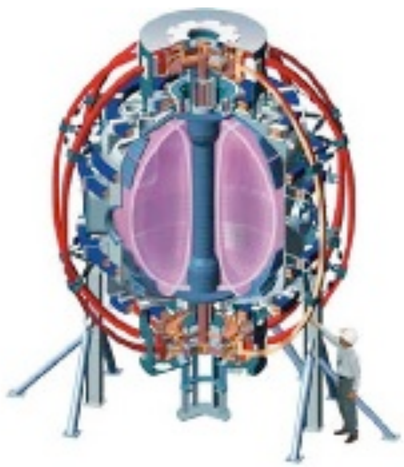
Three-wave interaction of fast ion driven modes in NSTX



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Abstract (Revised)

Three-wave interaction of fast ion driven modes in the National Spherical Torus Experiment[1]

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Fast ions generated in fusion plasmas by heating techniques, and alpha particles in burning plasmas, can excite many types of global modes that degrade fast ion confinement. While the dynamics of such modes have been well studied, nonlinear three-wave interactions have received little attention. Such interactions can transfer energy, leading to damping and excitation. Using core localized reflectometry and edge magnetic measurements in NBI-heated plasmas, we observe three-wave interactions coupling distinctly different types of modes. In particular, we observe interactions between low frequency (~ 15 – 30 kHz), low toroidal mode number (n = 1) energetic particle modes (EPMs) and higher frequency modes (HFMs) in the ~ 75 - 300 kHz range. The HFMs can be of various types, including toroidal Alfvén eigenmodes (TAEs), with mode number of n ≈ 3 - 7, and modes of another currently unknown type with mode numbers in the range n ≈ 5 - 15. The work presented here goes significantly further than previously reported similar observations of modes with frequencies and mode numbers suggesting such interaction [EJ Strait, Plas. Phys. & Cont. Fus. (1994)]. Three-wave interaction is verified using the standard tool, bicoherence. Further analysis is performed, revealing that these interactions typically organize the higher frequency modes into a toroidally localized wave-packet whose envelope is stationary in the EPM toroidal rotation frame.

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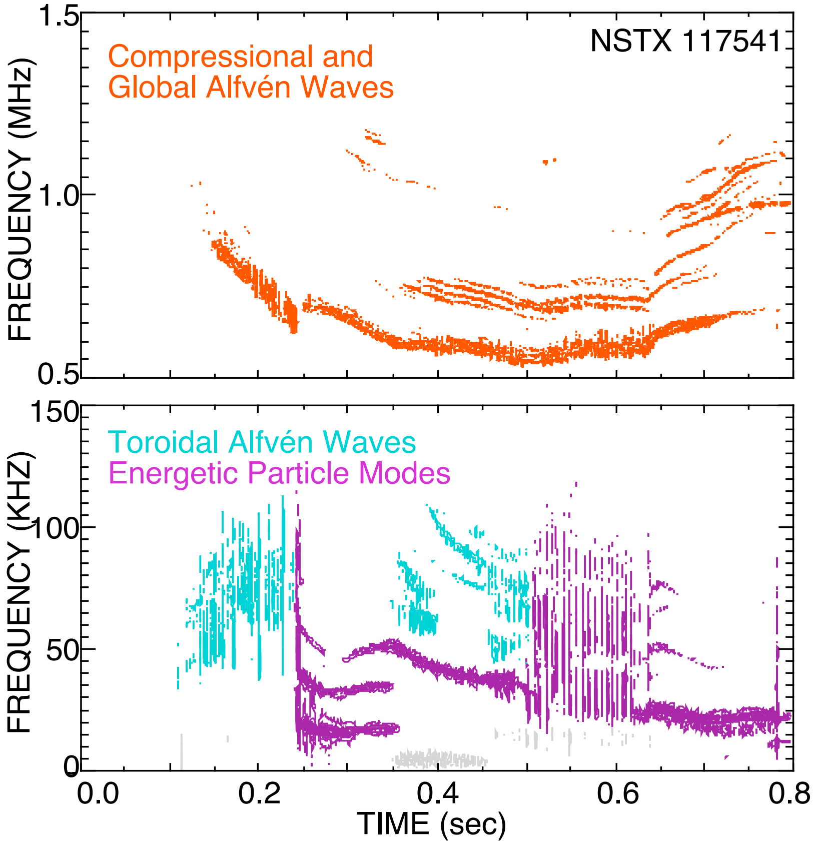
Motivation

- **Fast ions important in fusion plasmas - theoretical understanding desired**
  - Produced by
    - Heating techniques: neutral beams, radio frequency power
    - Fusion products: alpha particles
  - must be confined to heat plasma
  - may excite coherent modes
  - modes may modify orbits, degrading fast ion confinement
- **Three-wave interactions may play important role in fast ion mode dynamics**
  - transfer energy between modes: may excite or damp fast ion driven modes
  - may broaden mode spectrum - possible stochastic fast ion orbits
  - must be accounted for theoretically to predict mode dynamics and effects on fast ions
- **Measurements can probe effects of three-wave coupling**
  - reflectometry probes density perturbation in core.
  - toroidal arrays of Mirnov coils outside plasma - toroidal structure (i.e. mode number).
  - other diagnostics may probe perturbation structure (future work)
  - effects of three-wave coupling visible in perturbation structure

Summary of Results

- **Two categories of modes observed: energetic particle modes (EPMs) and higher frequency modes (HFMs)**
  - EPMs: Harmonics dominated by low frequency and toroidal mode number fundamental (fundamental EPM: f ~ 15 - 30 kHZ kHz, n = 1).
  - HFMs: ~ 75 - 300 kHz; uniformly spaced in frequency and mode number ($\Delta n = 1$):
    - toroidal Alfvén eigenmodes (TAEs), with mode number of n ≈ 3 - 7
    - modes of another currently unknown type with mode numbers in the range n ≈ 5 - 15
  - each type of mode exhibits both bursting and persistent dynamics
- **Three-wave interactions couple EPMs with HFMs**
  - each interacting triplets consists of the dominant EPM and a pair of HFMs
  - frequencies and mode numbers of triplet satisfy "three wave matching conditions"
  - such triplet of modes show strong bicoherence
- **Three-wave interactions organize HFMs and EPMs into coherent structures**
  - EPMs form slowly changing, rigidly toroidally rotating structure
  - HFMs form toroidally propagating, toroidally localized wave packet
  - wave packet envelope is stationary in EPM frame, but carrier is not
- **Three-wave coupled modes can causes fast ion loss**

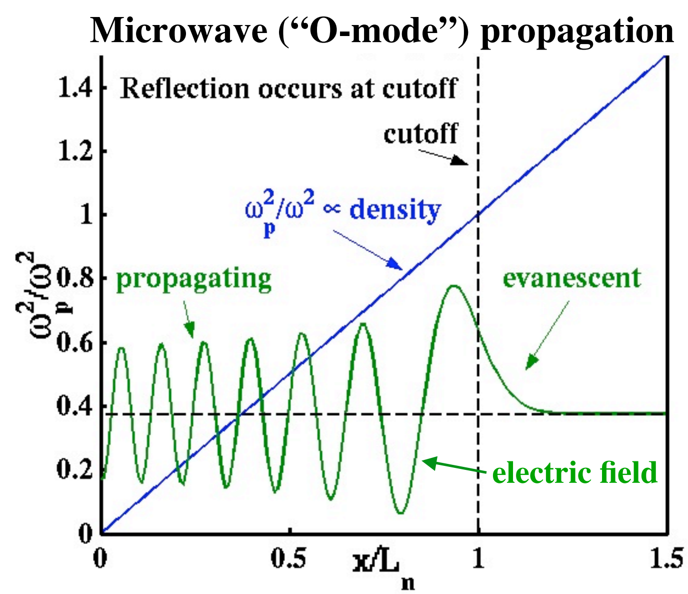
Fast Ion Modes dominate spectrum in NSTX



Fast Ion Modes can be sorted into three categories:

1. CAE/GAE modes (0.4 to > 2 MHz)
   - No evident fast ion losses
   - Fast ion driven; some effect likely
   - CAE also seen in DIII-D, ITER?
2. TAE-like modes (40 - 150 kHz)
   - Ion losses with strong bursts
   - Modes bursting, chirping
   - Present at high β
3. Energetic Particle Modes ($\lesssim$ 50 kHz)
   - Cause most fast ion loss events
   - Extend to TAE frequencies
   - Modes are kink-like
   - Include non-fishbones, n > 1

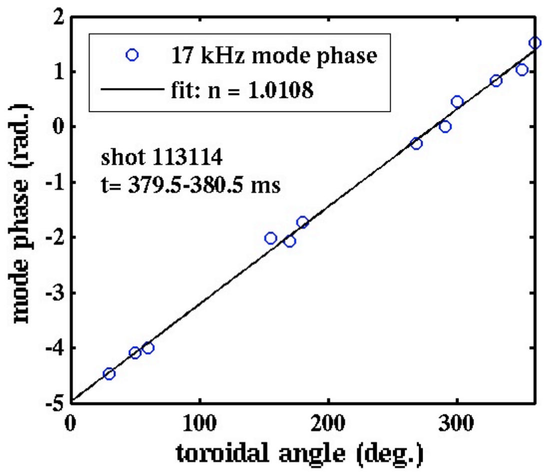
Reflectometer used to probe coherent modes in plasma



- Microwaves with low enough frequency ($\omega < \omega_p$) will reflect from "cutoff" layer in plasma
  - ω_p^2 is proportional to density: $\omega_p^2 = e^2 n_0/\varepsilon_0 m_e$
  - Dispersion relation for "ordinary mode" ("O-mode") microwaves: $\omega^2 = \omega_p^2 + c^2k^2$
  - $k \to 0$ as $\omega \to \omega_p$
  - Microwaves reflect at "cutoff" surface, where $\omega = \omega_p$, $k = 0$
- Reflectometer launches microwaves into plasma and determines relative phase of reflected and launched waves
- Wave propagation controlled by density $\Rightarrow$ phase fluctuations proportional to density fluctuations: $\delta n/n_0 \sim \delta\phi/(2k_{vac}L_n)$

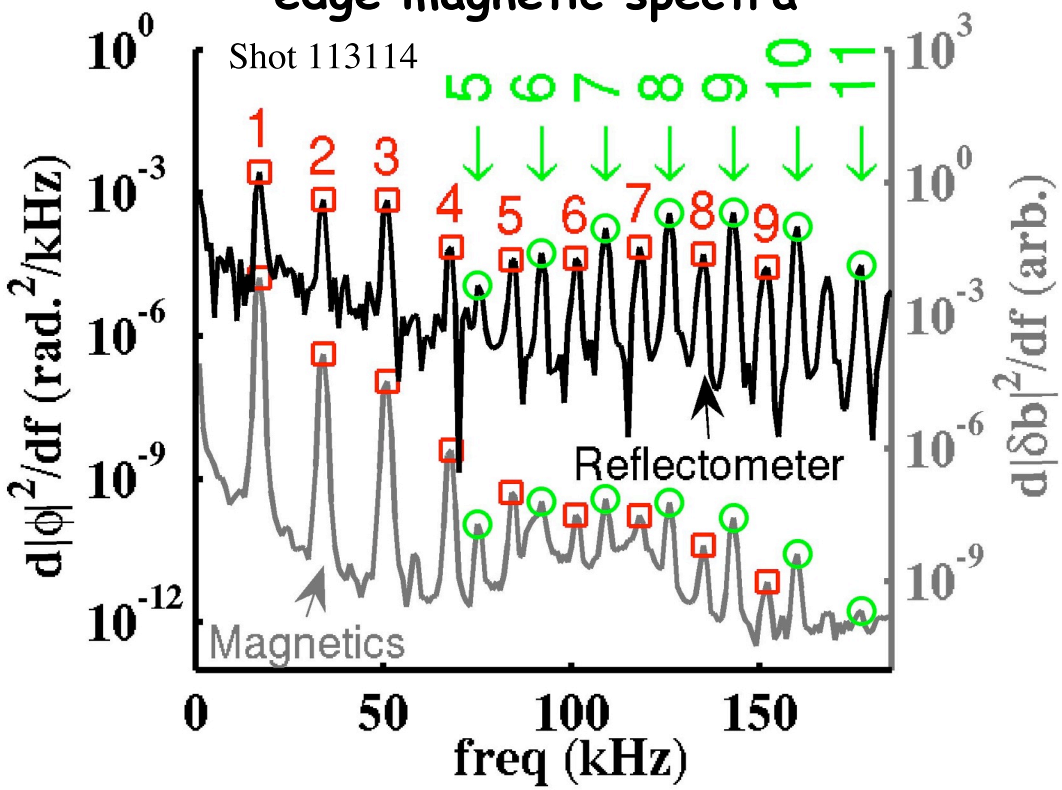
Toroidal mode numbers of coherent modes seen by reflectometer may be determined from Mirnov array

Example of Mode Number Determination



- **Toroidal array of Mirnov coils outside plasma**
  - measure magnetic perturbation
  - modes visible in spectrum
  - mode phase varies with coil position
    ⇒ measure toroidal mode number

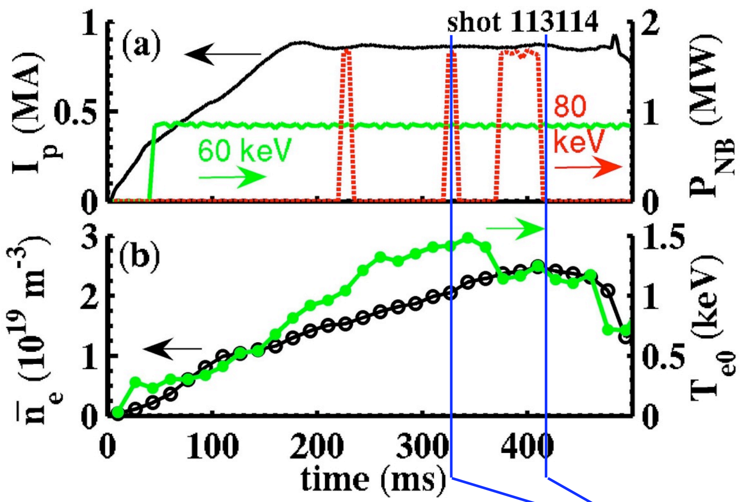
50 GHz reflectometer and edge magnetic spectra

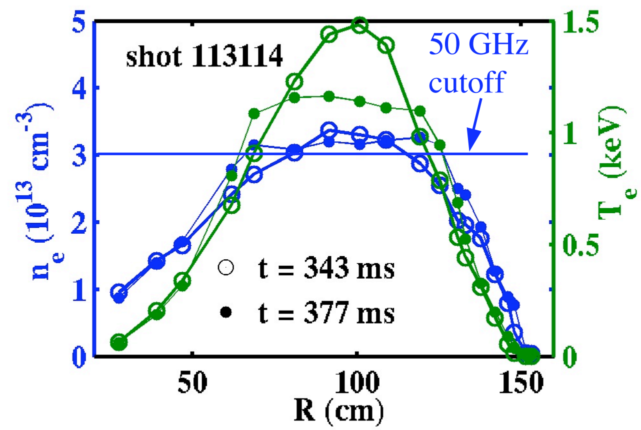


- **Many peaks in refl. spectrum also seen in magnetic spectrum**
  - if peaks in both spectra, mode number of refl. perturbation known
  - At high frequency, some peaks only appear in refl. spectrum: extrapolate mode number from lower frequency modes of same type

NSTX Shot 113114 allows study study three-wave interactions of fast ion modes

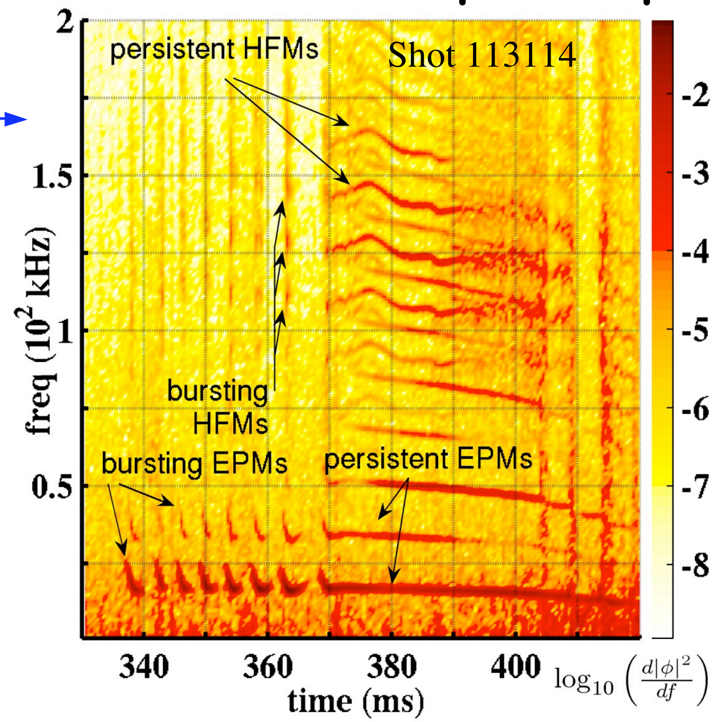
$B_T = 4.4$ kG, $\beta_{T,max} = 11.5\%$





$f_{ROT}(0) < 30$ kHz, $v_A(0) \sim 1 \times 10^6$ m/s

50 GHz reflectometer phase Spectrum

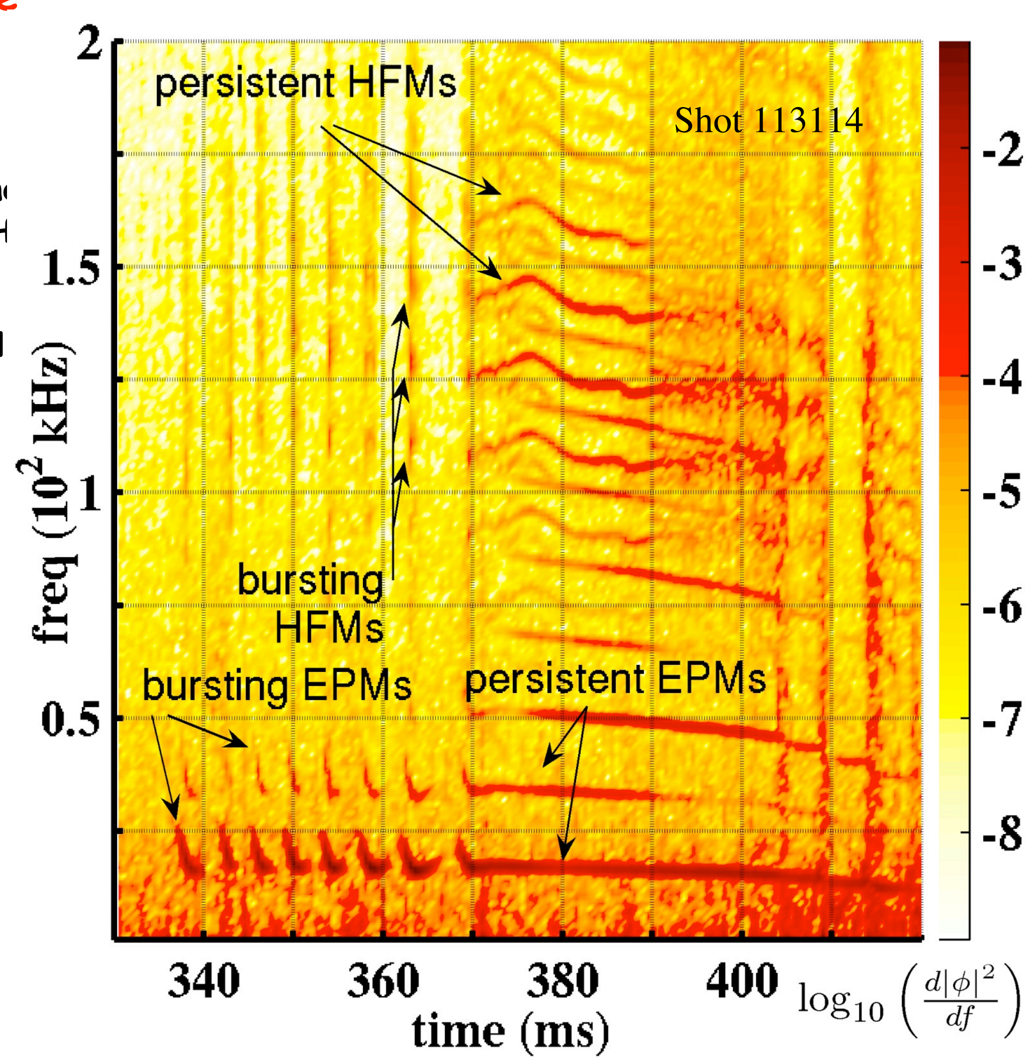


- Rich variety of coherent mode activity
- Different types of modes
- Different kinds of dynamics
- Mode spacing suggestive of three-wave coupling (discussed below)

Shot 113114: Two types of modes and two types of mode dynamics

- **Two types of modes: energetic particle modes (EPM) and higher frequency modes (HFM)**
  - EPM: Harmonics dominated by low frequency an toroidal mode number fundamental - (f ~ 17 kH n = 1).
  - HFM: dominated by high frequency and toroidal mode number modes (n too high for TAEs ⇒ mode identification is future work) (f ~ 125 and 142 kHz, n = 8 and 9).
- **Two types of mode dynamics**
  - before t = 380 ms: Bursting/Chirping
  - after t = 380 ms: Persistent
- **Mode dynamics change, but not mode type**
  - persistent modes emerge from modes of last burst/chirp
  - mode numbers don't change at transition

50 GHz reflectometer phase Spectrum



Three-wave interactions arise from nonlinearities

- Plasma evolution is nonlinear
  - for example, convective derivative in momentum equation (i.e. $v \cdot \nabla v$)

$$\partial v/\partial t + v \cdot \nabla v = \ldots$$

- Quadratic nonlinearities (e.g. $v \cdot \nabla v$) influence spectrum thru three-wave interaction
  - for example, equation for velocity spectrum (derived from momentum equation):

$$\langle |v_{\omega,k}|^2 \rangle = \mathrm{Re}\Big(\omega^{-1} \Sigma_{\omega',k'}\, k' \cdot \langle v_{\omega',k'} v_{\omega-\omega',k-k'} \cdot v^*_{\omega,k} \rangle\Big) + \ldots$$

($\langle\ \rangle$ denotes average)

Three-wave interaction term

- Possible interactions constrained
  - for waves (ω',k') and (ω'',k'') to drive (ω,k), must satisfy matching conditions:

"Energy conservation": $\omega = \omega' + \omega''$

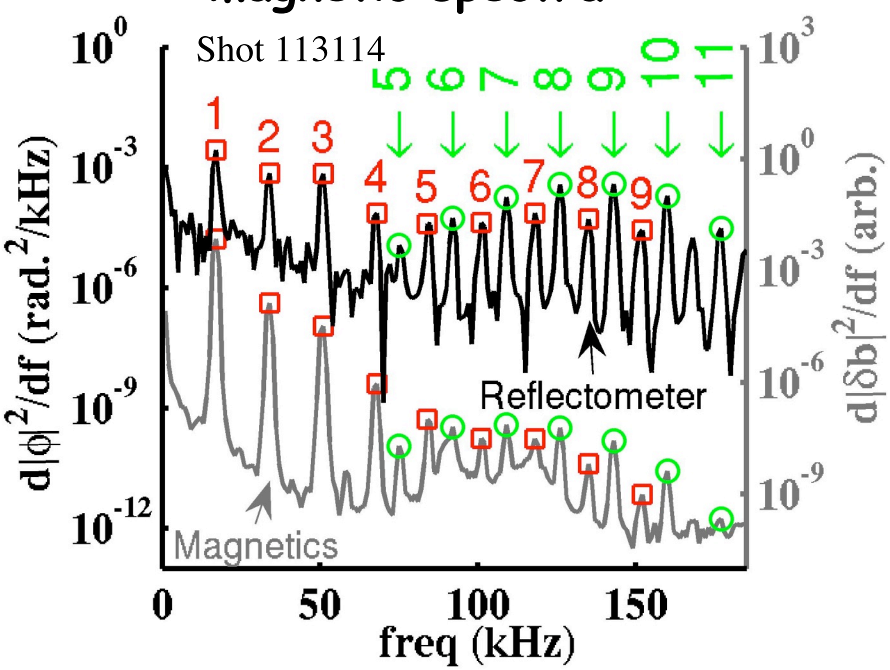
"Momentum conservation": $k = k' + k''$

Frequencies and mode numbers suggest three-wave interaction

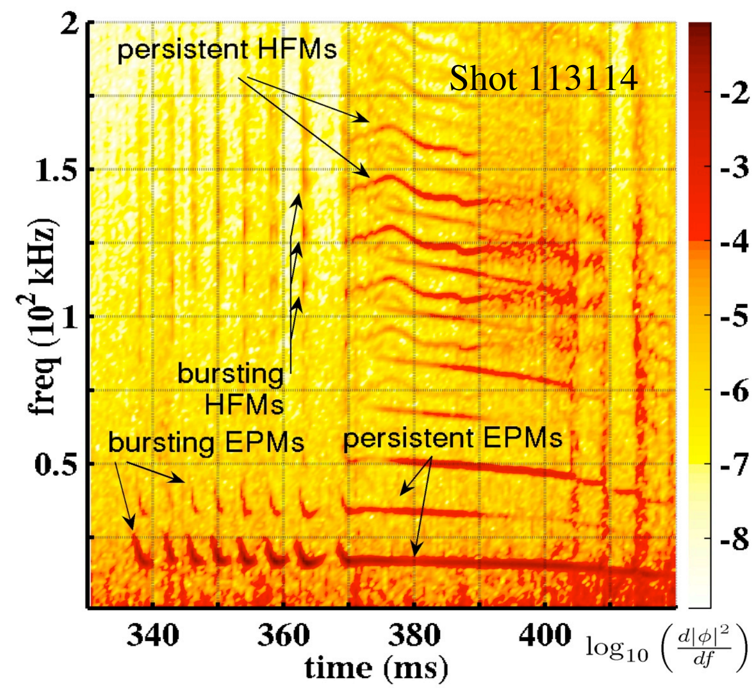
- **Certain mode triplets satisfy energy and momentum conservation:**
  - each triplet composed of dominant EPM and a pair of neighboring HFMs
  - frequency and mode number spacing of HFMs match frequency and mode number of dominant EPM (i.e. $\Delta f_{HFM} = f_{EPM}$ and $\Delta n_{HFM} = n_{EPM}$)
  - for pair of neighboring HFMs, (f,n) and (f',n'):

$$f' = f + \Delta f_{HFM} = f + f_{EPM} \text{ and } n' = n + \Delta n_{HFM} = n + n_{EPM}$$

50 GHz reflectometer and edge magnetic spectra



50 GHz reflectometer phase spectrum



"Bicoherence" tests for three-wave interactions

- **Modes "extracted" from point measurement, $x(t)$:**
  - determine mode frequency, $f(t)$, using Fourier transform in moving window
  - using $f(t)$ and filtering techniques, extract $\psi(t) = A(t)\exp(i\phi(t) + i2\pi\int_0^t f(t')\,dt')$ from $x(t)$, where $\phi(t)$, $A(t)$ are mode phase and amplitude.
- **Bicoherence determined for modes that satisfy "energy conservation"**
  - when mode frequency varies slowly over time, "energy conservation" is $f(t) \approx f'(t) + f''(t)$
  - for waves satisfying "energy conservation" ψ, ψ' and ψ'', bicoherence given by

$$B[\psi,\psi',\psi''] = |\langle\psi'\psi''\psi^*\rangle| / (\langle|\psi'\psi''|^2\rangle\langle|\psi|^2\rangle)^{1/2}$$

- **Bicoherence = coherence between a wave and the quadratic nonlinearity that excites or damps it**
  - bicoherence is coherence between $\psi'\psi''$ and ψ.
  - $B > 0$ implies net either exciting or damping
    - Whether exciting or damping depends on phase of "Bispectrum" $S[\psi,\psi',\psi''] = \langle\psi'\psi''\psi^*\rangle$
    - B ranges from 0 to 1 $\Rightarrow$ effective measure of "strength" of interaction

High bicoherence confirms three-wave interactions

- Mode triplets that satisfy matching condition show high bicoherence ⇒ confirms three-wave interaction
  - Mode frequencies determined with 1 ms resolution
  - Modes extracted with 5 kHz bandwidth around mode frequency
  - Bicoherence calculated from t = 369.5 to 394 ms
  - Bicoherence of white noise would be 1/sqrt(24.5 ms x 5 kHz) ~ .09

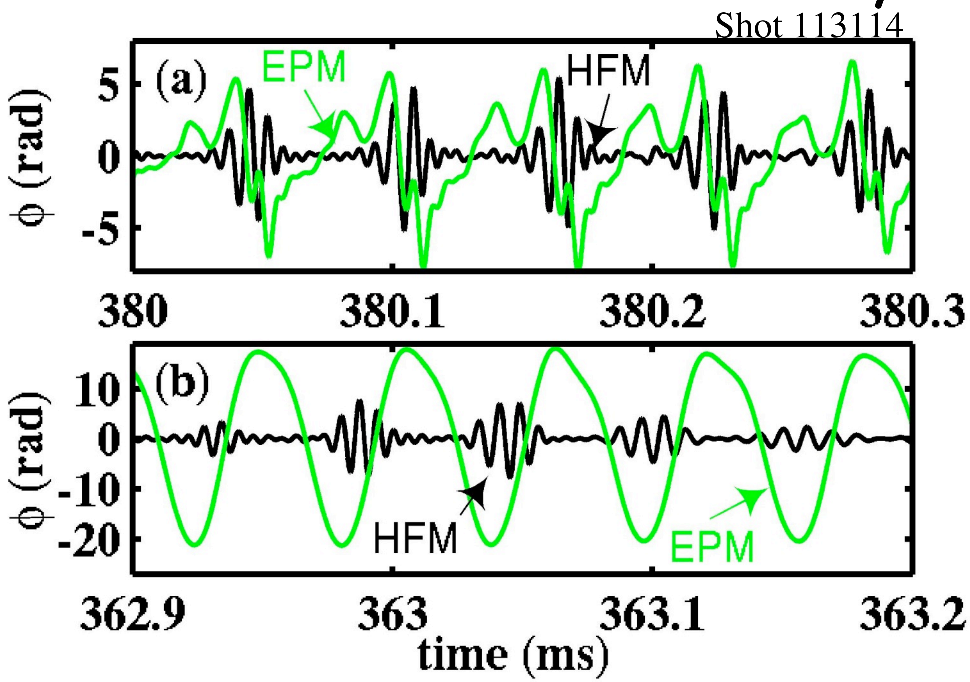
Bicoherence of mode triplets
(i.e. $B[HFM_1, EPM_{n=1}, HFM_2]$)

n of HFM_1	n of HFM_2	Bicoherence (50 GHz)	Bicoherence (42 GHz)
5	6	0.3117	0.4333
6	7	0.561	0.7691
7	8	0.6497	0.8816
8	9	0.6451	0.8841
9	10	0.6257	0.8458
10	11	0.6389	0.7182
11	12	0.4055	0.5985

(noise level ~ .09)

HFMs form "wave packet" in EPM rotation frame

Total HFM and total EPM as seen by 50 GHz reflectometer



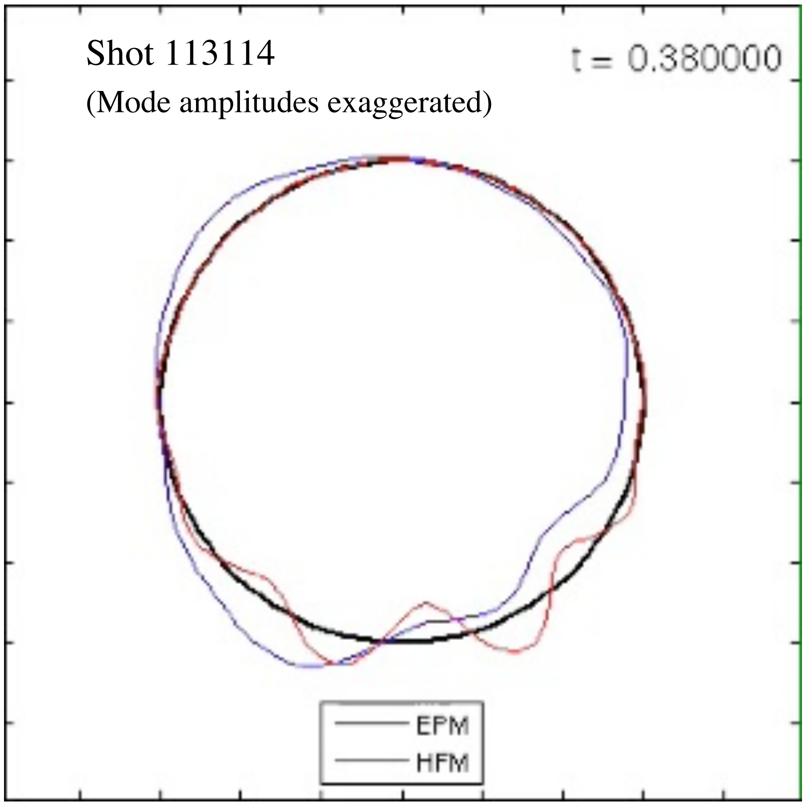
Persistent modes
(t > 380 ms)

Bursting/Chirping modes
(t < 380 ms)

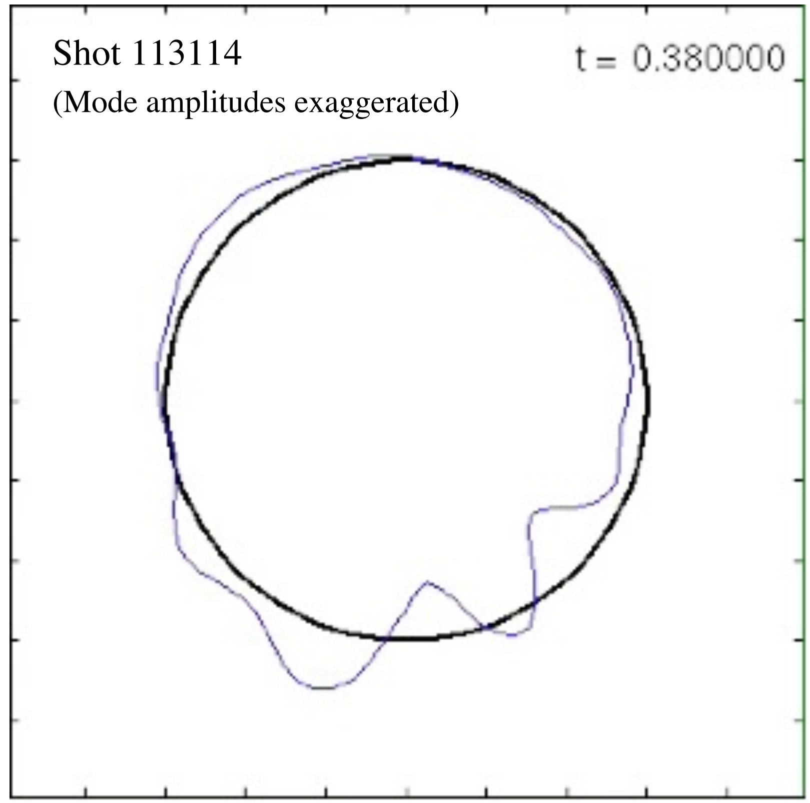
- **EPMs form slowly varying, rigidly toroidally rotating perturbation**
  - EPMs are toroidal and frequency harmonics ⇒ uniform phase velocity
- **HFMs form wave packet: envelope stationary in EPM frame, carrier not**
  - frequency and toroidal mode number spacing uniform ⇒ well defined group velocity
  - HFM group velocity same as EPM phase velocity (implied by three-wave interactions)
  - wave packet has "carrier" phase velocity which differs from group velocity

What do HFMs and EPMs look like in toroidal geometry?

Total HFM and total EPM

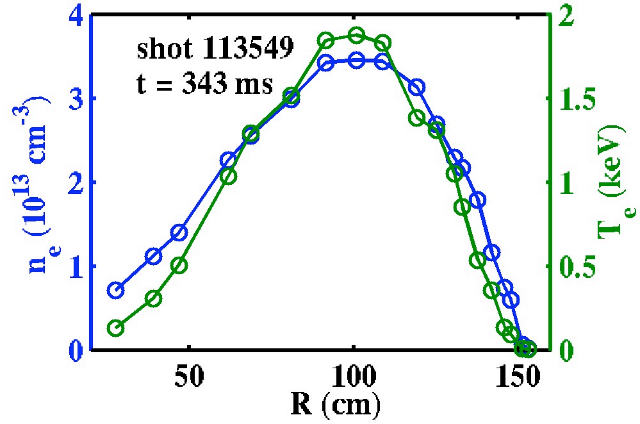


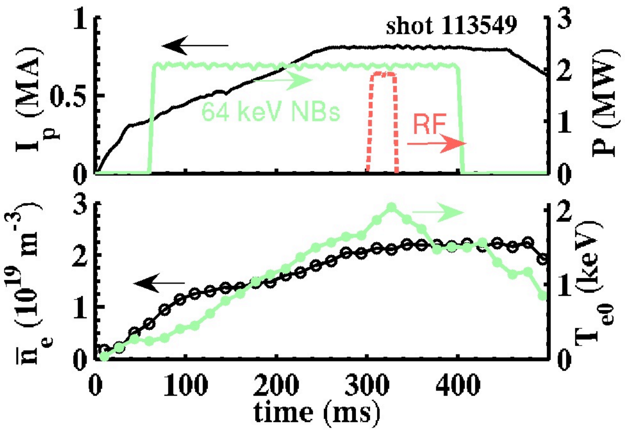
Total HFM + Total EPM
(total coherent perturbation)

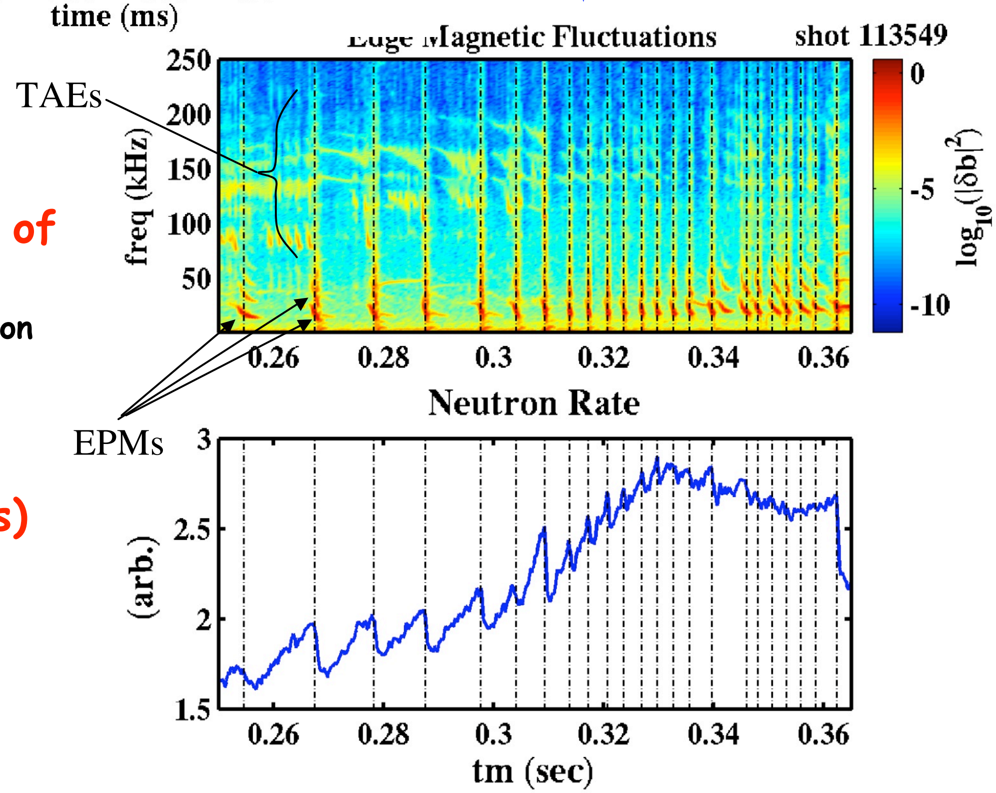


- **Left: Total EPM (blue) and total HFM (red)** [not to scale]
  - EPM is slowly changing, rigidly rotating perturbation
  - HFM is localized wave packet: envelope stationary in EPM frame, but carrier not
- **Right: Total EPM + Total HFM (total coherent perturbation)** [not to scale]

NSTX shot 113549 allows study of role of three-wave interactions in fast ion loss







- Fast ion mode activity can cause loss of fast ions
  - Neutron rate sensitive to neutral beam-fast ion fusion reactions: drop ⇒ loss of fast ions
- Bursts of mode activity include EPMs and toroidal Alfvén eigenmodes (TAEs)

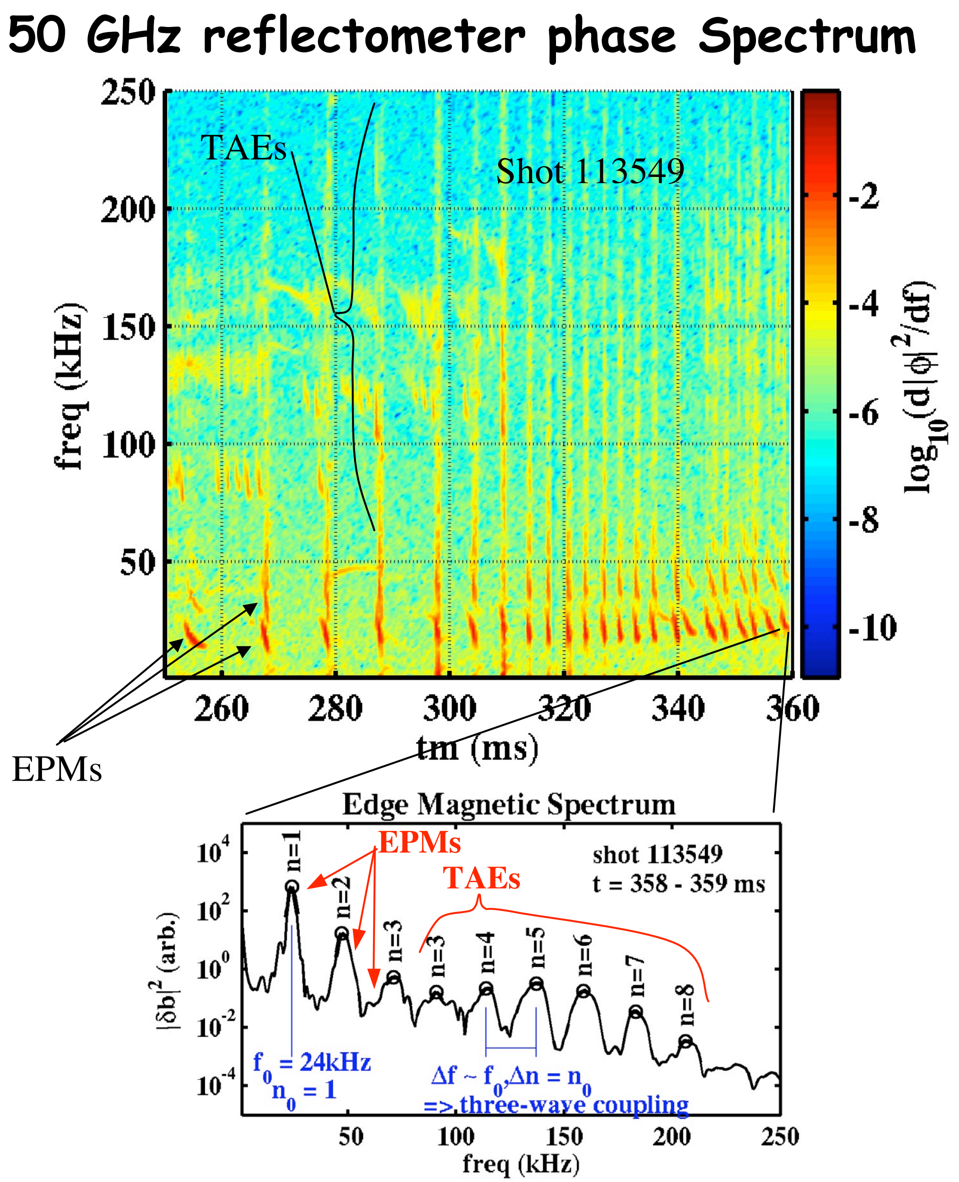
EPMs couple to TAEs thru three-wave interactions

- **Two types of modes: EPMs, TAEs**
  - EPM: Harmonics with low frequency and toroidal mode number: (f ~ 25 kHz, 50 kHz, n = 1,2).
  - TAEs: higher frequencies and mode numbers: (f ~ 75 - 225 kHz, n = 3 - 8);
    - uniformly spaced in frequency and mode number (Δf ~ 25 kHz, Δn = 1).
- **Three-wave interactions couple low frequency EPM (n = 1) to pairs of TAEs:**
  - Pairs of neighboring TAEs satisfy f and n matching requirements to couple with low frequency EPM
  - Triplet that satisfy matching requirements show high bicoherence

Bicoherence of mode triplets
(i.e. B[TAE_1,$EPM_{n=1}$, TAE_2])
(t = 345 - 360 ms; ~ 20 kHz bandwidth)

n of TAE_1	n of TAE_2	Bicoherence (50 GHz)	Bicoherence (42 GHz)
4	5	0.5587	0.3865
5	6	0.603	0.4423
6	7	0.5745	0.4341

(noise level ~ .06)



Conclusions

- **Two categories of modes observed: energetic particle modes (EPMs) and higher frequency modes (HFMs)**
  - EPMs: Harmonics dominated by low frequency and toroidal mode number fundamental (fundamental EPM: f ~ 15 - 30 kHZ kHz, n = 1).
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  - wave packet envelope is stationary in EPM frame, but carrier is not
- **Three-wave coupled modes can cause fast ion loss**

Future Work

- **More detailed structure mode measurement:**
  - use different microwaves frequencies simultaneously for radial structure
  - explore use of other diagnostics to determine poloidal structure
- **Test predictions of mode structure (EPMs and TAEs) against measurements**
- **Identify HFMs in shot 113114**
  - compare structure with theory - consistency? (test theory and identifies mode)
  - identify fast ions which excite modes
- **Determine direction of energy transfer in three-wave interactions:**
  - in absence of interactions, are either EPMs or HFMs/TAEs stable?
- **Determine effect of modes on equilibrium and fast ion confinement**

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