NSTX

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Multi-mode RWM Analysis of NSTX High Beta Plasmas

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J.M. Bialek[1], S.A. Sabbagh[1], A. Boozer[1],
L. Delgado-Aparicio[2], S.P. Gerhardt[2]

[1]Department of Applied Physics, Columbia University, NY, NY
[2]Plasma Physics Laboratory, Princeton University, Princeton, NJ

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Multi-mode RWM Analysis of NSTX High Beta Plasmas

The behavior of resistive wall modes (RWM) in the NSTX device has been examined with the multi-mode VALEN computer code. Experiments at very high-normalized beta reaching 7.4 in conditions with the ideal MHD no-wall limit near 4 were produced in NSTX and are analyzed to compare the multi-mode model with experiment. Multi-mode VALEN includes conducting structures, coils, and plasma response as an L-R circuit [1]. We summarize the multi-mode formulation and contrast it with single-mode VALEN [2]. We present multi-mode characteristics of RWM behavior for NSTX high beta plasmas including the mode-spectrum evolution for passive growth and the interaction of the mode spectrum with exterior fields typical of RWM feedback control.

[1] A.H. Boozer, Phys. Plasmas 10 (2003) 1458.

[2] J.M. Bialek et al. Phys. Plasmas 8 (2001) 2170.

Outline

- Control of the RWM is important
- VALEN equations for multi-mode vs. original single mode formulation
- NSTX conducting structure used in VALEN model
- NSTX discharge used for multi-mode calculations
- Comparison example of single mode vs. multi-mode analysis
- Detail of eigenmodes used in multi-mode VALEN RWM spectrum (“ballooning” mode, “divertor’ mode, etc.)
- Multi-mode VALEN RWM mode spectrum for passive growth and comparison of multi-mode VALEN RWM mode spectrum for passive growth vs. spectrum that is near marginal stability with mode rotation
- Effects of active feedback on the multi-mode spectrum
- Summary

Control of the Resistive Wall Mode is important

- Motivation
  - Achieve high β_N with sufficient physics understanding to allow confident extrapolation to ST applications
  - Sustain target β_N of ST applications with margin to reduce risk
  - Leverage unique ST operating regime to test physics models

- Physics Research Addressed
  - Resistive wall mode active control
  - Multi-mode RWM spectrum in high β_N plasmas

VALEN model of NSTX includes conducting structure, coils, and sensors

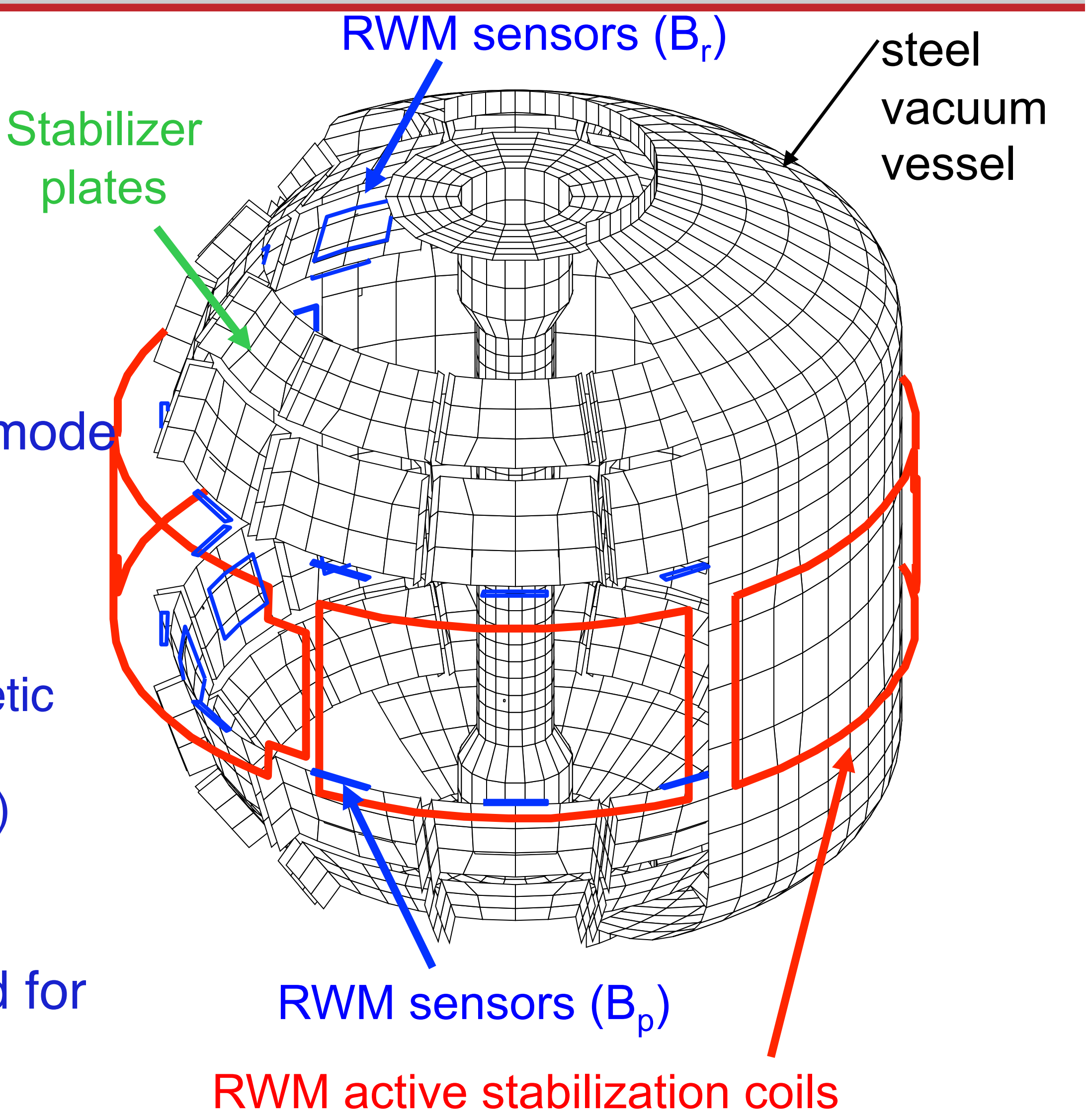
- High beta, low aspect ratio
  - R = 0.86 m, A > 1.27

  - $I_p < 1.5$ MA, $B_t = 5.5$ kG
  - $\beta_t < 40\%$, $\beta_N < 7.4$

- Copper stabilizer plates for kink mode stabilization

- Midplane control coils
  - n = 1 – 3 field correction, magnetic braking of ω_ϕ
  - n = 1 resistive wall mode (RWM) control

- Varied sensor combinations used for RWM feedback
  - 48 upper/lower B_p, B_r



The multi mode VALEN formulation includes many ideal plasma eigenfunctions in addition to the 3-D wall and coils

- VALEN formulation of the Resistive Wall Mode uses simple R-L circuit equations. The plasma response to an external perturbation is

$$\Phi_{plasma}^{total}(\phi,\theta) = \left[P\right] \Phi_{plasma}^{external}(\phi,\theta)$$

- [P] is the ‘plasma permeability’ [dimensionless], and Φ is the magnetic flux [v*s] on the unperturbed surface of the plasma

- In single mode VALEN [P] depends on only the unstable plasma mode

- In multi-mode VALEN [P] depends on multiple modes (unstable & stable)

- The conducting walls, control coils, and magnetic sensors are represented by L-R circuit equations.

$$\text{i.e.,}\quad [L]\left\{\dot{I}(t)\right\} + [R]\left\{I(t)\right\} = \left\{V(t)\right\}$$

$$\text{and}\quad \left\{\Phi^{sensors}(t)\right\} = [M]\left\{I(t)\right\}$$

VALEN equations, multi-mode & original formulation

FLUX on wall, coils, and unperturbed plasma surface from

$\{I_w(t)\}$ = wall currents, $\{I_c(t)\}$ = coil currents,

$\{I_d(t)\}$ = dissipative plasma circuit currents, $\{I_p(t)\}$ = plasma mode currents:

$$[L_{ww}]\{I_w(t)\}+[L_{wc}]\{I_c(t)\}+[L_{wp}]\{I_d(t)\}+[L_{wp}]\{I_p(t)\}=\Phi_w(t) \quad =\Phi_{WALL}(t)$$

$$[L_{cw}]\{I_w(t)\}+[L_{cc}]\{I_c(t)\}+[L_{cp}]\{I_d(t)\}+[L_{cp}]\{I_p(t)\}=\Phi_c(t) \quad =\Phi_{COIL}(t)$$

$$[L_{pw}]\{I_w(t)\}+[L_{pc}]\{I_c(t)\}+[L_{pp}]\{I_d(t)\}+[L_{pp}(t)]\{I_p\}=\Phi_p^{total}(t) \quad =\Phi_{PLASMA}^{total}(t)$$

$$[L_{pw}]\{I_w(t)\}+[L_{pc}]\{I_c(t)\}+[L_{pp}]\{I_d(t)\} \quad =\Phi_p^{external}(t)=\Phi_{PLASMA}^{external}(t)$$

$$\Phi_p^{external}(t) \quad + \quad [L_{pp}]\{I_p\}=\Phi_p^{total}(t)=[P]\Phi_p^{external}(t)$$

where: plasma permeability $[P]=[\Lambda][L_{pp}]^{-1}$ and $[\Lambda]^{-1}=2[G][\varepsilon][G]^t$

$[G]$ = gram schmidt transformation and $[\varepsilon]$ = ideal δW values (from DCON)

i.e., $\{f_i(\theta,\phi)\}=A\sum_\alpha[G_{i\alpha}]\ \{b_\alpha^{DCON-Bn}(\theta,\phi)\},\ \ and,\ \ \oint\frac{f_i(\theta,\phi)f_j(\theta,\phi)}{A}=\delta_{ij},\ \oint da=A$

VALEN equations, multi-mode & original formulation

single mode VALEN

$$[P]=\left(\frac{-1}{s}\right) \quad or \quad [\mathrm{P}]=\frac{\begin{bmatrix}-s & \alpha \\ -\alpha & -s\end{bmatrix}}{\left(s^2+\alpha^2\right)} \text{ (with rotation)}$$

$$\text{this is same as: } [P]=[\Lambda]\left[L_{pp}\right]^{-1}$$

$$\mathrm{s}=-\Lambda^{-1}L=-2G_{11}\varepsilon G_{11}$$

$$G_{11}=\frac{1}{A\left[\oint B^n B^n dA/A\right]^{1/2}}$$

$$s=-\frac{2}{A^2}\frac{1}{\left[\oint B^n B^n dA/A\right]^{1/2}}\varepsilon\frac{1}{\left[\oint B^n B^n dA/A\right]^{1/2}}L$$

$$s=\frac{-2L(\delta W)}{\Phi^2}=\frac{-\delta W}{LI^2/2}$$

multi-mode VALEN

$$[P]=[\Lambda]\left[L_{pp}\right]^{-1} \quad and \quad [\Lambda]=2[G][\varepsilon][G]^t$$

$$[\varepsilon]=\begin{bmatrix}
\delta W_1 & 0 & 0 & 0 & & 0 & \ldots & & \\
0 & \delta W_1 & 0 & 0 & & 0 & \ldots & & \\
 & & \delta W_2 & 0 & & 0 & \ldots & & \\
 & & 0 & \delta W_2 & & 0 & \ldots & & \\
 & . & & & & & & & \\
 & . & & & & & & & \\
0 & 0 & 0 & 0 & \ldots & 0 & \delta W_n & 0 & \\
0 & 0 & 0 & 0 & \ldots & 0 & 0 & \delta W_n &
\end{bmatrix}$$

or (with rotation)

$$[\varepsilon]=\begin{bmatrix}
\delta W_1 & \Gamma_1 & 0 & 0 & & 0 & \ldots & & \\
-\Gamma_1 & \delta W_1 & 0 & 0 & & 0 & \ldots & & \\
 & & \delta W_2 & \Gamma_2 & & 0 & \ldots & & \\
 & & -\Gamma_2 & \delta W_2 & & 0 & \ldots & & \\
 & . & & & & & & & \\
 & . & & & & & & & \\
0 & 0 & 0 & 0 & \ldots & 0 & \delta W_n & \Gamma_n & \\
0 & 0 & 0 & 0 & \ldots & 0 & -\Gamma_n & \delta W_n &
\end{bmatrix}$$

VALEN equations (continued), & post processing to obtain Bnormal from DCON modes on the plasma surface

single mode VALEN

solve for surface current potential:

$$g(\vec{r}) \quad or \quad g(\theta,\phi)$$

$$\vec{J} = \nabla \times g\,\hat{n}\,\delta\left(\vec{r} - \vec{r}_{surface}\right) \quad and$$

$$\vec{B}^{\substack{unstable\\DCON}}_{normal}(\vec{r}) = \frac{\mu_0}{4\pi}\nabla \times \oint_{\substack{\text{plasma}\\\text{surface}}} \frac{\vec{J}(\vec{r}\,')}{|\vec{r} - \vec{r}\,'|}$$

calculate one plasma L using $g(\vec{r})$

multi-mode VALEN

use gram schmidt vectors for plasma surface current potential

$$\vec{J}_i(\vec{r}) = \nabla \times f_i(\vec{r})\;\hat{n}_{surface}\;\delta\left(\vec{r} - \vec{r}_{surface}\right)$$

calculate all plasma L_{ik} using $f_i(\vec{r})$ & $f_j(\vec{r})$

multi-mode post processing

$$\{I_p\} = \left[L_{pp}^{\substack{plasma\\modes}}\right]^{-1}\left([P]-[1]\right)\left\{\left[L_{pw}^{plasma-wall}\right]\{I_w\} + \left[L_{pc}^{plasma-coil}\right]\{I_c\} + \left[L_{pp}^{\substack{plasma\\dissipative}}\right]\{Id\}\right\}$$

Bn (gram schmidt functions) →

$$\left\{\vec{B}_{plasma}(t)\cdot\hat{n}_{surface}(\theta,\phi)\right\}_i = \left\{\left[L_{pp}\right]_{ij}\left(\left(f_j(\theta,\phi)\right)I_j^{plasma}(t) + \left(f_j(\theta,\phi)\right)I_j^{\substack{plasma\\dissipative}}(t)\right)\right\}/A$$

$$\text{recall:} \quad f_j(\theta,\phi) = [G]_{jk}B^n_{\text{DCON\_mode}}(\theta,\phi)$$

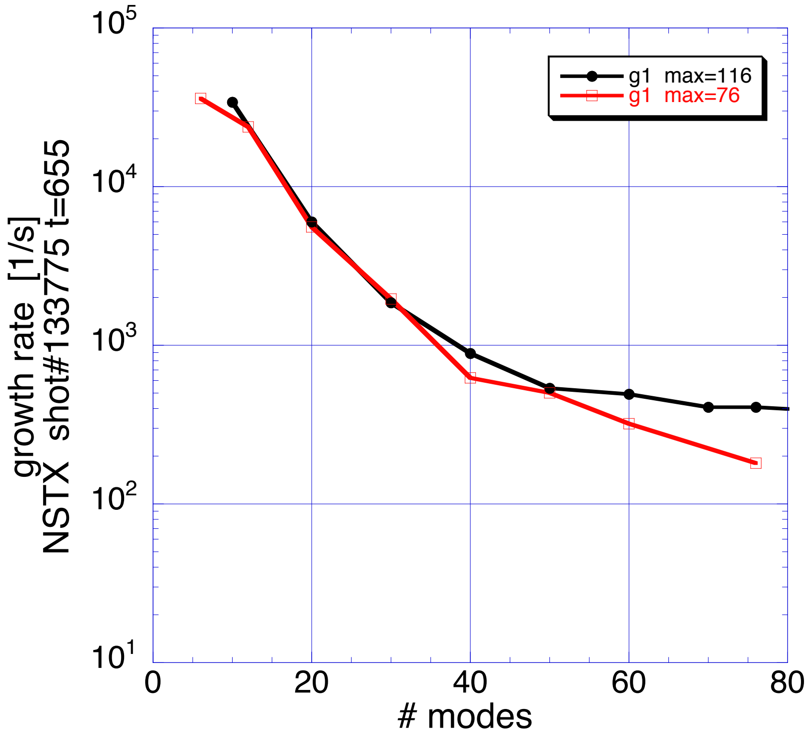
Bn (DCON eigenmodes) →

$$\left\{B^{normal}(\theta,\phi,t)\right\} = \left\{\left[L_{pp}\right]_{ij}\left(\left([G]_{jk}B^n_{\text{DCON\_mode}}(\theta,\phi)\right)I_j^{plasma}(t) + \left([G]_{jk}B^n_{\text{DCON\_mode}}(\theta,\phi)\right)I_j^{\substack{plasma\\dissipative}}(t)\right)\right\}/A$$

Convergence Studies: the number of ideal modes needed in multi mode VALEN

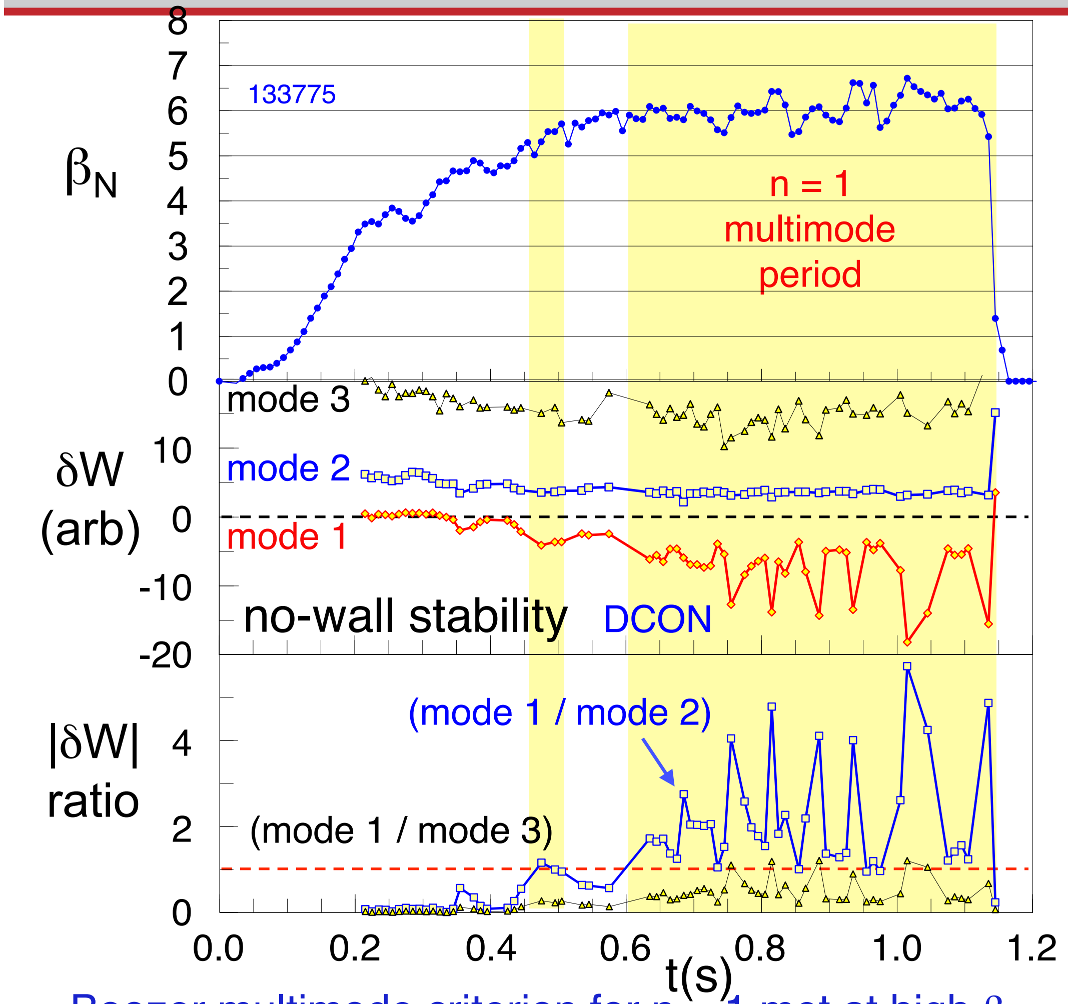
- Multi mode VALEN requires many modes to obtain a converged result
- Fewer modes required for larger aspect ratio plasmas
- NSTX problems also require extra modes from DCON analysis
- figure shows good convergence when we use >= 70 modes from DCON (out of 116), poor convergence when DCON provides fewer modes, i.e., 76 modes
- Not a large impact on VALEN

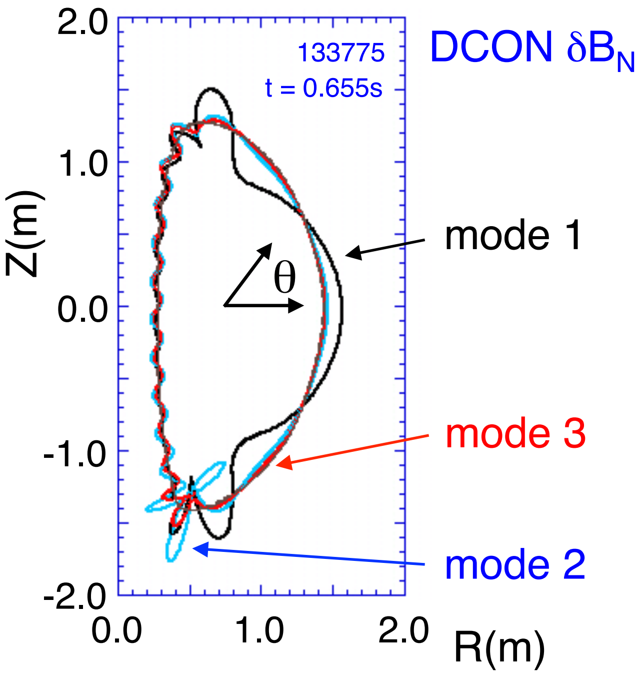
Growth rate vs. number of DCON modes used

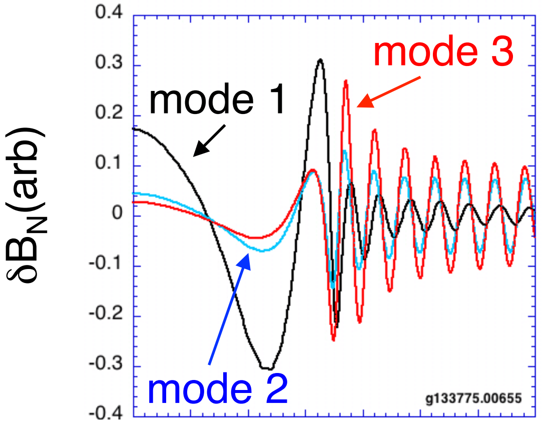


mmVALEN.studyconvergence09

Multimode response theoretically is expected to be significant at high β_N



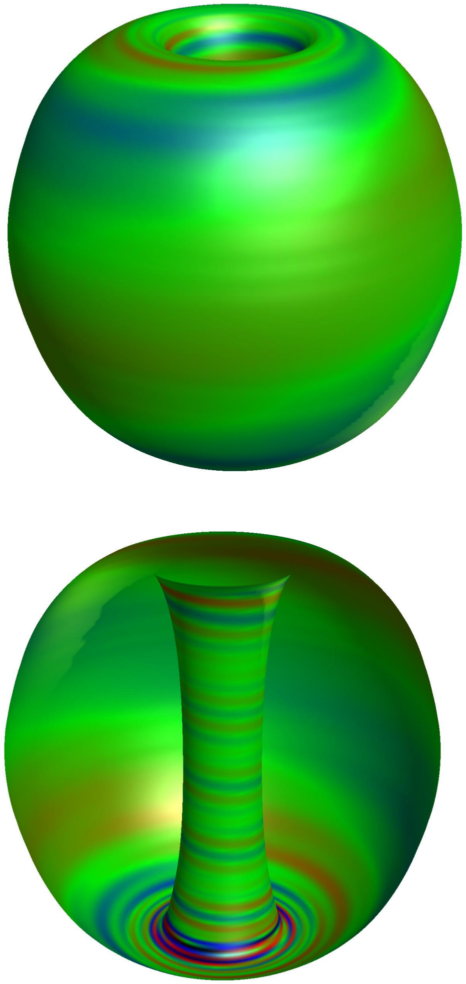




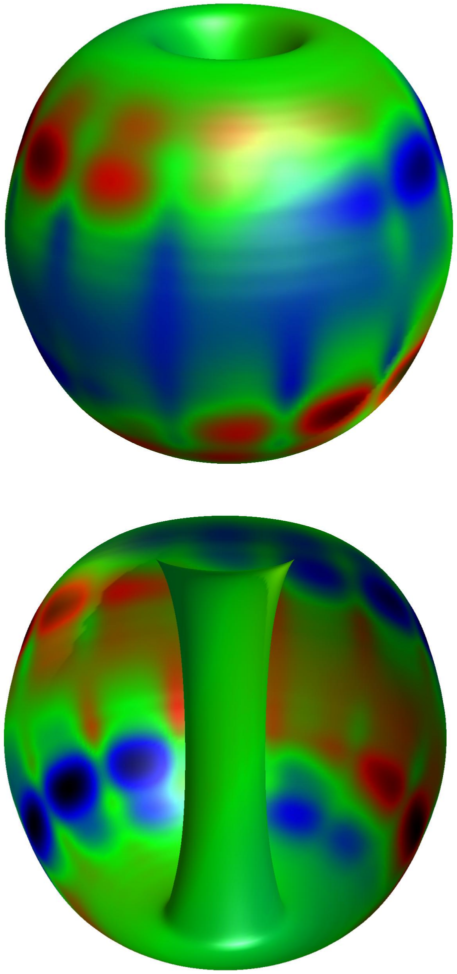
- Boozer multimode criterion for n = 1 met at high β_N
  - |δW| smallest for 2nd n = 1 eigenfunction (PoP 10 (2003) 1458.)
  - Ratio of |δW| for 3rd vs. 1st least stable mode sometimes also > 1

Passive growth @ $\beta_N = 5.54$ multi mode VALEN results for NSTX shot 133775.00495

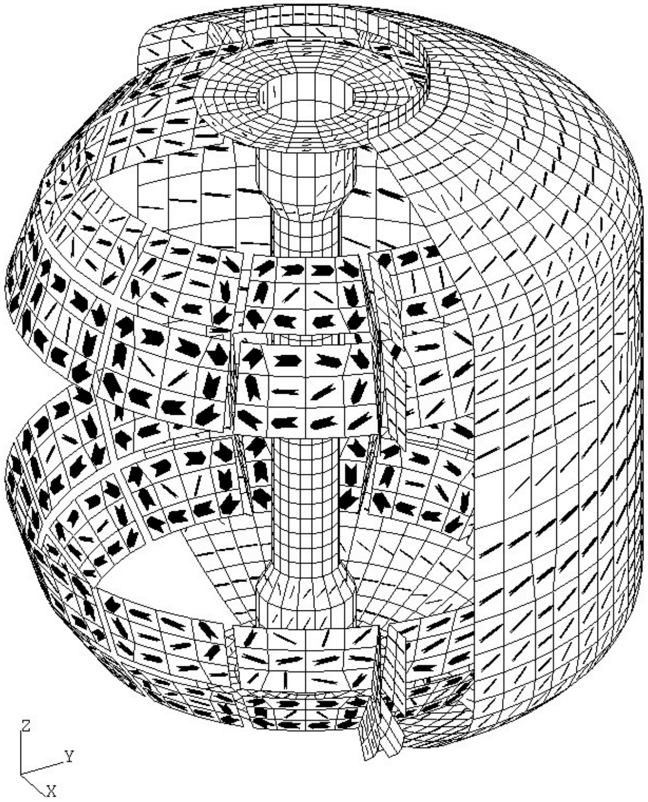
total B_n on plasma surface



B_n on plasma surface from only wall currents

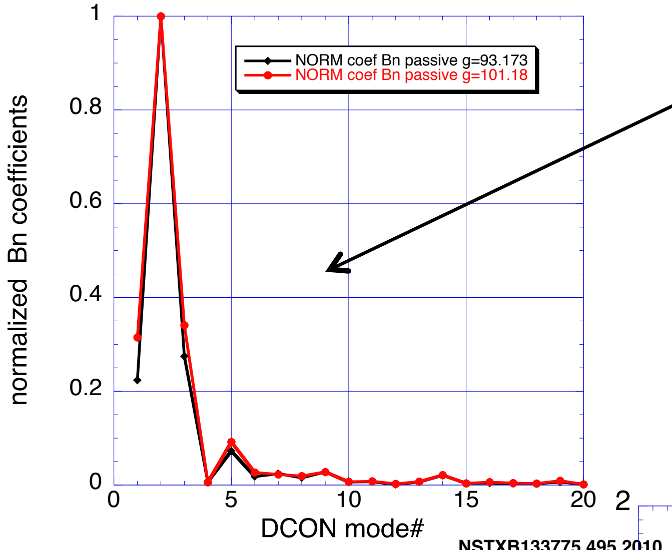


cut away view of
wall currents
$\beta_N = 5.54$
growth rates =93, 101 [1/s]



color contours: dark red ~big negative, green ~ zero, dark blue ~ big positive

Multi-mode spectra passive RWM in NSTX @ β_N=5.54



relative weight of DCON modes in passive growing RWM

NSTX shot #133775
t = 0.495 [s]

DCON mode weighted sums shown below

Illustration of B_normal signature for RWM mode

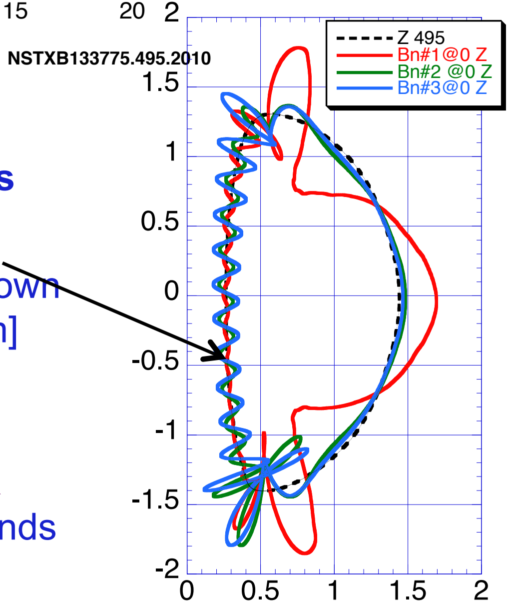
normal distance from plasma surface corresponds to B_normal

Illustration of DCON modes equal weights

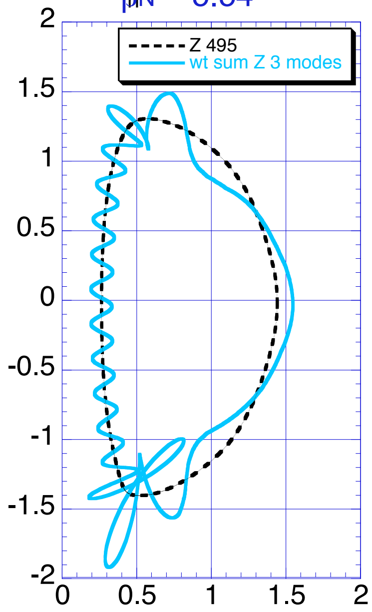
#1 thru #3,

each DCON mode shown with max B_n = 0.5[m]

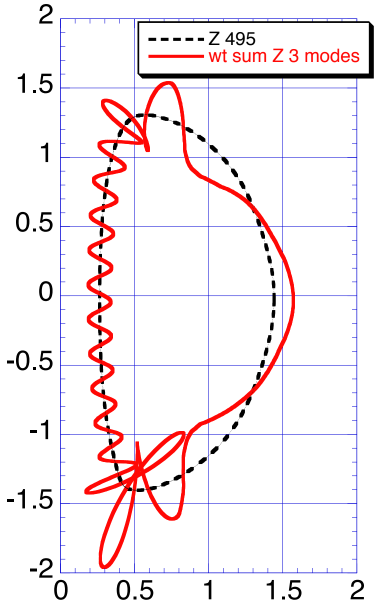
normal distance from plasma surface (black dashed line) corresponds to B_normal



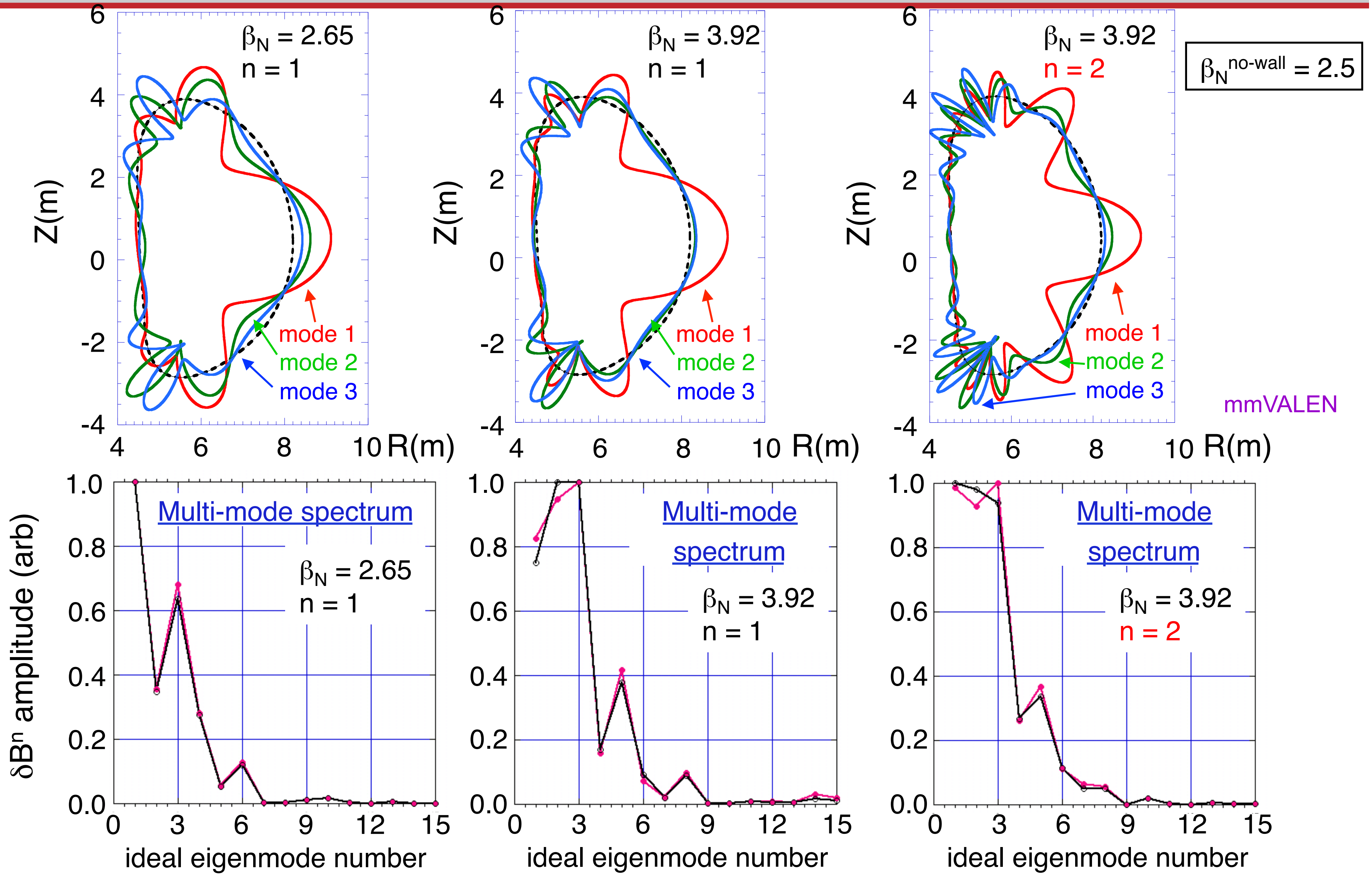
γ = 93.17 [1/s]
β_N = 5.54



γ = 101.18 [1/s]
β_N = 5.54

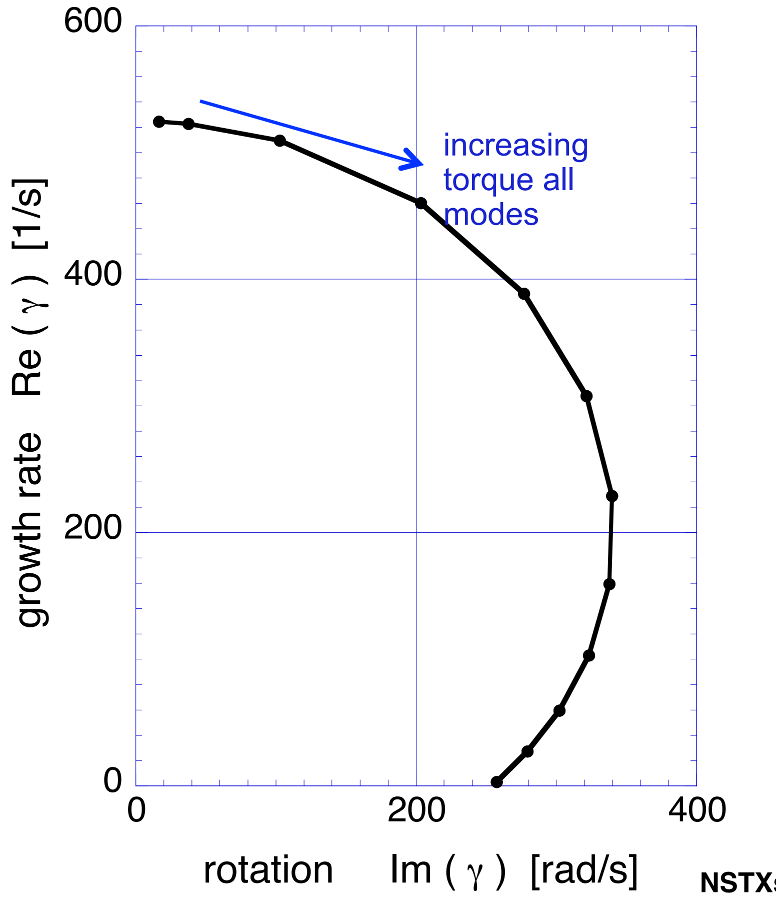


ITER Advanced Scenario IV: multi-mode RWM spectra computation shows significant ideal eigenfunction amplitude for several components



VALEN predicts RWM growth rate as torque is applied to all modes, mode rotation and growth rate varies, mode may be stabilized by rotation alone

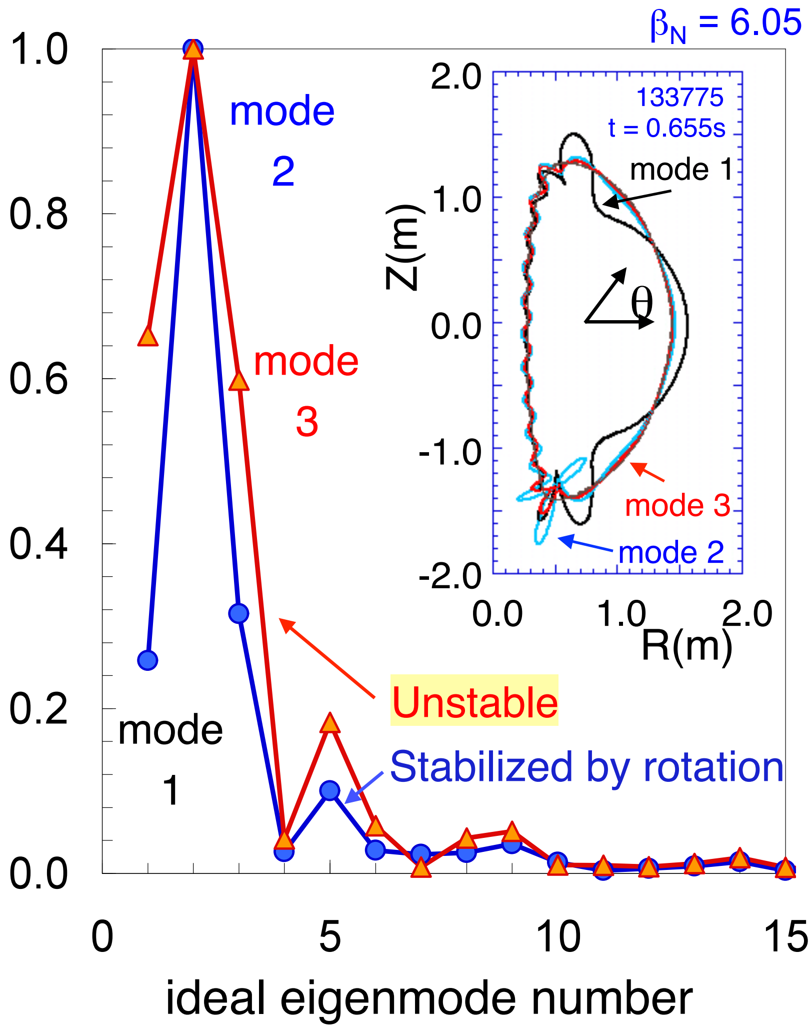
NSTX shot 133775
time = 0.655 [s]



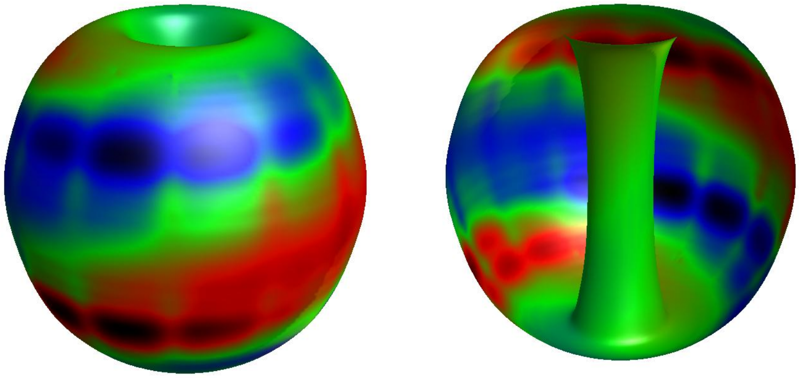
- NSTX RWM stabilized by ω_ϕ
- VALEN calculation of rotation needed for stabilization ~ 41 [Hz] is close to experimental value of ~ 30 [Hz]
- β_N = 6.05

VALEN multi-mode RWM computation shows 2^{nd} eigenmode component usually has dominant amplitude in NSTX at high β_N

δB^n RWM multi-mode composition



δB^n from wall, multi-mode response



- NSTX unstable RWM
  - Computed growth time consistent with experiment
  - 2^{nd} eigenmode ("divertor") has larger amplitude than ballooning eigenmode
- NSTX RWM stabilized by ω_ϕ
  - Ballooning eigenmode amplitude decreases relative to "divertor" mode

RWM feedback control: Mathematics of VALEN feedback for NSTX

- Analyze upper Bp sensor signals in NSTX for selected ‘n’ to identify RWM
- Compensate for control coil flux in sensors
- Scan phase angle between the RWM signal and the pattern of voltage applied to the control coils
- Choose gain for feedback [v/flux]
- Run multi-mode VALEN eigenvalue calculation and predict growth rate
- Feedback may cause the plasma modes to rotate

use classic least squares fitting to find $S \,\&\, C$:

$$S*\sin(n*\phi_1^{sensor}) + C*\cos(n*\phi_1^{sensor}) = \Phi_1^{sensor}$$

$$S*\sin(n*\phi_2^{sensor}) + C*\cos(n*\phi_2^{sensor}) = \Phi_2^{sensor}$$

$$\vdots$$

$$S*\sin(n*\phi_N^{sensor}) + C*\cos(n*\phi_N^{sensor}) = \Phi_N^{sensor}$$

or $[A]_{Nx2}\begin{Bmatrix} S \\ C \end{Bmatrix}_{2x1} = \left\{\Phi^{sensor}\right\}_{Nx1}$

where: sensors located at angles ϕ_i

and sensors read magnetic flux Φ_i

solution is:

$$\begin{Bmatrix} S \\ C \end{Bmatrix} = \left([A]^t[A]\right)_{2x2}^{-1}[A]_{2xN}\left\{\Phi^{sensor}\right\}_{Nx1}$$

voltages applied to coil are:

$$\begin{Bmatrix} V_1^{coil} \\ \vdots \\ V_C^{coil} \end{Bmatrix} = G_p \begin{bmatrix} \sin(\phi_1^{coil}+\delta) & \cos(\phi_1^{coil}+\delta) \\ \vdots & \vdots \\ \sin(\phi_C^{coil}+\delta) & \cos(\phi_C^{coil}+\delta) \end{bmatrix} \left([A]^t[A]\right)_{2x2}^{-1}[A]_{2xN}\left\{\Phi^{sensor}\right\}_{Nx1}$$

where: coils located at angles ϕ_i^{coil}

and we use phase δ

to provide compensation for coil to sensor coupling

replace $\left\{\Phi^{sensor}\right\}_{Nx1}$ by $\left\{\Phi^{sensor}\right\}_{Nx1} - \left[M_{sensor,coil}\right]_{NxC}\left\{I^{coil}\right\}_{Cx1}$

RWM feedback control:
VALEN results, scan in feedback phase for $\beta_N = 4.65$

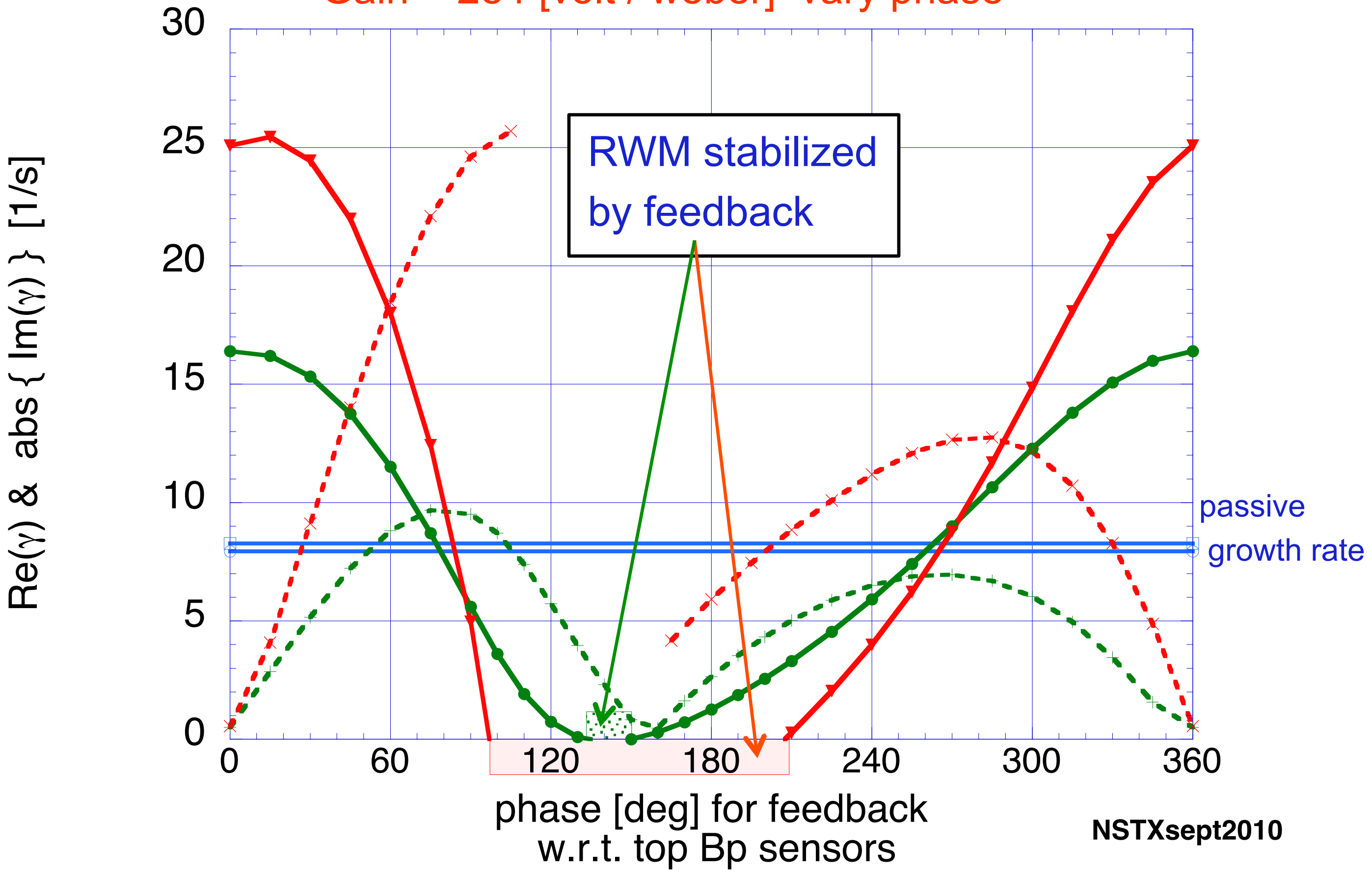
- Notes:
- solid lines show growth rate
- dashed lines show rotation
- shaded regions indicate RWM is stabilized by feedback
- horizontal lines show passive growth rate without feedback

NSTX shot #133775
t = 0.345 [s]

VALEN eigenvalue calcuations with feedback

Gain = 1e4 [volt / weber] vary phase

Gain = 2e4 [volt / weber] vary phase



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RWM feedback control:
VALEN results, scan in feedback phase for β_N = 5.54

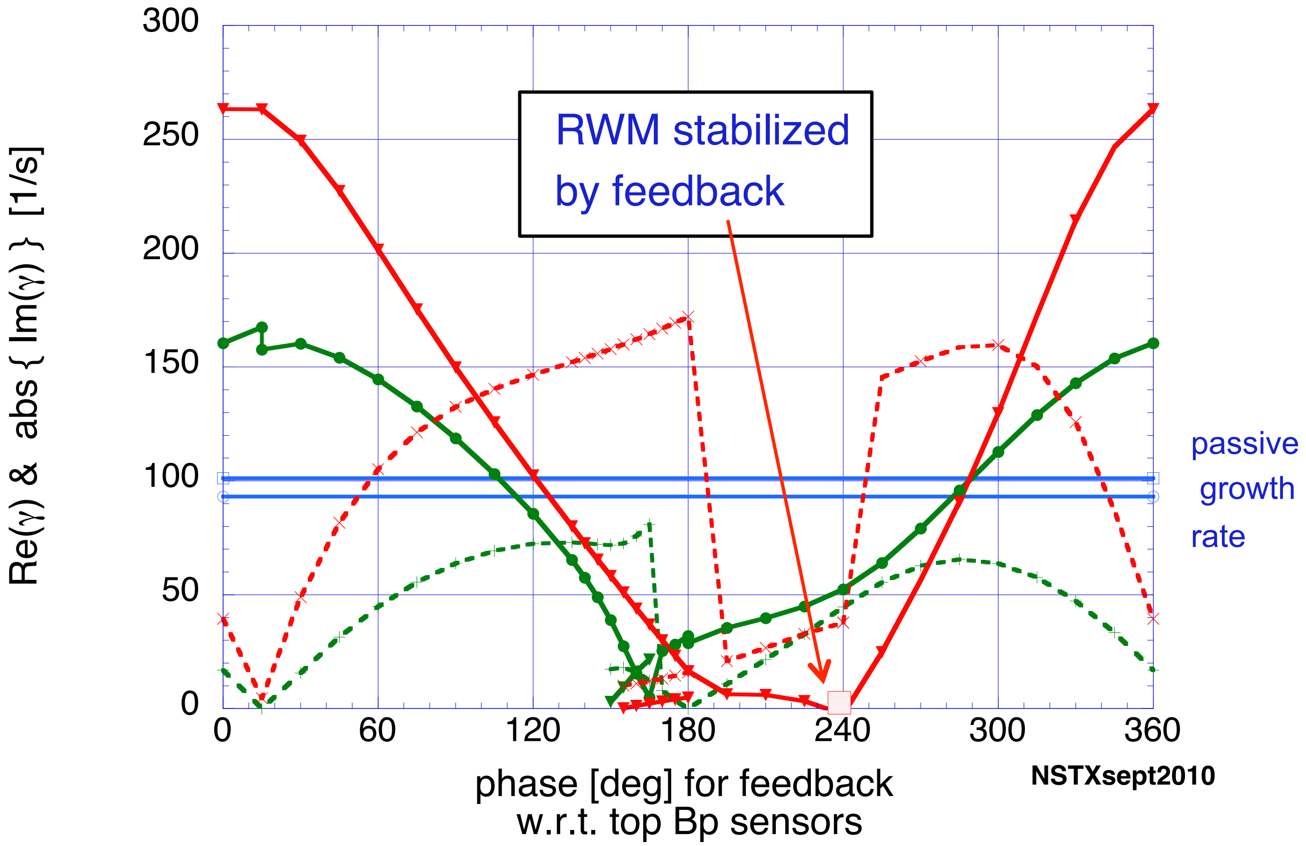
- Notes:
- solid lines show growth rate
- dashed lines show rotation
- horizontal lines show passive growth rate without feedback
- only a small region at G=1e5 near phase = 240 is stabilized when βn=6.079

NSTX shot #133775
t = 0.495 [s]

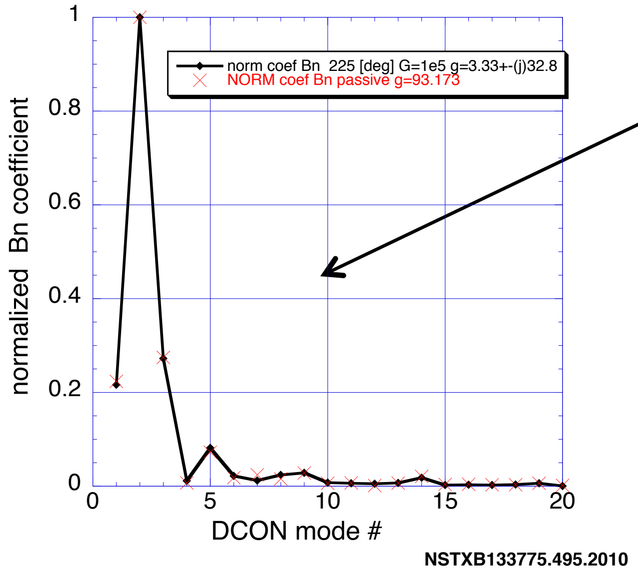
VALEN eigenvalue calcuations with feedback

Gain = 4e4 [volt / weber] vary phase

Gain = 1e5 [volt / weber] vary phase



Multi-mode spectra with RWM feedback in NSTX @ β_N = 5.54



- relative weight of DCON modes in RWM with feedback G=1e5, phase=225 [deg]

x relative weight of DCON modes for passive RWM growth

DCON mode weighted sums shown below

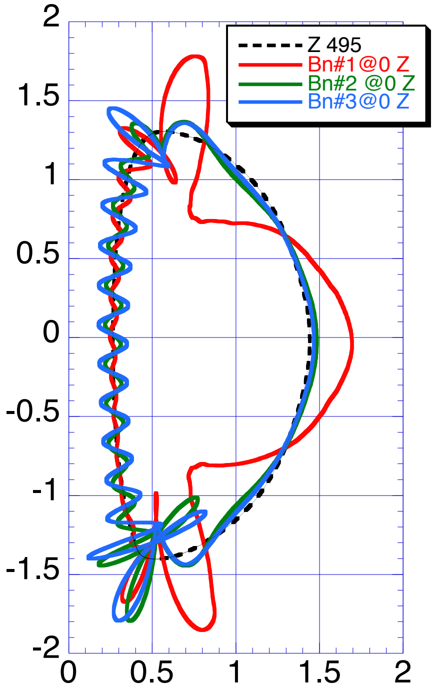
Illustration of B_normal signature for RWM mode with feedback

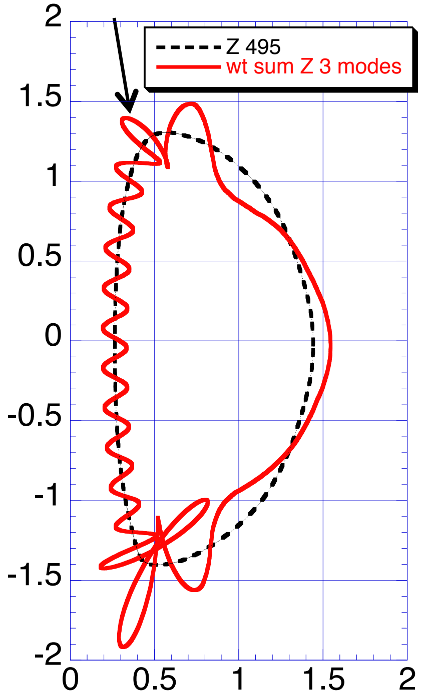
normal distance from plasma surface corresponds to B_normal

Illustration of DCON modes #1 thru #3,

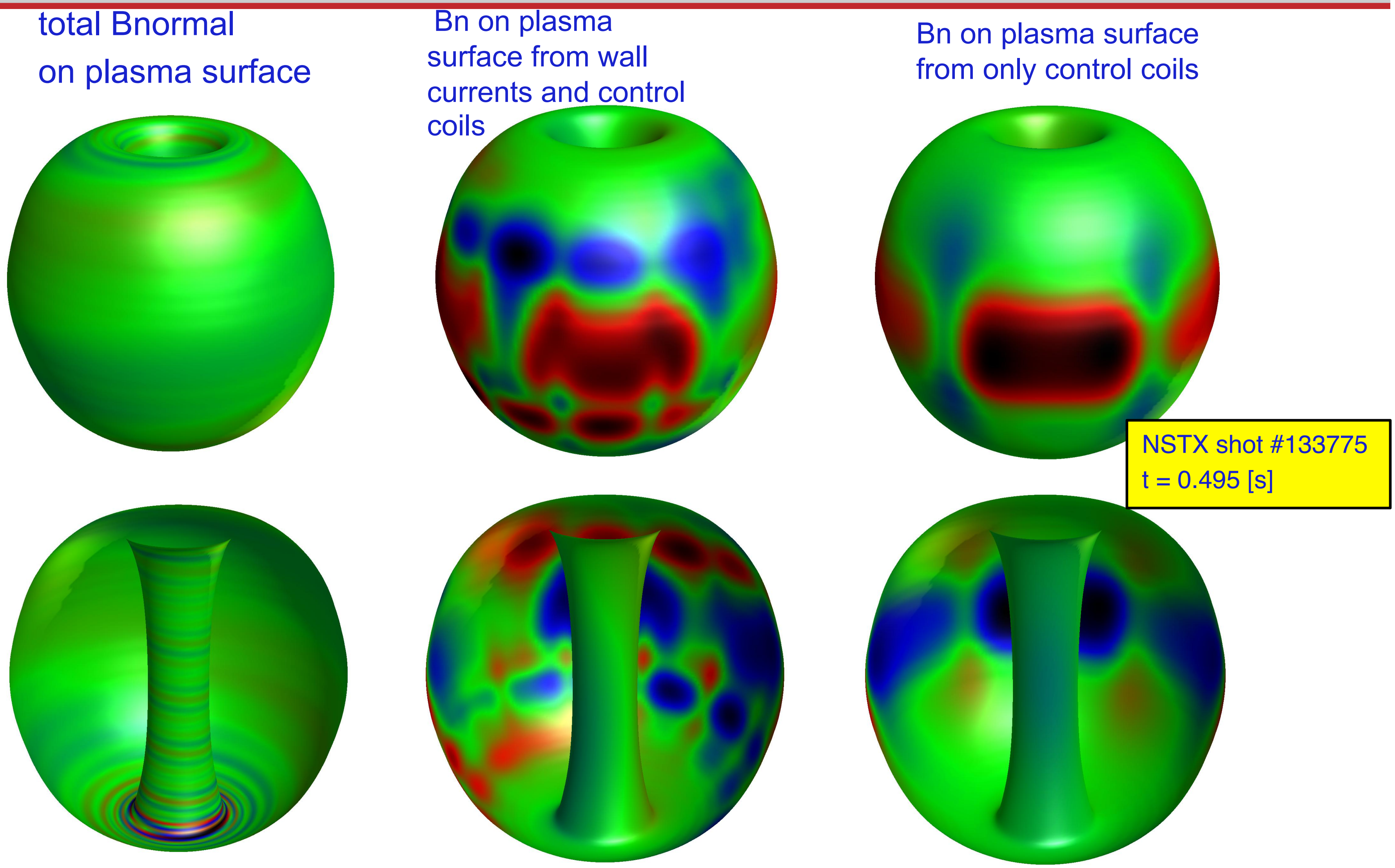
each DCON mode shown with max B_n = 0.5 [m] (equal weights)

normal distance from plasma surface (dashed black line) corresponds to B_normal





Bnormal on NSTX plasma surface with RWM feedback gain =1e5 [volt / weber] and phase = 225 [deg], β_N = 5.54



total Bnormal
on plasma surface

Bn on plasma surface from wall currents and control coils

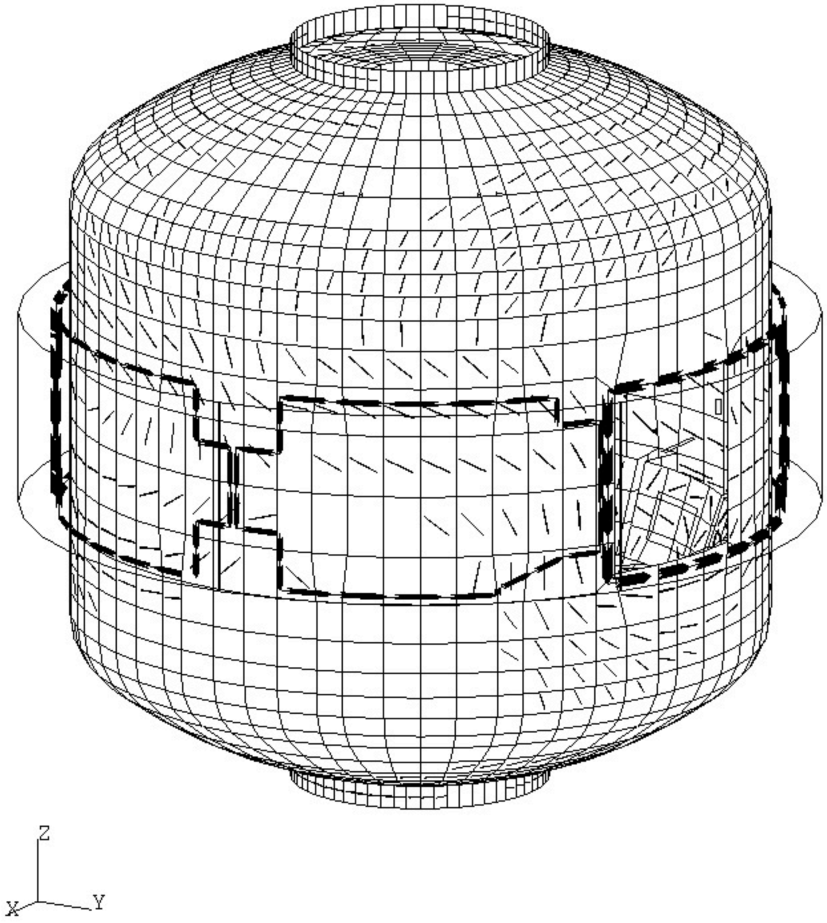
Bn on plasma surface from only control coils

NSTX shot #133775
t = 0.495 [s]

- color contours: dark red ~big negative, green ~ zero, dark blue ~ big positive

Coil and wall currents in NSTX with RWM feedback gain =1e5 [volt / weber] and phase = 225 [deg]

NSTX shot #133775
t = 0.495 [s]



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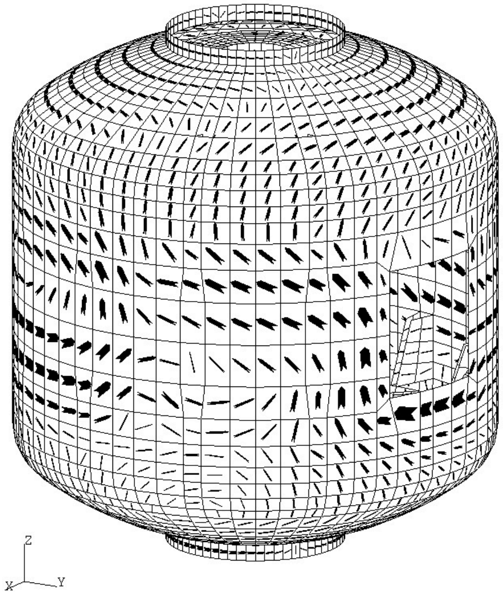

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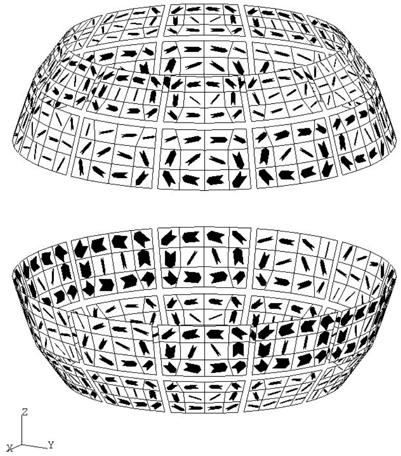


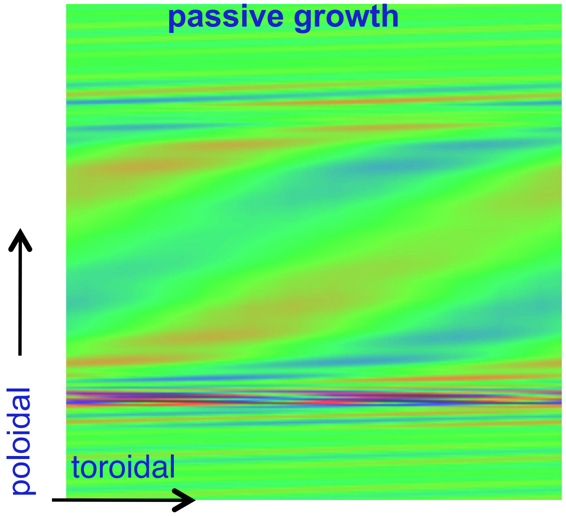
With feedback the RWM has growth rate 3.33 [1/s] and is rotating at 5.2 [Hz],

without feedback growth rate ~ 100 [1/s]

Comparison multi mode RWM Bnormal distribution in NSTX without feedback (passive) and with RWM feedback

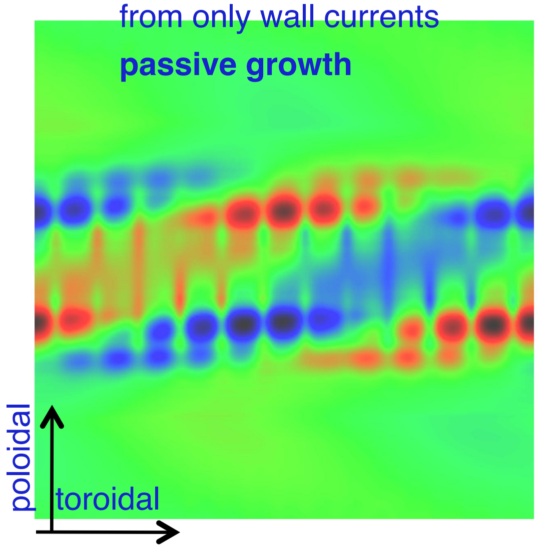
total Bn on plasma surface

passive growth



Bn on plasma surface
from only wall currents

passive growth

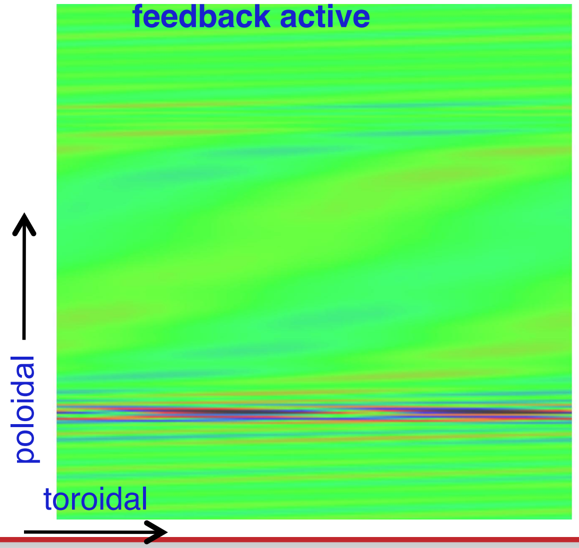


differences in Bn on the plasma surface with and without feedback are subtle

NSTX shot #133775
t = 0.495 [s]

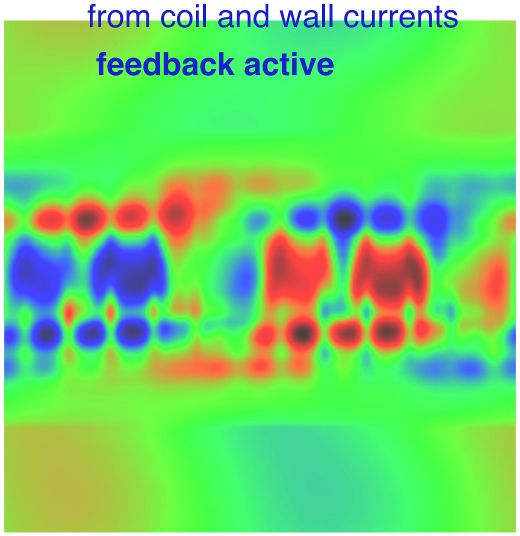
total Bn on plasma surface

feedback active



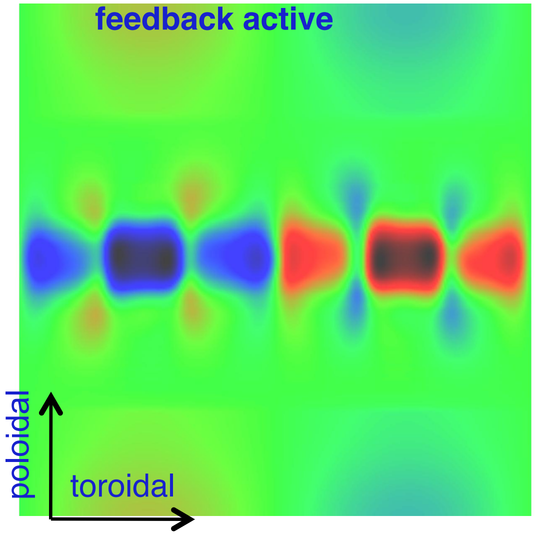
Bn on plasma surface
from coil and wall currents

feedback active



Bn on plasma surface
from only coil currents

feedback active



Summary - Conclusions

- Control of RWM is important, allows sustained access to high beta regimes

- Multi mode VALEN predictions, both passive growth and performance with applied feedback consistent with experimental observations in NSTX

- Multi mode VALEN calculations indicate lowest order modes dominate the RWM response, both passive and with active feedback
    - The second (stable) DCON mode has the greatest contribution for simple passive growth, passive growth with substantial mode rotation, and when feedback is applied

- Future work planned includes studies of error field amplification and application of advanced feedback logic both in the time domain and frequency domain.