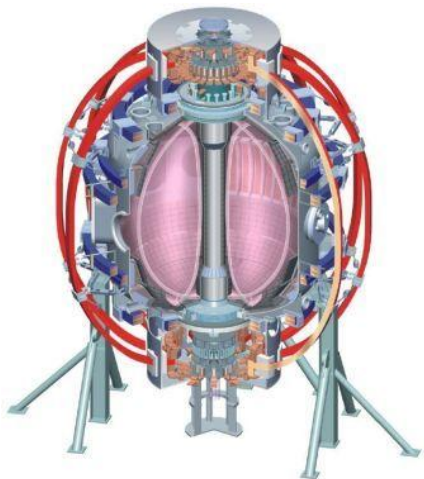


Precession Drift Frequency Benchmarking

Jack Berkery

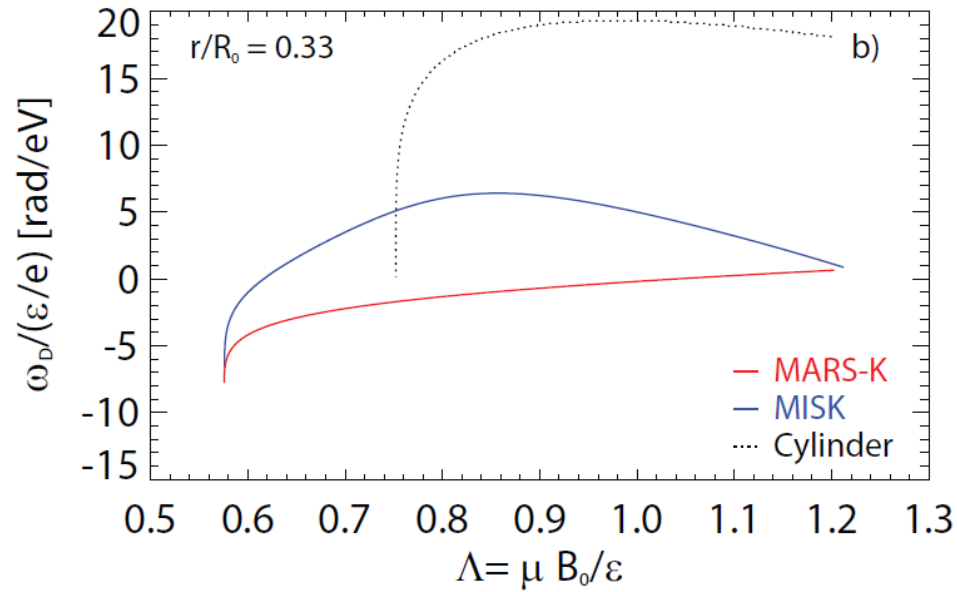
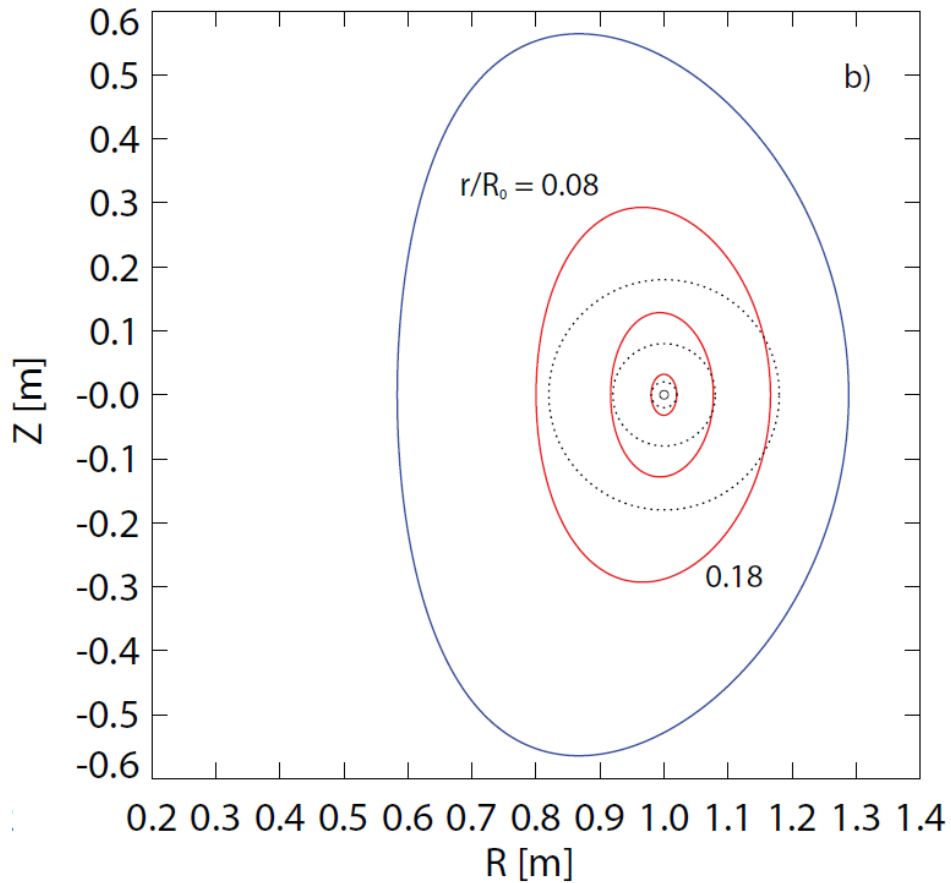
Department of Applied Physics, Columbia University, New York, NY, USA

College W&M
 Colorado Sch Mines
 Columbia U
 CompX
 General Atomics
 INL
 Johns Hopkins U
 LANL
 LLNL
 Lodestar
 MIT
 Nova Photonics
 New York U
 Old Dominion U
 ORNL
 PPPL
 PSI
 Princeton U
 Purdue U
 SNL
 Think Tank, Inc.
 UC Davis
 UC Irvine
 UCLA
 UCSD
 U Colorado
 U Illinois
 U Maryland
 U Rochester
 U Washington
 U Wisconsin



Culham Sci Ctr
 U St. Andrews
 York U
 Chubu U
 Fukui U
 Hiroshima U
 Hyogo U
 Kyoto U
 Kyushu U
 Kyushu Tokai U
 NIFS
 Niigata U
 U Tokyo
 JAEA
 Hebrew U
 Ioffe Inst
 RRC Kurchatov Inst
 TRINITY
 KBSI
 KAIST
 POSTECH
 ASIPP
 ENEA, Frascati
 CEA, Cadarache
 IPP, Jülich
 IPP, Garching
 ASCR, Czech Rep
 U Quebec

Solov'ev 3 equilibrium



Precession drift frequency

$$\omega_D = -\frac{1}{\tau} \frac{\varepsilon}{Z_j e} \int \frac{d\ell}{v_{\parallel}} \frac{\Lambda}{B_0} \frac{\partial B}{\partial \Psi} - \frac{1}{\tau} \frac{\varepsilon}{Z_j e} \int \frac{d\ell}{v_{\parallel}} \left(\frac{2}{B} - \frac{2\Lambda}{B_0} \right) \left(\frac{\partial B}{\partial \Psi} + \frac{\mu_0}{B} \frac{\partial p}{\partial \Psi} \right) - \frac{1}{\tau} \int_{\theta(t)}^{\theta(t')} \hat{q} d\theta$$

$$\omega_{B1} = \underbrace{-\frac{1}{\tau} \frac{\varepsilon}{Z_j e} \int \frac{d\ell}{v_{\parallel}} \frac{\Lambda}{B_0} \frac{\partial B}{\partial \Psi}}_{D_{\mu}} \underbrace{- \frac{1}{\tau} \frac{\varepsilon}{Z_j e} \int \frac{d\ell}{v_{\parallel}} \left(\frac{2}{B} - \frac{2\Lambda}{B_0} \right) \left(\frac{\partial B}{\partial \Psi} + \frac{\mu_0}{B} \frac{\partial p}{\partial \Psi} \right) - \frac{1}{\tau} \int_{\theta(t)}^{\theta(t')} \hat{q} d\theta}_{D_q}$$

$$\omega_{B1} = -\frac{\partial B}{\partial \Psi}$$

$$= -\frac{\nabla \Psi \cdot \nabla B^2}{2B |\nabla \Psi|^2}$$

$$= -\frac{1}{2B |\nabla \Psi|^2} \left(\nabla \Psi \cdot \nabla \left(\frac{F^2}{R^2} + \frac{\nabla \Psi^2}{R^2} \right) \right)$$

$$= -\frac{1}{2B |\nabla \Psi|^2} \left(-\frac{2B^2}{R} \frac{\partial \Psi}{\partial R} + \frac{2F}{R^2} \nabla \Psi \cdot \nabla F + \frac{1}{R^2} \nabla \Psi \cdot \nabla (|\nabla \Psi|^2) \right)$$

$$\omega_{B1\theta} = -\frac{1}{2BR^2} \frac{\nabla \Psi \cdot \nabla (|\nabla \Psi|^2)}{|\nabla \Psi|^2}$$

MARS and MISK find $\nabla(|\nabla\Psi|^2)$ differently

$$\omega_{B1\theta} = -\frac{1}{2BR^2} \frac{\nabla\Psi \cdot \nabla(|\nabla\Psi|^2)}{|\nabla\Psi|^2}$$

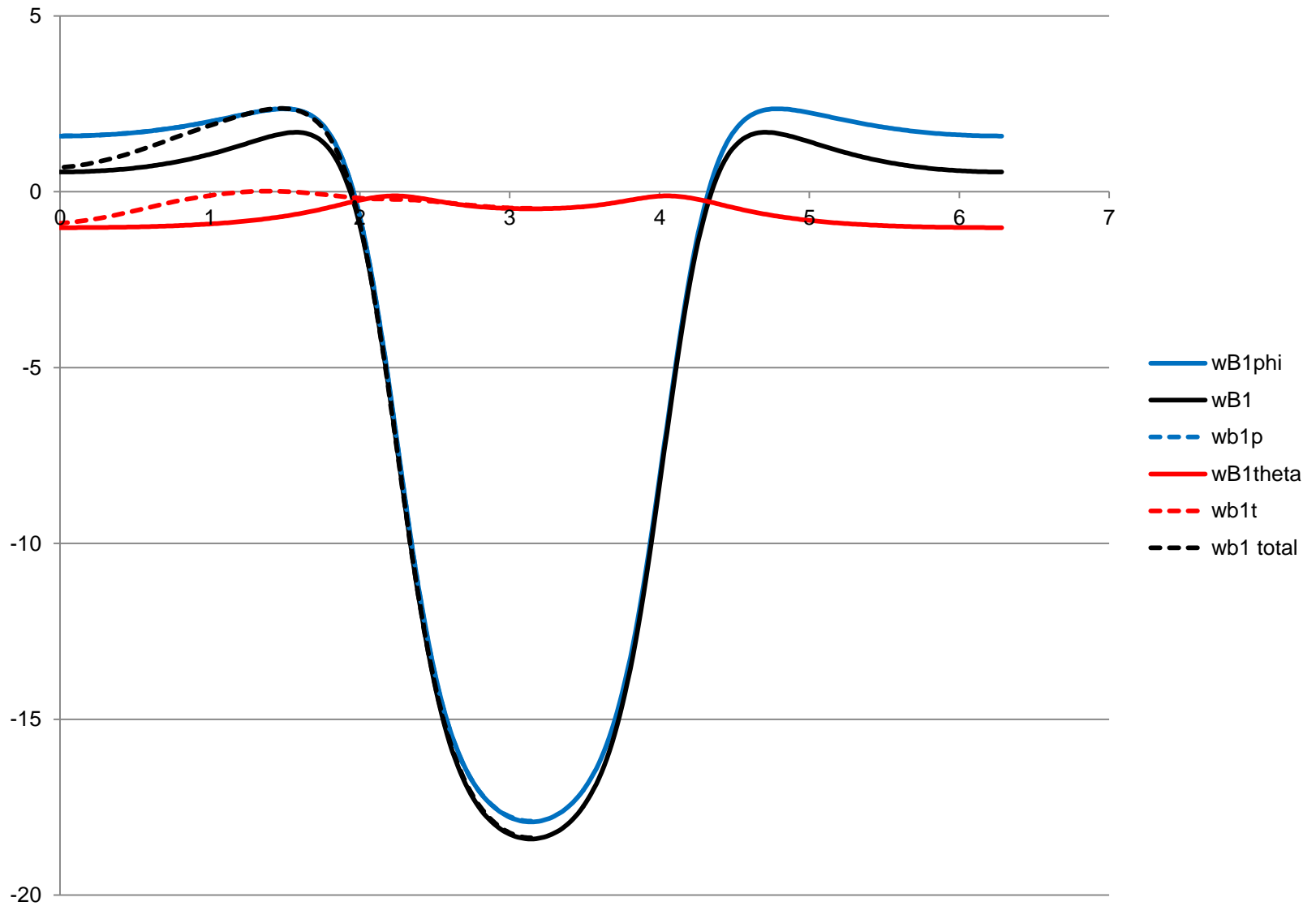
MARS:

$$\begin{aligned} \omega_{B1\theta} &= -\frac{1}{2BR^2} \frac{1}{|\nabla\Psi|^2} \left(\frac{\partial\Psi}{\partial R} \left(\frac{\partial|\nabla\Psi|^2}{\partial R} \right) + \frac{\partial\Psi}{\partial Z} \left(\frac{\partial|\nabla\Psi|^2}{\partial Z} \right) \right) \\ &= -\frac{1}{2BR^2} \frac{1}{\left(\frac{\partial\Psi}{\partial R}\right)^2 + \left(\frac{\partial\Psi}{\partial Z}\right)^2} \left[\frac{\partial\Psi}{\partial R} \frac{\partial}{\partial R} \left(\left(\frac{\partial\Psi}{\partial R}\right)^2 + \left(\frac{\partial\Psi}{\partial Z}\right)^2 \right) + \frac{\partial\Psi}{\partial Z} \frac{\partial}{\partial Z} \left(\left(\frac{\partial\Psi}{\partial R}\right)^2 + \left(\frac{\partial\Psi}{\partial Z}\right)^2 \right) \right] \end{aligned}$$

MISK:

$$\begin{aligned} \omega_{B1\theta} &= -\frac{1}{2BR^2 |\nabla\Psi|^2} \nabla\Psi \cdot \left(\frac{\partial|\nabla\Psi|^2}{\partial\Psi} \nabla\Psi + \frac{\partial|\nabla\Psi|^2}{\partial\theta} \nabla\theta \right) \\ &= -\frac{1}{2BR^2} \left(\frac{\partial|\nabla\Psi|^2}{\partial\Psi} + \frac{\partial|\nabla\Psi|^2}{\partial\theta} \frac{\nabla\Psi \cdot \nabla\theta}{|\nabla\Psi|^2} \right) \end{aligned}$$

$\omega_{B1\theta}$ is different



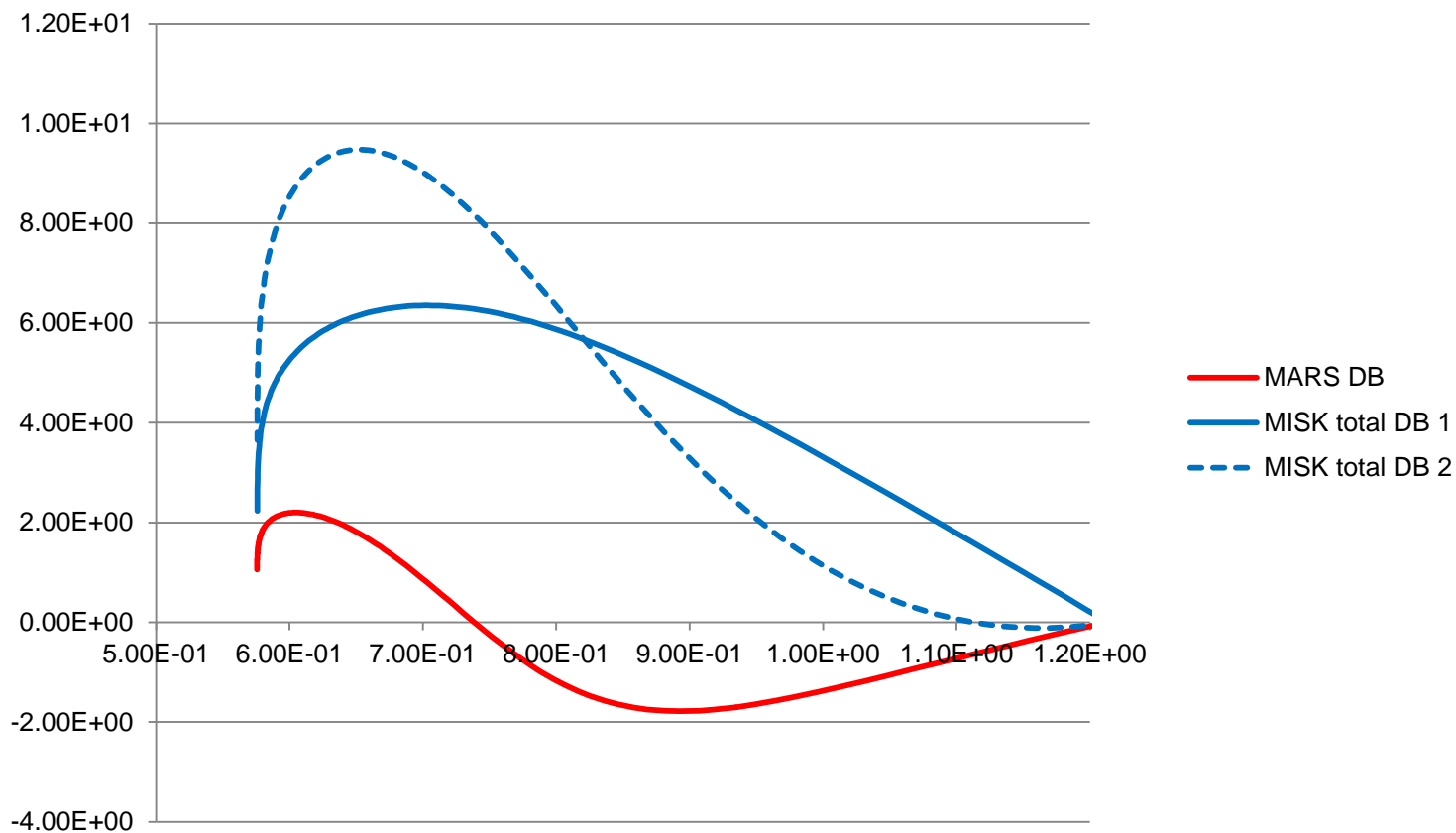
q terms should be the same, but they are not

$$D_{\hat{q}} = \frac{\varepsilon}{Z_j e \tau} \int \frac{d\ell}{v_{\parallel}} \frac{2v_{\parallel}^2}{v^2} \left[\frac{2}{B} \frac{\partial B}{\partial \Psi} - \frac{1}{B_{\theta}^2} \frac{\partial B_{\theta}^2}{\partial \Psi} - \frac{B_{\phi}^2}{B^2 B_{\theta}^2} \mu_0 \frac{\partial p}{\partial \Psi} - \frac{B_{\phi}^2}{B_{\theta}^2} \frac{1}{F} \frac{\partial F}{\partial \Psi} \right] = \frac{1}{\tau} \frac{\varepsilon}{Z_j e} \int_{\theta(t)}^{\theta(t')} \frac{\partial \hat{q}}{\partial \Psi} 2R \frac{v_{\phi}}{v^2} d\theta.$$

Slightly different



Very different!



XXX