Discrete Compressional Alfvén Eigenmode Spectrum in DIII-D and NSTX

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in collaboration with
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Sub-cyclotron Mirnov activity in NSTX have been identified as Compressional Alfvén Eigenmode (CAE) instability

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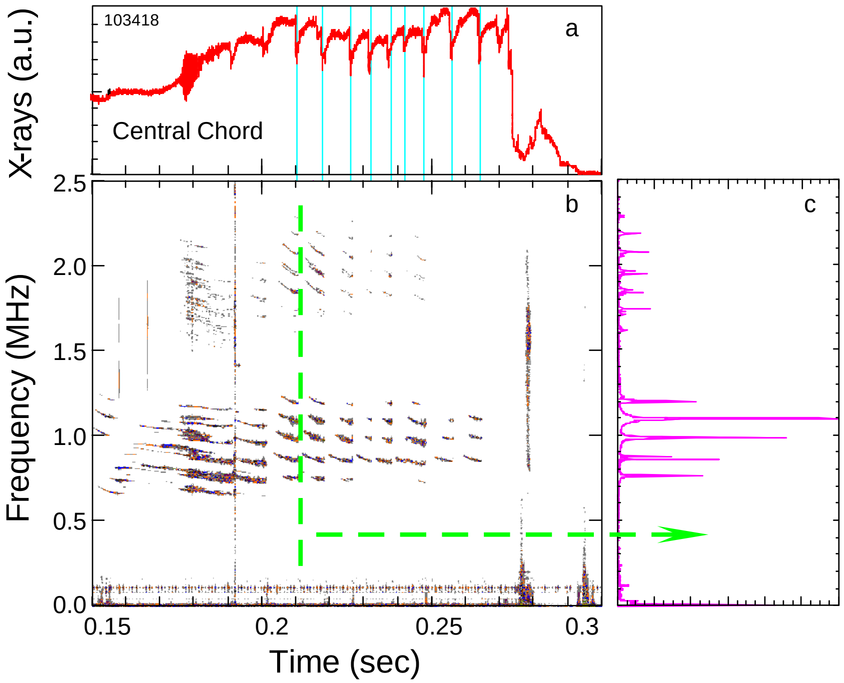

- CAEs (or Magnetosonic modes) and Ion Cyclotron Emission (ICE) have common physics: drive, cavity
- Discrete CAEs allow for detailed instability study (Fredrickson'01)
- CAEs can interact with thermal ions stochastically (Gates'01):
  - new way of ion heating
  - energy channeling: fast to thermal ions (Fisch'92)

ICE amplitude and neutron power correlate over six orders of magnitude (Cottrell'93) ⇒ ICE and CAEs are expected in ITER

Outline and motivations

1. Observations of high-frequency AEs in NSTX and DIII-D
   (a) Magnetic spectrum and mode numbers
   (b) Polarization measurements
2. Qualitative theoretical analysis of CAE structure
3. NOVA-K numerical modeling of CAE spectrum and structure in NSTX and DIII-D
   (a) CAE spectrum/properties with model density profile in DIII-D
   (b) realistic density profiles in NSTX and DIII-D
4. Comparison of results from NOVA with analytical theory and experiments

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Theoretical challenge: are these modes localized? Can we model them?
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Main findings:
Confirm theoretical predictions for the spectrum and poloidaly/radially localized low-*n* Compressional Alfvén Eigenmodes (CAEs)

High frequency modes are observed in both NSTX and low-B field DIII-D experiments (Heidbrink, ’05)

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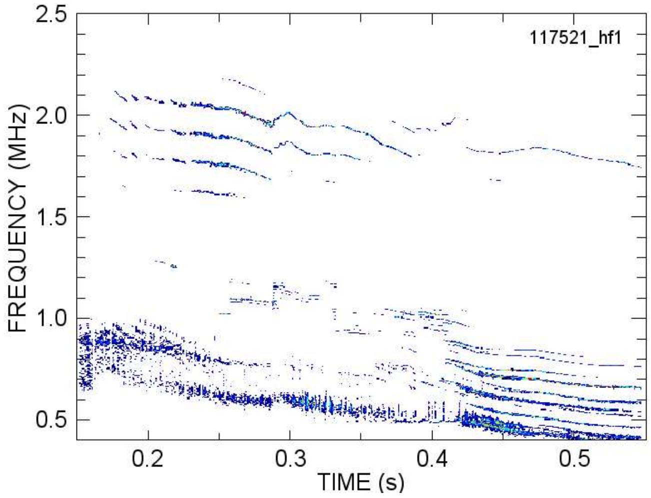
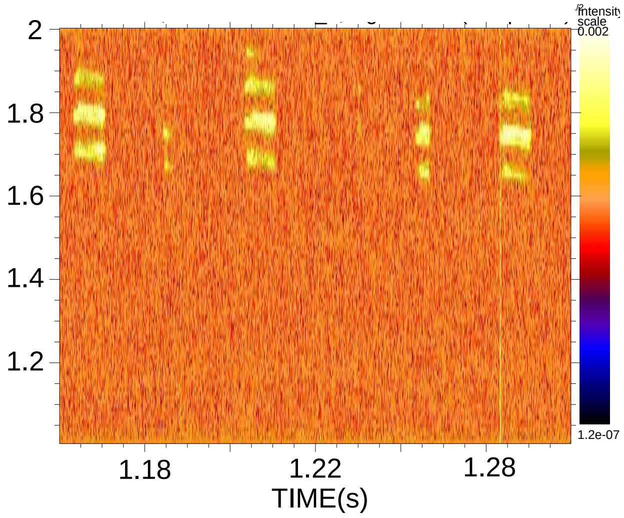
NSTX



DIII-D

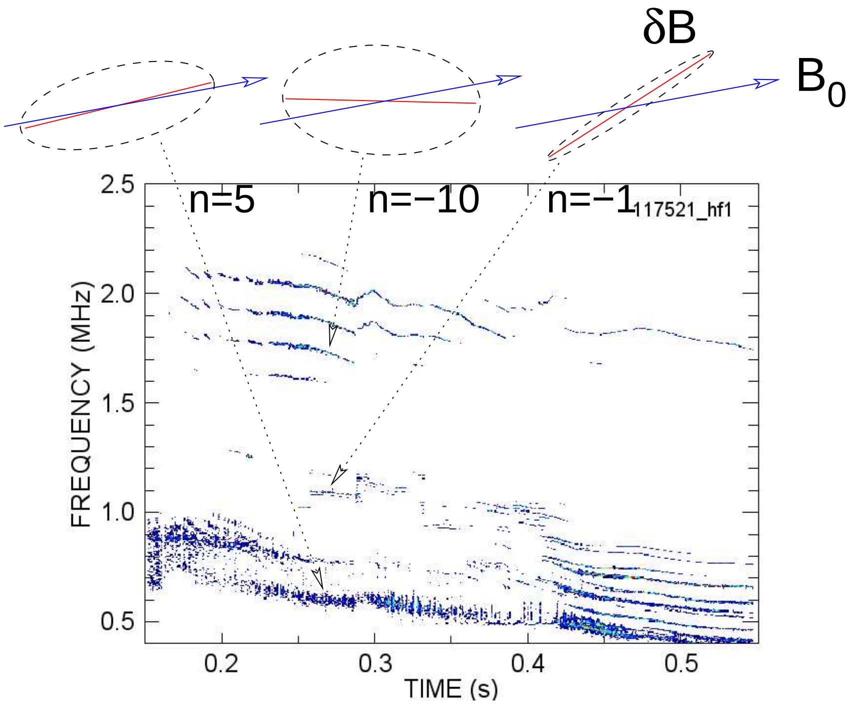


Plasma parameters in similarity experiments:

$B = 0.44T, R_0 = 0.85m$
$n_e = 5.83 \times 10^{13} cm^{-3}$
$f_{cD0} = 3.39MHz,$
$f_{cDedge} = 2.67MHz$

$B = 0.52T, R_0 = 1.68m$
$n_e = 4.11 \times 10^{13} cm^{-3}$
$f_{cD0} = 4MHz,$
$f_{cDedge} = 3.2MHz$

Magnetic polarization measurements identify CAEs

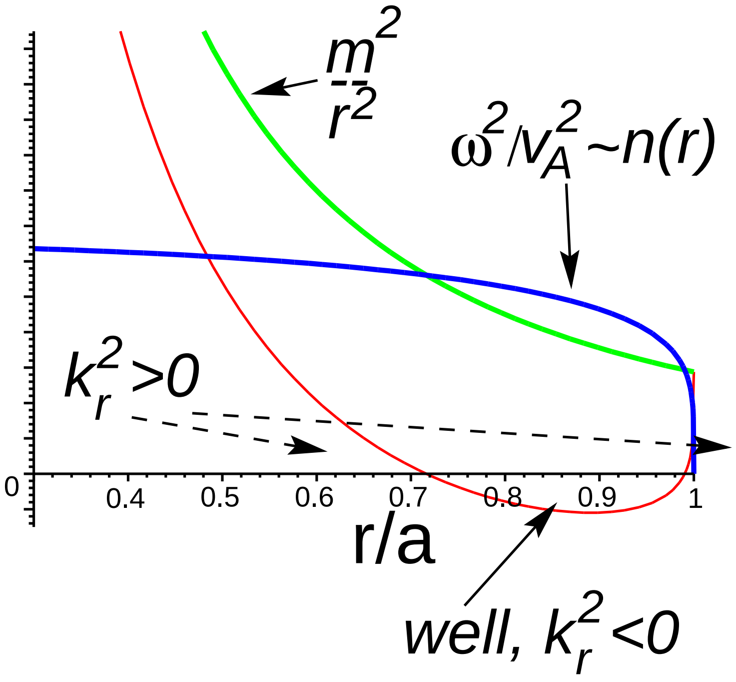


Summary of experimental observations of CAEs:

- Mode frequency correlates with Alfvén speed
- CAE signal lines do not cross in the spectrum
- $\delta B_{\parallel}$ is dominant for CAEs
- Spectra have fine structures
- Driven by fast ions $v_{b0}/v_A = 2-4$

Characteristics of each mode can be measured separately ⇒ use advantage to understand CAE instabilities.

Compressional Alfven Eigenmode (CAE) theory in cylindrical plasmas



- Mode dispersion, $k_\theta = im/r$,

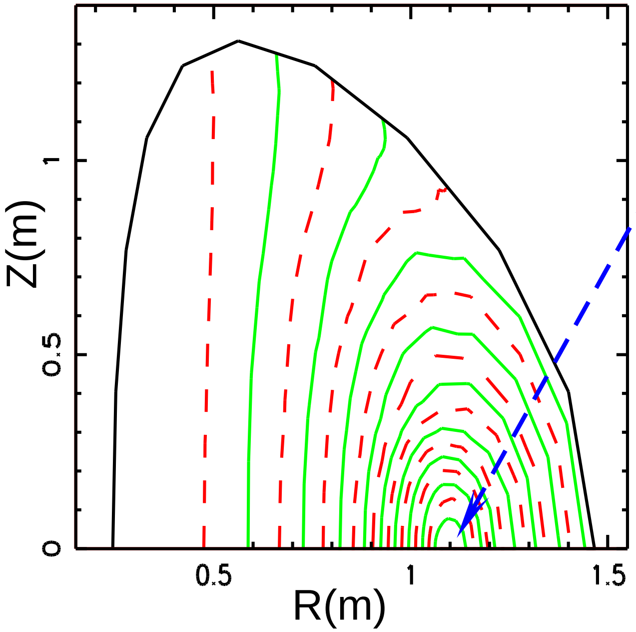
$$\omega^2 = k^2 v_A^2 \;\rightarrow$$

$$\rightarrow\; k_\varphi^2 + k_\theta^2 - \frac{\omega^2}{v_A^2} = -k_r^2 = \frac{\partial^2}{\partial r^2}.$$

- Radial “potential” well is formed by density variation, $v_A^{-2} \sim n_i$. (Mahajan *et.al.* 1983; Coppi *et.al.*1986)
- $\Rightarrow$three mode numbers for each mode: poloidal M, radial S, toroidal n:

$$\omega^2 \simeq \frac{M^2}{r^2} + \frac{S^2}{L_r^2} + \frac{n^2}{R^2}$$

CAE potential well in toroidal plasma



- CAEs are expected to be localized in 2D potential well (v_A^{-2} contours shown for NSTX) at LFS due to magnetic field variation $v_A^{-2} \sim n_i/B^2(\theta)$ (Gorelenkov, ’95,’02)
- Hall term breaks $\pm m$ symmetry (Penn’98, Kolesnichenko’98, Fulop ’00, Smith ’03).
- Experimental agreement with theory for the spectrum fine structure is poor

There is no systematic numerical study of CAE localization and spectra.

Application of LION and PENN codes for CAE problem shows poloidal localization only for high toroidal mode numbers (Hellsten ’03).

CAEs are simulated with global code NOVA

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NOVA (Cheng, JCP,'87) and NOVA-K (Cheng, PhR,'92) codes are used to compute various AEs spectra and stability:

- NOVA is interfaced with TRANSP output for plasma parameters
- Mode structure is computed within ideal MHD model (NOVA)
  - both shear and compressional branches are recovered
  - no FLR, ω/ω_c effects on mode structures at the moment
  - no Hall term in the model
- Perturbative kinetic mode analysis is performed with NOVA-K code
  - Fast ion drive includes with FOW and FLR effects
  - Damping mechanisms, including ion/electron Landau, radiative, radiative, trapped electron collisional, continuum damping

NOVA finds low-n CAE spectrum (DIII-D plasma)

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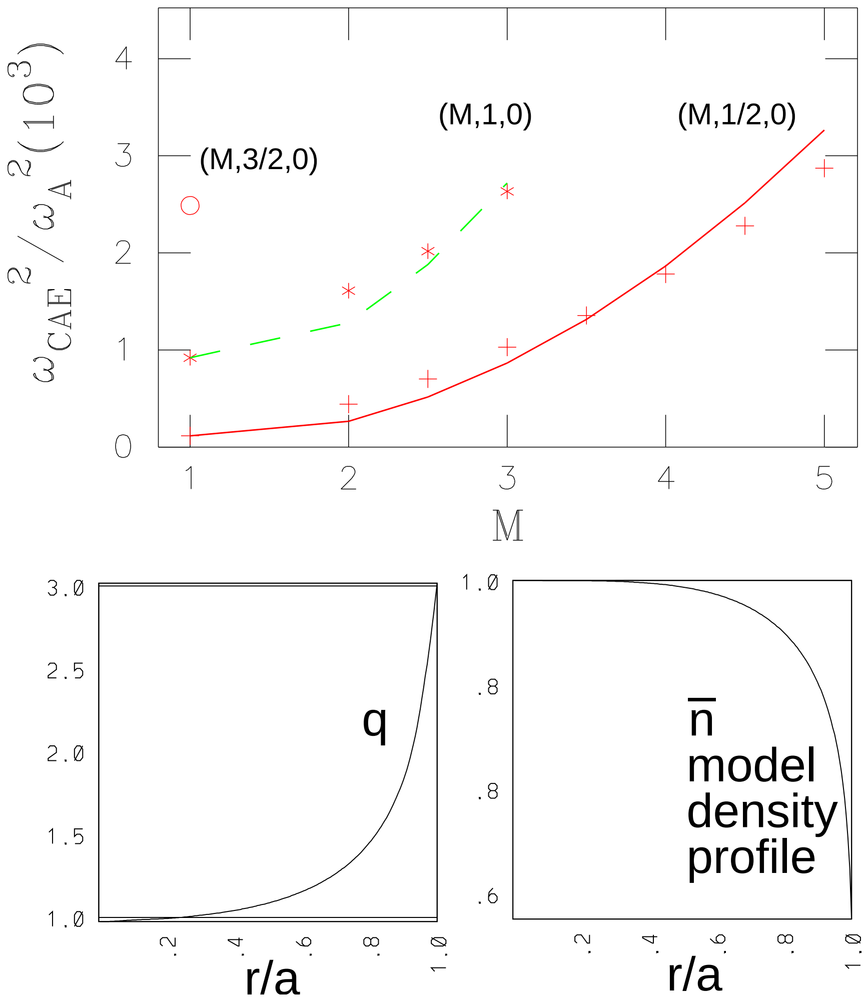

- Take simple monotonic density profile first
- Each mode has single combination of (M,S,n):
  - M is poloidal mode number
  - S is radial mode number
  - n is toroidal mode number
- $n=0$ in NOVA input, poloidal harmonic expansion, m, is used to identify each mode

Radial, poloidal and toroidal frequency scales

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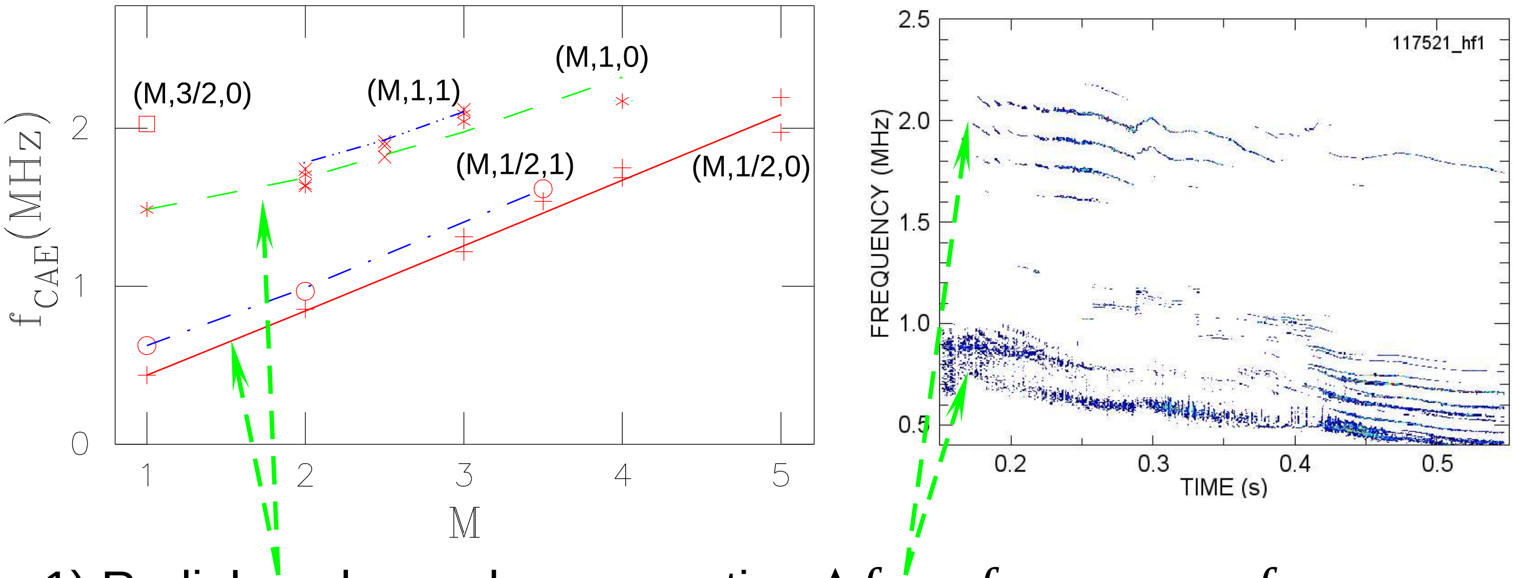

1) Radial mode number separation $\Delta f_S \equiv f_{M,S+1/2,n} - f_{M,S,n}$

Radial, poloidal and toroidal frequency scales

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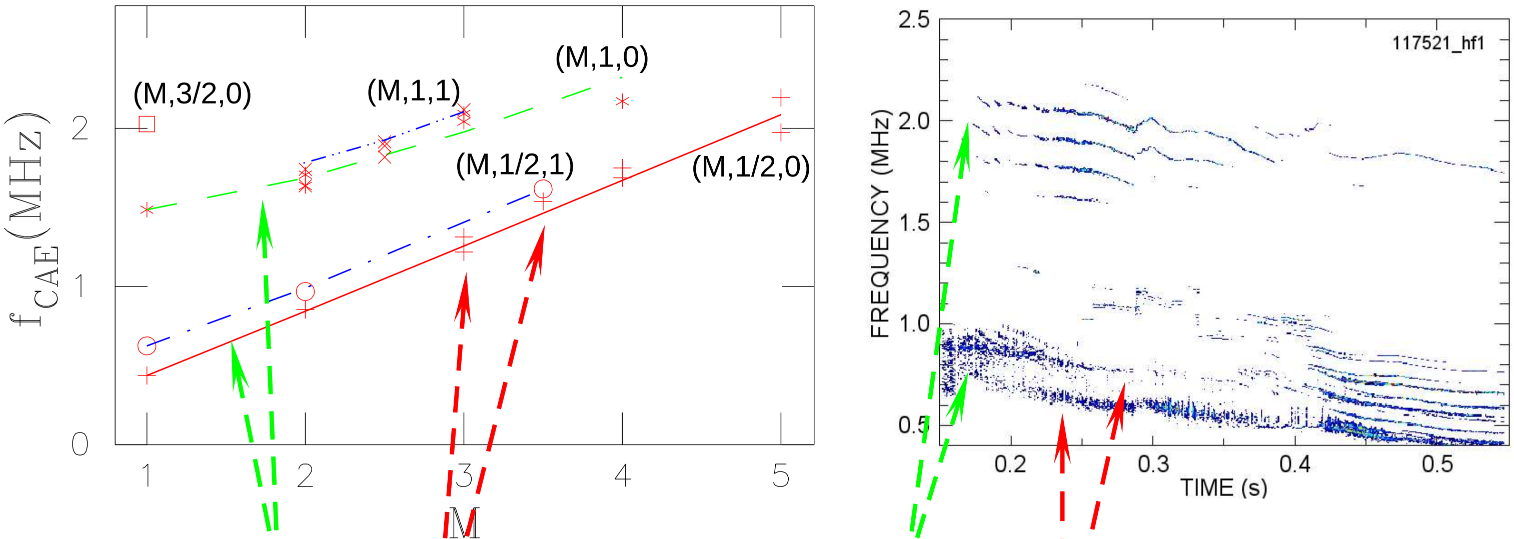

1) Radial mode number separation $\Delta f_S \equiv f_{M,S+1/2,n} - f_{M,S,n}$

2) Poloidal mode number separation $\Delta f_M \equiv f_{M+1/2,S,n} - f_{M,S,n}$

Radial, poloidal and toroidal frequency scales

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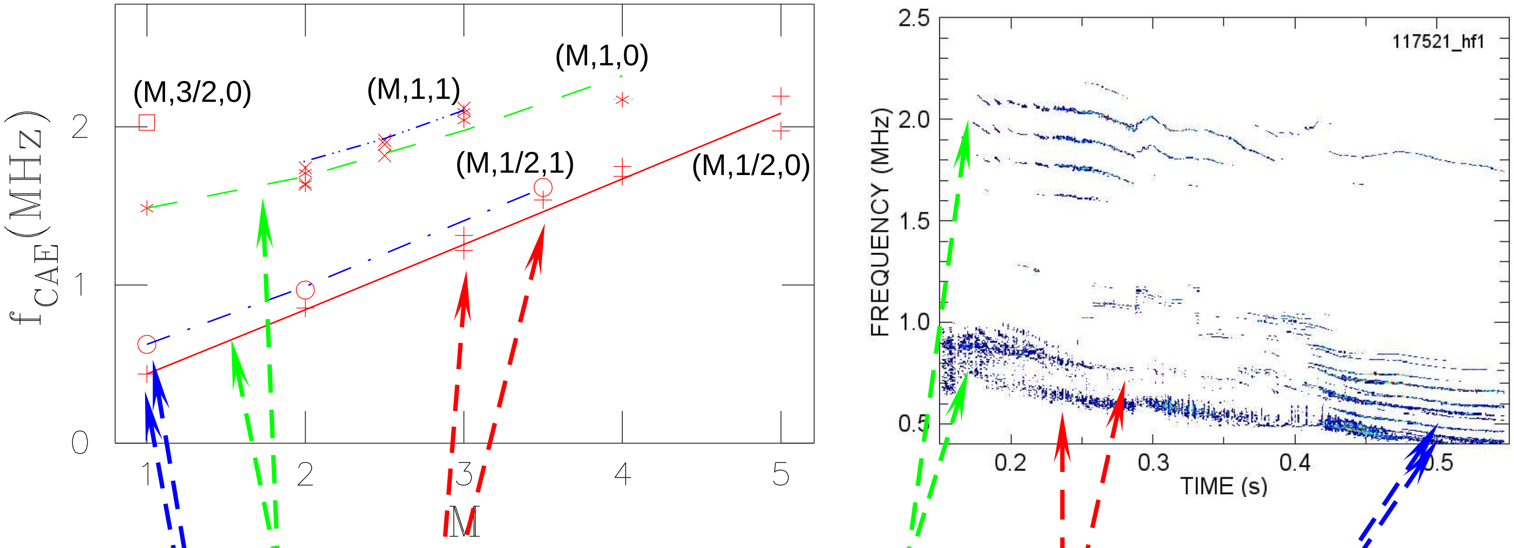

1) Radial mode number separation $\Delta f_S \equiv f_{M,S+1/2,n} - f_{M,S,n}$

2) Poloidal mode number separation $\Delta f_M \equiv f_{M+1/2,S,n} - f_{M,S,n}$

3) Toroidal mode number separation $\Delta f_n \equiv f_{M,S,n+1} - f_{M,S,n}$ (smallest)

Radial number determines largest frequency splitting (DIII-D)

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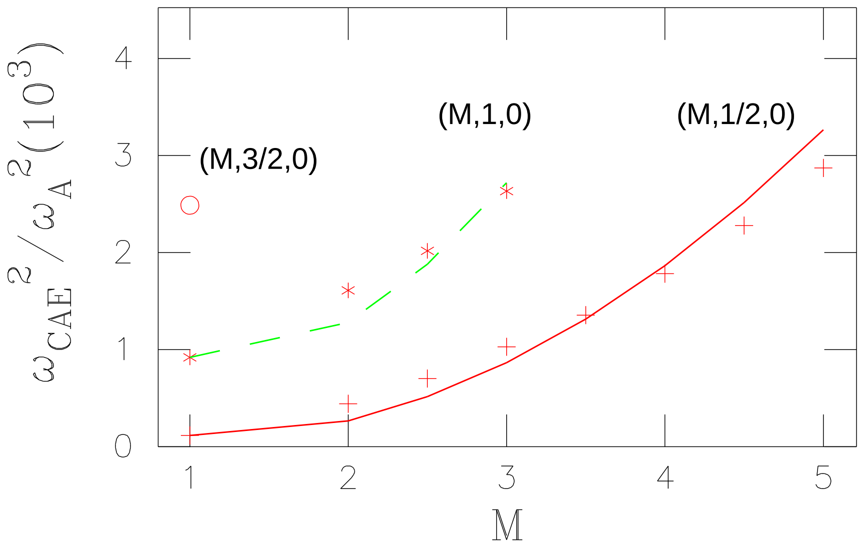
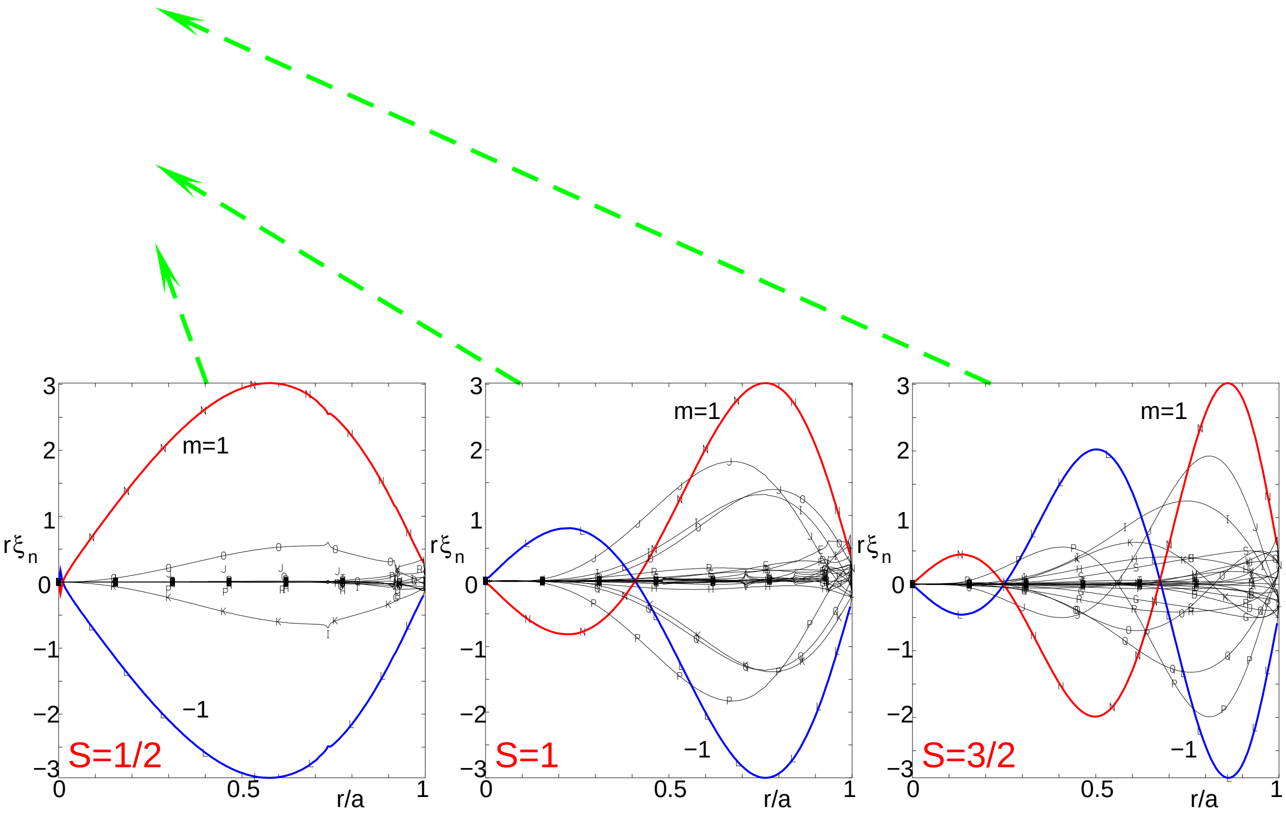


- ξ_n is normal component of the plasma displacement
- Shown are harmonics of ξ_n
- First poloidal number modes are shown $M = 1$ with dominant cylindrical $m = \pm 1$
- $m = -12$ to 12 used for convergence



3
2
1
0
−1
−2
−3
$r\xi_n$
m=1
−1
S=1/2
0
0.5
1
r/a
m=1
−1
S=1
m=1
−1
S=3/2

Poloidal number, M, splitting (DIII-D)

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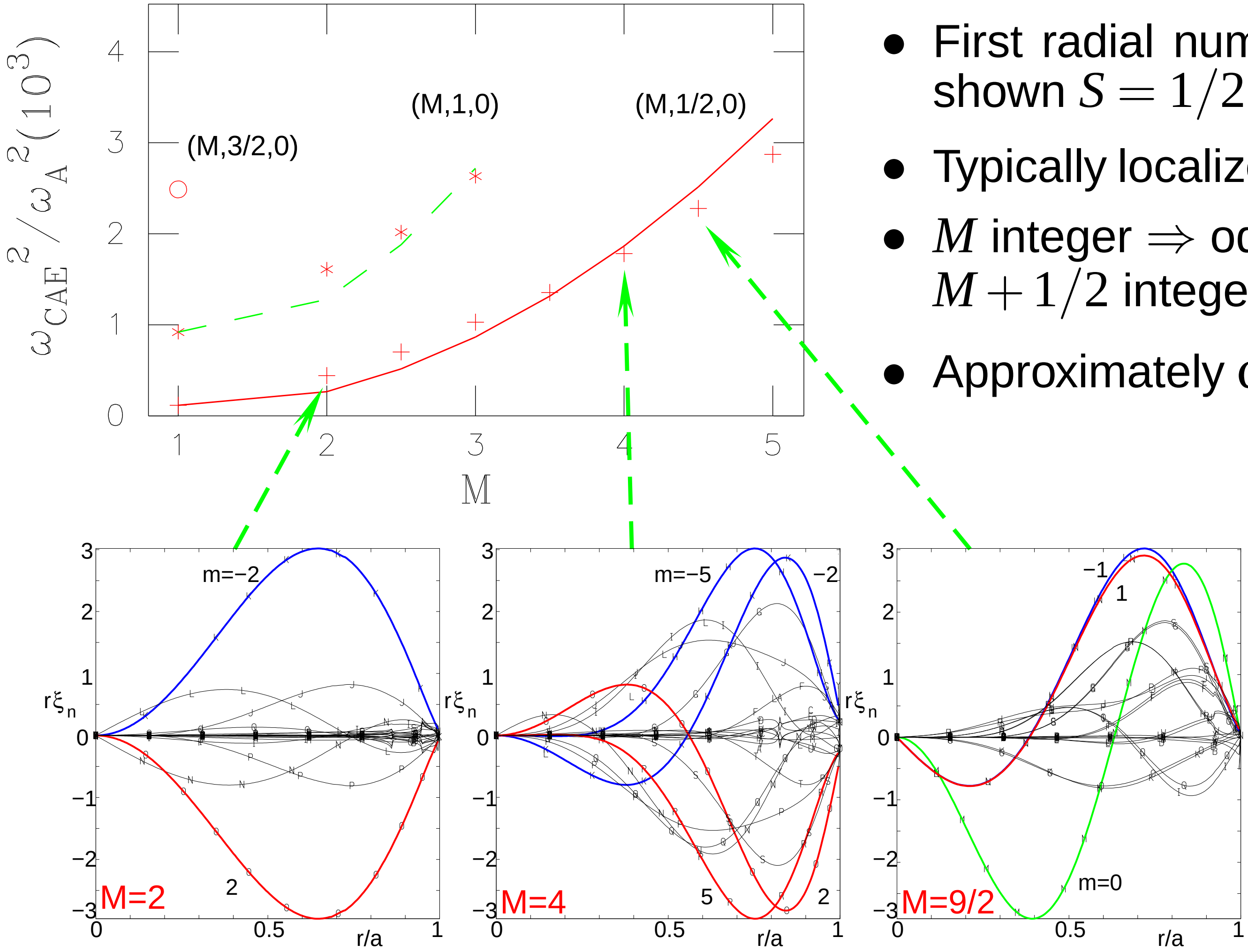

- First radial number modes are shown $S = 1/2$
- Typically localized at $r_0/a \simeq 0.7$
- M integer $\Rightarrow$ odd mode
  $M + 1/2$ integer $\Rightarrow$ even mode
- Approximately $\omega \sim M^2$ (shown)

CAEs are poloidaly and radially localized (DIII-D)

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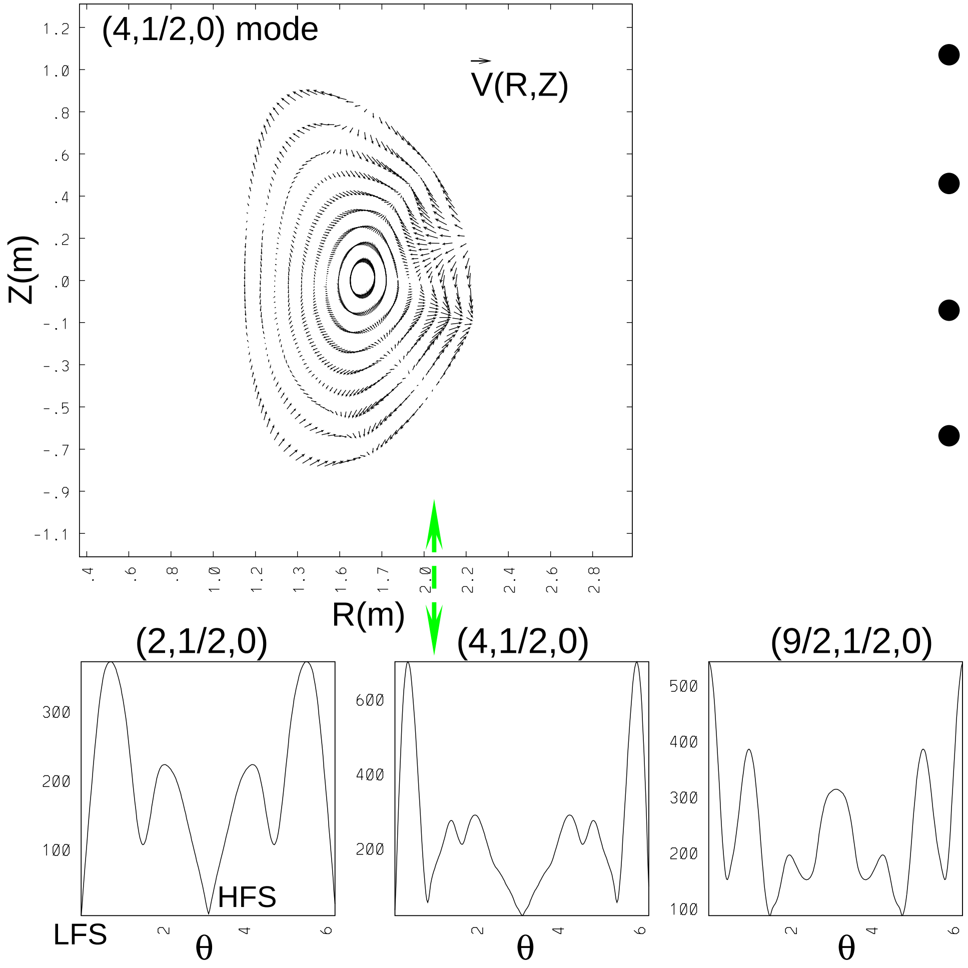
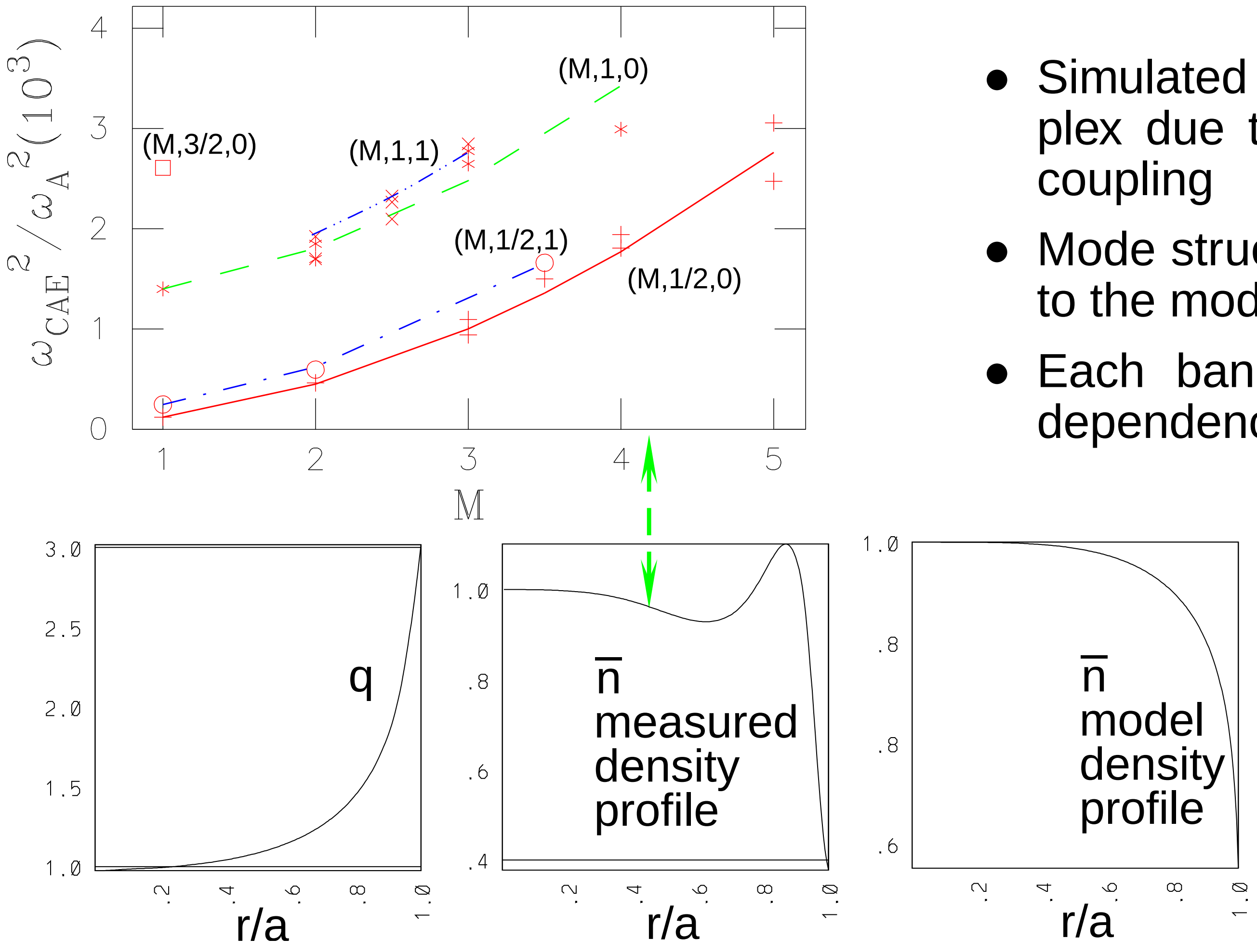
- First radial band is shown $S = 1/2$
- CAEs are localized poloidaly at Low Field Side (LFS) $\theta = 0$
- Typically high-f, high-M modes are more localized
- The same is true for studied $n = 1$ branches



$\Leftarrow$ rms of ξ_n (NOVA)

Realistic density profiles implies double potential well (DIII-D)



- Simulated spectra are complex due to multiple mode coupling
- Mode structures are similar to the model case
- Each band follows $\sim M^2$ dependence

Realistic density profile (non-monotonic) results in CAE radial sub-splitting

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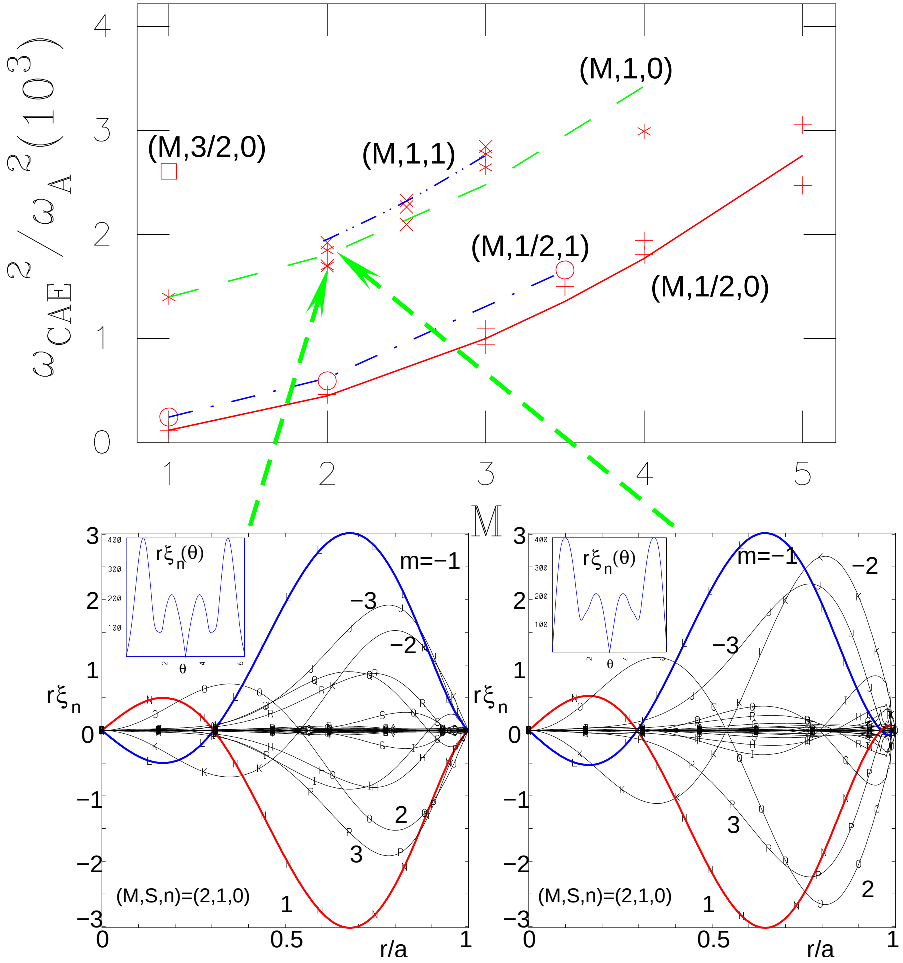
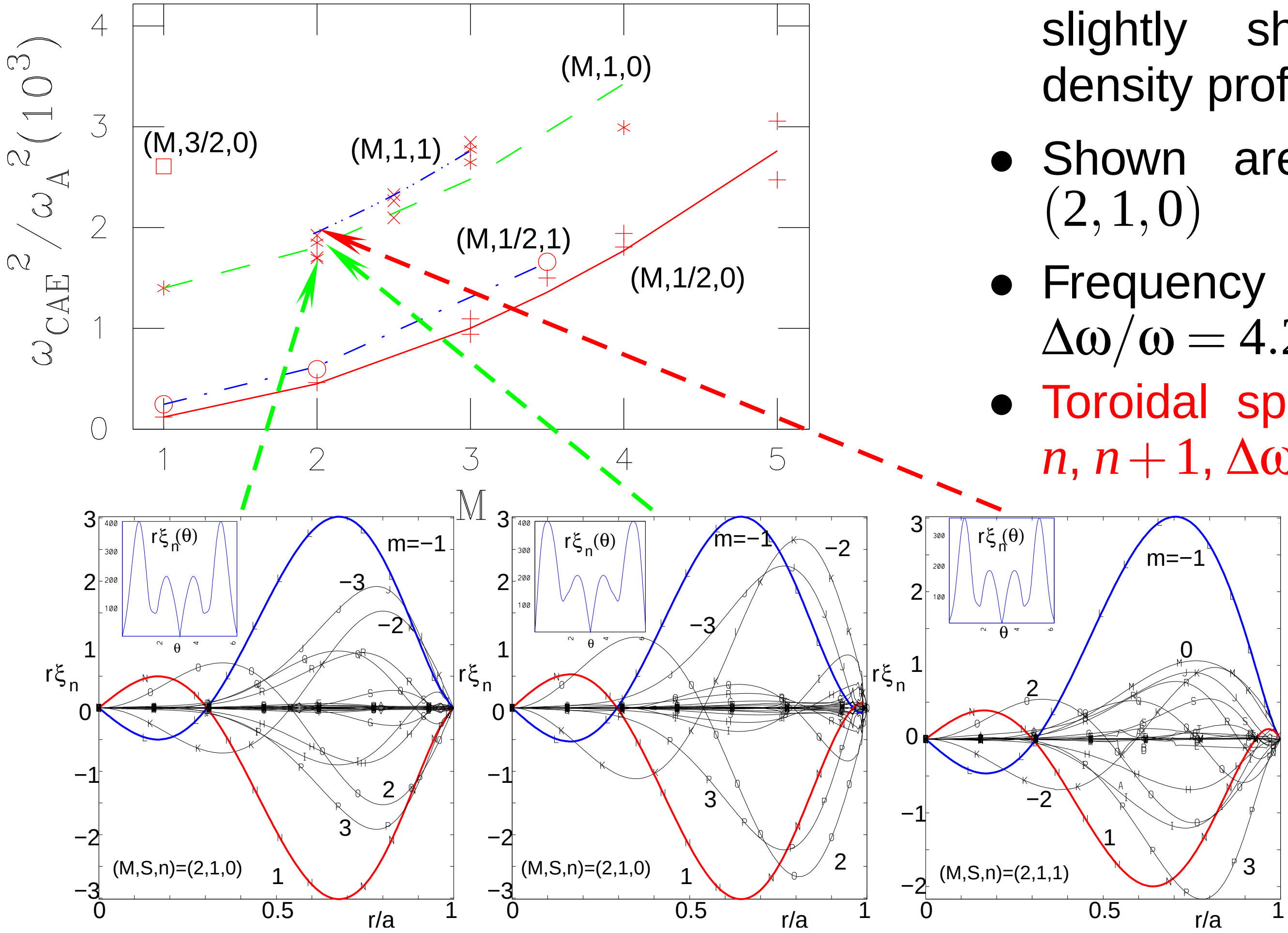


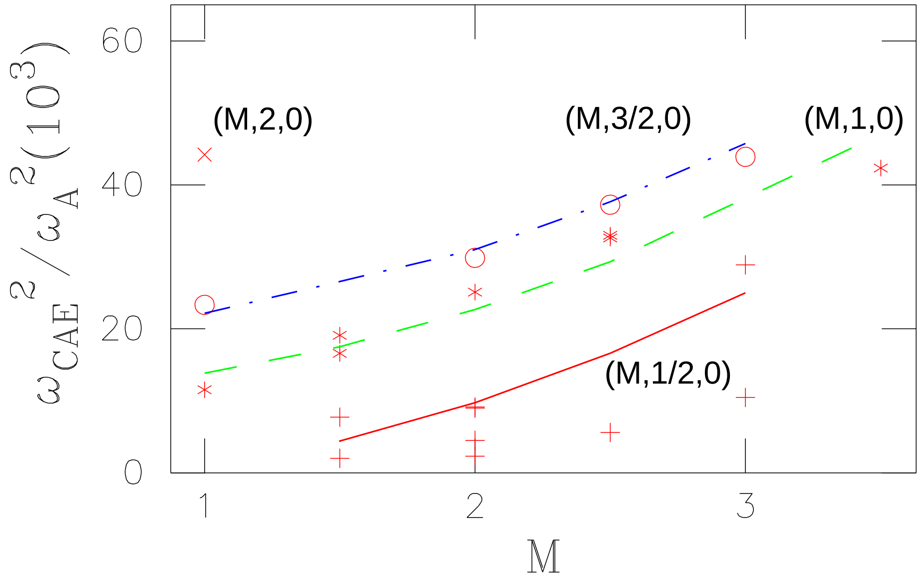
- Radial localization is slightly shifted due to density profile $\Delta r/a \simeq 3\%$
- Shown are modes with $(2,1,0)$
- Frequency “well” splitting $\Delta\omega/\omega = 4.2\%$ (shown)

Realistic density profile (non-monotonic) results in CAE radial sub-splitting

- Radial localization is slightly shifted due to density profile $\Delta r/a \simeq 3\%$
- Shown are modes with $(2,1,0)$
- Frequency “well” splitting $\Delta\omega/\omega = 4.2\%$ (shown)
- Toroidal splitting, between n, $n+1$, $\Delta\omega_n/\omega_{cD0} \simeq 1.3\%$



CAE spectrum in NSTX is rich in simulations



- Similar spectrum as in DIII-D: radial bands, but
  - numerically more challenging due to harmonic coupling
- Within one band sub-splitting even for the monotonic (measured) density
- Extra branch at low frequency is found: geometrical effect?
  - May be explained by cylindrical solutions with different dispersion (Gorelenkova'98)

CAEs are also poloidaly and radially localized in NSTX

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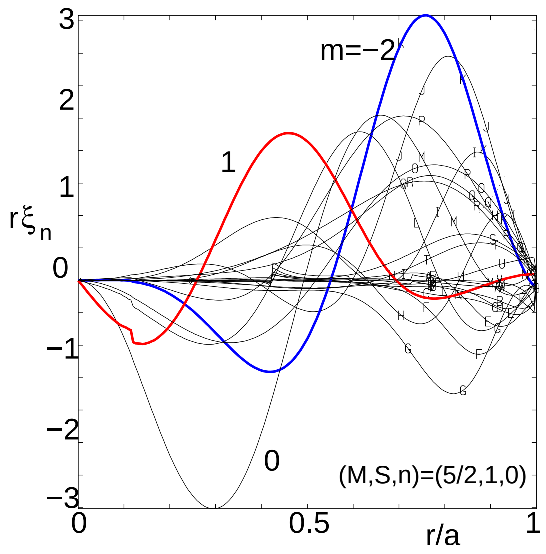

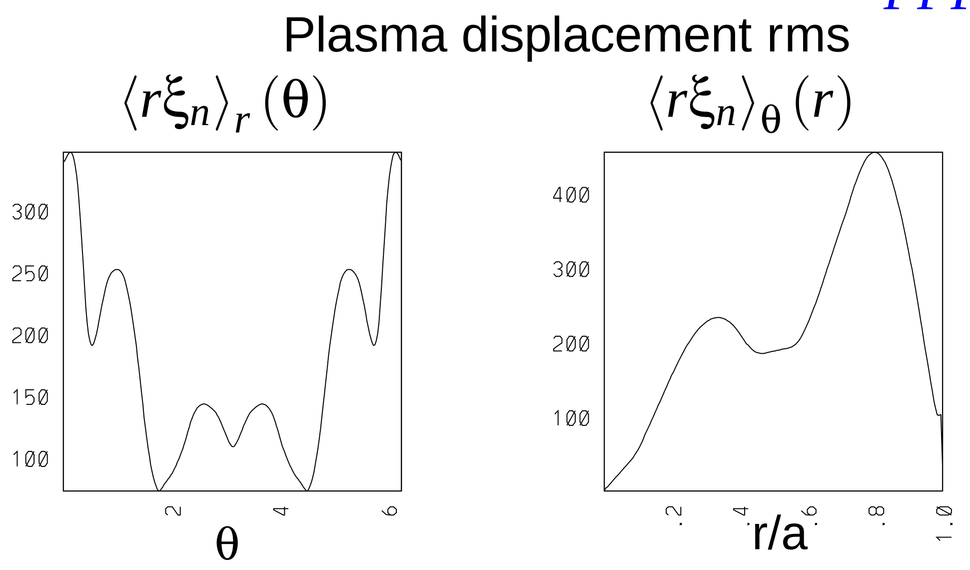
Plasma displacement rms

$\langle r\xi_n\rangle_r(\theta)$ $\langle r\xi_n\rangle_\theta(r)$

Stronger poloidal (cylindrical) harmonic coupling is usually found.

Convergence is still achieved at $m = -12$ to 12.

Reflectometer measurements can validate simulations and theory (NSTX)

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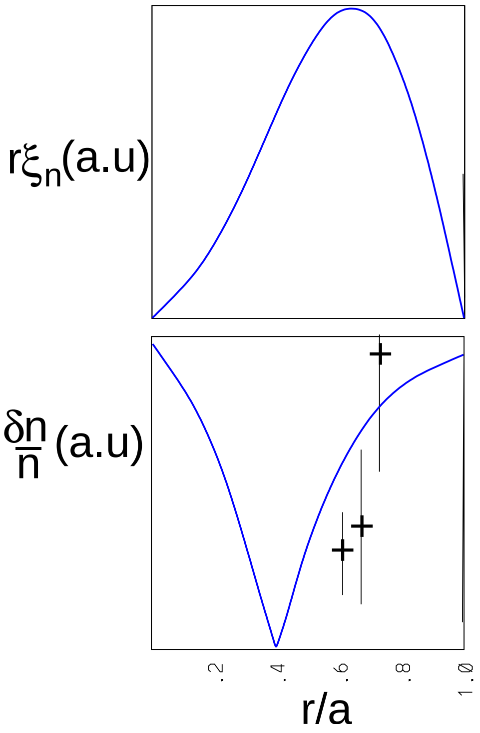

- Shown is internal oscillation measurements (+ marks) of $f = 0.81 MHz$ vs simulated CAE $f = 0.93 MHz$ ($f = 0.81 MHz$ CAE does not agree in structure)
- Hall effect may be needed for better agreement: frequency shift and radial structure change
- Compressional effects are critical: $\delta n/n \propto \xi_n$
- Error bars are large $20 - 60\%$

Simulated CAE spectrum vs. DIII-D experiment

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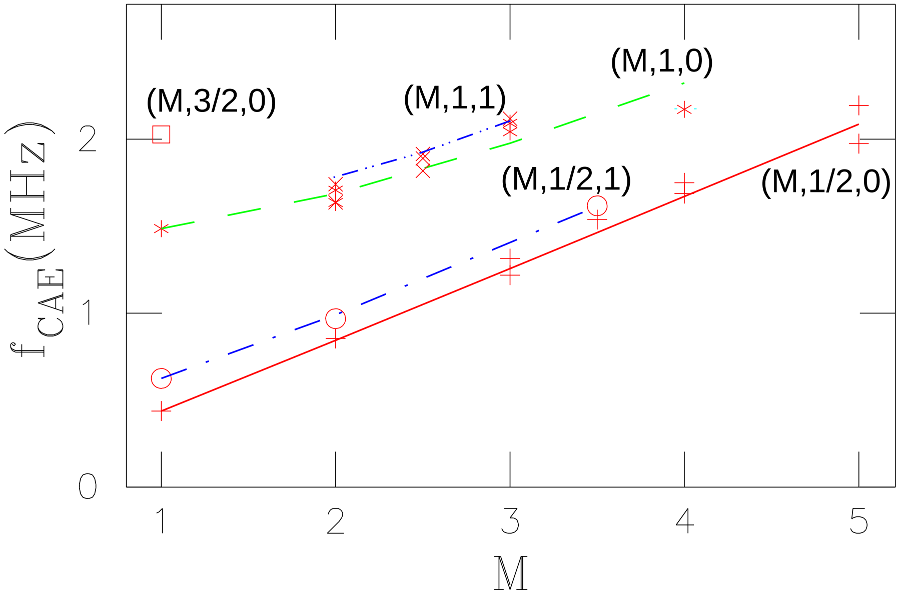

Predictable CAE properties (f separations):

- $\Delta f_S = 0.7 - 1\,MHz$
- $\Delta f_M = 200kHz$ for $S = 1/2$
  $\Delta f_M = 120kHz$ for $S = 1$
- $\Delta f_n = 50kHz$

Experiment (Heidbrink ’05):

- $\Delta f_S \simeq 1MHz$
- $\Delta f_M \simeq 130kHz$
- $\Delta f_n \simeq 30kHz + f_{rot}$,
  $f_{rot} \simeq 20kHz$

Experimental identification of CAEs mode numbers is required for robust comparison with simulations.

Simulated CAE spectrum vs. NSTX experiment

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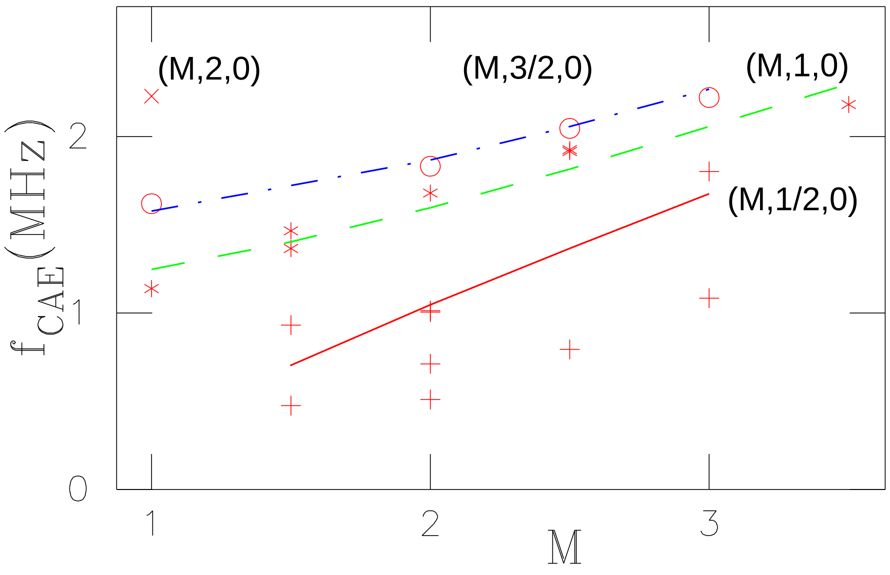

Simulated CAE f separations:

- $\Delta f_S = 0.5 - 1\,MHz$
- $\Delta f_M = 200kHz$
- $\Delta f_n = 50kHz$

Experiment shot #117521:

- $\Delta f_S \simeq 1MHz$
- $\Delta f_M \simeq 120kHz$
- $\Delta f_n \simeq 20kHz + f_{rot}$, $f_{rot} \simeq 20kHz$

Low-*n* number modes have been observed and identified as CAEs.

How theories agree with simulations and experiments?

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NSTX plasma

splitting	experiment	NOVA	Gorelenkov'02	Smith'03
Radial, Δf_S	$\sim 1 MHz$	$0.5-1 MHz$	$\sim 0.7 MHz$	$\sim 0.5 MHz$
Poloidal, Δf_M	$\sim 120 kHz$	$\sim 200 kHz$	$\sim 150 kHz$	$\sim 250 kHz$
Toroidal, Δf_n	$\sim 20 kHz$	$\sim 50 kHz$	$\sim 200 kHz$	$\sim 250 kHz$

- Main discrepancy with theories is in toroidal mode number splitting at low-n,m
- Simulations are in better agreement with the simple dispersion $\omega^2 \sim n^2/R^2$
- f separations of CAEs in DIII-D plasma are similar

HYM hybrid code for nonlinear GAE/CAE study

Model (Linearized and nonlinear):

- Background MHD + kinetic fast ions
- Fluid electrons + particle background ions + fast ions

Method:

- explicit time scheme, 2nd order accuracy
- 4th order spatial derivatives
- particles are treated with the δf scheme
- anisotropic equilibrium (Belova PoP,'03)

HYM simulations show localized CAE structure at high n

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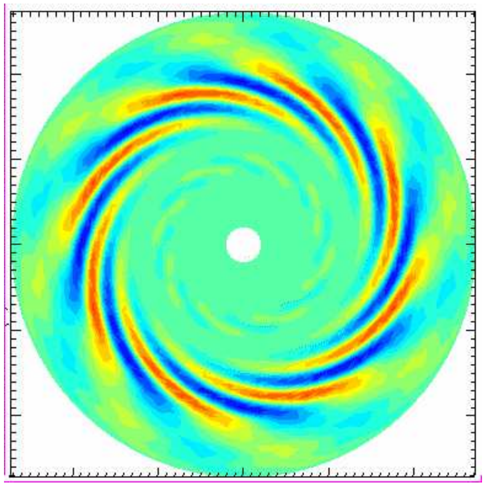
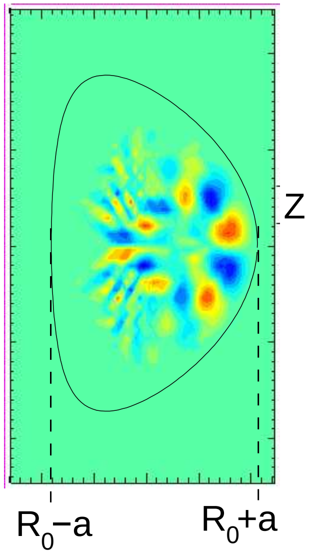


Z=0, $V_\phi\left(\vec{R}\right)$



δP

- Mode has characteristics of CAE with $n = 8, m = 8 - 10$
- $k_{\|} v_{b0}$ gives large Doppler shift $\sim \omega_c/2$
- Compressional polarization: $\delta B_{\|} > \delta B_{\perp}$
- Qualitatively the same poloidal, radial structure as in NOVA

Summary and Future Plans

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- NOVA is capable to confirm poloidal and radial localization of CAE modes in STs and tokamaks predicted by theory
  - Geometrical effects are important for the mode structure and spectrum
  - Simulations show that theory captures qualitative features of CAEs. Still some aspects such as toroidal mode number splitting are not well described.
- Observed sub-cyclotron oscillations in NSTX and DIII-D have been identified as CAEs
- Both hybrid HYM and NOVA codes compute similar CAE mode localization region
- NOVA-K can be used for the stability studies:
  - ω/ω_c and FLR effects need to be included
- HYM will be used for full nonlinear study to address stochastic heating with multiple CAEs