Gyrokinetic Studies of Neoclassical Poloidal Rotation with Finite Orbits

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Outline

A significant discrepancy of poloidal velocity from conventional theoretical predictions is found in global neoclassical drift-kinetic simulations of magnetic confinement fusion devices. The difference is identified to be due to the presence of large ion orbits. In the case of a large-aspect ratio tokamak configuration with steep toroidal flow profiles, a novel theoretical model which describes this nonlocal effect is presented and shown to explain the simulation results. The dominant nonlocal mechanisms captured by the model are associated with ion parallel flow modification due to the steep toroidal flow and radial electric field profiles. We compare simulation results with theoretical estimates based on the new model using profiles relevant for the National Spherical Torus Experiment (NSTX). The carbon poloidal velocity observed in the simulation is in good agreement with the neoclassical theory modified by the newly identified nonlocal effects.

Unlike-particle collision operator

- The linearised unlike-particle collision operator for species a colliding with species b is

$$C_{ab}[F_{0a}, \delta f_b] + C_{ab}[\delta f_a, F_{0b}] = C_{ab}^{TP}(\delta f_a) + C_{ab}^{FP}(\delta f_b)$$

- The field-particle operator is

$$C_{ab}^{FP}(\delta f_b) = (\mathcal{H}_{ab}(v)\delta N_{ab} + \mathcal{R}_{ab}(v)v_{||}\delta P_{ab} + \mathcal{Q}_{ab}(v)\delta E_{ab})$$

  – particle number, momentum and energy gained by species a field particles must equal that lost by species b test particles

$$\begin{aligned}
\delta N_{ab} &= -\int d^3v C_{ba}^{TP}(\delta f_b), \\
\delta P_{ab} &= -\int d^3v m_b v_{||} C_{ba}^{TP}(\delta f_b), \\
\delta E_{ab} &= -\int d^3v (m_b v^2/2) C_{ba}^{TP}(\delta f_b).
\end{aligned}$$

– linearised operator must have the correct null space

$$C_{ab}^{TP}(\delta f_a) + C_{ab}^{FP}(\delta f_b) = 0$$

when δf_a and δf_b are perturbations of Maxwellians with the same flow velocity and the same temperature perturbation:

$$C_{ab}^{TP}(m_a v_{||} F_{0a}) + C_{ab}^{FP}(m_b v_{||} F_{0b}) = 0,$$
$$C_{ab}^{TP}(m_a v^2 F_{0a}) + C_{ab}^{FP}(m_b v^2 F_{0b}) = 0.$$

- The conservation properties may be written as follows

$$\begin{aligned} \int d^3v C_{ab}^{FP}(\delta f_b) &= \delta N_{ab}, \\ \int d^3v m_b v_{||} C_{ab}^{FP}(\delta f_b) &= \delta P_{ab}, \\ \int d^3v (m_b v^2/2) C_{ab}^{FP}(\delta f_b) &= \delta E_{ab}. \end{aligned}$$

- With appropriate choices of multiplying factors, the functions $\mathcal{R}_{ab}$ and $\mathcal{Q}_{ab}$ are

$$\mathcal{R}_{ab}(v)v_{||}F_{0a} = \frac{C_{ab}^{TP}(m_a v_{||}F_{0a})}{\int d^3v m_a v_{||} C_{ab}^{TP}(m_a v_{||}F_{0a})},$$

$$\mathcal{Q}_{ab}(v)F_{0a} = \frac{C_{ab}^{TP}(m_a v^2 F_{0a})}{\int d^3v (m_a v^2/2) C_{ab}^{TP}(m_a v^2 F_{0a})}.$$

- Computing the Rosenbluth potentials gives

$$\mathcal{R}_{ab}(v) = \frac{3\sqrt{\pi}}{4n_a T}(1 + m_b/m_a)^{3/2} y_b^{-3/2} \phi(y_b),$$

$$\mathcal{Q}_{ab}(v) = \frac{\sqrt{\pi}}{2n_a T}(1 + m_b/m_a)^{3/2} y_b^{-1/2} \left(m_a/m_b - d/dy_b\right) \phi(y_b),$$

$$\mathcal{H}_{ab}(v) = 1 - \mathcal{Q}_{ab}(v).$$

where $\phi(y) = 2/\sqrt{\pi} \int_0^y e^{-t} \sqrt{t} dt$ and $y_b = v^2/v_b^2 = m_b v^2/(2T)$.

- The equations for the two marker weights become

$$\dot{w} = \frac{1-p}{F_{0s}}\left(-\frac{DF_{0s}}{Dt} + \sum_b C_{sb}^{FP}(\delta f_b)\right) - \eta(w - \overline{w}_s),$$
$$\dot{p} = \frac{1-p}{F_{0s}}\left(-\frac{DF_{0s}}{Dt}\right) - \eta(p - \overline{p}_s).$$

- The local shifted Maxwellian background distribution function is expressed as follows

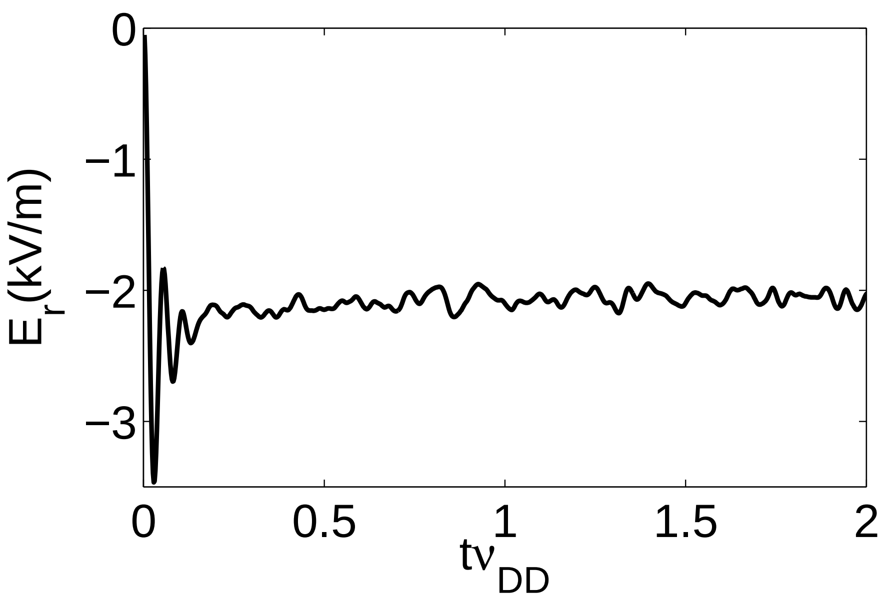
$$F_{0s} \equiv F_{0s}(n_s, T, U_{||})$$
$$= n_s\left(\frac{m_s}{2\pi T}\right)^{3/2} \exp\left[-\frac{m_s}{T}\left((v_{||} - U_{||})^2/2 + \mu B\right)\right].$$

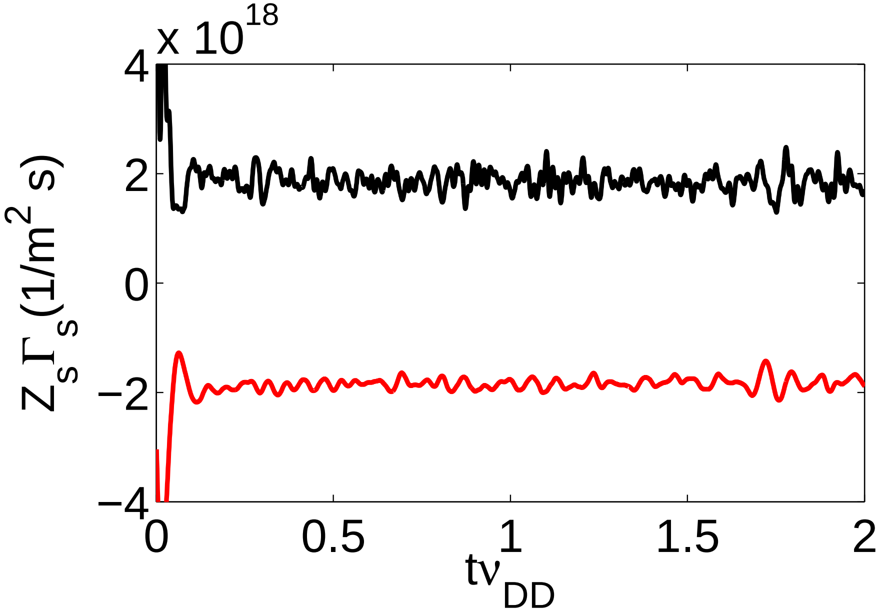
  – The difference in species' temperature $T(r)$ and parallel flow $U(r)$ profiles is captured by initial $\delta f_s(t=0)$

$$\delta f_s(t=0) = \frac{F_{0s}(n_s, T_s, U_{||s})}{F_{0s}(n_s, T, U_{||})} - 1,$$

  where $T_s(r)$ and $U_{||s}(r)$ are experimental profiles for species s.

- Time evolution of the radial electric field and radial guiding center currents $Z_s\Gamma_s$ for both **deuterium** and **carbon**





– Ambipolar radial electric field reaches steady state when the total radial guiding center particle current $\sum_s Z_s\Gamma_s = 0$ vanishes, while individual particle fluxes Γ_s stay finite.

Unlike-particle collision operator (residual errors)

- Residual errors will depend on the number of markers used in the simulation. Following the procedure developed by Satake, we rewrite the field-particle operator in the following form

$$C_{ab}^{FP}(\delta f_b) = (\mathcal{H}_{ab}(v)\delta N + \mathcal{R}_{ab}(v)v_{||}\delta P + \mathcal{Q}_{ab}(v)\delta E)$$

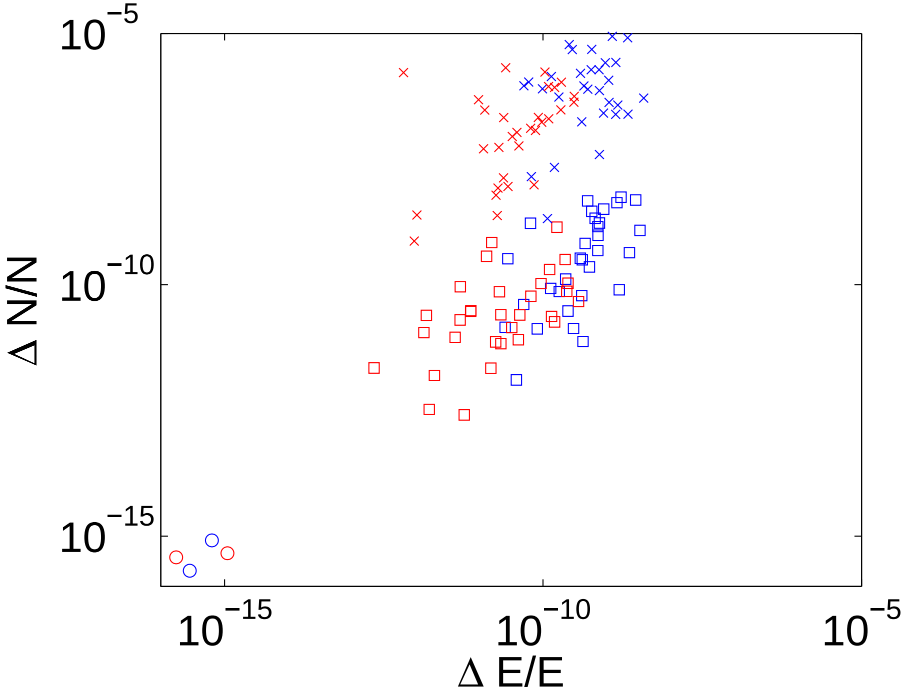
  – corrections δN, δP and δE are found by solving the equation

$$\sum_k (1-p_k) \begin{pmatrix} \mathcal{H}_{ab} & \mathcal{R}_{ab}v_{||} & \mathcal{Q}_{ab} \\ \mathcal{H}_{ab}v_{||} & \mathcal{R}_{ab}v_{||}^2 & \mathcal{Q}_{ab}v_{||} \\ \mathcal{H}_{ab}v^2 & \mathcal{R}_{ab}v^2 v_{||} & \mathcal{Q}_{ab}v^2 \end{pmatrix}_k \begin{pmatrix} \delta N \\ \delta P \\ \delta E \end{pmatrix}$$

$$= - \begin{pmatrix} \delta N_{ab} \\ \delta P_{ab}/m_a \\ 2\delta E_{ab}/m_a \end{pmatrix}$$

  – enforces precise conservation of number, momentum and energy locally. Residual error is at the rounding-error level for both deuterium and carbon, independent of the number of markers in the simulation.

- The blue and the red crosses (for deuterium and carbon) show the errors due to application of $(\mathcal{R}_{ab}(v)v_{||}\delta P_{ab} + \mathcal{Q}_{ab}(v)\delta E_{ab})$ operator;
- Application of the complete operator $(\mathcal{H}_{ab}(v)\delta N_{ab} + \mathcal{R}_{ab}(v)v_{||}\delta P_{ab} + \mathcal{Q}_{ab}(v)\delta E_{ab})$ (squares);
- Application of the new operator $(\mathcal{H}_{ab}(v)\delta N + \mathcal{R}_{ab}(v)v_{||}\delta P + \mathcal{Q}_{ab}(v)\delta E)$ (circles).



Neoclassical Poloidal Rotation

- The reason why the nonlocal effects are important for the poloidal flow U_p may be understood by constructing an appropriate combination of parallel $U_{||}$ and perpendicular $\boldsymbol{U}_\perp$ velocities to obtain

$$U_p = U_{||}\frac{B_p}{B} + \boldsymbol{U}_\perp \cdot \frac{\boldsymbol{B_p}}{B_p},$$

where $\boldsymbol{U}_\perp$ is the drift perpendicular to the magnetic field

$$\boldsymbol{U}_\perp = \frac{cT}{ZeB}\widehat{\boldsymbol{b}} \times \left(\frac{\partial \ln nT}{\partial \boldsymbol{r}} - \frac{Ze}{T}\boldsymbol{E_r}\right).$$

- $U_{||}$ can be evaluated as a sum of a local neoclassical result $U_{||0}$ and some unknown nonlocal correction $\Delta\mathcal{U}_{||}$

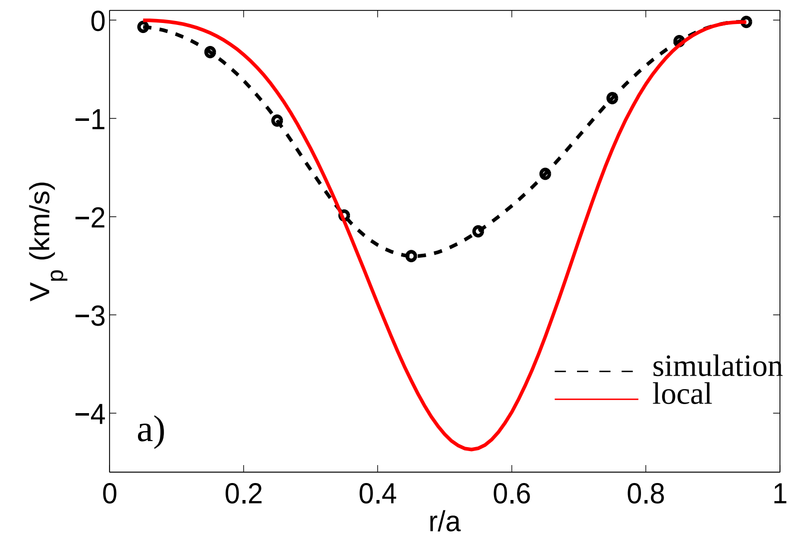
$$\begin{aligned} U_{||} &= U_{||0} + \Delta\mathcal{U}_{||}, \\ U_{||0} &= \frac{cIT}{ZeB\psi'}\left(-\frac{\partial \ln nT^{(1-k)}}{\partial r} + \frac{Ze}{T}E_r\right). \end{aligned}$$

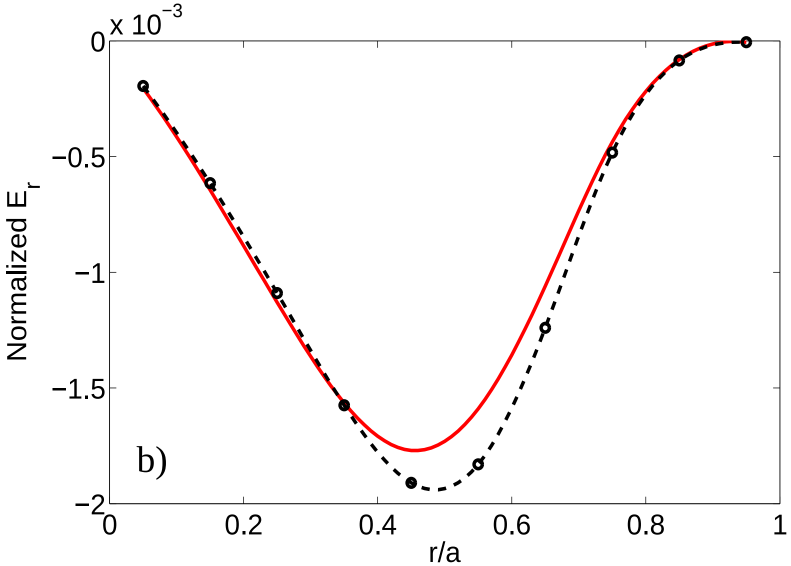
Here Ze and m are the ion charge and mass, B_p and B_t are poloidal and toroidal parts of the magnetic field, $I = RB_t$, $\psi' = \partial\psi/\partial r$, with ψ being the poloidal magnetic flux. k is collisionality dependent parameter. The toroidal rotation frequency is $\omega = U_{||}B/I$.

- The resulting ion poloidal velocity is

$$U_p = U_{p0} + \frac{B_p}{B}\Delta\mathcal{U}_{||},$$
$$U_{p0} = \frac{B_p}{B}k\frac{Ic}{ZeB\psi'}\frac{\partial T}{\partial r}.$$

- Most of the terms cancel, leading to the result that the standard neoclassical part of the poloidal flow U_{p0} only depends on the radial temperature gradient. While the nonlocal $\Delta\mathcal{U}_{||}$ correction might be small, its effect on the poloidal flow U_p, due to a cancellation, might be significant, especially in the case of a strong gradient in the toroidal rotation ω.





- Deuterium poloidal velocity
  - from **simulation** and **local neoclassical estimate**

- Radial electric field

Neoclassical Poloidal Rotation (*shear in the toroidal flow*)

- We start with the distribution function $f(\boldsymbol{x}, \epsilon, \mu)$ in the form

$$f = (m/2\pi T)^{3/2} N(\psi^\star) \exp\left(-a\epsilon - b\mu\right).$$

  - guiding center canonical angular momentum $\psi^\star = \psi - I v_{||}/\Omega$;
  - energy $\epsilon = (v_{||}^2 + v_\perp^2)/2 + Ze\Phi(\psi)/m$ and magnetic moment $\mu = v_\perp^2/2B$ with a and b being constants;
  - radial electric field $E_r = -\partial\Phi(\psi)/\partial r$.

- By substituting $N(\psi^\star) = N_0 \exp(-\beta\psi^\star - \gamma\psi^{\star 2})$ and equating the magnetic surface average of the coefficients in front of $v_{||}^2$ and $v_\perp^2$ we obtain

$$f = \left(\frac{m}{2\pi T}\right)^{3/2} N_0 \exp\left(-\beta\psi - \gamma\psi^2 - a\frac{Ze\Phi}{m} + \frac{m}{2T}U_{||}^2\right)$$
$$\exp\left(-\frac{m}{2T}\left[(v_{||} - U_{||})^2 + v_\perp^2\right]\right).$$

– The temperature and the parallel flow are defined to be

$$\frac{m}{T} = a + \frac{2\gamma I^2 m^2 c^2}{Z^2 e^2}\langle B^{-2}\rangle,$$
$$U_{||} = \frac{cTI}{ZeB}(\beta + 2\gamma\psi),$$

with $\langle\rangle$ being the magnetic surface average.

- Taking the density moment and using $\gamma = (Ze/2cT)\partial\omega/\partial\psi$ we obtain $U_{||} = U_{||0} + \Delta\mathcal{U}_{||1}$, where $U_{||0}$ is the local prediction with $\partial T/\partial r = 0$. The nonlocal contribution is calculated to be

$$\Delta\mathcal{U}_{||1} = -\frac{ZeI}{m^2 c\Omega}\langle\frac{I^2}{\Omega^2}\rangle\frac{\partial \ln nT}{\partial\psi}\frac{\partial\omega}{\partial\psi}.$$

Ion cyclotron frequency is $\Omega = ZeB/mc$.

- The mechanism underlying the nonlocal effect is generation of extra ion parallel velocity near steep toroidal flow gradient. As a result, extra poloidal velocity is also produced via the trivial relation $\Delta\mathcal{U}_{p1} = (B_p/B)\Delta\mathcal{U}_{||1}$.

Neoclassical Poloidal Rotation

(squeezing in the radial electric field)

- We utilize a quasi-equilibrium function of constants of the particle motion. We start with $f = \exp\left(\alpha(\psi^\star) - \beta(\psi^\star)\epsilon\right)$. Then the functions $\alpha(\psi^\star)$ and $\beta(\psi^\star)$ can be identified from the requirement that f must reduce to a shifted Maxwellian

$$
\begin{aligned}
f_{sm} &= n(\psi)\left(\frac{m}{2\pi T(\psi)}\right)^{3/2} \\
&\quad \exp\left(-\frac{m}{2T(\psi)}\left[(v_{||} - U_{||}(\psi))^2 + v_\perp^2\right]\right)
\end{aligned}
$$

in the limit of zero orbit width $\psi^\star - \psi \rightarrow 0$. This gives

$$
\begin{aligned}
\alpha(\psi) &= \ln\left(\frac{m}{2\pi T(\psi)}\right)^{3/2} + \frac{Ze\Phi(\psi)}{T(\psi)} \ln n(\psi), \\
\beta(\psi) &= \frac{m}{T(\psi)}.
\end{aligned}
$$

- The parallel flow will be determined by maximizing an entropy expression defined by

$$S = \frac{1}{n(\psi)} \int d^3\boldsymbol{v} f_{sm} (\ln f - \ln f_{sm}),$$

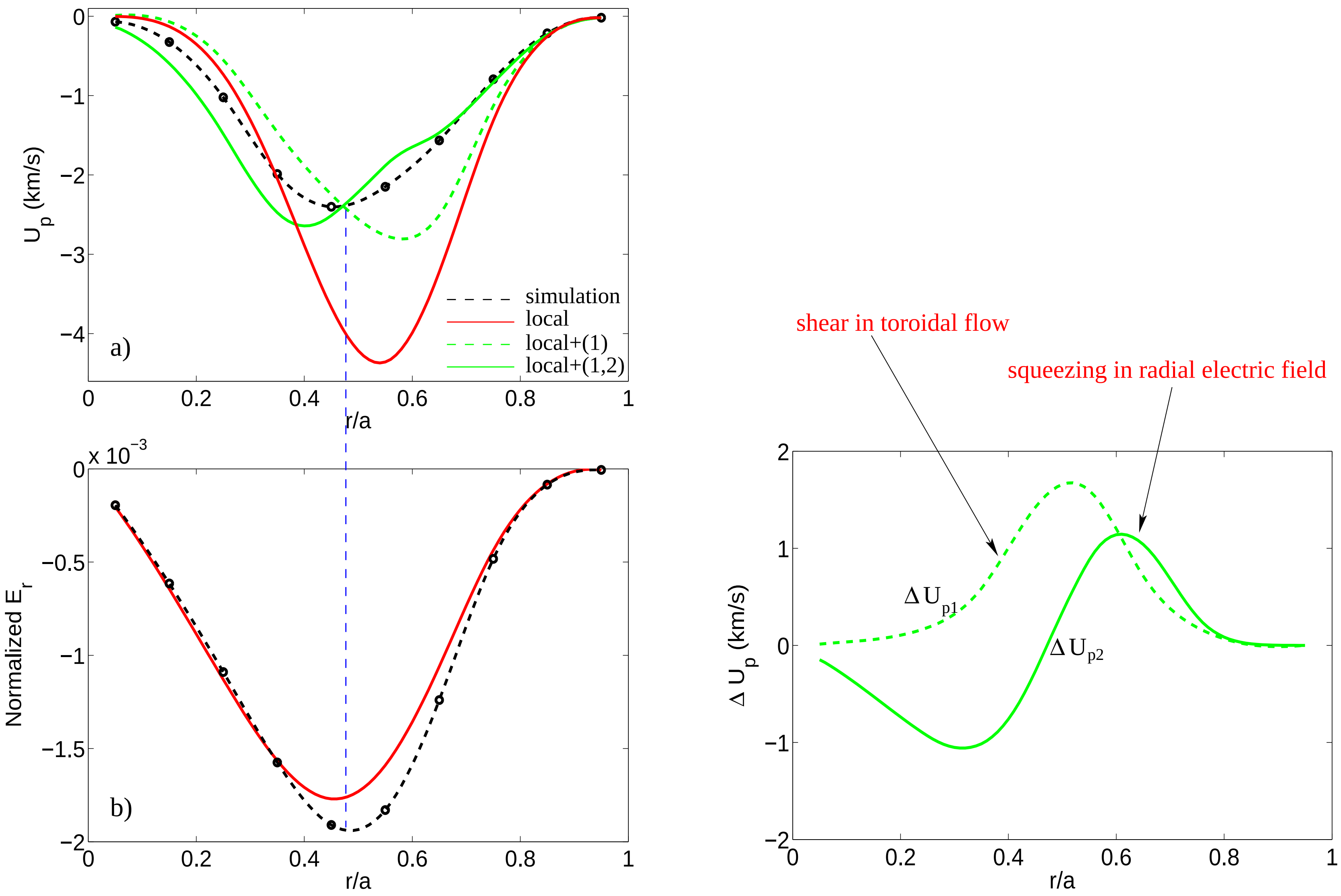
where according to the small width approximation

$$\begin{aligned} \ln f - \ln f_{sm} &= \alpha(\psi^\star) - \alpha(\psi) - [\beta(\psi^\star) - \beta(\psi)]\,\epsilon \\ &+ \beta(\psi)\left(-v_{||}U_{||} + U_{||}^2/2\right). \end{aligned}$$

The equation $\partial S/\partial U_{||} = 0$ gives a nonlocal correction to $U_{||0}$. The term proportional to $\partial E_r/\partial r$ is

$$\Delta\mathcal{U}_{||2} = -\frac{ZeI^3}{m^2\Omega^3} T \frac{\partial \ln nT}{\partial\psi}\frac{\partial^2\Phi}{\partial\psi^2}.$$

- As a result of ion orbit squeezing in the electric field, an extra parallel velocity $\Delta\mathcal{U}_{||2}$ and thus an extra poloidal velocity $\Delta\mathcal{U}_{p2} = (B_p/B)\Delta\mathcal{U}_{||2}$ are generated.

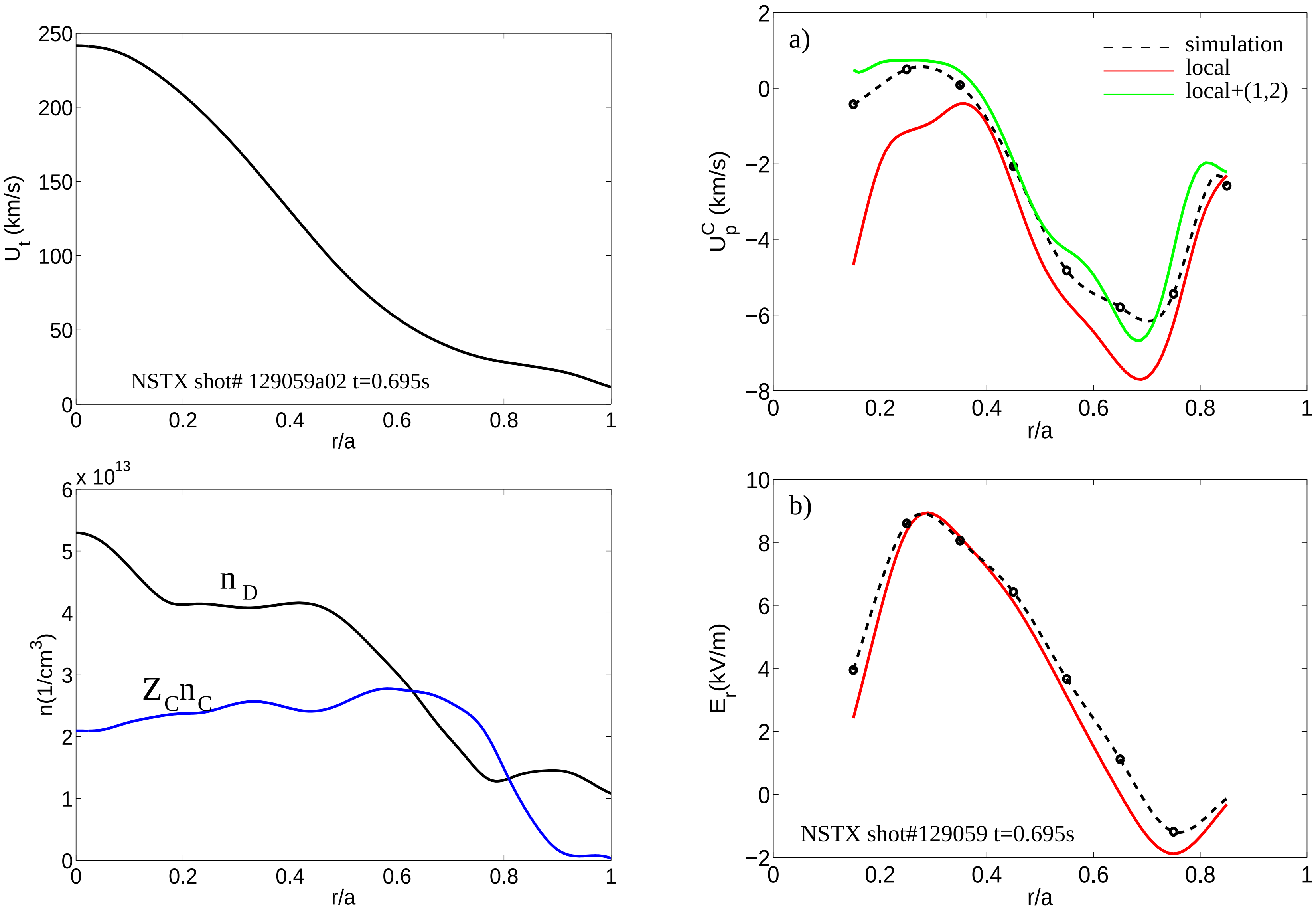


U_p (km/s)
0
−1
−2
−3
−4
simulation
local
local+(1)
local+(1,2)
a)
0
0.2
0.4
0.6
0.8
1
r/a
x 10^−3
Normalized E_r
0
−0.5
−1
−1.5
−2
b)
0
0.2
0.4
0.6
0.8
1
r/a
shear in toroidal flow
squeezing in radial electric field
Δ U_p (km/s)
2
1
0
−1
−2
Δ U_p1
Δ U_p2
0
0.2
0.4
0.6
0.8
1
r/a

Simulation of Neoclassical Poloidal Rotation in NSTX

- On the left is the density and toroidal angular frequency radial profiles for NSTX shot #129059 at $t = 0.695s$. Toroidal angular frequency as calculated by the TRANSP code is assumed to be the same (mass averaged) for both species. $\partial nT/\partial\psi \partial\omega/\partial\psi$ is strong near $r/a \sim 0.2 - 0.7$, where the first nonlocal effect is expected to be the most important.

- On the right is the sumulation results for the carbon poloidal flow and the radial electric field versus minor radius for this NSTX shot. In the upper panel, the dashed black curve is from the simulation, while the red curve is a local neoclassical estimate U_{p0}^{C} from the NCLASS code.


U_t (km/s)
r/a
NSTX shot# 129059a02 t=0.695s
x 10^13
n(1/cm^3)
n_D
Z_C n_C
a)
U_p^C (km/s)
simulation
local
local+(1,2)
b)
E_r (kV/m)
NSTX shot#129059 t=0.695s

- In order to understand the role of nonlocal effects, we write down the system of four equations for the poloidal velocity U_p^s and the poloidal heat flux Q_p^s for species $s = C, D$

$$\begin{pmatrix} \widehat{\mu}_1^s & \widehat{\mu}_2^s \\ \widehat{\mu}_2^s & \widehat{\mu}_3^s \end{pmatrix} \begin{pmatrix} U_p^s - B_p/B\Delta\mathcal{U}_{||}^s \\ Q_p^s \end{pmatrix}$$

$$= \sum_{b=C,D} \begin{pmatrix} l_{11}^{sb} & -l_{12}^{sb} \\ -l_{12}^{sb} & l_{22}^{sb} \end{pmatrix} \begin{pmatrix} \widetilde{U}^b - B_p/B\Delta\mathcal{U}_{||}^b \\ \widetilde{Q}^b \end{pmatrix}.$$

The superscripts $'C'$ and $'D'$ stand for carbon and deuterium, respectively. Here, $\widetilde{U}^s = V_{1s}B/\langle B^2\rangle + U_p^s$ and $\widetilde{Q}^s = V_{2s}B/\langle B^2\rangle + Q_p^s$. It is clear that the carbon poloidal velocity U_p^s will be affected by the nonlocal effects from both the carbon ($\Delta\mathcal{U}_{||}^C$) and deuterium ($\Delta\mathcal{U}_{||}^D$) species.

- To simplify the analysis, we notice that the finite orbit effect on the radial electric field primarily comes from the prevailing deuterium ions. Then, rewriting the radial force balance for carbon yields

$$\Delta\mathcal{U}_p^C \approx \frac{B_p}{B}\left(\Delta\mathcal{U}_{||}^C - \Delta\mathcal{U}_{||}^D\right).$$

- Having added analytical expressions for the new nonlocal effects $\Delta\mathcal{U}_p^C = \Delta\mathcal{U}_{p1}^C + \Delta\mathcal{U}_{p2}^C$ together with the local neoclassical estimate from the NCLASS code U_{p0}^C, we obtain the green curve which agrees better with the simulation result than the original local prediction.

Conclusions

We have performed a neoclassical drift-kinetic particle simulation of the poloidal velocity in a two ion-species system. New nonlocal effects due to shear in the toroidal flow and squeezing in the radial electric field are observed. An analytical model based on these effects has been proposed to explain the simulation results in a large aspect-ratio plasma with a steep toroidal flow profile. The newly discovered nonlocal effects are shown to fairly well account for the discrepancy between the simulation result for carbon poloidal flow in NSTX and the prediction from the local theory. Future work includes comparisons of simulation with measurement results for the carbon poloidal rotation in tokamak experiments. Our new model might offer insight into the role the nonlocal effects have on the poloidal velocity in realistic magnetic fusion devices.

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