NSTX-U

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Theory Improvements

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March 15, 2012

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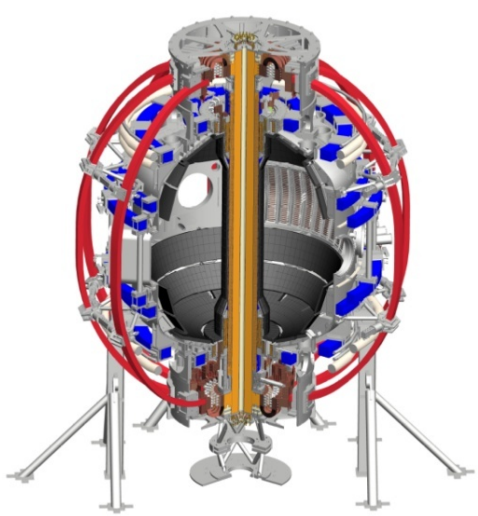

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Kinetic terms in the RWM dispersion relation enable stabilization; theory consistent with experimental results

Dissipation (Im(δW_K)) and restoring force (Re(δW_K)) from kinetic term enables stabilization of the RWM:

$$(\gamma - i\omega_r)\,\tau_w = -\frac{\delta W_\infty + \delta W_K}{\delta W_b + \delta W_K}$$

[B. Hu *et al.*, Phys. Plasmas **12**, 057301 (2005)]

$$\delta W_K = \sum_j \sum_{l=-\infty}^{\infty} 2\sqrt{2}\pi^2 \int\!\!\int\!\!\int \left[|\langle H/\hat{\varepsilon}\rangle|^2 \frac{(\omega - n\omega_E)\frac{\partial f_j}{\partial \varepsilon} - \frac{n}{Z_j e}\frac{\partial f_j}{\partial \Psi}}{n\langle\omega_D^j\rangle + l\omega_b^j - i\nu_{\text{eff}}^j + n\omega_E - \omega} \right] \frac{\hat{\tau}}{m_j^{\frac{3}{2}} B} |\chi| \hat{\varepsilon}^{\frac{5}{2}} d\hat{\varepsilon}\, d\chi\, d\mathbf{\Psi}, \qquad \chi = v_\parallel / v$$

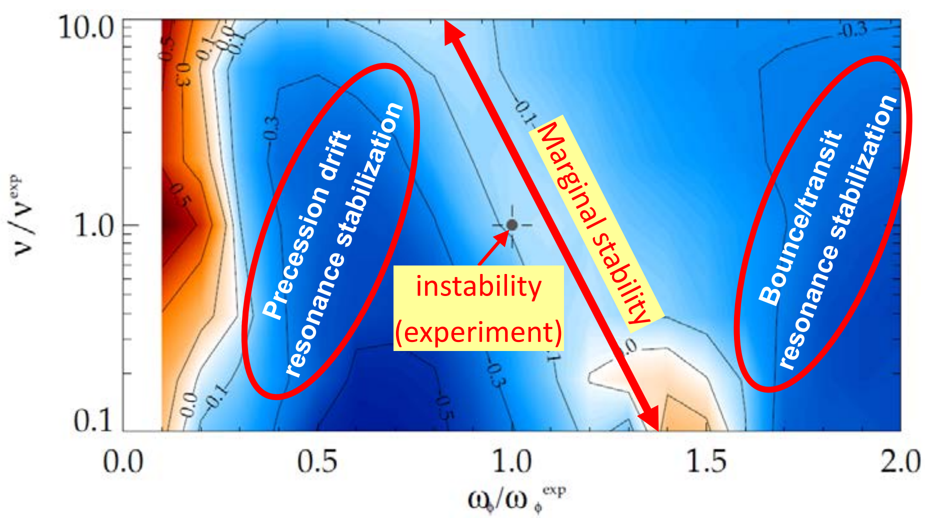
Precession Drift

Bounce

Collisionality

~ Plasma Rotation: $\omega_\phi \approx \omega_E + \omega_{*i}$

$\gamma\tau_w$ contours vs. ν and ω_ϕ

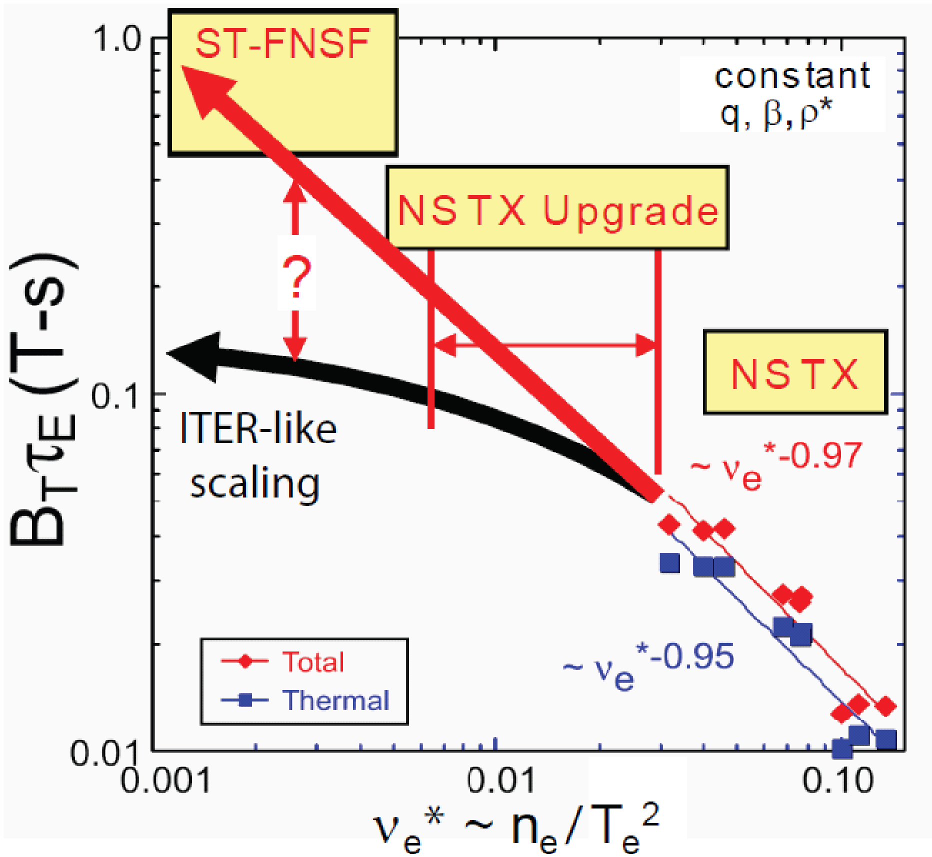


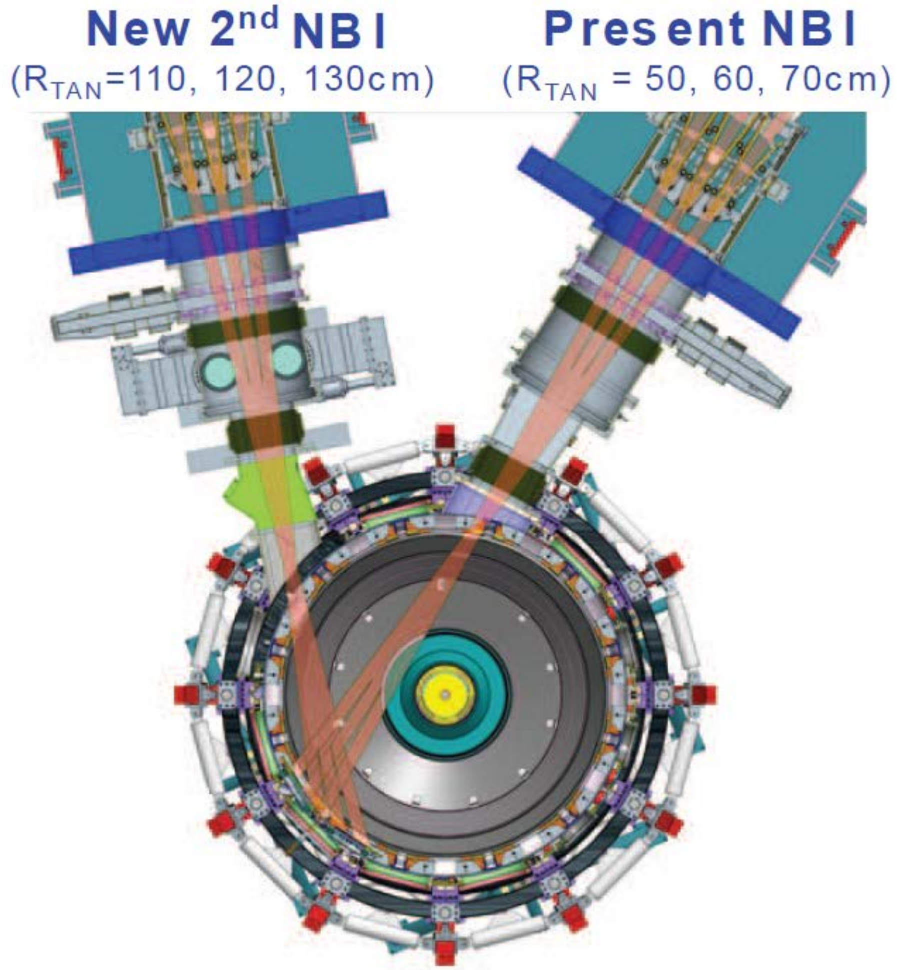
Theory Development

- Collisionality model improvements
- Anisotropy of energetic particles
- Further rotation effects (inc. poloidal)
- Eigenfunction modifications
- Neoclassical orbit modification? (with G. Kagan)

[J. Berkery *et al.*, Phys. Rev. Lett. **104**, 035003 (2010)]

NSTX-U will have lower collisionality and second, off-axis neutral beam





[J. Menard *et al.*, submitted to Nucl. Fusion (2011)]

EPs have a generally stabilizing effect that is independent of rotation; Anisotropic distribution impacts stability

$$\delta W_F = \frac{1}{2}\int\left\{\left(\underbrace{-\frac{|\tilde{\boldsymbol{B}}_\perp|^2}{\mu_0}}_{\text{shearAlfvén}} \underbrace{-\frac{B^2}{\mu_0}|\boldsymbol{\nabla}\cdot\boldsymbol{\xi}_\perp + 2\boldsymbol{\xi}_\perp\cdot\boldsymbol{\kappa}|^2}_{\text{fast magneto-acoustic}} + \underbrace{j_\parallel\left(\boldsymbol{\xi}^*_\perp \times \hat{\boldsymbol{b}}\right)\cdot\tilde{\boldsymbol{B}}_\perp}_{\text{kink}}\right) + \underbrace{2\left(\boldsymbol{\kappa}\cdot\boldsymbol{\xi}^*_\perp\right)\left(\boldsymbol{\xi}_\perp\cdot\boldsymbol{\nabla}p_{\text{avg}}\right)}_{\text{ballooning}}\right\} d\mathbf{V}, \quad (16)$$

Anisotropy effects fluid terms, mostly through ballooning term.

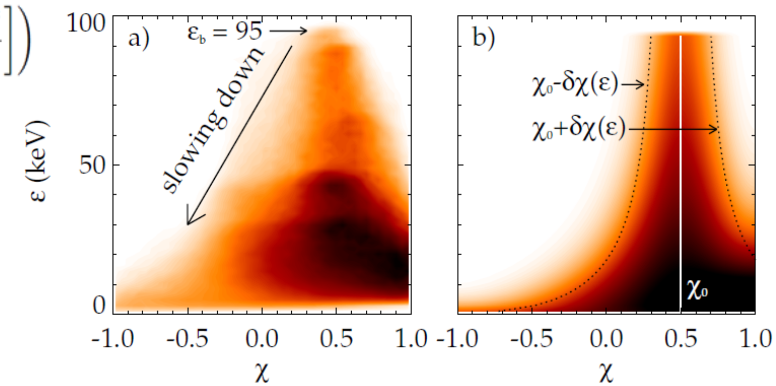
$$\delta W_A = \frac{1}{2}\int\left\{(\sigma-1)\left(\overbrace{-\frac{|\tilde{\boldsymbol{B}}_\perp|^2}{\mu_0}} \overbrace{-\frac{B^2}{\mu_0}|\boldsymbol{\nabla}\cdot\boldsymbol{\xi}_\perp + 2\boldsymbol{\xi}_\perp\cdot\boldsymbol{\kappa}|^2} + \overbrace{j_\parallel\left(\boldsymbol{\xi}^*_\perp \times \hat{\boldsymbol{b}}\right)\cdot\tilde{\boldsymbol{B}}_\perp}\right) \overbrace{- 2B\,|\boldsymbol{\nabla}\cdot\boldsymbol{\xi}_\perp + \boldsymbol{\kappa}\cdot\boldsymbol{\xi}_\perp|^2\,\frac{\partial p_{\text{avg}}}{\partial B}}\right\} d\mathbf{V},$$

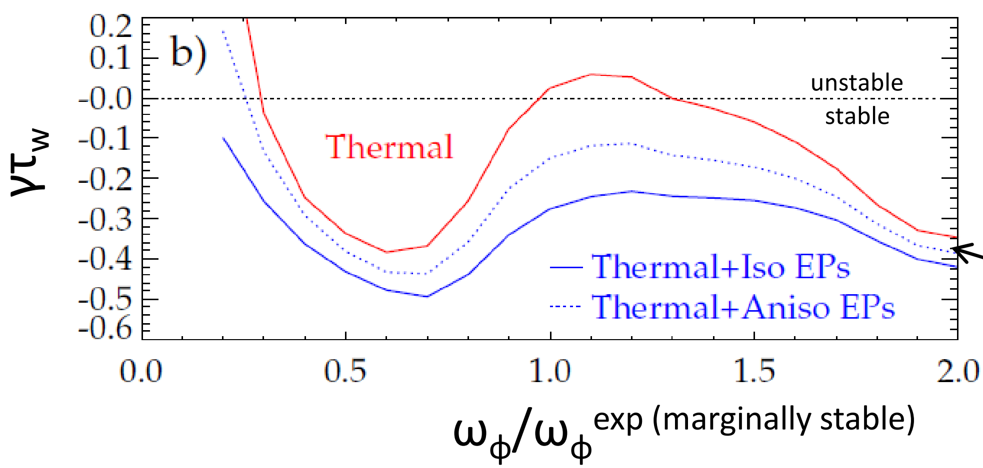
Also effects kinetic term, through pitch angle dependence of distribution function.

$$\delta W_K = \sum_j \sum_{l=-\infty}^{\infty} 2\sqrt{2}\pi^2 \int\!\!\int\!\!\int \left[|\langle H/\hat{\varepsilon}\rangle|^2 \frac{(\omega - n\omega_E)\frac{\partial f_j}{\partial \varepsilon} - \frac{n}{Z_j e}\frac{\partial f_j}{\partial \Psi}}{n\langle\omega_D^j\rangle + l\omega_b^j - i\nu_{\text{eff}}^j + n\omega_E - \omega}\right] \frac{\hat{\tau}}{m_j^{\frac{3}{2}} B}|\chi|\hat{\varepsilon}^{\frac{5}{2}}\, d\hat{\varepsilon}\, d\chi\, d\mathbf{\Psi},$$

$$f_j^b(\varepsilon,\Psi,\chi) = n_j A_b \left(\frac{m_j}{\varepsilon_b}\right)^{\frac{3}{2}} \frac{1}{\hat{\varepsilon}^{\frac{3}{2}} + \hat{\varepsilon}_c^{\frac{3}{2}}}\frac{1}{\delta\chi}\left(\exp\left[\frac{-(\chi-\chi_0)^2}{\delta\chi^2}\right] + \exp\left[\frac{-(\chi+2+\chi_0)^2}{\delta\chi^2}\right] + \exp\left[\frac{-(\chi-2+\chi_0)^2}{\delta\chi^2}\right]\right)$$

[N. Gorelenkov *et al.*, Nucl. Fusion **45**, 226 (2015)]

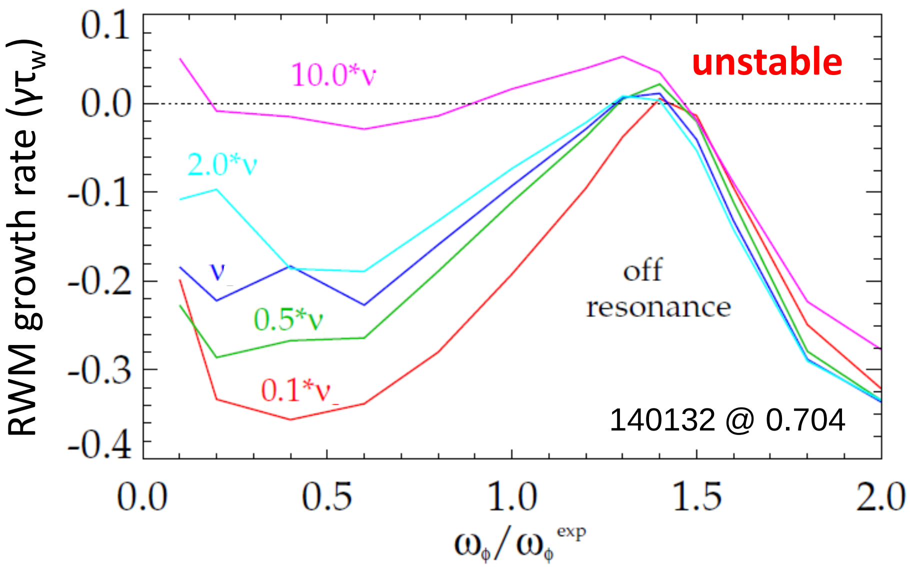




Addition of simple anisotropy model (χ_0 = 0.75, $\delta\chi$ = 0.25) reduces stabilizing effect, consistent with quantitative comparison to NSTX

[J.W. Berkery *et al.*, Phys. Plasmas **17**, 082504 (2010)]

Reduced collisionality (ν) is stabilizing for RWMs, but only near kinetic resonances



- NSTX-tested kinetic RWM stability theory: 2 competing effects at lower ν
  - Stabilizing collisional dissipation reduced (expected from early theory)
  - Stabilizing resonant kinetic effects enhanced (contrasts early RWM theory)

[J. Berkery *et al.*, Phys. Rev. Lett. **106**, 075004 (2011)]

MISK currently uses an energy-dependent collisionality, MARS-K uses a constant.

Possible improvements:

Particle, momentum, and energy conserving Krook operator for like-particle collisions (suggested by G. Hammett):

$$C(\tilde{f}_j) = -\nu_{\text{eff}}\tilde{f}_j + \nu_{\text{eff}} f_j \left[\frac{\tilde{n}_j}{n_j} + \frac{m_j u_\parallel v_\parallel}{T_j} + \frac{\tilde{T}_j}{T_j}\left(\hat{\varepsilon} - \frac{3}{2}\right)\right]$$

Lorentz operator with pitch angle dependence:

$$\nu_3(\varepsilon, \chi, \Psi) = \frac{1}{2}\nu_2\epsilon_r \left[Z_{\text{eff}} + \frac{1}{\sqrt{\pi\hat{\varepsilon}}}e^{-\hat{\varepsilon}} + \frac{1}{\sqrt{\pi}}\left(2 - \hat{\varepsilon}^{-1}\right)\int_0^{\sqrt{\hat{\varepsilon}}} e^{-t^2} dt\right] \frac{\partial}{\partial\chi}\left(1 - \chi^2\right)\frac{\partial}{\partial\chi}$$

Further exploration of the effect of plasma rotation

- Effect on equilibrium
- Including poloidal rotation
- Eigenfunction modification (next slide)

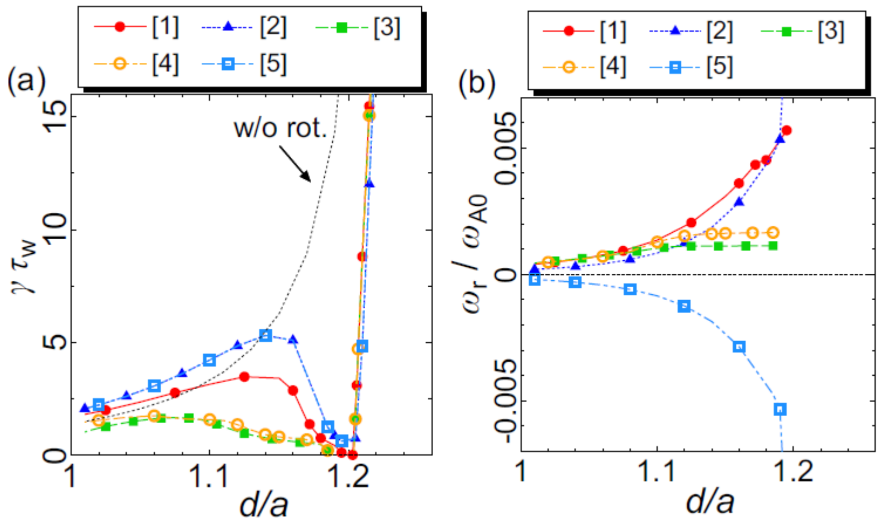


FIG. 4. (Color online) Dependence of (a) γ and (b) ω_r on d/a in the toroidal case when $(\Omega_\phi, \Omega_\theta)$=[1] $(\Omega_{\phi i}, 0)$, [2] $(\Omega_{\phi i}, 0.05\Omega_{\phi i}/q)$, [3] $(\Omega_{\phi i}, -0.05\Omega_{\phi i}/q)$, [4] $(1.05\Omega_{\phi i}, 0)$, and [5] $(-\Omega_{\phi i}, -0.05\Omega_{\phi i}/q)$, respectively. Note that in (a), the line that shows the result in case [2] overlaps with that in case [5] because the γ dependences on d/a in these cases are identical to each other.

[N. Aiba *et al.*, Phys. Plasmas **18**, 022503 (2011)]

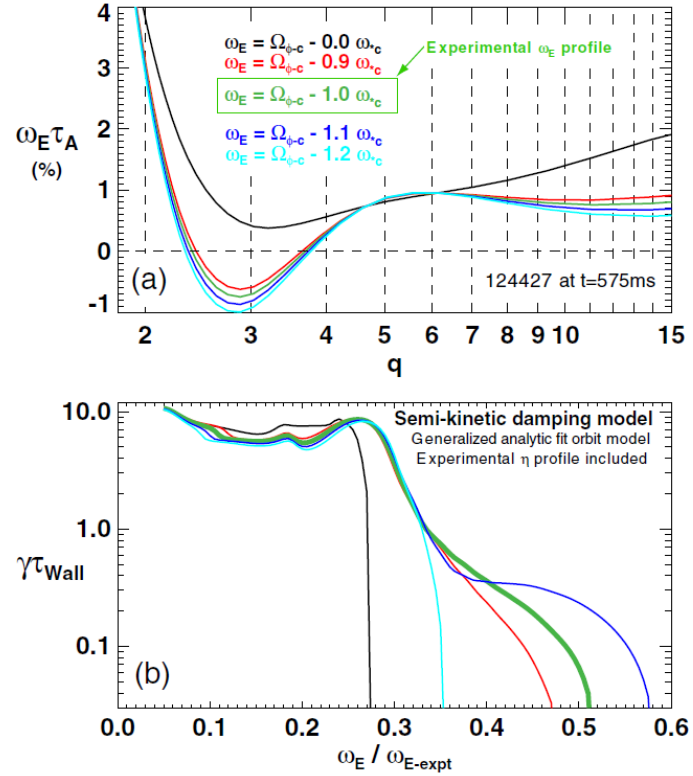
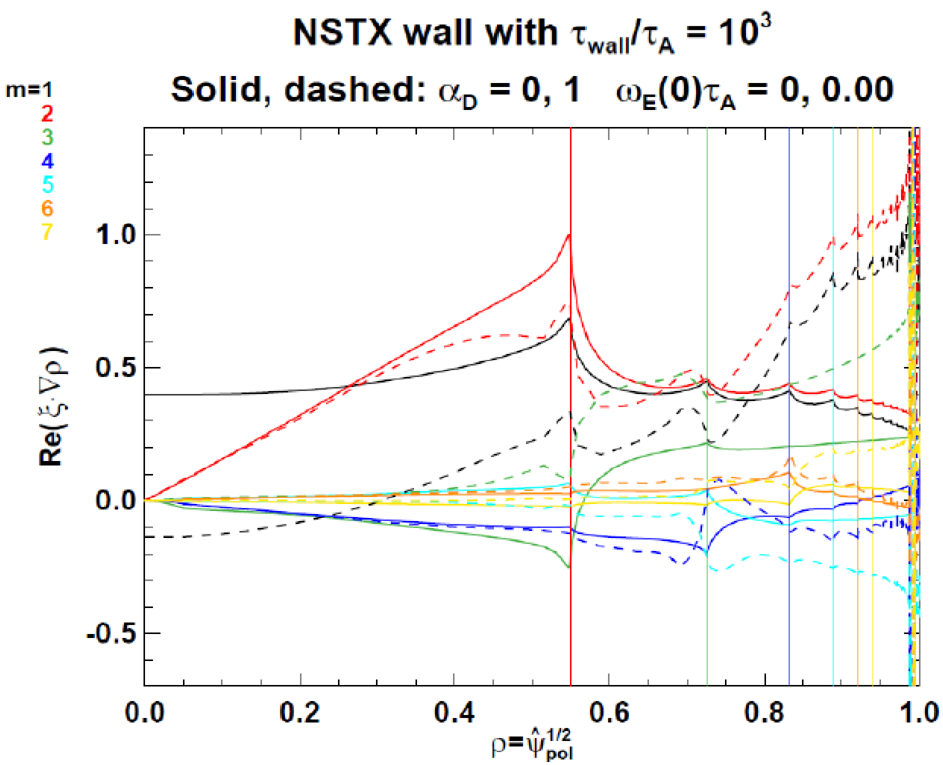


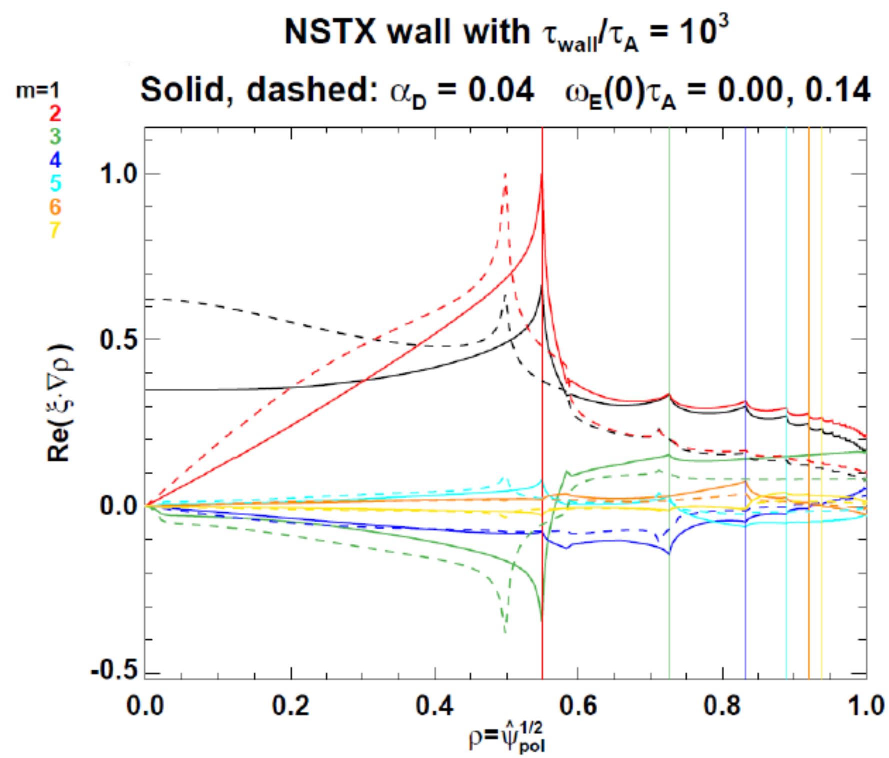
Figure 26. (*a*) Comparison of plasma ω_E profiles versus q for the RWM-unstable plasma excluding (black) and including (other colours) the carbon impurity diamagnetic rotation in the radial force balance equation for the calculation of the electrostatic potential profile $\Phi(\psi)$. (*b*) Comparison of growth rates of the $n = 1$ RWM computed with the MARS-F code plotted versus ω_E using the generalized-geometry analytic fit to the particle orbit times and including the neoclassical parallel resistivity profile for the plasma resistivity.

[J. Menard, APS 2010 and 2011]

[J. Menard *et al.*, Nucl. Fusion **50**, 045008 (2010)]

The RWM eigenfunction may be modified by several factors





[J. Menard and Y.Q. Liu, APS 2011]

Kinetic dissipation

Plasma rotation

The importance of eigenfunction modification and Alfven resonances at rational surfaces will come out of code benchmarking.

How can we approximate eigenfunction modification in MISK with an iterative approach?

1) We take the eigenfunction from the PEST marginally-stable eigenfunction: γ -> 0, δI -> 0, $\delta W_F = -\delta W_V$ (at the marginal wall position).
2) We can calculate a fluid growth rate with that fixed eigenfunction, the true wall position, and the assumption that the inertial term is still negligible.

$$\gamma_F \tau_w = -\delta W_\infty / \delta W_b$$

3) We can calculate a kinetic growth rate and mode rotation frequency by including kinetic effects and anisotropy corrections, but still assuming a fixed eigenfunction and negligible inertial term.

$$(\gamma - i\omega_r)\tau_w = -\frac{\delta W_\infty + \delta W_K}{\delta W_b + \delta W_K}$$

4) We can even iterate for corrected γ and ω_r, and solve for multiple roots. [J. Berkery et al., Phys. Plasmas **18**, 072501 (2011)]
5) Keeping the eigenfunction fixed, we could additionally try to include the inertial term (which actually involves multiple roots as well):

$$(\gamma - i\omega_r)\tau_w = -\frac{\delta W_\infty + \delta W_K + \delta I}{\delta W_b + \delta W_K + \delta I}$$

$$\delta I = \frac{1}{2}\sum_j \int \rho_0(\gamma + i(n\omega_\phi - \omega_r))(\gamma + i(n\omega_\phi - \omega_r) + i\omega_{*j})\left|\boldsymbol{\xi}_\perp\right|^2 d\mathbf{V}$$

6) Now, if the eigenfunction is allowed to change, how do we solve for a new one?

Rotation and kinetic damping may also affect the ideal no-wall stability limit

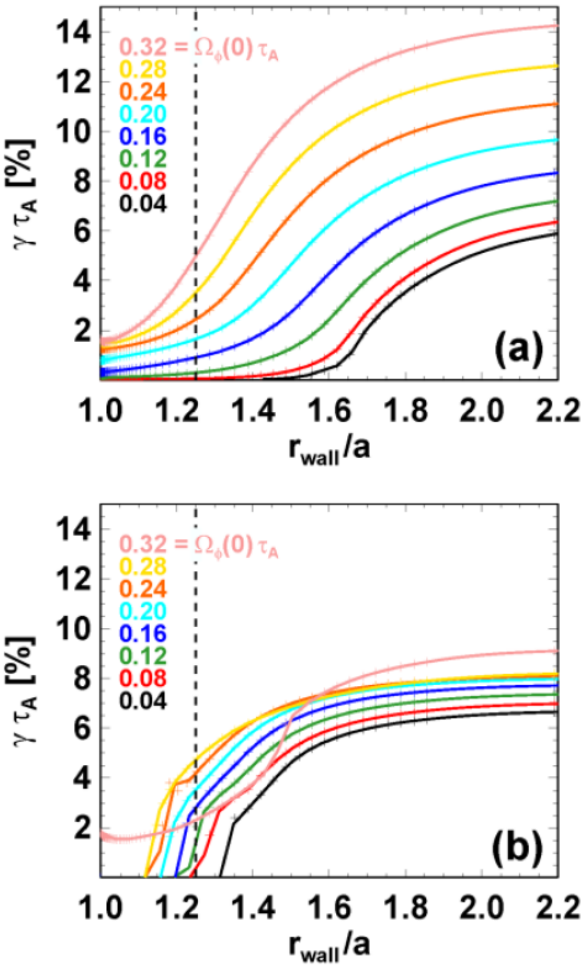


Figure 1 – *Growth rate of n=1 instability versus wall position and rotation (a) without and (b) with kinetic dissipation.*

[J. Menard and Y.Q. Liu, EPS 2012]

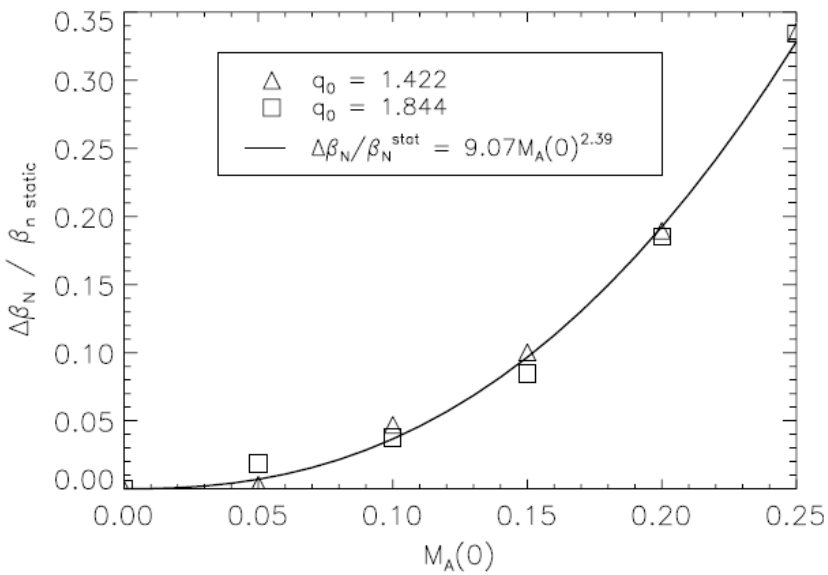


Figure 4. The fractional increase in β-limit versus $\mathcal{M}_A(0)$ for different q_0.

[I. Chapman *et al.*, Plasma Phys. Control. Fusion **53** 125002 (2011)]

Finite Larmor Radius Effects

Important Physics Is Given by the Evaluation of the Orbit Integral.

The general solution of the linearized Vlasov equation including zeroth and first order gyrophase-dependent terms is written as

$$\tilde{f} = -\tilde{\boldsymbol{\xi}}_\perp \cdot \nabla f + Z_j e \frac{\partial f}{\partial \mathscr{E}} \tilde{\zeta} + i m_j \left(\omega \frac{\partial f}{\partial \mathscr{E}} - \frac{\partial f}{\partial P_\phi} \right) [\tilde{\boldsymbol{\xi}}_\perp \cdot \mathbf{v} - \tilde{\eta}] + \frac{\partial f}{\partial \mu} \left(\tilde{\boldsymbol{\xi}}_\perp \cdot \nabla \mu - \frac{M(\tilde{\boldsymbol{\xi}}_\perp)}{B} \right)$$

$$\begin{aligned}
\tilde{\eta} = &-\frac{v_\parallel}{\Omega_{cj}} [\hat{b} \times \mathbf{v}_\perp \cdot (\hat{b} \cdot \nabla) \tilde{\boldsymbol{\xi}}_\perp + \hat{b} \cdot (\hat{b} \times \mathbf{v}_\perp) \cdot \nabla \tilde{\boldsymbol{\xi}}_\perp] \\
&+ \frac{1}{4\Omega_{cj}} [\mathbf{v}_\perp \times \hat{b} \cdot \mathbf{v}_\perp \cdot \nabla \tilde{\boldsymbol{\xi}}_\perp + \mathbf{v}_\perp \cdot (\mathbf{v}_\perp \times \hat{b}) \cdot \nabla \tilde{\boldsymbol{\xi}}_\perp] - \frac{v_\perp^2}{2\Omega_{cj}} \mathbf{B} \cdot \nabla \times \left(\frac{\tilde{\boldsymbol{\xi}}_\perp}{B} \right) \\
&+ \int_{-\infty}^{t} dt' \left\{ \left(\frac{v_\perp^2}{2} - v_\parallel^2 \right) \boldsymbol{\kappa} \cdot \tilde{\boldsymbol{\xi}}_\perp + \frac{v_\perp^2}{2} \nabla \cdot \tilde{\boldsymbol{\xi}}_\perp - \frac{Z_j e}{m_j} \tilde{\zeta} \right. \\
&\left. \qquad + \frac{v_\parallel v_\perp^2}{2\Omega_{cj}} \nabla \cdot \left[\frac{1}{2} (\hat{b} \cdot \nabla \ln B)(\hat{b} \times \tilde{\boldsymbol{\xi}}_\perp) - \tilde{\boldsymbol{\xi}}_\perp \times \hat{b} \cdot \nabla \hat{b} \right] \right\}
\end{aligned}$$

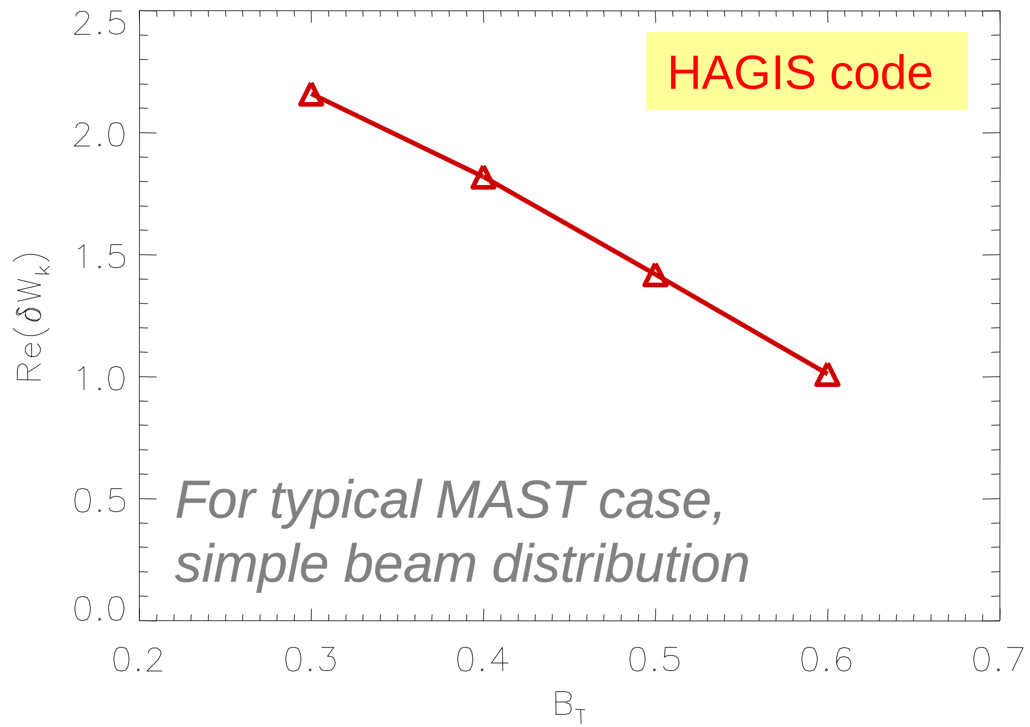
FLR effects

large for MHD modes at rational surfaces

PPPL THEORY

Finite Orbit Width Kinetic damping of RWMs

- Orbit widths can be very important for fast ions
- Use guiding-centre following code to capture this physics



- RWM passively stable in ITER Advanced Scenario due to kinetic damping
- Only capture these effects by including orbit widths
- Sensitive to rotation, so should not be relied upon!

Finite Orbit Width

Ian Chapman email:

"I need to talk to YQ and Jon Graves who probably have insight from the algebraic formulation with FOW included that they're working on."

IOP PUBLISHING PLASMA PHYSICS AND CONTROLLED FUSION

Plasma Phys. Control. Fusion **52** (2010) 055004 (19pp) doi:10.1088/0741-3335/52/5/055004

Neoclassical ion heat flux and poloidal flow in a tokamak pedestal

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