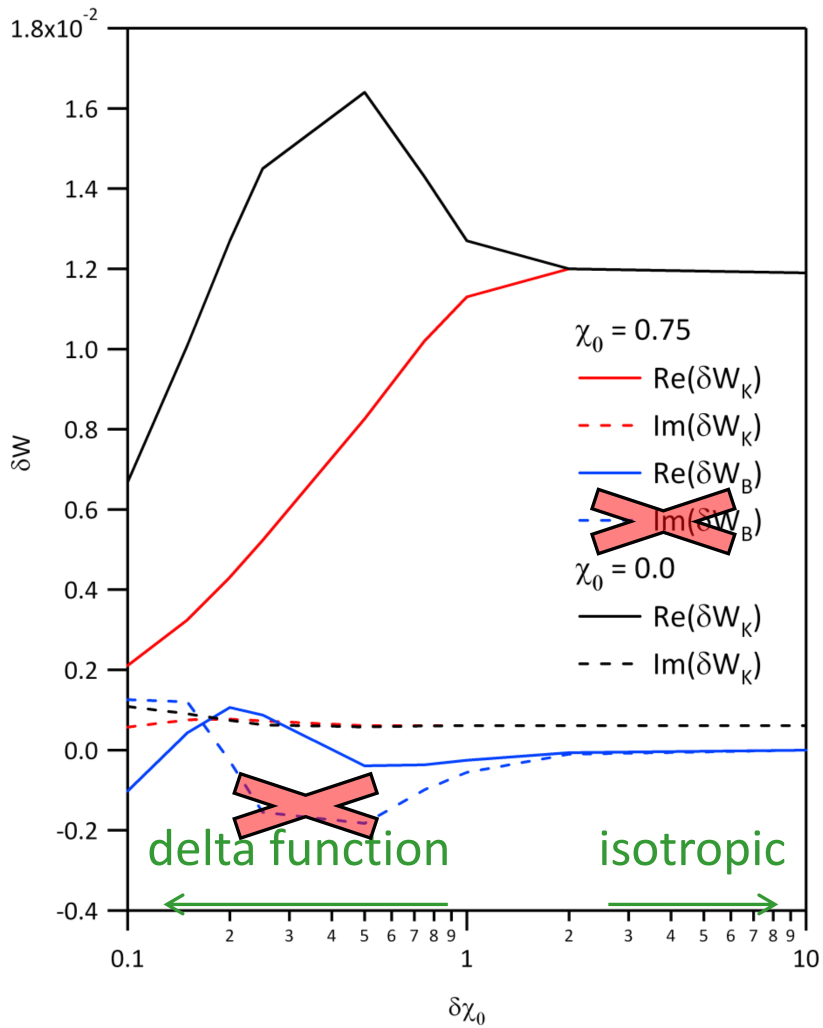


Keeping $\delta\chi$ constant, varying χ_0

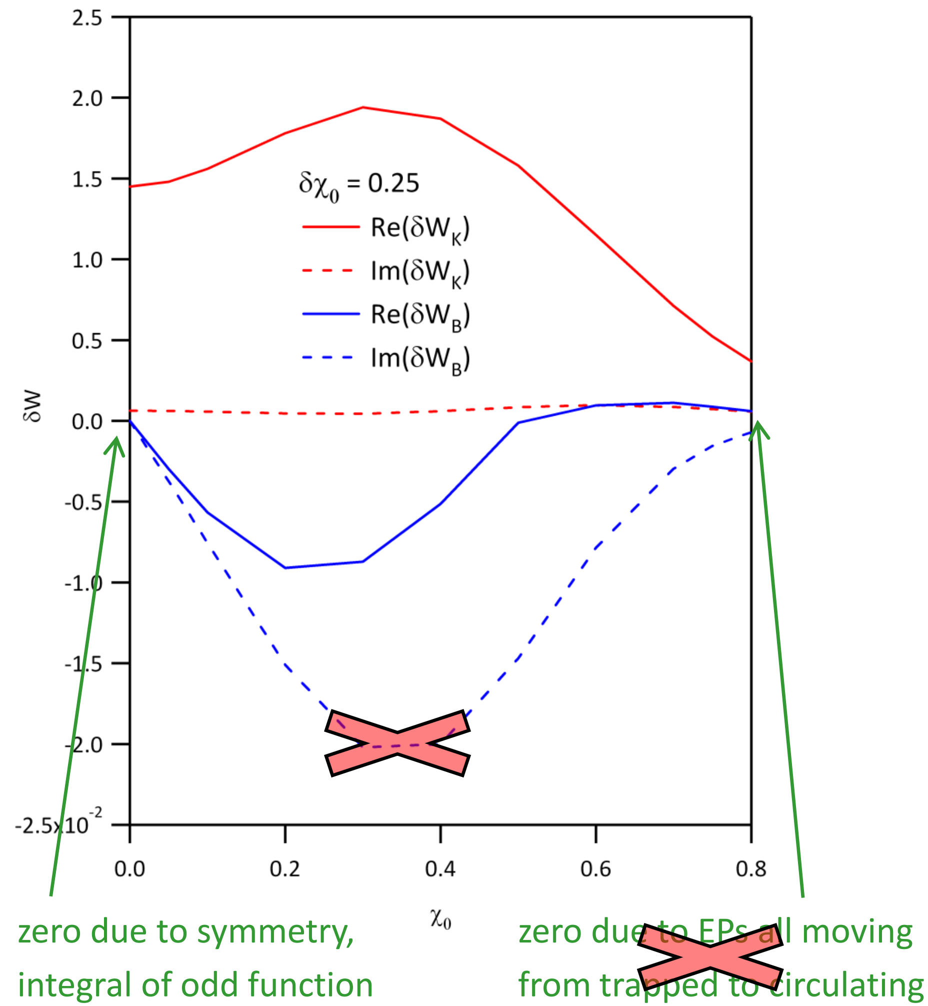


Previous: Re(δW_B) and Im(δW_B) for trapped EPs
New: Re(δW_B) for trap. and circ. EPs, with fixed formula

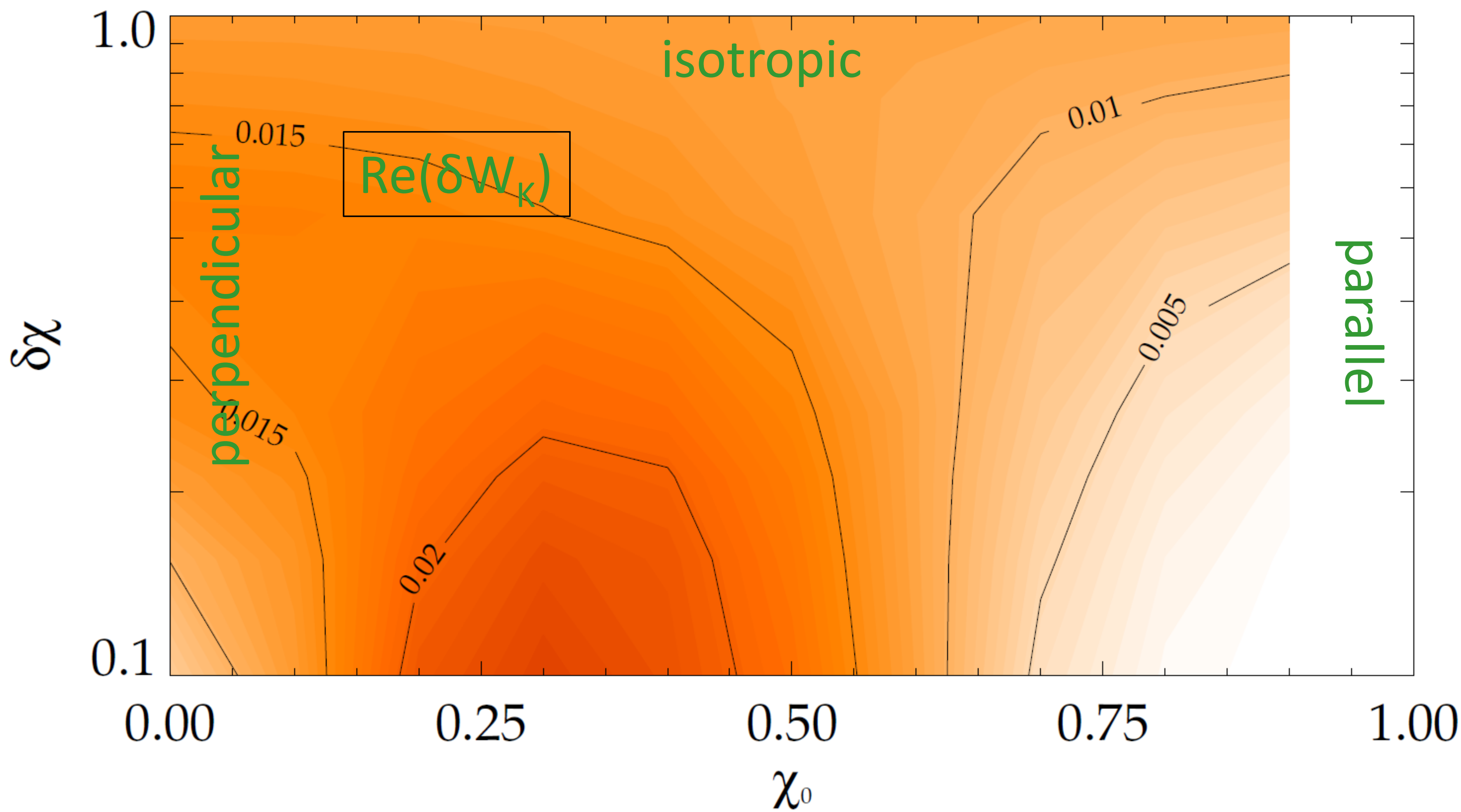
Keeping χ_0 constant, varying $\delta\chi$



Keeping $\delta\chi$ constant, varying χ_0



Anisotropy of the EP distribution can actually be more or less stabilizing than isotropy



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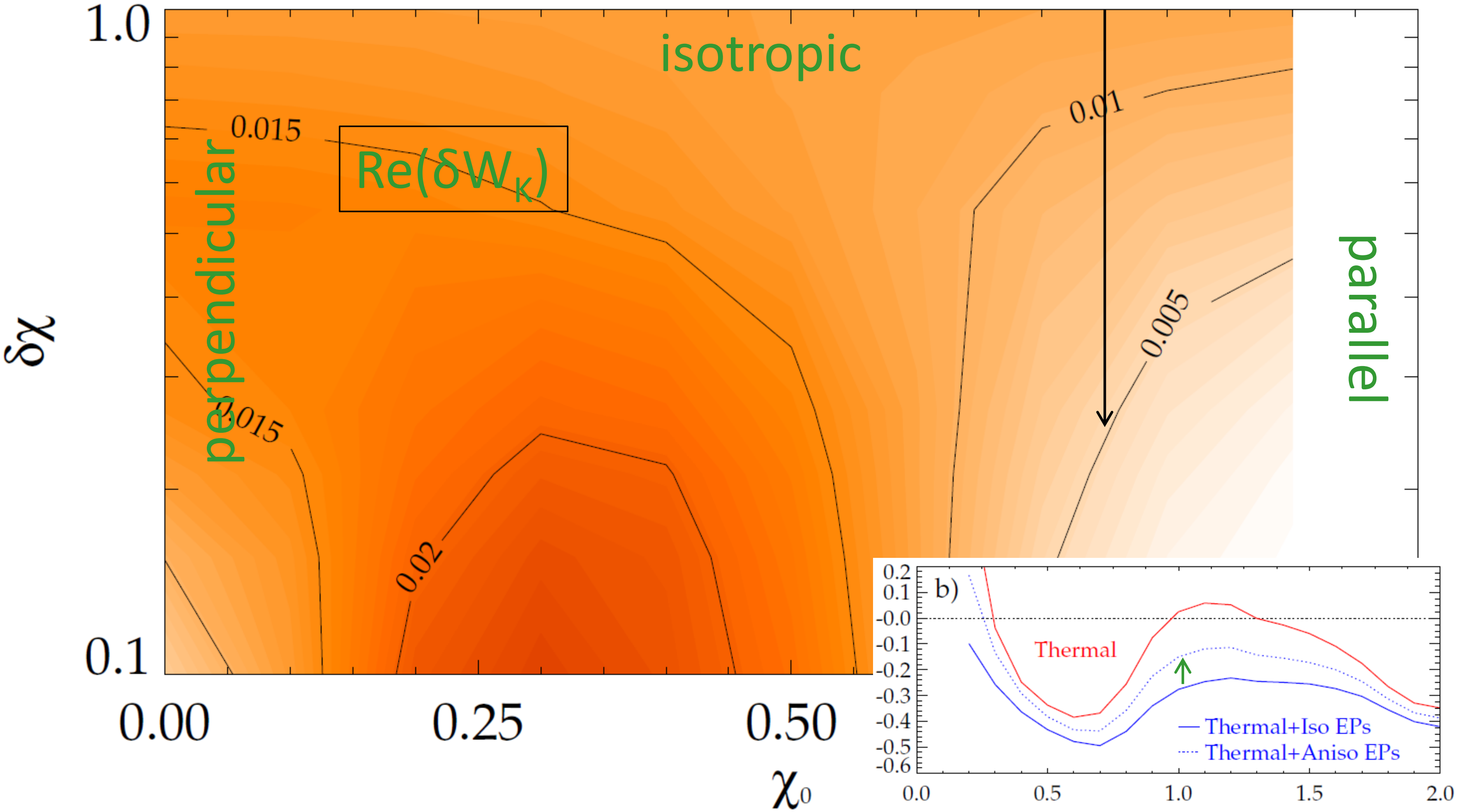
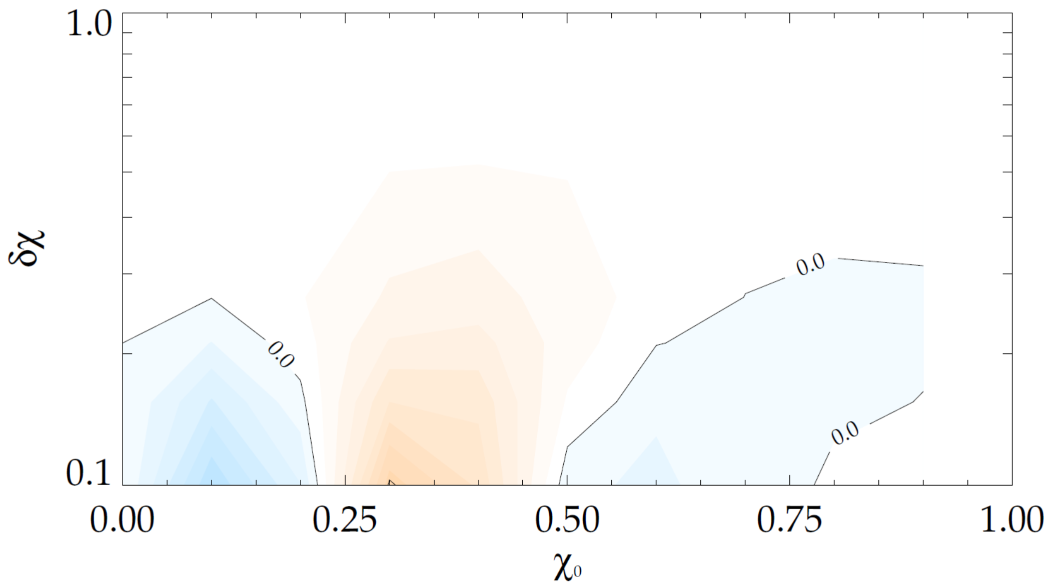
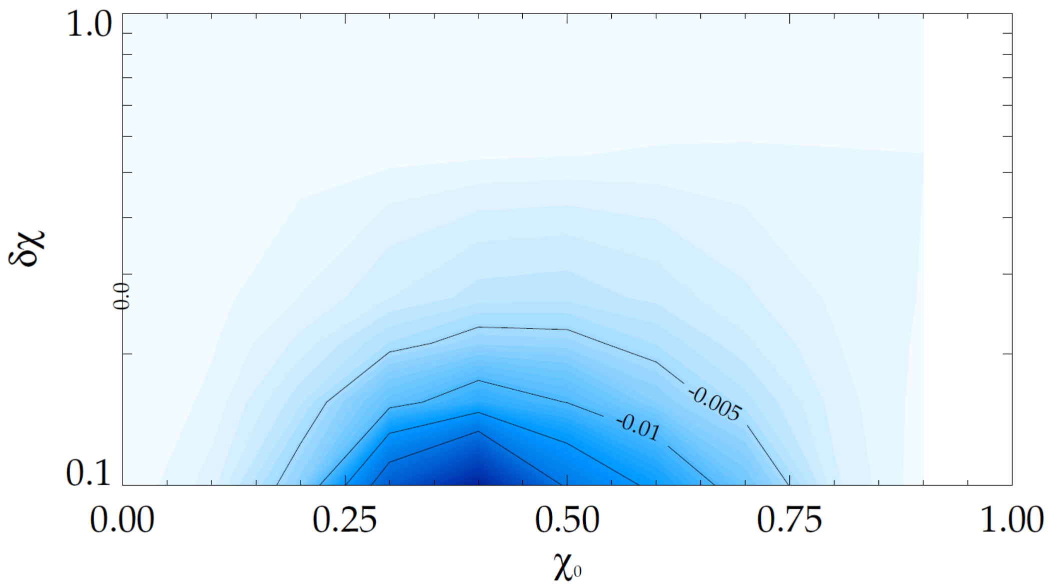


The overall effect of δW_B is dominated by circulating EPs

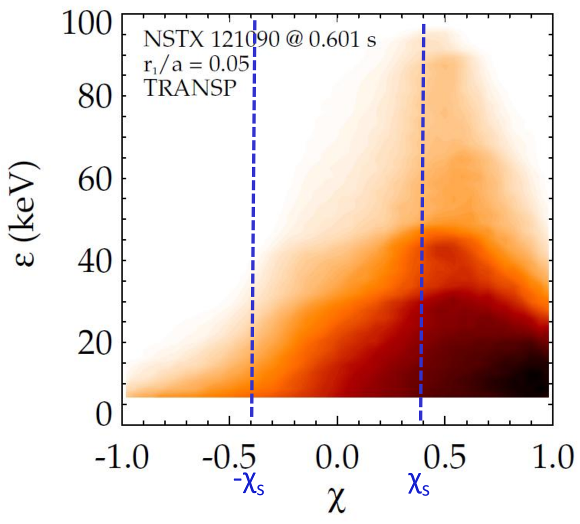


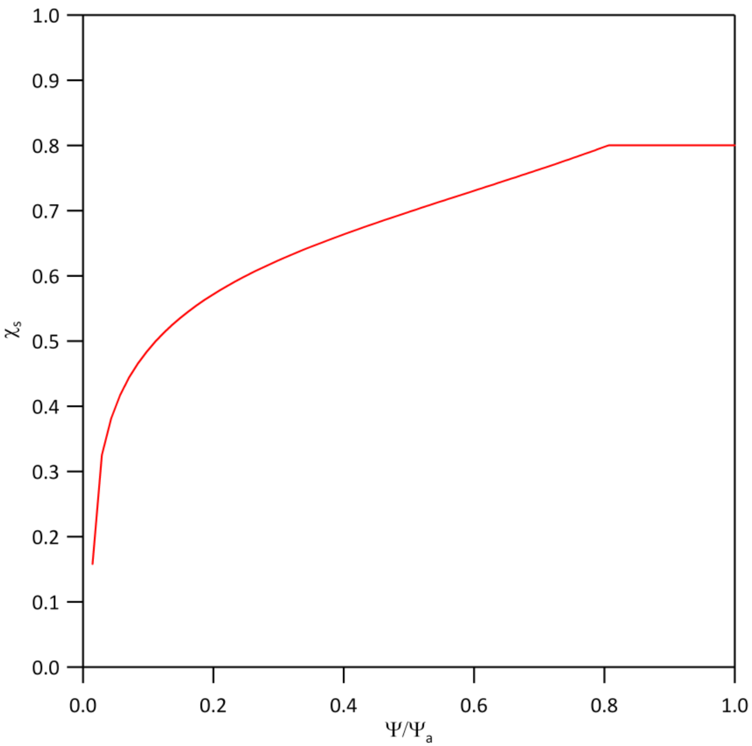
δW_B (trapped)



δW_B (circulating)

A large portion of NSTX beam EPs are circulating



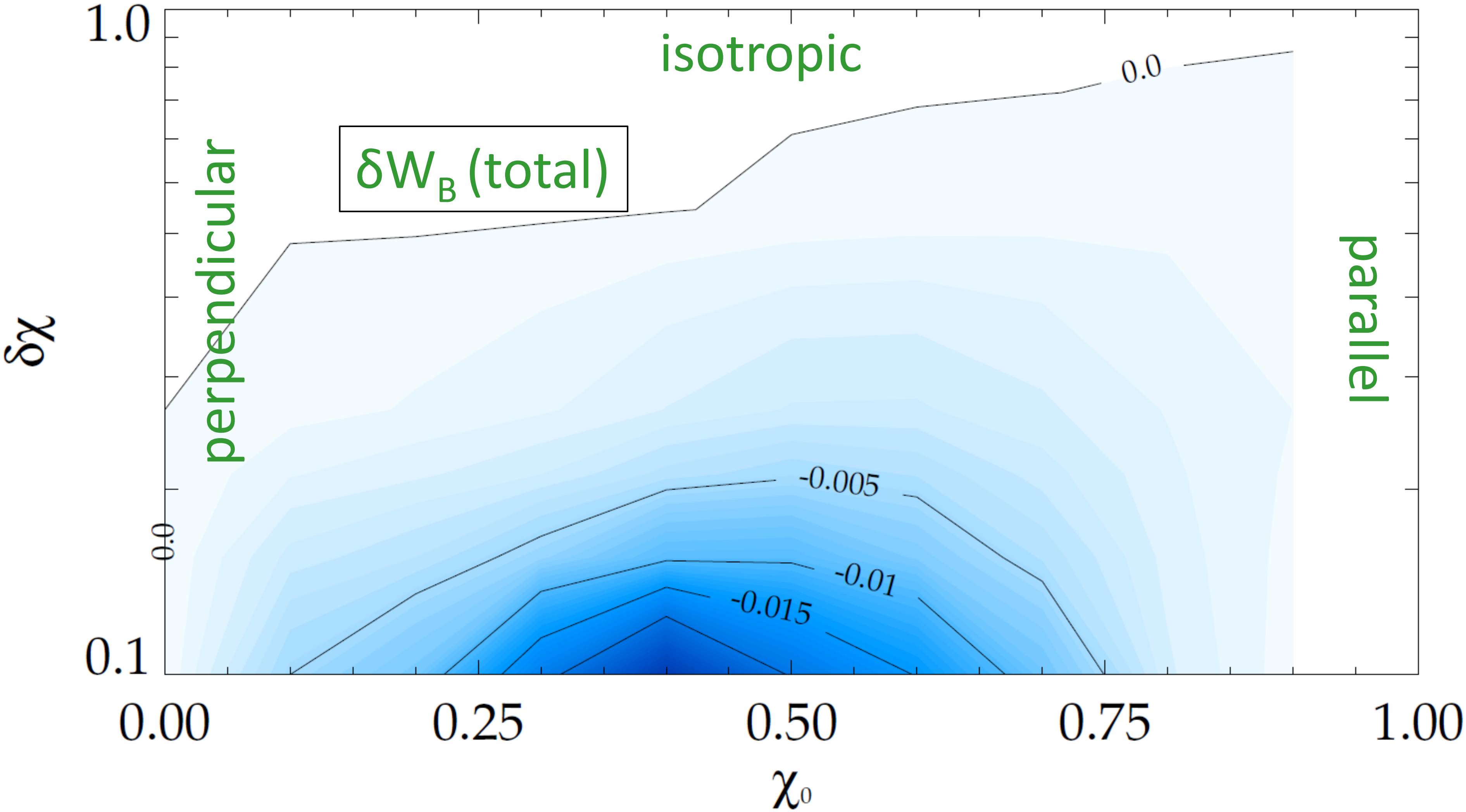


Should we also be including the δW_K term for circulating EPs?

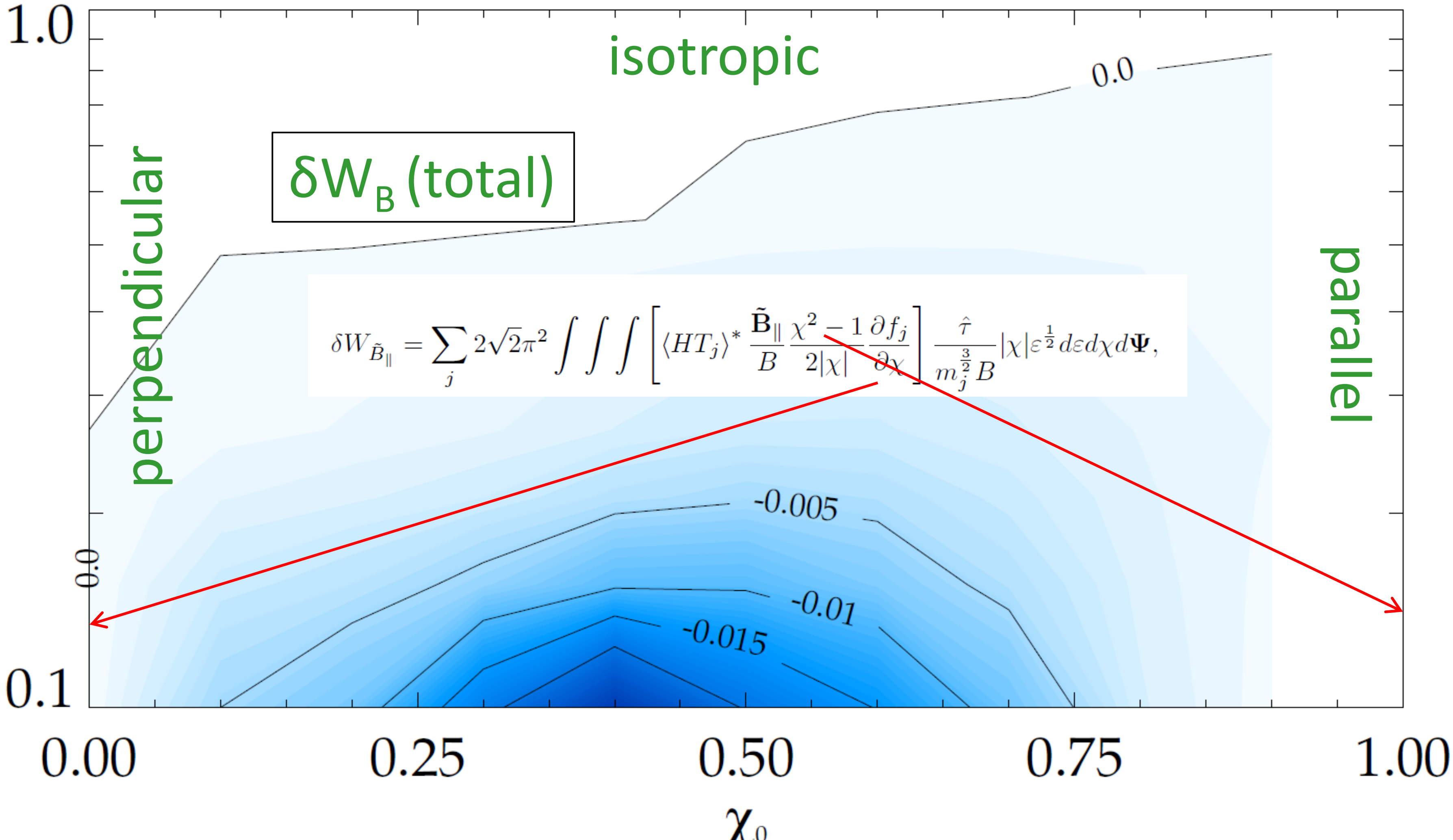
$$\delta W_H = \sum_j \sum_{l=-\infty}^{\infty} 2\sqrt{2}\pi^2 \int\int\int \left[|\langle HT_j \rangle|^2 \frac{(\omega - n\omega_E)\frac{\partial f_j}{\partial \varepsilon} - \frac{n}{Z_j e}\frac{\partial f_j}{\partial \Psi}}{n\langle \omega_D^j \rangle + l\omega_b^j - i\nu_{\text{eff}}^j + n\omega_E - \omega} \right] \frac{\hat{\tau}}{m_j^{\frac{3}{2}} B} |\chi| \varepsilon^{\frac{1}{2}} d\varepsilon d\chi d\mathbf{\Psi},$$

replaced by $\sigma(l-q)\omega_t$

δW_B will significantly alter δW_K for EPs for narrow Gaussian distributions not symmetric to perp. or parallel pitch

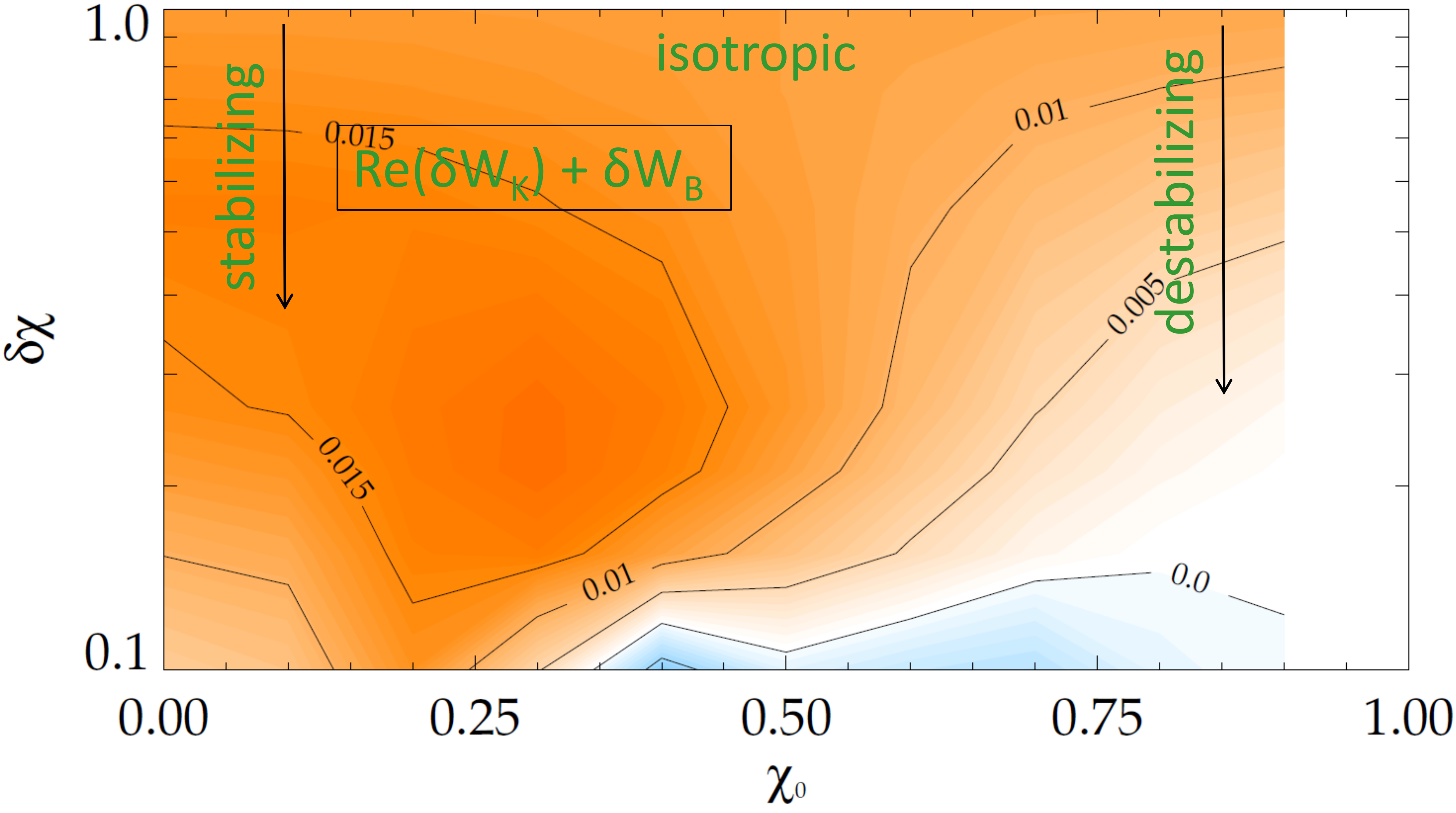


δW_B probably always has this general pattern, but the sign could change

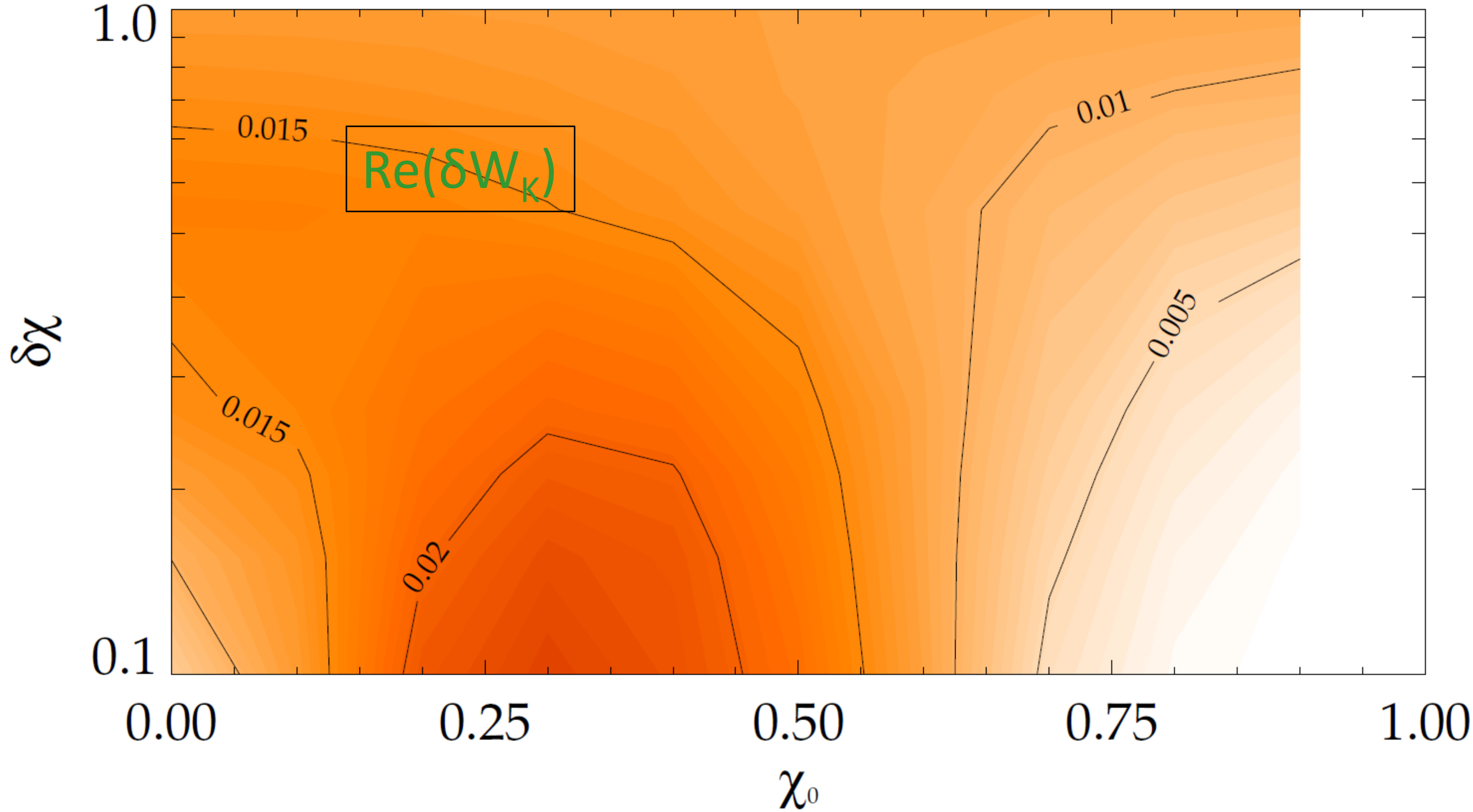


$$\delta W_{\tilde{B}_\parallel} = \sum_j 2\sqrt{2}\pi^2 \int\int\int \left[\langle HT_j \rangle^* \frac{\tilde{\mathbf{B}}_\parallel}{B} \frac{\chi^2 - 1}{2|\chi|} \frac{\partial f_j}{\partial \chi} \right] \frac{\hat{\tau}}{m_j^{\frac{3}{2}} B} |\chi| \varepsilon^{\frac{1}{2}} d\varepsilon d\chi d\Psi,$$

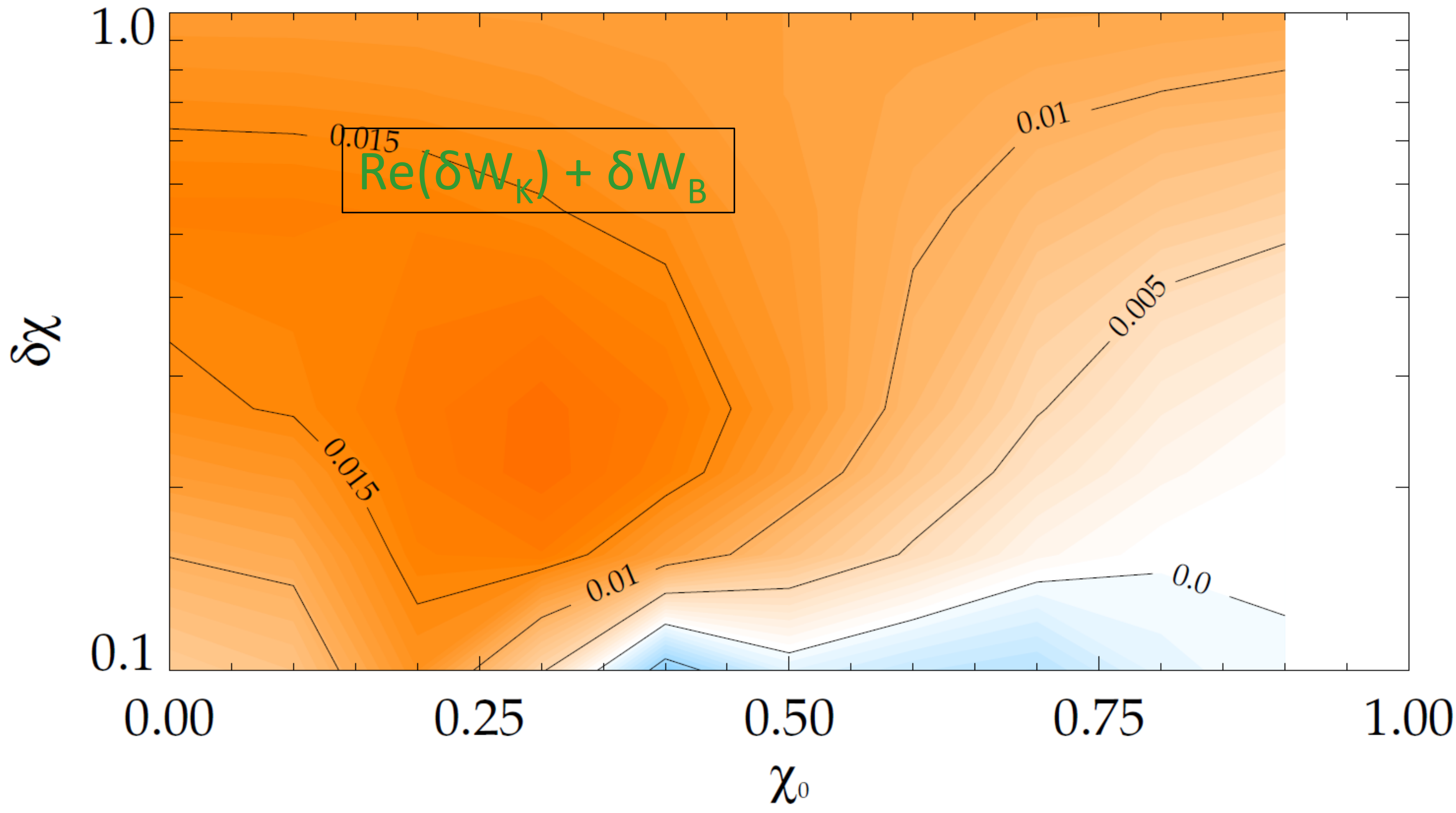
The total effect of anisotropy can be stabilizing or destabilizing compared to isotropy, depending on χ_0



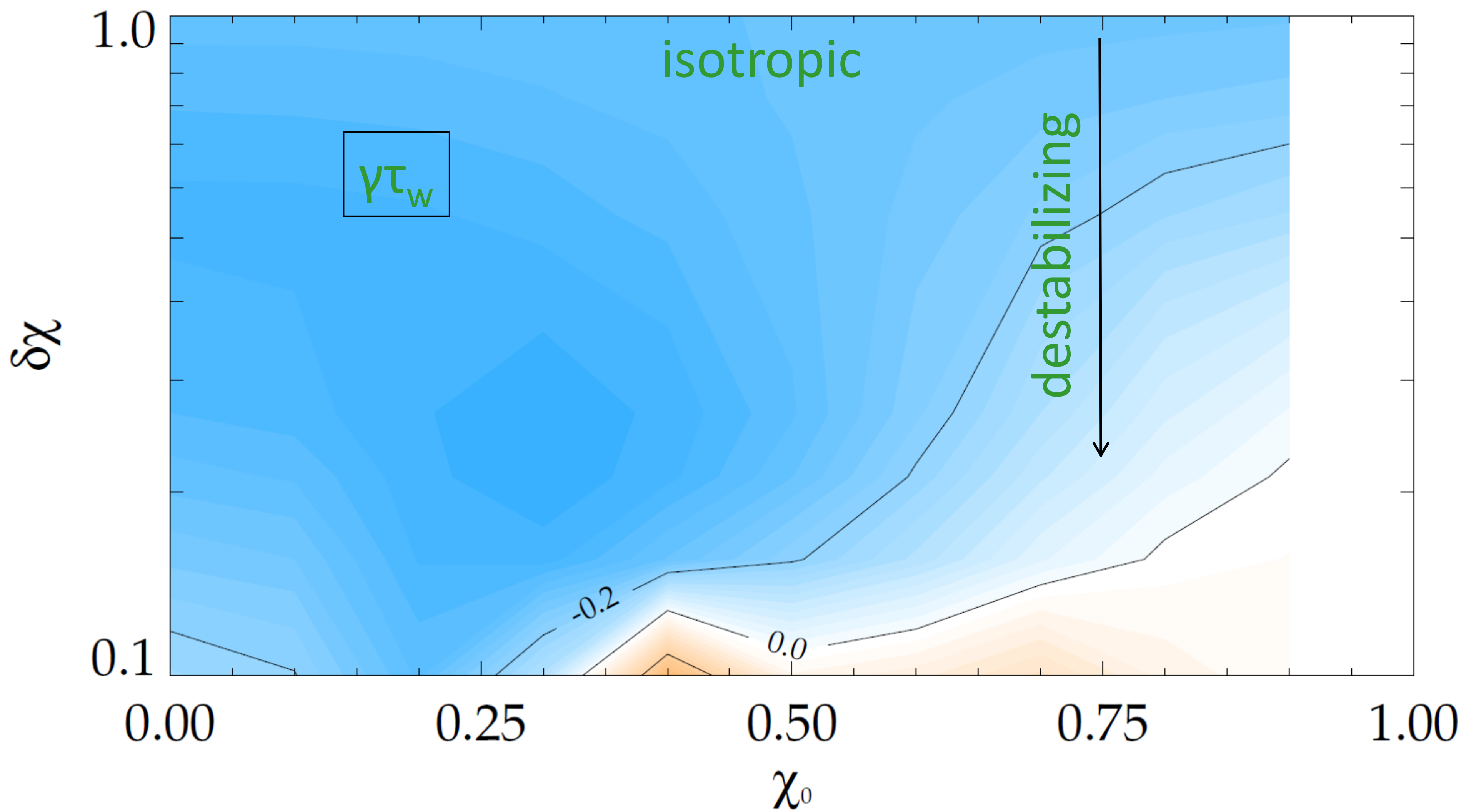
The δW_B term can make a significant difference



The δW_B term can make a significant difference



The effect of anisotropy on the growth rate is, generally, to make it closer to marginal than isotropy does



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Reminder of MISK formulae, previous status, and what's new

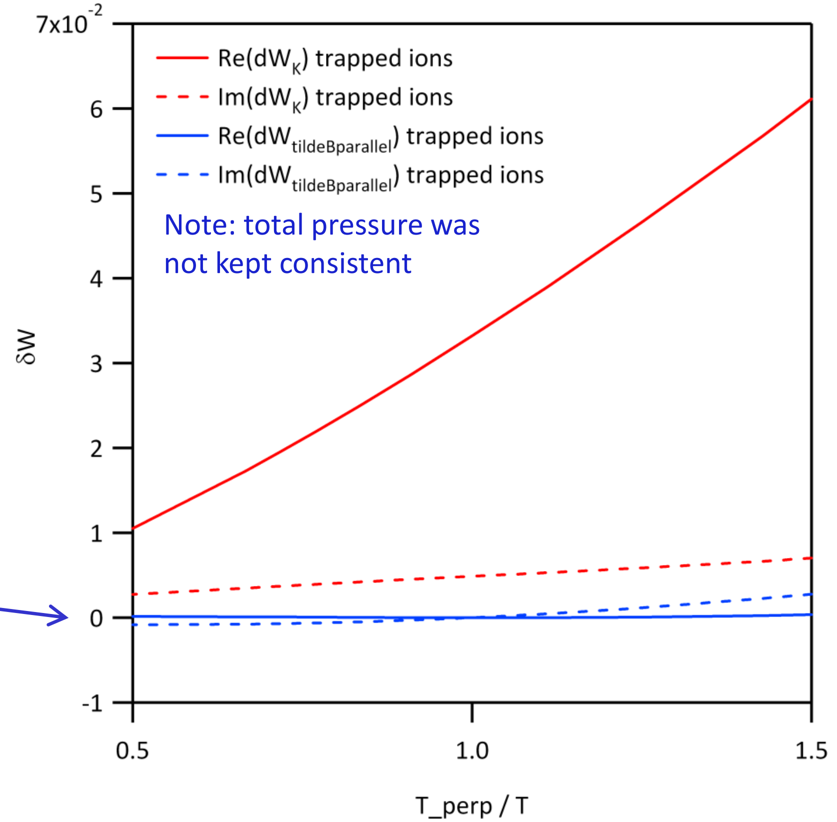
$$\delta W_H = \sum_j \sum_{l=-\infty}^{\infty} 2\sqrt{2}\pi^2 \int\int\int \left[|\langle HT_j \rangle|^2 \frac{(\omega - n\omega_E)\frac{\partial f_j}{\partial \varepsilon} - \frac{n}{Z_j e}\frac{\partial f_j}{\partial \Psi}}{n\langle \omega_D^j \rangle + l\omega_b^j - i\nu_{\mathrm{eff}}^j + n\omega_E - \omega} \right] \frac{\hat{\tau}}{m_j^{\frac{3}{2}} B} |\chi| \varepsilon^{\frac{1}{2}} d\varepsilon d\chi d\mathbf{\Psi},$$

$$\delta W_{\tilde{B}_\parallel} = \sum_j 2\sqrt{2}\pi^2 \int\int\int \left[\langle HT_j \rangle^* \frac{\tilde{\mathbf{B}}_\parallel}{B} \frac{\chi^2 - 1}{2|\chi|} \frac{\partial f_j}{\partial \chi} \right] \frac{\hat{\tau}}{m_j^{\frac{3}{2}} B} |\chi| \varepsilon^{\frac{1}{2}} d\varepsilon d\chi d\mathbf{\Psi},$$

- Bi-Maxwellian for thermal ions
  - fixed so that the pressure is consistent, and circulating ions included in $\delta W_{B_\parallel}$
- Simple Gaussian for EPs
  - $\partial f/\partial \chi$ expression was wrong in $\delta W_{B_\parallel}$! Now fixed.
  - Circulating EPs are now included in $\delta W_{B_\parallel}$
  - Only $\mathrm{Re}(\delta W_{B_\parallel})$ used
  - Many more cases were run

Previous: A bi-Maxwellian distribution function introduces anisotropy to the thermal particles; δW_B term small

$$f_j^{bM}(\varepsilon,\Psi,\chi) = n_j\left(\frac{m_j}{2\pi}\right)^{\frac{3}{2}}\frac{1}{T_{j\perp}T_{j\parallel}^{\frac{1}{2}}}e^{-\varepsilon\chi^2/T_{j\parallel}}e^{-\varepsilon(1-\chi^2)/T_{j\perp}}$$



$$\delta W_{\tilde{B}_\parallel}^{bM} = \sum_j \frac{15\pi}{8}\int\int n_j T_{j\parallel}\left(\frac{T_{j\parallel}}{T_{j\perp}}\right)\left(\frac{T_{j\parallel}}{T_{j\perp}}-1\right)\frac{\hat{\tau}}{B}\left[\langle HT_j/\varepsilon\rangle^*\frac{\tilde{\mathbf{B}}_\parallel}{B}\right]$$
$$|\chi|\left(\chi^2-1\right)\left[\chi^2+\left(1-\chi^2\right)\left(\frac{T_{j\parallel}}{T_{j\perp}}\right)\right]^{-\frac{7}{2}}d\chi d\mathbf{\Psi}$$

$\delta W_H \rightarrow$

$$=\sum_j\sum_{l=-\infty}^{\infty}\sqrt{\pi}\int\int\int n_j T_{j\parallel}\left(\frac{T_{j\parallel}}{T_{j\perp}}\right)\frac{\hat{\tau}}{B}|\chi|\hat{\varepsilon}_\parallel^{\frac{5}{2}}e^{-\hat{\varepsilon}_\parallel\chi^2}e^{-\hat{\varepsilon}_\parallel(1-\chi)^2(T_{j\parallel}/T_{j\perp})}d\hat{\varepsilon}_\parallel d\chi d\mathbf{\Psi}$$
$$\left[|\langle HT_j/\varepsilon\rangle|^2\frac{n\left(\omega_{*N_\parallel}^j+\left(\hat{\varepsilon}_\parallel\chi^2-\frac{1}{2}\right)\omega_{*T_\parallel}^j+\left(\hat{\varepsilon}_\parallel(1-\chi^2)\left(\frac{T_{j\parallel}}{T_{j\perp}}\right)^2-\frac{T_{j\parallel}}{T_{j\perp}}\right)\omega_{*T_\perp}^j+\omega_E\right)-\omega}{n\langle\omega_D^j\rangle+l\omega_b^j-i\nu_{\text{eff}}^j+n\omega_E-\omega}\right]$$

New: A bi-Maxwellian distribution function introduces anisotropy to the thermal particles; δW_B term ???

$$f_j^{bM}(\varepsilon,\Psi,\chi) = n_j \left(\frac{m_j}{2\pi}\right)^{\frac{3}{2}} \frac{1}{T_{j\perp}T_{j\|}^{\frac{1}{2}}} e^{-\varepsilon\chi^2/T_{j\|}} e^{-\varepsilon(1-\chi^2)/T_{j\perp}}$$

$$\delta W_{\tilde{B}_\|}^{bM} = \sum_j \frac{15\pi}{8} \int\int n_j T_{j\|} \left(\frac{T_{j\|}}{T_{j\perp}}\right) \left(\frac{T_{j\|}}{T_{j\perp}} - 1\right) \frac{\hat{\tau}}{B} \left[\langle HT_j/\varepsilon\rangle^* \frac{\tilde{\mathbf{B}}_\|}{B} \right]$$
$$|\chi| \left(\chi^2 - 1\right) \left[\chi^2 + (1-\chi^2)\left(\frac{T_{j\|}}{T_{j\perp}}\right)\right]^{-\frac{7}{2}} d\chi d\mathbf{\Psi}$$

$\delta W_H \rightarrow$

$$= \sum_j \sum_{l=-\infty}^{\infty} \sqrt{\pi} \int\int\int n_j T_{j\|} \left(\frac{T_{j\|}}{T_{j\perp}}\right) \frac{\hat{\tau}}{B} |\chi| \hat{\varepsilon}_\|^{\frac{5}{2}} e^{-\hat{\varepsilon}_\|\chi^2} e^{-\hat{\varepsilon}_\|(1-\chi)^2 (T_{j\|}/T_{j\perp})} d\hat{\varepsilon}_\| d\chi d\mathbf{\Psi}$$
$$\left[|\langle HT_j/\varepsilon\rangle|^2 \frac{n\left(\omega_{*N_\|}^j + \left(\hat{\varepsilon}_\|\chi^2 - \frac{1}{2}\right)\omega_{*T_\|}^j + \left(\hat{\varepsilon}_\|(1-\chi^2)\left(\frac{T_{j\|}}{T_{j\perp}}\right)^2 - \frac{T_{j\|}}{T_{j\perp}}\right)\omega_{*T_\perp}^j + \omega_E\right) - \omega}{n\langle\omega_D^j\rangle + l\omega_b^j - i\nu_{\mathrm{eff}}^j + n\omega_E - \omega} \right]$$

XXX

XXX