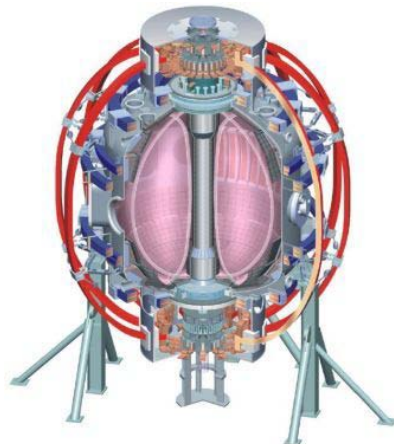


# Investigation of Power Balance and Excess Ion Heating in the National Spherical Torus Experiment

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# Abstract

- Neutral Beam modulations were used to create a transient heat pulse to investigate power balance of thermal ions in NSTX discharges. The primary effect of the beam modulation is a change in the ion temperature, with very little electron response. Classical neutral beam heating is expected to preferentially heat electrons with only 1/3 of the power going to the ions. Grad-Shafranov reconstructions of the plasma were performed using Thomson scattering, CHERS and MSE to measure the plasma temperature profiles and constrain the current profile. The ion power balance was calculated using the TRANSP time dependent transport calculation code. Assuming ion neoclassical transport, TRANSP calculates excess ion heating for some plasma conditions. This heating is strongest in the presence of high frequency MHD modes. The predicted peak of these modes correlates well with the location of strongest heating. Full orbit calculations of thermal ions in the presence of these modes show that stochastic heating can be sufficient to explain the excess heating.

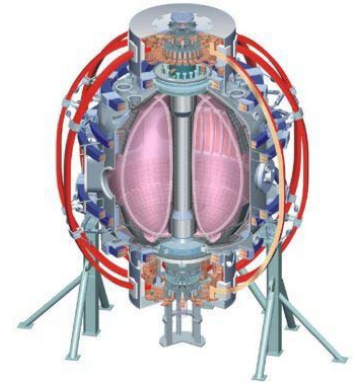
This work was supported by DoE contract No. DE-AC02-09CH11466.

# Outline

- Review of NSTX properties and diagnostics
- Review of Past Excess Heating Theory and Calculations
- Power Balance Overview
- Excess Heating in NSTX
  - Excess heating observed in NSTX
  - High frequency MHD modes are ubiquitously present
  - CAE eigenmode equation matches the radial deposition profile for excess heating
  - The high-k diagnostic is able to observe the CAE perturbations
    - This information can be used to get an estimate of the size of the CAE modes
- Full Orbit Calculations Show that High Frequency Modes Can Heat the Plasma

# NSTX Properties

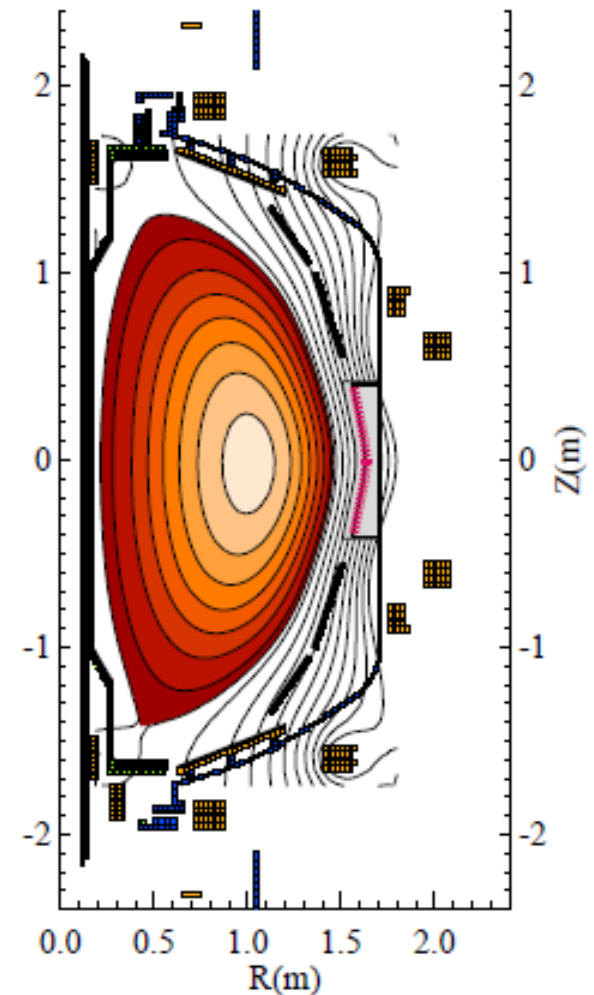
- NSTX Properties:
  - Deuterium plasmas
  - Electron and Ion Temperatures are 0.5-2 keV
  - Densities are  $\sim 10^{13} \text{ cm}^{-3}$
  - B field  $\sim 0.5$  Tesla on axis
- NSTX is heated by 3 Neutral Beam Sources.
  - Neutral beams are created by accelerating deuterium ions to 90 keV
  - The ions are then neutralized and passed into the plasma
  - The neutral particles then charge-exchange with plasma ions
  - Each beam contributes  $\sim 2$  MW of energy to the plasma
- TRANSP is a self consistent transport code, which takes the plasma temperature and density and the magnetic equilibrium as inputs and calculates power balance terms.



# NSTX Diagnostics and Reconstruction

- Relevant Power Balance Diagnostics
  - Thomson Scattering (30 radial channels)
    - 60 Hz laser system
    - Measures  $T_e$  and  $n_e$
  - CHERS
    - Measures  $T_i$
  - MSE diagnostic
    - Measures radial magnetic field pitch (constraining the  $q$  profile)
  - Magnetic Pickup Coils (Mirnov Coils)
    - Measure the rate of change of the magnetic field, used for magnetic reconstruction
  - Edge Neutral Density Diagnostic
    - Measures the edge neutral density profile for power balance calculations

from \LRDfit09, Shot 128820, time=397ms



# Theoretically, MHD modes can Stochastically Heat Thermal Particles, but Can This be Observed in Experiment?

- MHD modes are electromagnetic “waves” of sloshing plasma.
- Excess ion heating was observed as early as 2002.
- A theory was proposed that CAE modes could stochastically heat the plasma.
- Slab geometry full orbit calculations show a measurable effect.

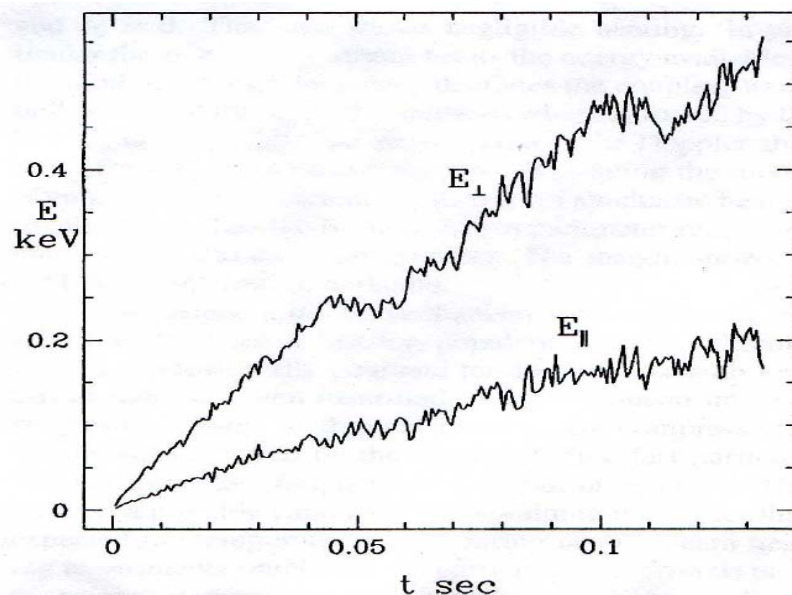


FIG. 1. Ion energy in keV vs time, for  $\delta B_y/B = 6.0 \times 10^{-4}$ .

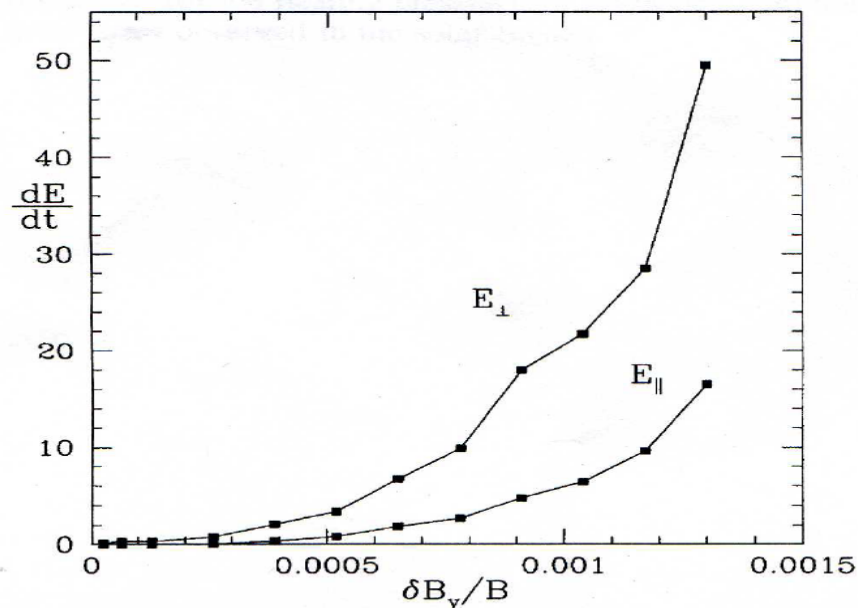


FIG. 2. Heating rate (keV/s) vs  $\delta B_y/B$ .

The MHD modes heat the thermal ions perpendicularly. In the absence of collisions, the perpendicular energy saturates. Increased  $\delta B$  leads to increased heating rates. (Gates, et al, Phys. Rev. Lett. **87**, 205003 (2001).

# Power Balance Overview

In its simplest form, power balance can be written as

$$P_{in} = P_{out}$$

where  $P_{in}$  is the heat we apply to the plasma, and  $P_{out}$  is everywhere that the heat can go. If we assume that neutral beam heating is the only source of ion heating, we get:

$$P_{nb} = \dot{W} + P_{ie} + P_{nc} + P_{loss}$$

$P_{nb}$  - Power Deposited from neutral beams

$P_{nc}$  - Neoclassical Heat transport

$P_{ie}$  - Heat transfer from ions to electrons

$\dot{W}$  - Thermal heating of the ions

$P_{loss}$  - Other loss terms, such as MHD induced power loss, charge exchange, etc.



# Power Balance Continued

If there is a source of excess heating, then the power balance equation becomes

$$P_{nb} + P_{excess} = P_{nc} + P_{ie} + \dot{W}$$

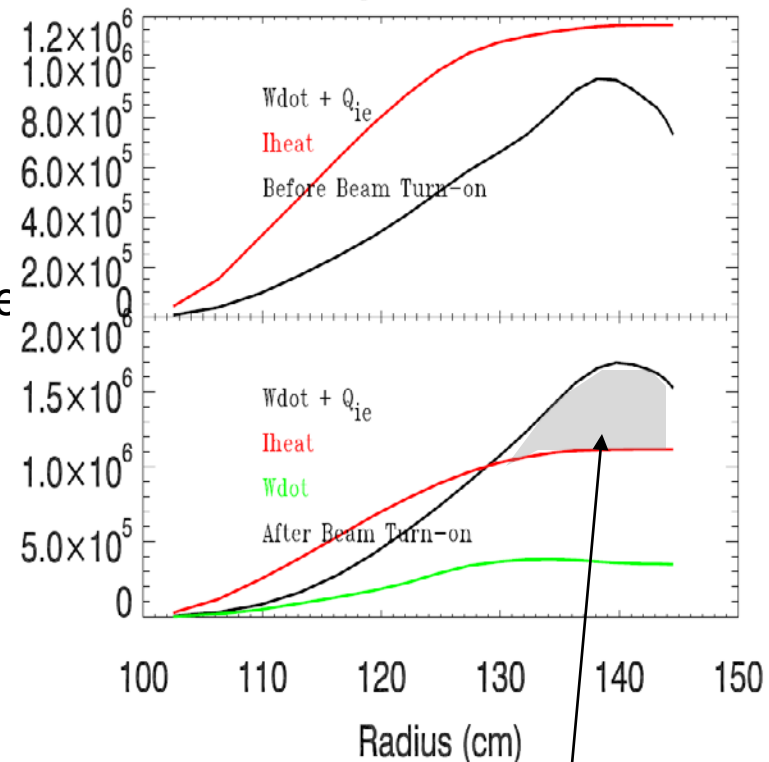
To determine the excess heating of the plasma, we subtract the neutral beam power from the other terms.

$$P_{excess} = P_{nc} + P_{ie} + \dot{W} - P_{nb}$$

This determines a lower bound on the excess heating. If other loss mechanisms are included, the excess heating will increase.

Note: These equations depend on the accuracy of models used, such as neoclassical transport in NSTX.

Volume integrated Power Balance



Before the 3<sup>rd</sup> beam turns on, the heating power is above the loss. After the 3<sup>rd</sup> beam turns on, the loss dominates the heating, implying an excess heating mechanism



# Beam Modulation Was Used to Modify Plasma Input Power

## Two Modulation Speeds

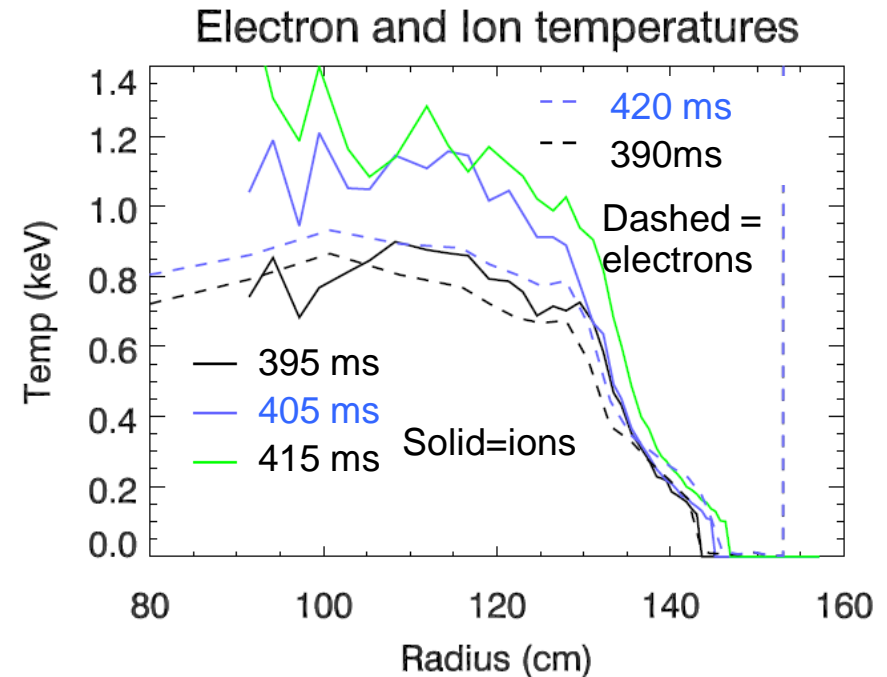
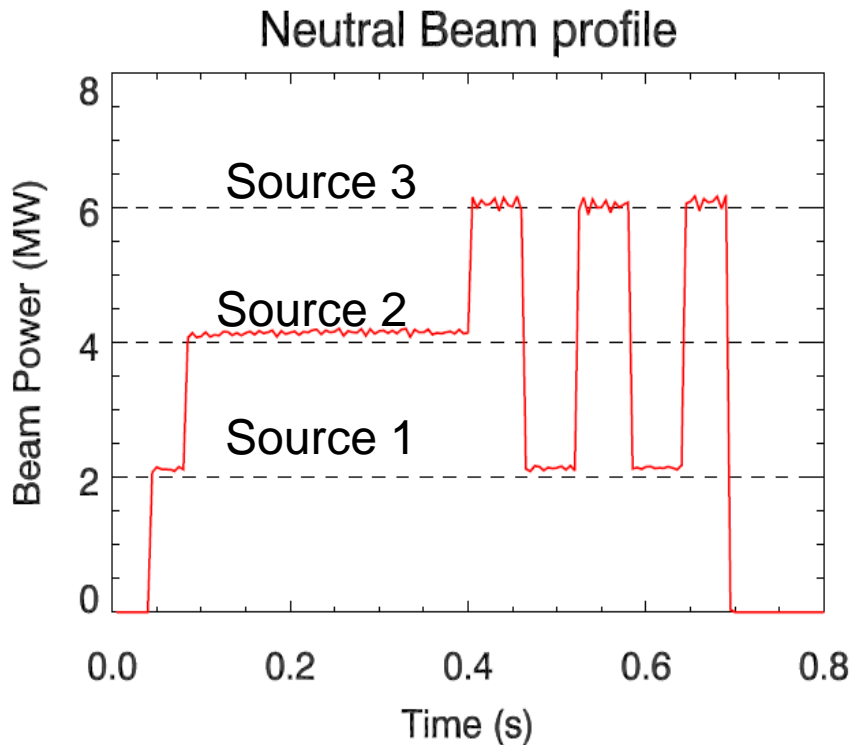
### Fast Modulations:

- Used to observe “Transient” behavior (behavior not related to fast particles slowing down)
- On the order of the beam particle slowing down time
  - Modulations: 30 ms
  - Slowing down: 30 ms
  - $E = 50$  ms
- Should not significantly impact thermal ion population unless excess heating is present

### Slow Modulations:

- Used to observe global behavior changes due to power loss
- On the order of the energy confinement time
  - Modulations: 60 ms
  - Slowing down: 30 ms
  - $E = 50$  ms
- By the end of the modulation, the thermal population should show changes due to fast ion population

# The Third Beam significantly heats the ions, but does not heat the Electrons as much



- The Neutral Beam profile
- The Third beam turns on at 400 ms
- This coincides with the MHD free period

Electron (dashed) and ion (solid) temperatures. The ion temperature rises quickly after the beam turn on at 400 ms. The electron temperature rises only slightly.

# Excess Heating Increases the Power to Ions Compared to Classical Calculations from 30% to 41%

## TRANSP calculations

- Total Power to Ions (classical):  $1.35 \times 10^6$
- Total Power to Electrons (classical):  $3.11 \times 10^6$

Percent of Total Power to Ions (classical): **30.3%**

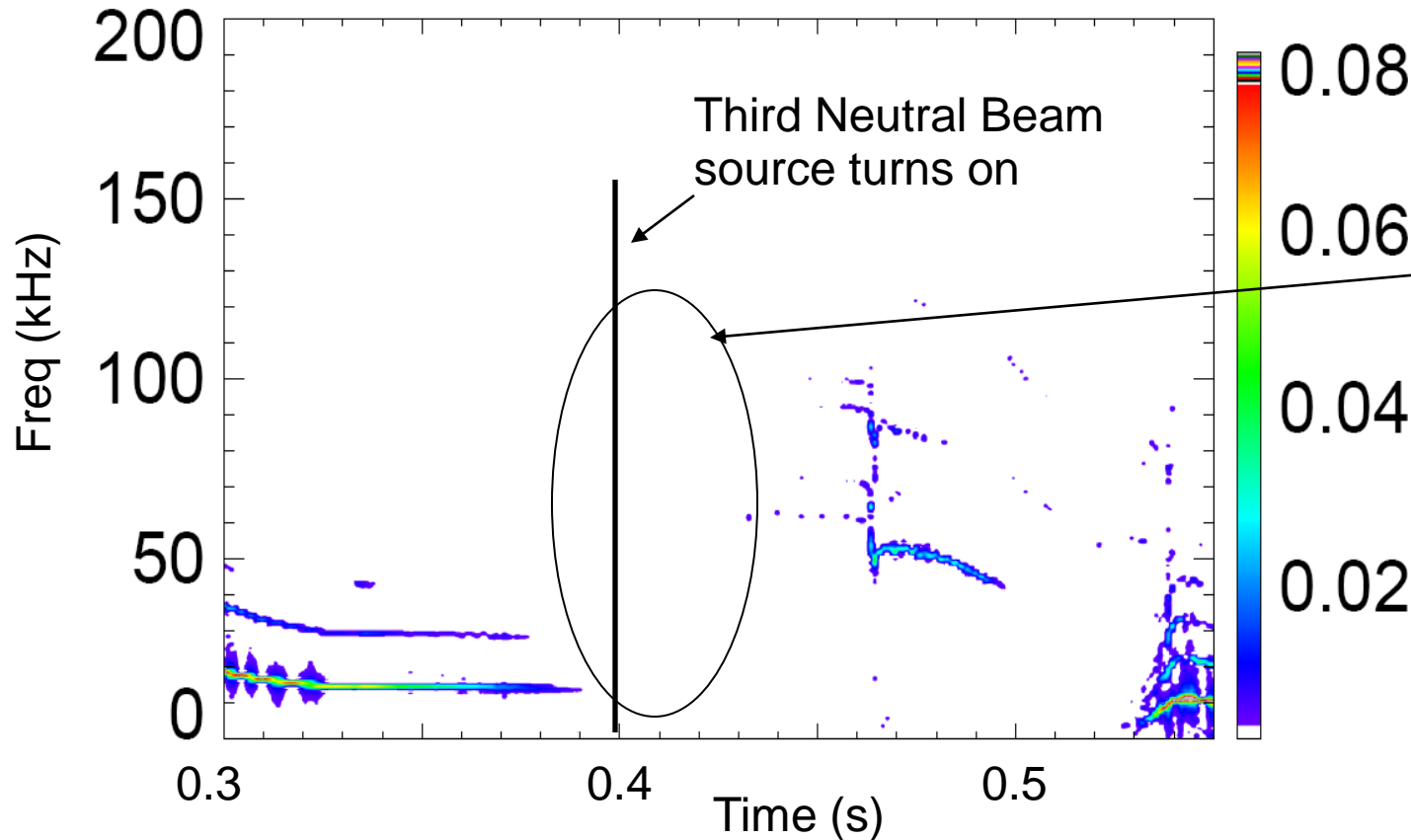
## Excess Heating Calculations:

- Excess Heating Power to Ions:  $5 \times 10^5$
- Total Heating of Ions:  $1.85 \times 10^6$
- Total Power to Electrons:  $3.11 \times 10^6$

Percent of Total Power to Ions: **41.5%**

# A Quiescent Low Frequency MHD Period Was Necessary to Study Power Balance

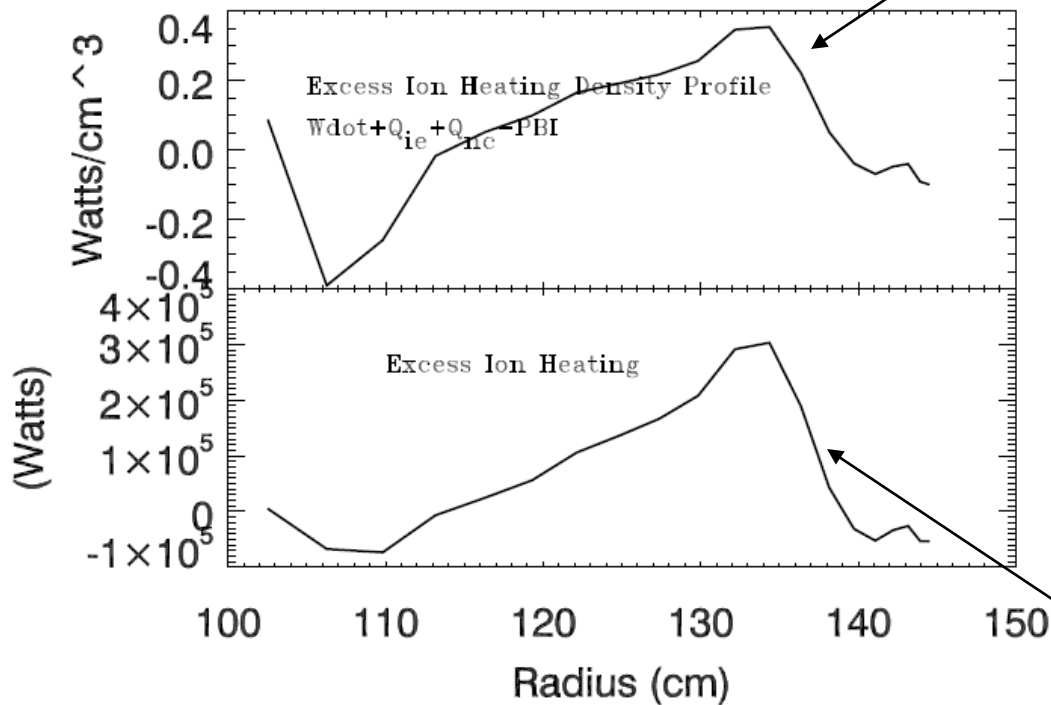
Low frequency MHD is associated redistributions of beam ions. In order to accurately measure the fast ion distribution, The modulation was performed in a quiescent low frequency MHD period.



The modulations were timed to coincide with the MHD free window

# Excess Heating required peaks around 130 cm

Transp Run 128820P07



Excess Heating Profile

The excess heating is calculated by

$$P_{excess} = P_{nc} + P_{ie} + \dot{W} - P_{nb}$$

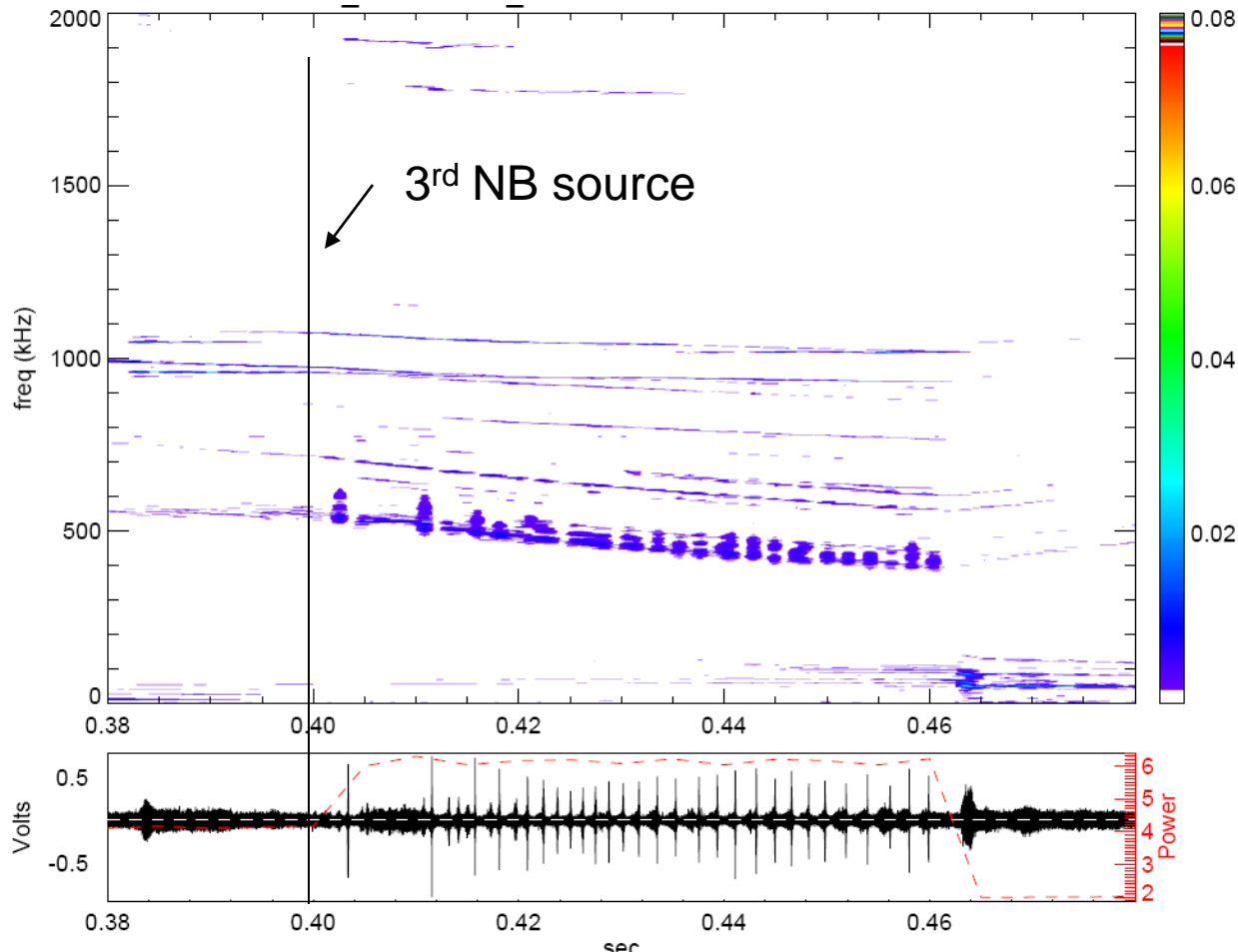
Since this equation is satisfied locally it is possible to solve for the excess heating at each point radially. The total ion heating peaks at 134 cm.

P\*dVol profile

Note: Most discharges do not require excess heating to satisfy the power balance equation. However, it was prevalent during these conditions.

# Bursting High Frequency MHD Appears with 3<sup>rd</sup> Source

## High Frequency MHD shows Bursting Modes



- Mid frequency MHD 'bursting' activity occurs with 3<sup>rd</sup> source.
- Modes are driven by fast ions
- It is not clear why these modes are bursting.
- Bursting modes have a duty cycle of ~3%

# The Theory of CAE Modes Predicts Eigenfunctions

- These MHD modes have been identified as Compressional Alfvén Eigenmodes. (Gorelenkov et al, Nuclear Fusion **42** 977 (2002))
  - This identification is currently under dispute
- Possible to relate magnetic field fluctuations to density fluctuations
- Can then use high-k signal as an interferometer to estimate MHD fluctuation size

$$E_{\theta} = E_0 \phi_m \left( \frac{\sqrt{2}\theta}{\Theta} \right) \phi_s \left( \frac{\sqrt{2}(r-r_0)}{\Delta} \right) e^{i(n\phi - \omega t)}$$

$$E_r = \frac{i\omega_c}{\omega} E_{\theta}$$

$$\phi_s(x) = \frac{e^{-x^2/2} H_s(x)}{\sqrt{s! 2^s \sqrt{\pi}}}$$

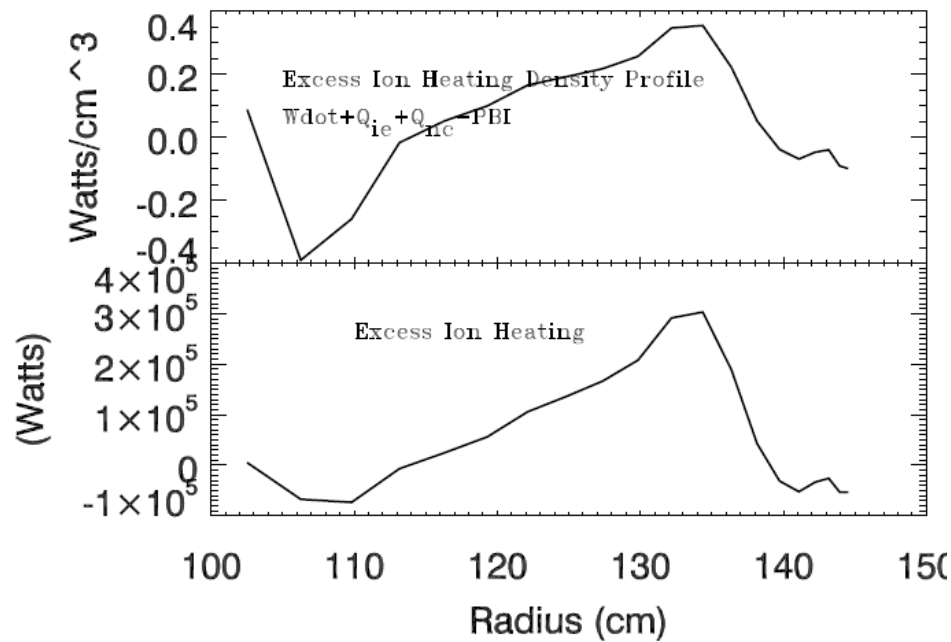
$$\Delta^2 = a^2 \frac{\sqrt{2\sigma_i/(1+\sigma_i)}}{m(1+\sigma_i)(1+\epsilon_0)}$$

See Gorelenkov, Cheng, Fredrickson, *Phys. Plasmas*, **9**, 3483 (2002).



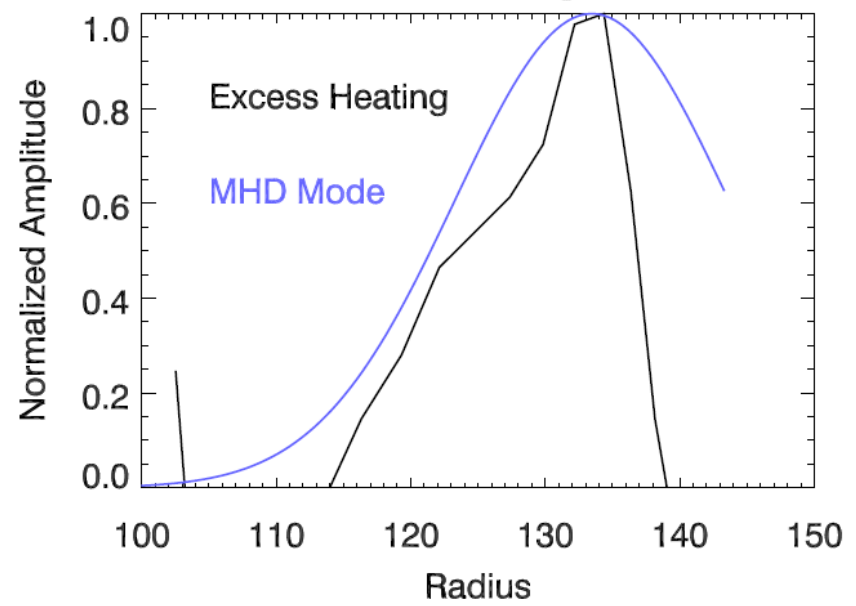
# Good Spatial Correlation Between Peak in Excess Heating and the Peak in the Radial Eigenfunction of the CAE's

**Excess Heating  
Transp Run 128820P07**



Excess heating as calculated by Transp. (Top is heating density, bottom is total heating)

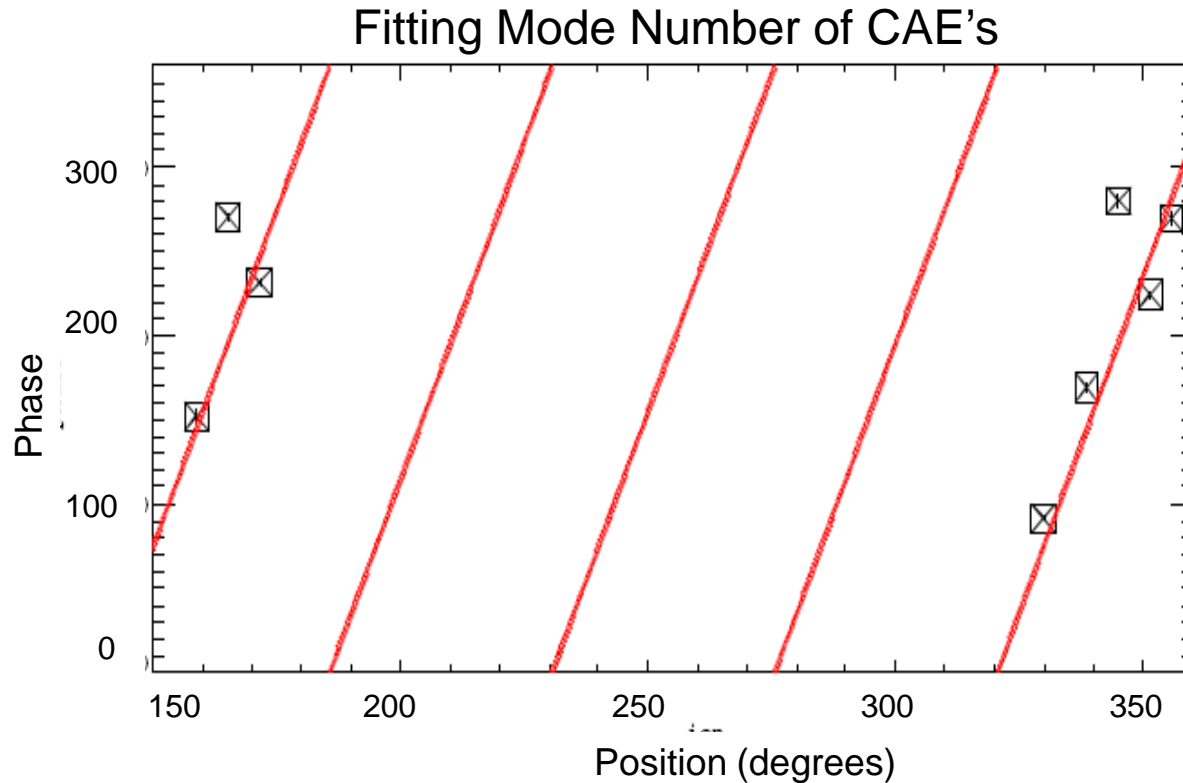
**Mode and Heating Profiles**



High frequency Alfvén modes (radial mode = 0)

The required excess heating peaks around 134 cm, which is near the peak amplitude of the high frequency Alfvén modes.

# Toroidal Mode Numbers of CAE's Can Be Determined by Fitting the Phase of Mirnov Coils



High frequency modes normally have toroidal mode numbers in the range of 7-9. Each mode must be analyzed specifically to determine mode numbers.

## Correlating $\delta n/n$ and $\delta B_{\parallel}/B$

Start with the continuity equation: 
$$\frac{\partial \tilde{n}}{\partial t} + \nabla \cdot (\vec{v}n) = 0$$

The total derivative becomes 
$$\frac{dn}{dt} = \frac{\partial n}{\partial t} + \vec{v} \cdot \nabla n = -n \nabla \cdot \vec{v} \quad (1)$$

Note:  $\tilde{B} \sim e^{i\omega t} e^{i\vec{k} \cdot \vec{r}} \rightarrow \nabla \cdot \tilde{B} = \vec{k} \cdot \tilde{B} = 0$

Lowest damped modes have  $k_{\perp} \gg k_{\parallel}$ , which gives  $\vec{k}_{\perp} \cdot \tilde{B} = 0 \rightarrow \tilde{B} = B_{\parallel}$

This means that the magnetic field perturbation is strictly parallel.

The velocity is related to the displacement by 
$$\vec{v} = \frac{\partial \vec{\xi}}{\partial t}$$

All quantities vary in time like  $e^{i\omega t}$ , so equation (1) becomes

$$\tilde{n} = -\vec{\xi} \cdot \nabla n - n \nabla \cdot \vec{\xi} \quad (2)$$

$$\delta n/n = -\delta B_{\parallel}/B$$

Since the field lines are “locked into” the plasma, equation (2) becomes

$$\tilde{n} = -\xi_r \frac{\partial n}{\partial r} - n \nabla \cdot \vec{\xi} \quad (3)$$

The displacement is calculated from ExB drift:  $\xi_r = \frac{iE_{\theta}c}{B\omega}$ ,  $\nabla \cdot \vec{\xi} = \frac{\tilde{B}_{\parallel}}{B}$

This makes equation (3) become 
$$\frac{\tilde{n}}{n} = -\frac{iE_{\theta}}{B\omega} \frac{\partial \ln n}{\partial r} - \frac{\tilde{B}_{\parallel}}{B} \quad (4)$$

If we assume the mode frequency  $\omega \ll \omega_c$ , then  $E_{\theta} \sim \frac{B\omega^2}{\omega_c}$  (R. White, unpublished)

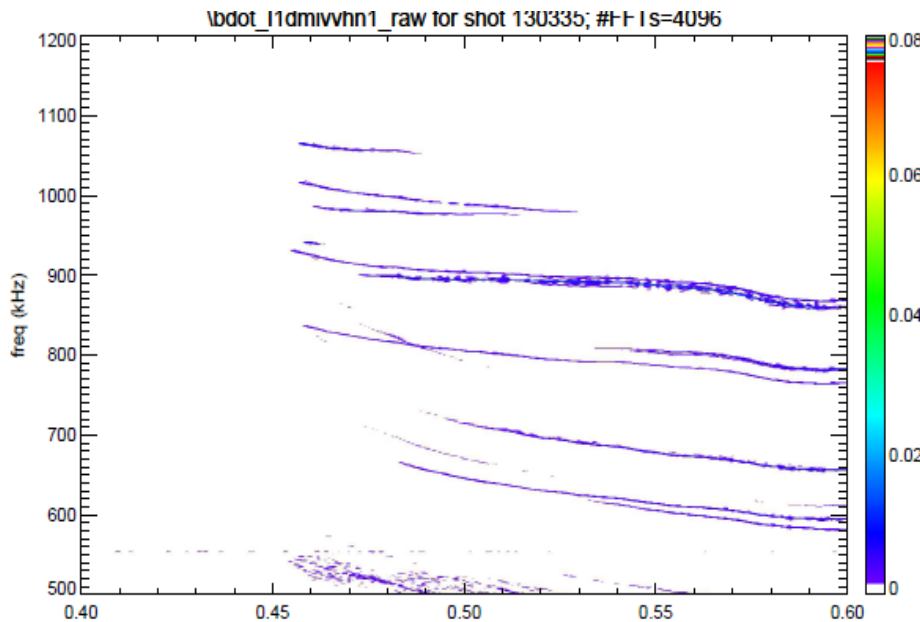
This means that only the 2<sup>nd</sup> term of equation (4) survives, giving

$$\frac{\tilde{n}}{n} = -\frac{\tilde{B}_{\parallel}}{B}$$

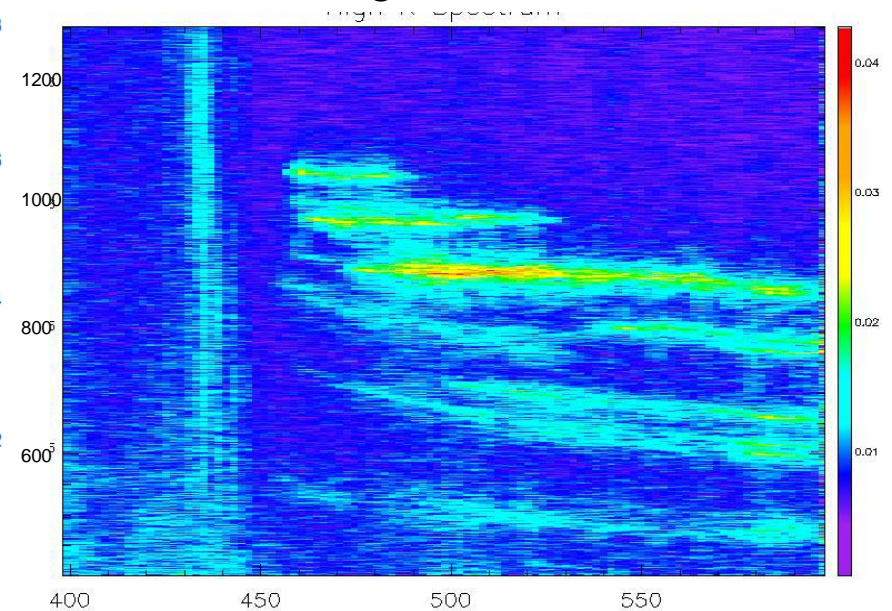
This analysis was reproduced from N. Gorelenkov, 2008 (unpublished)

# The High-k diagnostic can get an amplitude for Magnetic Perturbations

## FFT of Mirnov Coil Signal

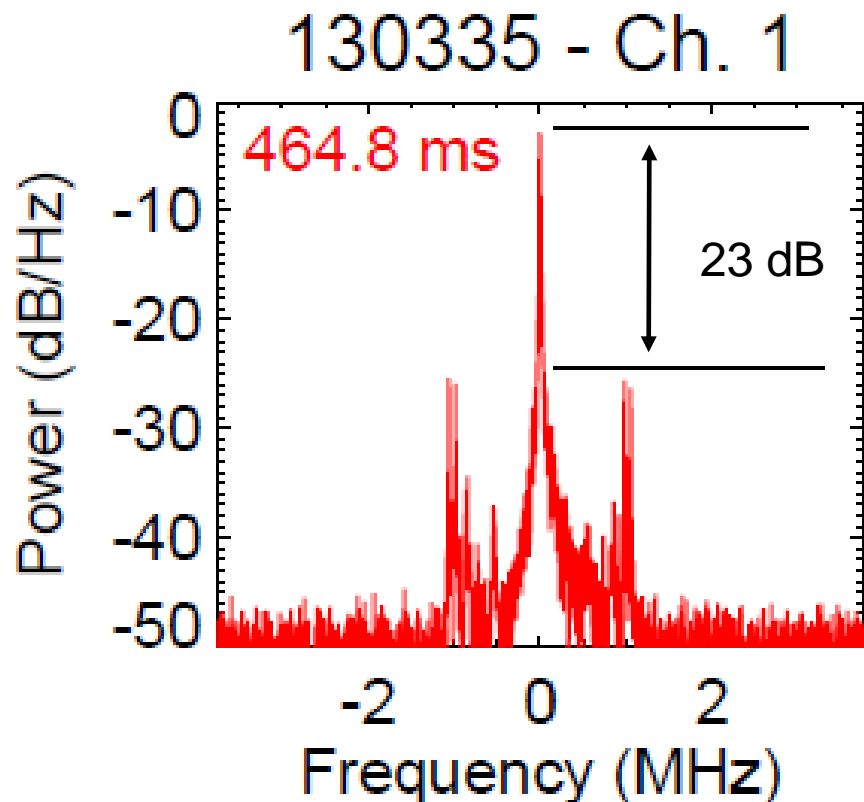


## FFT of high-k Channel 1



The high-k diagnostic shows the same perturbations as the Mirnov pickup coils. This means the high-k diagnostic can be used as an interferometer to match perturbation amplitudes with plasma fluctuations

# Line Integrated Phase Shift = $7.07e-2$



The signal amplitude goes as

$$\begin{aligned} e^{i\phi} &= e^{i(\phi_0 + \tilde{\phi})} \\ &= e^{i(\phi)} e^{i(\tilde{\phi})} \\ &= A e^{i(\tilde{\phi})} \\ &\approx A(1 + i\tilde{\phi}) \end{aligned}$$

Unshifted      Shifted

The phase shift is then the ratio of the shifted to unshifted amplitudes. To avoid phase problems, we use power and take the square root.

$$\tilde{\phi}^2 = -23dB = 5e-3$$

$$\tilde{\phi} = 7.07e-2$$

# Integrated Density Perturbation $\sim 1.6e13$

$$\tilde{\phi} = k \int \tilde{N} dl$$

Where N is the index of refraction, and k is the wave number of the beam.

For an electromagnetic wave in a plasma,

$$N = \sqrt{1 - \frac{\omega_p^2}{\omega^2}} \sim 1 - \frac{1}{2} \frac{\omega_p^2}{\omega^2} = 1 - \frac{1}{2} \frac{\omega_p^2 \lambda^2}{(2\pi c)^2}$$

where  $\omega_p$  is the plasma frequency, and  $\omega$  is the frequency of the beam. This assumes  $\omega_p \ll \omega$

$$N = 1 - \frac{1}{2} \frac{4\pi e^2}{m(kc)^2} \langle n \rangle - \frac{1}{2} \frac{4\pi e^2}{m(kc)^2} \langle \tilde{n} \rangle$$

The first 2 terms relate to the phase shift of the unperturbed beam through the plasma. The perturbation is only related to the third term. Thus

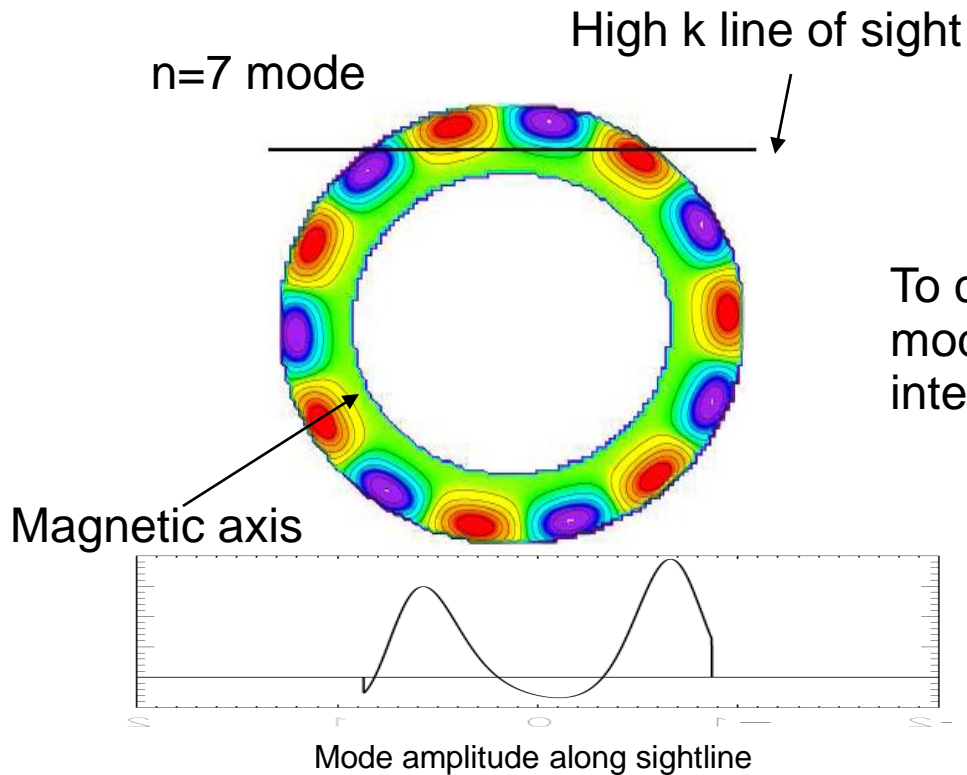
$$\tilde{\phi} = k \int \frac{1}{2} \frac{4\pi e^2}{m(kc)^2} \tilde{n}_e dl = 4.48 \times 10^{-15} \int \tilde{n}_e dl \Rightarrow \int \tilde{n}_e dl = 1.56 \times 10^{13}$$

$$\frac{\int \tilde{n}_e dl}{\int n_e dl} = \frac{1.56 \times 10^{13}}{6.0 \times 10^{15}} = 2.6 \times 10^{-3}$$



# Mode Amplitude $\delta n/n = 7.9e-3$

Using the CAE mode profile above, a line integral was performed to determine the amplitude of the CAE. The mode is assumed to vary toroidally as  $\cos(n\phi)$ .



Integrating an MHD mode with amplitude "A", and  $n=8$  gives

$$\int \tilde{n}_{norm} dl = 0.33A$$

To calculate the amplitude of the MHD mode in the plasma, we divide the integrated density by this integration factor

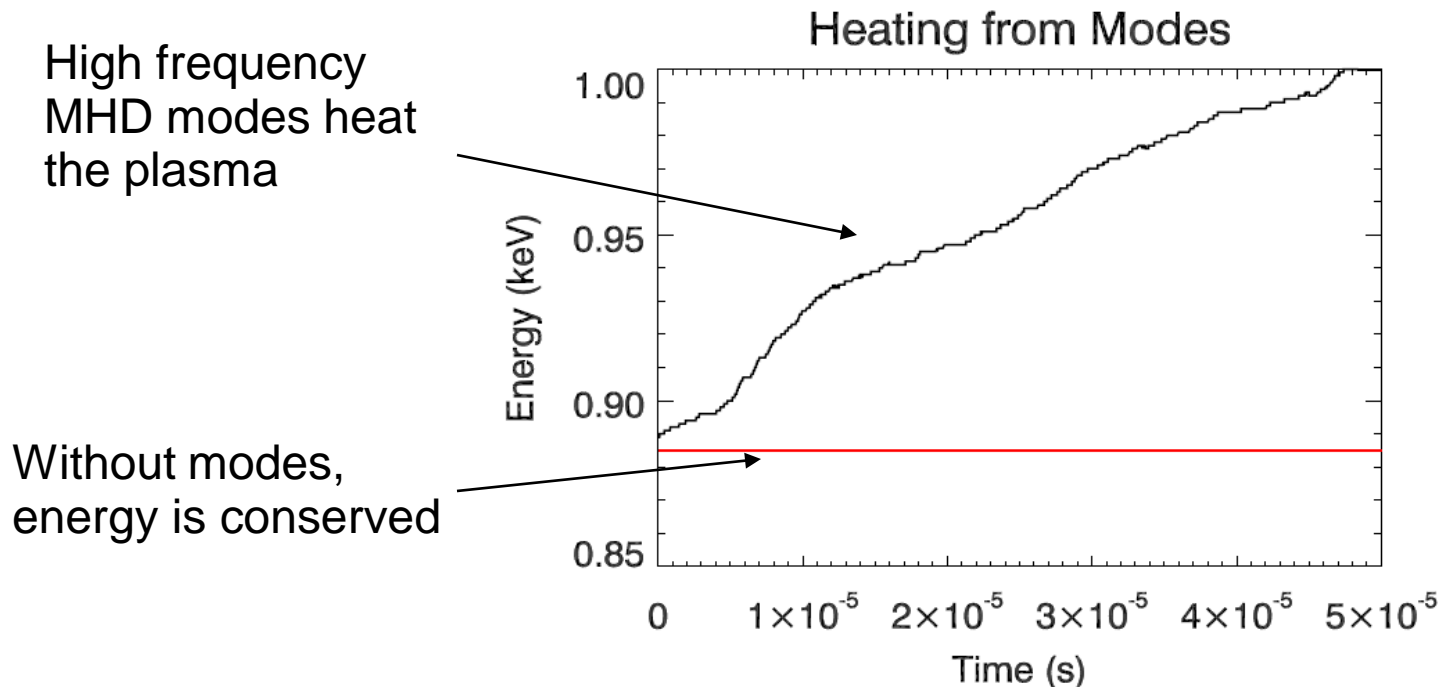
$$\frac{\int \tilde{n}_{measured}}{\int \tilde{n}_{calculated}} = \frac{0.0026}{0.33} = \frac{\tilde{B}_{\parallel}}{B} = 0.0079$$

This gives

$$\frac{\tilde{B}_{\parallel}}{B} = \frac{\tilde{n}_e}{n_e} = 0.0079$$

# Full Gyroorbit Calculations were performed using MHD Perturbations

- ‘Gyrox’ Full orbit (toroidal geometry) code with 12 modes, radial mode number 0.
- Mode amplitudes were measured by scaling the Mirnov coil signal to the High-k amplitude
- 5000 particles placed at the location of the peak amplitude of the modes.
- Energy of the particles was calculated as a function of time.



# The Calculated Mode Heating Match TRANSP Heating

$$P_{tot} = \frac{\text{Energy}}{\text{particle sec}} * \frac{\text{particles}}{m^3} * m^3$$

$$P_{tot} = \frac{0.1 \text{ keV}}{5e-5 \text{ sec}} * 4e19 = 1.2e7 \text{ Watts}$$

$$\text{Duty Cycle} = 3\%$$

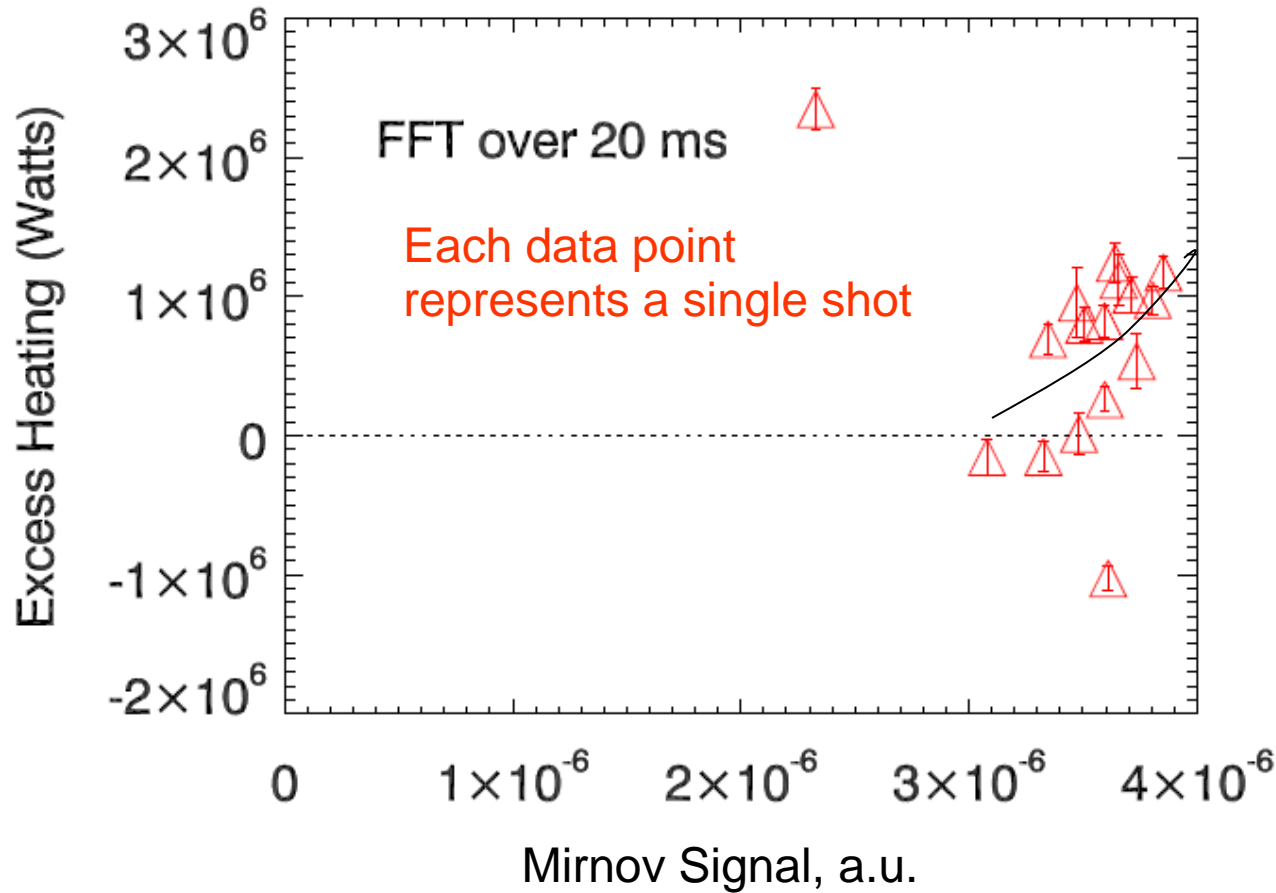
$$P_{tot} = 1.2e7 \text{ Watts} \times 0.03 = 3.6e5 \text{ Watts}$$

- Particles placed at the peak amplitude.
- Different poloidal mode number (m numbers) heat different amounts
- No diagnostics measure “m” number

The MHD Modes can account for the excess heating.

# High Frequency MHD Amplitude Correlates with Observed Excess Heating

## Excess Ion Heating Required



- Sum over FFT of the Mirnov coil signals
- A correlation exists between the RMS amplitudes and the excess heating as calculated by TRANSP
- The shape of the plot is similar to that computed in a slab geometry
- A threshold may exist for heating

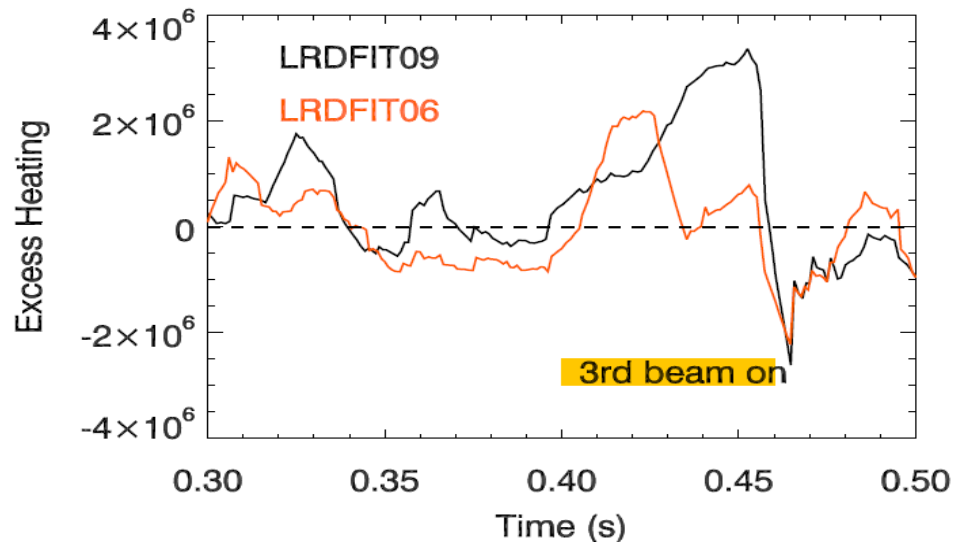
# Error Analysis in the Power Balance Calculations

The power balance analysis was performed using LRDFIT09 and LRDFIT06 reconstructions for the Grad-Shafranov reconstructions.

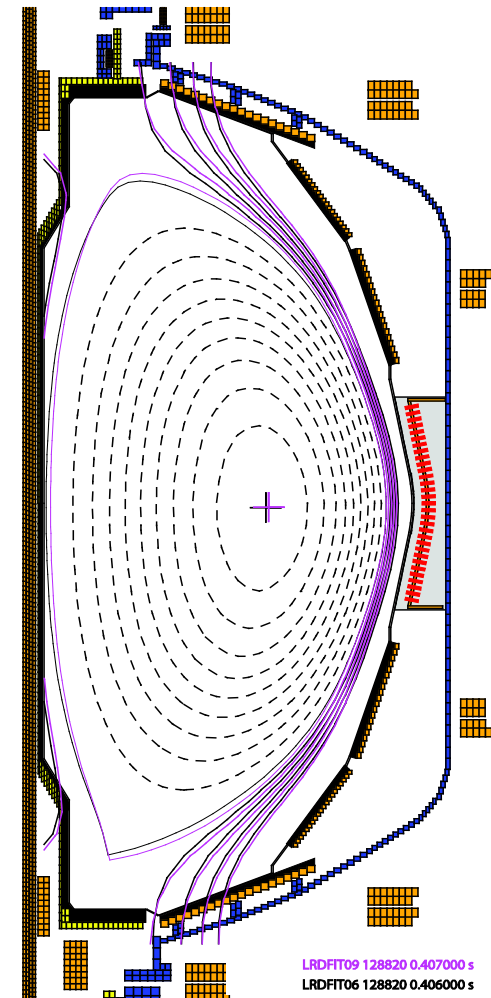
LRDFIT06 – Magnetics+Pitch angle +  $T_{e\_iso}$

LRDFIT09 – LRDFIT06 + Rotation effects

LRDFIT 06 vs. 09



Excess heating is present in both LRDFIT06 and 09, but strongest in LRDFIT09.



# Conclusions

- Beam modulations were successfully used to investigate Power Balance in NSTX.
- During some discharges, excess heating was required to match the power balance equation. This was not true for all discharges or all times during the discharge. It was most noticeable with the turn on of a third neutral beam source.
- During these discharges, the neutral beam modulations excited high frequency MHD modes.
- The profile of the CAE eigenfunctions matches the profile of the excess heating.
- Interferometric data has been used to connect the MHD mode amplitudes to the observed density perturbations.
- Full orbit calculations show that MHD modes can heat thermal ions enough to account for this excess heating. A final step to confirm this will be to put into the code a distribution of particles similar to the density of the plasma to observe the effects.