Improved Mode Number Identification of Low-frequency MHD Activity in NSTX

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NSTX MHD Considerations

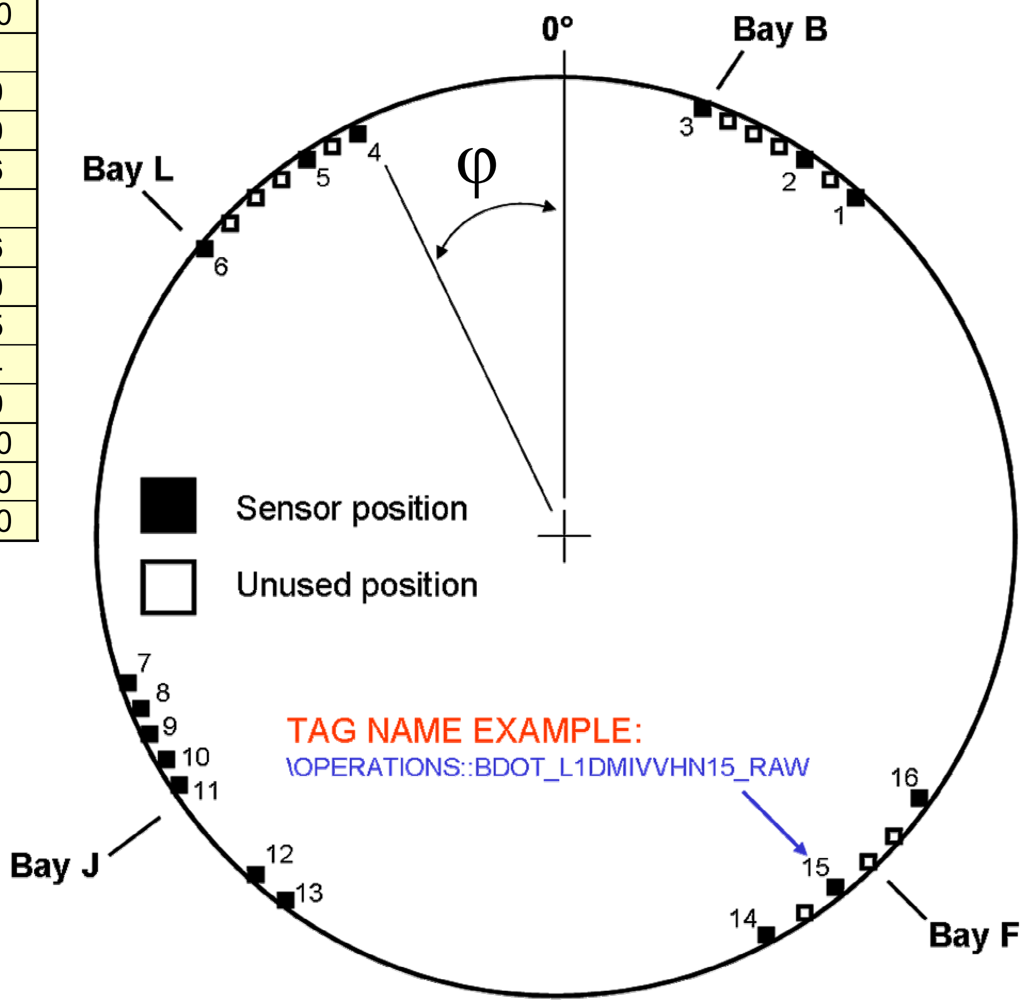
- Instabilities lead to the growth of MHD ‘modes’ – perturbations in the equilibrium magnetic configuration.
- If the plasma rotates, we can measure the perturbations with arrays of Mirnov coils positioned around the device.
- We have developed algorithms superior to FFT that calculate the temporal and spatial phases of the signals.
- The spatial phase can be characterized by the Fourier poloidal and toroidal mode numbers, m and n.
- The non-circular cross section of NSTX presents difficulties for determining m – thus we revert to numerical techniques.

Layout of Mirnov Coil Arrays

NSTX

INDEX	ANGLE
1	300.0
2	310.0
3	330.0
4	0.0
5	10.0
6	30.0
7	60.6
8	64.1
9	67.6
10	71.0
11	74.5
12	88.4
13	91.9
14	180.0
15	190.0
16	205.0

Toroidal Array



Poloidal Array

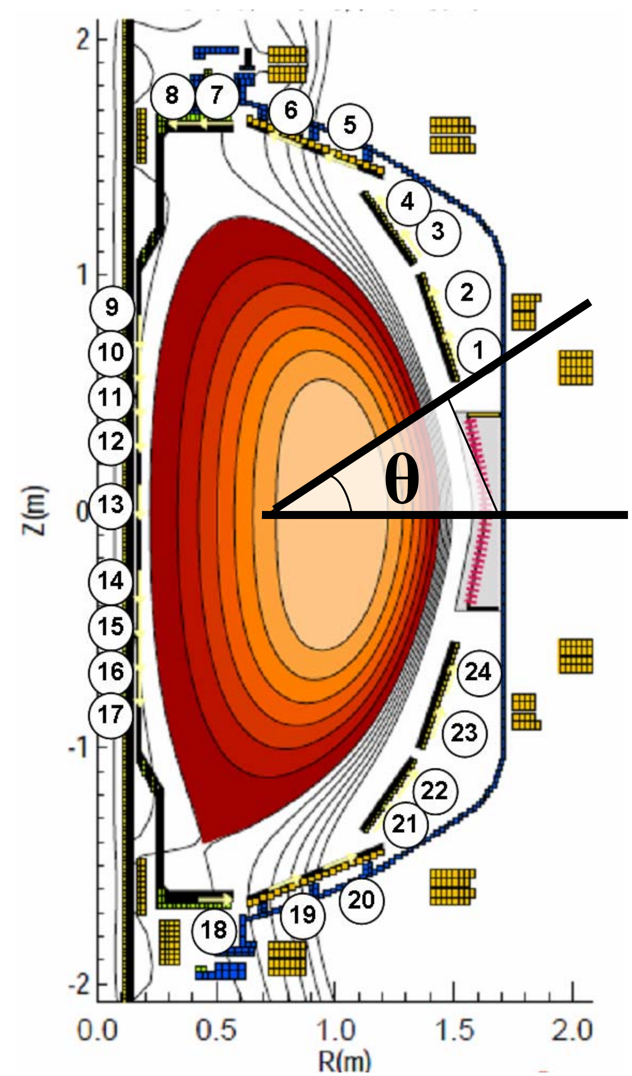
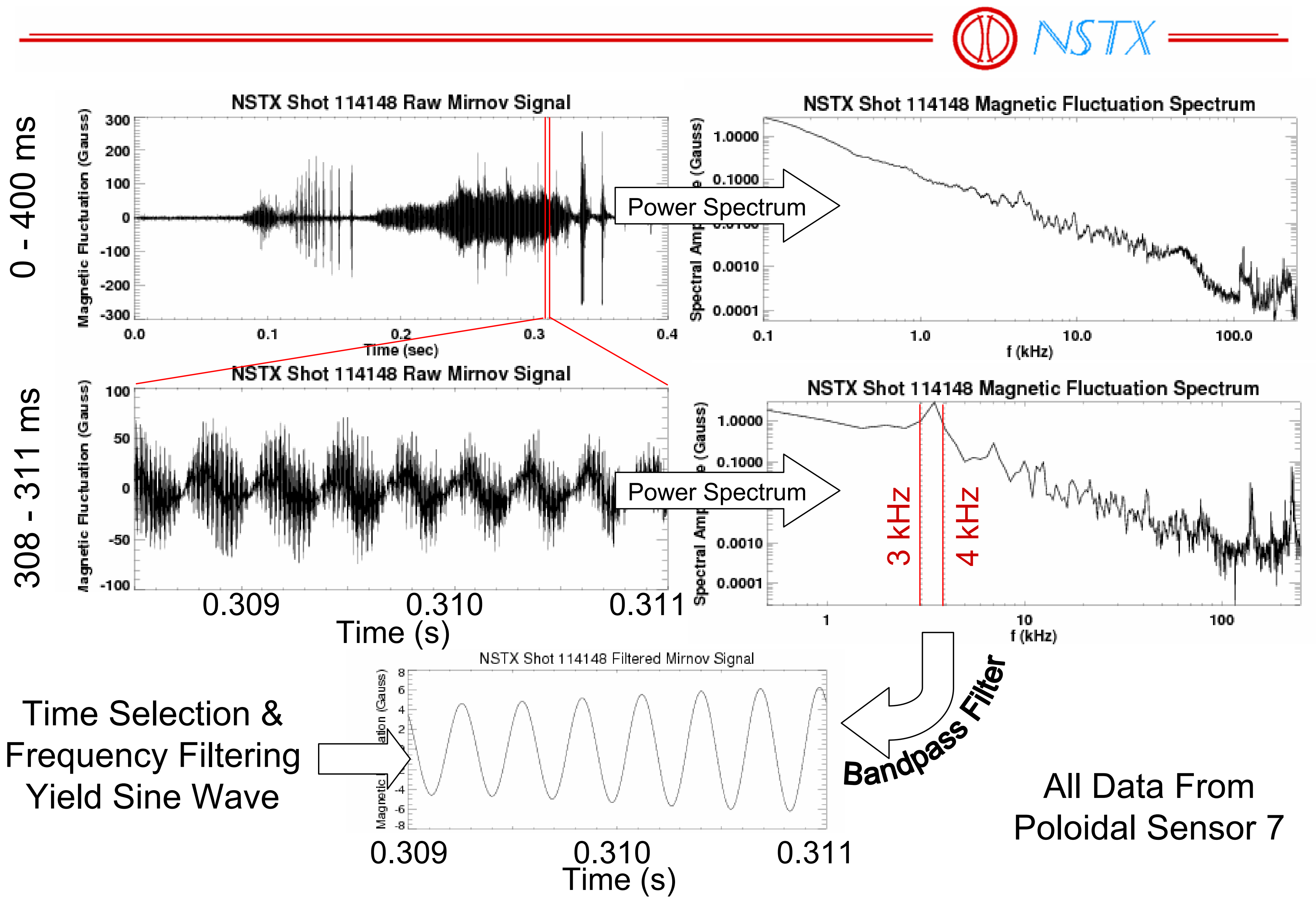


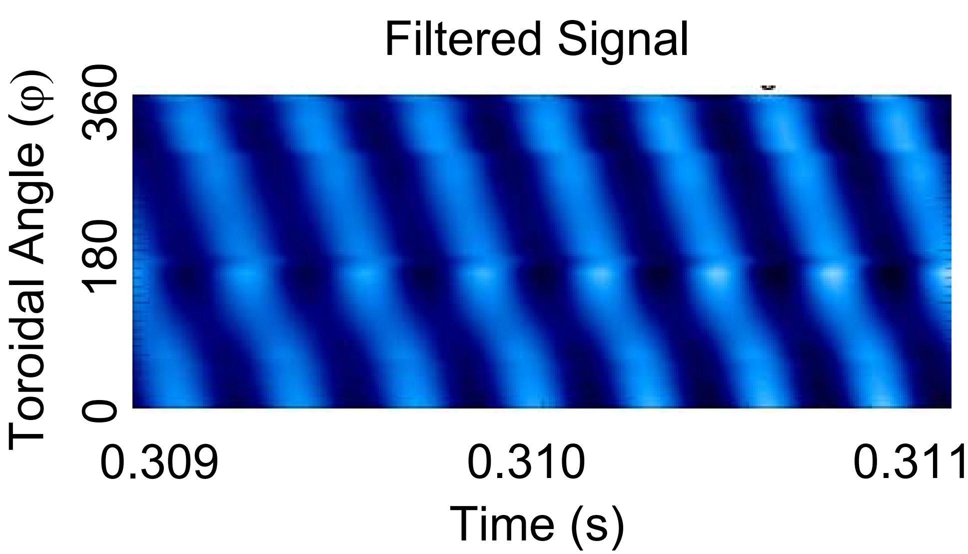
Diagram and Table created by J. Menard

Filtered Signal is Sinusoidal



Visualization & Models of Full Arrays

Toroidal Array

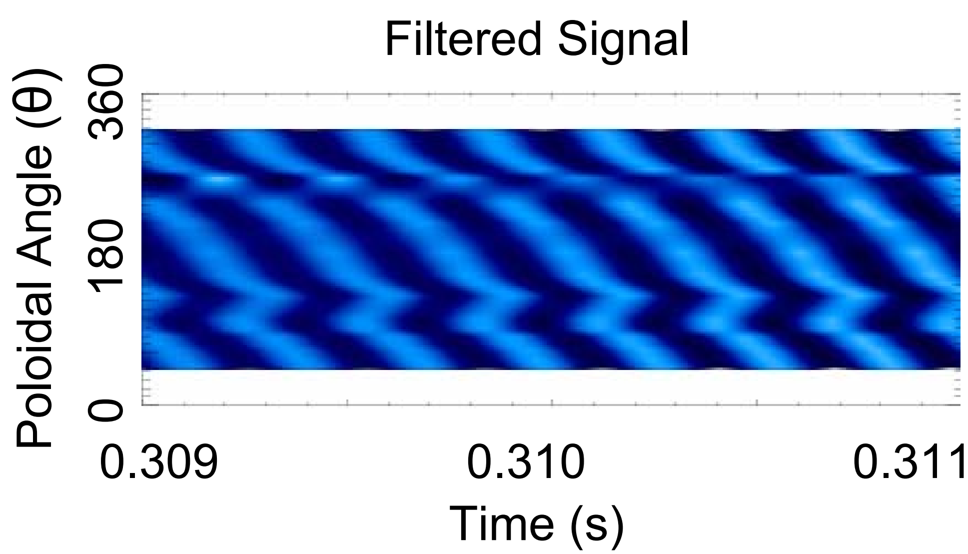


The toroidal signals can be modeled as

$$\widetilde{B}(\phi, t) = B_0 e^{\gamma t} \sin(\omega(t)t - n\phi)$$

- Here, n=1, by inspection.
- Need quantification of the error.

Poloidal Array

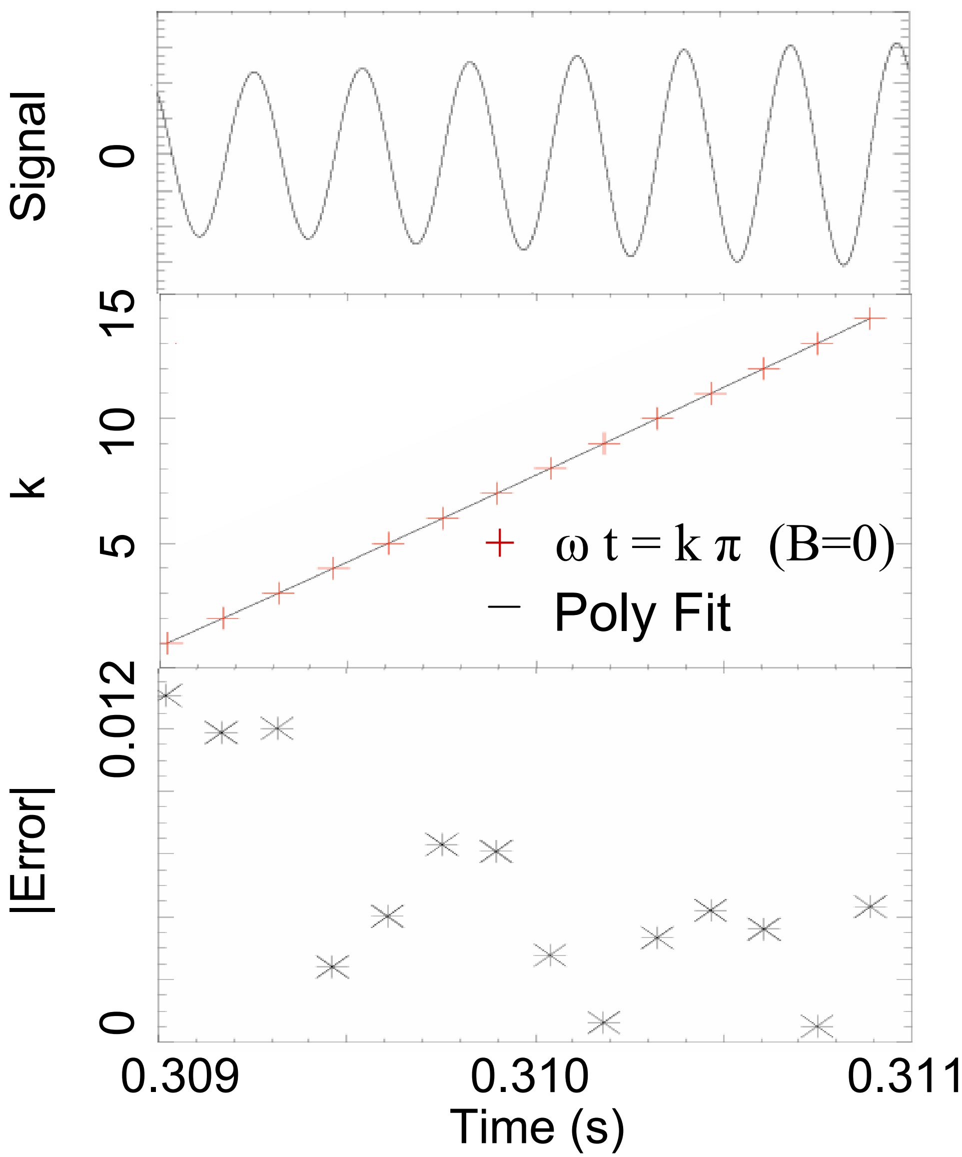


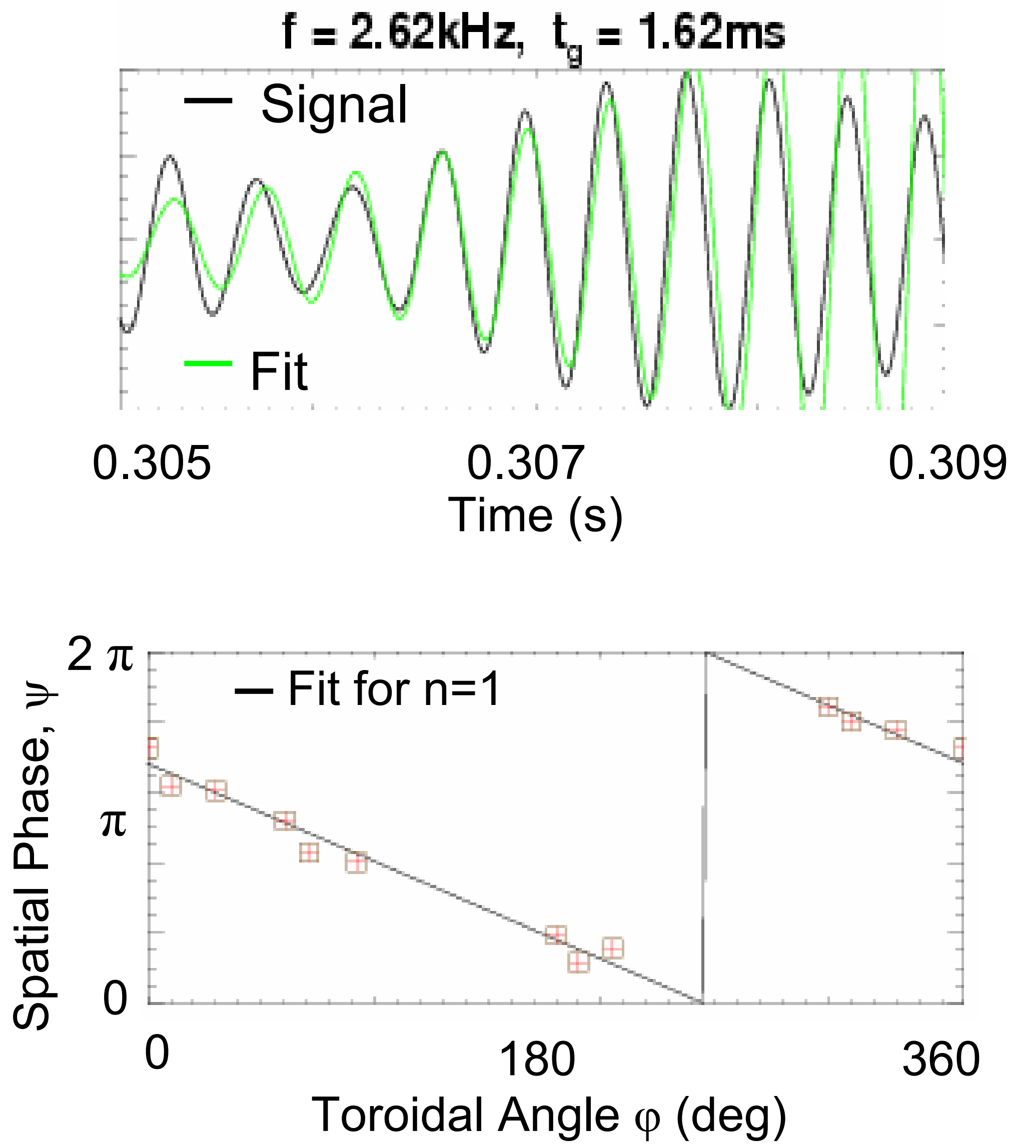
The poloidal signals can be modeled as

$$\widetilde{B}(\theta, t) = B_0 e^{\gamma t} \sin(\omega(t)t - m\theta^*(\theta))$$

- What are m and θ^*?
- What are γ and ω?

Determining ω and n





Associated Error = 0.19

Determining Oscillation Parameters

The model equation of the perturbations is

$$\widetilde{B}_s(t, \phi_s, \theta_s) = B_0 e^{\gamma_g t} \sin(\omega(t)t - \psi_s). \qquad (1)$$

The temporal phase, ωt, can be determined by fitting a polynomial function to the times of zero $\widetilde{B}$, which will yield a possibly time dependent $\omega(t)$. If, for a single sensor, the total errors between the zero $\widetilde{B}$'s and the temporal fit are greater than 3 times the average (over all sensors) total error then the sensor is subsequently ignored. The ignored sensors (only 2-3, if any) are usually in the divertor regions, where there is a smaller $\widetilde{B}$ to detect. *This method of determining the temporal phase is superior to using the phase from an FFT, because the FFT is constrained*

Determining Oscillation Parameters (2)

to look at fixed ω's: the FFT phases will be polluted if the frequency of the perturbation changes in time.

The spatial phase, ψ_s, of a sensor, s, is the average difference between the temporal phase of s and the average temporal phase of all of the sensors. Since the temporal phases mentioned before were more accurate than those obtained from an FFT *the spatial phases will be more accurate as well.*

The growth rate, γ_g, is obtained by taking an appropriate linear fit of the form

$$\ln\left(\frac{B_s}{B_0 \sin(\omega t)}\right) = \gamma_g t, \text{ where } \sin(\omega t) > 0.7.$$

Spatial Phase: Finding m or n

Express the spatial part of Eq. (1) as $\mu_s = (B_0)_s e^{i\psi_s}$. As mentioned, theory predicts that the measurements (when a single MHD mode can be isolated) should behave like $\eta_s = be^{i[n\phi_s \text{ or } m(\theta^*)_s]}$, where b is complex-valued and is found below, $m[n]$ is the poloidal [toroidal] mode number, ϕ_s is the toroidal angle of s, and $(\theta^*)_s$ is the poloidal angle of s measured from the horizontal ray extending from the center of the mode in the outboard direction including toroidicity effects. For NSTX, because all of the poloidal sensors are at the same toroidal angle, and vice versa, m and n can be calculated independently.

Spatial Phase: Finding m or n (2)

Minimizing the error between μ_s and η_s yields

$$b_{n,m} = \frac{1}{N_{sens}} \sum_s \eta_s e^{i(n,m)\phi_s}$$

with a reduced error of

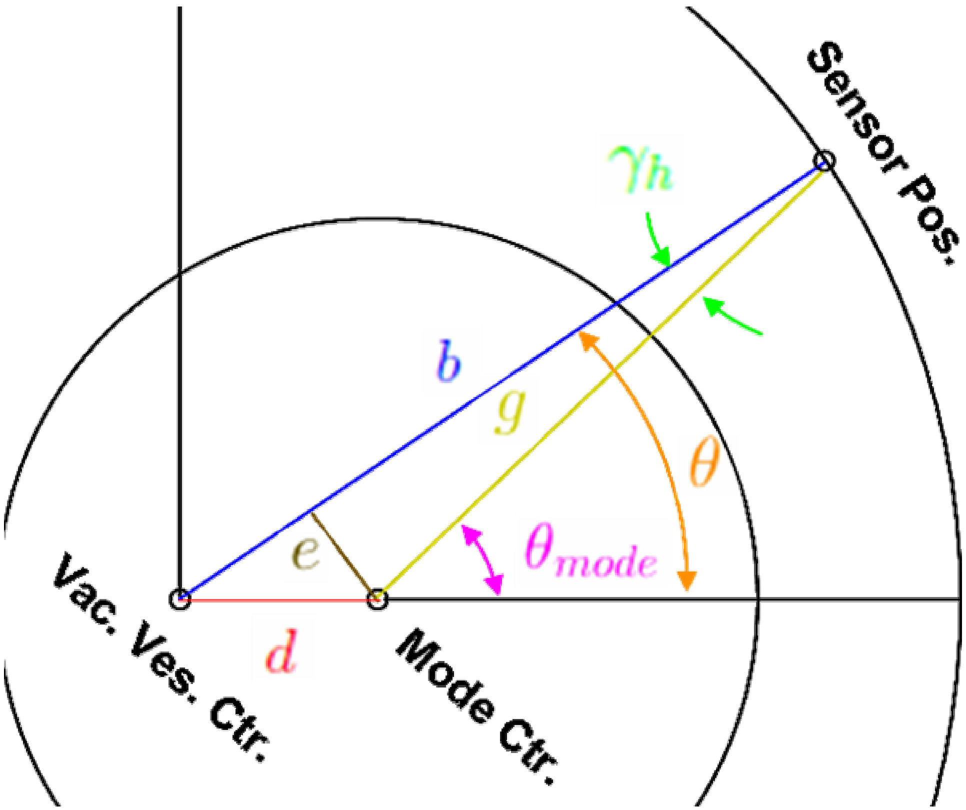
$$\hat{\varepsilon}_{n,m} = \left(1 - \frac{N_{sens}|b_{n,m}|^2}{\sum_s |\eta_s|^2}\right)^{1/2}, \qquad (2)$$

where N_{sens} is the number of sensors being considered. The subscript n, m denotes the mode number, so the lowest error over the range of mode numbers tried corresponds to the best fit mode number.

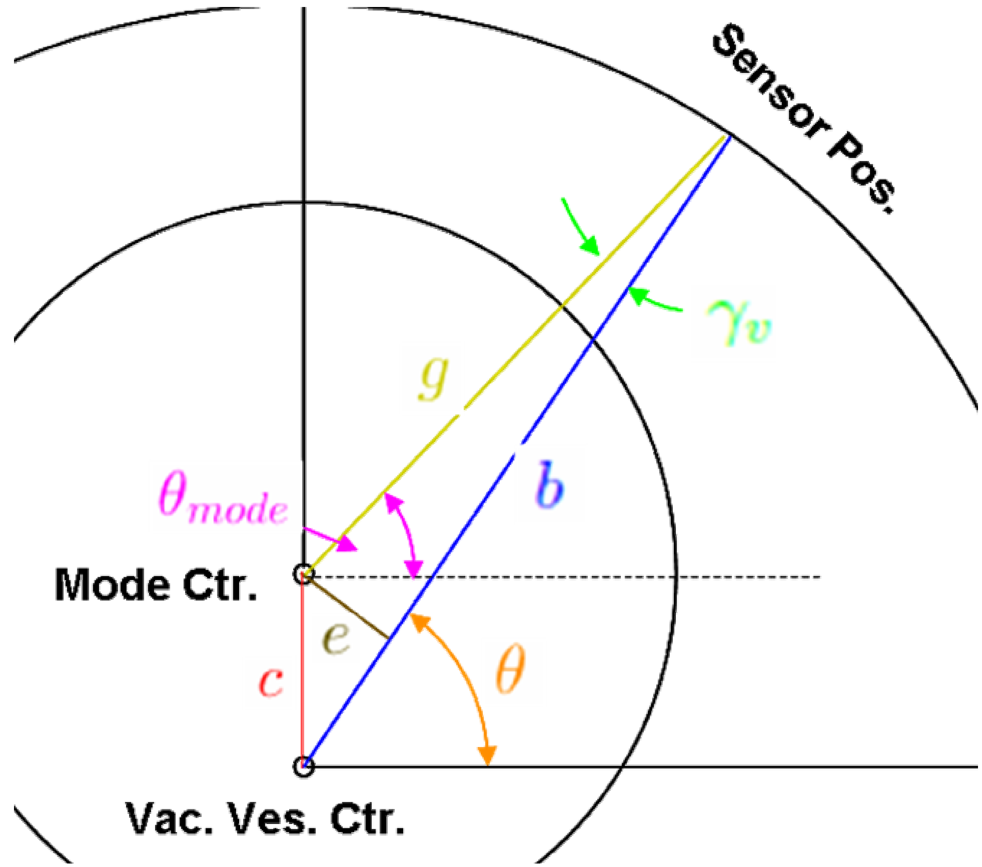
Off-Center Geometry

NSTX

Horizontal Displacement, d



Vertical Displacement, c



Toroidicity Effects on θ^*

The first toroidal effect by θ_s^* was derived by Merezhkin[1] as

$$\theta^*(\theta_M) = \theta_M - \lambda \sin(\theta_M), \tag{3}$$

where $\lambda = (\beta_p + l_i/2 + 1)\frac{a}{R}$, β_p is the poloidal β, l_i is the internal inductance, a/R is the inverse aspect ratio, and θ_M is the angle of a sensor measured from the horizontal relative to the center of the mode. Denote θ as the angle of a sensor measured from the horizontal, relative to the center of the vacuum vessel. If the center of the mode is displaced a horizontal distance, d, and a vertical distance, c, from the vacuum vessel center (which is a distance b from a sensor), then an expansion for (3), where $y = d/b,\ z \approx c/b$

[1]Merezhkin, V.G., Sov. J. Plasma Phys. **4** (1978) 152.

Toroidicity Effects on θ^* (2)

yields

$$\theta^* = \theta + \sum_L \alpha_L \sin(L\theta) + \sum_K \beta_K \cos(K\theta) + O(y^3, z^3), \qquad (4)$$

where

$$\alpha_0 = 0 \qquad \beta_0 = \frac{\lambda z}{2}$$

$$\alpha_1 = \frac{\lambda}{8}(y^2 + 3z^2) - \lambda + y \qquad \beta_1 = -\frac{\lambda z^2}{4} - z$$

$$\alpha_2 = \frac{y^2 - z^2 - \lambda y}{2} \qquad \beta_2 = \frac{\lambda z}{2}$$

$$\alpha_3 = \frac{3\lambda}{8}(z^2 - y^2) \qquad \beta_3 = \frac{\lambda z^2}{4} \quad .$$

Coefficients $\alpha,\ \beta$

Although NSTX has an elongation of $\kappa \approx 2$ and the previous calculations were for a circular plasma, we have assumed that any noncircular effects could be approximately incorporated into θ^* by a combination of sines and cosines as in Eq. (4). The coefficients, α_L and β_K are determined by an automated computational search algorithm (see following slides) to find the values with the lowest error, as computed in Eq. (2). This lowest error also corresponds to an appropriate mode number, m.

Algorithm to Find θ^*

$$\theta^* = \theta + \sum_{L=1}^{L_{max}} \frac{\alpha_L}{L} \sin(L\theta) + \sum_{K=1}^{K_{max}} \frac{\beta_K}{K} \cos(K\theta)$$

1. Initialize the ranges of α_L and β_K as α_{r_L} and β_{r_K} around the best values, $\alpha_{La},\ \beta_{La}$, which are initially 0. Set $L_{ind} = 1$
2. Vary α_1 and α_2 simultaneously over their respective ranges to find the best values so far for α_{1a} and α_{2a}.
3. If $L_{ind} = L_{max}$, then goto 7. $(L_{max} \leq 5)$
4. Decrease α_{r_L}, only for $L = 1 \ldots L_{ind}$.
5. Set $L_{ind} = L_{ind} + 1,\ L = L_{ind}$.
6. Vary α_L over α_{r_L} to get α_{La}. If $L{-}1 = 2$, goto 2, otherwise repeat 6 with $L = L - 1$.

Algorithm to Find θ^* (2)

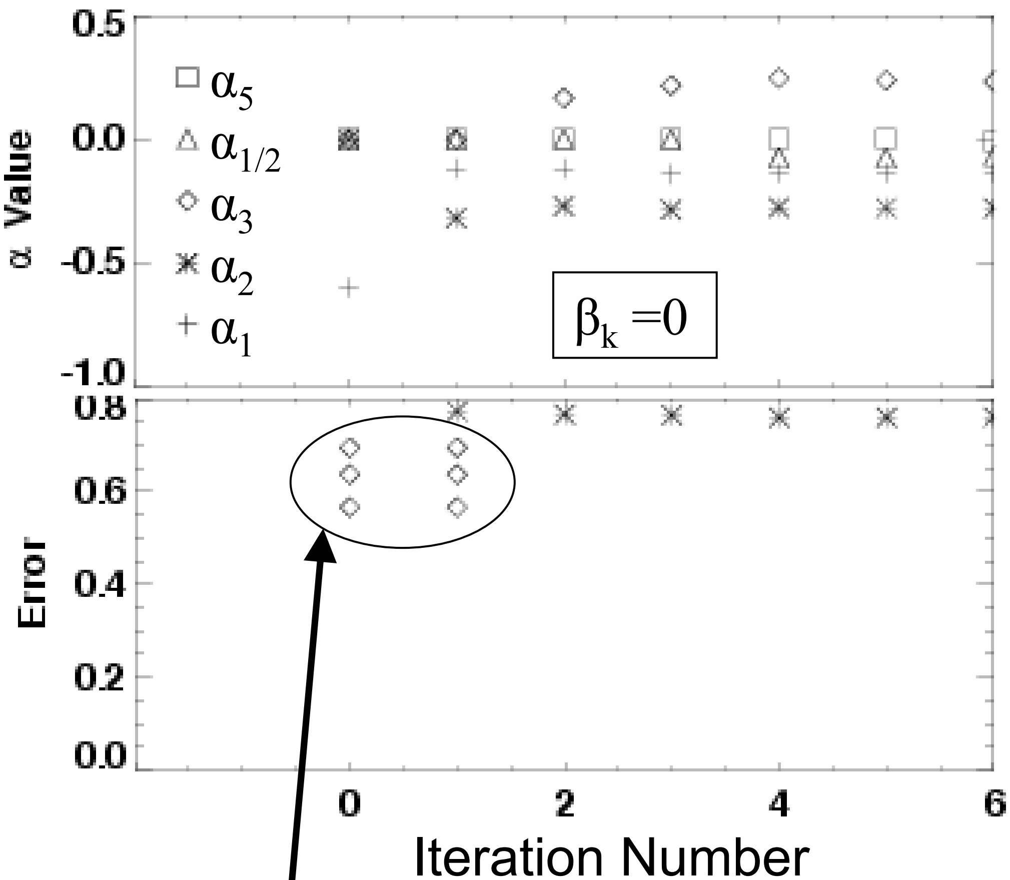
7. If $K_{ind} = K_{max}$ then goto 11.
8. If $K_{ind} > 0$ then decrease β_{r_K}, only for $K = 1 \ldots K_{ind}$
9. Set $K_{ind} = K_{ind} + 1$, $K = K_{ind}$
10. Vary β_K over β_{r_K} to get β_{Ka}. If K=1, goto 6, otherwise repeat 10 with $K = K - 1$.
11. If $\hat{\varepsilon}_m > \hat{\varepsilon}_{max}(\approx 0.4)$ for α_a and β_a, then get rid of the phase of the sensor with the worst error and recalculate $\hat{\varepsilon}_m$. Repeat this step up to $N_{ditch}(\leq 4)$ times.

Constraints on $\alpha_L,\ \beta_K$, & the Algorithm

- $\alpha_{1,2}$ are looked at first, because, in NSTX,
  - there is very little vertical displacement
  - the center of the vacuum vessel cross section is ill-defined, necessitating a possible horizontal displacement.
- $L = \frac{1}{2}$ is allowed
- The derivative $\dfrac{d\theta^*}{d\theta} > 0$ is demanded.
- Based on experience, supported by the equations following Eq. (4), $\alpha_{1,2} < 0$ always (since $y < \lambda$ always).
- Ditching a given sensor will not drastically change $\alpha_a,\ \beta_a$, or the best m, but will improve the error.
- The inboard midplane sensor can not be ditched.

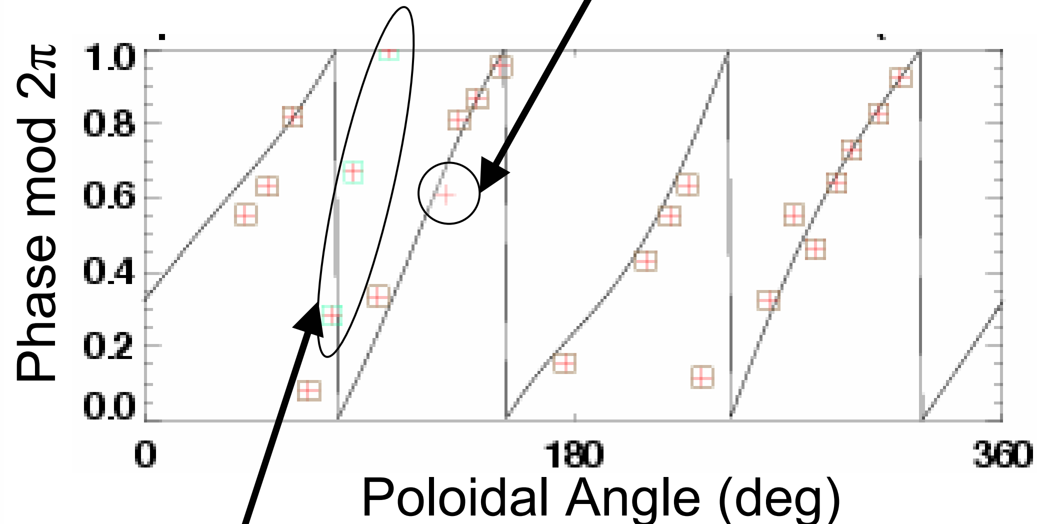
Search Algorithm: α,β & Errors

Shot 116087: 360ms, 3.1 kHz, m=-4, error = 0.566.



'Ditching' to Get Lower Error

Rejected by Temporal Phase



'Ditched' by Spatial Phase

Rotation and Frequency

The plasma rotates in the toroidal direction with a velocity, v_{pl}. A given perturbation of $n = 1$ will then appear to have a frequency of $\omega_{app} = \dfrac{2\pi R_{pl}}{v_{pl}}$, where R_{pl} is the major radius of the plasma, or for arbitrary n, $\omega_{app} = n\dfrac{2\pi R_{pl}}{v_{pl}}$.

Utilizing Charge Exchange Recombination Spectroscopy (CHERS) along with EFIT, one can measure the velocity profile $v_{pl}(R)$ to compare the calculated mode frequency to the rotation frequency of the plasma at the flux surface location corresponding to the computed helicity, $\frac{m}{n}$.

Shot Parameters at the Times of Interest

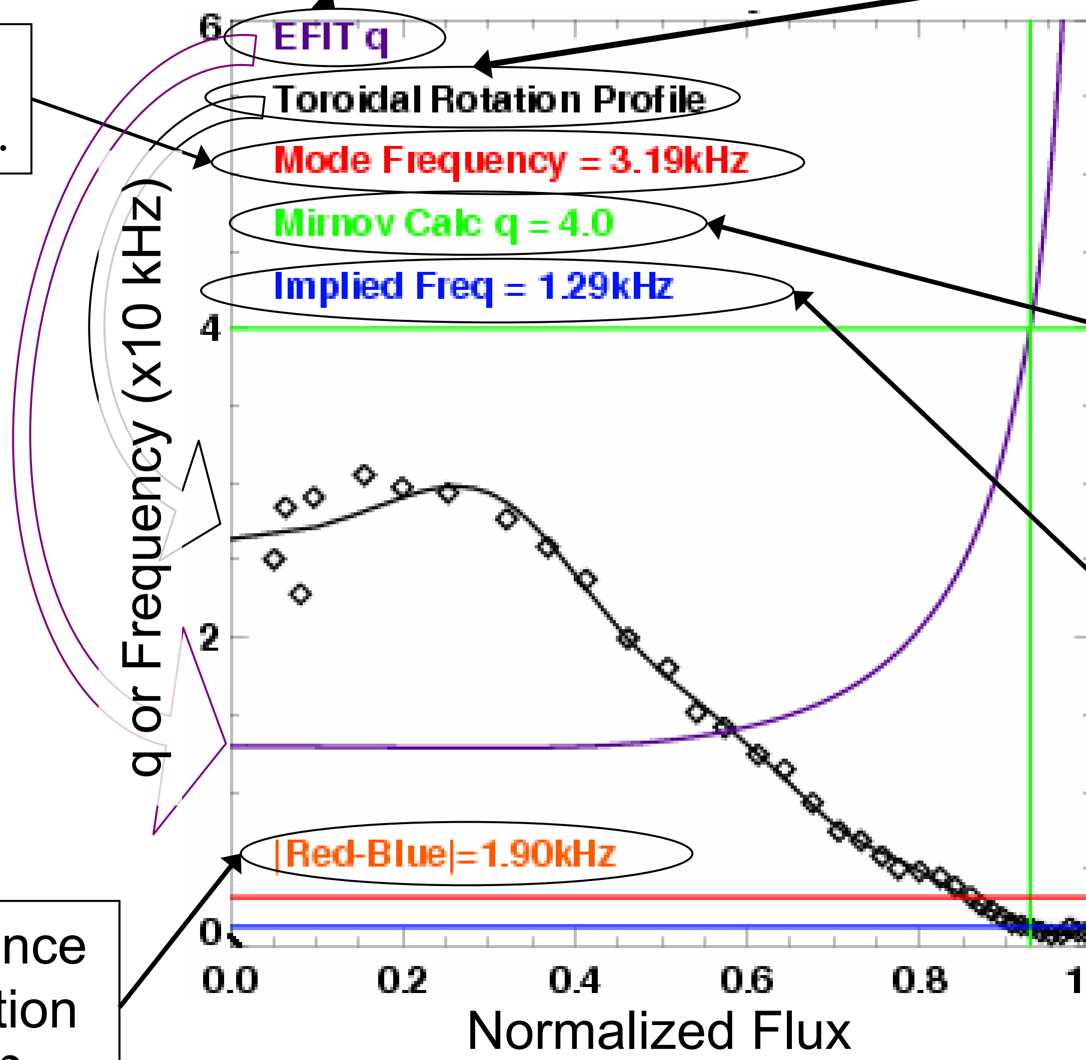
	114148	114184	116087
I_p (MA)	1.2	0.5	0.8
B_T (T)	-0.31	-0.32	-0.44
β_T (%)	36	7	9
κ	2.2	1.9	1.9
δ(upper)	0.49	0.44	0.29
q_{min}	1.35	0.9	1.3

Sample m=4/n=1 Frequency vs CHERS

q profile from EFIT

Rotation profile from CHERS

This is $\omega/2\pi$ from the temporal phase.

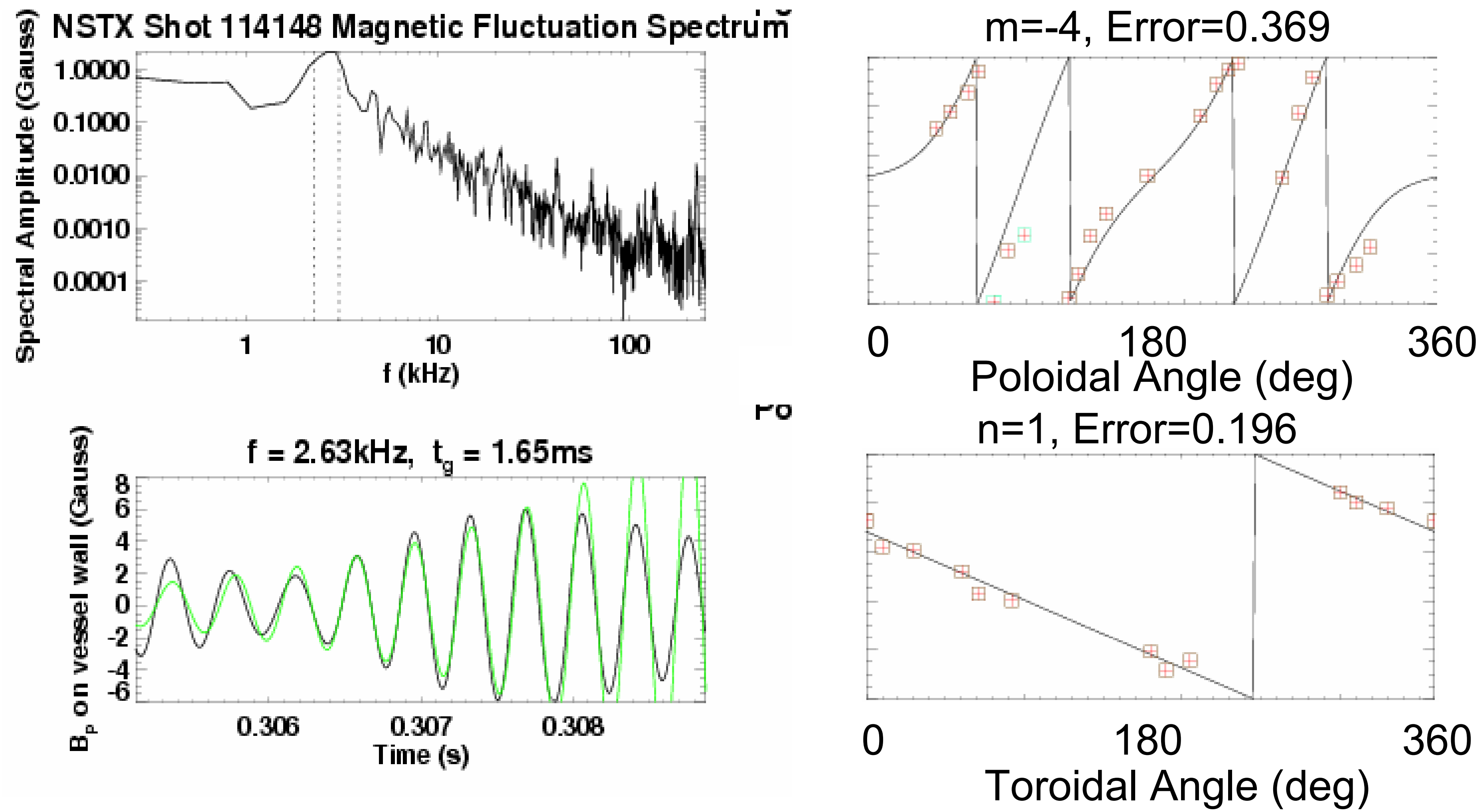


EFIT indicates where the flux surface of q=m/n resides.

This is the difference between the rotation frequency and $\omega/2\pi$.

This is the rotation frequency of the m/n flux surface.

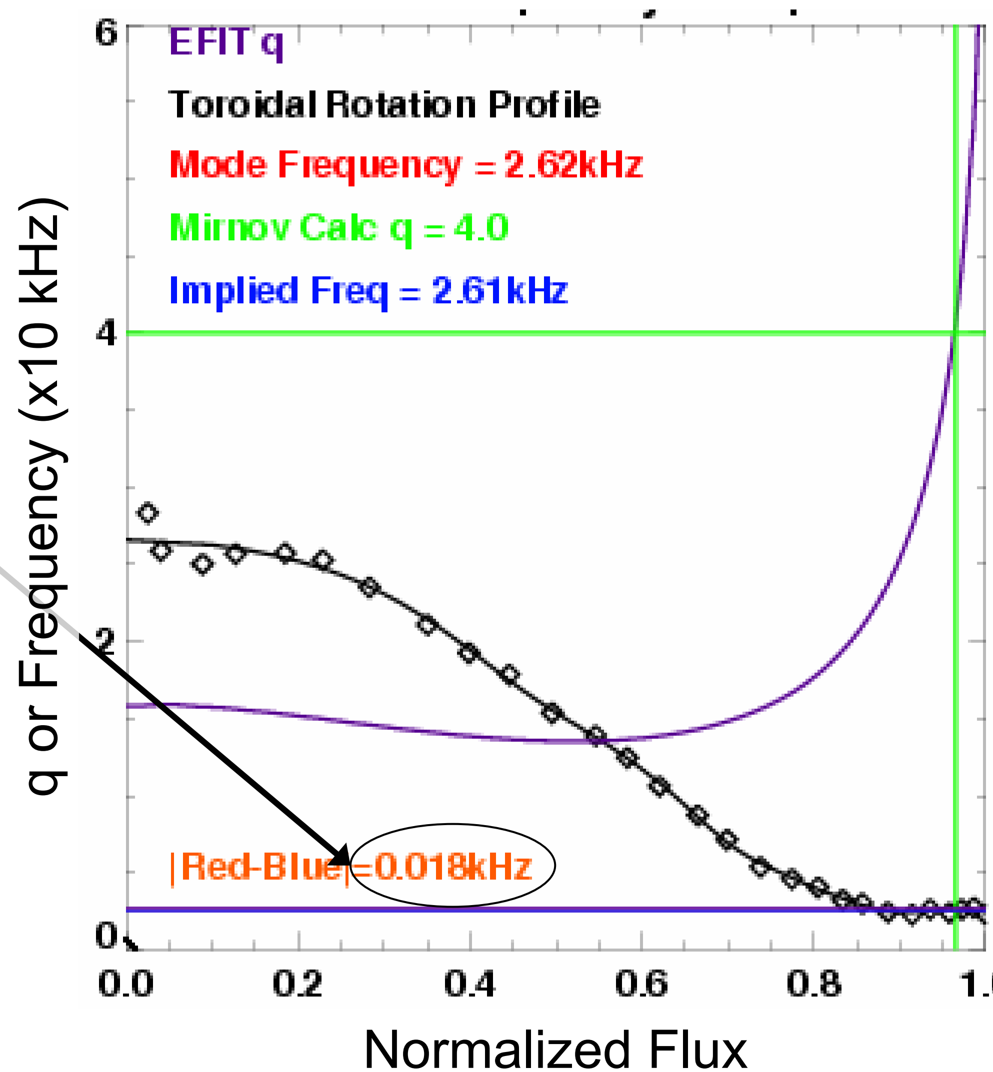
Common m=4/n=1 mode for high β_T



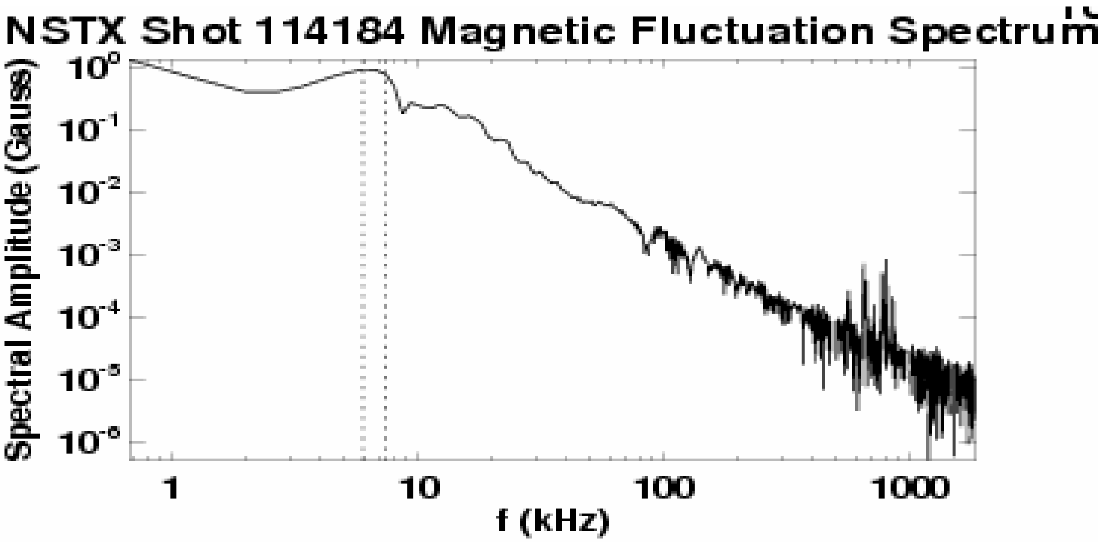
Shot 114148

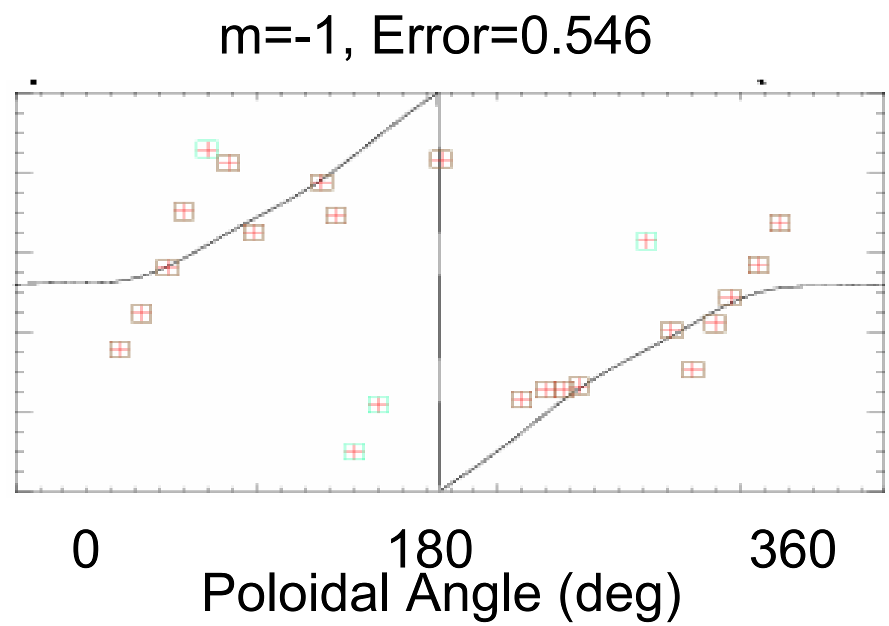
Common m=4/n=1 mode for high β_T (2)

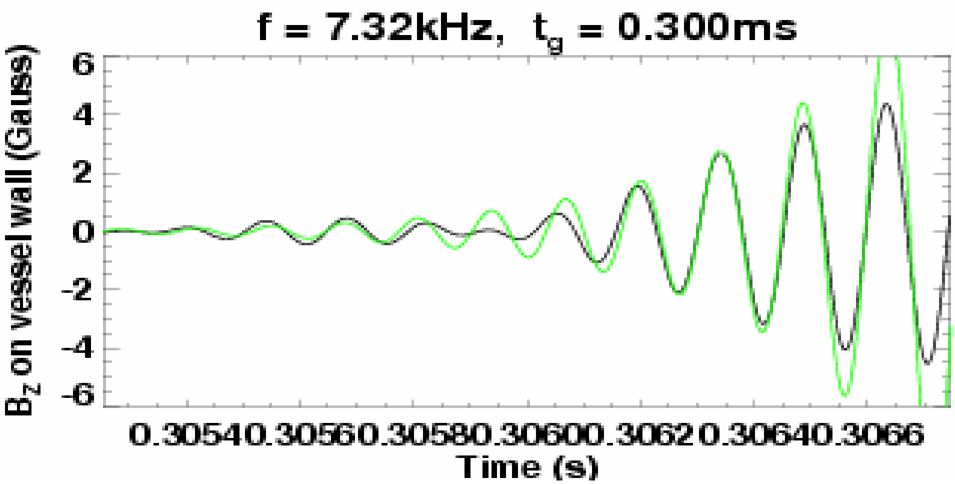
There is very good agreement here between the calculated frequency and the rotational frequency of the m/n surface.

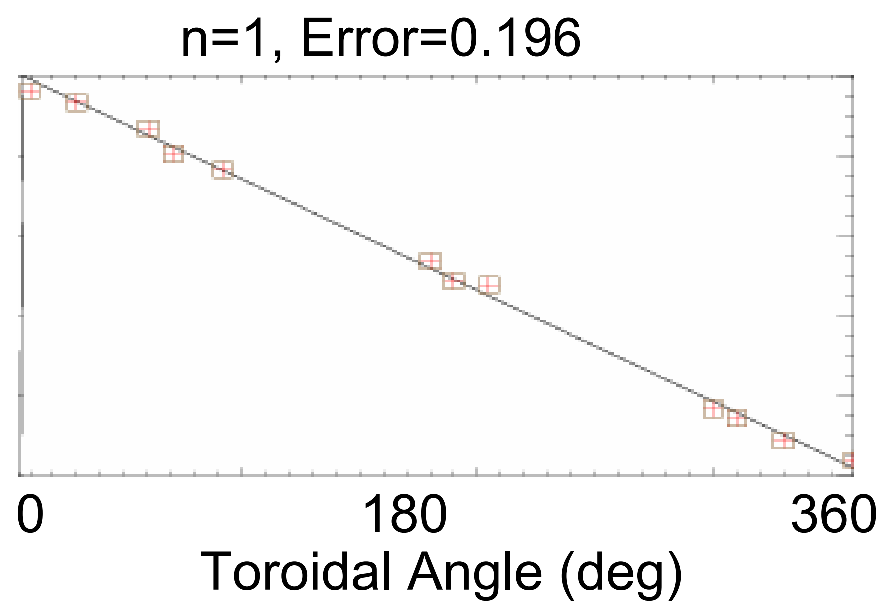


Possible core m=1/n=1 mode



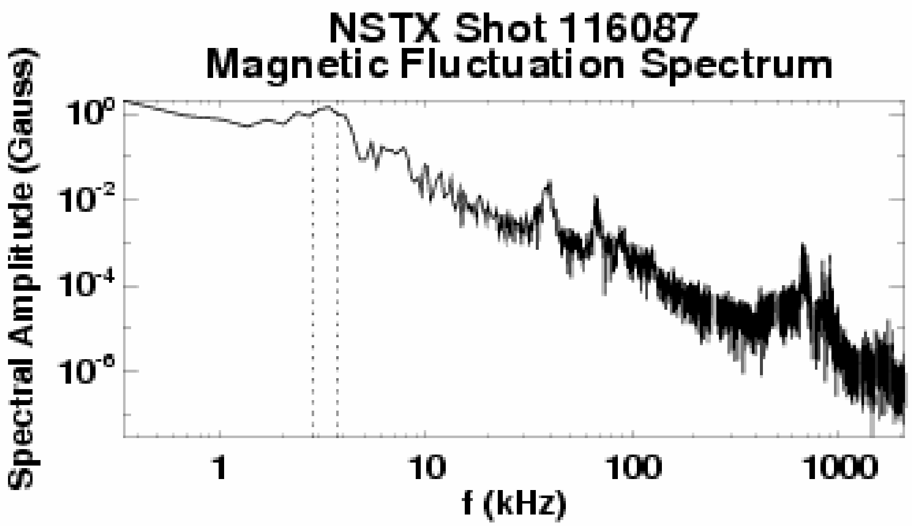


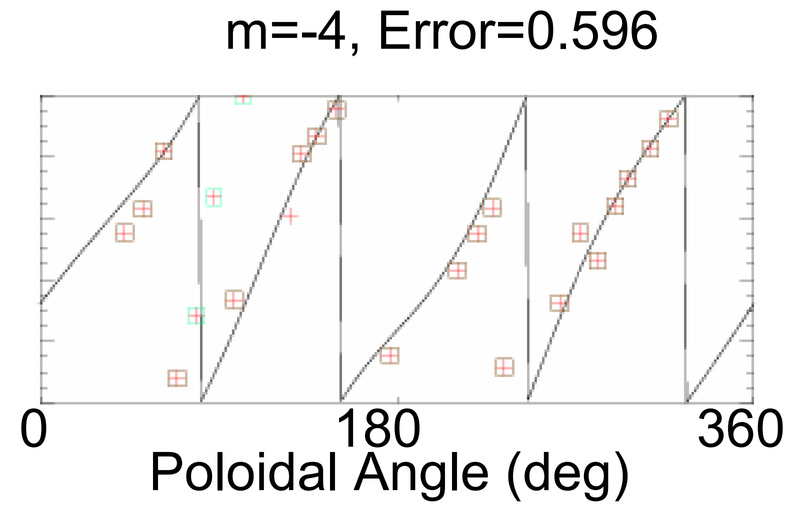


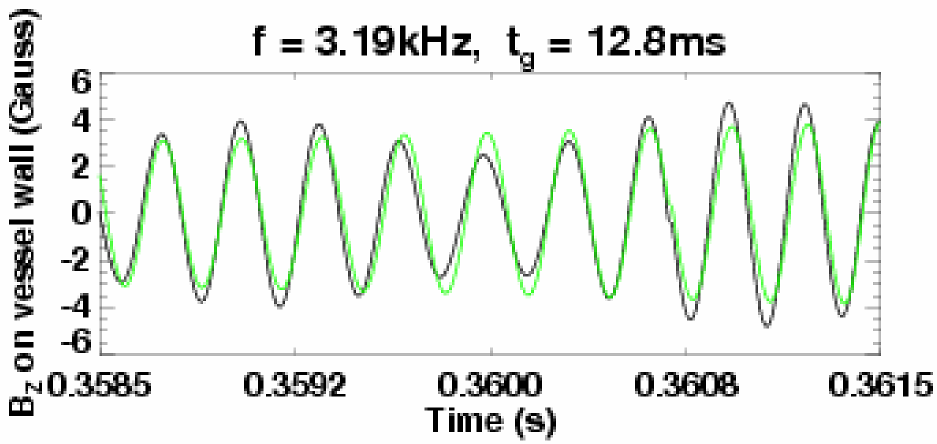


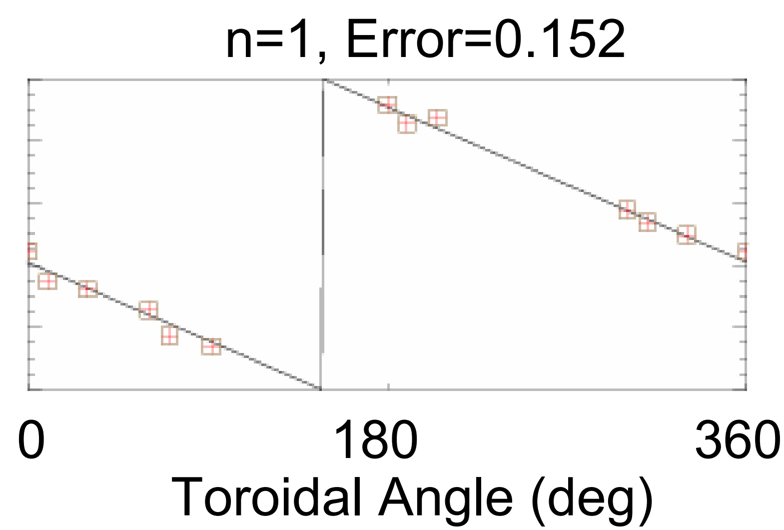
Shot 114184: No rotation data.
Need to compare to other diagnostics.

Another m=4/n=1 mode (Low β)









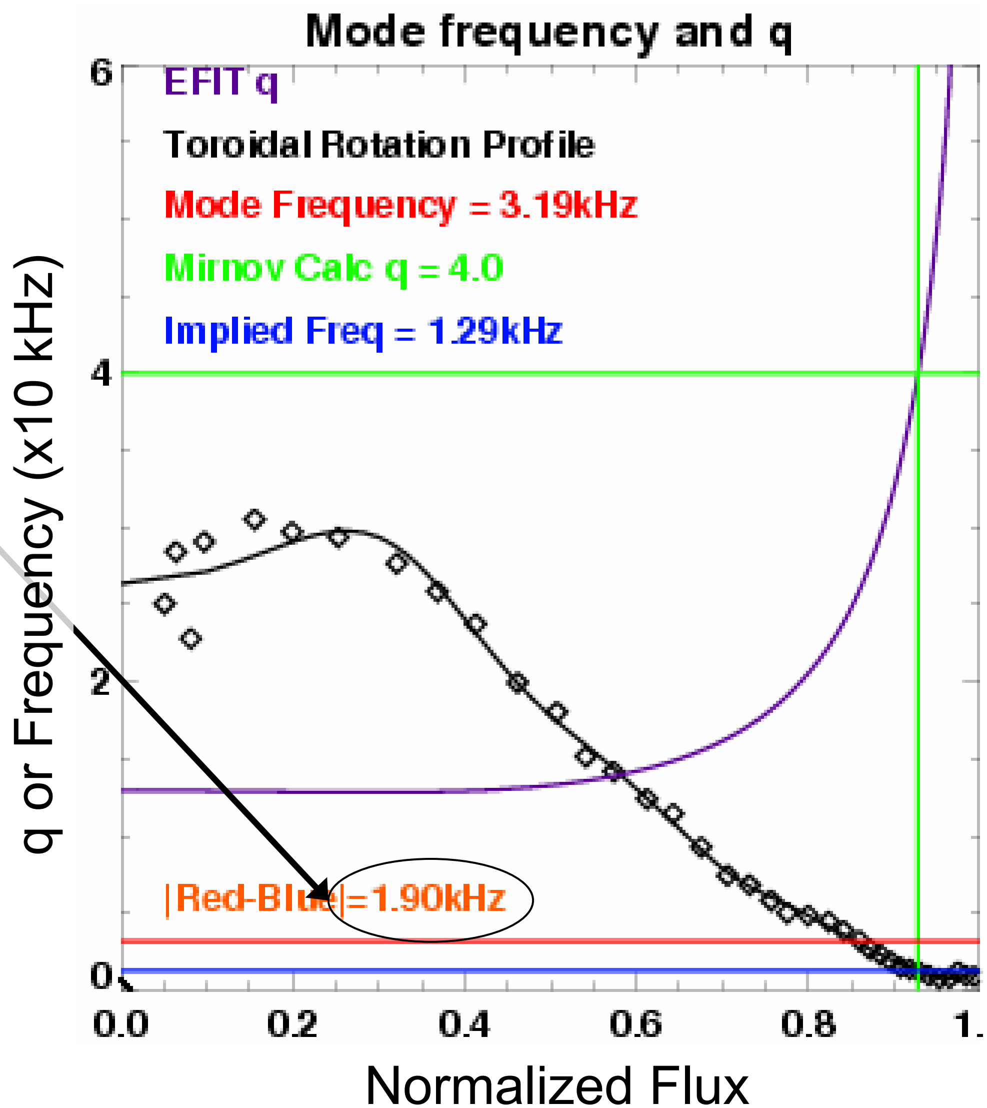
Shot 116087

Another m=4/n=1 mode (Low β)   (2)

There is not as good of an agreement between frequencies, which is also reflected in the higher value of the error of m, 0.596.

This might have something to do with the mode already being saturated.

Thus, there are limitations as to how well this algorithm will work.



Shot 116087

Conclusions & Future Work

- Rotation of NSTX plasmas allows the measurement of magnetic perturbations by Mirnov coils.
- We developed algorithms that are applicable to the shaped, high beta plasmas in NSTX to determine the modes':
  - Growth Rate
  - Frequency (possibly time dependent)
  - Toroidal Mode Number, and
  - Dominant Poloidal Mode Number
- Our calculations of these characteristics agrees well with CHERS and EFIT.
- Further work is needed to resolve m of core modes.