Modeling Polarization of Propagating Electromagnetic Waves in NSTX

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and the NSTX Research Team

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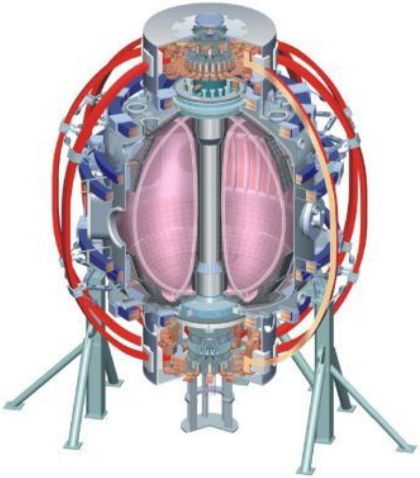

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Motivation for plasma polarimetry on NSTX

- Polarimetry measures change of wave polarization caused by magnetized plasma
- **Polarimetry can help equilibrium reconstruction**
  - magnetic field structure
  - current distribution
- **Polarimetry can potentially measure magnetic fluctuations**
  - Alfvén eigenmodes
  - tearing modes

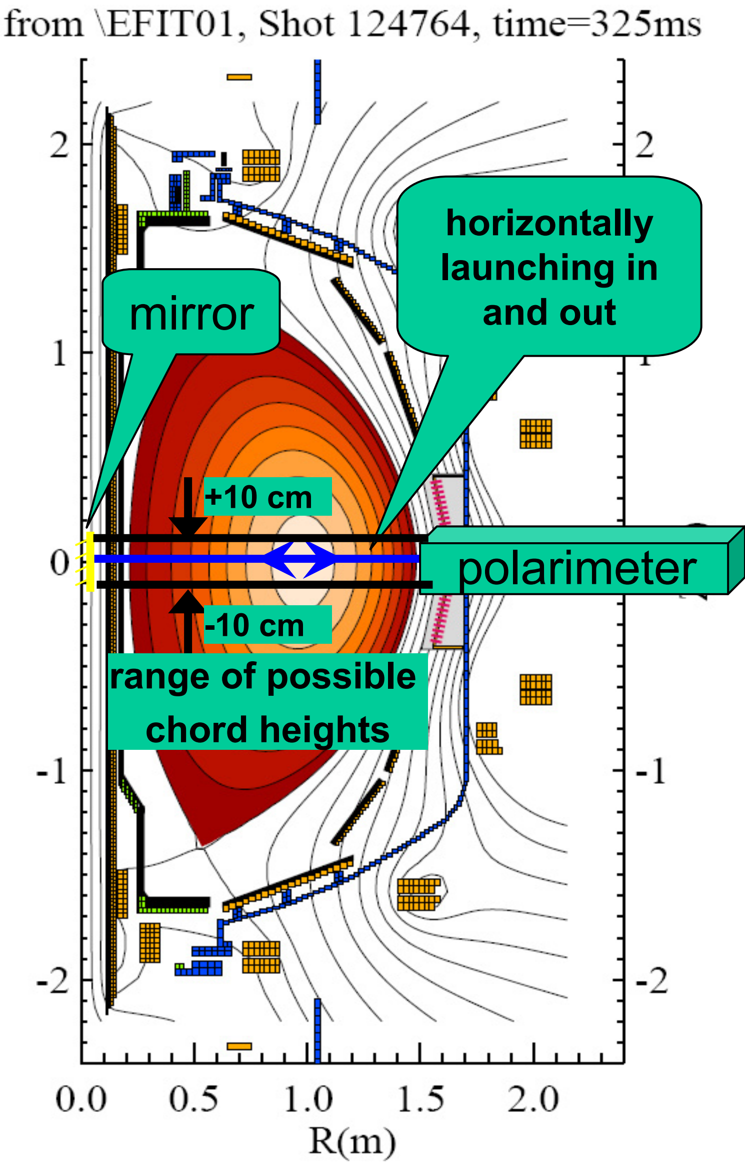
Modeling used to understand & optimize diagnostic technique

Summary of interesting results

- Experimental polarimetry scenario on NSTX is unique
- For a radial chord not at mid-plane
  - plasma central region contributes the most to Faraday rotation
  - high field region contributes the most to Cotton-Mouton effect
- Retroreflection does not cause cancellation of either effect
  - effects accumulate on inbound and outbound paths
- Real equilibria can cause complications
  - $\vec{B}_0 \cdot d\vec{l}$ can change sign along propagation path
  - can lead to difficulty in interpreting Faraday rotation fluctuation data
- Faraday rotation & Cotton-Mouton interact with each other
  - e.g. pure Cotton-Mouton also causes polarization rotation
  - this interaction decreases with increasing microwave frequency

Experimental polarimetry scenario on NSTX is unique

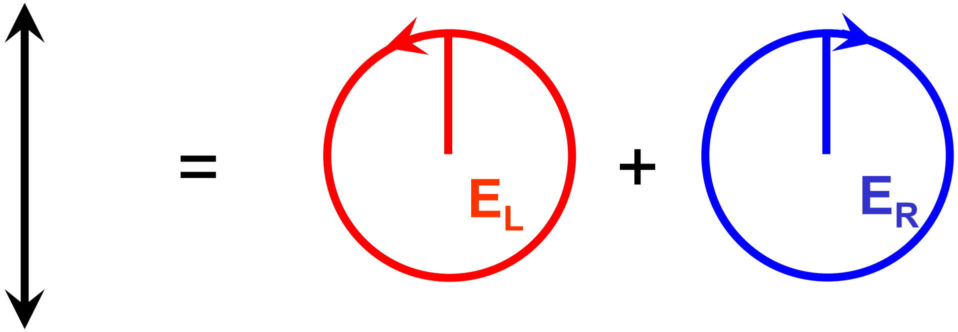
- Polarimetry has long history in toroidal devices, i.e. conventional tokamak, RFP
- Spherical tokamak – different from other toroidal devices
  - large edge B pitch angle (~35°) (*vs.* conventional tokamak)
  - large B_{TOR} variation (*vs.* conventional tokamak, RFP)
  - relatively weak B, $\omega_{pe} >> \omega_{ce}$ (*vs.* conventional tokamak)
- Operates at *f* = *288 GHz* ($\lambda \sim 1$ mm)
  - low *f* compared to historical systems
  - simpler hardware (e.g. solid state source) than higher *f*
- Radial propagation near mid-plane & retroreflection from inside wall
  - tangential & vertical chords common

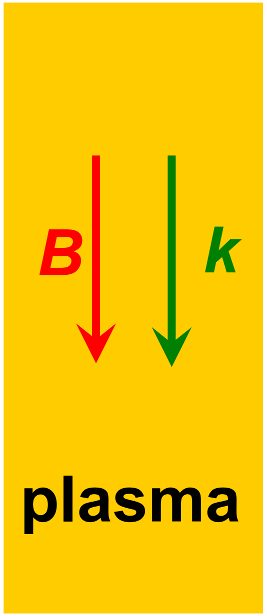


Faraday rotation is the rotation of linearly polarized EM-wave in a magnetized plasma

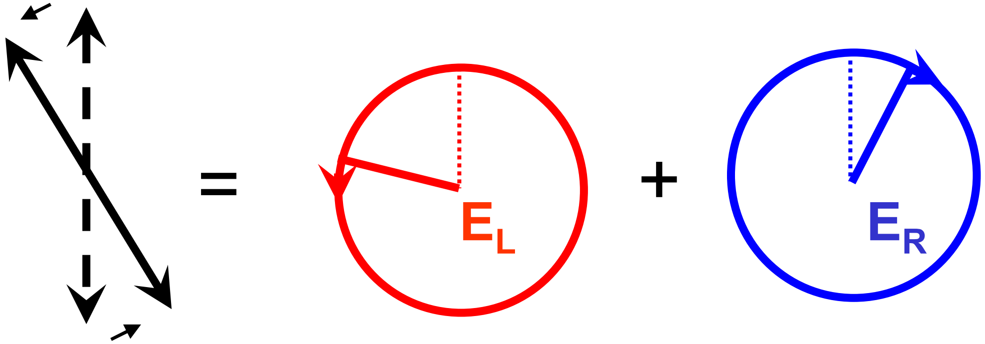
$\vec{k} \parallel \vec{B}$

- Linearly polarized wave is sum of Left- and Right-handed circular polarized waves





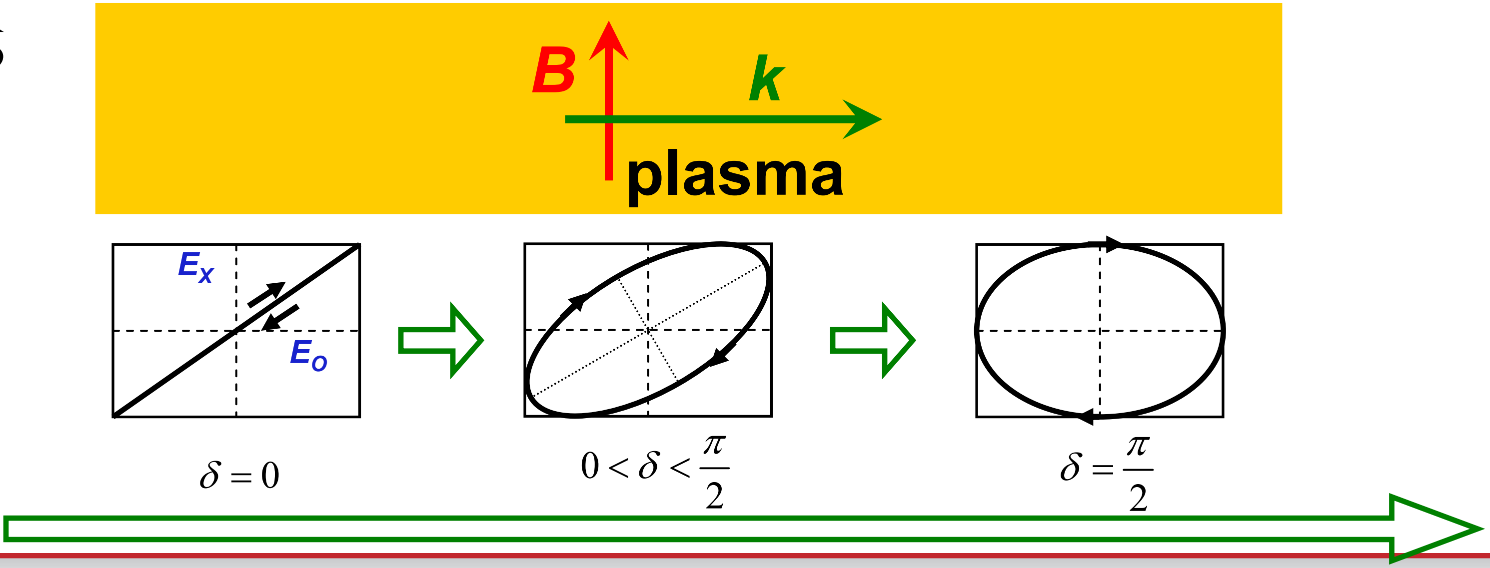
- In plasma, L-wave phase advances faster than R-wave causing *rotation of the linear polarization*



Cotton-Mouton effect is caused by birefringence of plasma

- When a linearly polarized wave propagates perpendicular to plasma magnetic field, it can become elliptized
- Linearly polarized wave is some combination of *in-phase* X & O modes
- X & O waves propagate with different phase velocities causing elliptization of the emerging beam
  - as wave propagates, phase difference (δ) between E_X & E_O increases

$$\vec{k} \perp \vec{B}$$



Model calculates polarization evolution along wave path

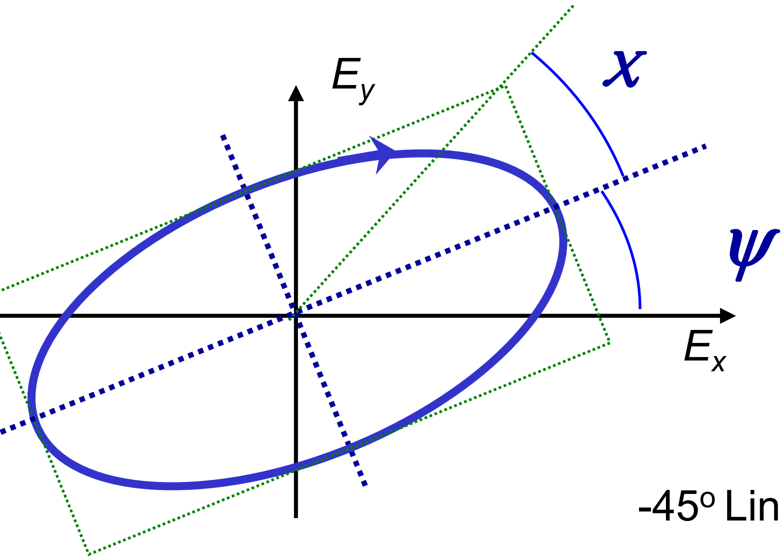
- WKB (G. Wentzel, H.A. Kramers, and L. Brillouin) approximation used
  - plasma parameters does not vary much within a wavelength

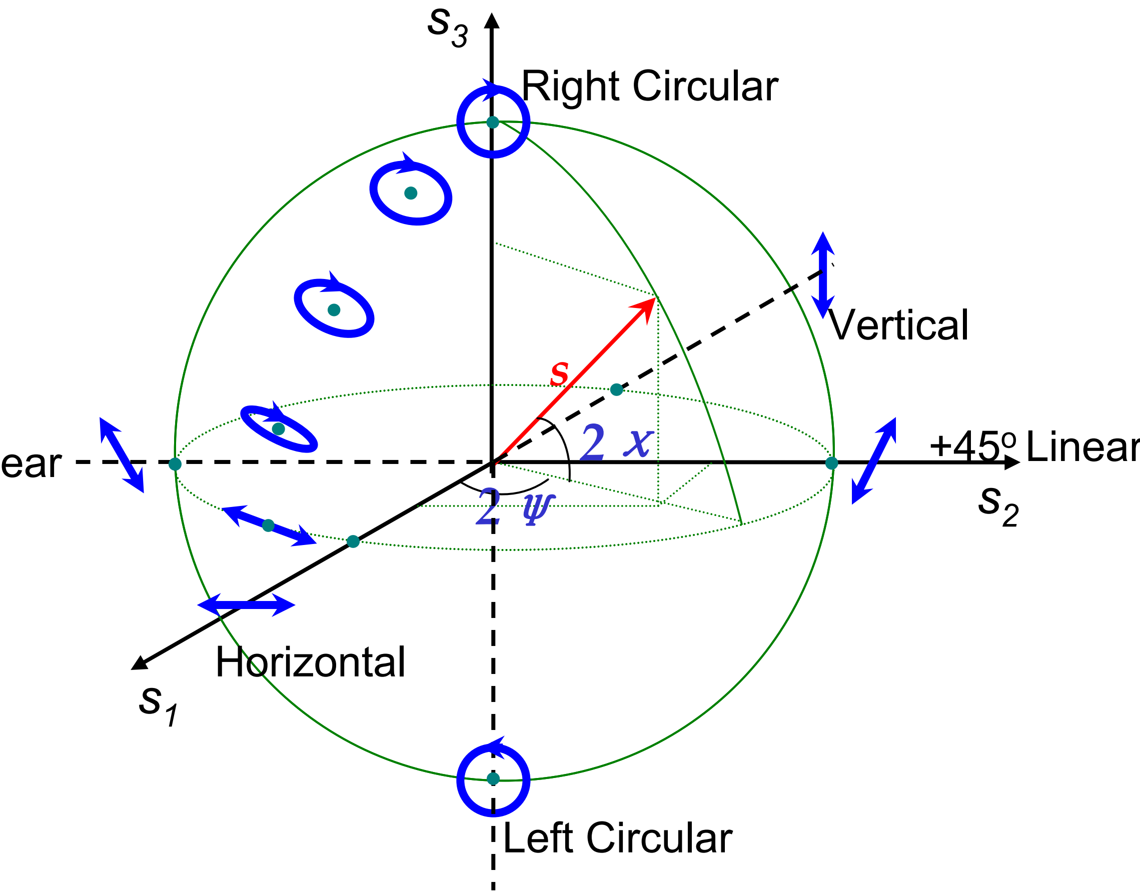
$$\left|\vec{B}\right| \gg \left|\frac{1}{k}\frac{d\vec{B}}{dz}\right|, |n| \gg \left|\frac{1}{k}\frac{dn}{dz}\right|$$

- Cold plasma assumed
  - absorption of wave is negligible
- Ion motion ignored

$$\omega_{pe} \ll \omega$$

Polarization can be represented by point on Poincaré sphere

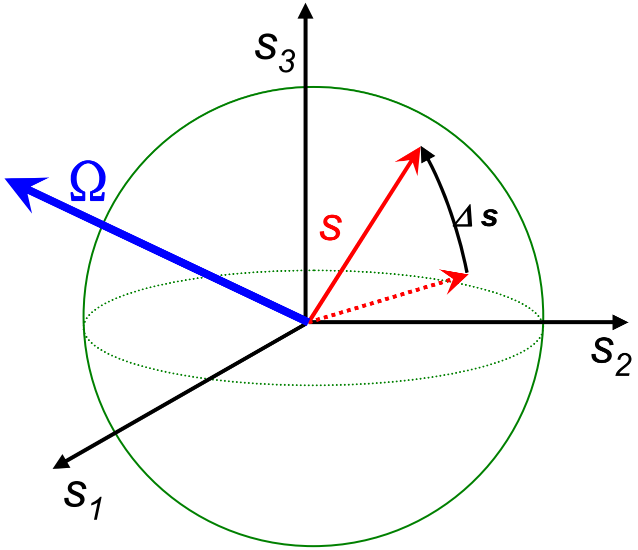




$$\begin{cases} s_1 = s_0 \cos 2\chi \cos 2\psi \\ s_2 = s_0 \cos 2\chi \sin 2\psi \\ s_3 = s_0 \sin 2\chi \end{cases}$$

Poincaré sphere

Evolution of polarization in a plasma is represented by a trajectory on the Poincaré Sphere

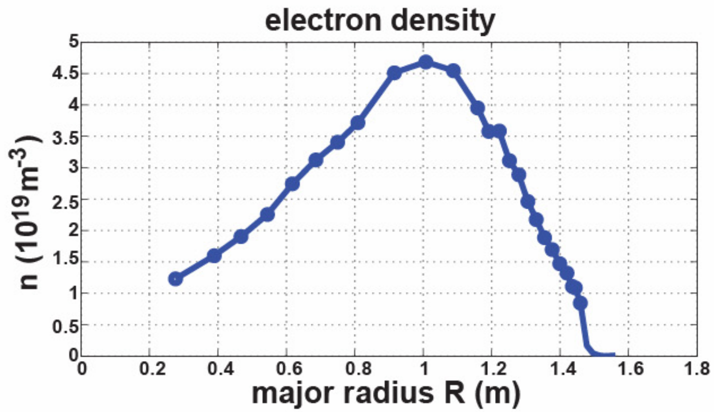


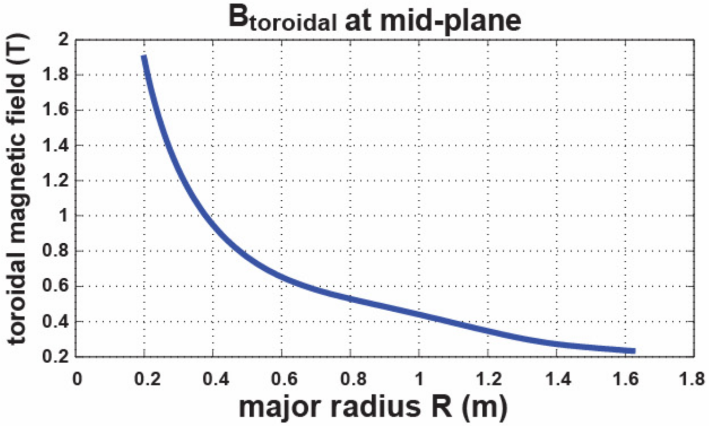
$$\frac{d\vec{s}(z)}{dz} = \vec{\Omega}(z)\times\vec{s}(z)$$

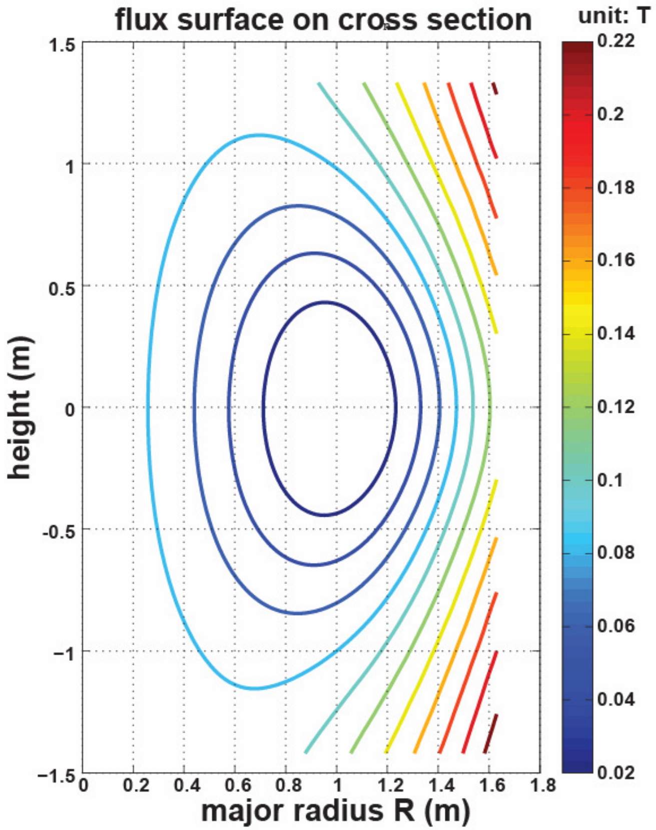
$$\vec{\Omega} = \frac{\omega}{2c}\left(\frac{\omega_{pe}}{\omega}\right)^2 \frac{1}{\left(\frac{\omega}{\omega_{ce}}\right)^2 - 1} \begin{pmatrix} (B_x^2 - B_y^2)/B^2 \\ 2B_xB_y/B^2 \\ 2\left(\frac{\omega}{\omega_{ce}}\right)B_z/B \end{pmatrix} \begin{matrix} \left.\vphantom{\begin{matrix}a\\b\end{matrix}}\right\} \text{Cotton-Mouton effect} \\ \text{— Faraday Rotation} \end{matrix}$$

- Each portion of trajectory is a rotation of *s vector* about an *axis vector Ω*
- *Axis vector Ω* depends on local plasma parameters (electron density, B-field)
- Calculations using *full NSTX equilibrium* result in the Faraday rotation & Cotton-Mouton effects *interacting* with each other to determine the final polarization state

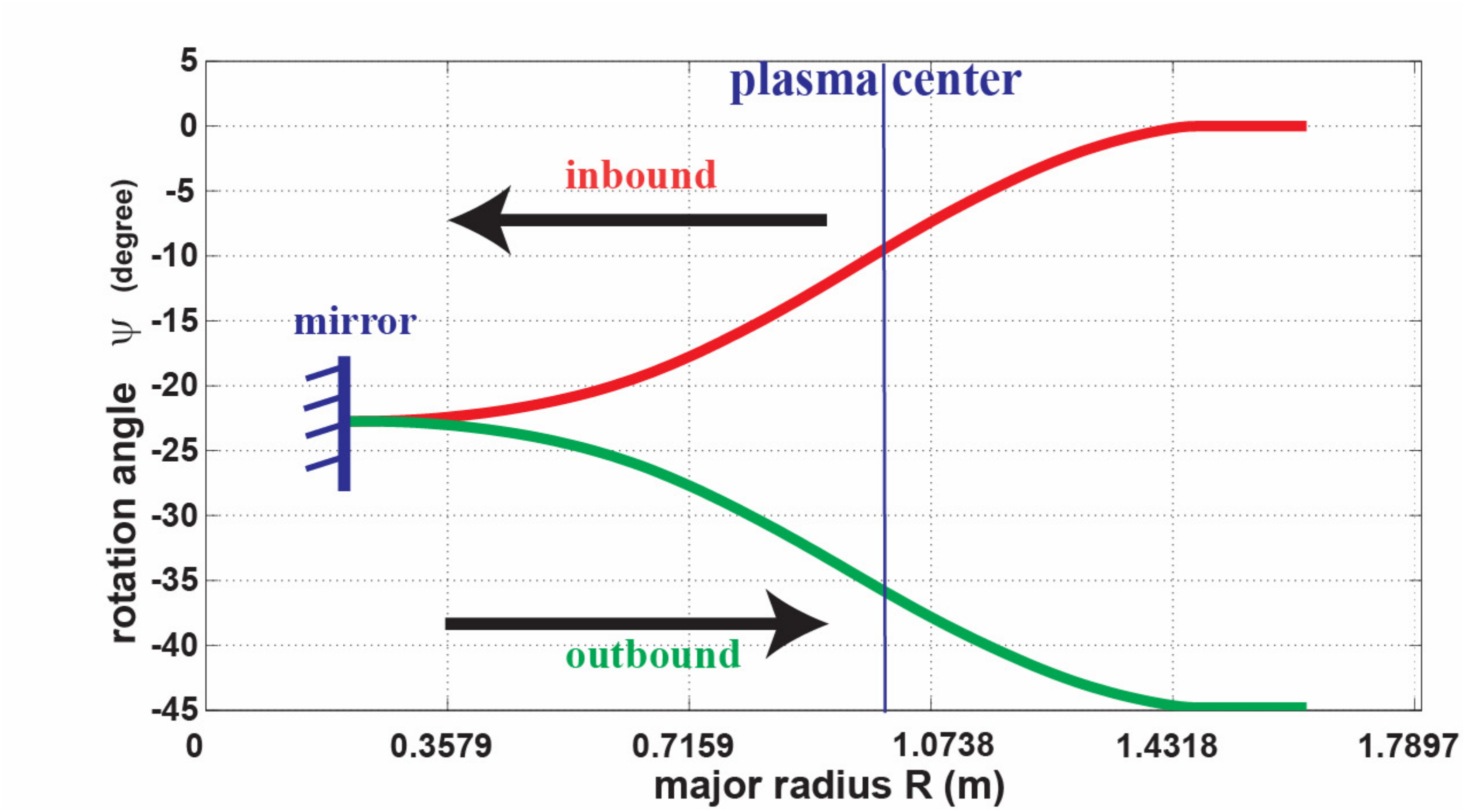
NSTX density and magnetic field profiles used



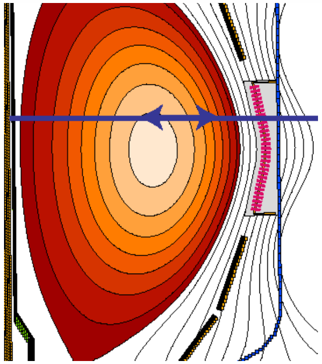




Central plasma contributes most to Faraday rotation

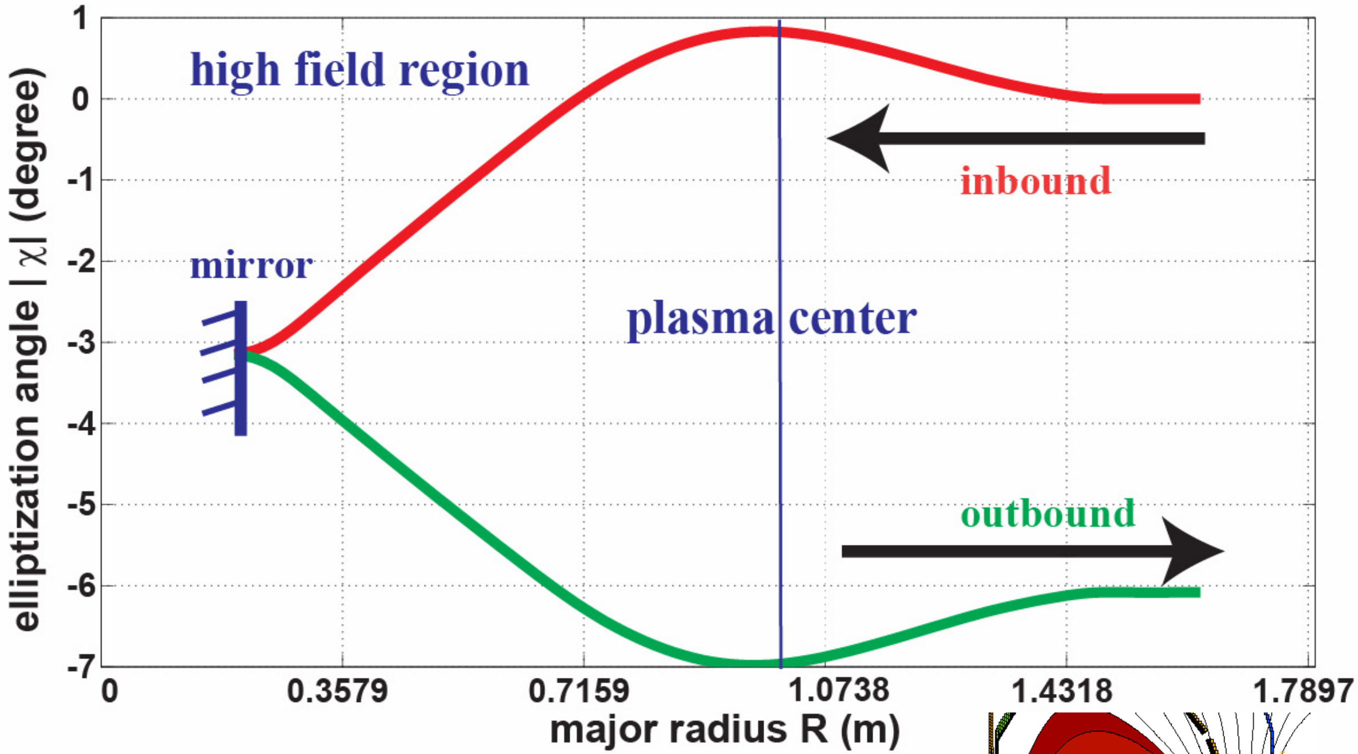


- Inbound and outbound rotation are *additive*
- Density is greater towards the central plasma
- B_{horiz} also peaks towards the central plasma

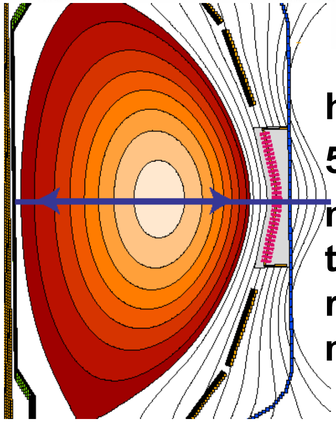


horizontal chord 20 cm above mid-plane

High field region contributes most to Cotton-Mouton effect

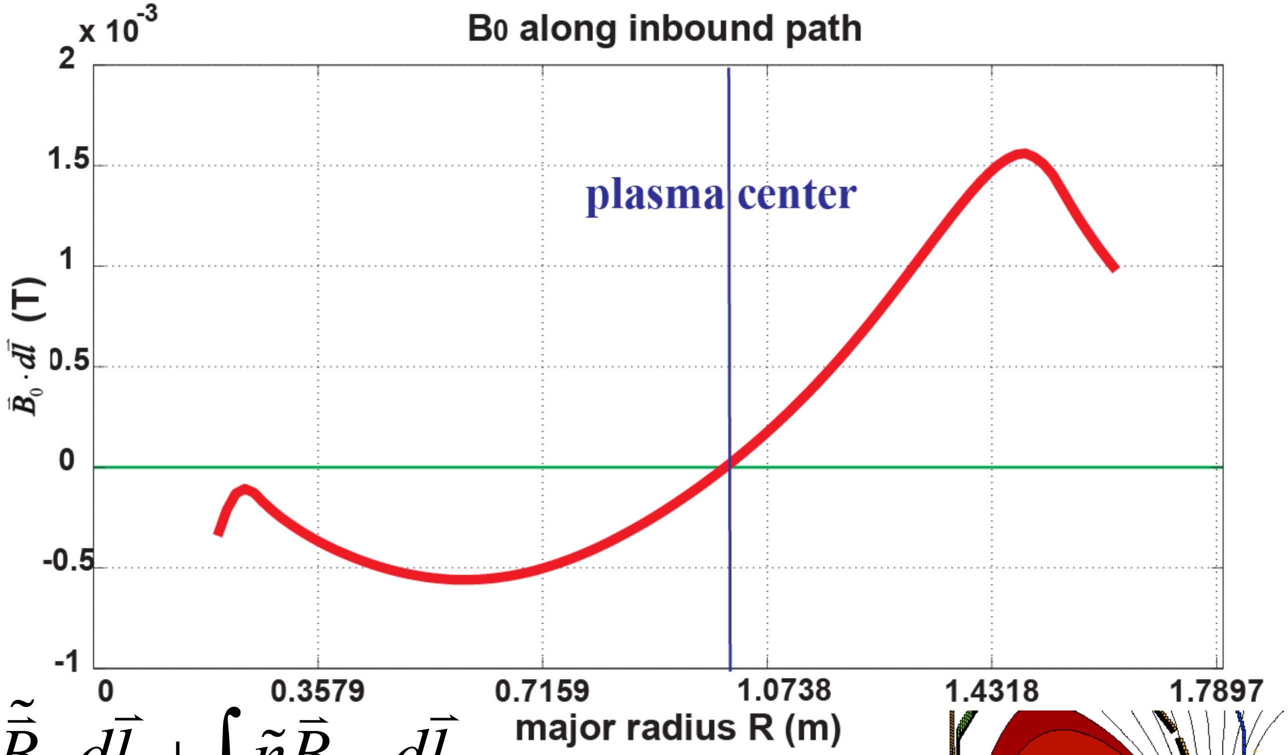


- Inbound and outbound elliptization are *additive*
- Transverse B-field (including B_{TOR} and B_{vert}) causes Cotton-Mouton effect
- B_{TOR} dominant in high field region



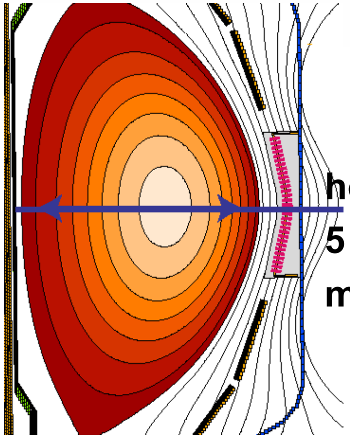
horizontal chord 5 mm below mid-plane, where total Faraday rotation is minimized

Up-down asymmetry complicates interpretation of Faraday rotation fluctuation measurement



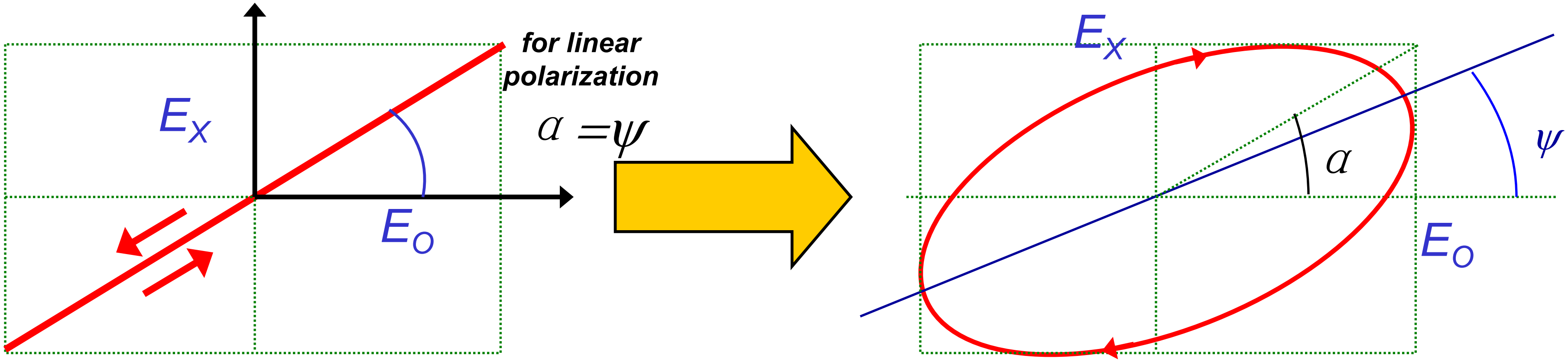
$$\tilde{\psi} \propto \int n_0 \tilde{\vec{B}} \cdot d\vec{l} + \int \tilde{n} \vec{B}_0 \cdot d\vec{l}$$

- *Total* Faraday rotation angle approximates 0 at about 5 mm below mid-plane
- Determination of magnetic fluctuation is simpler if $\vec{B}_0 \cdot d\vec{l}$ is uniformly zero



horizontal chord
5 mm below
mid-plane

Pure Cotton-Mouton effect may also cause small rotation



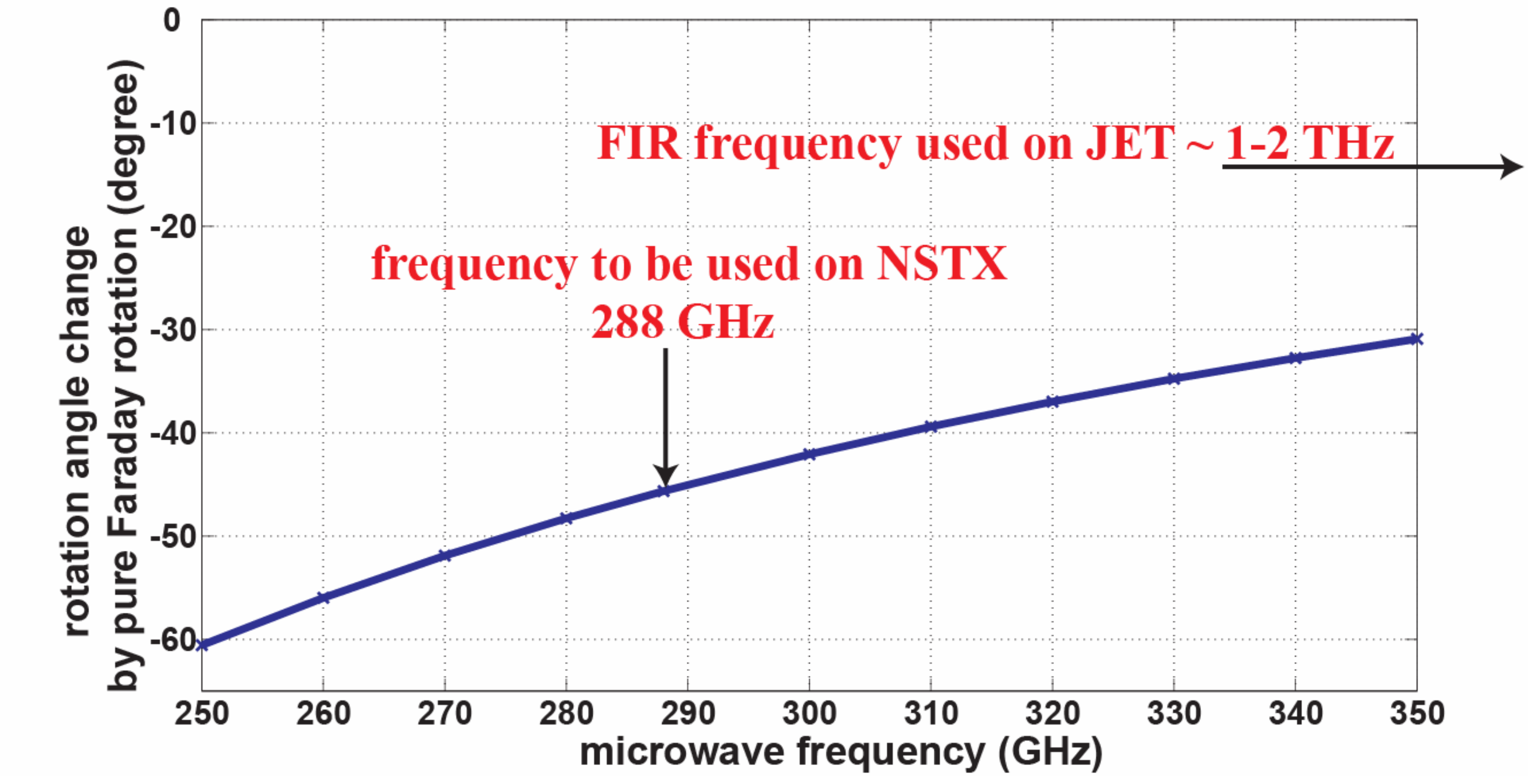
$\delta = 0$

$0 < \delta < \pi/2$

- Pure Cotton-Mouton effect causes increasing phase difference (δ) between E_X & E_O
- For wave other than X or O characteristic mode, it may experiencing a small polarization rotation
  - d_{FR}: change due to Faraday rotation
  - d_{CM}: change due to Cotton-Mouton effect

$$\frac{d\psi}{dz} = \frac{d_{FR}\psi}{dz} - \frac{1}{2}\sin 2\psi \cos 2\psi \tan\delta \frac{d_{CM}\delta}{dz}$$

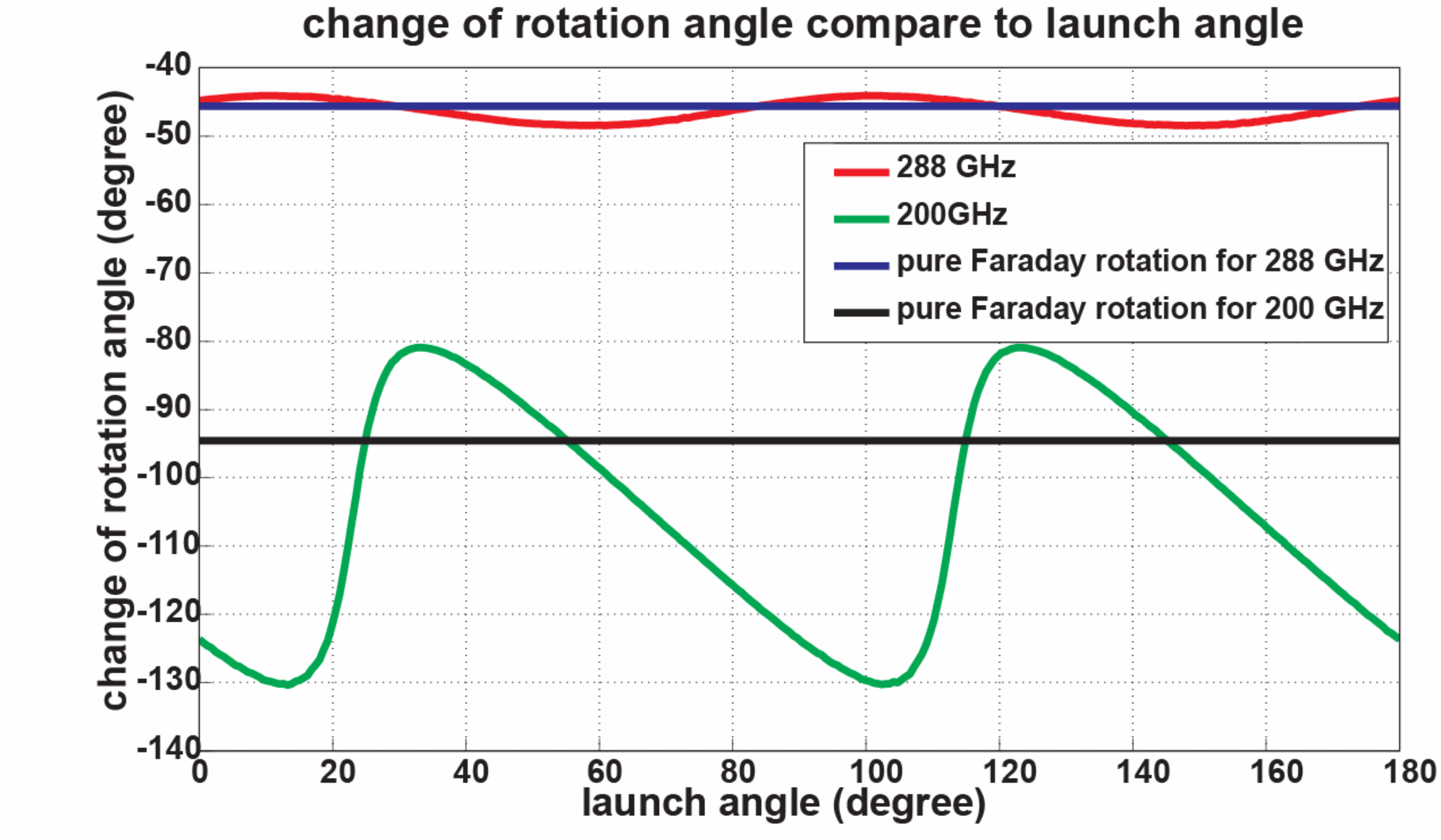
Pure Faraday rotation decreases as *f* increases



- As microwave frequency increases, change of Faraday rotation angle decreases, approximately ~ $1/f^2$
- Characteristic frequencies on NSTX

ω_{pe} ~ 50 GHz, ω_{ce} ~ 13 GHz

Interaction between Faraday rotation & Cotton-Mouton effect deceases as *f* increases



- The distortion is NOT symmetric within a rotating period
- It's valid to ignore the interaction, when using far-infrared laser (*f ~ 1-2 THz*) to measure Faraday rotation

Summary

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Future work

- Study equilibrium with chords near mid-plane (+/- 10 cm) where Faraday rotation is small
  - distinguish these two effects, to study different regions of plasma (central and high field region)
- Contrast with conventional tokamak, e.g. DIII-D, & RFP
  - expecting relatively larger distortion of Faraday rotation from Cotton-Mouton effect
- Simulating actual diagnostic system, i.e. synthetic diagnostic
- Add in magnetic fluctuations and try to identify magnetic modes
  - Alfvén eigenmodes
  - tearing modes

Abstract*

- Magnetized plasma has an anisotropic index of refraction. For propagation perpendicular to the magnetic field, this leads to polarization elliptization via the Cotton-Mouton effect. In contrast, for propagation parallel to the field, the axis of the polarization ellipse rotates: this is known as Faraday rotation. In fusion plasmas millimeter-waves typically experience a combination of these two effects. To date, little attention has been given to the evolution of polarization for radial propagation in a spherical tokamak where a much greater variation of magnetic pitch angle and field strength exists in comparison to conventional tokamaks. This work investigates the polarization modification of millimeter-waves propagating radially in the National Spherical Torus eXperiment. Typical NSTX density and magnetic field profiles are utilized. The calculations provide the basis for optimization of the performance of a planned radial chord polarimeter. Future analysis will assess the sensitivity of polarization modifications to magnetic perturbations such as Alfvén eigenmodes and tearing modes.
- *Supported by US DOE Contracts DE-FG03-99ER54527 and DE-AC02-09CH11466

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Combination of CM and FR

- In fusion plasmas millimeter-waves typically experience a combination of these two effects.

$$d = d_{FR} + d_{CM}$$

- For pure Cotton-Mouton effect:

$$\begin{cases} d_{CM}\alpha = 0 \\ d_{CM}\delta \neq 0 \end{cases}$$

- For pure Faraday Rotation:

$$\begin{cases} d_{FR}\chi = 0 \\ d_{FR}\psi \neq 0 \end{cases}$$

- The combination:

$$\begin{cases} d\chi = \dfrac{1}{2}\sin 2\psi \, d_{CM}\delta \\ d\psi = d_{FR}\psi - \dfrac{1}{2}\tan 2\chi \cos 2\psi \, d_{CM}\delta \end{cases}$$

- Or, more explicitly to

show the entanglement:

$$\begin{cases} d\psi = d_{FR}\psi - \dfrac{1}{2}\sin 2\psi \cos 2\psi \tan\delta \, d_{CM}\delta \\ d\delta = d_{CM}\delta - \dfrac{\sin 2\delta}{\tan 2\psi} d_{FR}\psi \end{cases}$$

NSTX profiles used (B-field decomposed to 3 directions)