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Bispectral Analysis of the L-H transition as seen in the NSTX GPI data

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Bispectral Analysis

- Bispectral analysis is a useful technique for studying systems that contain a quadratic nonlinearity -- the simplest example for plasma physics would be the Navier-Stokes equation.

- The bicoherence, a measure of the amount of phase coherence between three Fourier modes in a system, was applied to plasma physics for the study of drift wave turbulence by Kim and Powers in 1979 and has been used by many researchers since then.

- The bicoherence can be calculated using an FFT method, as was done in this project. One can also use wavelet analysis (Van Milligan 1995)

Bispectral Analysis

Bispectrum

$$B(f_1, f_2) =< \Phi(f_1)\Phi(f_2)\Phi^*(f_3) > \qquad f_1 + f_2 = f_3$$

Squared Bicoherence

$$b^2(f_1, f_2) = \frac{|B(f_1, f_2)|^2}{< |\Phi(f_1)\Phi(f_2)|^2 >< |\Phi(f_3)|^2 > +\epsilon} \qquad 0 \leq b^2(f_1, f_2) \leq 1$$

The bicoherence is a measure of phase coherence between three Fourier modes in the system

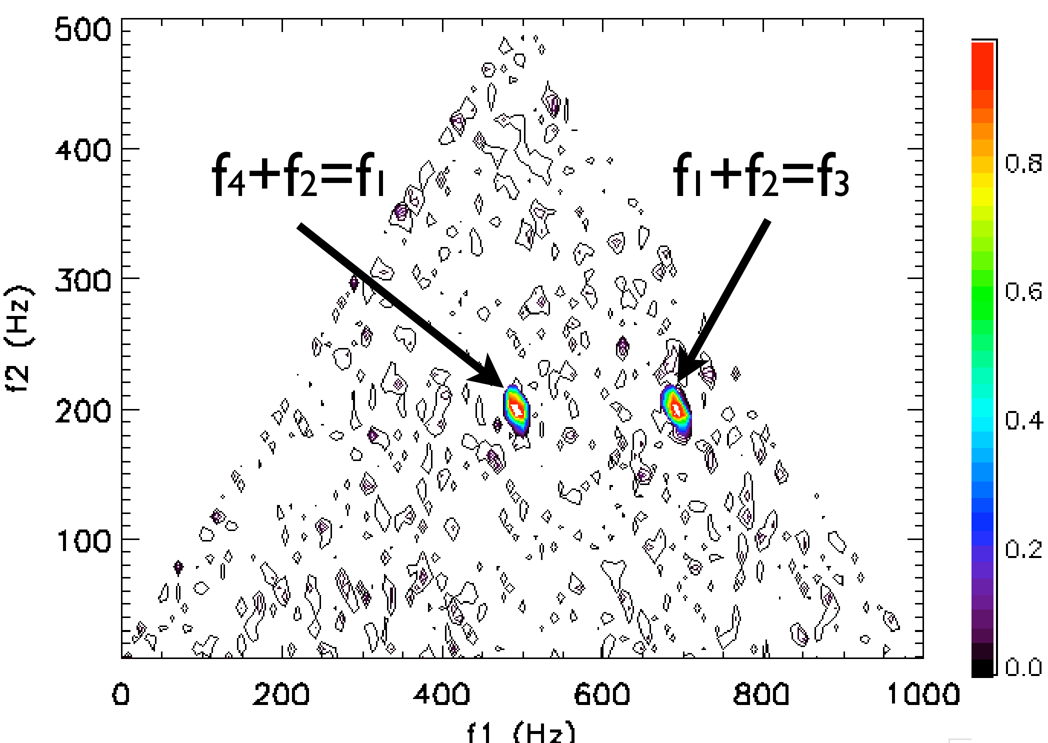
Interpretation of $b^2(f_1, f_2)$ contour plots

$$g(t) = sin(f_1 t + \phi_1) + sin(f_2 t + \phi_2) + sin(f_3 t + \phi_3) + sin(f_4 t + \phi_4)$$

$$f_1 + f_2 = f_3 \qquad f_1 - f_2 = f_4 \qquad \phi_{1,2} = random[-\pi, \pi]$$

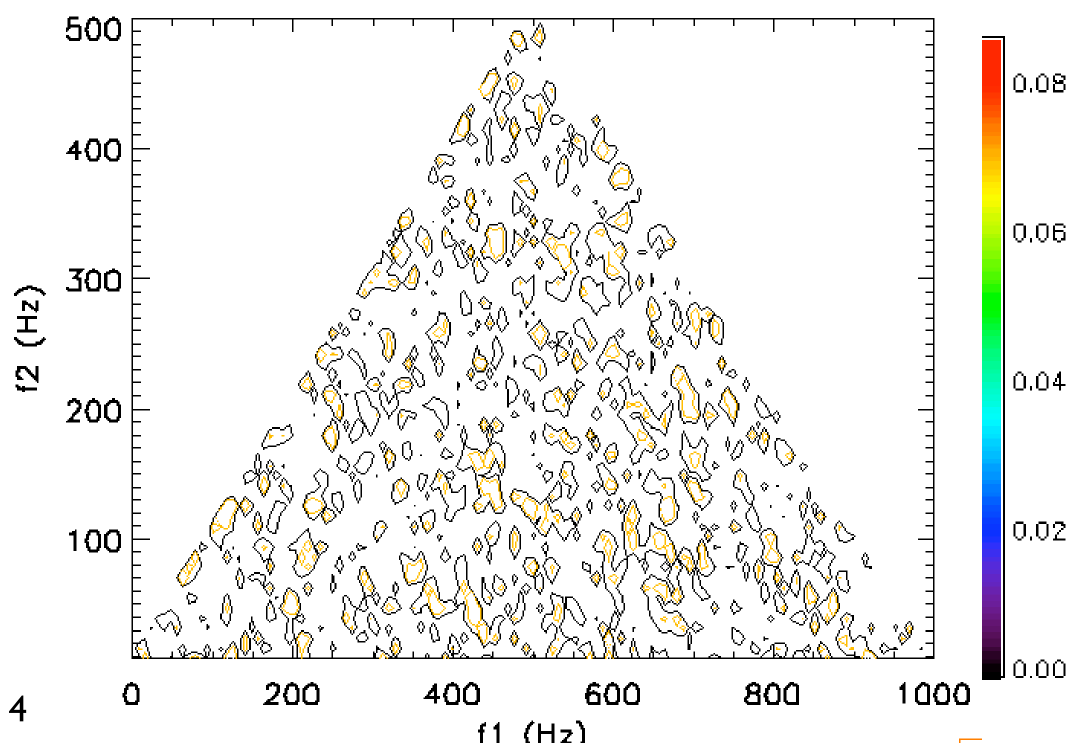
Coupled

$$\phi_1 + \phi_2 = \phi_3$$
$$\phi_1 - \phi_2 = \phi_4$$



Uncoupled

$$\phi_3 = random[-\pi, \pi]$$
$$\phi_4 = random[-\pi, \pi]$$



Interpretation of bicoherence figures

Summed bicoherence gives a measure of the strength of the coupling at the sum frequency f_3 with respect to all other sum frequencies

$$b^2(f_3) = \sum_{f_1+f_2=f_3} b^2_{f_3}(f_1, f_2)$$

Total bicoherence gives a measure of the amount of coupling in the signal summed over all frequencies

$$b^2 = \sum_{f_1}\sum_{f_2} b^2_{f_3}(f_1, f_2)$$

Definitions given by Van Milligan (1995) and Tynan (2001)

Background and Motivation

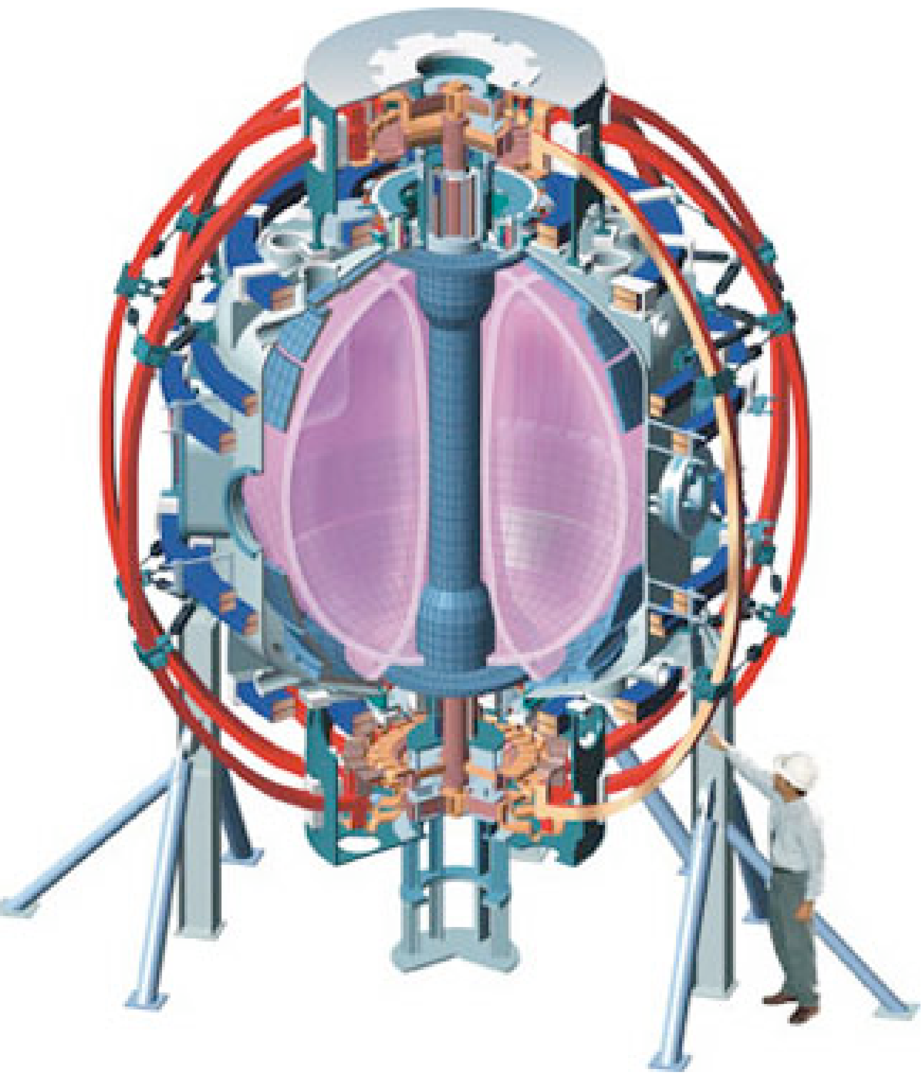
- Present understanding of the L-H transition says that the turbulence is sheared by poloidal plasma flows, -- breaking it up, causing the transient decrease in fluctuation amplitude -- this leads to the transport barrier

- One type of model for how this shearing occurs maintains that the *turbulence* generates the mean poloidal flows (shear/zonal flows) which break up the turbulence -- Reynold’s stress driven flow hypothesis.

- Using a single point time-series measurement of the turbulence, the auto-bicoherence can be calculated in order to search for this coupling between low frequency fluctuations (associated with shear/zonal flows) and high frequency turbulent fluctuations. (Tynan 2001)

- An increase in the coupling between low and high frequencies immediately prior to the L-H transition has been observed in previous work on DIIID (Moyer 2001)

National Spherical Torus Exp't (NSTX)



typical parameters

Global

R=0.85

a=0.68

B=0.3 T

I ≈ 0.8 MA

P ≈2-4 MW

$\beta_T \approx 10\%$

Outer Edge

n ~ 0.2-2x10^{13} cm^{-3}

$T_e \sim$ 5-50 eV

$L_\perp \sim$ 3-6 cm

$L_{||} \sim$ 5 m

$\rho_s \sim$ 0.2 cm

$\beta_e \sim 10^{-3}$

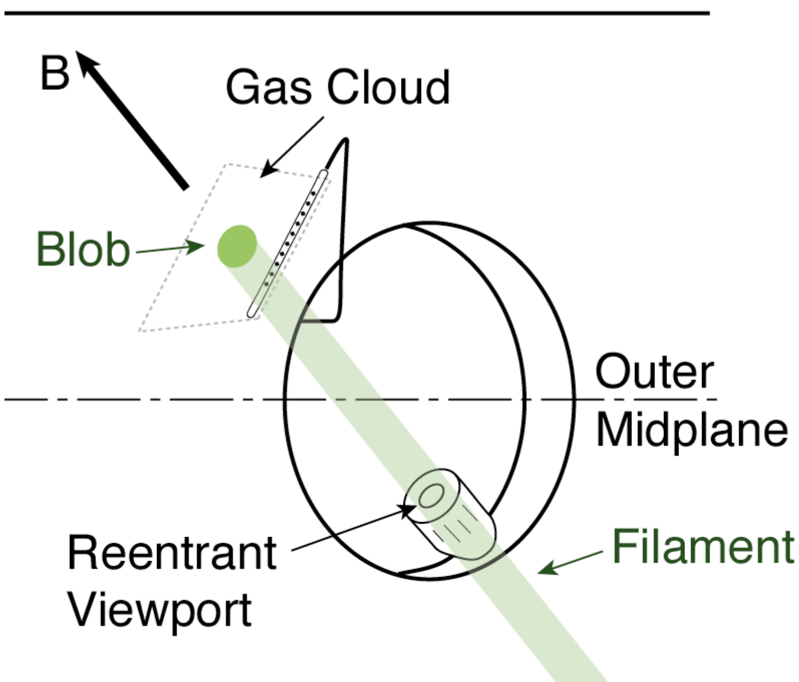
NSTX edge plasma is similar to most other tokamak edge plasmas

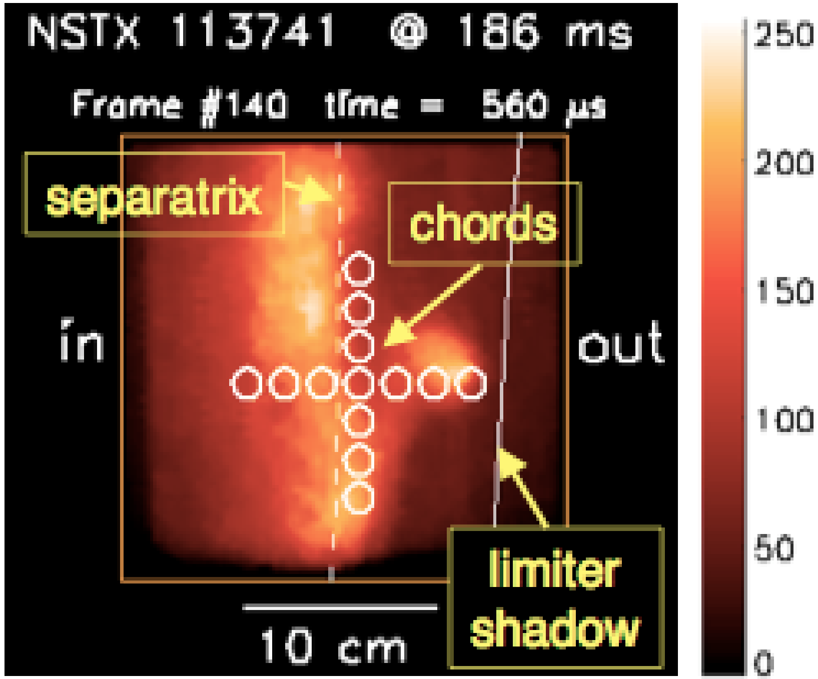
Slide content from S. Zweben APS 2005

Gas Puff Imaging (GPI) on NSTX

- Looks at D_α or HeI light from gas puff $I \propto n_o n_e f(n_e,T_e)$
- View ≈ along B field line to see 2-D structure ⊥ B
- 2-D video data from as intensified ultra-fast camera.
- Time-series data from an array of 13 discrete chords digitized at 500kHz

→ ***Use the chord data for bispectral analysis***





Figures from S. Zweben APS 2005

Viewing region of GPI chord data is near the separatix

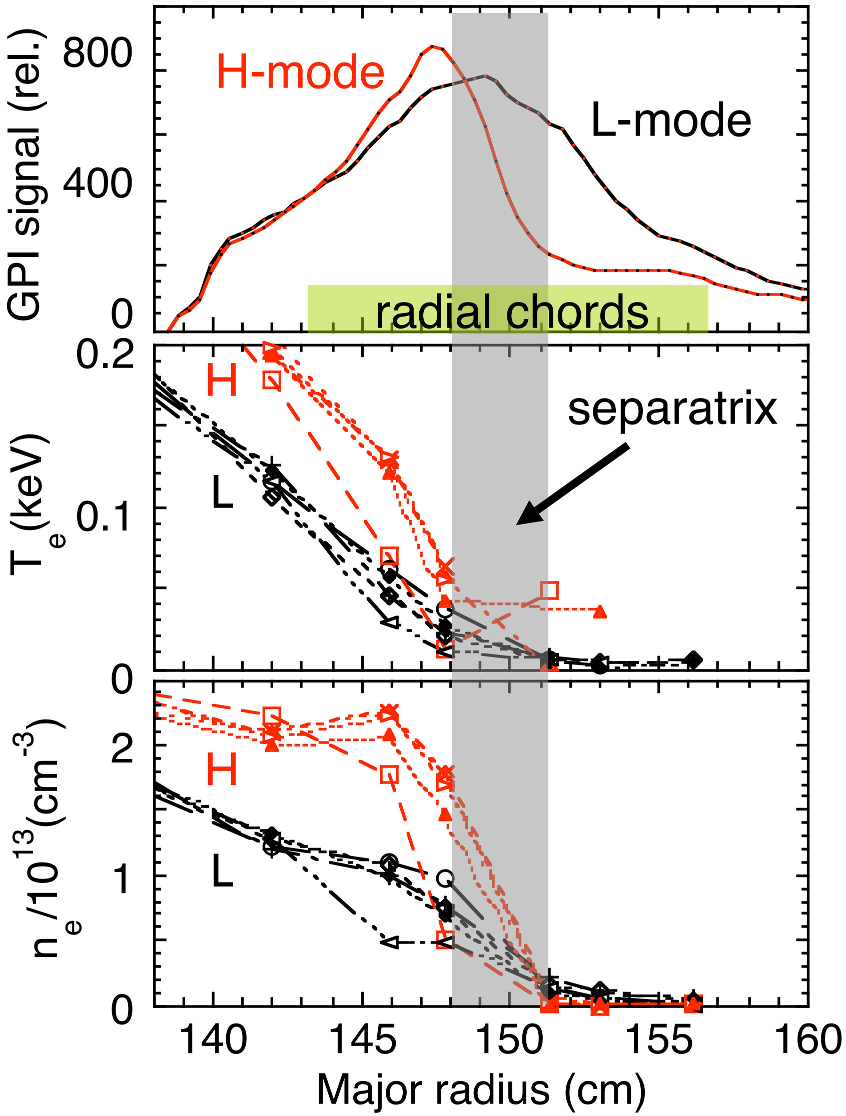


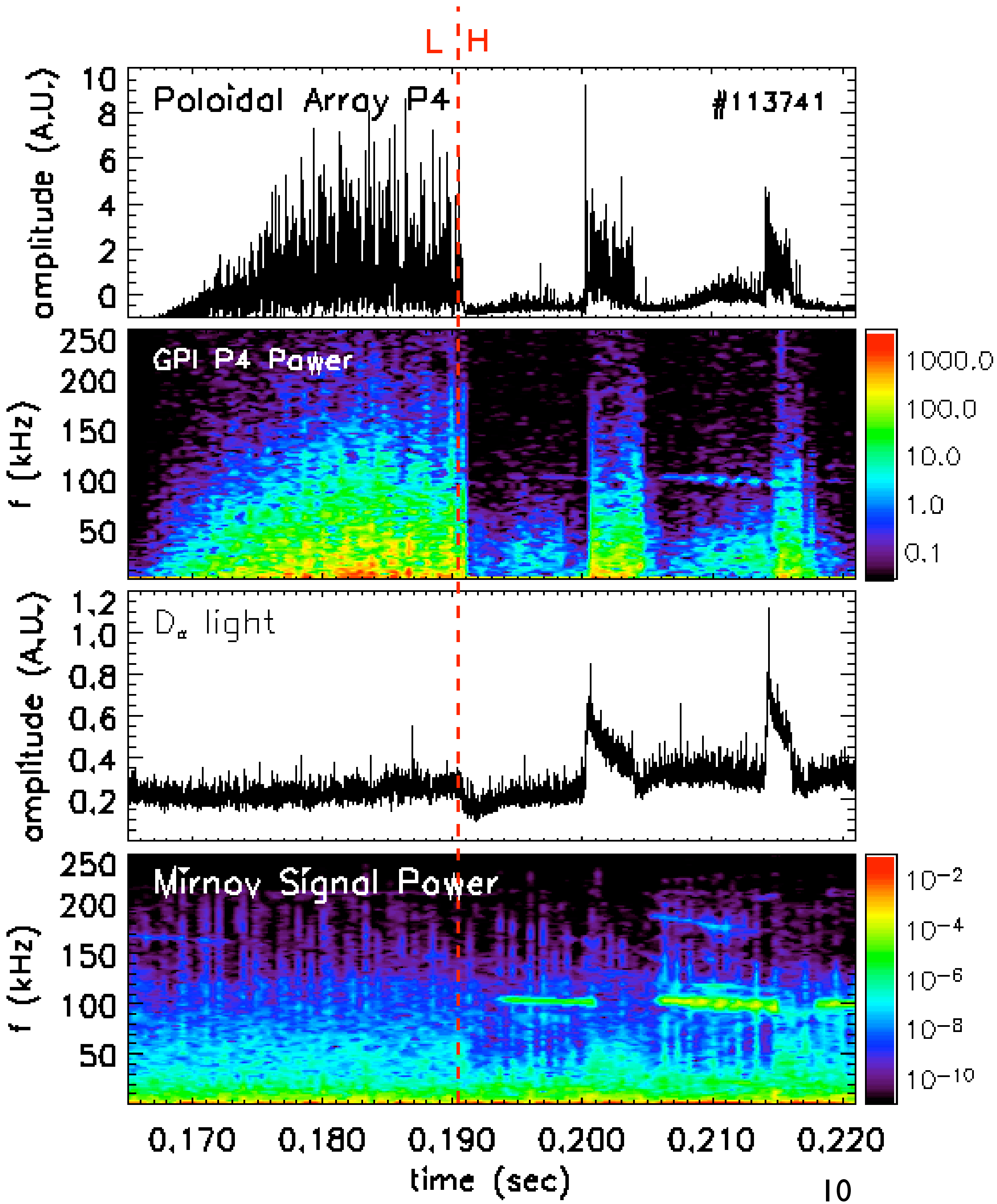
Figure from S. Zweben APS 2005

The GPI poloidal detector array on NSTX was located at R=149 cm

Increase in nonlinear coupling observed prior to the L-H transition in previous work on DIIID occurred just inside the separatrix*

*R.A. Moyer et al "Increased Nonlinear Coupling between Turbulence and Low-Frequency Fluctuations at the L-H Transition" PRL 87, 135001, (2001)

GPI Chord Data



Fluctuation amplitude decreases ~ 100 microsec after the L-H transition

GPI spectrum exhibits broadband turbulence

Long-lived coherent mode only shows up in H-mode -- identifiable as an MHD mode

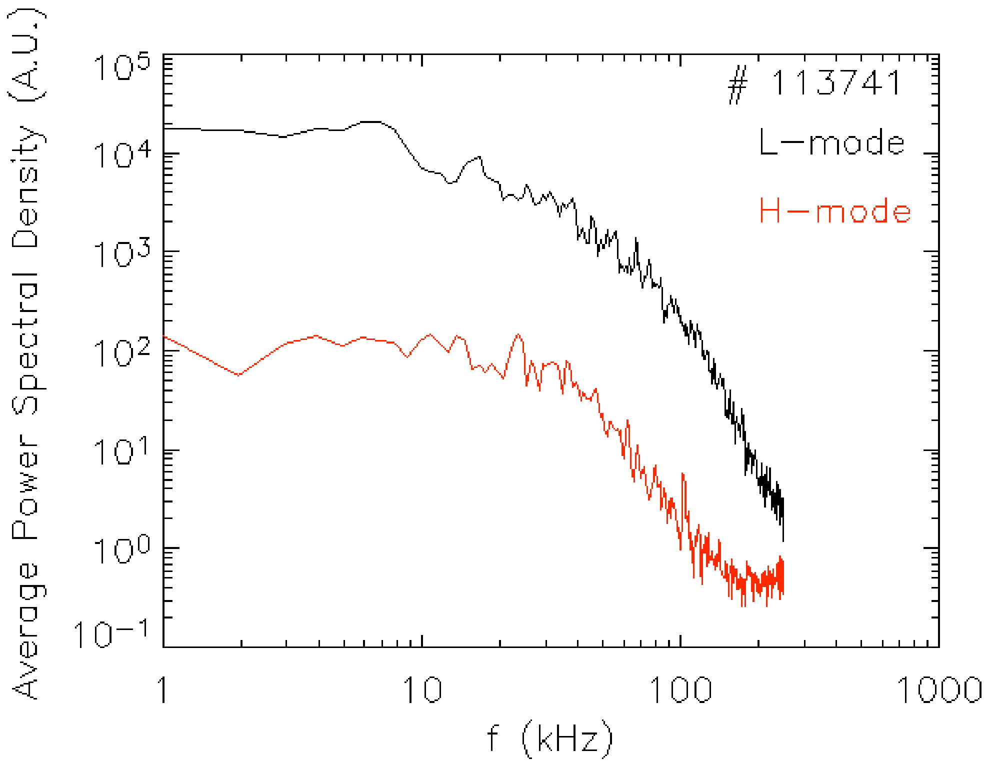
Turbulence properties such as correlation length can be studied across the L-H transition*

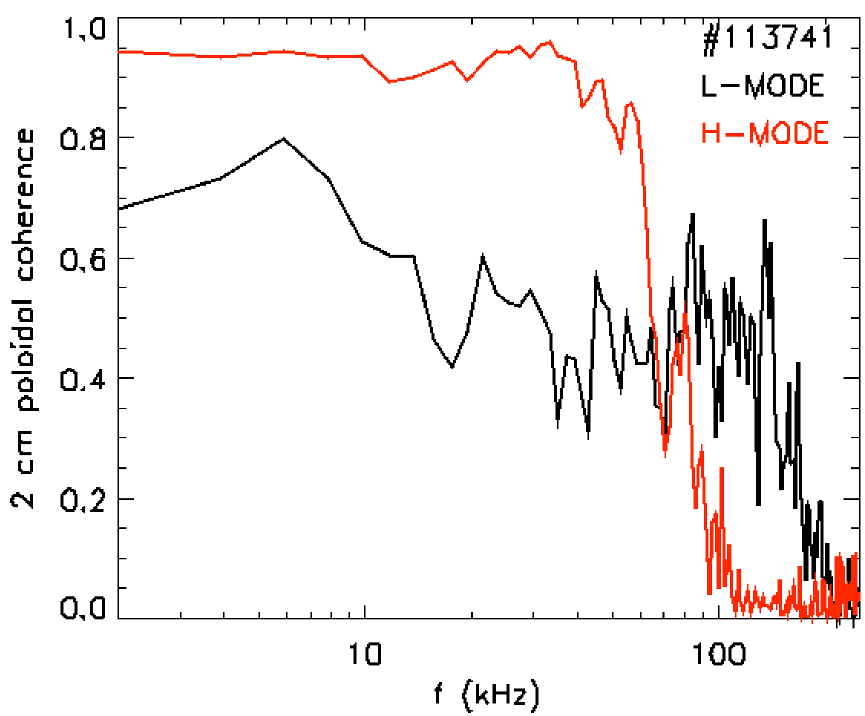
*S. Zweben Invited Talk APS 2005

Linear spectral analysis of NSTX GPI chord data

Average power spectra show lower amplitude during the H-mode

Coherence between poloidally separated detectors shows that coherence increases for 10 < f <100 kHz and decreases for f >100 kHz during the H-mode





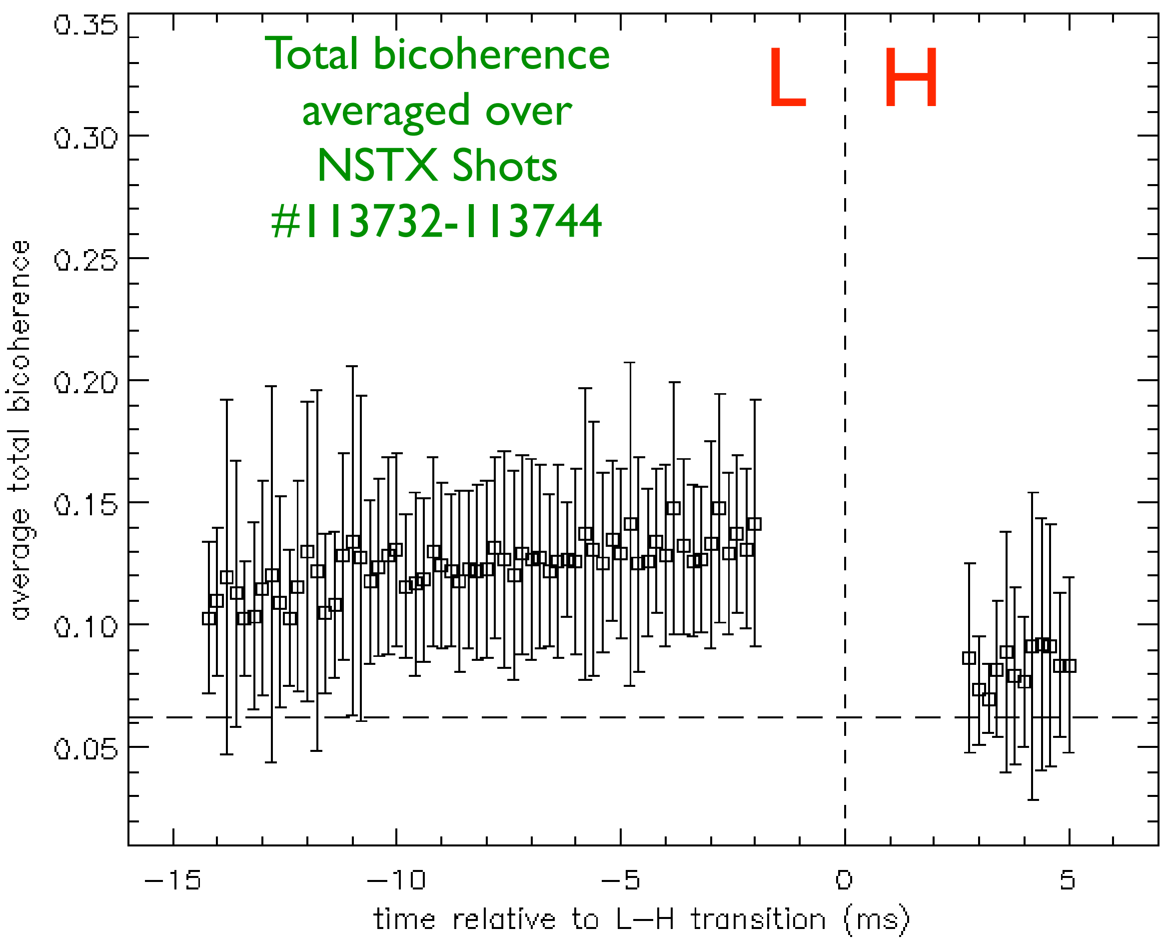
Application of bispectral analysis to NSTX GPI chord data

- Calculate the total bicoherence before and after the L-H transition --- *Does the total amount of coupling change in time?*

- Calculate profiles of the bicoherence from the GPI chord data ---*Does the amount of coupling vary radially and poloidally?*

- Calculate the squared bicoherence and the summed bicoherence -- *What frequencies take part in the coupling?*

In NSTX GPI chord data the amount of coupling *does not increase* significantly prior to the L-H transition



Averaging over 13 shots with an L-H transition shows no significant increase in total bicoherence within 15 msec before the transition.

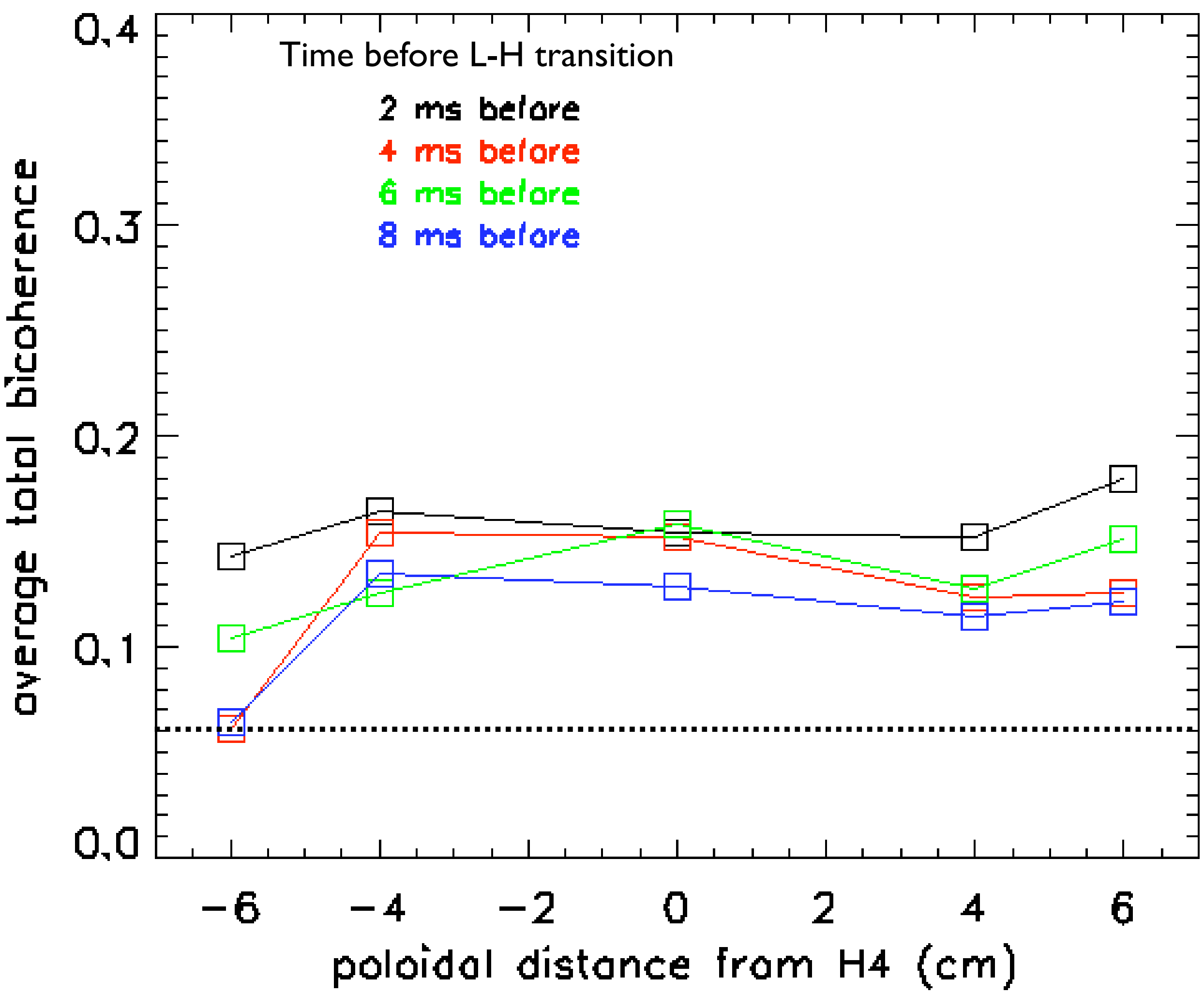
Error bars represent standard error in the mean total bicoherence calculated from 13 shots

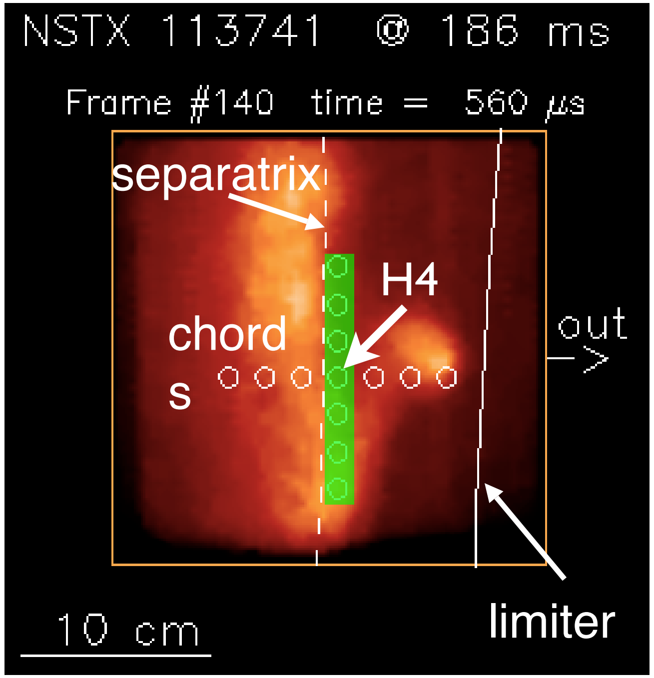
Horizontal line is statistical significance level = 0.0625

Last data point is at ~2ms before transition because data interval over which bicoherence is calculated is ~4ms long and ends at the transition time

~4ms windows (2048 points) were divided up into 16 records of 128 points for calculation of each value in plot

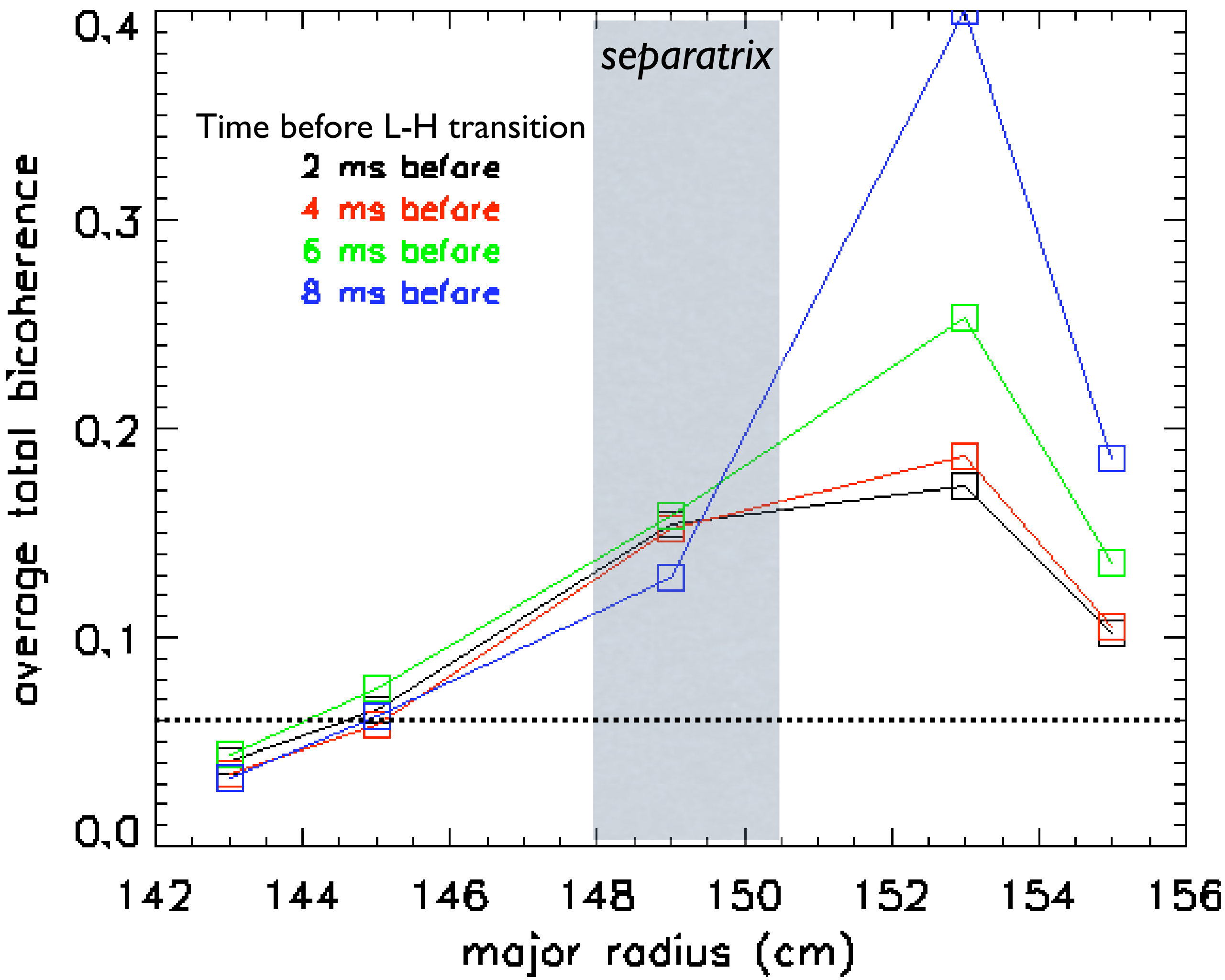
Amount of coupling in L-mode does not vary greatly with poloidal position

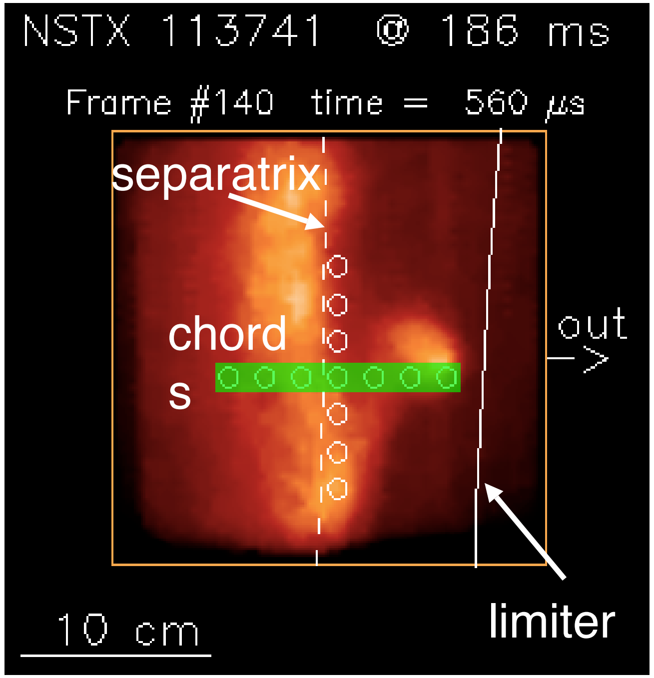




NSTX Shots
113737
113739
113741
113743
113744

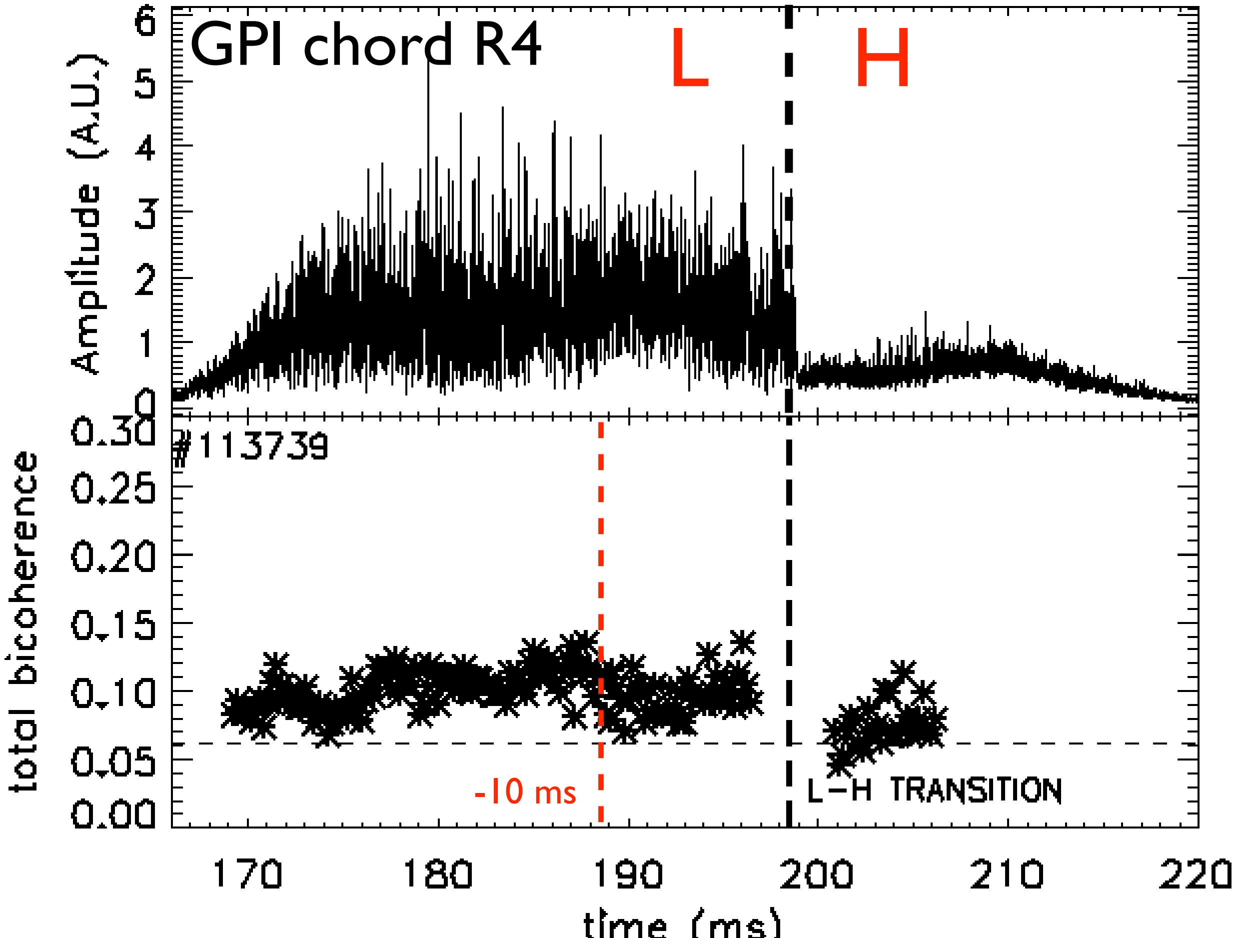
Amount of coupling in L-mode increases with radial position





NSTX Shots
113737
113739
113741
113743
113744

Most shots exhibited no increase in the bicoherence before the L-H transition

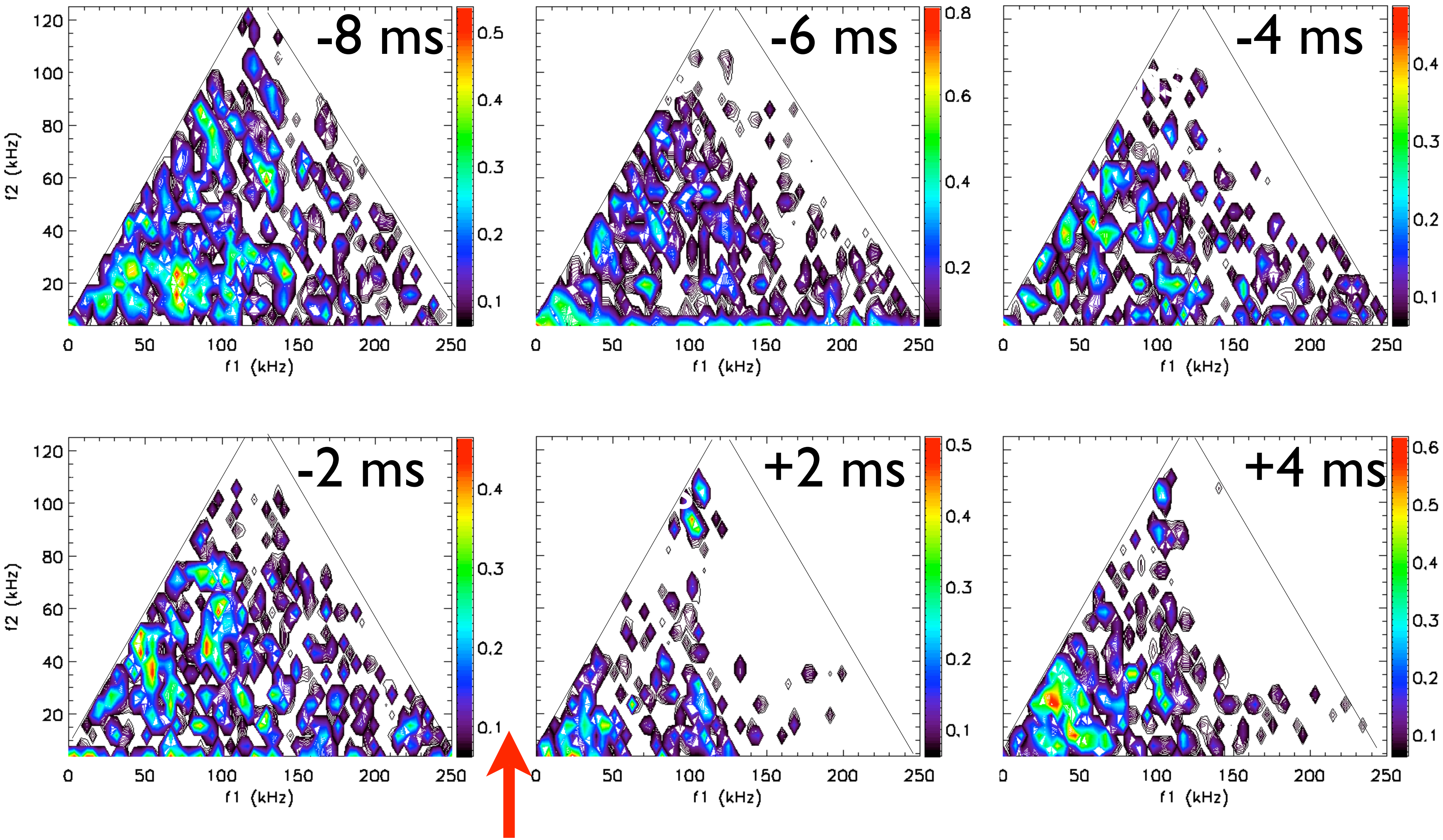


R4 chord view is the center view in the array and is nearest the separatrix

For most shots,
no increase in the total amount of coupling is seen before the transition

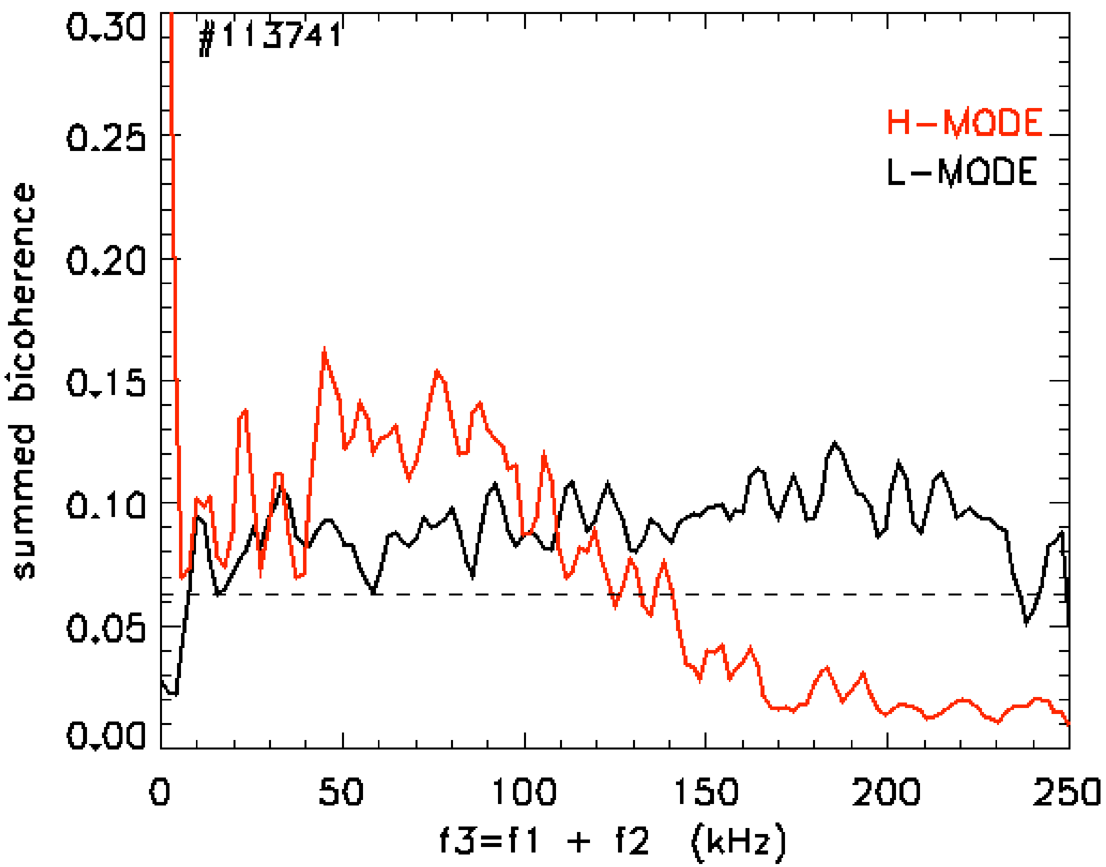
During the H-mode the coupling is localized mainly to low sum frequencies

#113739



L-H TRANSITION

The summed bicoherence indicates that coupling shifts to low sum frequencies following the L-H transition



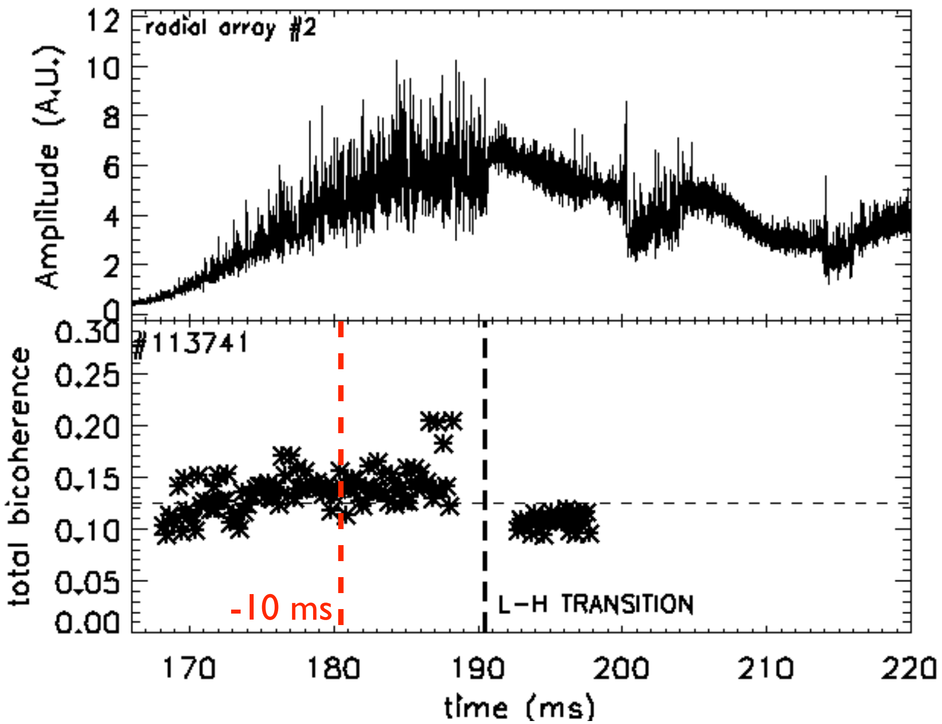
Increased bicoherence at low sum frequencies f3 indicates

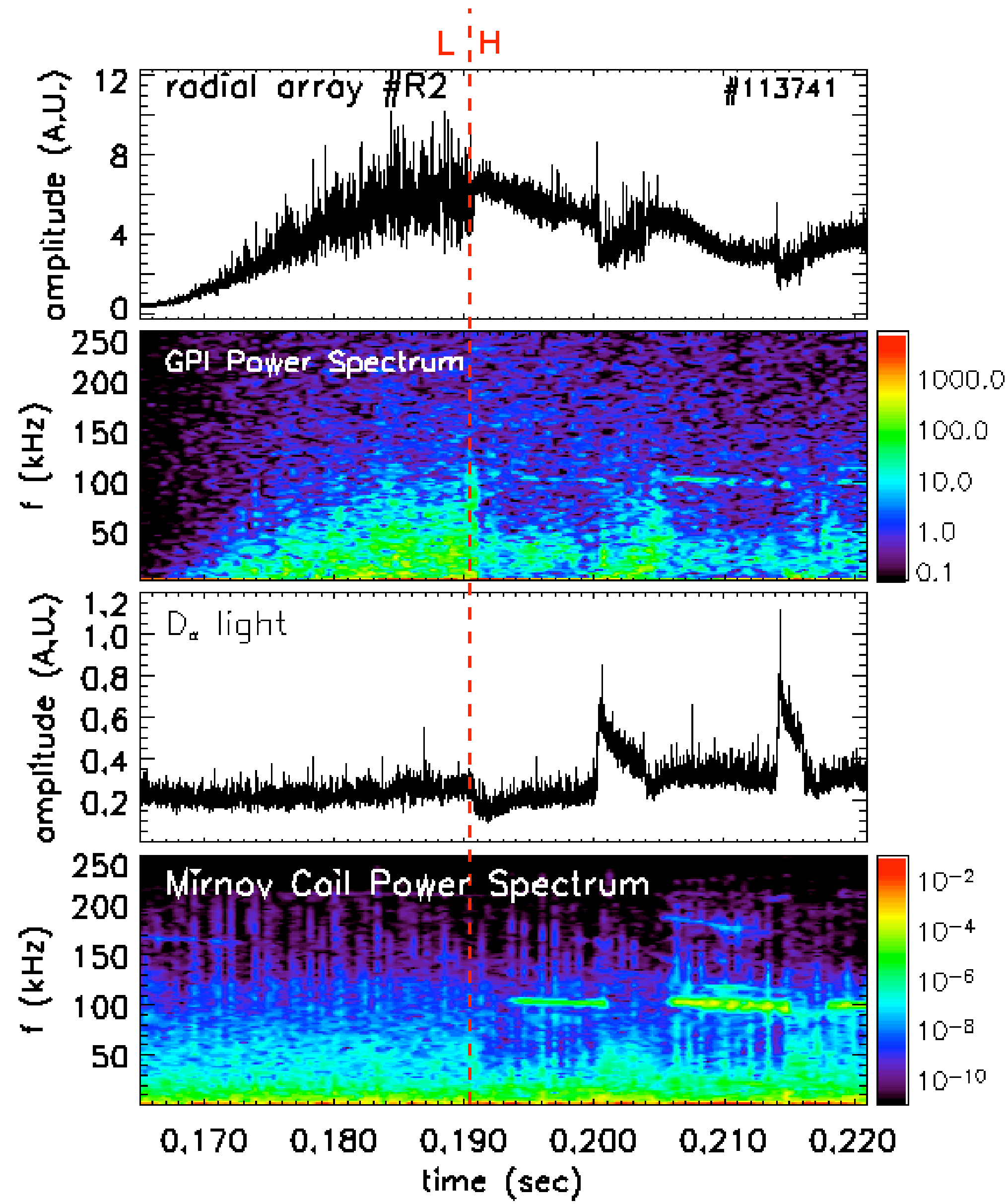
difference coupling between a low and a high frequency

or

sum coupling between two low frequencies

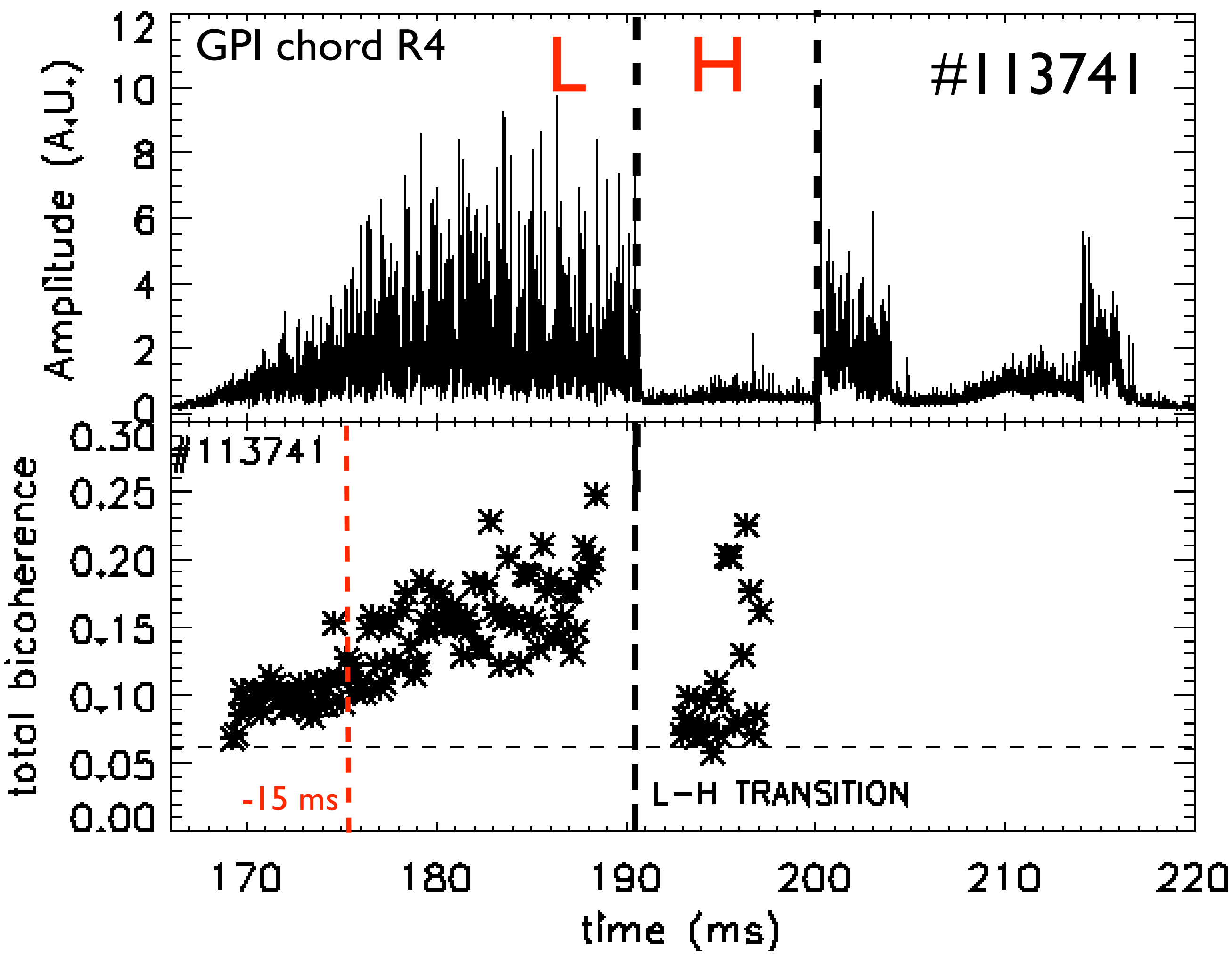
The total amount of coupling decreases further inside the separatrix





R2 chord view is located 4 cm further into plasma than R4

Some shots exhibited a very slight increase in total bicoherence long before the transition



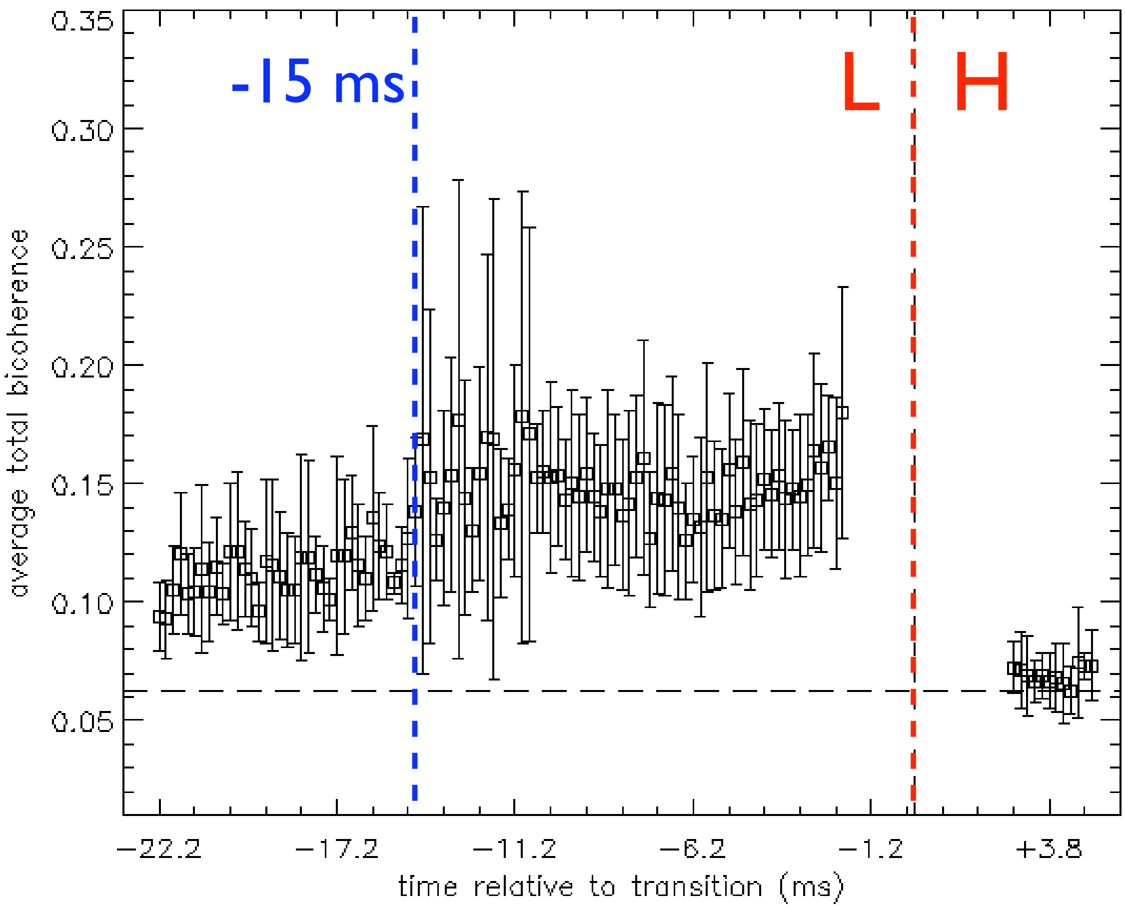
R4 chord view is the center view in the array and is nearest the separatrix

A factor of ~1.5 increase occurs nearly 15 ms before the transition in this shot

The scatter of the total bicoherence values also increases during this time

For the five NSTX shots with very stationary H-modes, the total bicoherence in the GPI chord data increases slightly about 15 msec before the transition

Total bicoherence averaged over 5 NSTX Shots: 113737, 113739, 113741, 113743, 113744

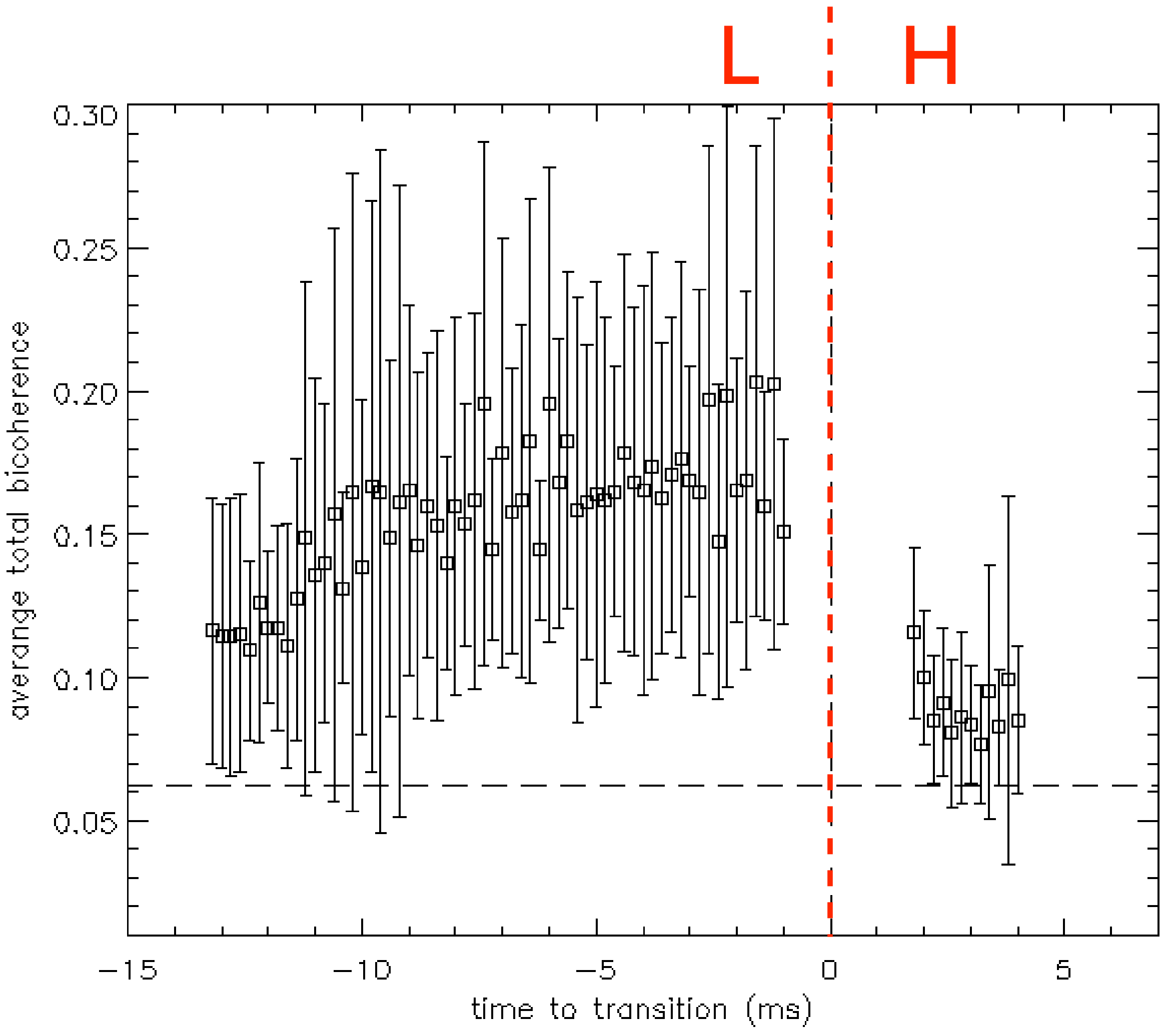


Averaging over 5 shots with the longer portions of data during the L-mode shows that a slight increase in the total bicoherence occurs nearly 15 msec before the transition.

The increase is less than a factor of 1.5 and appears to be within the standard error

No sudden and significant increase is seen just prior to the transition for any of the shots

Using smaller record lengths in FFT analysis in order to achieve better time resolution results in larger standard error for bicoherence results -- still no increase is seen just prior to the transition



Total bicoherence averaged over NSTX Shots #113732-113744

Averaging over 13 shots with an L-H transition shows no significant increase in total bicoherence just prior to the transition.

Error bars represent standard error in the mean total bicoherence calculated from 13 shots

Horizontal line is statistical significance level = 0.0625

Last data point is at ~1ms before transition because data interval over which bicoherence is calculated is ~2ms long and ends at the transition time

~2ms windows (1024 points) were divided into 16 records of 64 points each for calculation of each value in the plot

Summary of NSTX results and Conclusions

- Averaging over 13 NSTX shots indicates there is no significant increase in the amount of coupling immediately before the L-H transition

- During H-mode, coupling occurs mostly at low sum frequencies (f3 < 100 kHz)

- The amount of coupling is independent of poloidal position and increases with increasing radial position

- For a few shots a slight increase in the total bicoherence was seen *long before* the L-H transition (~15 msec before). The increase appears to be within the standard error.

- The amount of coupling typically decreases going from the L-Mode to the H-Mode

Correcting for low power in the signal

The low power correction is recommended by authors to minimize false positives in bicoherence results due to low power in the signal at any frequency.

$$b^2(f_1, f_2) = \frac{|B(f_1, f_2)|^2}{< |\Phi(f_1)\Phi(f_2)|^2 >< |\Phi(f_3)|^2 > +\epsilon}$$

Where $\epsilon = [min(< |\Phi(f)|^2 >)]^3$

see J.W.A. Fackrell *et al*, ***Applied Signal Processing***, 2, 186-199, (1995) and
W. B. Collis *et al*, ***Mechanical Systems and Signal Processing***, 12, 375-394 (1998)

Frozen Flow Hypothesis (FFH)

FFH maintains that the turbulence does not evolve significantly during the time it passes by two measurement points in the plasma. The propagation time $\delta t = d/v_\theta$ must be much less than the autocorrelation time τ_A of the turbulence

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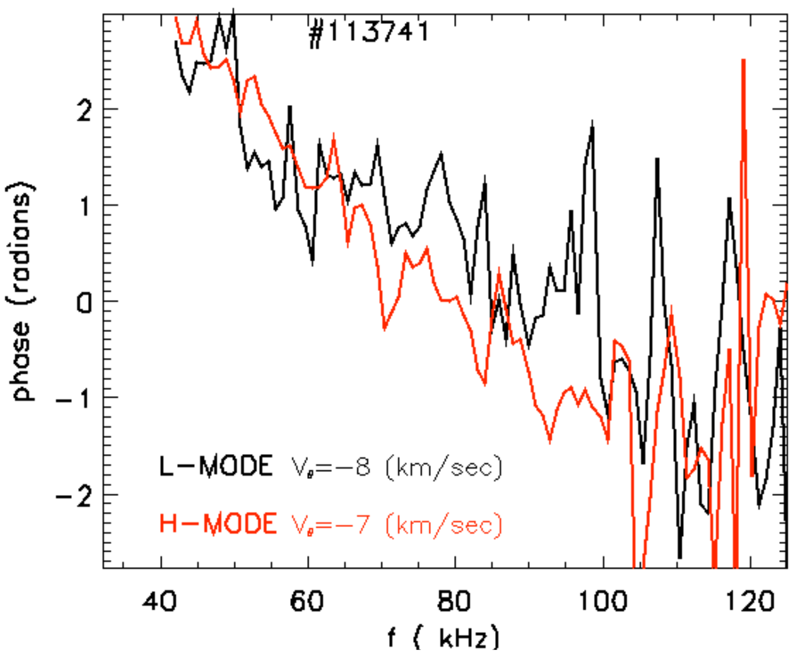
P7 and P4 are 6cm apart

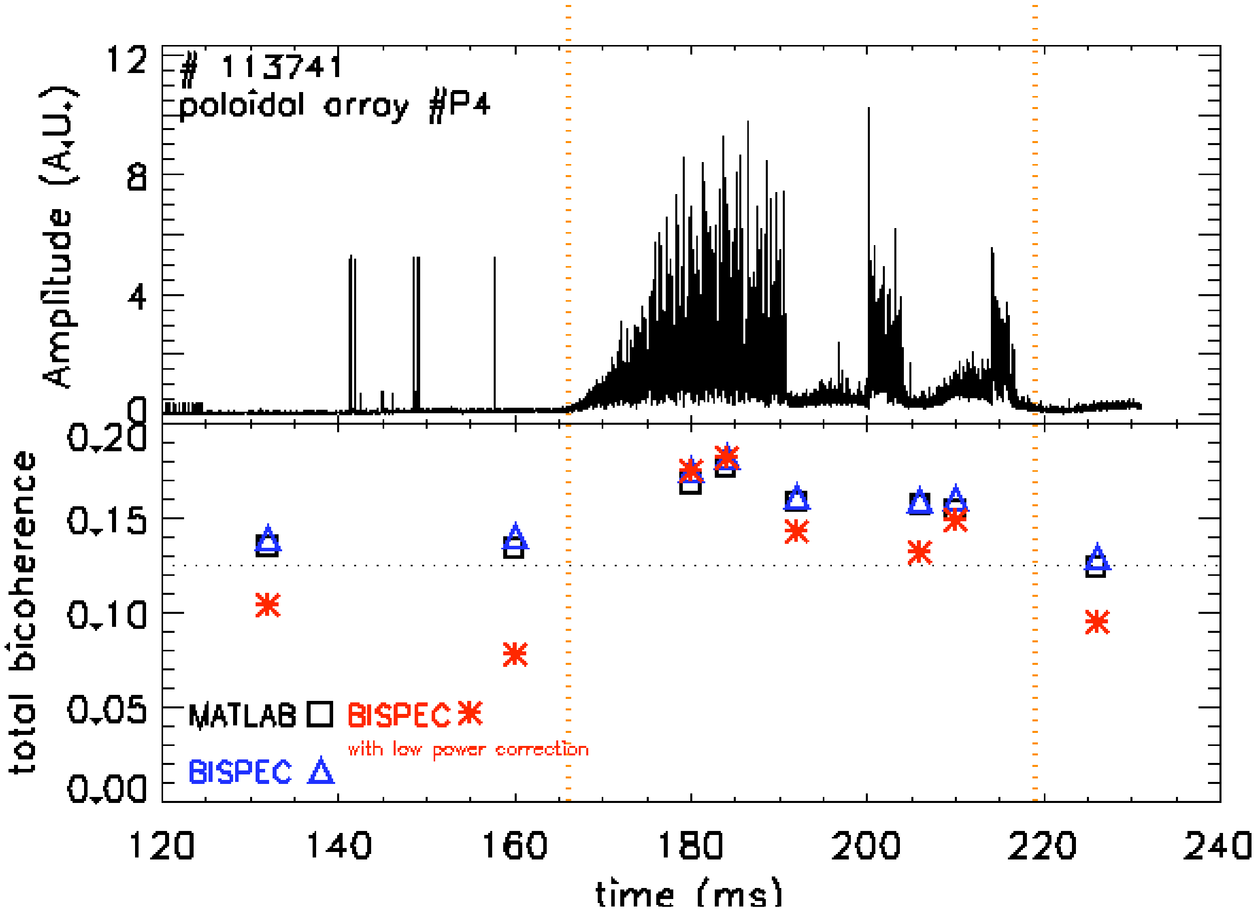


The FFH allows one to relate the wave number, ***k***, domain directly to the frequency domain when calculating the auto-bicoherence

Using 6 cm poloidally spaced chord data the propagation time is estimated to be
$\delta t \approx 7 - 9\mu sec$

The autocorrelation time of the turbulence is $\tau_A \approx 25 - 30\mu sec$

Bicoherence results from BISPEC code written in IDL agree well with the MATLAB routine



Without the low power correction BISPEC and MATLAB agree nearly perfectly.

The low power correction was used in NSTX data analysis presented in this poster because it reduces instances of false coupling

Identifying artificial results from bicoherence analysis

Type of problem with data	Result in bicoherence
Gradient in signal amplitude (delta function, square wave)	False positive for coupling mainly at low frequencies
RPA does not hold	False positive for coupling at the specific mode frequencies
Data did not have linear trend removed	False positive across all frequencies
Low power at any frequency in the signal	False positives appear as bands at k, across all l and at all k+l
Signal amplitude increasing in envelope	No false positives detected
FFT window too small, not enough frequency resolution	Peaks are broadened, false coupling at neighboring frequencies
Overlapping FFT records in ensemble average	violates RPA, leads to false coupling
Bent signal: mean is changing	False positives detected across all frequencies
SNR < 5	false negatives detected in perfectly coupled signals

Future Work

- Further investigation of bicoherence radial profile
- Apply wavelet analysis to compute the bicoherence of GPI data with better time resolution and compare to FFT method
- Compare bicoherence results with 2-D camera images.
- Work on theoretical/simulation predictions of spatial and temporal scales of coupling

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