New MHD modes below geodesic acoustic frequency and beam driven instabilities in NSTX

N. N. Gorelenkov[†]

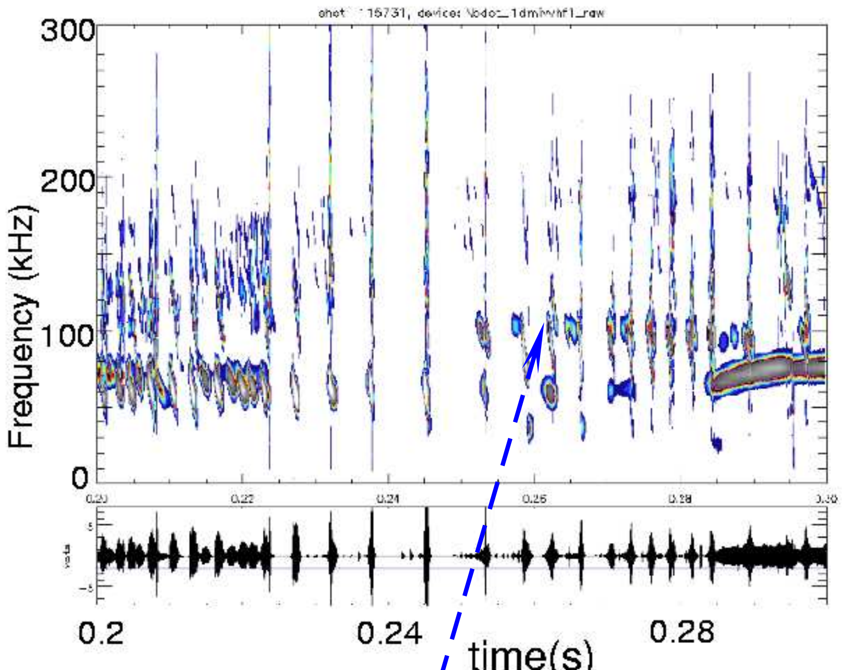
in collaboration with
H. L. Berk[‡], E. Fredrickson[†]

[†]*Princeton Plasma Physics Laboratory, Princeton*
[‡]*Institute for Fusion Studies, Austin, Texas*

2006 48th DPP APS meeting, 1 November, 2006, Philadelphia

Multiple MHD instabilities are routinely observed in NSTX

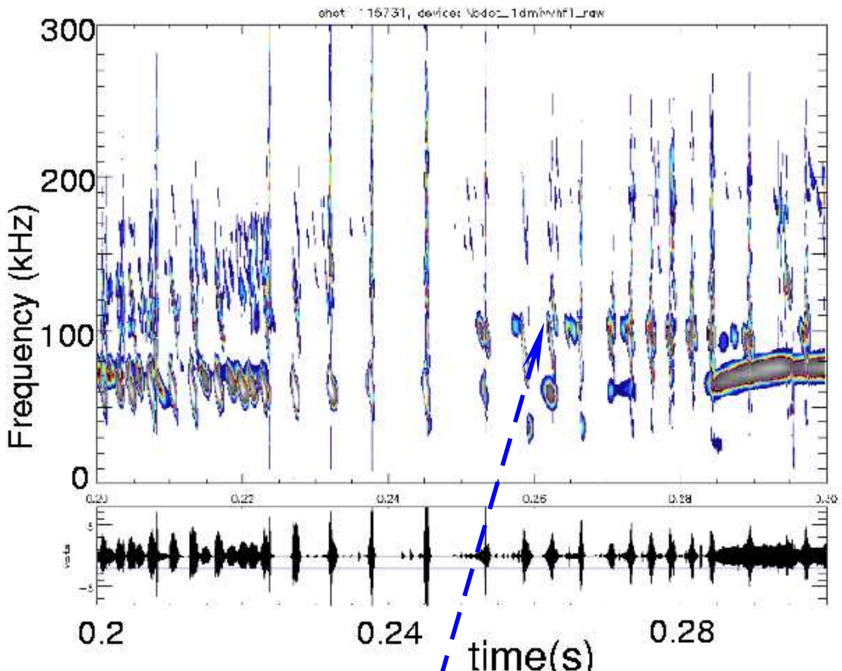
Mirnov activity @ reversed shear #115731 ($f_{TAE} = v_A/2qR \simeq 90kHz$)



At $t = 0.262sec$, $n = 2$ mode frequency $f_{lab} \simeq 103\,kHz$,
$f_{pl} \simeq f_{lab} - n \times 30 = 43\,kHz$, $\Rightarrow$ too low for TAE for all observed modes.

Multiple MHD instabilities are routinely observed in NSTX

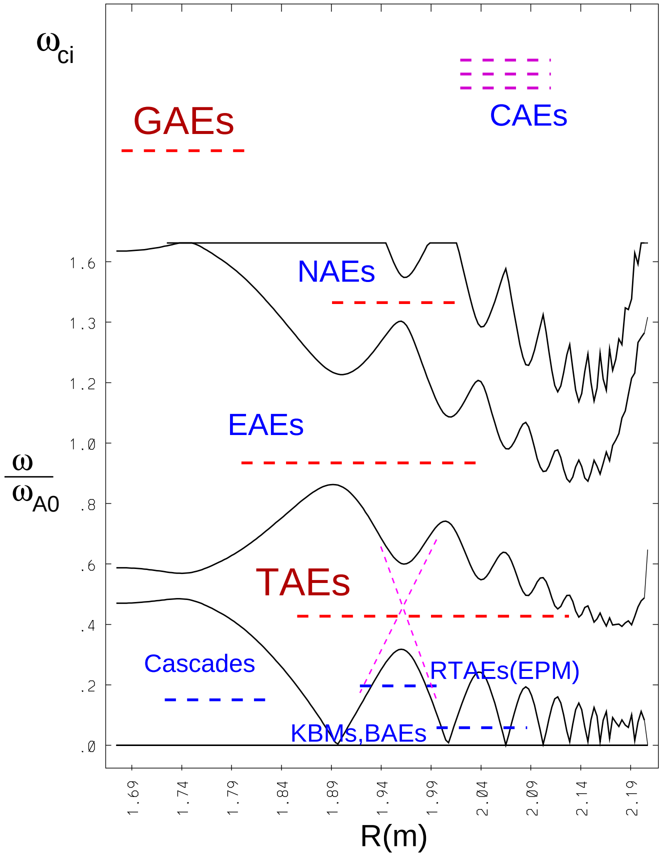
Mirnov activity @ reversed shear #115731 ($f_{TAE} = v_A/2qR \simeq 90kHz$)



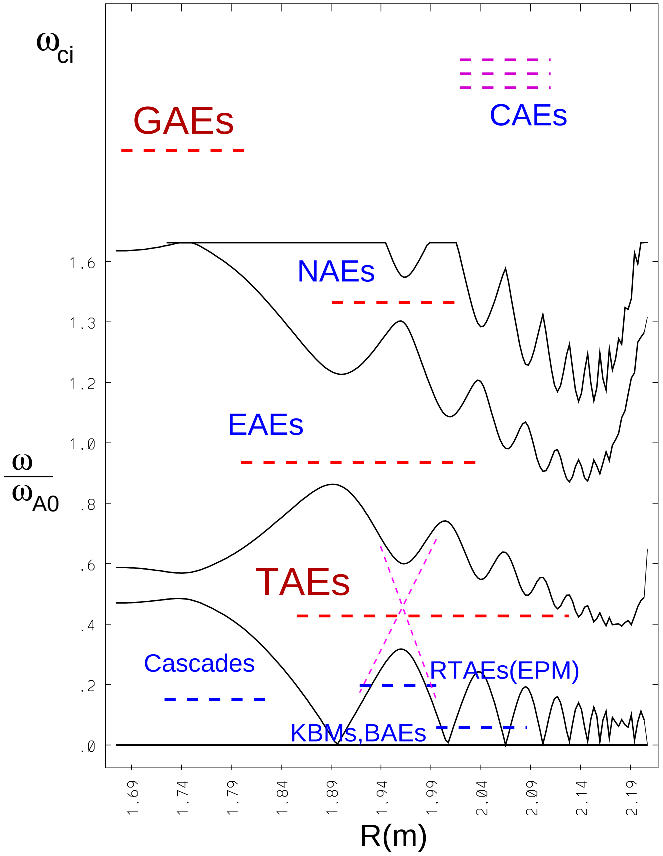
At $t = 0.262sec$, $n = 2$ mode frequency $f_{lab} \simeq 103\,kHz$,
$f_{pl} \simeq f_{lab} - n \times 30 = 43\,kHz$, $\Rightarrow$ too low for TAE for all observed modes.

What are these modes: EPMs, fishbones, KBM, TAEs?

Typical AE spectrum



Typical AE spectrum



- Simple, commonly used estimate

$$\omega_{TAE} = \frac{v_A}{2qR} \Leftarrow k_{\|m} = -k_{\|m+1}$$

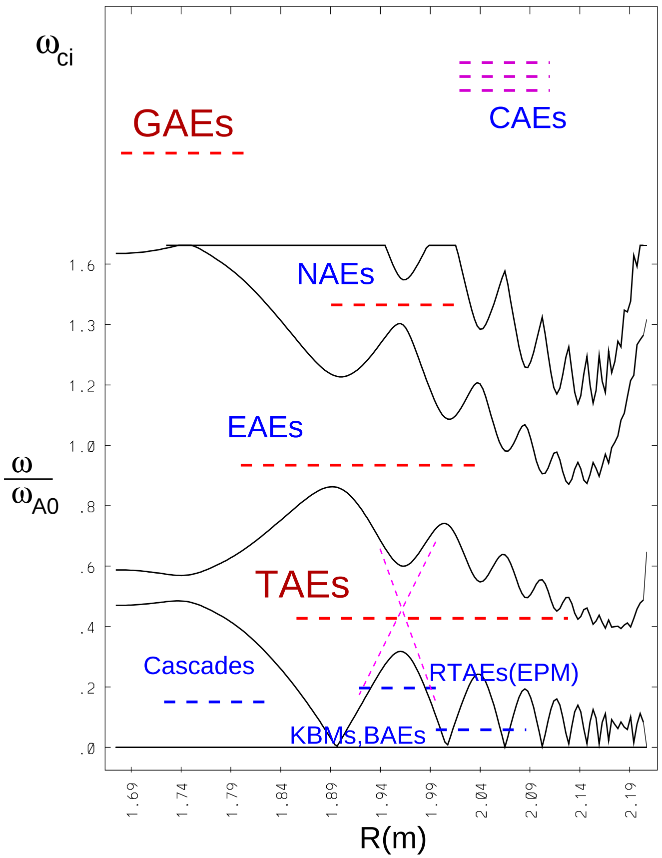
- For NSTX #115731 with reversed q-profile, $q_{min} = 1.3$, we find

$$\omega_{TAE} \simeq 90kHz.$$

- Rotation is important factor in mode identification in NSTX

$$f_{pl} = f_{lab} - n f_{rot}$$

Typical AE spectrum



- Simple, commonly used estimate

$$\omega_{TAE} = \frac{v_A}{2qR} \Leftarrow k_{\|m} = -k_{\|m+1}$$

- For NSTX #115731 with reversed q-profile, $q_{min} = 1.3$, we find

$$\omega_{TAE} \simeq 90kHz.$$

- Rotation is important factor in mode identification in NSTX

$$f_{pl} = f_{lab} - n f_{rot}$$

- In general low frequency modes are more effective in radial EP transport.

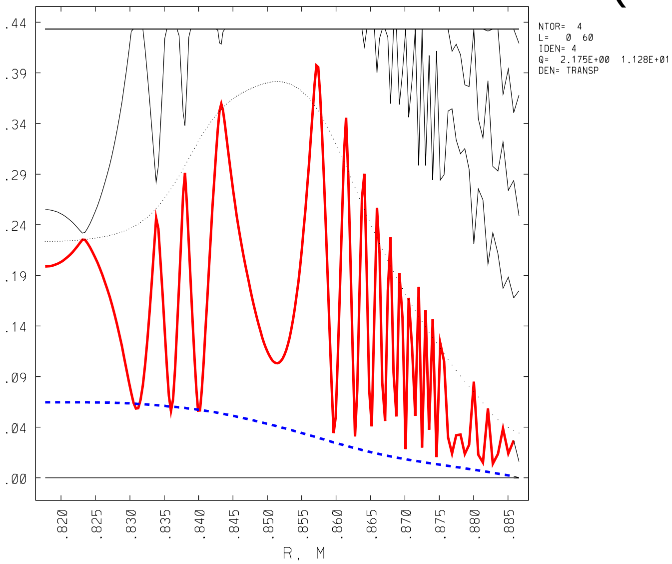
Finite continuum structure at high aspect ratio, low-β

Example NSTX reversed shear profile with $R_0 = 0.806m$, but with the aspect ratio $A = 10$, $a = 0.081m$, $\beta \equiv 2p/B_0^2 = 0.3\%$, $q_a = 11.3$, $q(0) = 2.16$, $q_{min} = 1.3$, $\gamma = 5/3$ (if applied)

Finite continuum structure at high aspect ratio, low-β

Example NSTX reversed shear profile with $R_0 = 0.806m$, but with the aspect ratio $A = 10$, $a = 0.081m$, $\beta \equiv 2p/B_0^2 = 0.3\%$, $q_a = 11.3$, $q(0) = 2.16$, $q_{min} = 1.3$, $\gamma = 5/3$ (if applied)

Acoustic filtered continuum (Chu'92)

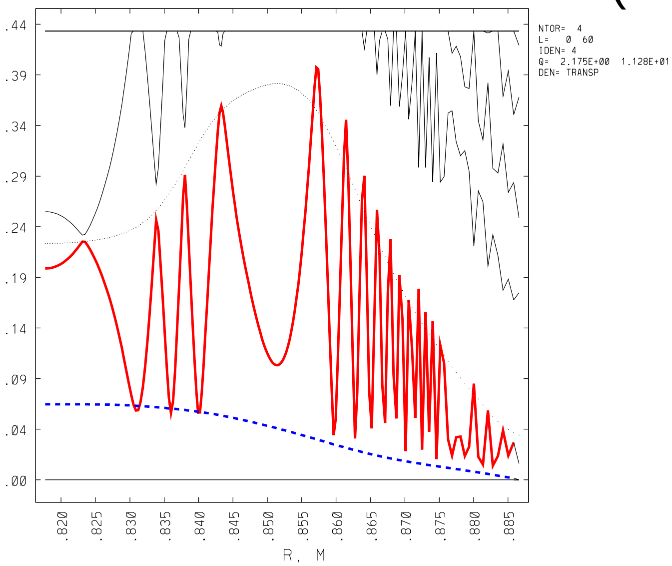


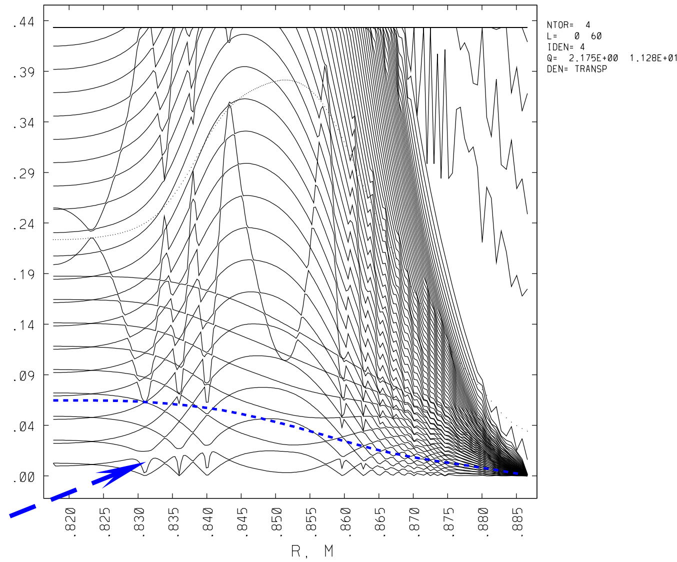
Numerical analysis is very much simplified with filtering (singularities are avoided).

Finite continuum structure at high aspect ratio, low-β

Example NSTX reversed shear profile with $R_0 = 0.806m$, but with the aspect ratio $A = 10$, $a = 0.081m$, $\beta \equiv 2p/B_0^2 = 0.3\%$, $q_a = 11.3$, $q(0) = 2.16$, $q_{min} = 1.3$, $\gamma = 5/3$ (if applied)

Acoustic filtered continuum (Chu'92) vs Full MHD continuum





Numerical analysis is very much simplified with filtering (singularities are avoided).

With compressibility, gaps emerge at the bottom of BAE gap $f \sim \sqrt{\beta}$ (Hausmans'95, Mikhailovski'75,'98)

Theory for Alfven/acoustic continuum in low-β large aspect ratio equilibrium

Simplifi ed shear Alfvén and acoustic equations (from Cheng, Chance '86):

$$\begin{aligned} \Omega^2 y + \partial_\parallel^2 y \quad & + \gamma\beta \sin\theta z & = 0 \\ \Omega^2 \left(1 + \frac{\gamma\beta}{2}\right) z + \frac{\gamma\beta}{2} \partial_\parallel^2 z \quad & + 2\Omega^2 \sin\theta y & = 0, \end{aligned} \qquad (1)$$

where $\Omega \equiv \omega R_0 / v_A$, $y \equiv \xi_s \varepsilon / q$, $\xi_s \equiv \vec{\xi} \cdot \frac{[\mathbf{B} \times \nabla\psi]}{|\nabla\psi|^2}$ and $z \equiv \nabla \cdot \vec{\xi}$, $\hat{k}_\parallel \equiv i\partial_\parallel$.

Various solutions follows:

- Pure Alfvénic branch $\Omega^2 = k_\parallel^2 + \gamma\beta \left(1 + 1/2q^2\right)$ (Chu'92, Breizman'05, Berk'06, Turnbull '92).
- Pure acoustic modes (AMs) $\Omega^2 = \frac{1}{2}\gamma\beta k_\parallel^2$. (Goedbloed'75)
- GAMs: $\Omega^2 = \gamma\beta \left(1 + 1/2q^2\right)$ (Winsor'68, Breizman'05, Berk'06) in the assumption of $\Omega^2 \geq \gamma\beta$.
- Modifi ed shear Alfvén branch exist for $\Omega^2 \ll \gamma\beta$.

Alfvén/acoustic continuum contains gaps due to geodesic curvature coupling

- Alfvén-like continuum branch emerges at low frequency: $\Omega^2 = k_0^2 / \left(1 + 2q^2\right)$, (Mikhailovskii,'75,'98, Smolyakov'06).
- It couples with two acoustic sidebands at $\Omega^2 = \gamma\beta k_{\pm1}^2 / 2\left(1 + \delta\right)$ and creates a new gap.

Alfvén/acoustic continuum contains gaps due to geodesic curvature coupling

- Alfvén-like continuum branch emerges at low frequency: $\Omega^2 = k_0^2/\left(1+2q^2\right)$, (Mikhailovskii,'75,'98, Smolyakov'06).
- It couples with two acoustic sidebands at $\Omega^2 = \gamma\beta k_{\pm 1}^2/2\left(1+\delta\right)$ and creates a new gap.

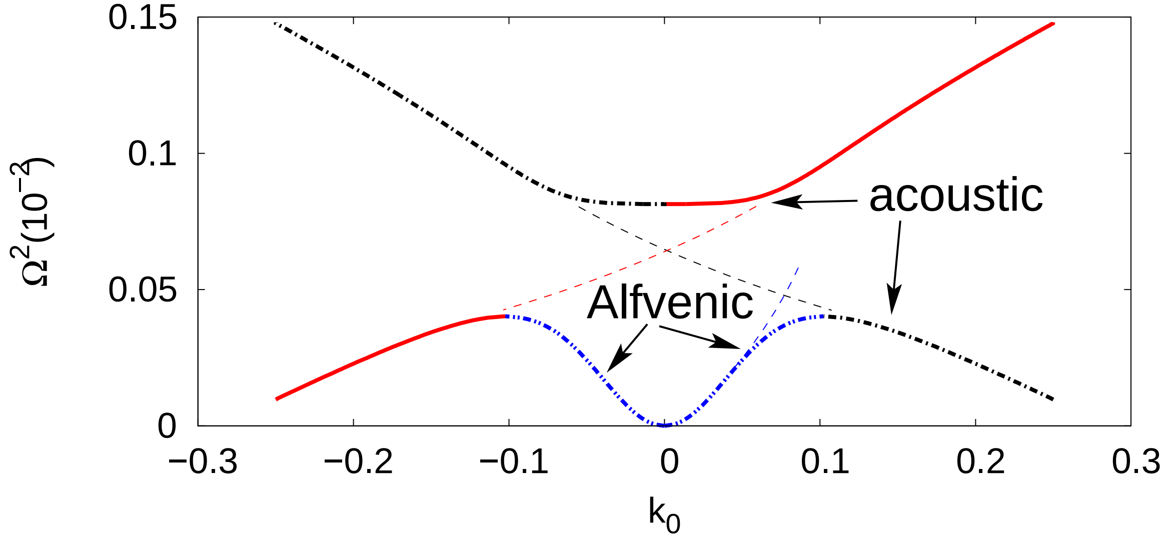


Figure 1: Local dispersion (thick lines) in the vicinity of rational surface at $q = 1.75$, $\beta = 0.3\%$.

Analytic solution for the center of the gap

Solution can be obtained by keeping mth Alfvén harmonic and two $m\pm 1$ acoustic harmonics via qubic equation:

$$\left(\Omega^2 - k_0^2\right)\left(\Omega^2(1+\delta) - \delta k_{+1}^2\right)\left(\Omega^2(1+\delta) - \delta k_{-1}^2\right) \quad - \tag{2}$$

$$-\delta\Omega^2\left(2\Omega^2(1+\delta) - \delta k_{+1}^2 - \delta k_{-1}^2\right) \quad = \quad 0. \tag{3}$$

Near resonant surface at high n, m

$$k_j^2 = (j/q + k_0)^2 \simeq j^2/q^2 + 2k_0 j/q. \tag{4}$$

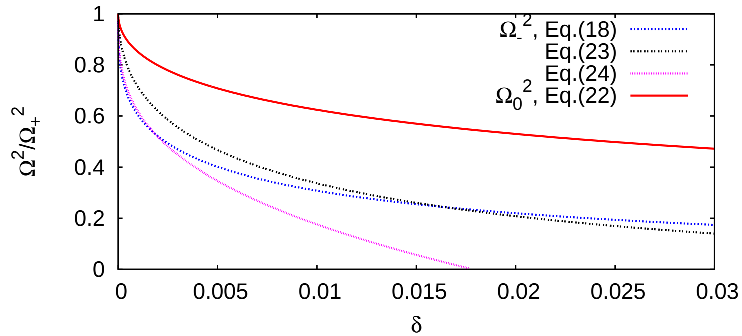
Resolving $k_j(\Omega)$ we find with $\Omega_+^2 = \delta/q^2(1+\delta)$ for the gap center:

$$\Omega_0 = \Omega_+ / \left(1 + \sqrt{\frac{\delta(1+2q^2)}{1+\delta}}\right)$$

Analytic solution for the gap width

Another analytic solution is found when $2\delta \ll \Omega^2(1+\delta) - \delta/q^2$ (typically $\delta < 0.002$)

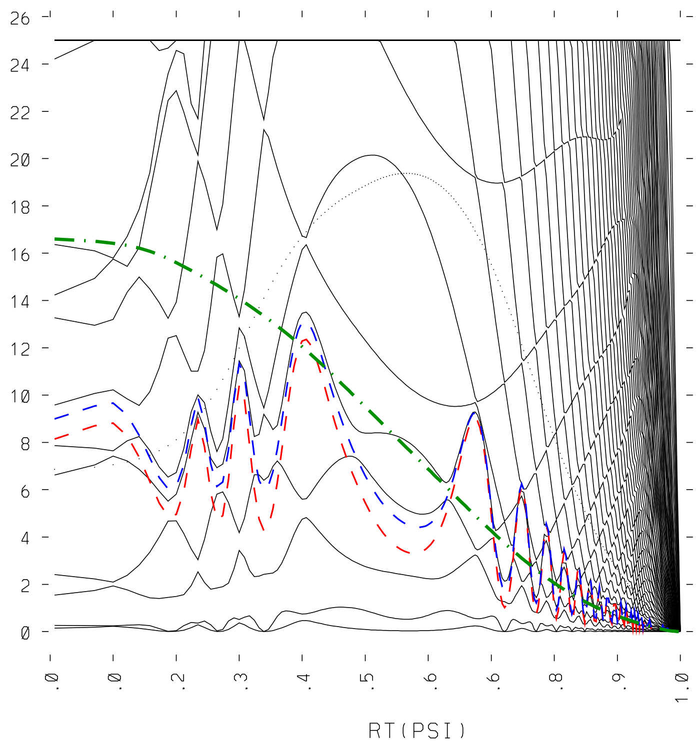
$$\Omega_-^2 = \Omega_+^2 \left[1 - (32q\delta)^{1/3}\right].$$



Gap width is comparable to the frequency.
Global modes can exist just like in TAE or any other gap.

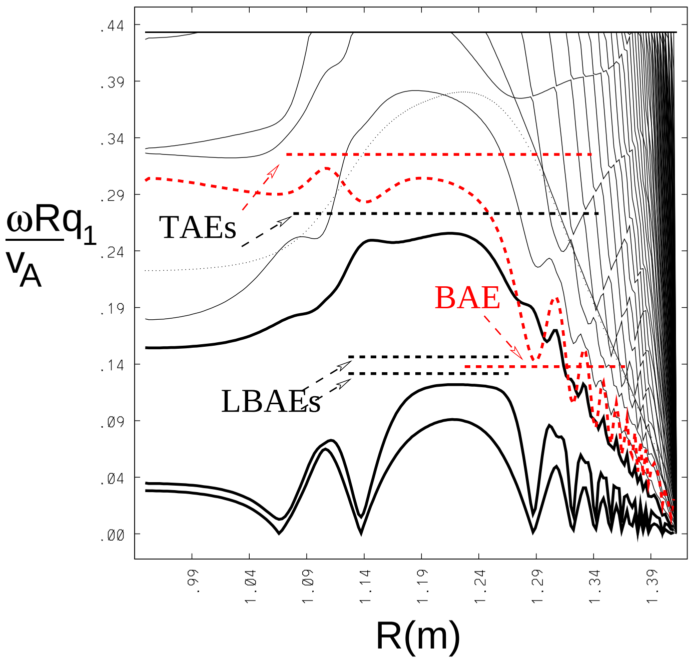
Alfvén/acoustic gap low boundary, Ω_-^2 and the center of the gap vs. pressure parameter δ.

NSTX medium $\beta = 5\%$ equilibrium from TRANSP @q (MSE)



- Overlaid red is for acoustic mode filtering scheme (Chu'92)
- Overlaid blue continuum includes q correction $\left(1+1/2q^2\right)$.
- Black is for full ideal MHD NOVA continuum. Influence on gaps from the acoustic modes.
- Global AMs are more likely at high β since frequency spacing for the acoustic continuum is $\Delta\Omega \sim \sqrt{\gamma\beta/2}$.
- Harder to find the mode at lower beta due to stronger interaction with the continuum.

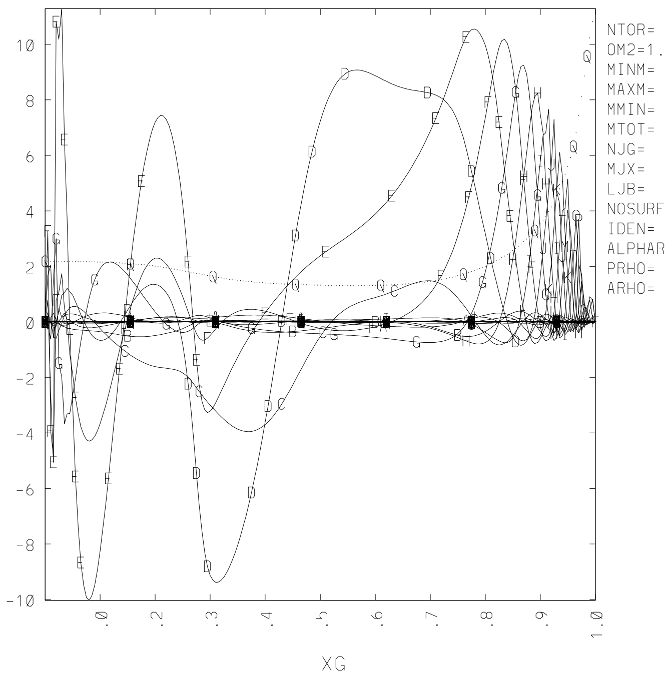
TRANSP high $\beta = 21\%$ *equilibrium case*



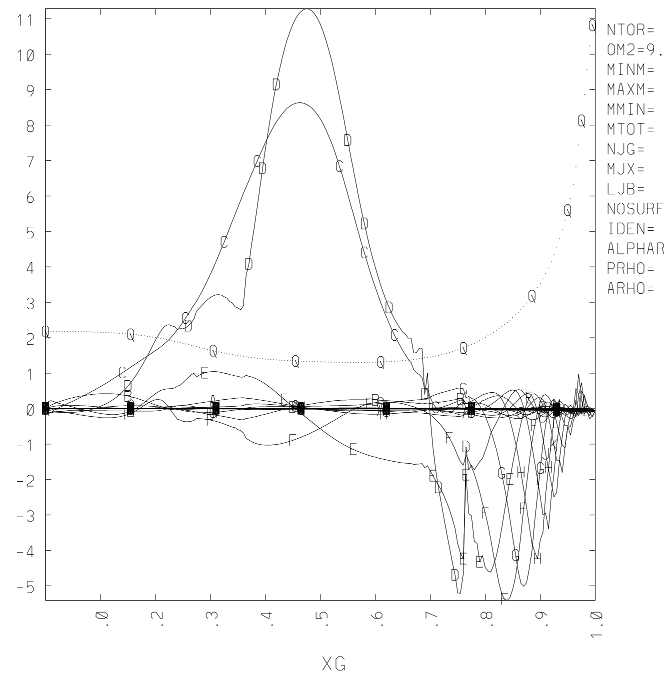
- Red is for acoustic mode filtering scheme (Chu'92): TAE and BAE (Turnbull '93) modes are found but frequencies are shifted.
- Black is for full ideal MHD NOVA continuum.
- Global modes are more likely to exist at high β since frequency spacing for the acoustic continuum is $\Delta\Omega \sim (\gamma\beta)^{3/4}/q$.
- Two Alfvén/acoustic continuum branches are found.
- AMs interacts strongly with the Alfvén-like branch.
- New global modes, Low shear beta-induced Alfvén Eigenmodes (LBAE), are formed above the continuum.

TAE structure with and without filtering

Filtered continuum opens the gap



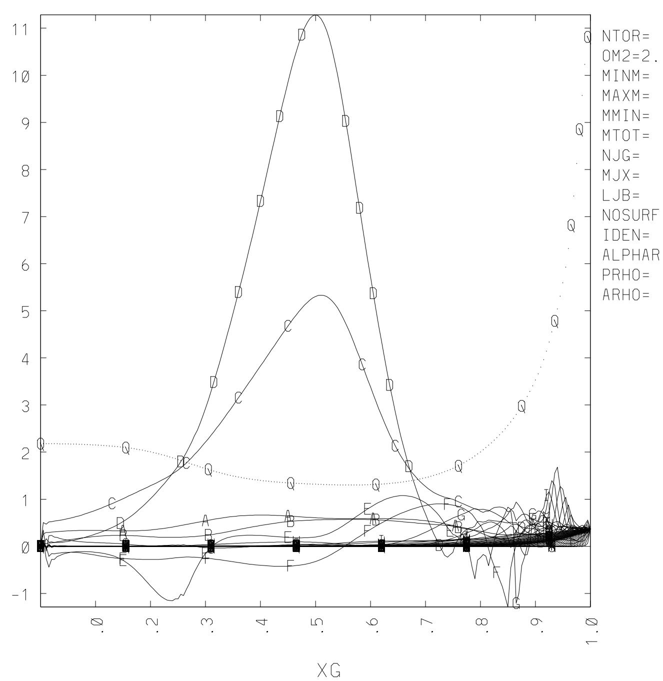
Full continuum limits TAE localization

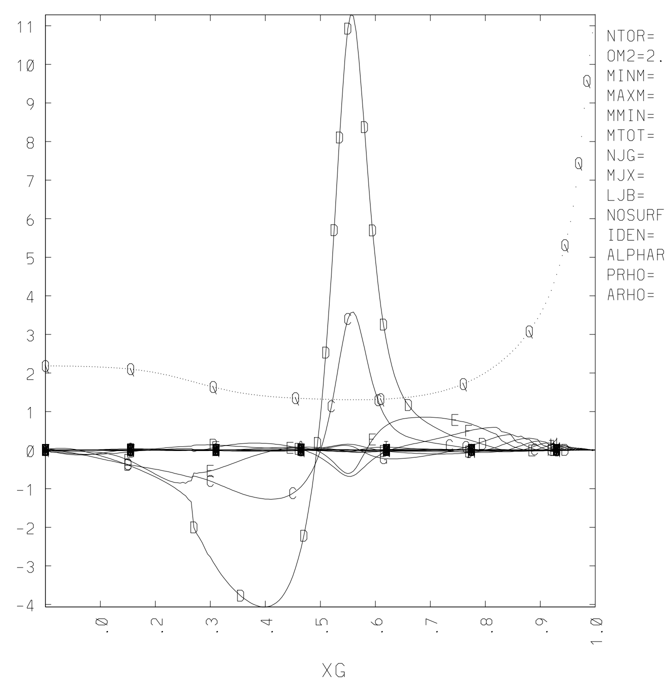


Mode structures are notably different in two models

Interaction with the continuum is stronger without filtering. Is continuum damping proportional to β?

First radial LBAE (higher $f = 35kHz$) Second radial LBAE $f = 33.8kHz$

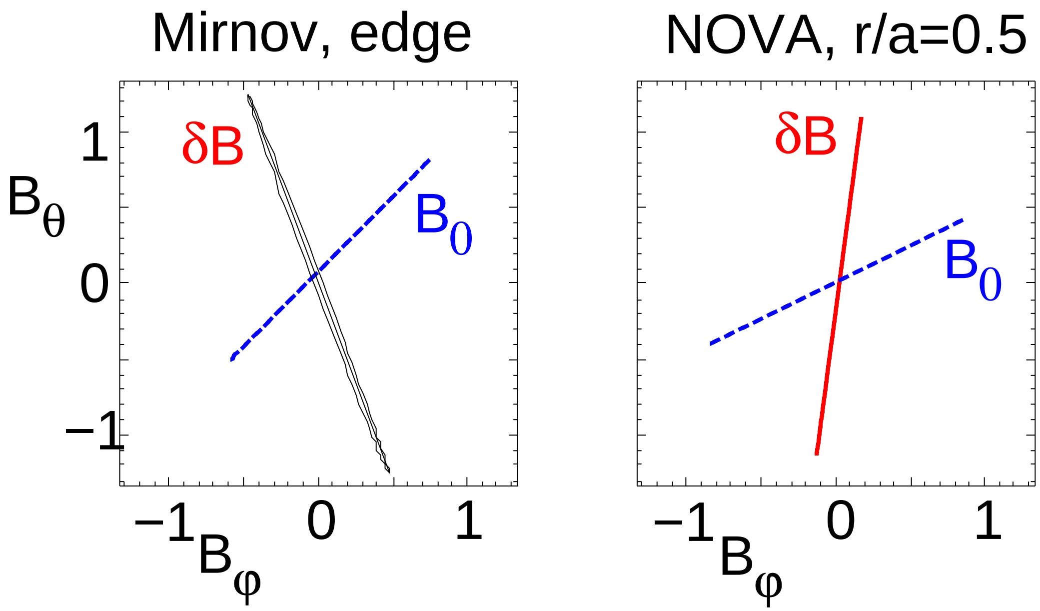




Two dominant harmonics, $m = 2, 3/n = 2$, are present due to $nq_{min} = 2.6$.

Measured and simulated LBAE polarization exhibit parallel magnetic field perturbation

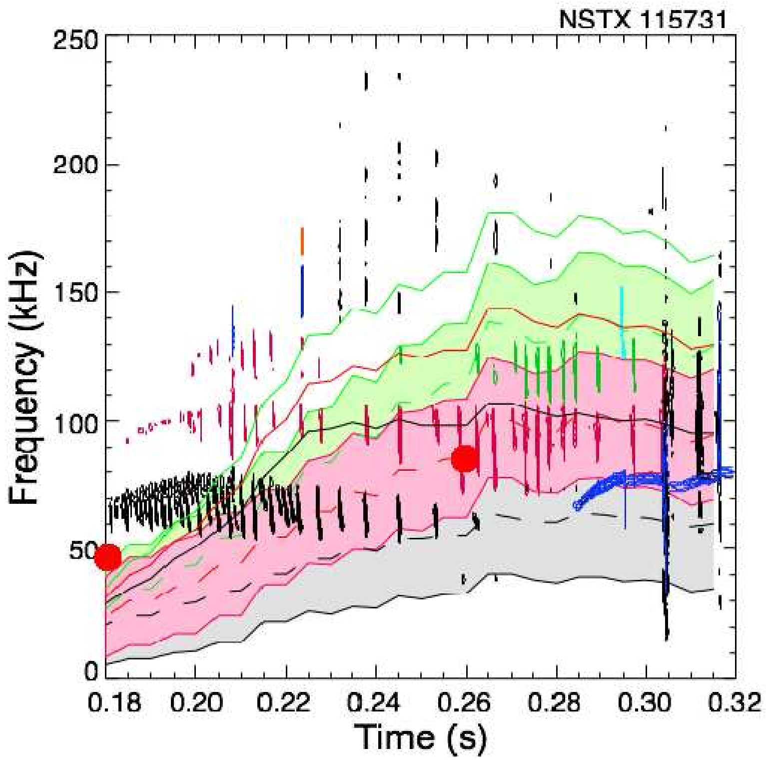
First radial LBAE (higher $f = 35kHz$)



NOVA model can not be used for polarization simulation at the edge due to multiple singularities intersections.

Significant $\delta B_{||}$ is measured and computed.

How LBAE frequency compares with NSTX data?



- Black, red, green shaded areas are for theoretical scaling of LBAE with rotation and $n = 1, 2, 3$.
- Black is for full ideal MHD NOVA continuum.
- Upper black, red and green are core n=1,2 and 3 TAE frequencies.
- Red dots are predictions by NOVA for $n = 2$.
- Discrepancy is due to strong rotation $f_{rot} \simeq 30kHz$ at $t = 0.26sec$ (local).
- Possible causes of the discrepancy:
  - toroidal rotation is strongly sheared and may affect the mode localization
  - EPM effects push TAE frequency down to merge into LBAE solution. r-TAE to r-KBM transition like effect (Cheng '95, Gorelenkov '03).

SUMMARY

- Theoretical analysis shows the existence of low-n global toroidicity-induced Alfvén/acoustic eigenmodes, we call low shear BAE (LBAE) in NSTX.
- LBAE can exist in high beta plasma within wider BAE gap.
- LBAEs are different from BAE of Heidbrink-Turnbull-Chu interpretation as they require compressibility effect, i.e. sound wave coupling
- TAEs are pushed higher in frequency due to beta effect.
- The $n = 2$ LBAE frequency computed by NOVA, $35kHz$, is very close to the observed frequency $43kHz$ after deducting the toroidal rotation Doppler shift for #115731 shot.
- Kinetic modification of MHD theory may be important issue for new class of modes (parallel phase velocity is still close to the Alfvén speed).