NSTX

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Kinetic RWM Stability Theory Needs for NSTX-U

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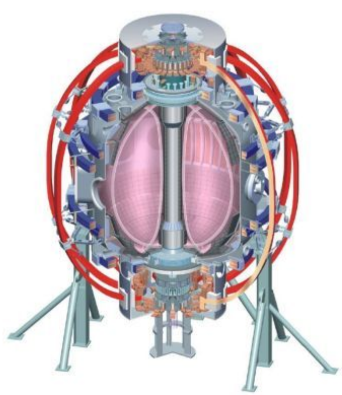

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Kinetic terms in the RWM dispersion relation enable stabilization; theory consistent with experimental results

Dissipation (Im(δW_K)) and restoring force (Re(δW_K)) from kinetic term enables stabilization of the RWM:

$$(\gamma - i\omega_r)\,\tau_w = -\frac{\delta W_\infty + \delta W_K}{\delta W_b + \delta W_K}$$

[B. Hu *et al.*, Phys. Plasmas **12**, 057301 (2005)]

$$\delta W_K = \sum_j \sum_{l=-\infty}^{\infty} 2\sqrt{2}\pi^2 \int\int\int \left[|\langle H/\hat{\varepsilon}\rangle|^2 \frac{(\omega - n\omega_E)\frac{\partial f_j}{\partial \varepsilon} - \frac{n}{Z_j e}\frac{\partial f_j}{\partial \Psi}}{n\langle\omega_D^j\rangle + l\omega_b^j - i\nu_{\text{eff}}^j + n\omega_E - \omega} \right] \frac{\hat{\tau}}{m_j^{\frac{3}{2}} B} |\chi| \hat{\varepsilon}^{\frac{5}{2}} d\hat{\varepsilon}\, d\chi\, d\Psi, \qquad \chi = v_{\|}/v$$

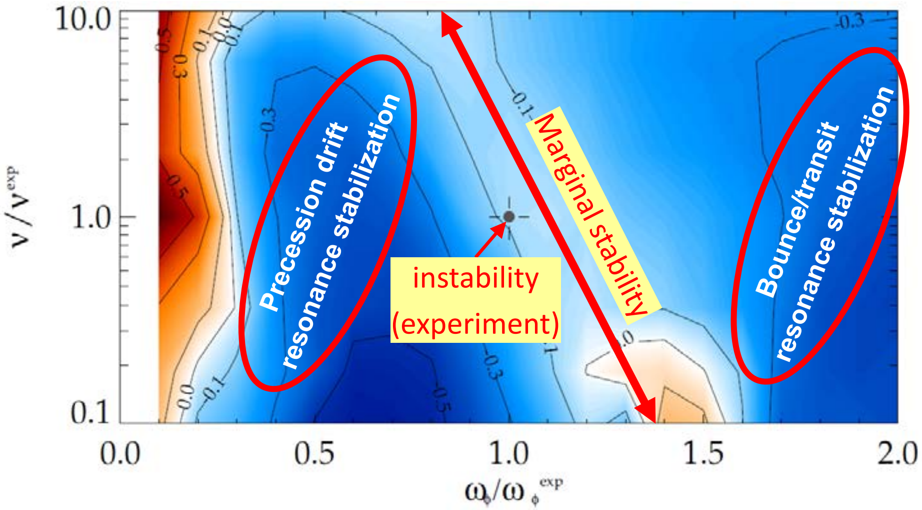
Precession Drift

Bounce

Collisionality

~ Plasma Rotation: $\omega_\phi \approx \omega_E + \omega_{*i}$

$\gamma\tau_w$ contours vs. ν and ω_ϕ

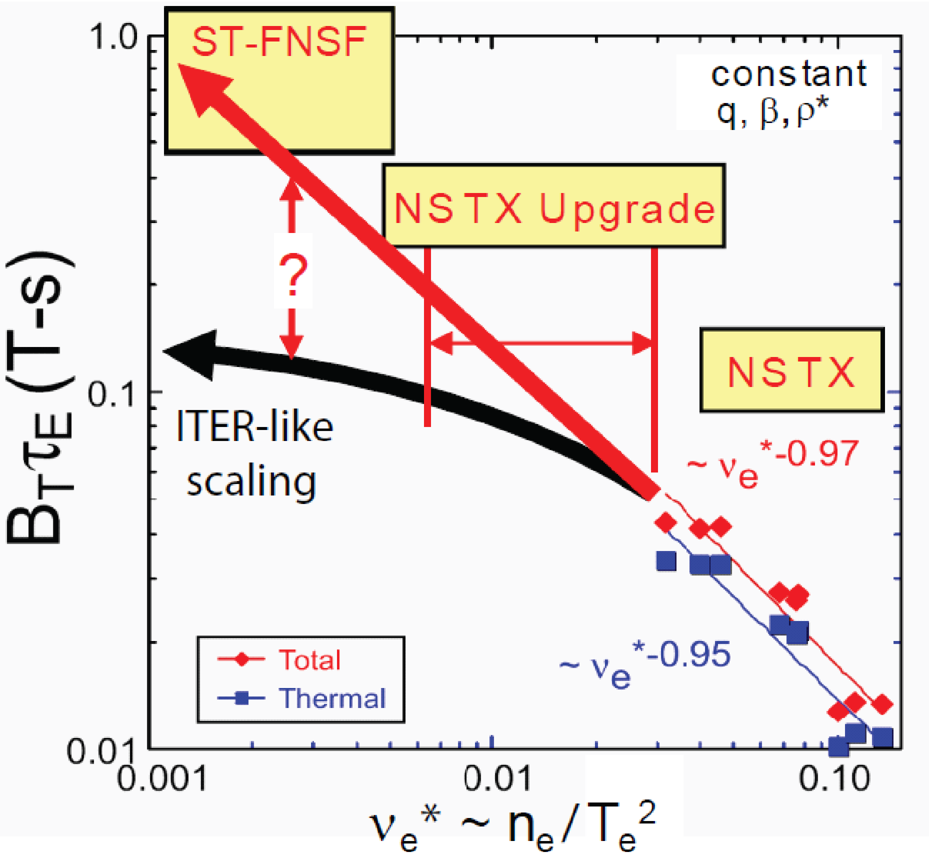


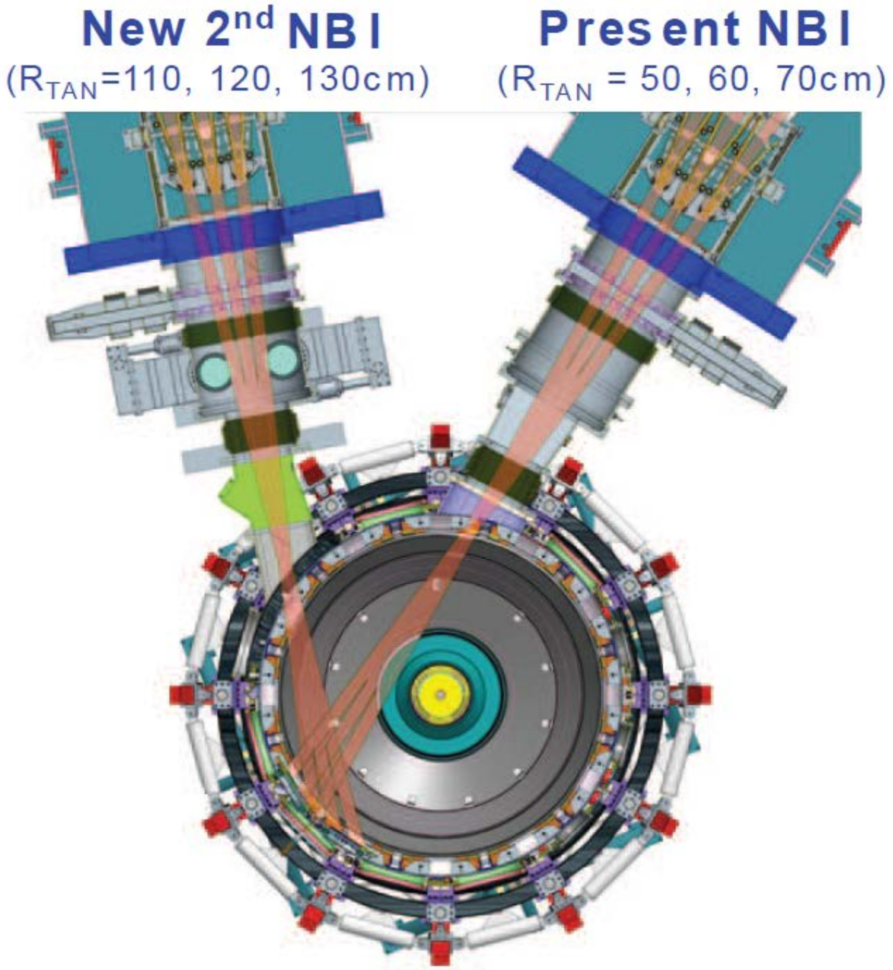
Theory Development

- Collisionality model improvements
- Anisotropy of energetic particles
- Further rotation effects (inc. poloidal)
- Eigenfunction modifications
- Neoclassical orbit modification? (with G. Kagan)

[J. Berkery *et al.*, Phys. Rev. Lett. **104**, 035003 (2010)]

NSTX-U will have lower collisionality and second, off-axis neutral beam





[J. Menard *et al.*, submitted to Nucl. Fusion (2011)]

EPs have a generally stabilizing effect that is independent of roation; Anisotropic distribution impacts stability

$$\delta W_F = \frac{1}{2}\int\Bigg\{\underbrace{\left(-\frac{|\tilde{\boldsymbol{B}}_\perp|^2}{\mu_0}\right.}_{\text{shearAlfvén}}\underbrace{-\frac{B^2}{\mu_0}|\boldsymbol{\nabla}\cdot\boldsymbol{\xi}_\perp + 2\boldsymbol{\xi}_\perp\cdot\boldsymbol{\kappa}|^2}_{\text{fast magneto-acoustic}}\underbrace{\left.+\, j_\parallel\left(\boldsymbol{\xi}_\perp^*\times\hat{\boldsymbol{b}}\right)\cdot\tilde{\boldsymbol{B}}_\perp\right)}_{\text{kink}}\underbrace{+\,2\left(\boldsymbol{\kappa}\cdot\boldsymbol{\xi}_\perp^*\right)\left(\boldsymbol{\xi}_\perp\cdot\boldsymbol{\nabla}p_{\text{avg}}\right)}_{\text{ballooning}}\Bigg\}\,d\mathbf{V}, \qquad (16)$$

Anisotropy effects fluid terms, mostly through ballooning term.

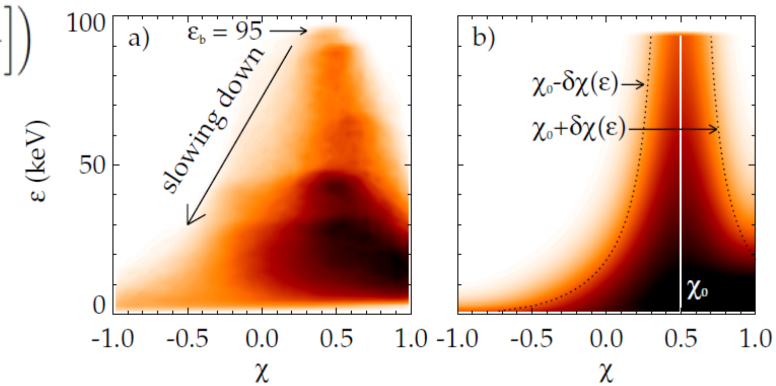
$$\delta W_A = \frac{1}{2}\int\Bigg\{(\sigma-1)\left(-\frac{|\tilde{\boldsymbol{B}}_\perp|^2}{\mu_0} - \frac{B^2}{\mu_0}|\boldsymbol{\nabla}\cdot\boldsymbol{\xi}_\perp + 2\boldsymbol{\xi}_\perp\cdot\boldsymbol{\kappa}|^2 + j_\parallel\left(\boldsymbol{\xi}_\perp^*\times\hat{\boldsymbol{b}}\right)\cdot\tilde{\boldsymbol{B}}_\perp\right) - 2B\,|\boldsymbol{\nabla}\cdot\boldsymbol{\xi}_\perp + \boldsymbol{\kappa}\cdot\boldsymbol{\xi}_\perp|^2\,\frac{\partial p_{\text{avg}}}{\partial B}\Bigg\}\,d\mathbf{V},$$

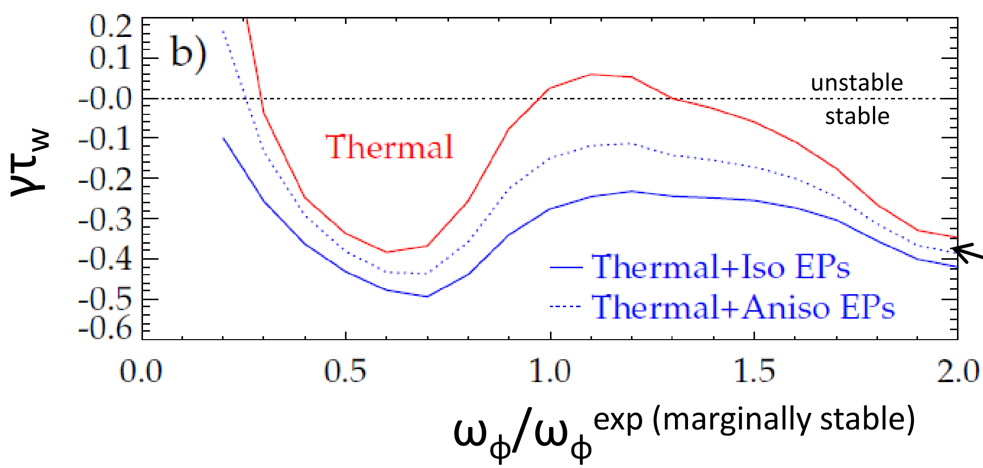
$$\delta W_K = \sum_j\sum_{l=-\infty}^{\infty} 2\sqrt{2}\pi^2\int\!\!\int\!\!\int\left[|\langle H/\hat{\varepsilon}\rangle|^2\,\frac{(\omega - n\omega_E)\frac{\partial f_j}{\partial\varepsilon} - \frac{n}{Z_j e}\frac{\partial f_j}{\partial\Psi}}{n\langle\omega_D^j\rangle + l\omega_b^j - i\nu_{\text{eff}}^j + n\omega_E - \omega}\right]\frac{\hat{\tau}}{m_j^{\frac{3}{2}}B}|\chi|\hat{\varepsilon}^{\frac{5}{2}}\,d\hat{\varepsilon}\,d\chi\,d\mathbf{\Psi},$$

Also effects kinetic term, through pitch angle dependence of distribution function.

$$f_j^b(\varepsilon,\Psi,\chi) = n_j A_b\left(\frac{m_j}{\varepsilon_b}\right)^{\frac{3}{2}}\frac{1}{\hat{\varepsilon}^{\frac{3}{2}} + \hat{\varepsilon}_c^{\frac{3}{2}}}\,\frac{1}{\delta\chi}\left(\exp\left[\frac{-(\chi-\chi_0)^2}{\delta\chi^2}\right] + \exp\left[\frac{-(\chi+2+\chi_0)^2}{\delta\chi^2}\right] + \exp\left[\frac{-(\chi-2+\chi_0)^2}{\delta\chi^2}\right]\right)$$

[N. Gorelenkov *et al.*, Nucl. Fusion **45**, 226 (2015)]

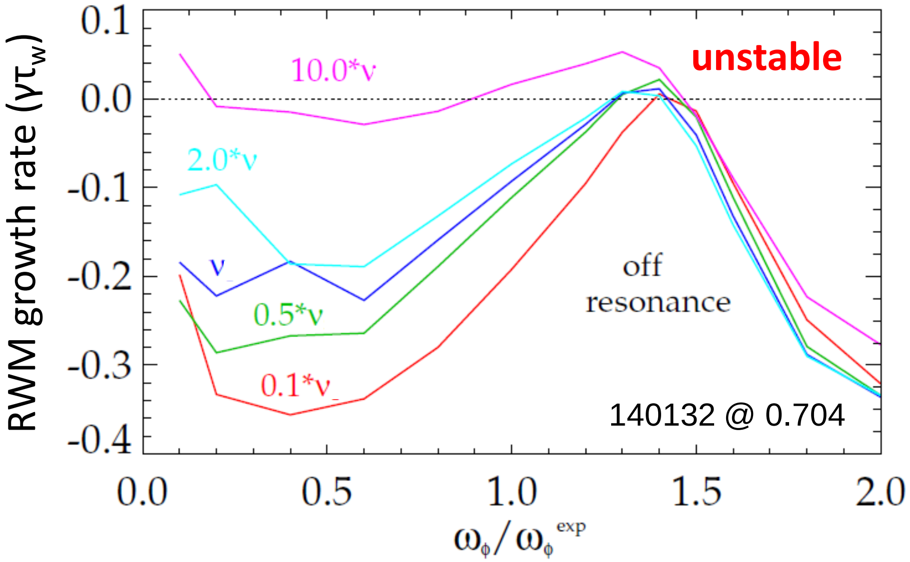




Addition of simple anisotropy model ($\chi_0 = 0.75$, $\delta\chi = 0.25$) reduces stabilizing effect, consistent with quantitative comparison to NSTX

[J.W. Berkery *et al.*, Phys. Plasmas **17**, 082504 (2010)]

Reduced collisionality (ν) is stabilizing for RWMs, but only near kinetic resonances



MISK currently uses an energy-dependent collisionality, MARS-K uses a constant.

Possible improvements:

Particle, momentum, and energy conserving Krook operator for like-particle collisions (suggested by G. Hammett):

$$C(\tilde{f}_j) = -\nu_{\text{eff}}\tilde{f}_j + \nu_{\text{eff}} f_j \left[\frac{\tilde{n}_j}{n_j} + \frac{m_j u_\parallel v_\parallel}{T_j} + \frac{\tilde{T}_j}{T_j}\left(\hat{\varepsilon} - \frac{3}{2}\right)\right]$$

Lorentz operator with pitch angle dependence:

$$\nu_3(\varepsilon, \chi, \Psi) = \frac{1}{2}\nu_2\epsilon_r \left[Z_{\text{eff}} + \frac{1}{\sqrt{\pi\hat{\varepsilon}}}e^{-\hat{\varepsilon}} + \frac{1}{\sqrt{\pi}}\left(2 - \hat{\varepsilon}^{-1}\right)\int_0^{\sqrt{\hat{\varepsilon}}} e^{-t^2}dt\right]\frac{\partial}{\partial\chi}\left(1 - \chi^2\right)\frac{\partial}{\partial\chi}$$

- NSTX-tested kinetic RWM stability theory: 2 competing effects at lower ν
  - Stabilizing collisional dissipation reduced (expected from early theory)
  - Stabilizing resonant kinetic effects enhanced (contrasts early RWM theory)

[J. Berkery *et al.*, Phys. Rev. Lett. **106**, 075004 (2011)]

Further exploration of the effect of plasma rotation

- Effect on equilibrium
- Including poloidal rotation
- Eigenfunction modification (next slide)

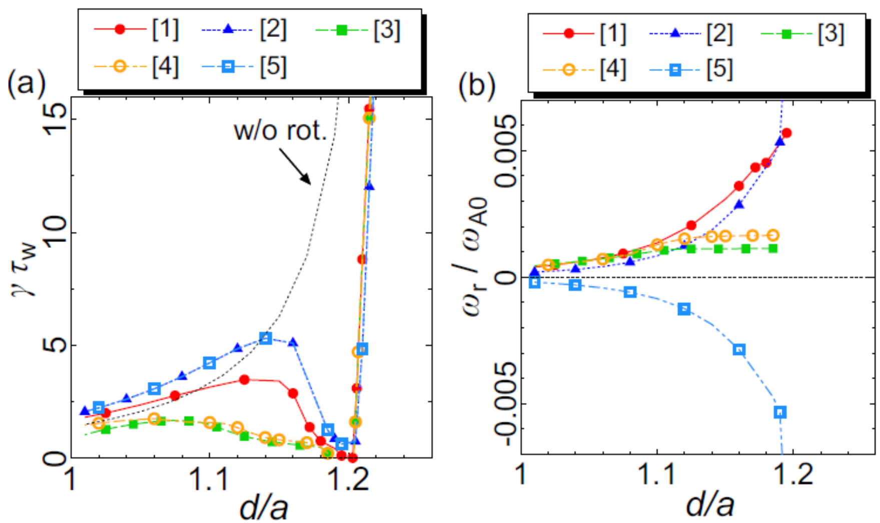


FIG. 4. (Color online) Dependence of (a) γ and (b) ω_r on d/a in the toroidal case when $(\Omega_\phi, \Omega_\theta)$=[1] $(\Omega_{\phi i}, 0)$, [2] $(\Omega_{\phi i}, 0.05\Omega_{\phi i}/q)$, [3] $(\Omega_{\phi i}, -0.05\Omega_{\phi i}/q)$, [4] $(1.05\Omega_{\phi i}, 0)$, and [5] $(-\Omega_{\phi i}, -0.05\Omega_{\phi i}/q)$, respectively. Note that in (a), the line that shows the result in case [2] overlaps with that in case [5] because the γ dependences on d/a in these cases are identical to each other.

[N. Aiba *et al.*, Phys. Plasmas **18**, 022503 (2011)]

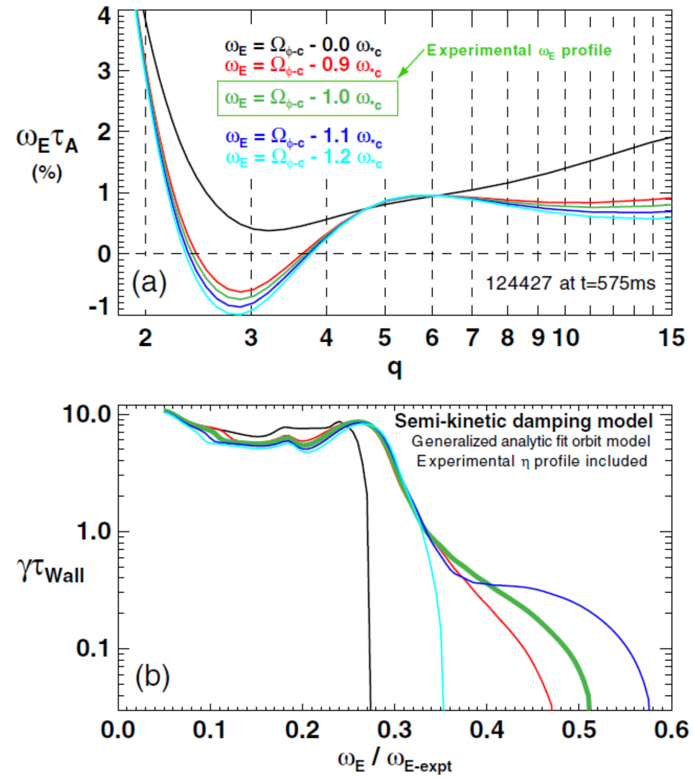
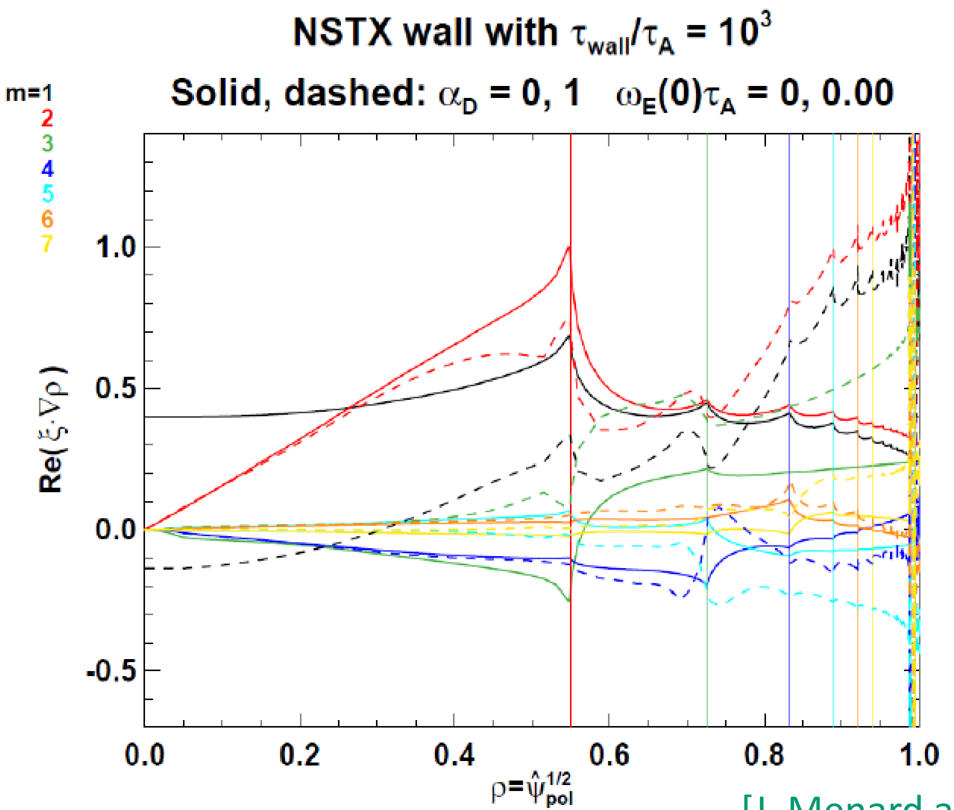


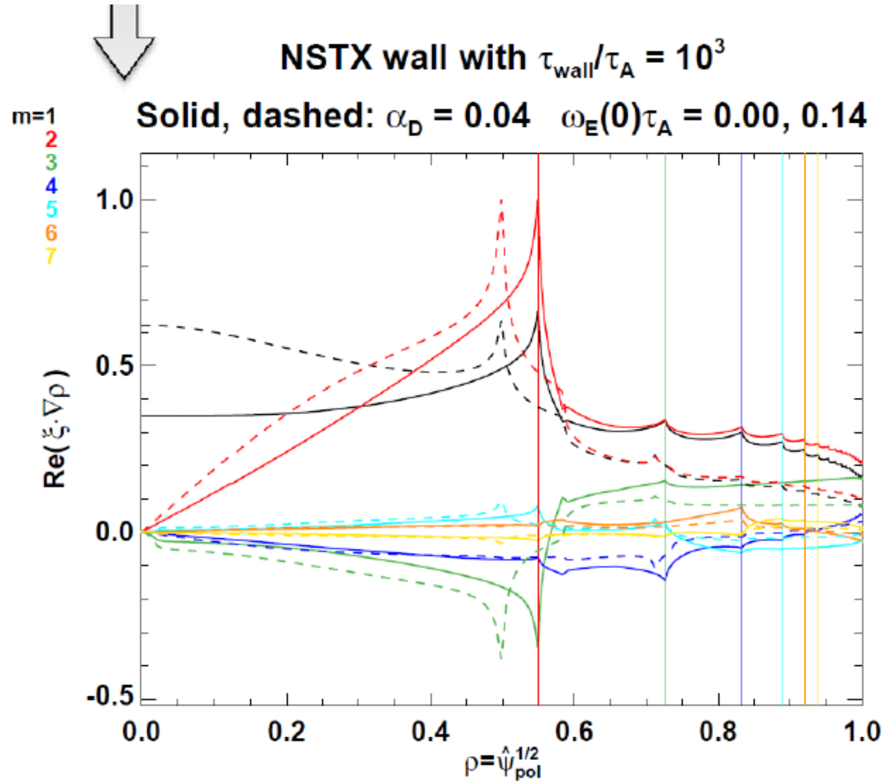
Figure 26. (*a*) Comparison of plasma ω_E profiles versus q for the RWM-unstable plasma excluding (black) and including (other colours) the carbon impurity diamagnetic rotation in the radial force balance equation for the calculation of the electrostatic potential profile $\Phi(\psi)$. (*b*) Comparison of growth rates of the $n = 1$ RWM computed with the MARS-F code plotted versus ω_E using the generalized-geometry analytic fit to the particle orbit times and including the neoclassical parallel resistivity profile for the plasma resistivity.

[J. Menard, APS 2010 and 2011]

[J. Menard *et al.*, Nucl. Fusion **50**, 045008 (2010)]

The RWM eigenfunction may be modified by several factors





[J. Menard and Y.Q. Liu, APS 2011]

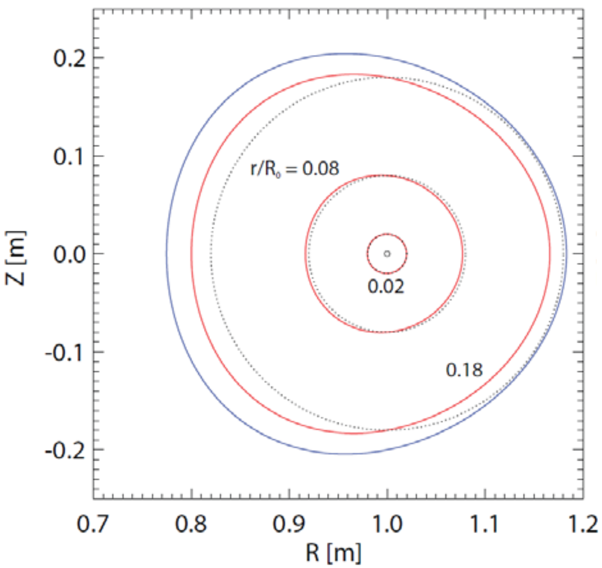
Kinetic dissipation

Plasma rotation

The importance of eigenfunction modification and Alfven resonances at rational surfaces will come out of code benchmarking.

The kinetic physics in MISK is currently being benchmarked with MARS-K and HAGIS through the ITPA's MDC-2

Solov'ev case 1 (near-circular)



$$\delta W_K = -\frac{\sqrt{\pi}}{2}\int_0^{\Psi_a}\frac{nT}{B_0}\int_{B_0/B_{\max}}^{B_0/B_{\min}}\tau\sum_l\left|\langle H/\hat{\varepsilon}\rangle\right|^2 I_{\hat{\varepsilon}}\,d\Lambda\, d\Psi.$$

Perturbed Lagrangian

$$\langle H/\hat{\varepsilon}\rangle(\Psi,\Lambda,l) = \frac{1}{\tau}\oint\frac{1}{\sqrt{1-\frac{\Lambda B}{B_0}}}\left[\left(2-3\frac{\Lambda B}{B_0}\right)(\boldsymbol{\kappa}\cdot\boldsymbol{\xi}_\perp)-\left(\frac{\Lambda B}{B_0}\right)(\boldsymbol{\nabla}\cdot\boldsymbol{\xi}_\perp)\right]e^{-il\omega_b t}d\ell.$$

Energy integral of the frequency resonance fraction

$$I_\varepsilon(\Psi,\Lambda,l) = \int_0^\infty\frac{\omega_{*N}+\left(\hat{\varepsilon}-\frac{3}{2}\right)\omega_{*T}+\omega_E-\omega_r-i\gamma}{\omega_D+l\omega_b+\omega_E-i\nu_{\text{eff}}-\omega_r-i\gamma}\hat{\varepsilon}^{\frac{5}{2}}e^{-\hat{\varepsilon}}d\hat{\varepsilon}.$$