Rotational Control of Plasma in NSTX*

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NSTX

Introduction

In an effort to assist the continuous extraction of fusion energy, a plasma model and a controller are developed for the spherical tokamak in the National Spherical Torus Experiment (NSTX). The model is aimed to capture the rotational (toroidal) momentum transport inside the tokamak. The neutral beam injection and the neoclassical toroidal viscosity are considered in the model for their uses as actuator forces. Based on the proposed model, a feedback controller is designed to sustain the toroidal momentum of the plasma in a stable fashion and to achieve desirable plasma geometry. The model reduction and control results are presented.

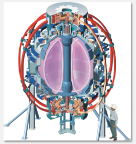


Figure 1. Setup of the NSTX device.

Toroidal Momentum Balance

In order to capture the momentum transport inside the tokamak, we consider the toroidal momentum equation for rotational frequency (angular velocity, ω) with axisymmetric assumption [1]:

$$\begin{aligned}
&\sum_i n_i m_i \langle R^2 \rangle \frac{\partial \omega}{\partial t} + \omega \langle R^2 \rangle \sum_i m_i \frac{\partial n_i}{\partial t} \\
&+ \sum_i n_i m_i \omega \frac{\partial \langle R^2 \rangle}{\partial t} + \sum_i n_i m_i \langle R^2 \rangle \omega \left(\frac{\partial V}{\partial \rho} \right)^{-1} \frac{\partial}{\partial t} \frac{\partial V}{\partial \rho} \\
&= \left(\frac{\partial V}{\partial \rho} \right)^{-1} \frac{\partial}{\partial \rho} \left[\frac{\partial V}{\partial \rho} \sum_i n_i m_i \chi_\phi \langle R^2 (\nabla \rho)^2 \rangle \frac{\partial \omega}{\partial \rho} \right] \\
&- \left(\frac{\partial V}{\partial \rho} \right)^{-1} \frac{\partial}{\partial \rho} \left[\frac{\partial V}{\partial \rho} \sum_i n_i m_i \omega \langle R^2 (\nabla \rho)^2 \rangle \frac{v_\rho}{|\nabla \rho|} \right] \\
&- \sum_i n_i m_i \langle R^2 \rangle \omega \left(\frac{1}{\tau_{\phi cx}} + \frac{1}{\tau_{c\delta}} \right) + \sum_j T_j
\end{aligned} \qquad \text{(Eq 1)}$$

It is observed that geometric features of the plasma can be approximated by time invariant functions.

$$\langle R^2 \rangle, \quad \langle R^2 (\nabla \rho)^2 \rangle, \quad \frac{\partial V}{\partial \rho}, \quad T(t,\rho)/\mathrm{max}_\rho T_j(t,\rho) \;\approx\; \mathrm{fnc}(\rho)$$

Pinch term and momentum loss due to charge exchange and field ripple are neglected.

Model

The present model is derived from approximations based on discussion in the previous section:

$$\sum_i n_i m_i \langle R^2 \rangle \frac{\partial \omega}{\partial t} = \left(\frac{\partial V}{\partial \rho} \right)^{-1} \frac{\partial}{\partial \rho} \left[\frac{\partial V}{\partial \rho} \sum_i n_i m_i \chi_\phi \langle R^2 (\nabla \rho)^2 \rangle \frac{\partial \omega}{\partial \rho} \right] + \sum_j T_j \qquad \text{(Eq 2)}$$

- 1-D linear parabolic PDE (diffusion equation with forcing)
- Neumann ($\rho = 0$) and Dirichlet ($\rho = 1$) BCs
- Geometric variable approximated by curve fits; only func of space
- For control studies, stable profiles of $\chi_\phi(\rho,t)$ and $\Sigma_i n_i m_i(\rho,t)$ are supplied from TRANSP (Figure 2)

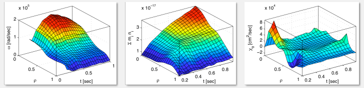
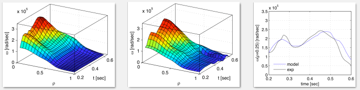


Figure 2. Representative time and space variation profiles (rotational frequency, density, and diffusivity) for NSTX shot 128820.

Validation

The model PDE (Eq 2) is solved numerically with adaptive time integration following the method of Skeel and Berzins [2]. Starting with a rotational frequency at some reference time from experiment as an initial condition, the model can be compared with measurements.

The model is able to capture the rotational physics well as shown in Figure 3. The current model represents momentum balance but cannot account for transition or instability.



(model) (experiment) (comparison)

Figure 3. Comparison of rotational frequency from model and experiment for NSTX shot 128820.

Rotational Control

The objective is to maintain steady rotational frequency with a desired spatial profile, ω_d. Three actuators are considered for shaping the plasma (Figure 4):

- Neutral Beam Injections (blue, green)
- Neoclassical Toroidal Viscosity [3,4] (red)

$$\sum_i n_i m_i \mu \frac{B_{\mathrm{eff}}^2}{B^2} \left(\langle R^2 \omega \rangle - \langle R^2 \omega^* \rangle \right)$$

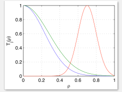


Figure 4. Example spatial profiles for torque inputs (actuators).

The feedback control is designed with linear quadratic regulator (LQR) for the weight function of

$$J = \int_{t_0}^{\infty} (\omega^T Q \omega + T^T R T) dt$$

For a chosen set of weighs Q and R, LQR determines the optimal feedback gain.

Control Results

Given a desired profile measured by discrete number of sensors, the feedback controller modifies the plasma profile to provide the minimum weight function J. In Figure 5, the controlled plasma shapes and steady states are shown for increasing value of ||Q||/||R||.

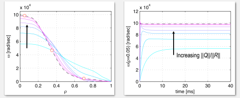


Figure 5. [Left]: Controlled plasma shapes (color) with desired profile (---). [Right]: Time trace of plasma control.

The plasma reaches steady state in a short time (< 10 ms) in the present model. Time delay due to collision is likely to increase the required time for control. Figure 6 exhibits the required torque input for desired error level.

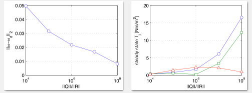


Figure 6. [Left]: Error between steady-state plasma and desired profile for increasing ||Q||/||R||. [Right]: Corresponding torque requirement.

Summary

- A one-dimensional PDE model that captures the rotational (toroidal) momentum balance is presented and validated against experimental data from NSTX.
- Optimal feedback control designed with LQR is considered for maintaining the plasma at a steady state.
- Plasma shape can be modified to a desired profile with actuation to any suitable error level.
- Neutral beam injection and neoclassical toroidal viscosity are selected as the means of actuation.

Ongoing Work:

- Inclusion of time delay due to collision in the model.
- Estimator design for sensors.
- Control for time varying profile of desired plasma.

References

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