PPPL PRINCETON PLASMA PHYSICS LABORATORY

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NSTX

Internal Magnetic Field Structure of Perturbed Tokamak Equilibria

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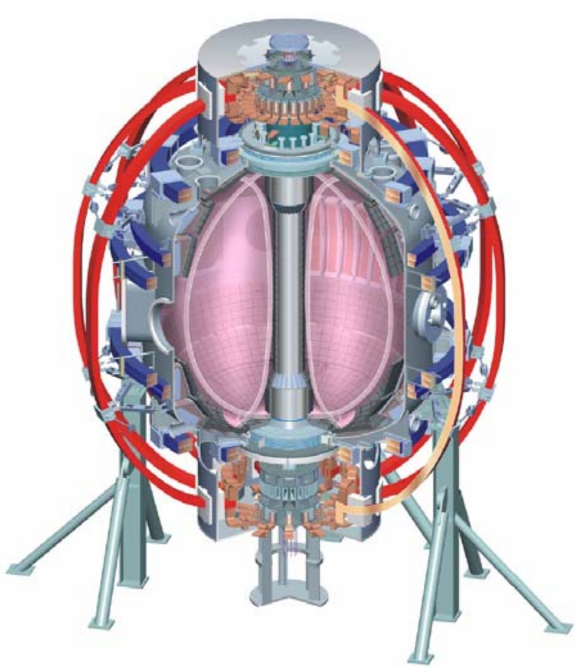
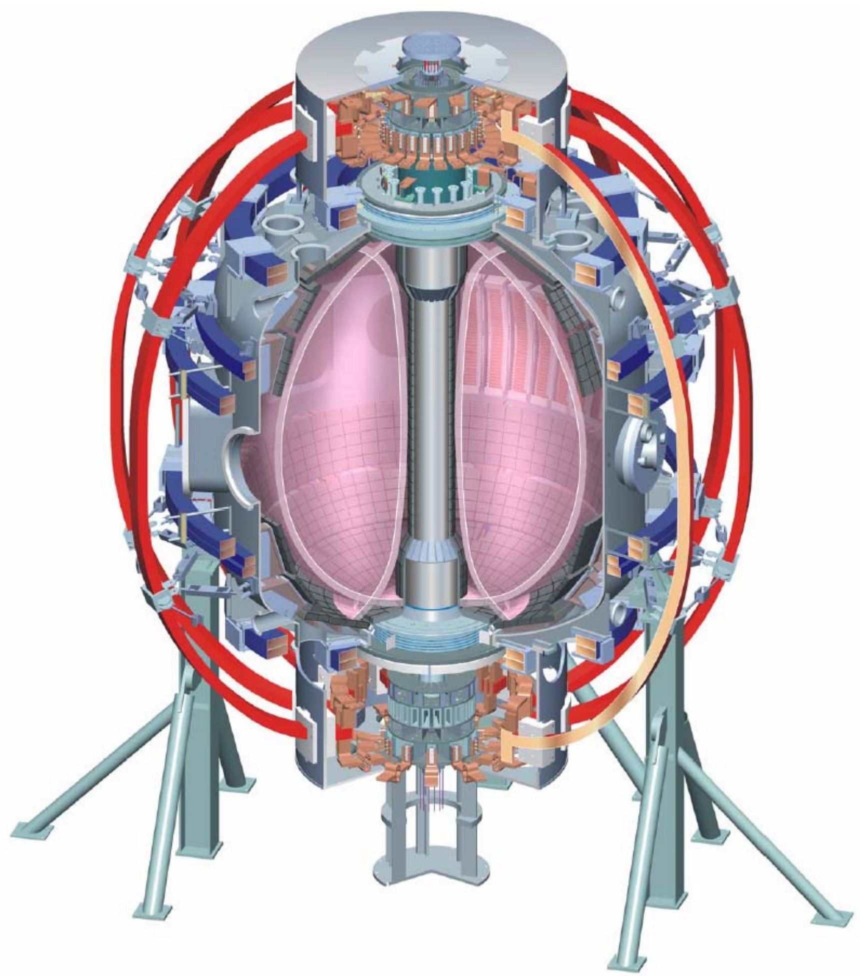
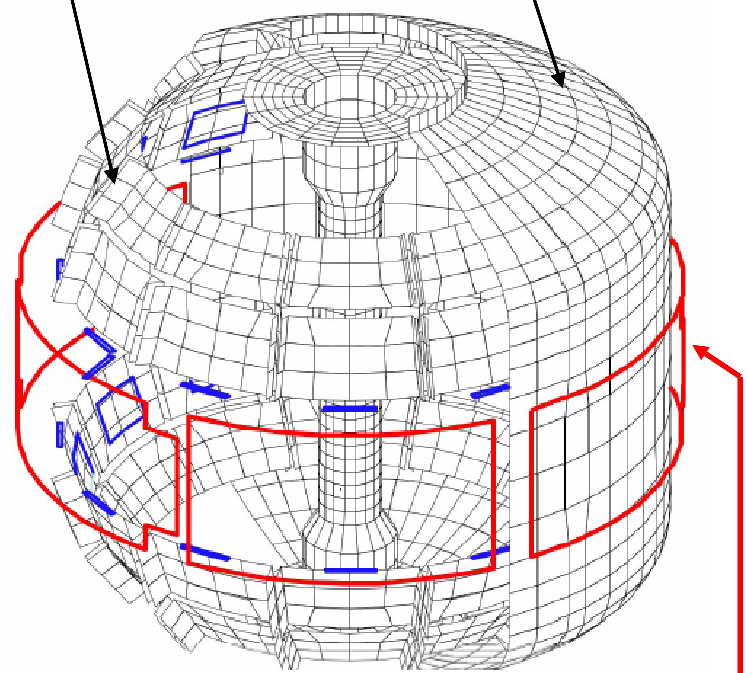


Outline

- Sensitivity of tokamak plasmas to 3D error fields
  - Error field effects in NSTX plasmas
  - Error field identification without plasma response
- Plasma response model
  - Effective plasma permeability and inductances
- Computation of perturbed equilibrium
  - Perturbed equilibrium and ideal MHD stability analysis
  - DCON, VACUUM and IPEC code
- Numerical examples of plasma response
  - Accuracy test for computation
  - Amplification in the plasma response
  - Internal magnetic field structure with the plasma response
  - Opening of magnetic island
- Future work

Configuration of coils in NSTX including "RWM or EF coils"

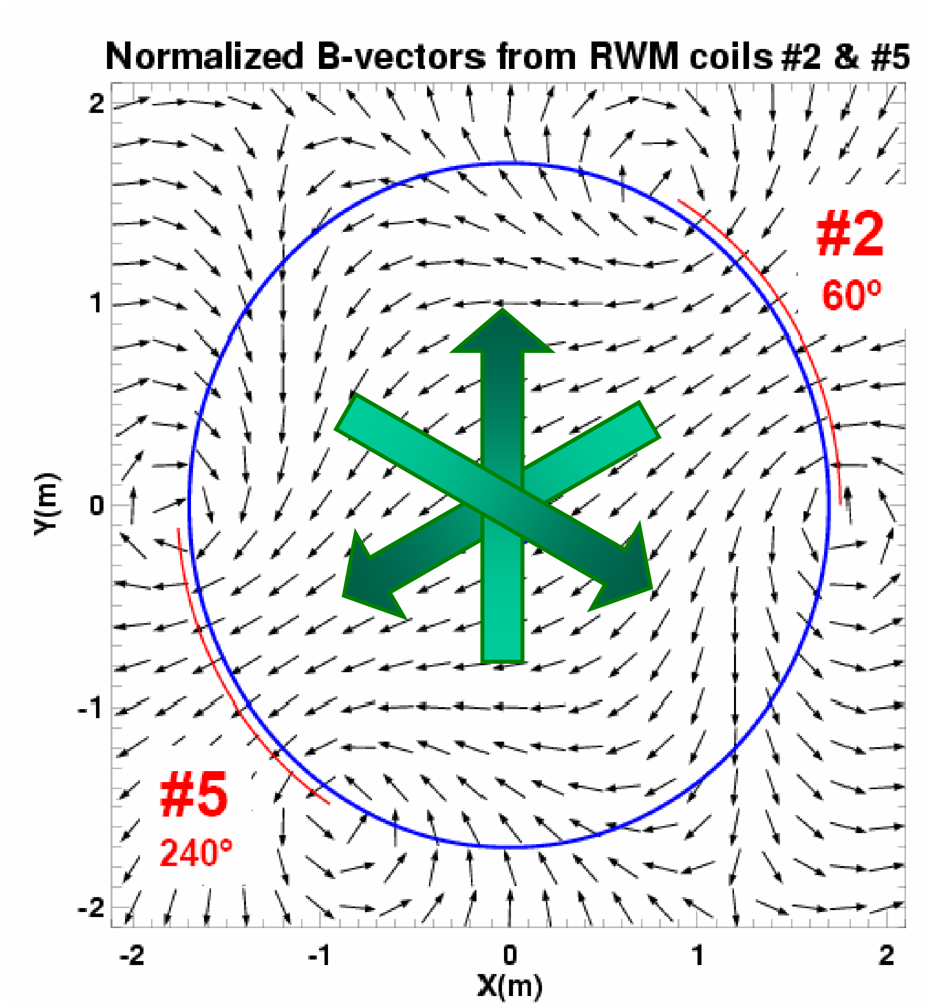


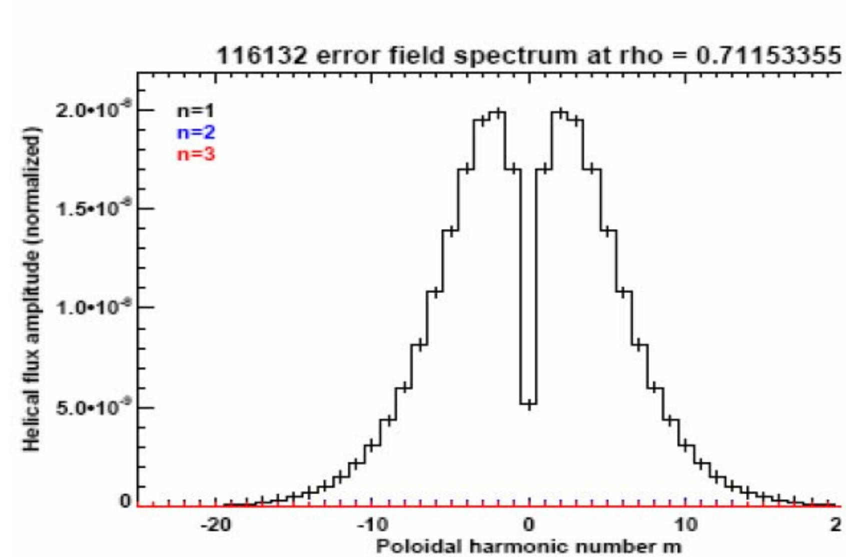


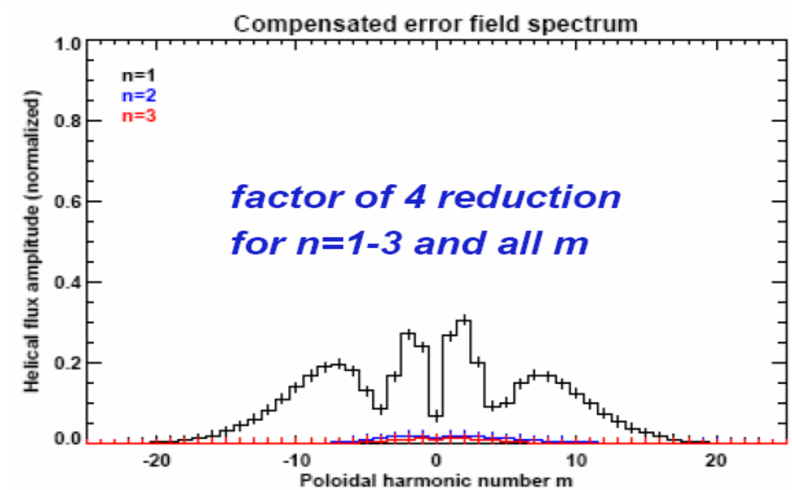
VALEN Model of NSTX (Columbia Univ.)

6 ex-vessel midplane control coils
+ 24 B_R and 24 B_P in-vessel sensors

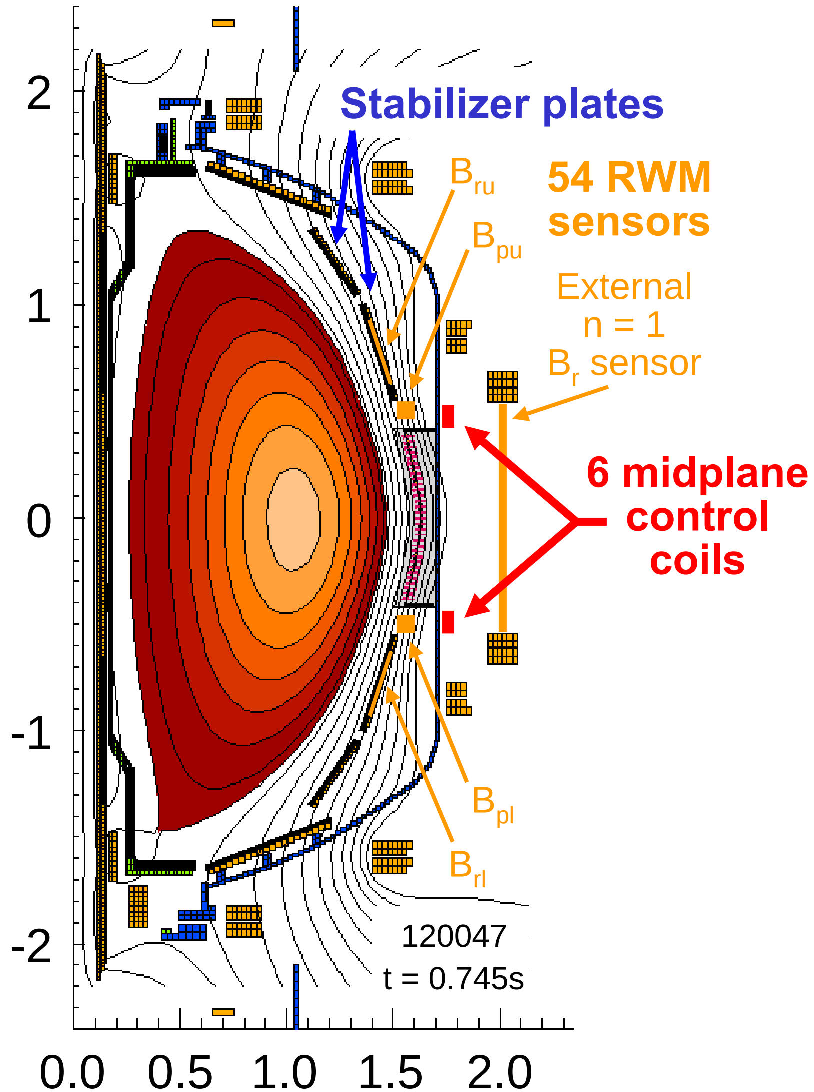
Error field generation and compensation by EF coils



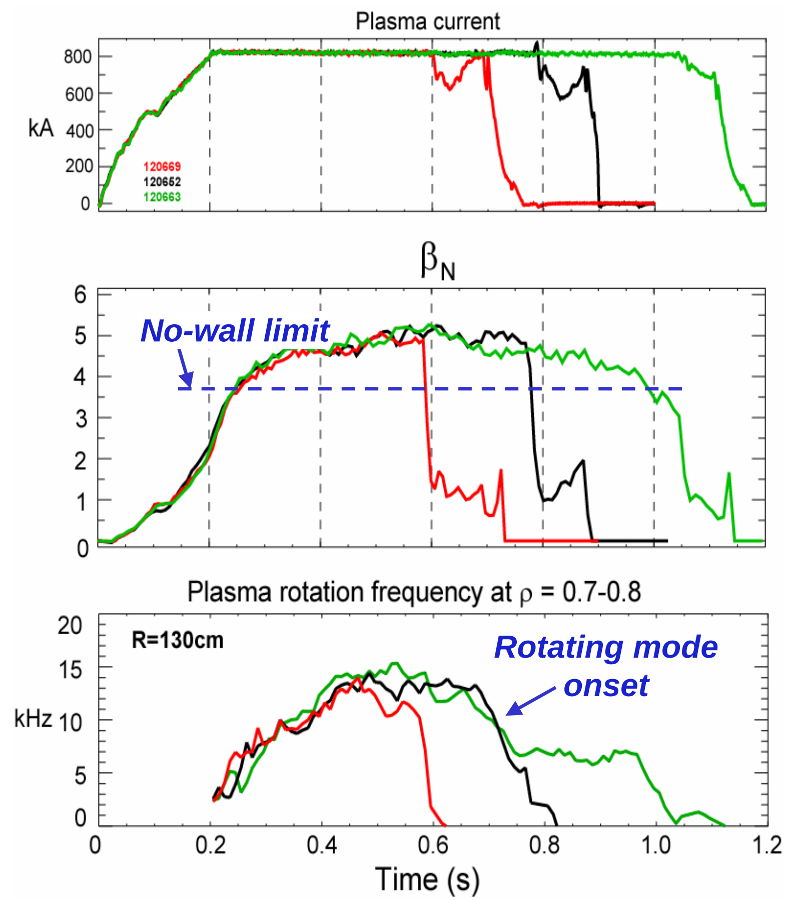




The right gain and phase is needed for the best performance

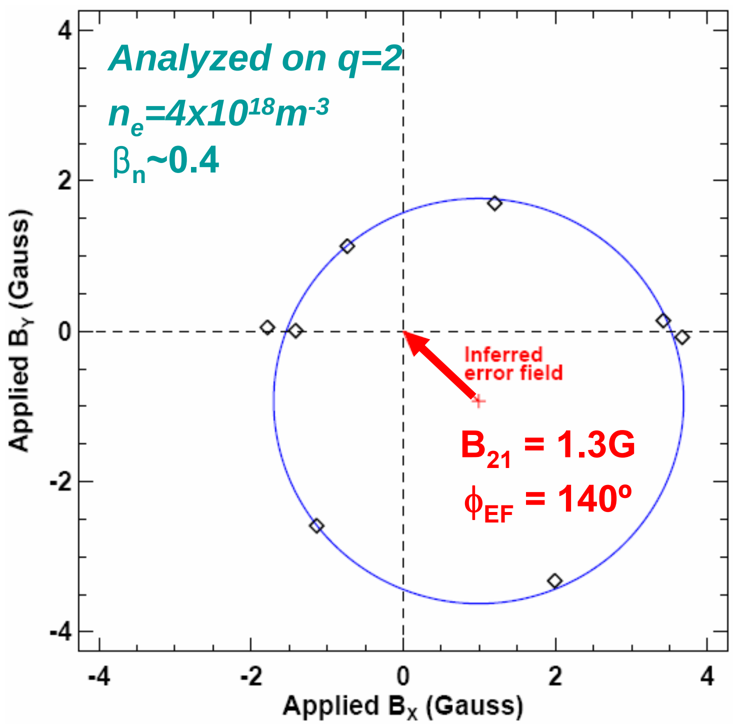


- Empirical way to choose the right gain and phase for the best performance

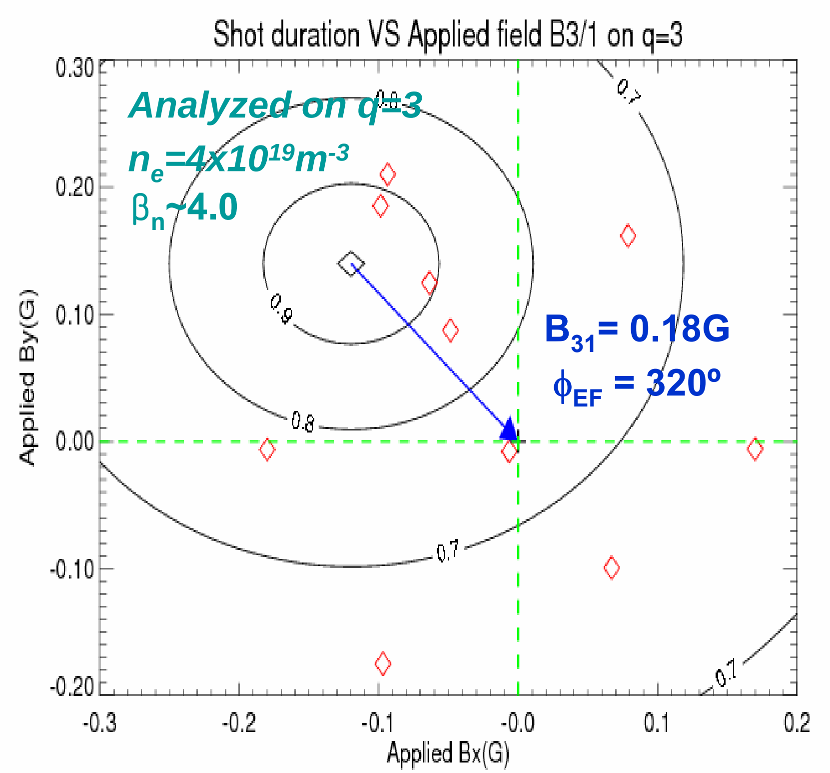


Examples of the right gain and phase

- **Error field can cause LM(Locked Mode) in low-β plasma, in which plasma response may be weak enough to be neglected**
- **Error field can affect RWM(Resistive Wall Mode) in high-β plasma, in which the actual error field may be significantly different by plasma response**

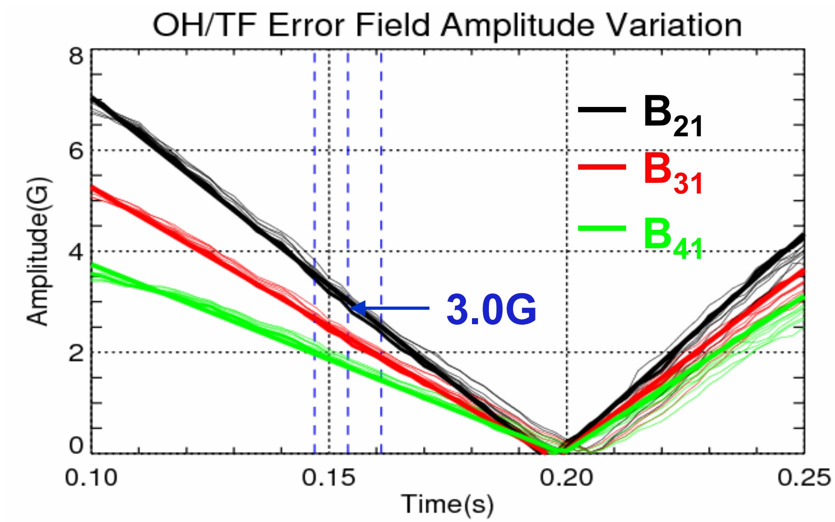


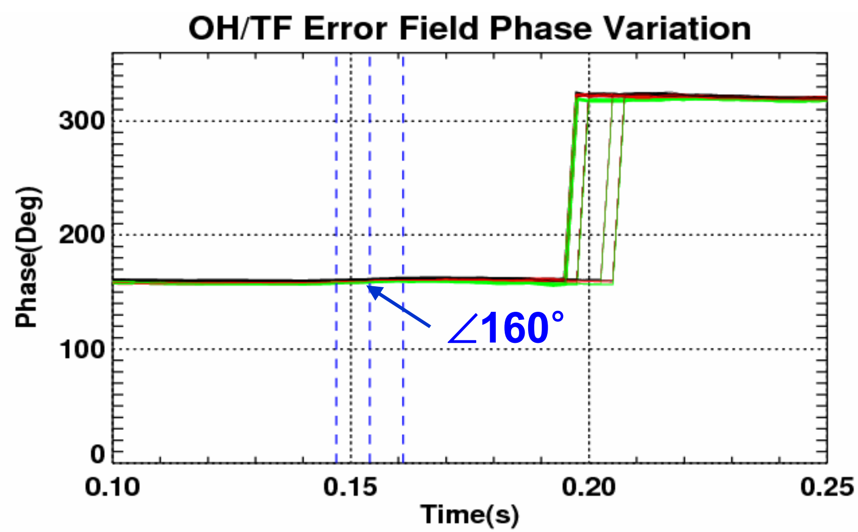
Inferred EF in Low-β



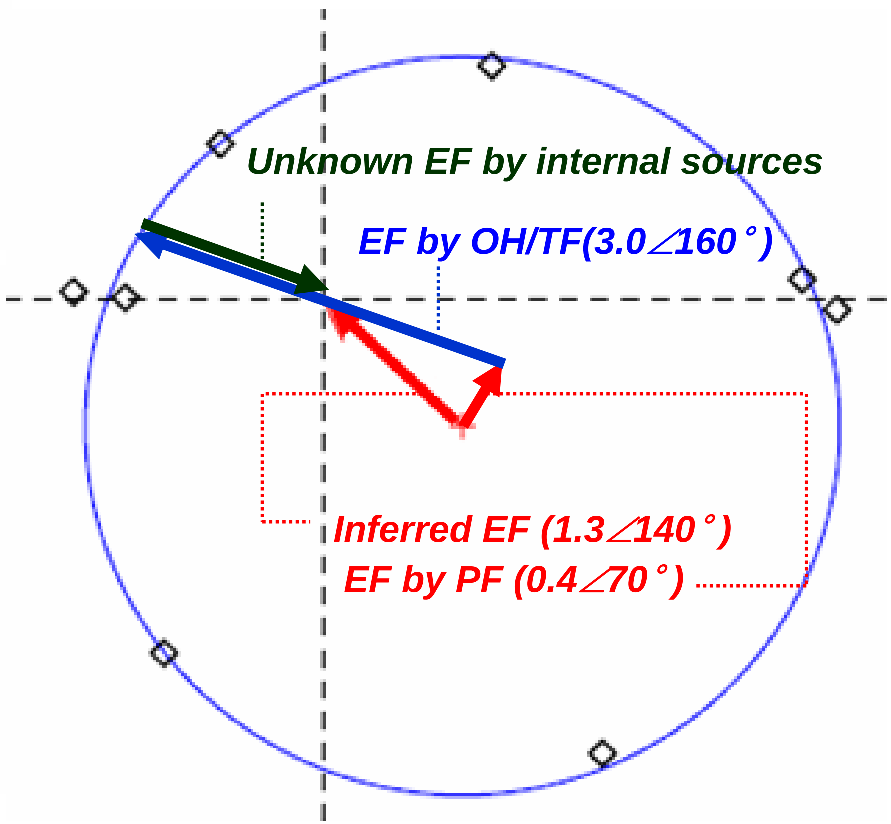
Inferred EF in High-β

Analysis for the LM experiments without plasma response

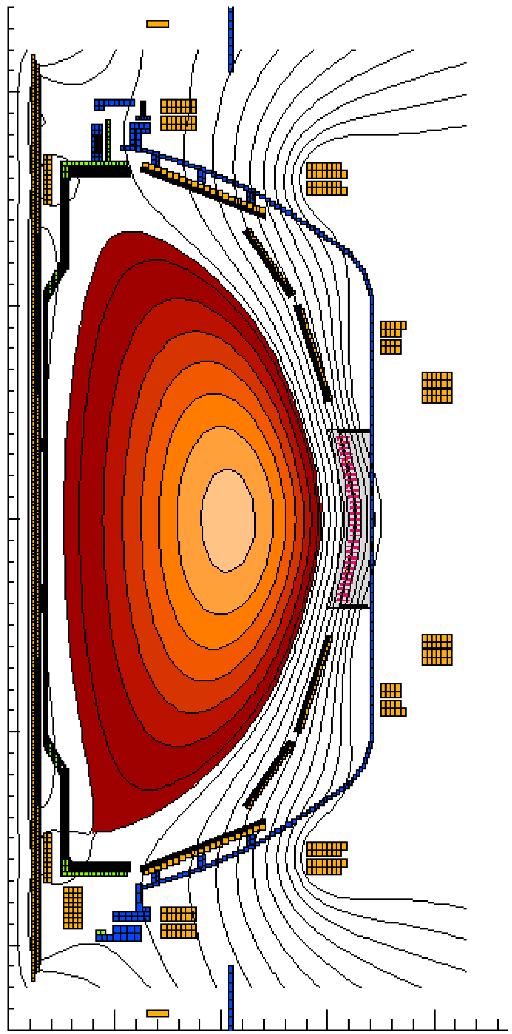




- Average external EF at relevant time is in same phase with inferred EF, but still need more to explain apparent different magnitude
  : Need to know plasma response



Ideal plasma response by perturbed plasma equilibria



- **Perturbed equilibrium in ideal MHD**

$$\vec{F} = \vec{j}_0 \times \vec{b} + \vec{j} \times \vec{b}_0 - \nabla p$$

$$\vec{b} = \nabla \times (\vec{\xi} \times \vec{b}_0) \qquad \vec{j} = \nabla \times \vec{b} \qquad \nabla \cdot \vec{b} = 0$$

$$p = -\vec{\xi} \cdot \nabla p_0 - \gamma p_0 (\nabla \cdot \vec{\xi})$$

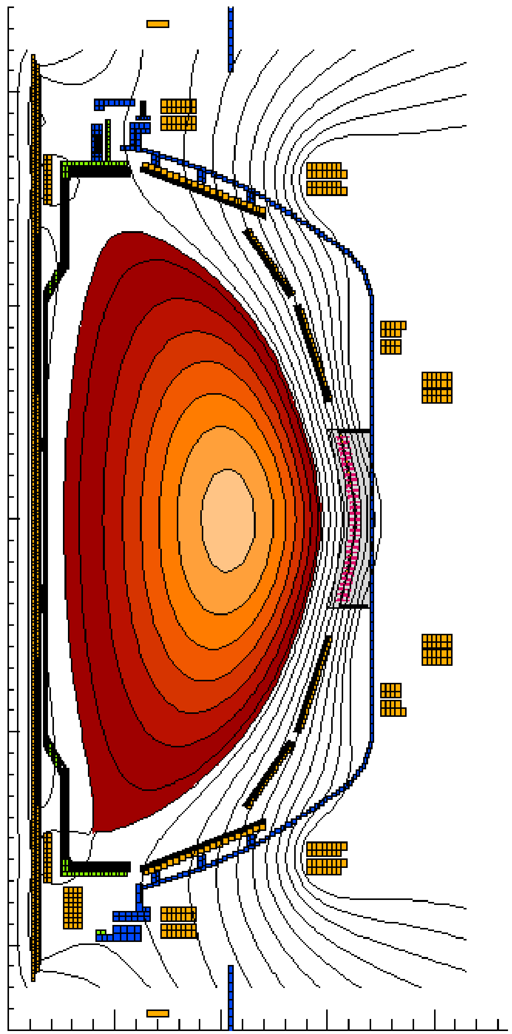
- **Difficult to solve the total system because of the complicated coils and conductors**

One problem all together

Two separate problems

1. **The external system + plasma with a permeability P on the control surface**
2. **Perturbed equilibrium with the actual field on the control surface without the external system**

Effective plasma permeability



- The effect on plasma by external currents is given by 'external field' $(\vec{b}^x \cdot \hat{n})(\theta,\zeta)$ on the control surface
- The effect on external system by plasma is given by 'plasma field' $(\vec{b}^p \cdot \hat{n})(\theta,\zeta)$ on the control surface
- 'Actual field' is given by

$$(\vec{b} \cdot \hat{n})(\theta,\zeta) = (\vec{b}^x \cdot \hat{n})(\theta,\zeta) + (\vec{b}^p \cdot \hat{n})(\theta,\zeta)$$

- Permeability gives the relation from the external field to the actual field

$$(\vec{b} \cdot \hat{n})(\theta,\zeta) = \hat{P}(\vec{b}^x \cdot \hat{n})(\theta,\zeta)$$

- Knowing the actual field on the control surface, we can use DCON stability code to obtain the plasma displacement $\vec{\xi}(\vec{x})$ inside, which gives the perturbed equilibrium of plasma

Method for obtaining the plasma permeability

1. Set the system with plasma, vacuum and wall at infinity
2. Choose the plasma-vacuum boundary as the control surface
3. Put perturbed normal magnetic fields on the control surface
4. Compute the actual surface currents and plasma inductance

$$\vec{\Phi} = \Lambda \cdot \vec{J} \qquad (\vec{b}\cdot\vec{n})(\theta,\zeta) = Re\left(\sum_m \Phi_m \sqrt{w} e^{i(m\theta - n\zeta)}\right) \quad \kappa(\theta,\zeta) = Re\left(\sum_m J_m \sqrt{w} e^{i(m\theta - n\zeta)}\right)$$

5. Eliminate the plasma inside and put the same perturbed fields
6. Compute the external surface currents and surface inductance

$$\vec{\Phi} = L \cdot \vec{J}^x \qquad (\vec{b}\cdot\vec{n})(\theta,\zeta) = Re\left(\sum_m \Phi_m \sqrt{w} e^{i(m\theta - n\zeta)}\right) \quad \kappa^x(\theta,\zeta) = Re\left(\sum_m J_m^x \sqrt{w} e^{i(m\theta - n\zeta)}\right)$$

7. Note that the same surface currents can be used instead of fields

$$\vec{\Phi} = \Lambda \cdot \vec{J} \qquad \vec{\Phi}^x = L \cdot \vec{J}$$

8. Compute the permeability matrix and the plasma response

$$P = \Lambda \cdot L^{-1} \qquad \vec{\Phi} = P \cdot \vec{\Phi}^x$$

The surface current on the control surface

- **Perturbed equilibrium. Vs. Stability analysis**

$$\delta W_{tot} = -\frac{1}{2}\int_p \vec{\xi}\cdot\vec{F}(\vec{\xi})d^3x \qquad \vec{F} = \vec{j}_0\times\vec{b} + \vec{j}\times\vec{b}_0 - \nabla p$$

Integrating by parts of $\vec{\xi}\cdot(\vec{j}\times\vec{b}_0)$ *And if the unperturbed pressure smoothly goes to zero,*

$$\delta W_{tot} = \delta W_p + \int_e \frac{|\vec{b}|^2}{2\mu_0}d^3x - \int_e \frac{1}{2}\vec{j}\cdot\vec{a}d^3x$$

- **The surface current exists to carry the perturbation energy**

If the surface current satisfy the conditions of $\vec{j}\cdot\nabla p = 0 \quad \nabla\cdot\vec{j} = 0$

Then the surface current potential can be used $\vec{j} = \delta(\psi-\psi_c)\nabla\kappa(\theta,\zeta)\times\nabla\psi$

$$\delta W_{tot} = \delta W_p + \int_e \frac{|\vec{b}|^2}{2\mu_0}d^3x - \int_c \frac{1}{2}\kappa\vec{b}\cdot d\vec{s}$$

Neglecting the kinetic energy, the total potential energy should be conserved as zero,
Therefore, if we put an external perturbation through the last term then,

$$\delta W \equiv \int_c \frac{1}{2}\kappa\vec{b}\cdot d\vec{s} = \delta W_p + \int_e \frac{|\vec{b}|^2}{2\mu_0}d^3x$$

Magnetic scalar potentials

- **The tangential magnetic field on either side gives the surface current**
- **The magnetic scalar potential for each side is useful**

For external vacuum, $\vec{b}^{(e)} = \nabla\chi^{(e)}$ *So,* $\int_c \frac{1}{2}\kappa\vec{b}\cdot d\vec{s} = \delta W_p - \int_c \frac{1}{2\mu_0}\chi^{(e)}\vec{b}\cdot d\vec{s}$

It implies that there should be an effective magnetic potential for the plasma part

$$\int_c \frac{1}{2}\kappa\vec{b}\cdot d\vec{s} = \int_c \frac{1}{2\mu_0}\chi^{(p)}\vec{b}\cdot d\vec{s} - \int_c \frac{1}{2\mu_0}\chi^{(e)}\vec{b}\cdot d\vec{s} \qquad \mu_0\kappa = \chi^{(p)} - \chi^{(e)}$$

The surface current term can be divided into each part,

$$\begin{aligned} \delta W_p &= \int_c \frac{1}{2\mu_0}\chi^{(p)}\vec{b}\cdot d\vec{s} \\ -\int_e \frac{|\vec{b}|^2}{2\mu_0}d^3x &= \int_c \frac{1}{2\mu_0}\chi^{(e)}\vec{b}\cdot d\vec{s} \end{aligned}$$

- **The magnetic scalar potential can be calculated by perturbed quantities as results from the stability analysis**

$$\begin{aligned} \hat{n}\times\nabla\chi^{(p)} &= \hat{n}\times\left(\vec{b}^{(p)} + \left((\vec{\xi}\cdot\hat{n})(\mu_0\vec{j}_0\times\hat{n})\right)^{(p)}\right) \\ \hat{n}\times\nabla\chi^{(e)} &= \hat{n}\times\vec{b}^{(e)} \end{aligned}$$

- **Inner vacuum case can be treated in the same way**

DCON, VACUUM and IPEC code

1. Do the stability analysis by using DCON/VACUUM code to obtain the perturbed eigenmodes for the given equilibrium
2. Calculate the normal magnetic fields on the orthonormal bases

$$(\vec{b}\cdot\vec{n})(\theta,\zeta) = Re\left(\sum_m \Phi_m \sqrt{w} e^{i(m\theta-n\zeta)}\right) \qquad w = 1/\sqrt{\mathcal{J}|\nabla\psi|}$$

3. Calculate the magnetic scalar potentials

$$\chi^{(x)}(\theta,\zeta) = Re\left(\sum_m X_m^{(x)} \sqrt{w} e^{i(m\theta-n\zeta)}\right)$$

$$X_m^{(p)} = -\frac{(\vec{C}\cdot(\partial\vec{x}/\partial\zeta))_m}{in}$$

$$X_m^{(e)} = -\frac{(\vec{b}^{(e)}\cdot(\partial\vec{x}/\partial\zeta))_m}{in}$$

$$X_m^{(i)} = -\frac{(\vec{b}^{(i)}\cdot(\partial\vec{x}/\partial\zeta))_m}{in}$$

4. Calculate the surface current potentials

$$\kappa(\theta,\zeta) = Re\left(\sum_m J_m \sqrt{w} e^{i(m\theta-n\zeta)}\right)$$

$$\begin{aligned}\mu_0\vec{J} &= \vec{X}^{(p)} - \vec{X}^{(e)}\\ \mu_0\vec{J}^v &= \vec{X}^{(i)} - \vec{X}^{(e)}\end{aligned}$$

DCON/VACUUM/IPEC code

5. Check the accuracy. If it is not enough, keep the large mode numbers

$$\delta W_p = \frac{1}{4\mu_0}(\vec{X}^{(p)\dagger} \cdot \vec{\Phi} + \vec{\Phi}^{\dagger} \cdot \vec{X}^{(p)})$$

$$-\int_e \frac{|\vec{b}|^2}{2\mu_0} d^3x = \frac{1}{4\mu_0}(\vec{X}^{(e)\dagger} \cdot \vec{\Phi} + \vec{\Phi}^{\dagger} \cdot \vec{X}^{(e)})$$

6. Calculate each inductances and permeability matrix by gathering all the previous eigenfunctions

$$(\Lambda^{-1})_{mm'} = 2\sum_{\alpha}(\Phi^{-1})_{m\alpha}\epsilon_{\alpha}((\Phi^{-1})^{\dagger})_{\alpha m'}$$

$$\Lambda_{mm'} = Re(\sum_{\alpha}\Phi_{m\alpha}(J^{-1})_{\alpha m'})$$

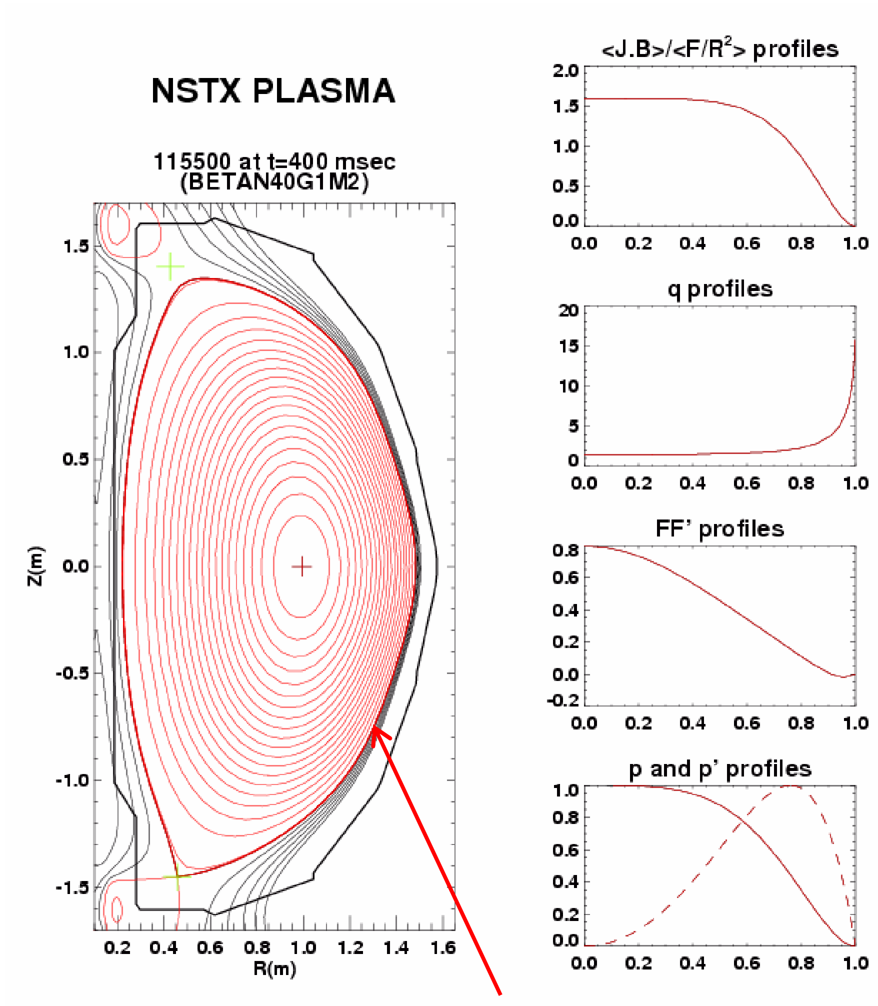
$$L_{mm'} = Re(\sum_{\alpha}\Phi_{m\alpha}(J^{(v)-1})_{\alpha m'})$$

$$P_{mm'} = \sum_{m''}(\Lambda)_{mm''}(L^{-1})_{m''m'}$$

$$\Phi_m = \sum_{m'} P_{mm'}(\Phi^{x})_m$$

7. Calculate the actual field on the control surface with the given external magnetic field, and calculate internal field with the given actual field by manipulating DCON code

Application to the NSTX plasmas



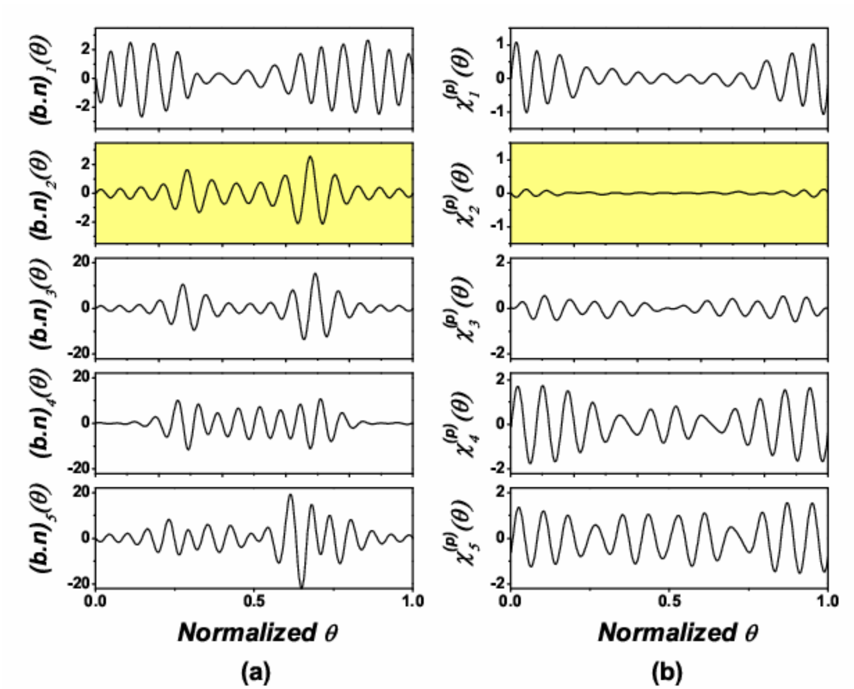
Control surface

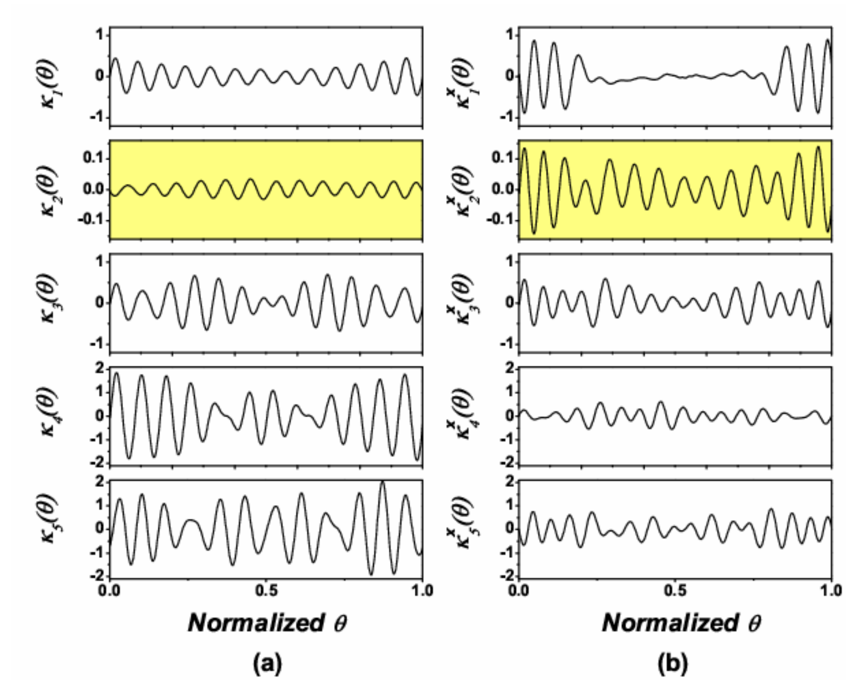
- **The almost worst case for the plasma response model**
- **High qedge(~13) makes large errors in equilibrium and stability analysis**
- **Computation needs high accuracy nearby control surface**
- **For simplicity, zero-edge current and pressure profile are used in the analysis**
- **n=1 mode considered**

Examples of the eigenfunctions

- **The five least stable eigenmodes for the equilibrium including the unstable mode, marginally stable mode, other stable modes**

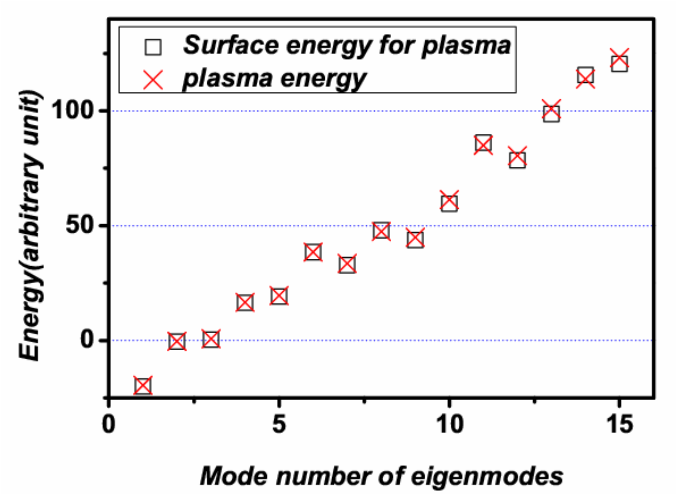
Cosine(or real) part of n=1 mode as a function in Hamada coordinates, for example, $(\vec{b}\cdot\vec{n})_{\alpha}(\theta)cos(\zeta)$

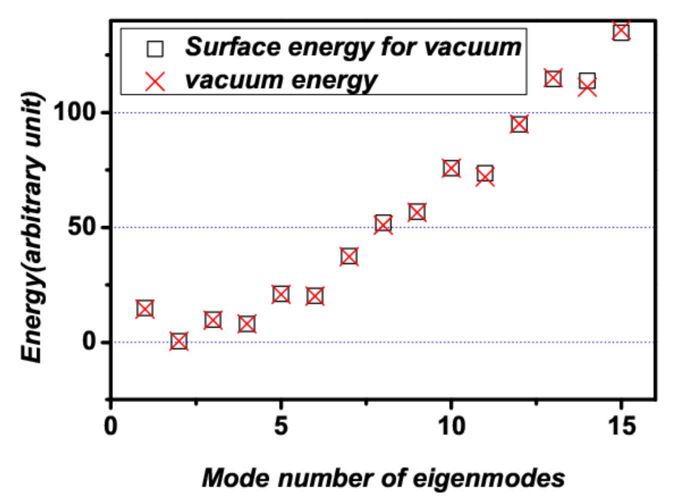




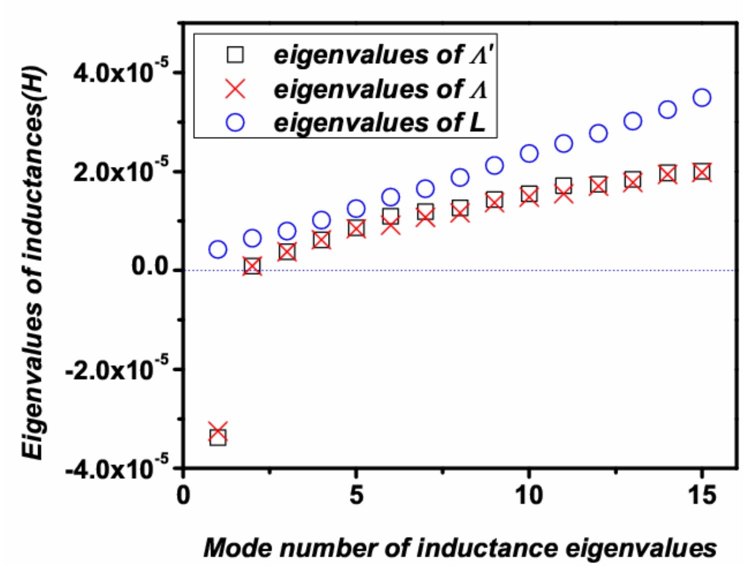
Normal magnetic field	*Magnetic scalar potential for plasma*	*The actual surface current*	*The external surface current*

The accuracy of the code





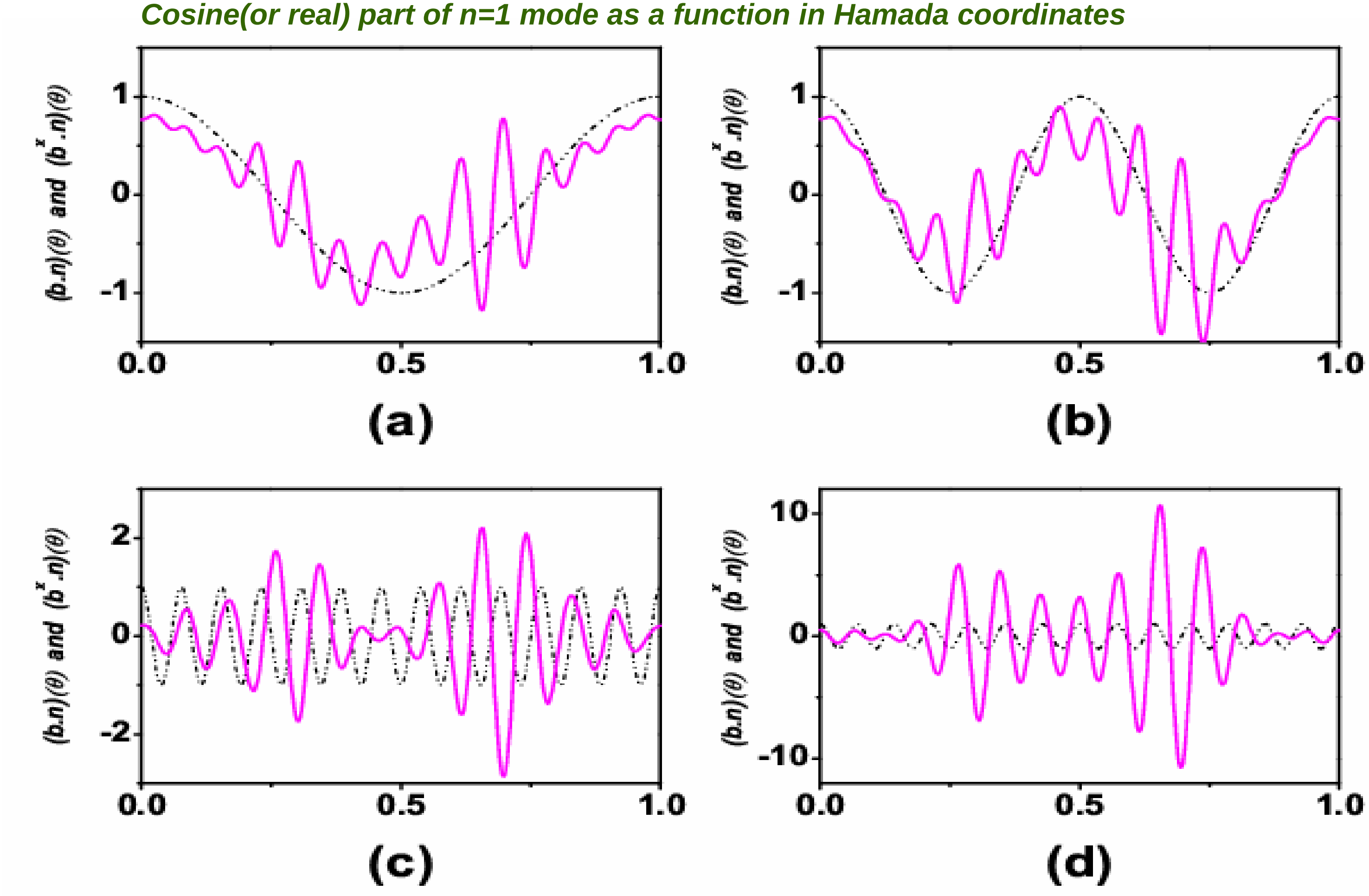
- **Surface current carries all the perturbation energy in high accuracy for plasma and vacuum**
- **Two different ways to compute the plasma inductance show a good agreement**



The plasma response on the control surface

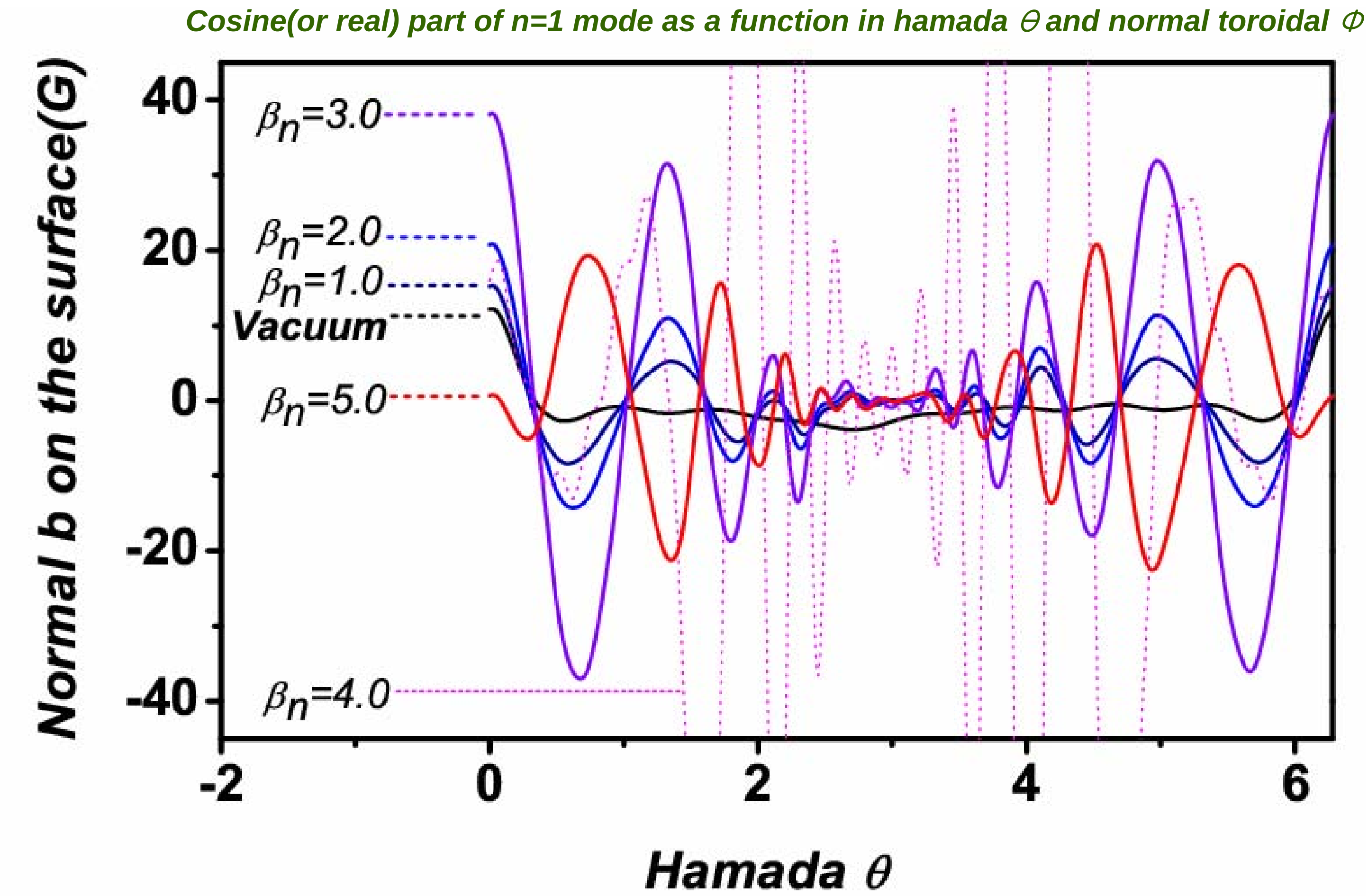
- **Plasma tends to pick up the marginally stable structure on the control surface, with amplification depending on the external perturbation**

Cosine(or real) part of n=1 mode as a function in Hamada coordinates



The plasma response to the external error field

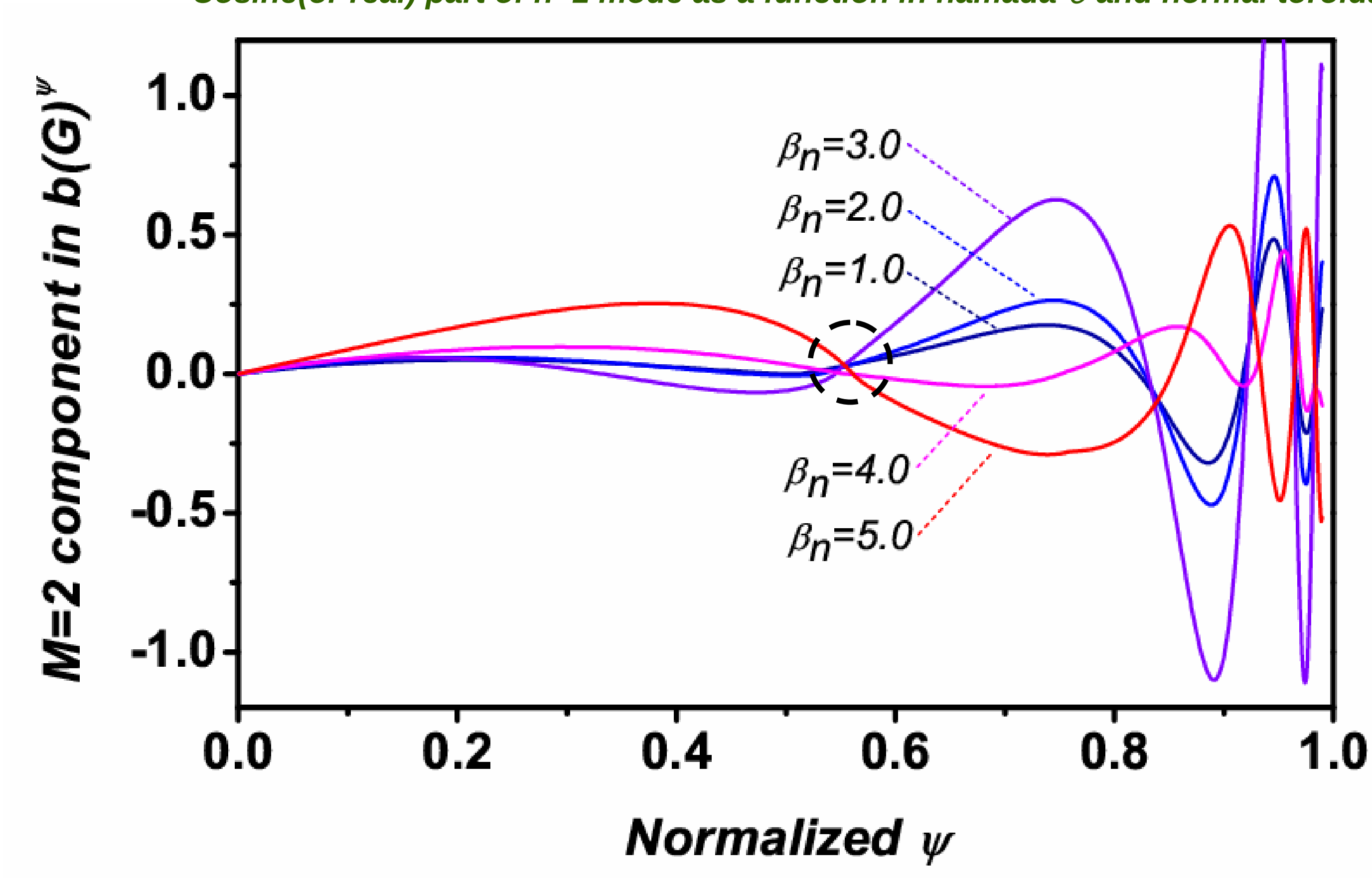
- **Amplification effect is the largest nearby the marginally stable point**



Internal structure of perturbed magnetic field

- **Internal structure of the perturbed magnetic field shows the similar amplification effect as well**

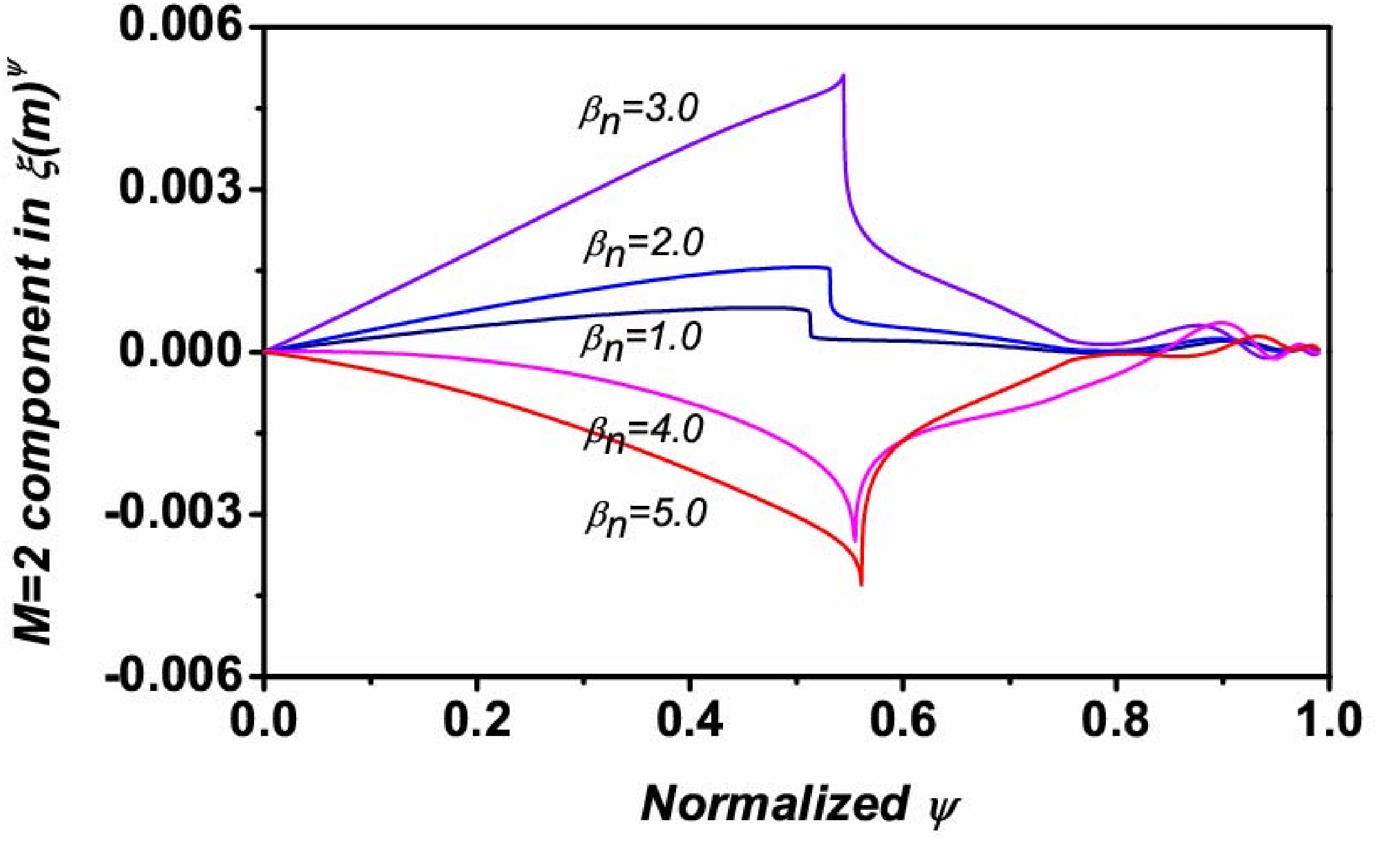
Cosine(or real) part of n=1 mode as a function in hamada θ and normal toroidal Φ



Internal structure of perturbed plasma displacement

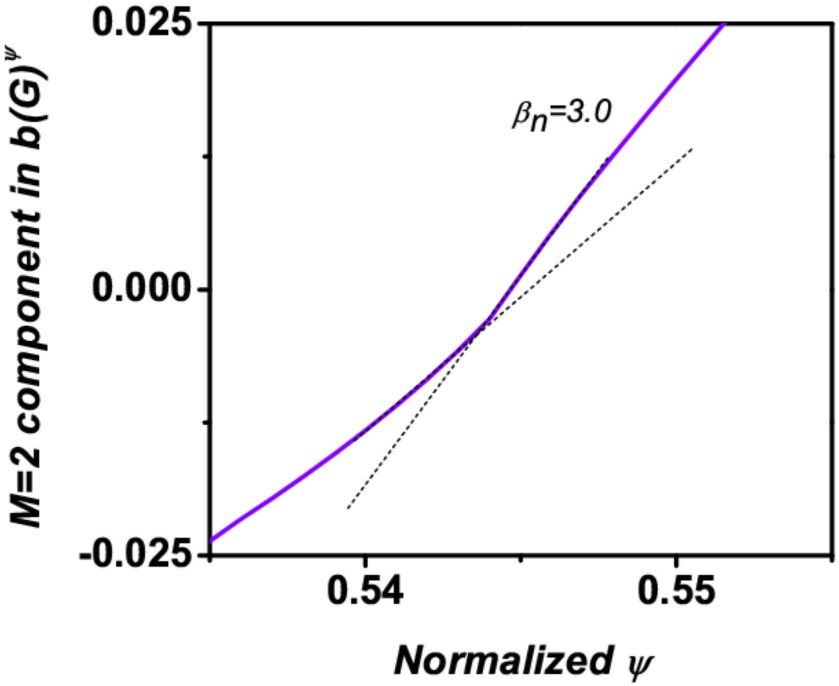
- **Internal structure of the perturbed plasma displacement shows the very large distortion of plasma nearby the resonance surface with the similar amplification tendency**

Cosine(or real) part of n=1 mode as a function in hamada Θ and normal toroidal Φ



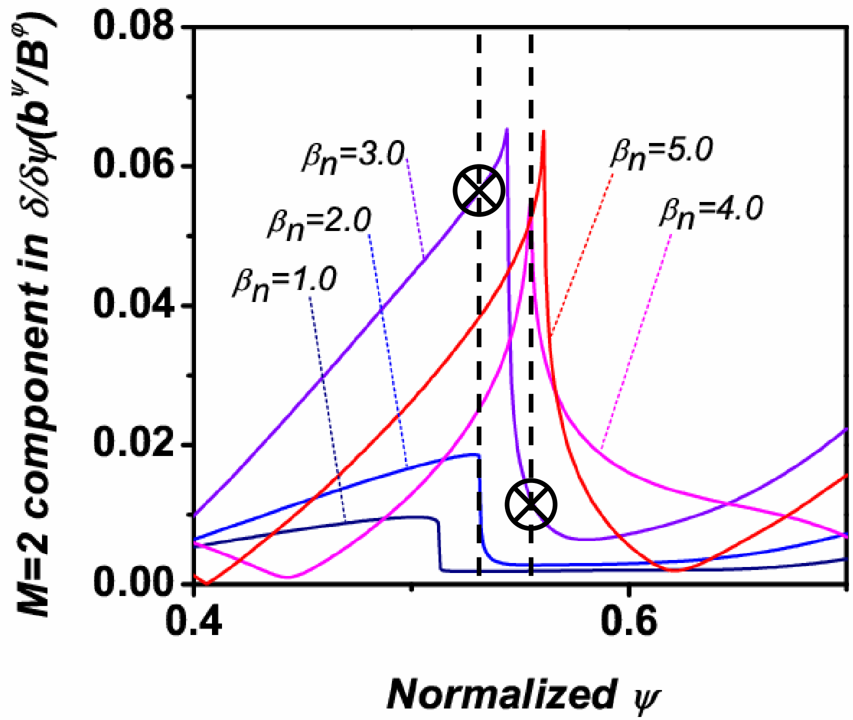
Internal structure of the derivative of perturbed magnetic field

- **The jump of the derivative of perturbed magnetic field is related to the magnetic island width in the resistive layer**



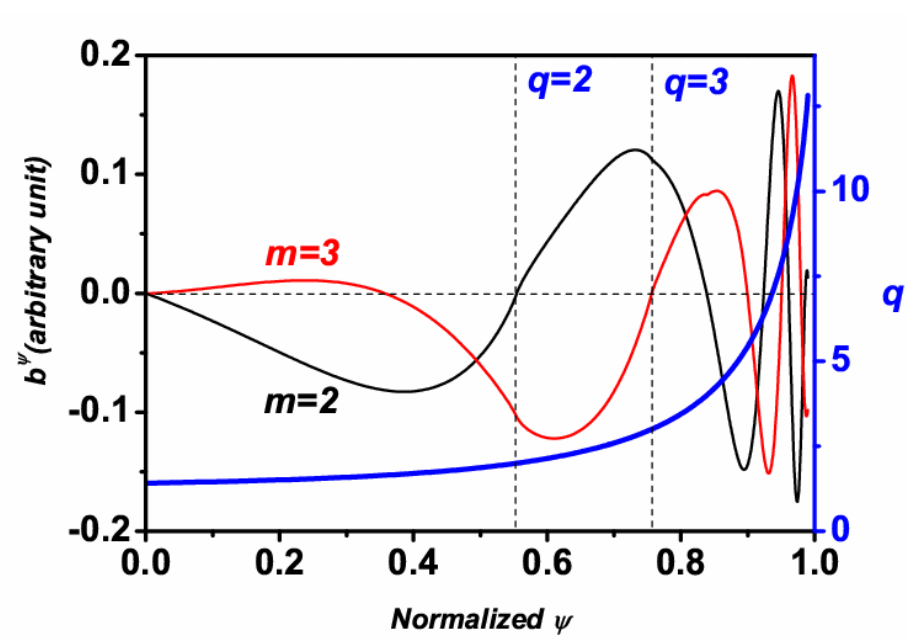
- **Delta quantity is defined by**

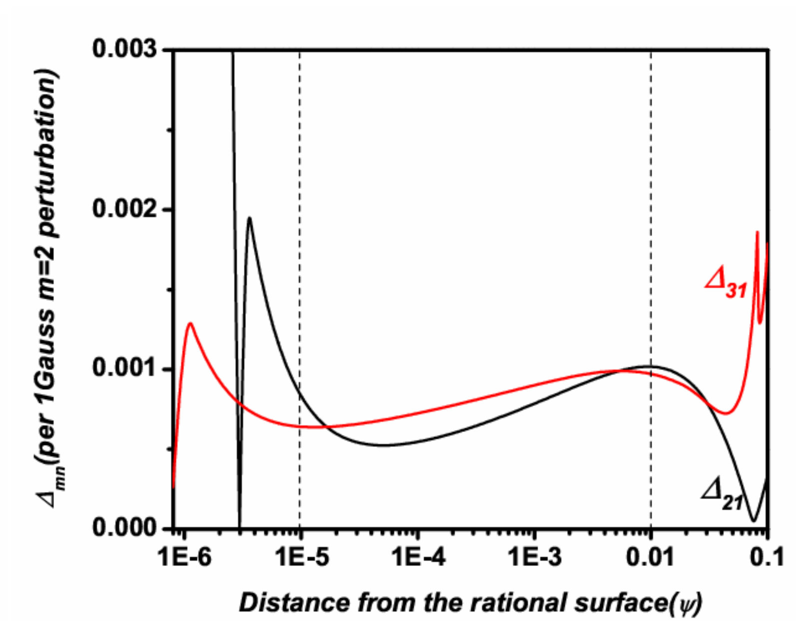
$$\Delta_{mn} = \left[\frac{\partial}{\partial \psi} \left(\frac{\vec{b} \cdot \nabla \psi}{\vec{b}_0 \cdot \nabla \zeta} \right) \right]_{mn}$$



The ideal MHD approximation for the magnetic island consideration

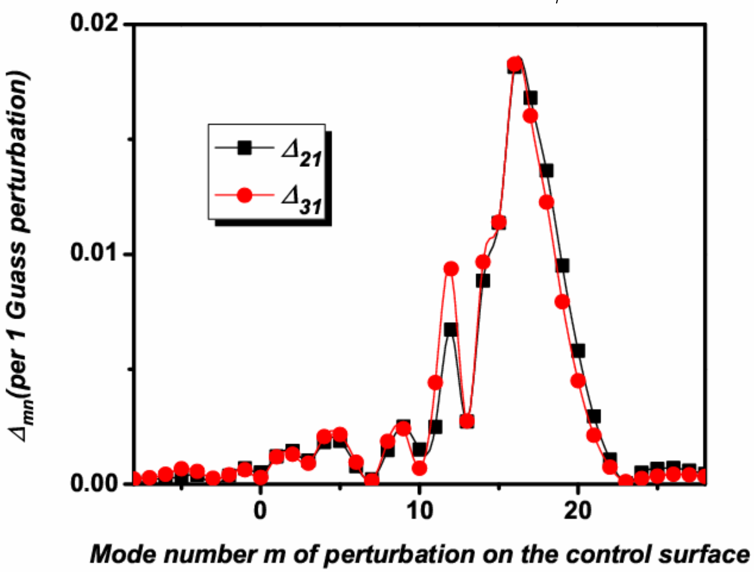
- Issue is how to define the legitimate distance from the exact rational surface in ideal MHD approximation
- Ideal DCON crosses each rational surface by eliminating each large solution component in the ideal limit
- There exists a quasi-asymtotic value of the delta in very wide region around the each rational surface



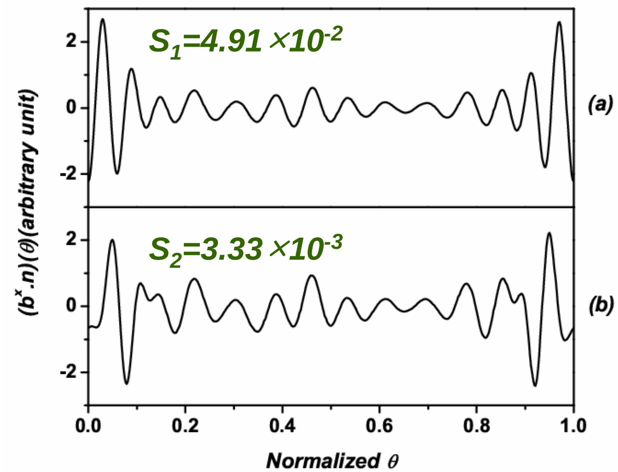


The control of the magnetic island opening

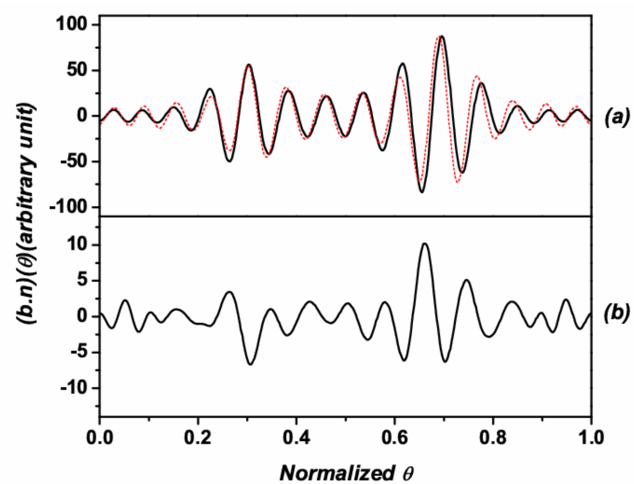
Take the distance away from the rational surface as $\psi = \pm 10^{-3}$



SVD



Permeability matrix



- **The broad spectrum for the external modes can be understood by SVD analysis about the coupling matrix**

$$\vec{\Delta} = \mathcal{K} \cdot \vec{\Phi}^x$$

Future work

- More benchmarking study
  - Surface consideration with non-zero edge current/pressure plasma equilibrium
  - Application to the DIII-D/ITER
  - Direct comparison with other stability codes
- More study for characteristics of the plasma response
  - β or shape effect on the plasma response in the different coordinate system
  - Mode jump on the rational surface
- Application for the various purposes
  - Computation of plasma rotation damping
  - Feedback routines for the resistive wall mode control
- Implementation of interface with experimental devices
  - The gain and phase of control coils to obtain the best performance during NSTX plasma operation