Abstract

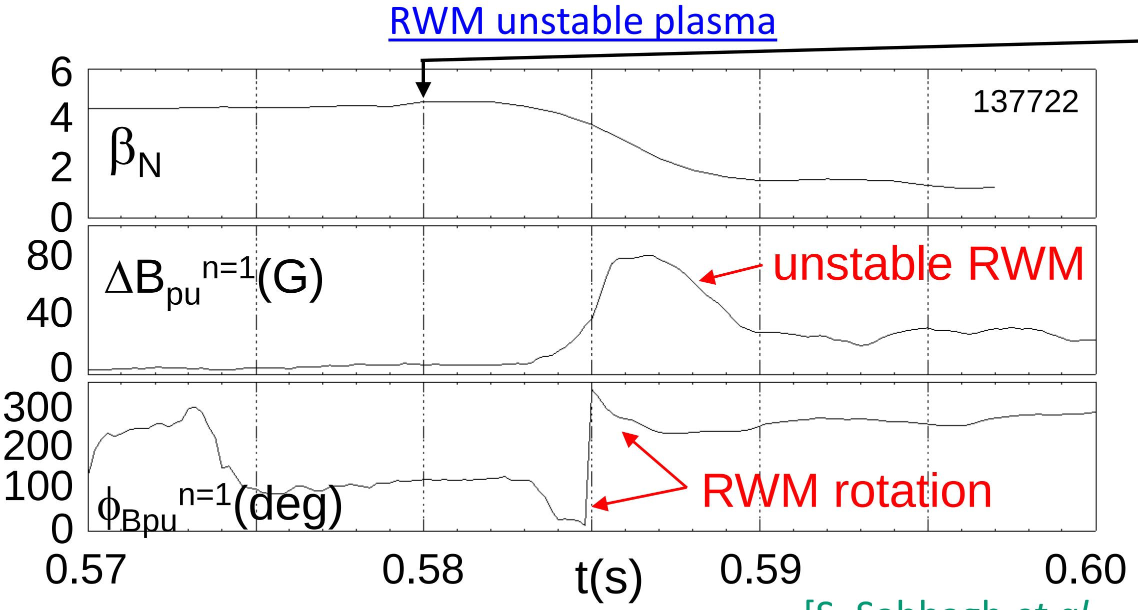
The theory of kinetic modification of ideal MHD stability has the potential to explain the physics of resistive wall mode (RWM) stability in high-beta tokamaks. The observation of unstable RWMs at plasma rotation frequencies between the stabilizing bounce and precession drift frequency resonances in NSTX is well explained by the kinetic theory[1], while energetic particles provide a stabilizing effect that is independent of plasma rotation. A description of the physics of RWM stability which may unify results between various devices is proposed. In certain cases large energetic particle stabilization may be preventing the RWM from going unstable except when triggered by a sudden loss of energetic particles. In NSTX, smaller energetic particle stabilization may be allowing the mode to go unstable more often, and for thermal resonances to be more clearly seen. This hypothesis is applied to analysis with the MISK code of plasmas from NSTX and other devices exhibiting RWMs with various levels of rotation and energetic particle content.

[1] J. Berkery et al., Phys. Rev. Lett. 104, 035003 (2010).

The resistive wall mode (RWM) is disruptive; it is important to understand the physics of its stabilization

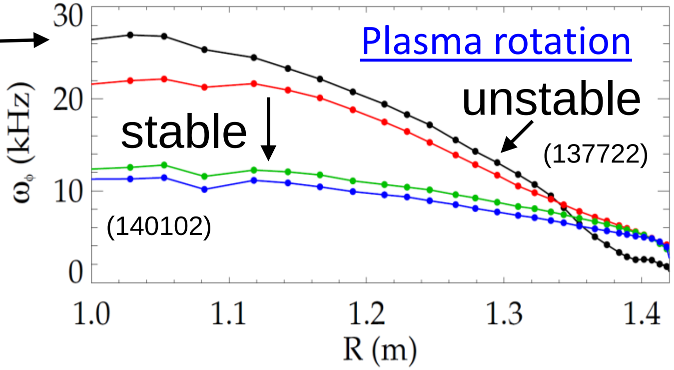
- Motivation
  - Physics of RWM stabilization is key for extrapolation to: disruption-free operation of a low rotation, low collisionality burning plasma (ITER).
- Outline
  - Comparison of kinetic RWM stability theory and NSTX experimental results shows intermediate ω_ϕ with weakened stability due to rotational resonances of thermal particles.
  - Reduced collisionality in future machines will enhance this effect
  - Dedicated NSTX and DIII-D experiments examined the stabilizing role of energetic particles.
  - Kinetic stability theory has the potential to unify results between machines.

Low plasma rotation level is insufficient to ensure RWM stability, which depends on ω_ϕ *profile*



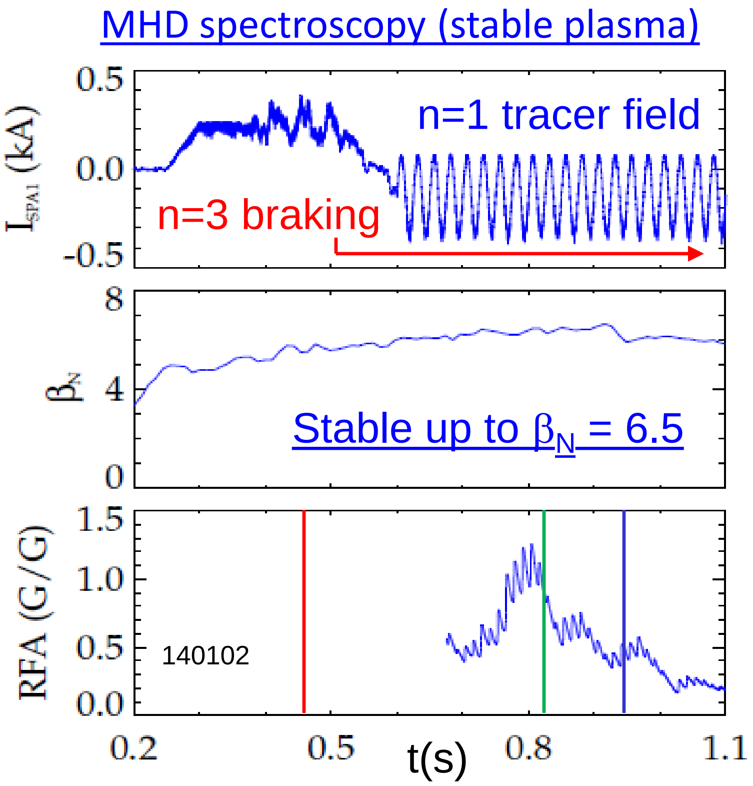
RWM unstable plasma

[S. Sabbagh *et al.*, IAEA FEC 2010 EXS/5-5]



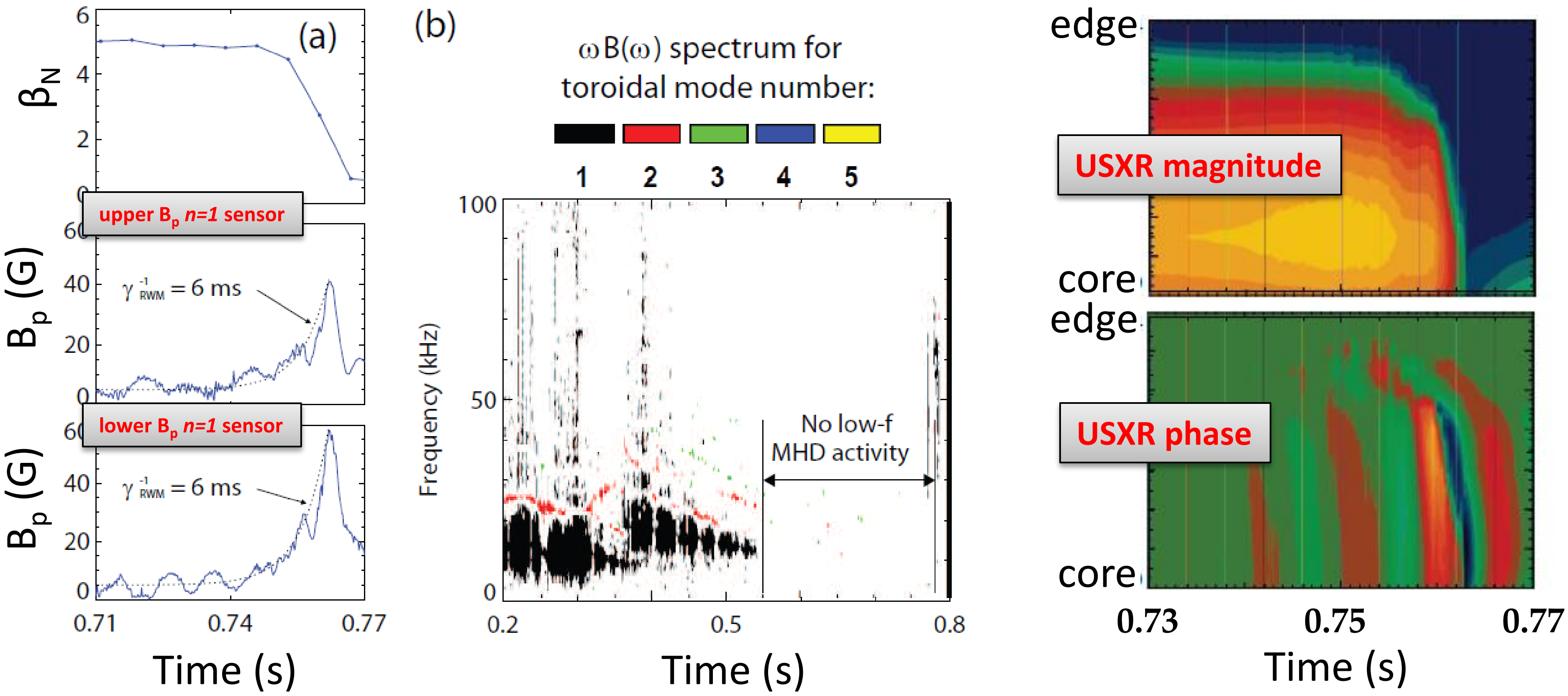
Plasma rotation

- RWM unstable plasma
  - Instability occurs at relatively high rotation level, and not at highest β_N (4.7)
- RWM stable plasma
  - MHD spectroscopy: increased resonant field amplification (RFA) indicates reduced stability
  - Plasma moves to more stable regime (lower RFA) at lower rotation (β_N up to 6.5)



MHD spectroscopy (stable plasma)

The RWM is identified in NSTX by a variety of observations



- Growing signal on low frequency poloidal magnetic sensors
- Global collapse in USXR signals, with no clear phase inversion
- Causes a collapse in β and disruption of the plasma

Kinetic δW_K term in the RWM dispersion relation provides dissipation that enables stabilization

A momentum balance:

$$\rho \frac{d\mathbf{v}}{dt} = \mathbf{j} \times \mathbf{B} - \boldsymbol{\nabla} \cdot \mathbb{P}$$

leads to an energy balance:

$$-\frac{1}{2}\int \rho\omega^2 |\boldsymbol{\xi}_\perp|^2 d\mathbf{V} = \frac{1}{2}\int \boldsymbol{\xi}_\perp^* \cdot \left[\tilde{\mathbf{j}} \times \mathbf{B_0} + \mathbf{j_0} \times \tilde{\mathbf{B}} - \boldsymbol{\nabla}\tilde{p}_F - \boldsymbol{\nabla} \cdot \tilde{\mathbb{P}}_K\right] d\mathbf{V}$$

The change in potential energy due to the perturbed kinetic pressure is:

$$\delta W_K = -\frac{1}{2}\int \boldsymbol{\xi}_\perp^* \cdot \left(\boldsymbol{\nabla} \cdot \tilde{\mathbb{P}}_K\right) d\mathbf{V}$$

Dissipation from kinetic term enables stabilization of the RWM:

$$(\gamma - i\omega_r)\,\tau_w = -\frac{\delta W_\infty + \delta W_K}{\delta W_b + \delta W_K}$$

[B. Hu *et al.*, Phys. Plasmas **12**, 057301 (2005)]

Calculation of δW_K with the MISK code includes:

- Trapped Thermal Ions and Electrons
- Circulating Thermal Ions
- Alfven Layers (analytic)
- Trapped Energetic Particles

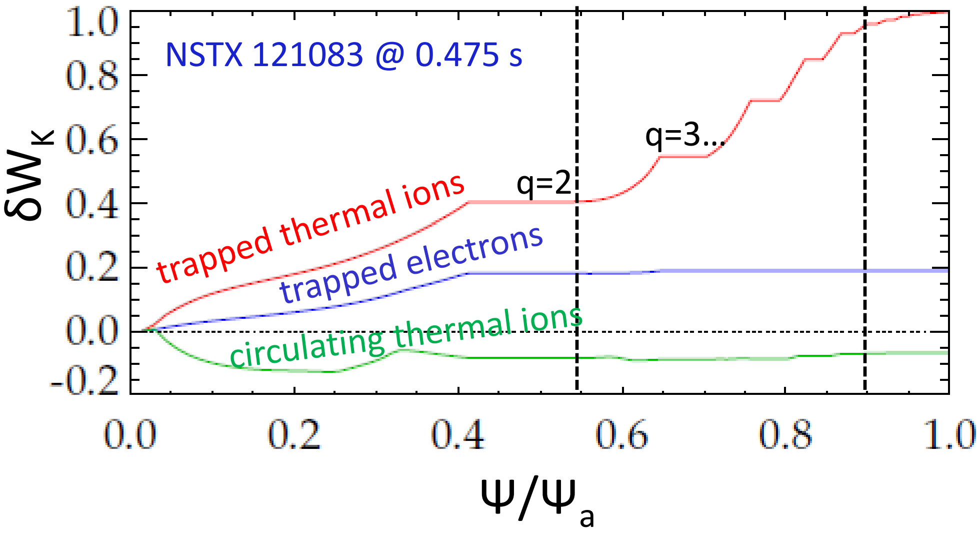
Full MISK calculation shows that trapped thermal ions are the most important contributors to stability

$$\delta W_K = \sum_j \sum_{l=-\infty}^{\infty} 2\sqrt{2}\pi^2 \int_0^{\Psi_a} \frac{d\boldsymbol{\Psi}}{m_j^{\frac{3}{2}} B} \int_{-1}^{1} |\chi| d\chi$$

$$\int_0^\infty \left[\frac{(\omega - n\omega_E)\frac{\partial f_j}{\partial \varepsilon} - \frac{n}{Z_j e}\frac{\partial f_j}{\partial \Psi}}{n\langle\omega_D^j\rangle + l\omega_b^j - i\nu_{\text{eff}}^j + n\omega_E - \omega} \right] \varepsilon^{\frac{1}{2}} d\varepsilon$$

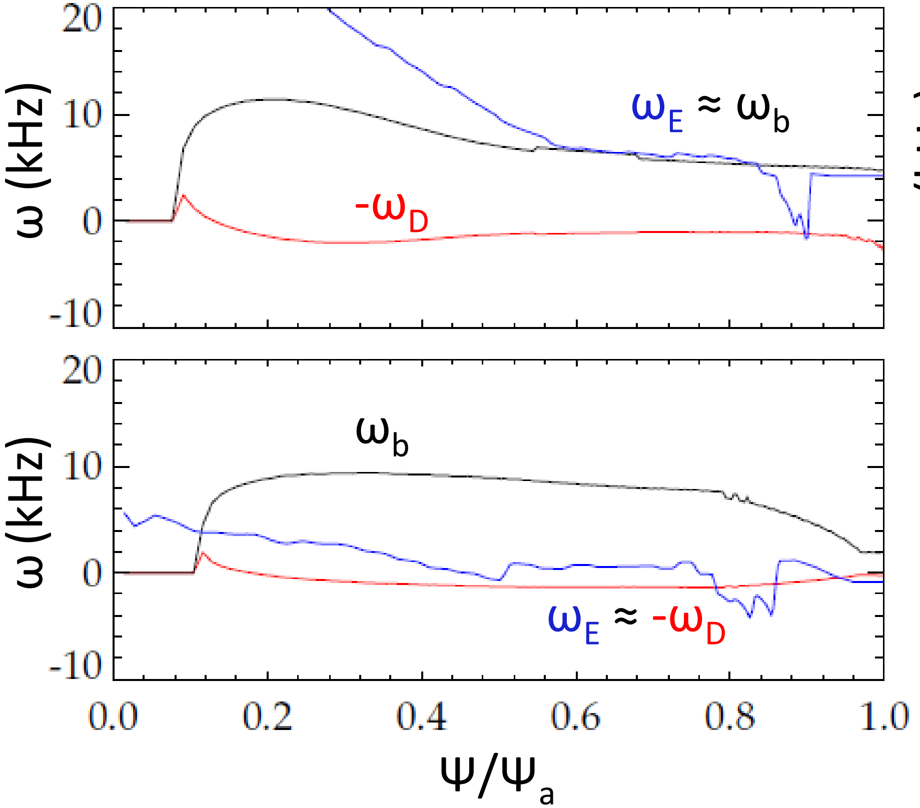
$$\left|\left\langle (3\chi^2-1)\,\boldsymbol{\kappa}\cdot\boldsymbol{\xi}_\perp - (\chi^2-1)\,\boldsymbol{\nabla}\cdot\boldsymbol{\xi}_\perp \right\rangle\right|^2$$

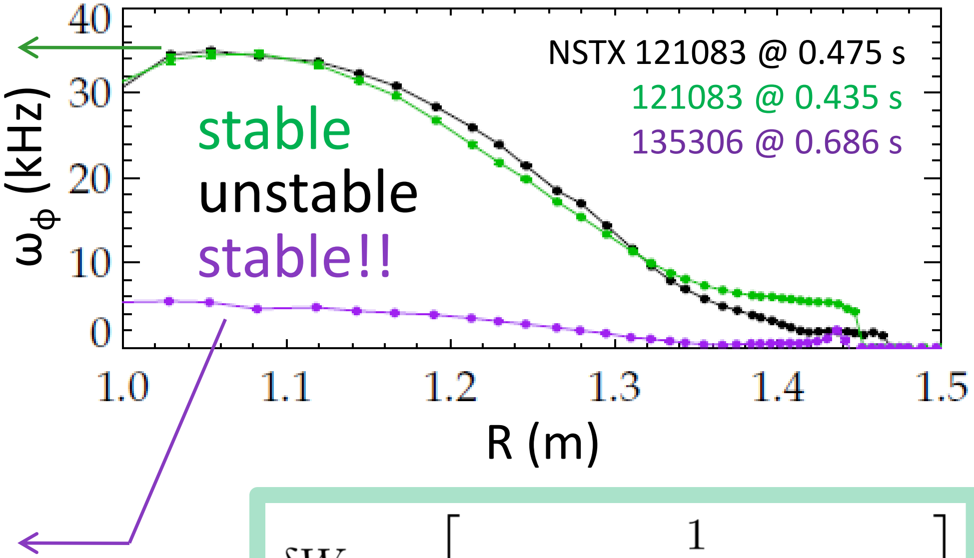
Full δW_K eqn. for general *f*



- Examine δW_K from each particle type vs. Ψ
  - Thermal ions are the most important contributor to stability.
  - Flat areas are rational surface layers (integer q ± 0.2).
- Entire profile is important, but q > 2 contributes ~60%
  - RWM eigenfunction and temperature, density gradients are large in this region.

When the rotation is in resonance, the plasma is stable



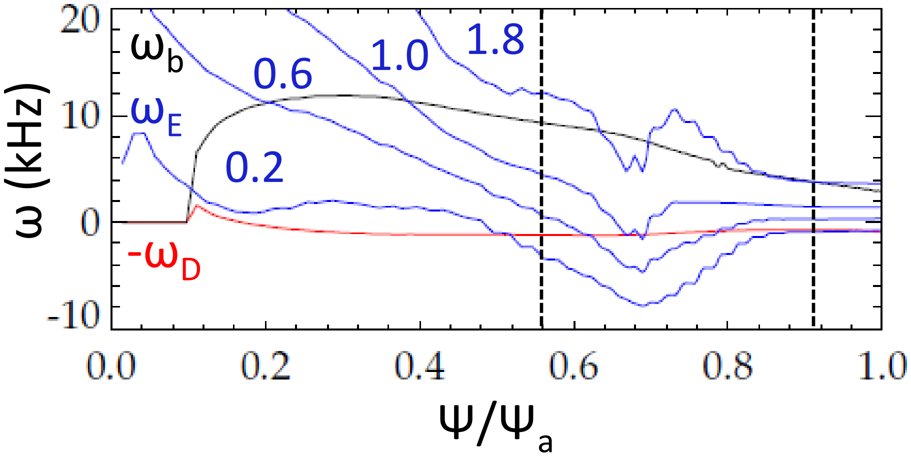


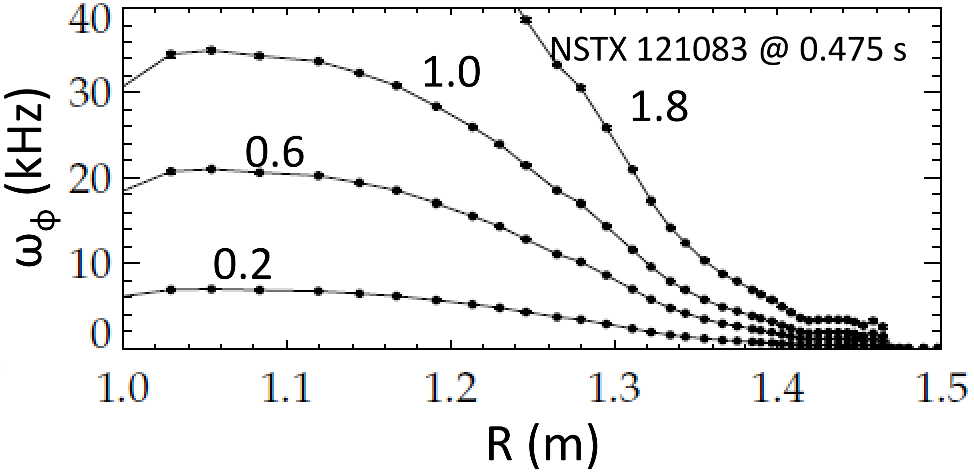
$$\delta W_K \sim \left[\frac{1}{\langle\omega_D\rangle + l\omega_b - i\nu_{\text{eff}} + \omega_E}\right]$$

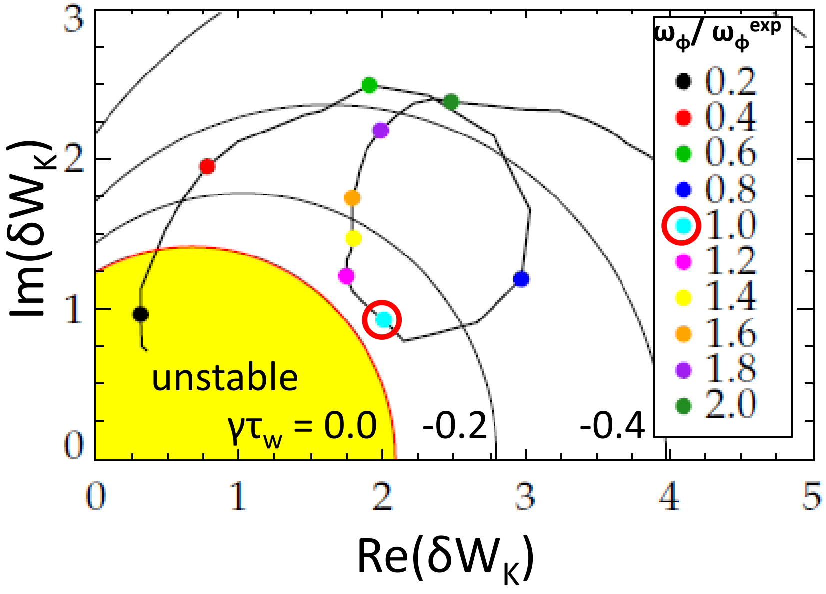
$$\omega_E = \omega_\phi - \omega_{*i}$$

- Stable cases in bounce resonance at high rotation
- Stable cases in precession drift resonance at low rotation

Scaling the experimental rotation profile illuminates the complex relationship between rotation and stability







[J. Berkery *et al.*, Phys. Rev. Lett. **104**, 035003 (2010)]

- Rotation profile scan:
  - 0.2: Instability at low rotation.
  - 0.6: Stable: ω_D resonance.
  - 1.0: Marginal: in-between resonances (actual experimental instability).
  - 1.8: Stable: ω_b resonance.

Collision frequency is very high for electrons, very low for energetic particles, but in the right range for ions

Three collisionality models:

$\nu_0 = 0$ (MARS)

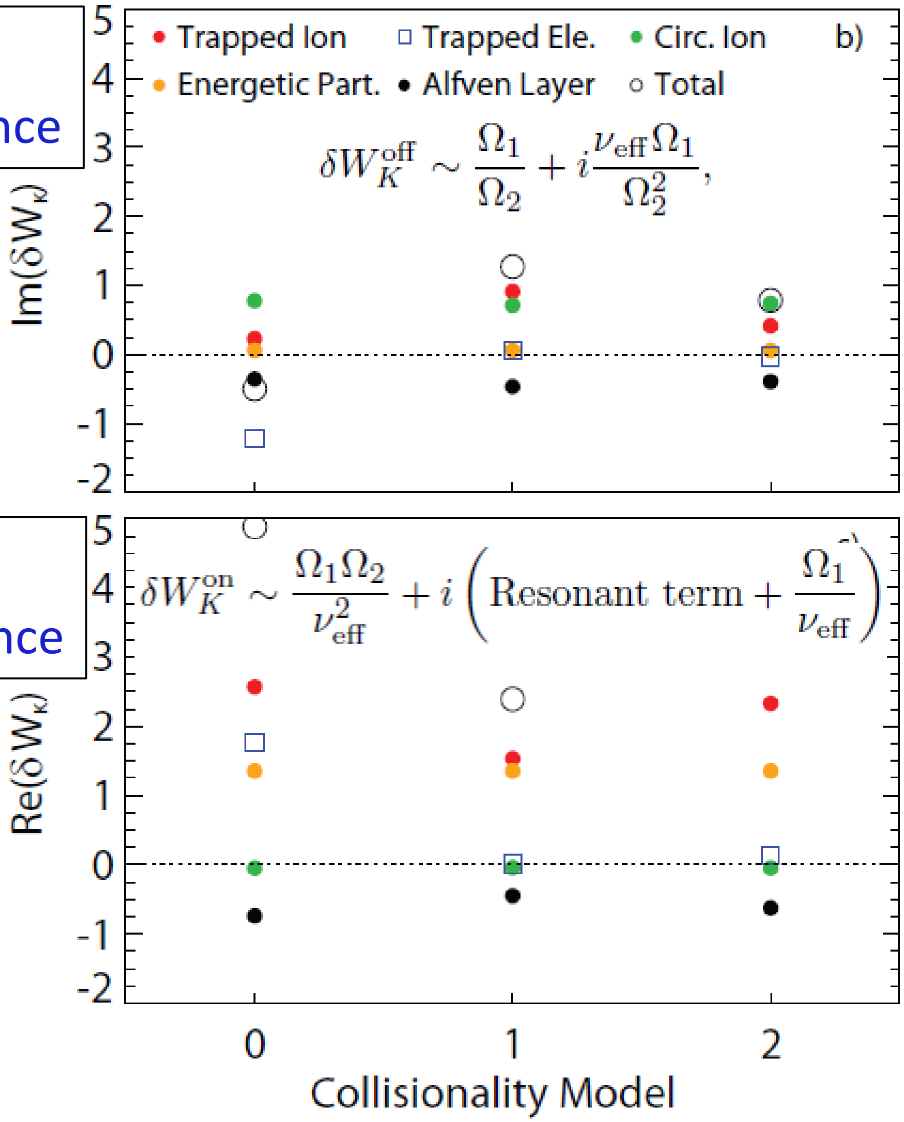
$$\nu_1(\Psi) = \frac{\sqrt{2} n_i m_{ji}^{\frac{1}{2}} Z_i^2 Z_j^2 e^4 \ln\Lambda}{12\pi^{\frac{3}{2}} \epsilon_0^2 m_j T_j^{\frac{3}{2}}} \epsilon_r^{-1}$$

$\nu_2(\Psi, \varepsilon) = \nu_1 \hat{\varepsilon}^{-\frac{3}{2}}$ (MISK)

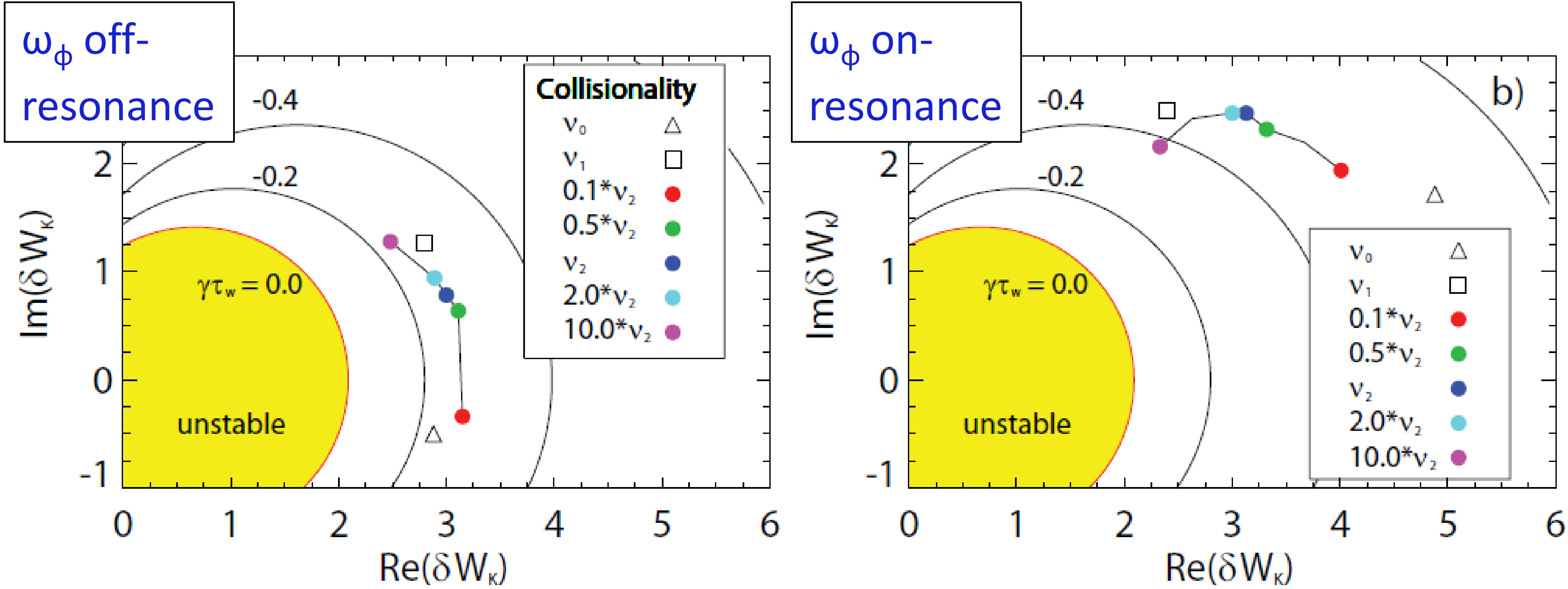
- Energetic Particles are unaffected by collisionality.
- As soon as ν ≠ 0, Electron component → 0.
- Trapped Ions *are* affected by collisionality model.

ω_ϕ off-resonance

ω_ϕ on-resonance



Reduced collisions allow resonant kinetic stabilizing effects to be more powerful, but also reduce collisional dissipation



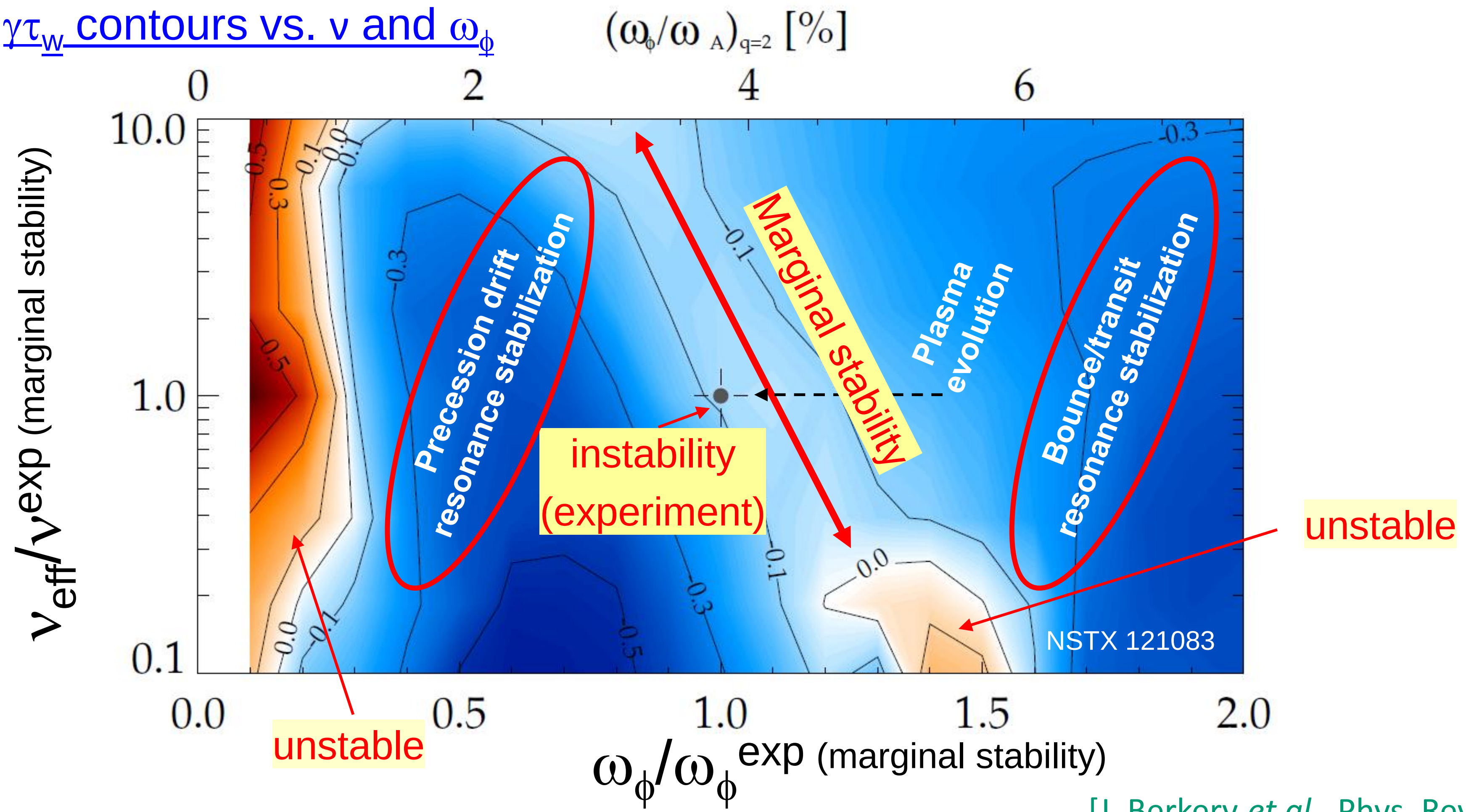
ω_ϕ off-resonance

ω_ϕ on-resonance

- The effect of reduced collisionality in future machines:
  - Lower stability off-resonance plasmas even less stable
  - Higher stability on-resonance plasmas even more stable
  - Makes it all the more important to mitigate the effects of reduced stability at off-resonance ω_ϕ.

MISK calculations consistent with RWM destabilization at intermediate plasma rotation; stability altered by collisionality

$\gamma\tau_w$ contours vs. ν and ω_ϕ



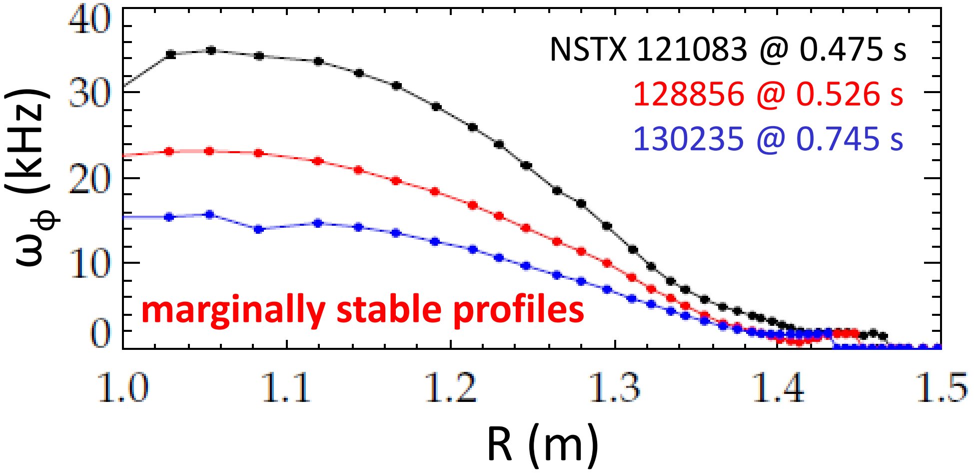
- Destabilization appears between precession drift resonance at low ω_ϕ, bounce/transit resonance at high ω_ϕ
- Destabilization moves to increased ω_ϕ as ν decreases

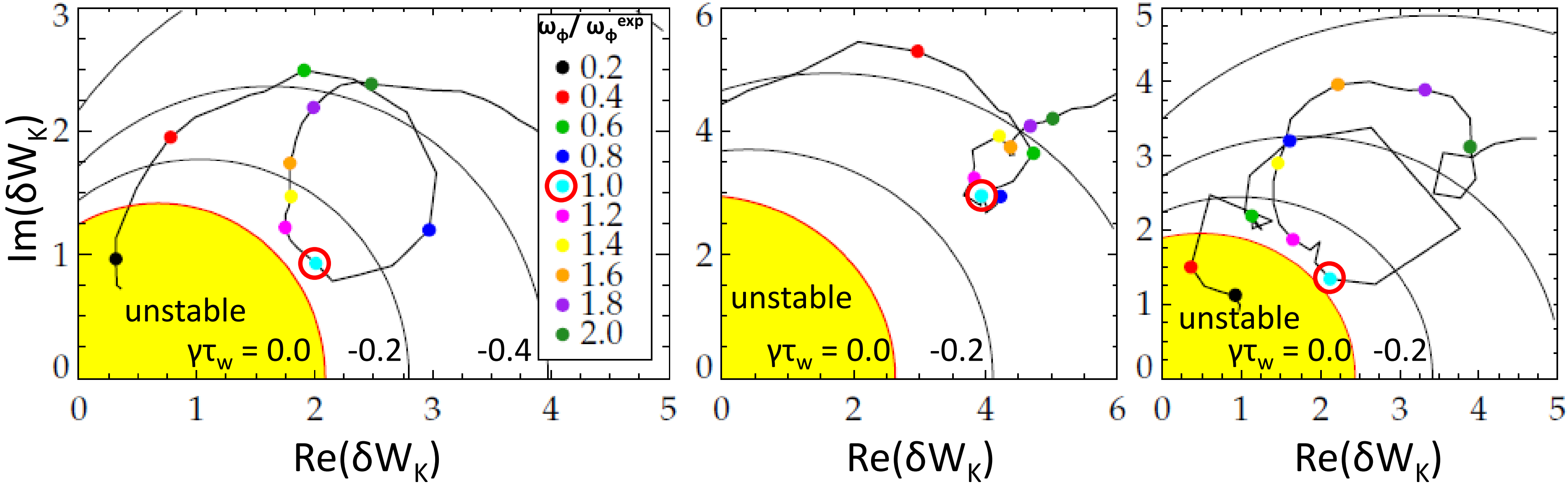
[J. Berkery *et al.*, Phys. Rev. Lett. **104**, 035003 (2010)]

[S. Sabbagh *et al.*, Nucl. Fusion **50**, 025020 (2010)]

Widely different experimentally marginally stable rotation profiles each are in the gap between stabilizing resonances

- Sometimes the stability reduction is not enough to quantitatively reach marginal
- Investigating sensitivities to inputs.



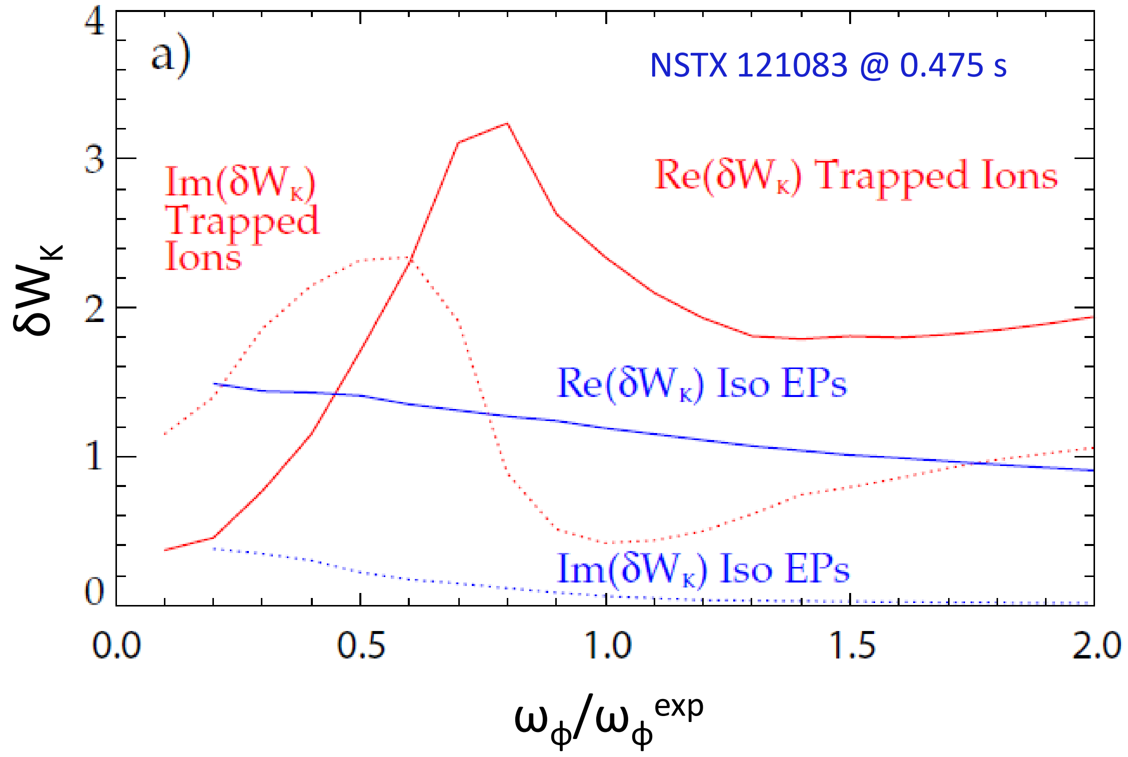


121083 @ 0.475 s

128856 @ 0.526 s

130235 @ 0.745 s

Energetic particles provide a stabilizing force that is nearly independent of rotation and collisionality



for energetic particles:

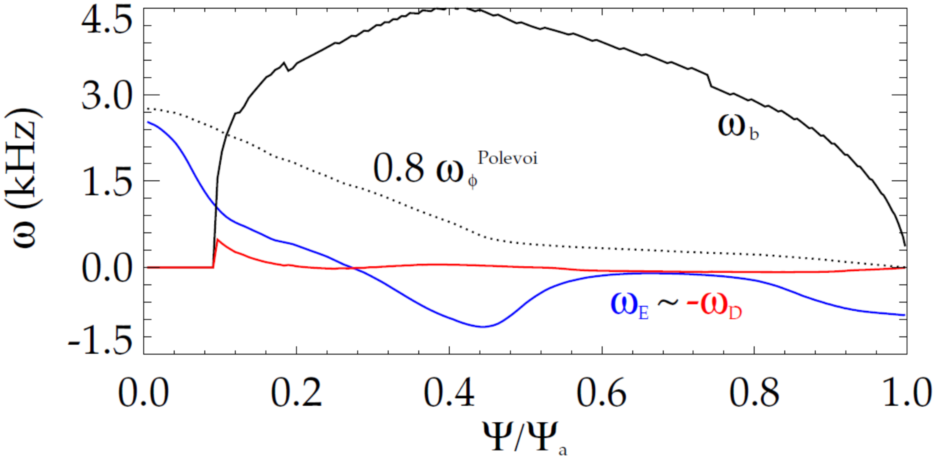
$$\delta W_K \sim \left[\frac{1}{\langle\omega_D\rangle + l\omega_b \underbrace{- i\nu_{\mathrm{eff}} + \omega_E}_{\text{small}}}\right]$$

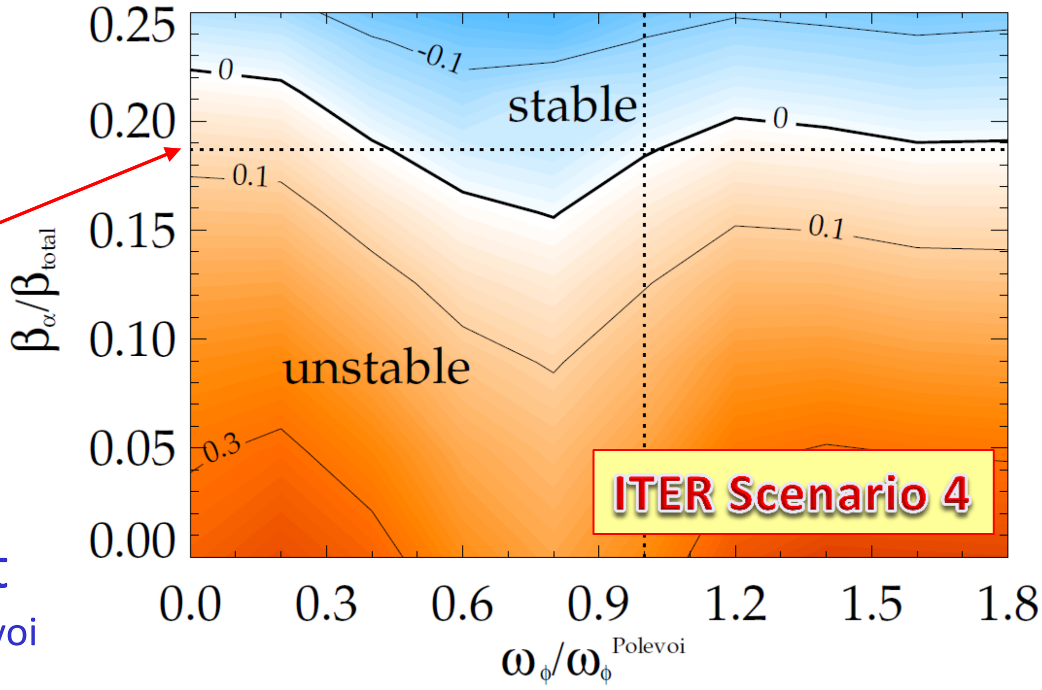
[J. Berkery *et al.*, Phys. Plasmas **17** 082504 (2010)]

- Significant Re(δW_K), but nearly independent of ω_ϕ
- Energetic particles are <u>not</u> in mode resonance
- Effect is not energy dissipation, but rather a restoring force

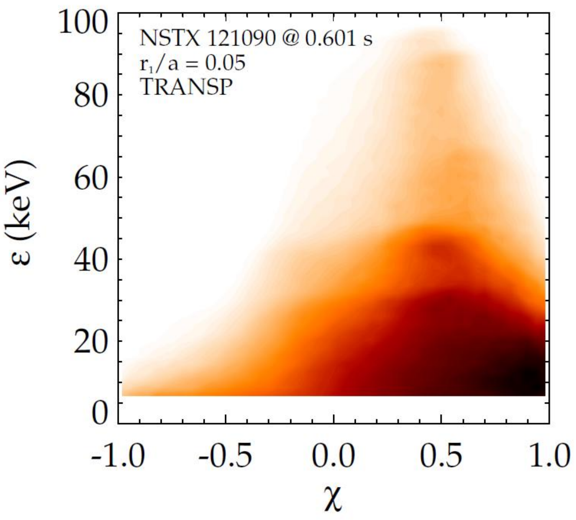
MISK computed RWM stability of ITER scenario 4 including energetic particles near marginal at $\beta_N = 3$

- ITER advanced scenario 4
  - With $\beta_N = 3$ (20% above n = 1 no-wall limit)
  - Polevoi rotation profile
- Energetic particle effect
  - Isotropic slowing down distribution of alphas
  - Near RWM marginal stability at expected β_a/β_{total}
- Plasma rotation effect
  - Stabilizing precession drift resonance $\omega_\phi = 0.8\ \omega_\phi^{Polevoi}$

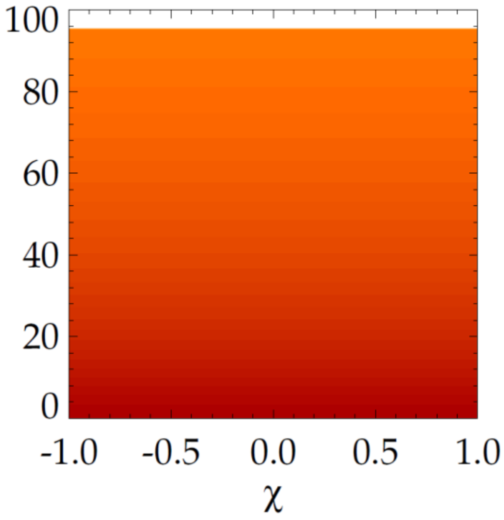




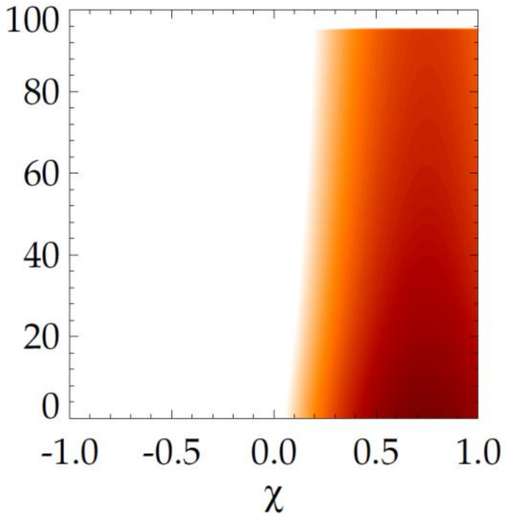
Anisotropic distribution function of beam ions impacts stability; work continues on improving model



Real (from TRANSP)



Isotropic

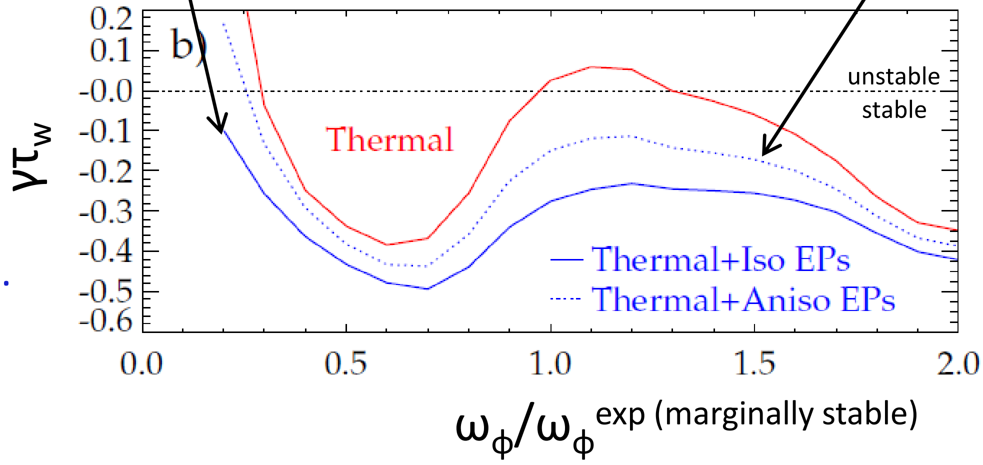


Simple anisotropic test case

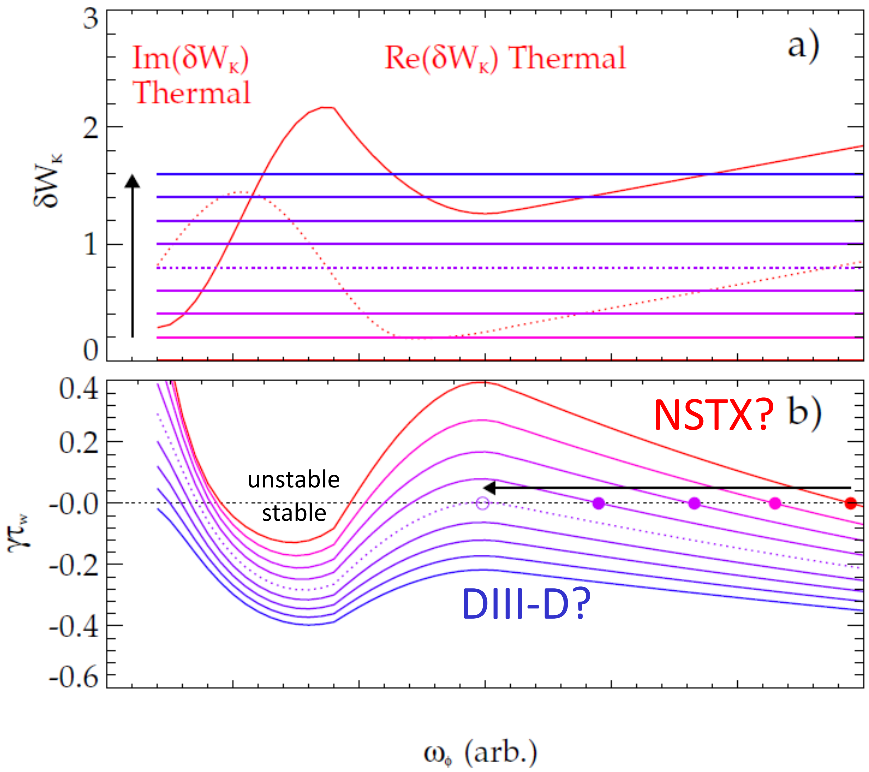
$\chi_0 = 0.75$, $\delta\chi = 0.25$

$$f\left(\varepsilon, \Psi, \chi\right) = \frac{C\left(\Psi\right)}{\varepsilon^{\frac{3}{2}} + \varepsilon_c^{\frac{3}{2}}} \frac{e^{-(\chi-\chi_0)^2/\delta\chi^2}}{\delta\chi}$$

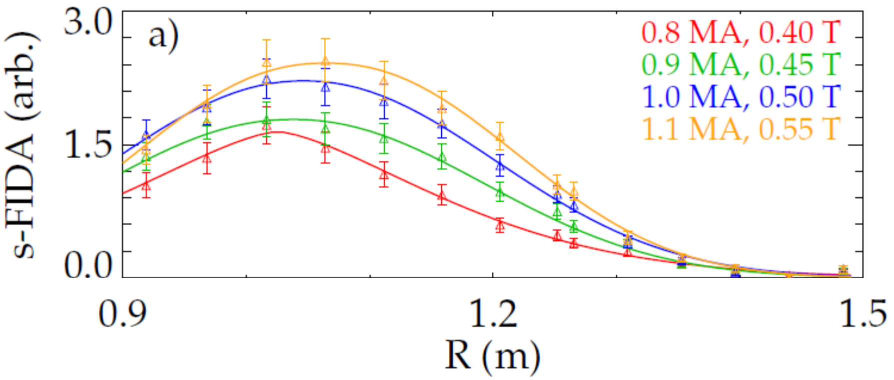
- Work towards analytical model of the TRANSP energetic particle distribution function continues.
- Complicated by multiple sources, energy components and deposition surfaces.

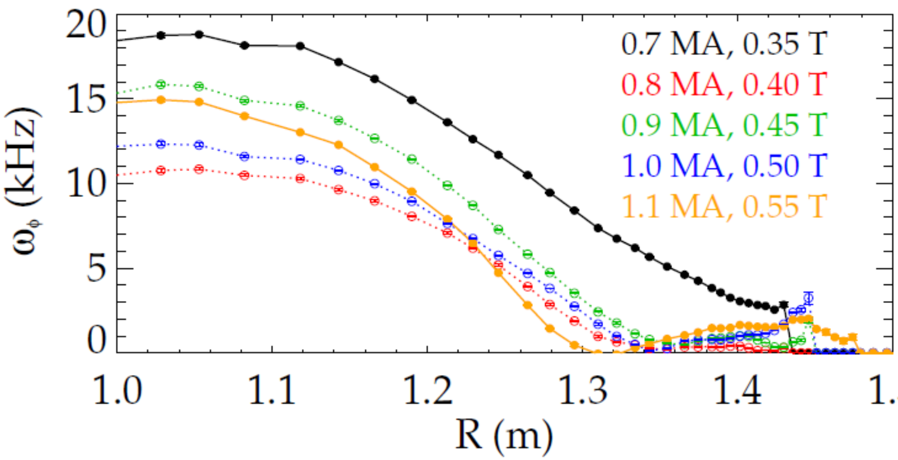


An NSTX experiment examined the role of energetic particles in RWM stability



Illustrative example



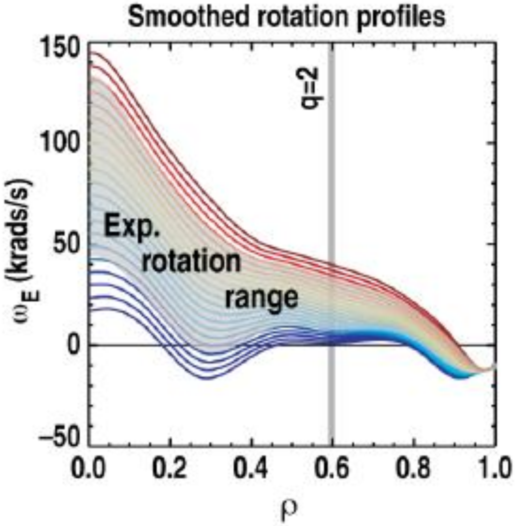


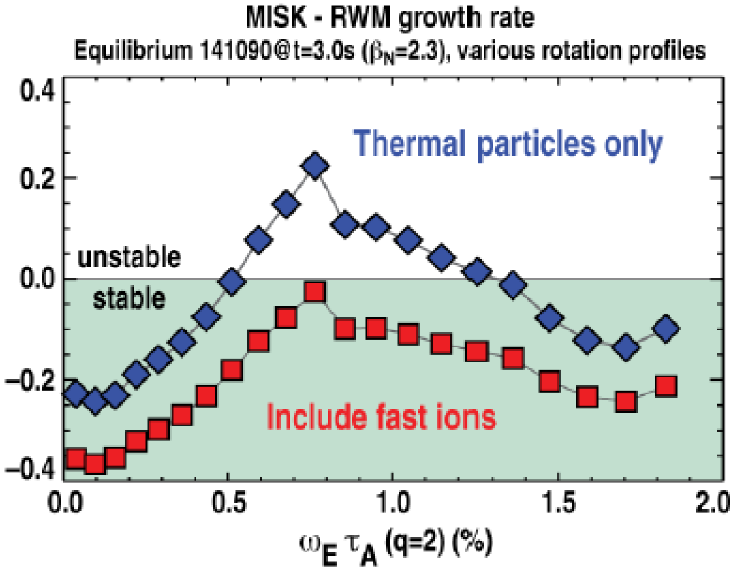
- Present model, with improvements, can explain exps.
  - NSTX: low EP: unstable more often, rotational resonances seen
  - DIII-D: higher EP: mode stable except when triggered by fishbones

MISK calculations are consistent with MHD spectroscopy results from DIII-D when EPs are included

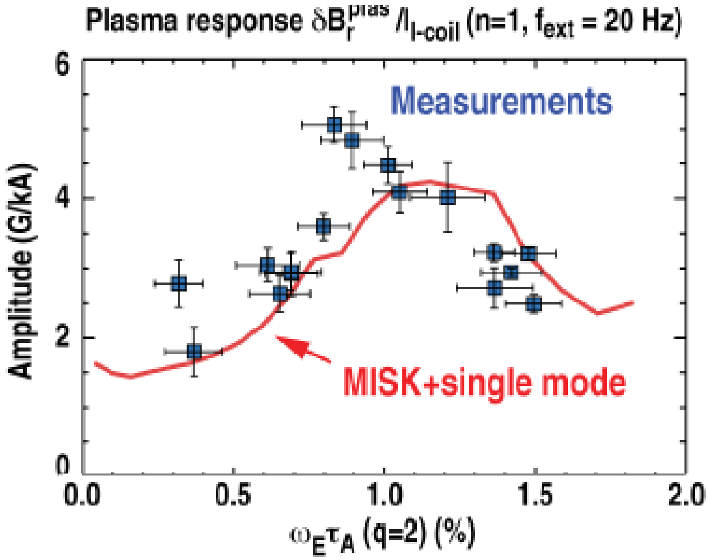
[H. Reimerdes *et al.*, IAEA FEC 2010, EXS/5-4]

- Kinetic model has to include EPs to explain experimental stability

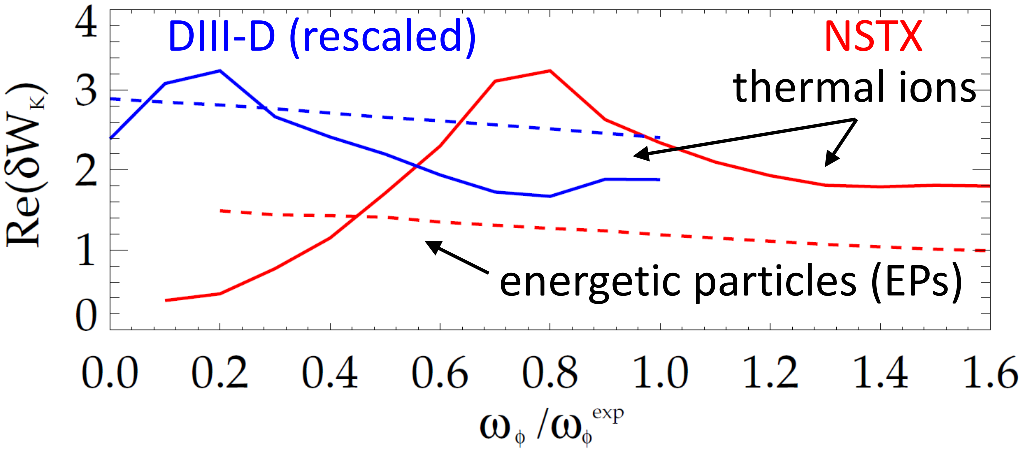


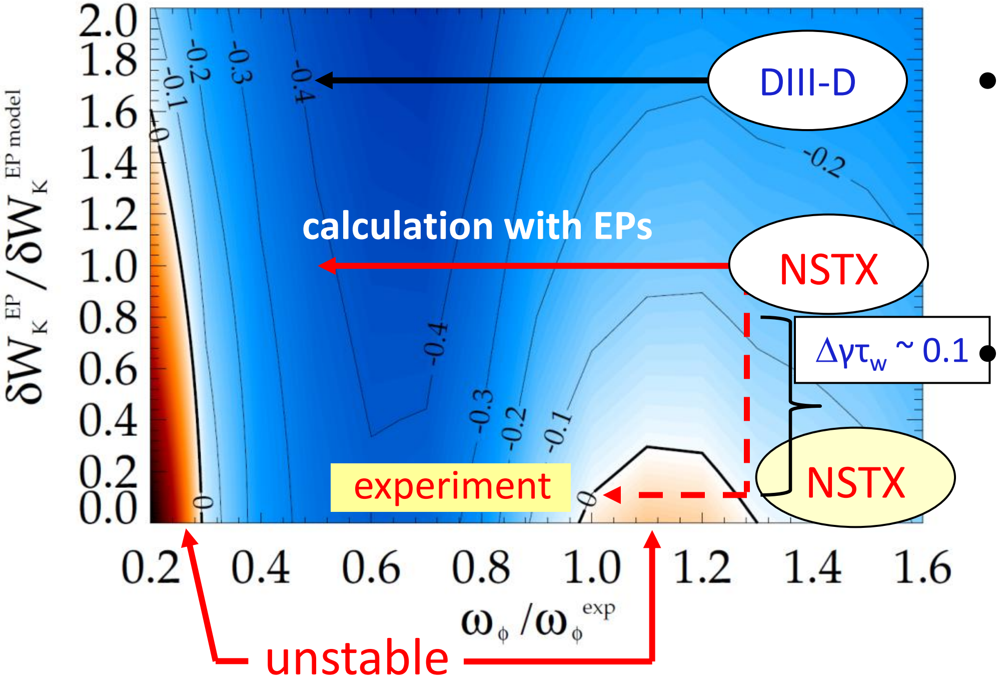


- Measured rotation dependence of the plasma response to a quasi-static external *n*=1 field reveals the characteristics of the resonances with the trapped particle precession frequency
  - Direct evidence for the relevance of kinetic resonances for RWM stability



Model of kinetic modifications to ideal stability can unify RWM stability results between devices



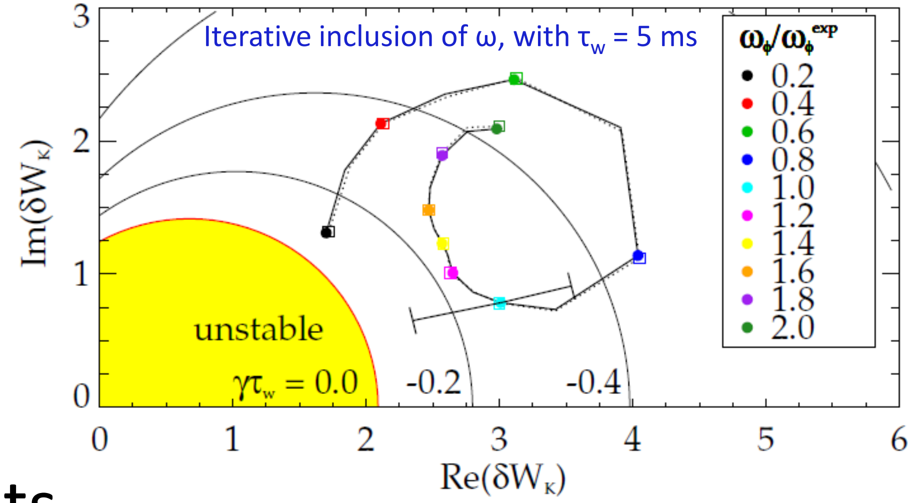


- NSTX
  - Less EP stability: RWM can cross marginal point as ω_ϕ is varied
- DIII-D [H. Reimerdes *et al.*, IAEA FEC 2010, EXS/5-4]
  - More EP stability (~2x NSTX): RWM stable at all ω_ϕ
  - RWM destabilized by events that reduce EP population
- ITER (advanced scenario IV)
  - RWM unstable at expected rotation
  - Only marginally stabilized by alphas at $\beta_N = 3$
- Stability overpredicted with EPs – model development continues
  - Improve NBI anisotropic dist.
  - Examine effects originally thought small

[S. Sabbagh *et al.*, IAEA FEC 2010, EXS/5-5]

RWM stabilization model is being carefully assessed for sensitivities

- Perturbative vs. self-consistent approaches
  - Is ξ changed by kinetic effects?
  - Non-linear inclusion of ω



- Sensitivity of code to inputs
  - Calculation is sensitive to profiles. Also, the select of Δq for analytic treatment around rational surfaces (shown above)
- Zero banana width approximation
  - Since RWM is a global mode, this effect may be minimal

These effects are not enough to explain quantitative disagreement.

Improvements to theoretical model are needed…

Advancements in the theoretical model continue

Electrostatic effect

The electrostatic component of the perturbed distribution function contributes to δW. This effect is likely to be small, however.

$$\delta W_\Phi = -\frac{1}{2}\int e^2 \left|\tilde{\mathbf{\Phi}} + \boldsymbol{\xi}_\perp \cdot \nabla \Phi_0\right|^2 \sum_j Z_j^2 \frac{n_j}{T_j} d\mathbf{V}$$

[B. Hu *et al.*, Phys. Plasmas **12**, 057301 (2005)]

Additional anisotropic term

In addition to the effect of anisotropy on δW_K, when *f* is anisotropic an additional term arises that is proportional to $\tilde{\mathbf{B}}_\|$:

$$\delta W_{\tilde B} = \sum_j \frac{1}{2} \int \int \langle HT_j \rangle^* \mu \frac{\tilde{\mathbf{B}}_\|}{B} \frac{\partial f_j}{\partial \mu} d^3\mathbf{v} d\mathbf{V}.$$

Centrifugal destabilization

This fluid force term is usually neglected, but it is always destabilizing, and could be important if the plasma rotation Mach number is significant, or for alpha particles rotating at higher frequency ~ $\omega_{*\alpha}$.

$$\delta W_C = -\frac{1}{2} \sum_j \int \boldsymbol{\xi}_\perp^* \cdot \left[\tilde{\rho} \mathbf{v_0} \cdot \nabla \mathbf{v_0}\right] d\mathbf{V}$$

Other possibilities:

- Inclusion of plasma inertia term in the dispersion relation.
- Effect of poloidal rotation on ω_E (small).
- Use of a Lorentz collisionality model instead of current ad-hoc inclusion of collisionality.

$$C\left(\tilde{f}\right) = \frac{1}{2} \nu \Pi_\varepsilon \frac{\partial}{\partial \chi} (1-\chi)^2 \frac{\partial \tilde{f}}{\partial \chi}$$

Kinetic model can unify RWM marginal stability results between machines

- The MISK code is used to calculate RWM stability with kinetic effects, including EPs, for NSTX, DIII-D and ITER.
- Thermal ion resonances can explain the complex relationship between plasma rotation and stability.
  - Effect will be enhanced by low ν in future machines.
- Computations indicate that energetic particles have a stabilizing effect, consistent with experiments.
  - Addition of EPs can unify NSTX and DIII-D results; further work needed for quantitative agreement of NSTX marginal point.
- Improvements to modeling continue, particularly an improved beam ion anisotropic distribution.

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Extra slides

NSTX

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Comparison of Resistive Wall Mode Kinetic Stabilization Theory and Experiment

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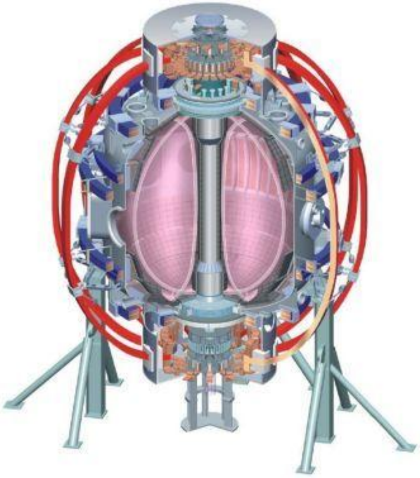

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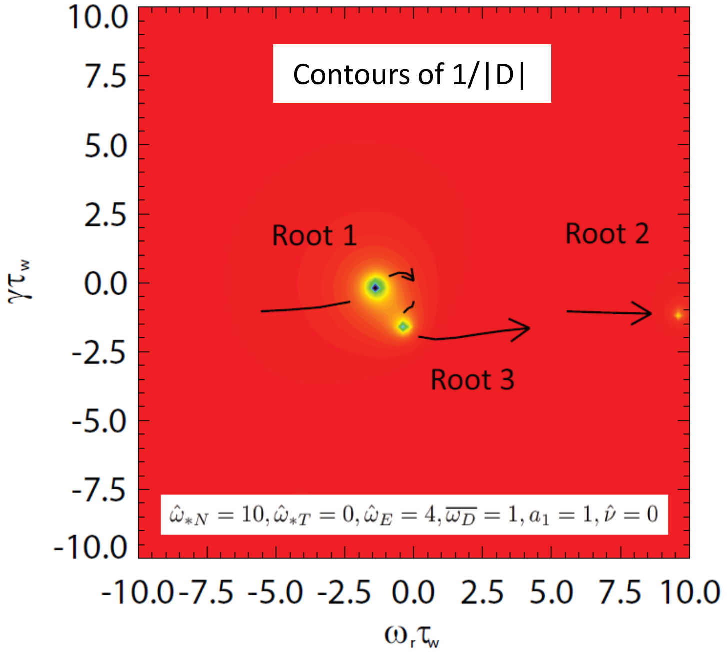
The RWM dispersion relation has three roots, but in realistic scenarios only one is near $\omega_r = 0$, $\gamma = 0$

The RWM dispersion relation, $D = (\hat{\gamma} - i\hat{\omega}_r)\left(\hat{\gamma}_f^{-1} + C\right) - 1 + C$, has three roots.

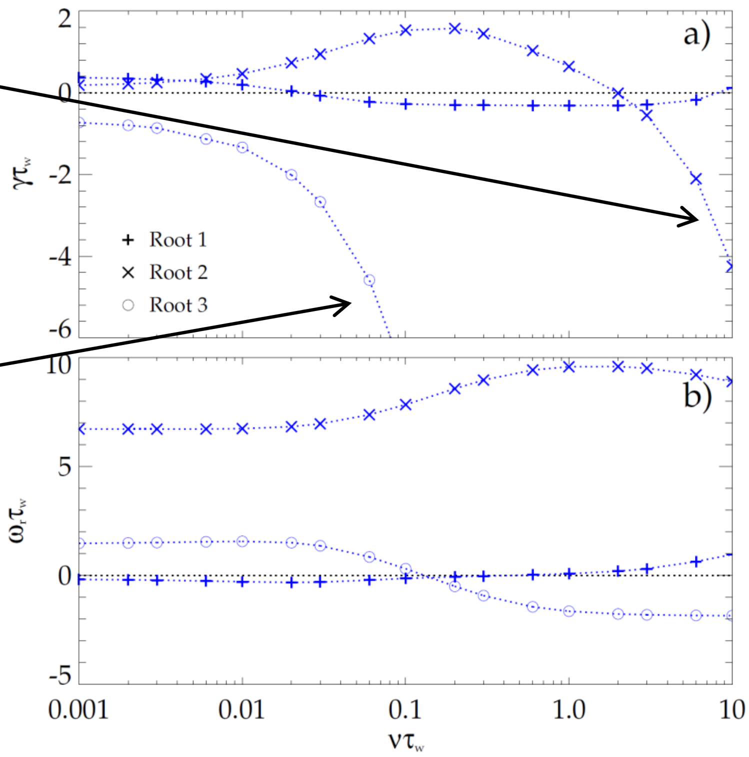
[Y. Liu *et al.*, Phys. Plasmas **16** 056113 (2009)]

$$C = c\frac{8}{15\sqrt{\pi}}\int_0^\infty \left[\frac{\hat{\omega}_{*N} + \hat{\omega}_E - \hat{\omega}_r - i\hat{\gamma}}{\overline{\omega_D}\hat{\varepsilon}^{a_1} - i\hat{\nu} + \hat{\omega}_E - \hat{\omega}_r - i\hat{\gamma}} + \frac{-\hat{\omega}_{*N} + \hat{\omega}_E - \hat{\omega}_r - i\hat{\gamma}}{-\overline{\omega_D}\hat{\varepsilon}^{a_1} - i\sqrt{2m_i/m_e}\hat{\nu} + \hat{\omega}_E - \hat{\omega}_r - i\hat{\gamma}}\right]\hat{\varepsilon}^{\frac{5}{2}}e^{-\hat{\varepsilon}}d\hat{\varepsilon}$$



Root stabilized by ion collisions

Root stabilized by electron collisions



- Arrows show trajectories of roots as $\omega_E = 0 \rightarrow 10$
- Root 1 becomes stationary with respect to the wall
- Roots 2 and 3 rotate with the plasma