

Gyrokinetic analysis of thermal transport scaling in NSTX and MAST

Walter Guttenfelder¹, S.M. Kaye¹, J. Candy²,
R.E. Bell¹, B.P. LeBlanc¹, G.W. Hammett¹,
D.R. Mikkelsen¹, H. Yuh³, A.R. Field⁴,
M. Valovič⁴, W.M. Nevins⁵, E. Wang⁵

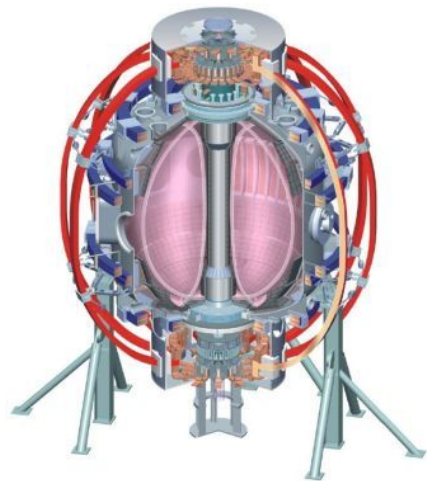
¹PPPL, USA

²General Atomics, USA

³Nova Photonics Inc., USA

⁴Culham Centre for Fusion Energy, UK

⁵LLNL, USA



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Overview

- Favourable $\tau_{E,th} \sim v_*^{-(0.7-0.9)}$ dependence in STs
 - Physical origin unclear, could influence design of next-generation device at low v_*
 - Microtearing modes found to be unstable in experimental v_* scans
 - Scaling of linear growth rates $\gamma_{lin} \sim v_e$ – potential candidate to explain experimental confinement trend
 - Linear thresholds exist in $v_e, \beta_e, a/L_{Te}$
- ⇒ **First non-linear microtearing simulations in NSTX**
- Require relatively fine radial resolution ($\Delta x \approx 0.2\rho_s, nx=400$) to capture physics
 - Significant transport predicted without $E \times B$ shear
 - Dominated by electromagnetic contribution ($\delta A_{||}$) → stochastic field lines
- ETG also a possible transport mechanism
 - Unstable in some regions, can drive significant transport
 - Transport from nonlinear ETG simulations decreases with increasing v_e – opposite to experimental trend

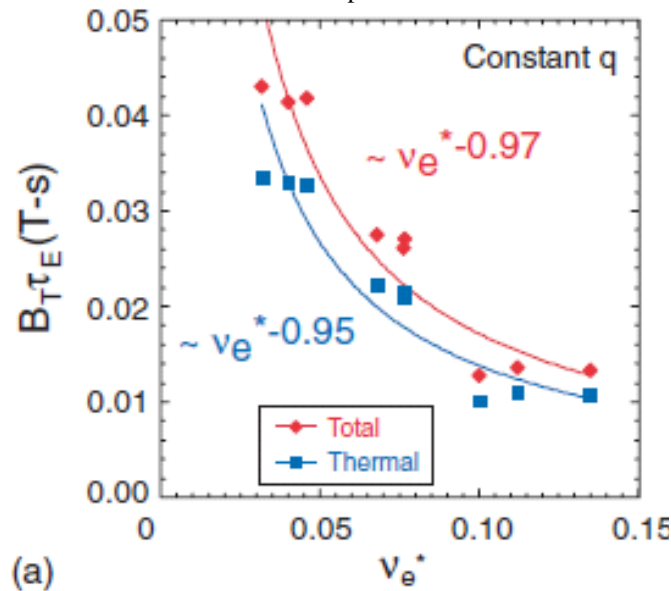
Experimental motivation - strong collisionality scaling in STs

- Next generation ST (high heat flux, CTF, ...) likely to be at lower v_*
- Present ST confinement scaling with v_* favorable \Rightarrow will it hold at lower v_* ?

- Large $(I_p + B_T)$ scaling related to strong v_* scaling (assuming gyroBohm ρ_*^{-3})
 \Rightarrow What physical mechanisms are important?

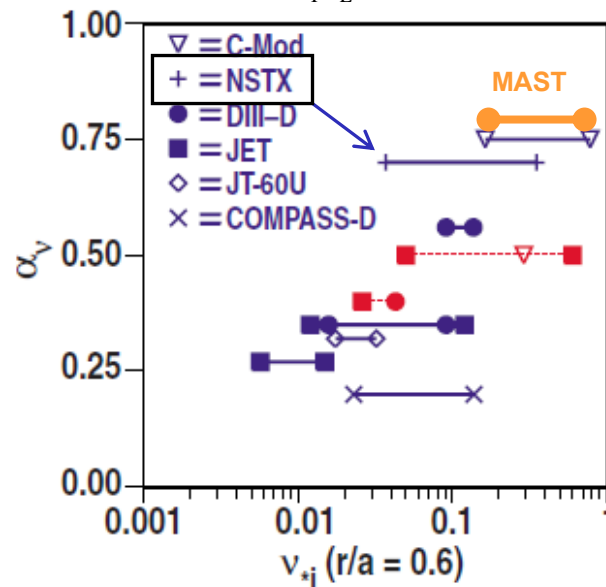
Kaye et al. (2007)

$$\tau_E^{\text{th}} \sim I_p^{0.6} B_T^{1.0} n_e^{0.35} P_L^{-0.6}$$



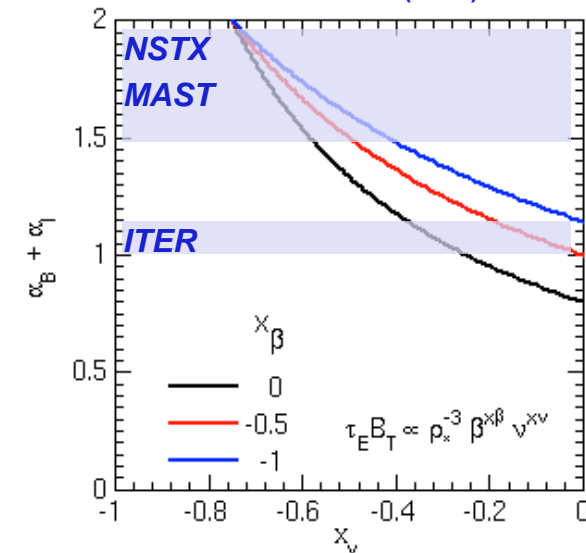
Petty (2008)

$$B_T \tau_E^{\text{th}} \sim v_*^{-\alpha_v}$$



$$\alpha_B + \alpha_I = \frac{(2 - 2x_\beta)}{(5/2 - x_\beta + 2x_v)}$$

Valovič et al. (2010)



Dimensionless v^* scans – basis of microstability analysis

$$q \sim I_p / B \quad I_p \sim B$$

$$\rho_* \sim T^{1/2} / B \quad \Rightarrow \quad T \sim B^2$$

$$\beta \sim nT / B^2 \quad n \sim B^0$$

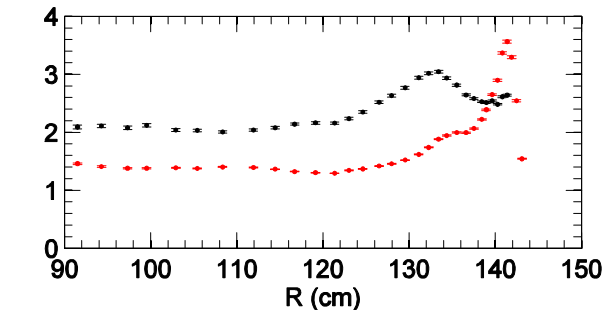
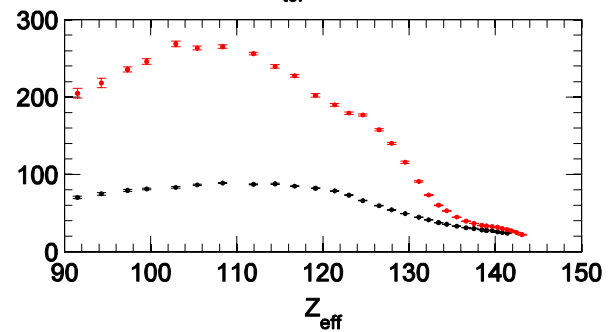
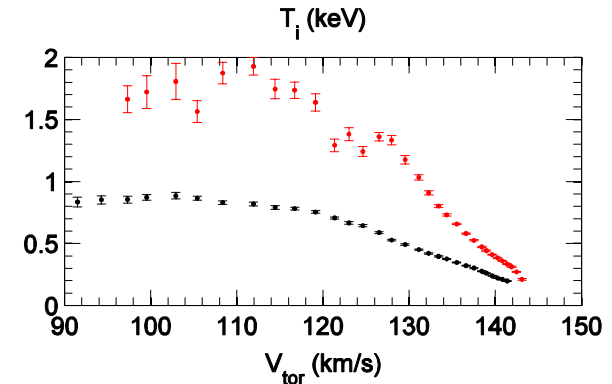
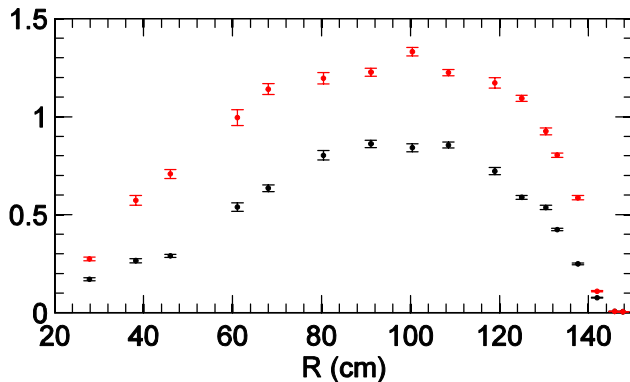
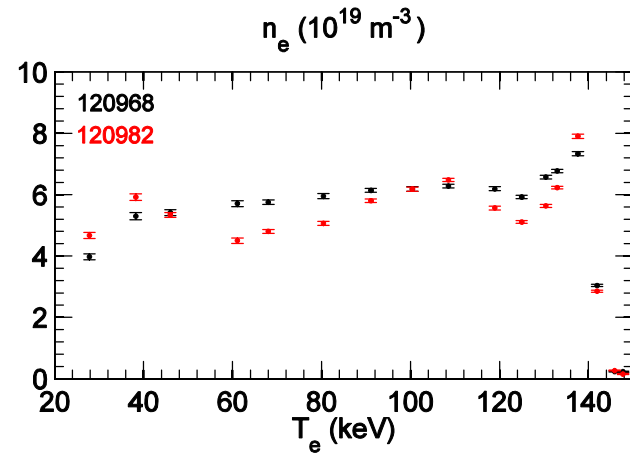
$$v_* \sim B^{-4}$$

NSTX 120968 $I_p / B_T / P_{NBI}$ - 0.7 MA / 0.35 T / 4 MW

NSTX 120982 $I_p / B_T / P_{NBI}$ - 1.1 MA / 0.55 T / 4 MW

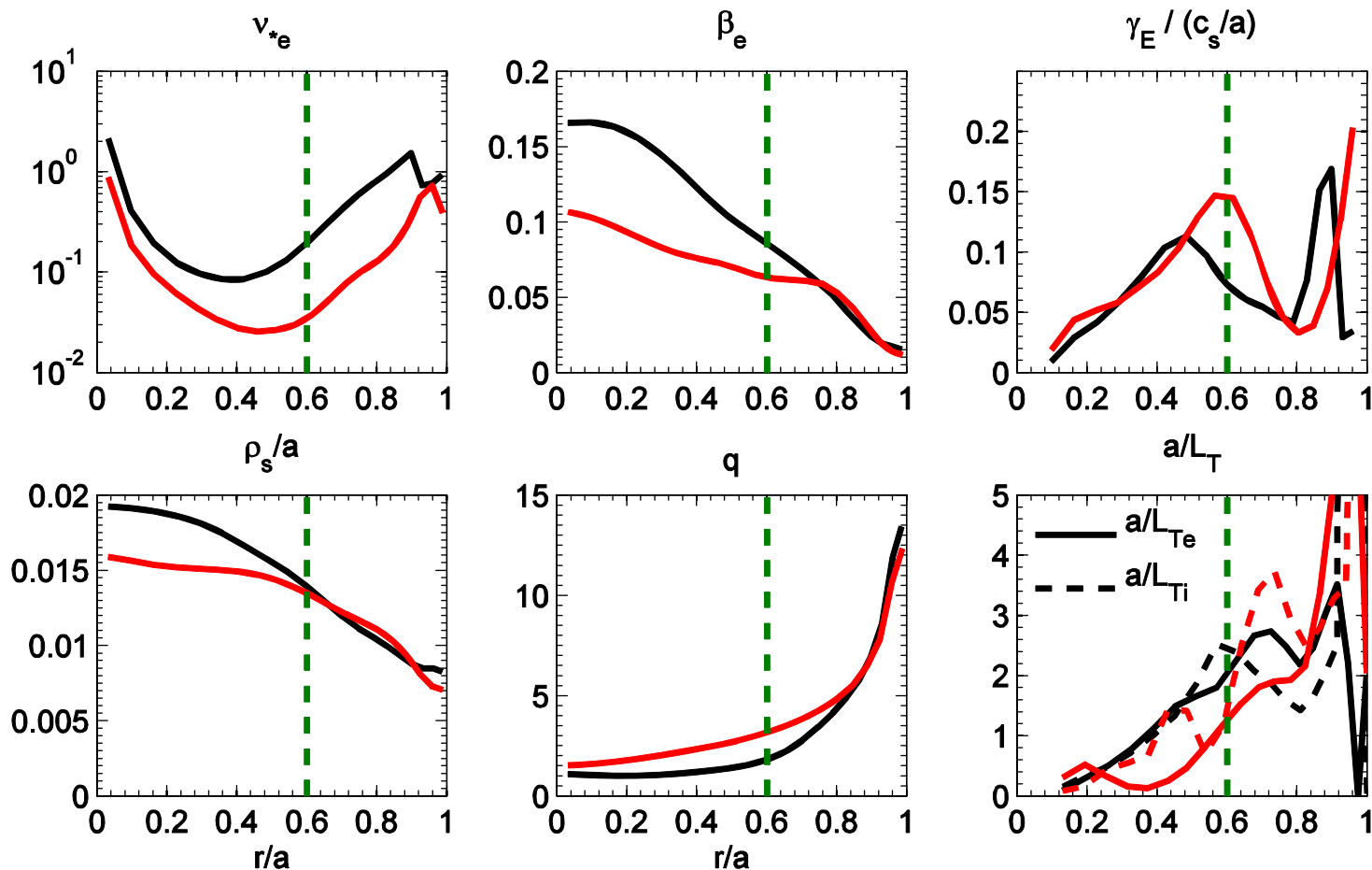
Strong rotation and rotation shear

Ion transport well described by neoclassical (NCLASS)



Experimental profiles of dimensionless parameters

Factor ~ 5 variation in v_* , additional (non-ideal) variation in other dimensionless parameters



The following simulations are based on high v_* NSTX discharge 120968 (mostly $r/a=0.6$)
 Calculations were also performed for MAST discharges with similar results (not shown)

Scaling of linear microtearing instability

GYRO* used for gyrokinetic simulations

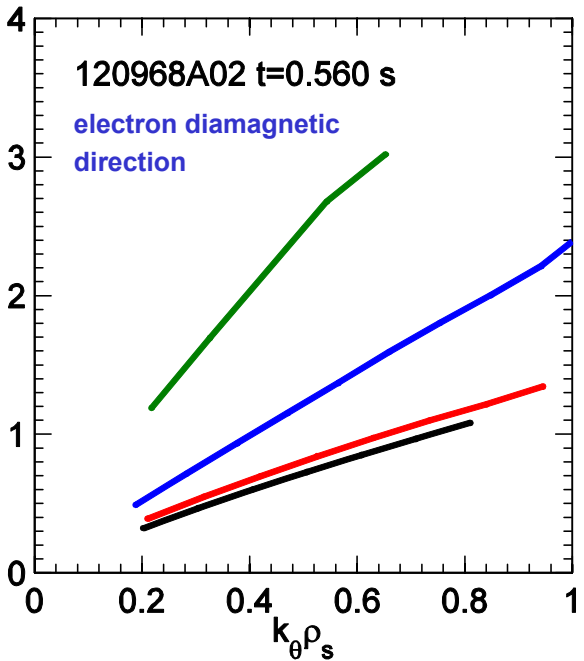
- Eulerian solver of gyrokinetic-Maxwell equations
- Can use experimental profile variations, $T(r)$, $n(r)$, $q(r)$, etc... (likely important in ST, $\rho_s/a \sim 1/100-1/50$)
- Fully collisional & electromagnetic (both important in NBI heated ST)
- Freedom to include toroidal flow and flow shear (important in NBI heated ST)
- Substantial user-friendly documentation*
- Can be run in the local flux-tube limit ($\rho/a \rightarrow 0$, flat profiles, similar to FULL, GS2, GENE, GKW, etc...) for
 - Code benchmarking
 - Comparing “local” limit (flat profiles, $\rho_* \rightarrow 0$) with “global” (experimental profiles)
- *Following linear calculations performed in the local, flux-tube limit*

*J. Candy & R.E. Waltz, Phys. Rev. Lett **91**, 045001 (2003); J. Comp. Physics **186**, 545 (2003); <https://fusion.gat.com/theory/Gyro>

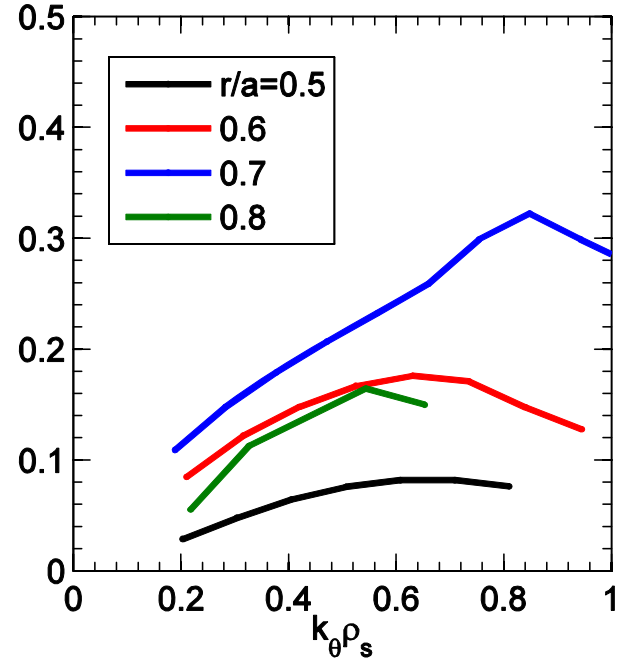
Microtearing modes found to be unstable in high v_* discharge

- Microtearing dominates $k_\theta \rho_s < 1$ in outer half-radius ($r/a=0.5-0.8$)
 - Resonant tearing parity in A_\parallel ($\delta B_r = -ik_\theta A_\parallel$)
 - Extended potential eigenfunctions in ballooning space
 - Real frequencies in electron diamagnetic direction
- ETG becomes unstable at outermost locations ($r/a=0.7-0.8$, not shown)

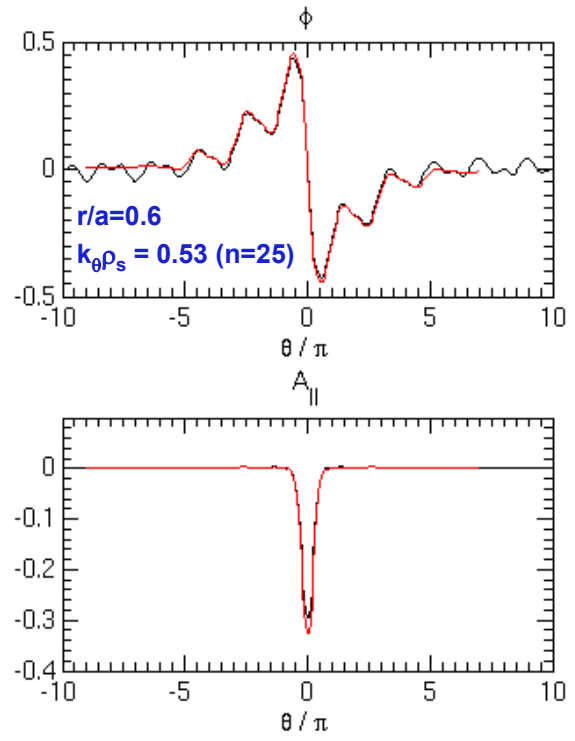
real frequencies
 ω_r (c/a)



growth rates
 γ (c/a)



Eigenfunctions in "ballooning" space

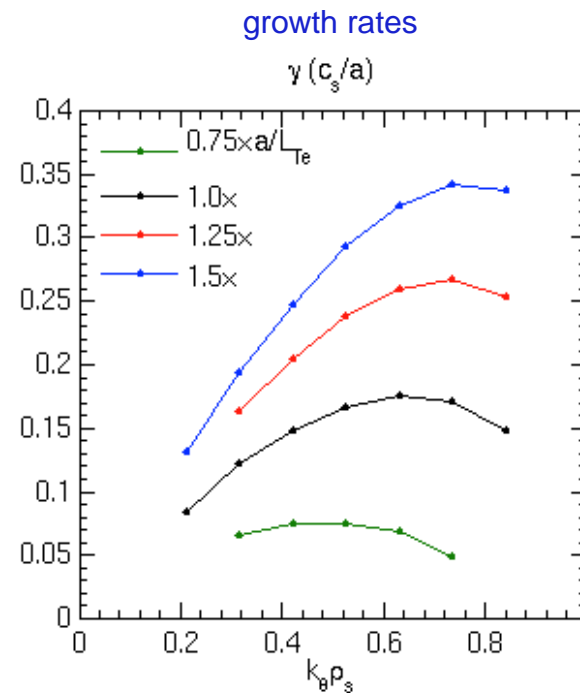
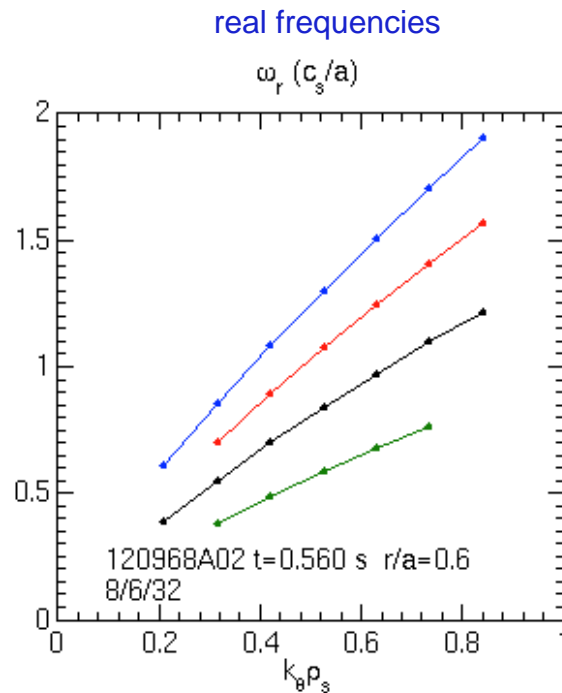


Linear microtearing instability

- High- m tearing mode around a rational $q(r_0)=m/n$ surface ($k_{\parallel}(r_0)=0$)
- Driven by ∇T_e with parallel thermal force, *requires collisionality*
(Classical tearing mode stable for large m , $\Delta' \approx -2m/r < 0$)
- Imagine helically resonant ($q=m/n$) δB_r perturbation $\delta B_r \sim \cos(m\theta - n\phi)$
- δB_r leads to radially perturbed field line, finite island width $w = 4 \left(\frac{\delta B_r}{B} \frac{Rq^2}{mq'} \right)^{1/2}$
- ∇T_e projected onto field line gives parallel gradient $\nabla_{\parallel} T_{e0} = \frac{\vec{B} \cdot \nabla T_{e0}}{B} = \frac{\delta B_r}{B} \nabla T_{e0}$
- Parallel thermal force ($R_{T\parallel} \approx -n_e \nabla_{\parallel} T_e$) drives parallel electron current that reinforces $\delta B_r \rightarrow$ instability
- Requires ∇T_e , finite β , positive magnetic shear (dq/dr) & energy dependent collision operator

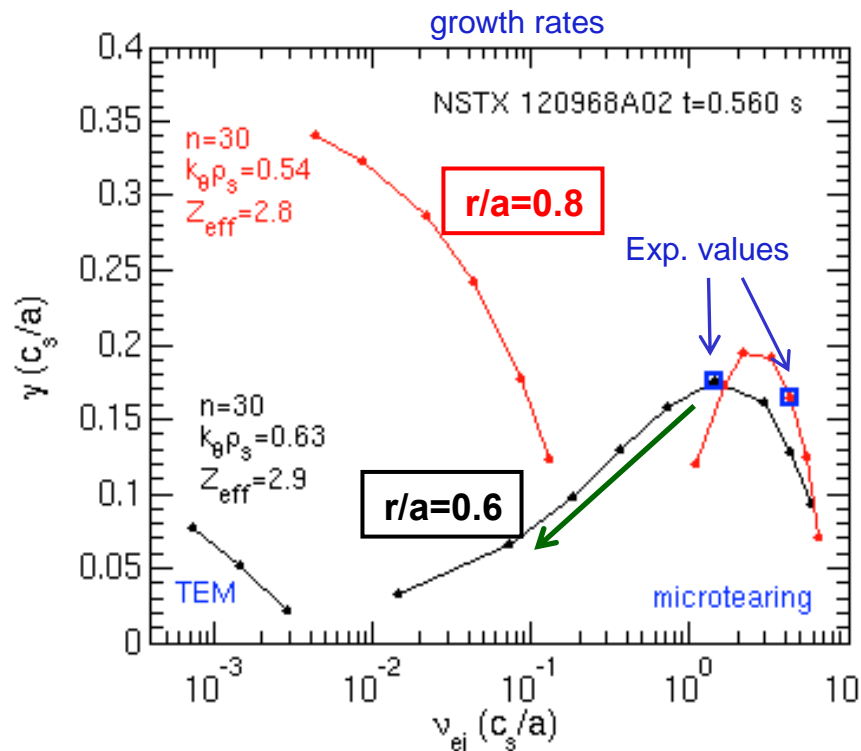
Microtearing instability exhibits a threshold in temperature gradient

- Growth rates increase with a/L_{Te} , **apparent threshold $(a/L_{Te})_{\text{threshold}} \approx 1.3-1.5$**
- $(a/L_{Te})_{\text{threshold}} \sim 0.5$ in Wong et al. (2008) (NSTX discharge 116313 $r/a = 0.5$)
- ω_r proportional with a/L_{Te} (and a/L_n) $\omega \approx \omega_{*e} = (k_{\theta}\rho_s) \cdot (a/L_n + a/L_{Te}) \cdot (c_s/a)$



At $r/a=0.6$, linear scaling with v_e consistent with global τ_E scaling trend

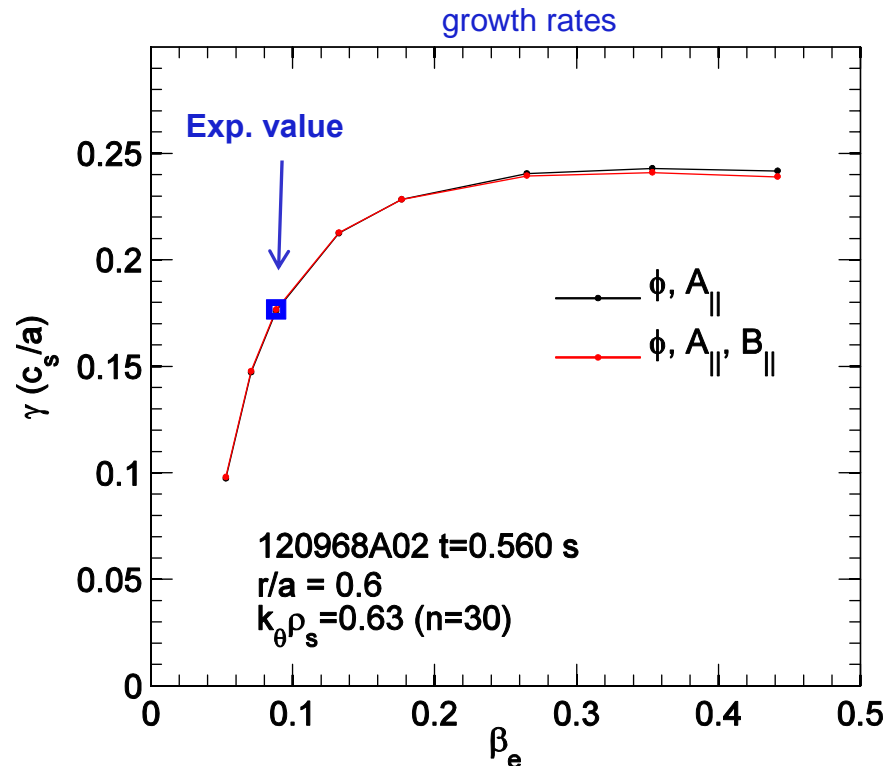
- Growth rates decrease with $v_e < v_{e,exp}$ (in the direction of the experimental v_* scan)
- Scaling with v_e not simply monotonic – transition to TEM at very low v_e
- Farther out ($r/a=0.8$) larger a/L_{Te} , a/L_n , and trapping (ϵ) leads to larger TEM growth rate at low collisionality
- Transition to microtearing occurs at higher v_e but still in experimental range



Similar scaling calculated for MAST 22769 $r/a=0.6$

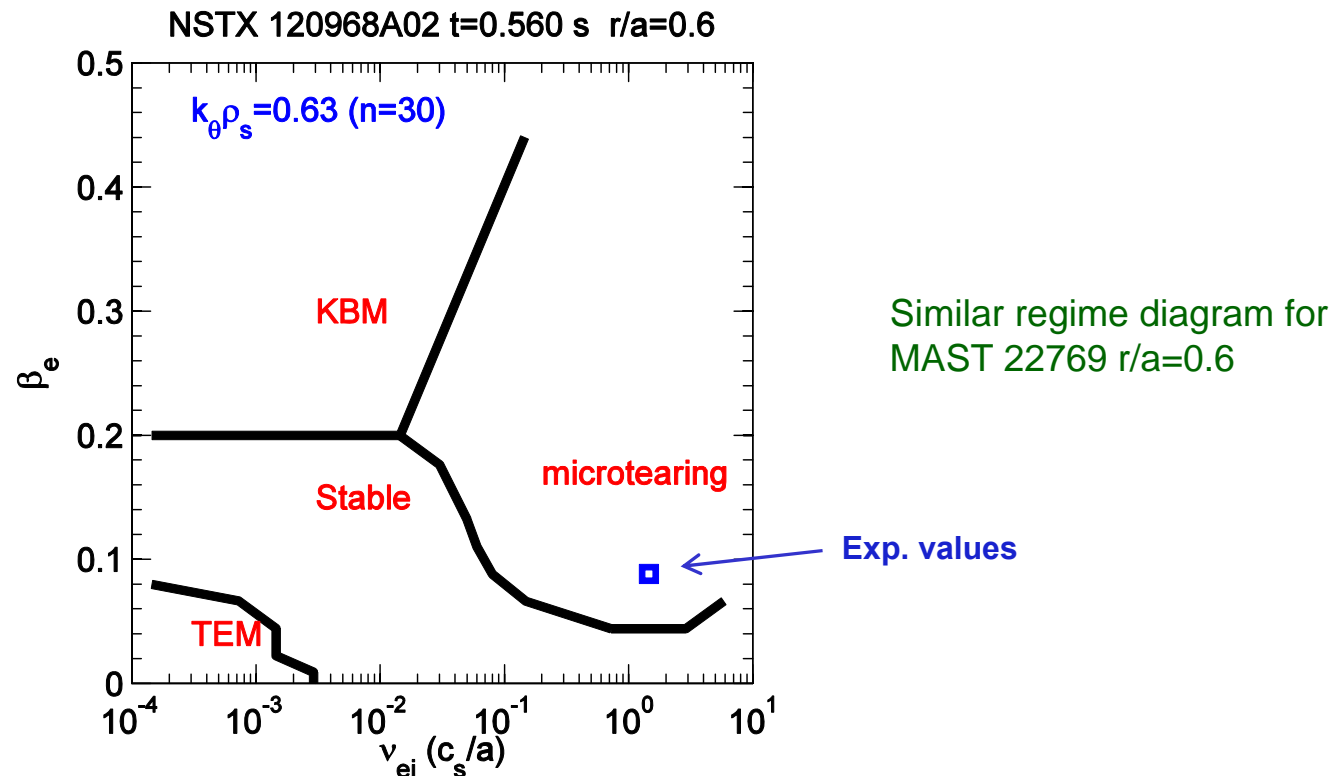
Finite beta critical to instability

- At experimental ν_e , lowering beta stabilizes microtearing \rightarrow no instability remains
- KBM becomes unstable at much larger β_e (not shown)
- Microtearing dynamics insensitive to compressional magnetic perturbations (δB_{\parallel})



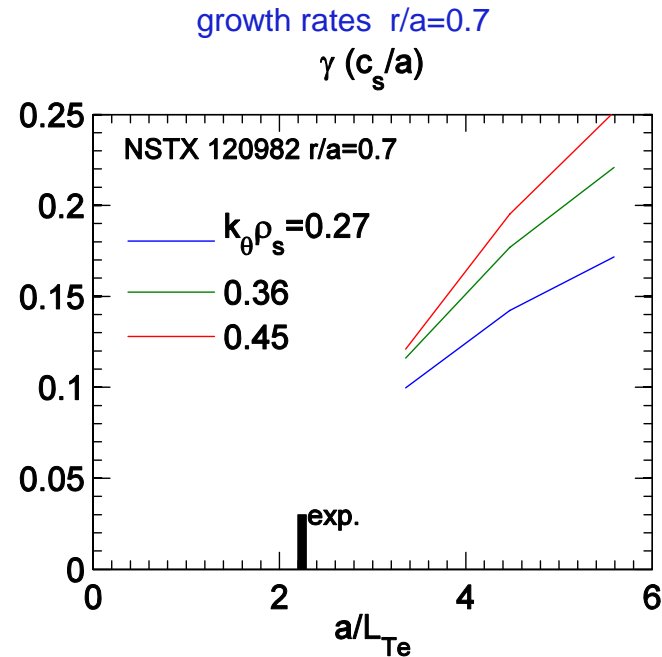
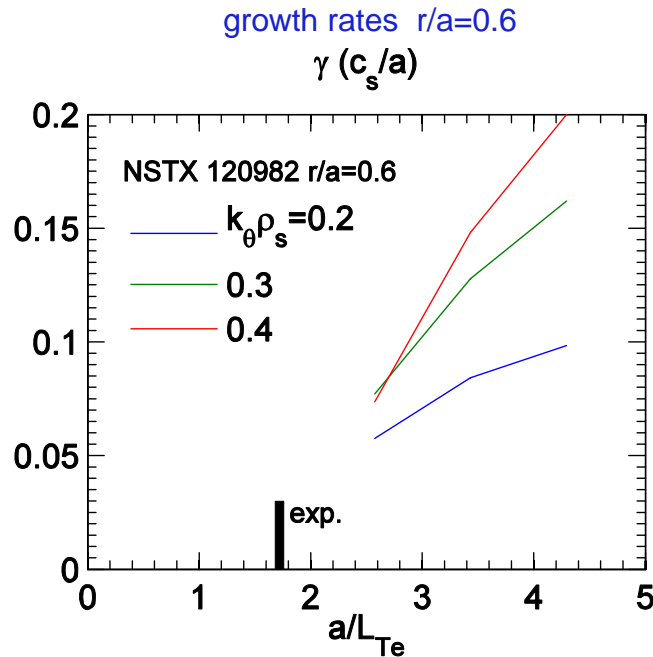
Mapping out stability space via v_e - β_e regime diagram

- Artificially varying v_e and β_e (inconsistently holding $\nabla\beta_{eq}$ constant) for fixed $k_\theta\rho_s=0.63$ ($n=30$)
- At this location, microtearing mode dominates over wide range of v_e and β_e
- There appears to be a broad stable region at much lower collisionality
- Onset of KBM is sensitive to $\delta B_{||}$ (included below)



Low v_* discharge near microtearing ∇T_e threshold

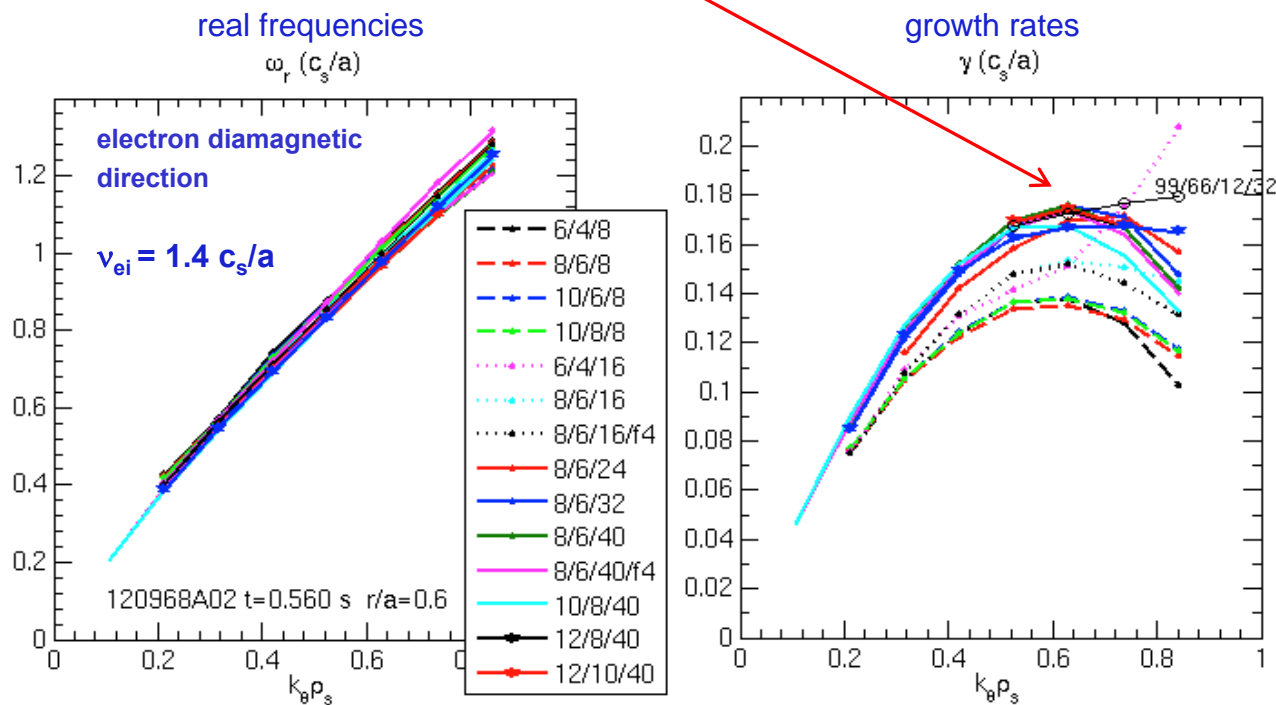
- Calculations for low v_* discharge (NSTX 120982) are stable ($r/a=0.5$) or near marginal ($r/a=0.6-0.8$)
- Consistent with above $v_e-\beta_e$ regime diagram, but other parameters changing



Structure of linear microtearing modes

Linear convergence sensitive to resolving narrow inner layer

- Growth rates most sensitive to **radial resolution** (Δx) at higher $k_\theta \rho_s$
- “Semi-collisional” regime of microtearing modes $v_{ei} \sim |\omega|$ (Drake & Lee, 1977)
 - Inner layer width Δ_d theoretically determined by balance of drift frequency ω_{*e} with Doppler shift due to parallel electron diffusion ω_d
 - $\Rightarrow \omega_{*e} \sim \omega_d = (k_{\parallel} v_{te})^2 / \nu$, $k_{\parallel} = k_y \Delta_d / L_s \Rightarrow \Delta_d \sim 0.07 \rho_s$ (for $k_\theta \rho_s = 0.6$)
 - Linear growth rate converged for $\Delta r \approx 0.03 \rho_s$ (for $k_\theta \rho_s = 0.6$)**



Linear mode structure in perpendicular (r,α) plane illustrates microtearing mode dynamics

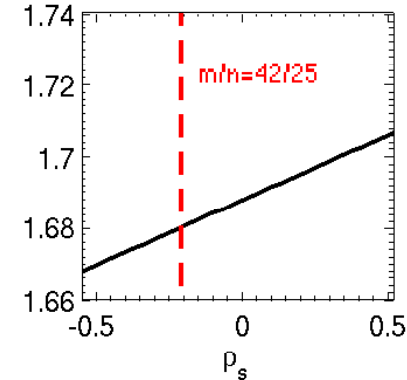
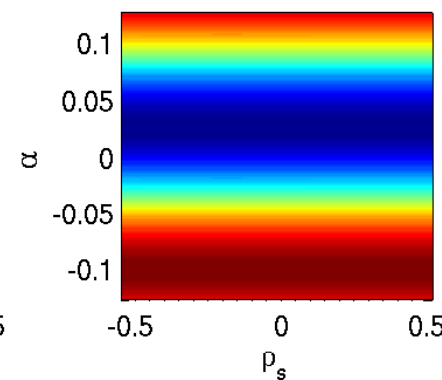
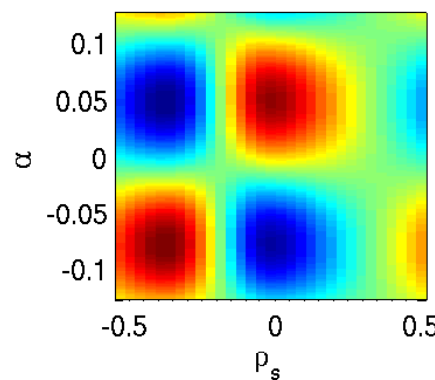
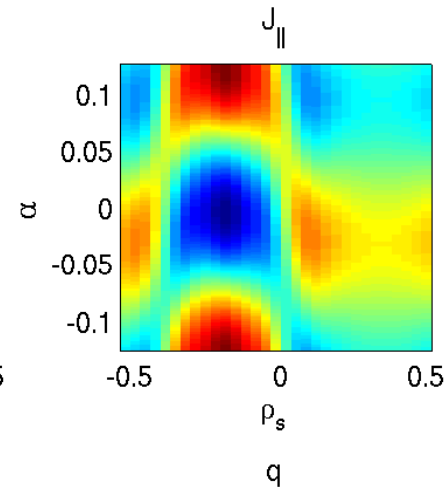
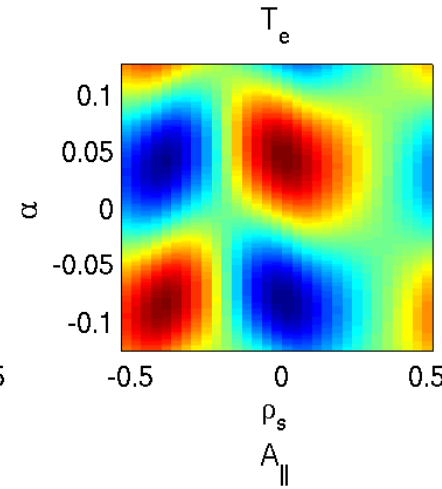
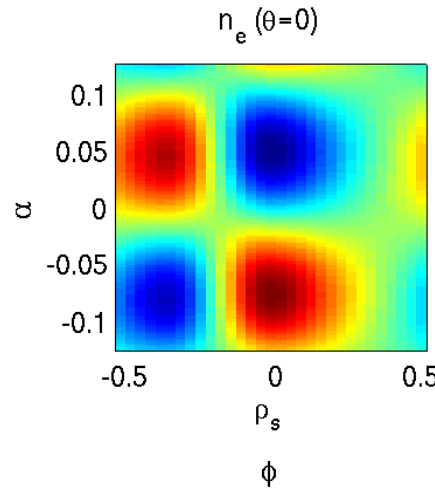
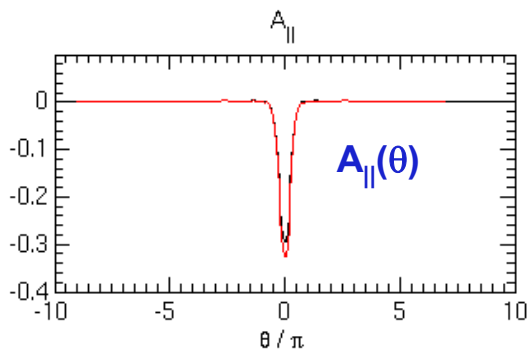
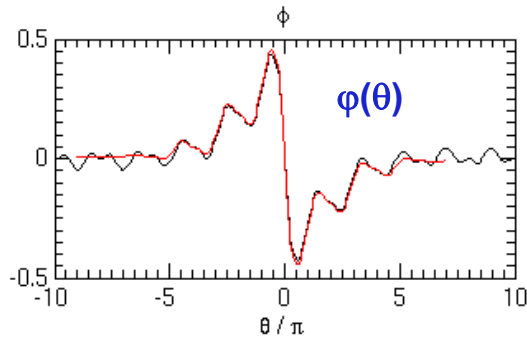
- Narrow resonant current channel ($\approx 0.3\rho_s$) centered on rational surface (compare to simple slab $\Delta_d \sim 0.07\rho_s$ estimate above)
- “Constant ψ ” ($A_{||}$), resonant tearing parity
- Nearly unmagnetized/adiabatic ion response $\Rightarrow \frac{\tilde{n}}{n_0} \approx -\left(\frac{e\tilde{\phi}}{T_i}\right)$
- Narrow potential, density, T_e perturbations

r - α perpendicular plan ($\theta=0$)

$\alpha = \phi - q\theta$ for low β circular surfaces

“ballooning” space

$$k_r(\theta) = \hat{s}k_\theta(\theta - \theta_0)$$



Field line integration used to map island

- δB_r in linear run (arbitrary) determines $w_{\text{island}} \sim 0.4 \rho_s$
- Slab/cylindrical island width estimate does not work well (δB_r strongly ballooning)

$$\left| \frac{\delta B_{r,mn}}{B} \right| = 1.8 \cdot 10^{-7}$$

$$w = 4 \cdot \left[\frac{\delta B_{r,mn}}{B} \frac{rR}{n\hat{s}} \right]^{1/2} = \underline{0.03 \rho_s}$$

- Estimate using rms δB_r gets closer

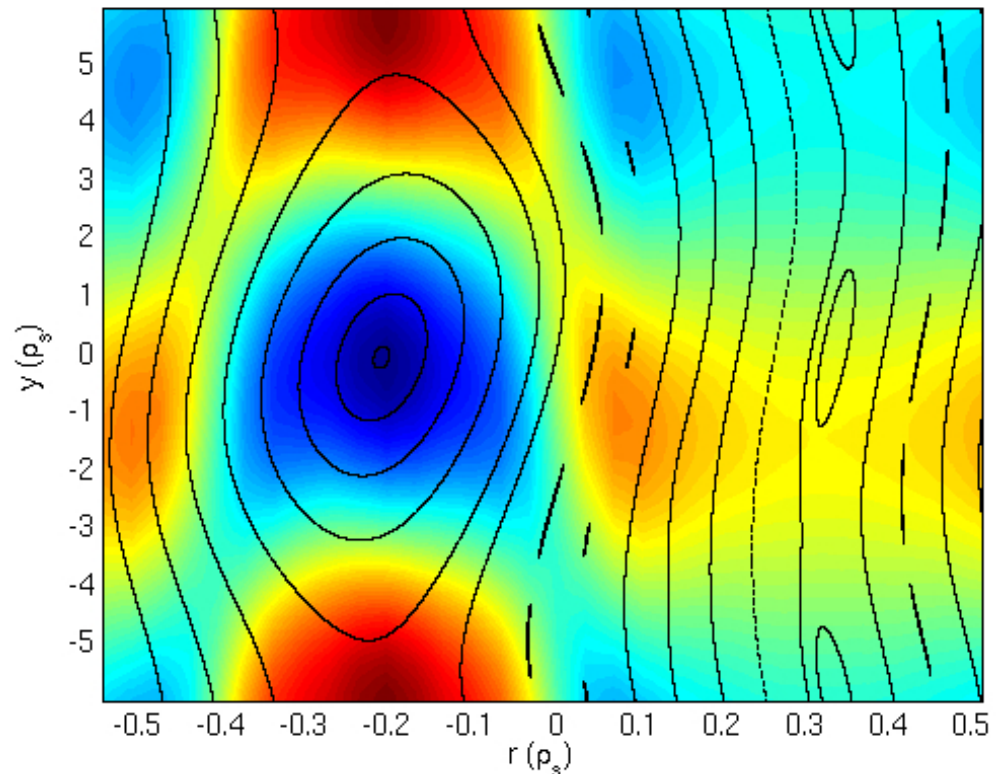
$$\left\langle \frac{\delta B_r^2}{B^2} \right\rangle_{\alpha, \theta}^{1/2} = 2.5 \cdot 10^{-5}$$

$$w = 4 \cdot \left[\left(\frac{\delta B_r}{B} \right)_{\text{rms}} \frac{rR}{n\hat{s}} \right]^{1/2} = \underline{0.39 \rho_s}$$

- $w_{\text{island}}/L_{Te} \approx 8 \cdot 10^{-3}$ but
 $\max(\delta T_e/T_e) \approx 4.5 \cdot 10^{-4}$

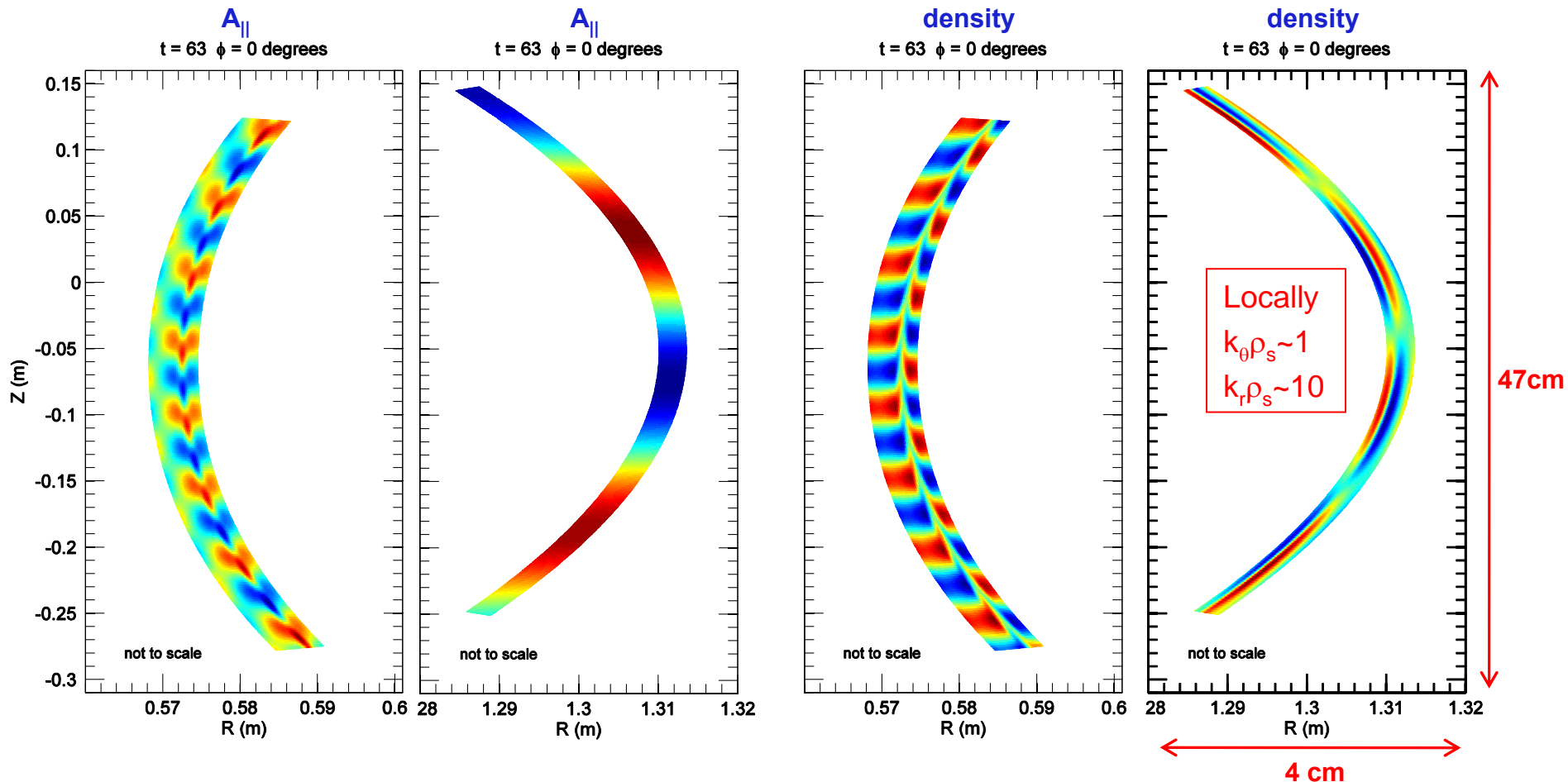
⇒ Influence of perpendicular drift dynamics

Poincare plot & contours of parallel current



Linear mode structure in toroidal (R,Z) plane

- Nonuniform poloidal structure (comparing inboard and outboard perturbations)
 - Density perturbations radially narrow, extended vertically on outboard side
- ⇒ “High-k” scattering diagnostic (see adjacent poster by Y. Ren) well suited for $k_r \gg k_\theta$



First *nonlinear* microtearing mode simulations in NSTX

Summary of first nonlinear microtearing simulations in NSTX

- Local, flux-tube simulations (flat profiles) at $r/a=0.6$ (NSTX 120968) where only microtearing unstable (no ETG)
 - Electromagnetic (ϕ, A_{\parallel}) and collisional (v_e)
 - Varying $E \times B$ shear
- Significant transport predicted, depending on γ_E/γ_{lin}
 - Fine radial resolution required ($\Delta x \leq 0.2 \rho_s$)
 - Transport dominated by electromagnetic component
 - Field lines are stochastic
 - Transport reduced with significant $E \times B$ shear

$L_x \times L_y = 80 \times 60 \rho_s$
 $n_x \times n_y = 400 \times 8$ ($\Delta x = 0.2 \rho_s$)
 $n_{\theta} = 14$ (parallel orbits)
 $n_{\lambda} = 12, n_E = 8$ (velocity space)

Fine radial resolution required to capture *linear* resonant layers

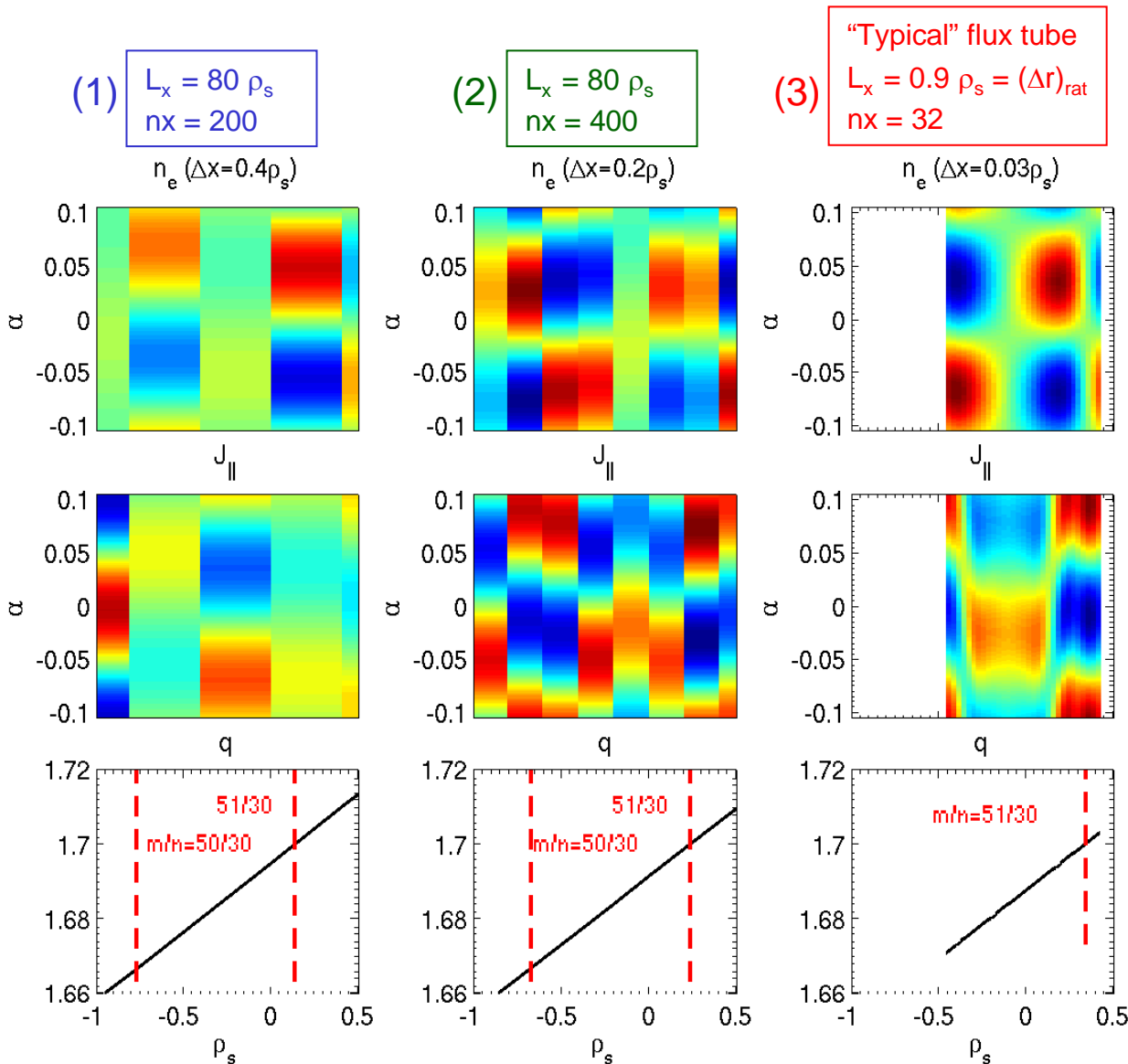
Calculating linear growth rate for single mode ($k_\theta \rho_s = 0.63$, $n=30$) using box width and resolution of nonlinear simulations

$L_x = 80 \rho_s$, $\Delta x = 0.4$ & $0.2 \rho_s$

(1) $\Delta x = 0.4 \rho_s$ is barely small enough to distinguish resonant layers

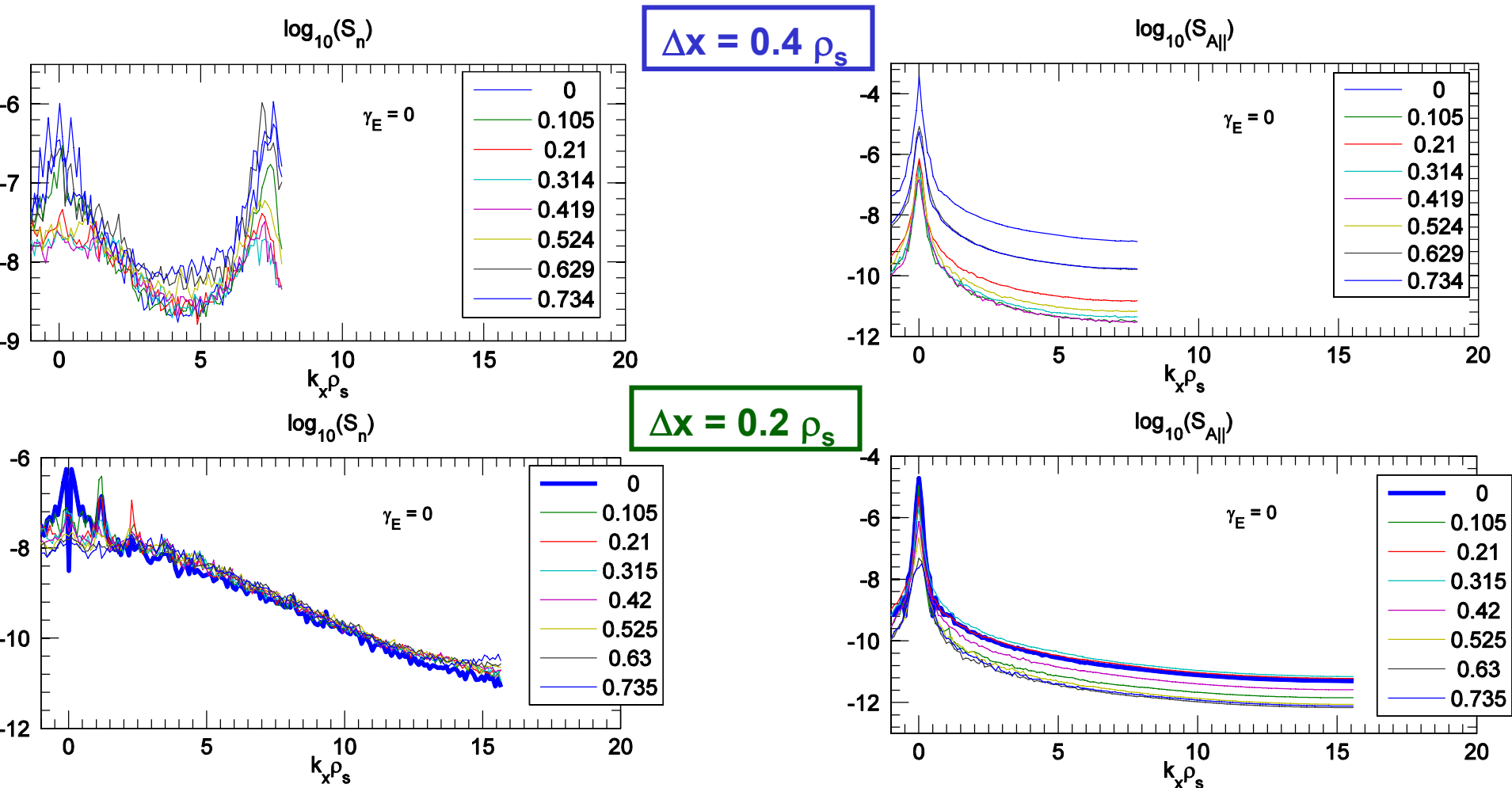
(2) $\Delta x = 0.2 \rho_s$ resembles the...

(3) high resolution flux-tube case



Fine radial resolution required for resolved *nonlinear* spectra

- k_x spectra completely different for $\Delta x = 0.4 \rightarrow 0.2 \rho_s$
- Insufficient resolution leads to peaking at high k_x similar to GS2 simulations in Applegate Ph.D. thesis (2007, Imperial College London)

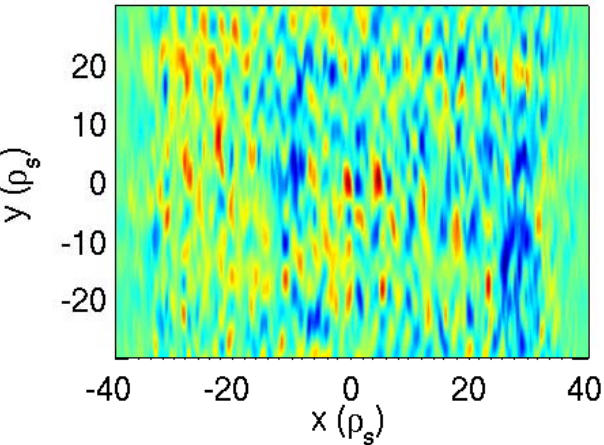


Narrow density perturbations remain in nonlinear simulations

- Narrow radial n , ϕ , j_{\parallel} structures need to be resolved but A_{\parallel} very broad
- $\delta B_r/B \sim 8.7 \times 10^{-4} \sim \rho_e/L_{Te} = 3.4 \times 10^{-4}$
- $\delta B_r/B \sim \rho_e/L_{Te}$ analytic approximation from Drake et al. PRL 1980; used for NSTX in Wong et al. PRL 2007

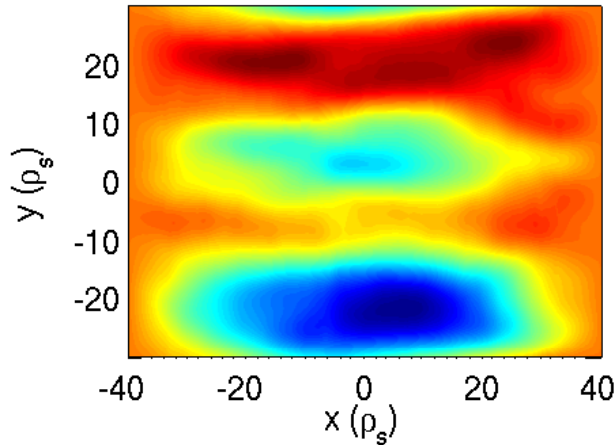
$\delta n/n \approx 0.5\%$

δn



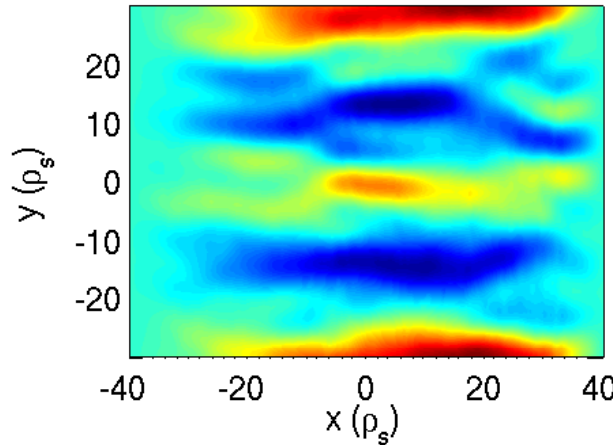
$\delta A_{\parallel}/c_s T_e \approx 0.8\%$

δA_{\parallel}



$\delta B_r/B \approx 0.09\%$

δB_r



$\delta T_e/T_e \approx 2\%$

$\delta v_{e,\parallel}/c_s \approx 6\%$

Perturbed field lines are stochastic

- $w_{\text{island}}(n) > \Delta r_{\text{rat}}(n)$, island overlap \rightarrow perturbed field line trajectories are stochastic
- \Rightarrow **~97% of transport from EM contribution $\chi_{e,EM}$**
- $\chi_{e,EM}$ well described by *collisionless* Rechester-Rosenbluth ($\lambda_{\text{mfp}}=25$ m, $qR=1.6$ m)
- \rightarrow see invited talk by Eric Wang, NI2:03 Wed. 10:30 am

$$w_{\text{island}} = 4 \sqrt{\frac{\delta B_{r,n}}{B} \frac{rR}{n\hat{s}}} \quad (\Delta r)_{\text{rat}} = \frac{1}{nq'} = \frac{1}{k_{\theta}\hat{s}}$$

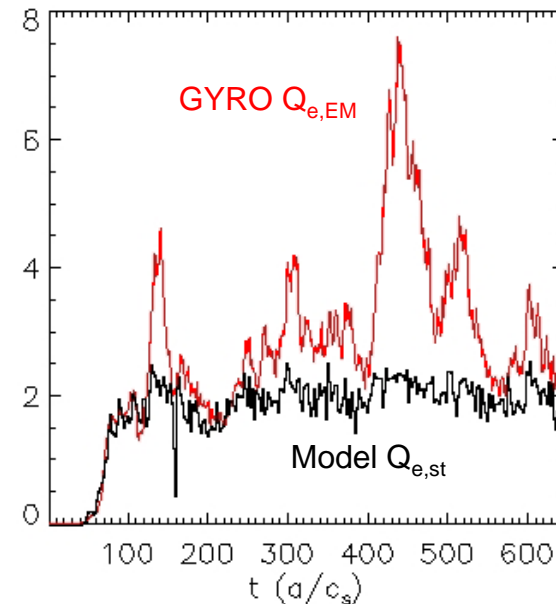
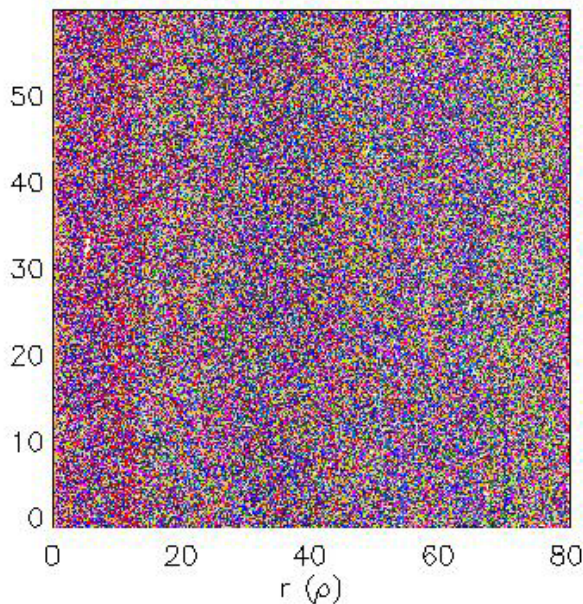
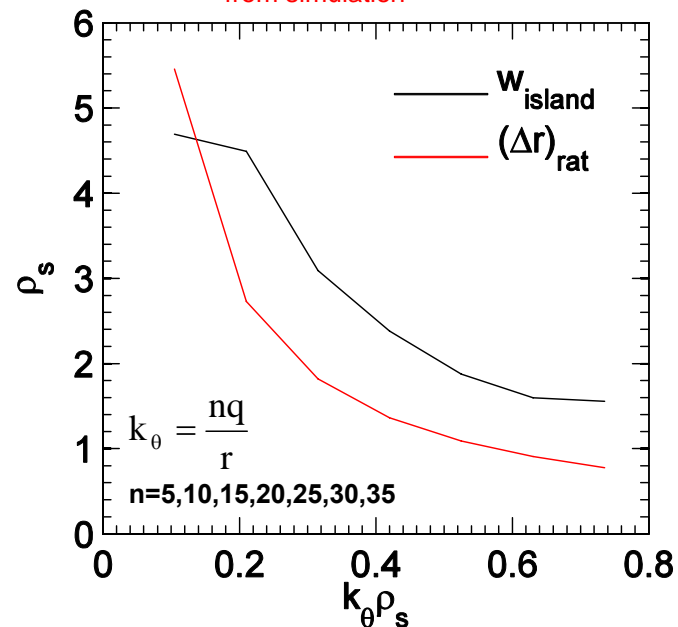
from simulation

integrating dI/B
 3000 transits for each
 of 100 field lines
 PoincareFieldmap[r] ()

$$\chi_{\text{st}} \approx 2 \left(\frac{2}{\pi} \right)^{1/2} D_{\text{st}} v_{te} f_p$$

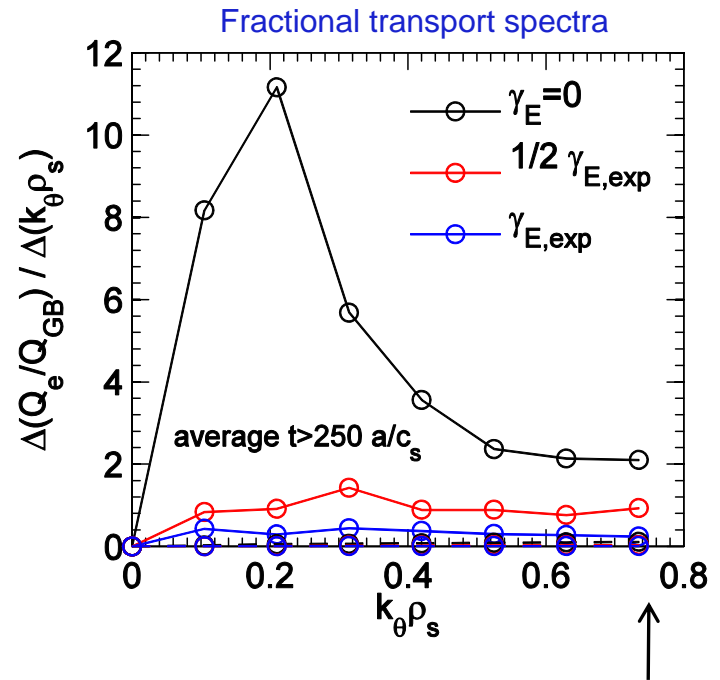
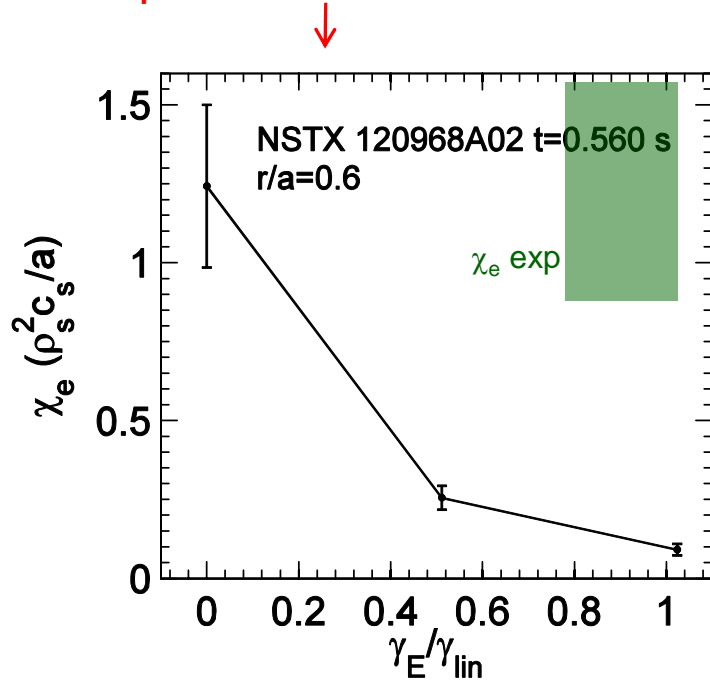
$f_p \approx 50\%$ passing particles

$$D_{\text{st}} = \lim_{s \rightarrow \infty} \frac{\langle [r_i(s) - r_i(0)]^2 \rangle}{2s}$$



Nonlinear microtearing transport sensitive to γ_E/γ_{lin}

- With no $E \times B$ shear predicted transport ($1.2 \rho_s^2 c_s/a$) comparable to experimental transport ($1.0-1.6 \rho_s^2 c_s/a$)
- Transport reduced when increasing γ_E to local experimental value



- Simulations are underway to investigate (1) convergence with binormal resolution ($k_\theta \rho_{s,max}$) and (2) sensitivity to a/L_{Te} , β_e , ν_e
- Above are local simulations, but $\rho_s/a=0.08$ & physical domain $r/a=0.3-0.9$ → have not investigated influence of profile variations, e.g. $a/L_{Te}(r)$, $\gamma_E(r)$, $q(r)$

Unclear how χ_e will scale with v_e

- Linear growth rate scaling $\gamma_{\text{lin}} \sim v_e$ may (or may not) influence saturated $\delta B_r/B$ and resulting D_{st}
 - Similar trend may (or may not) hold from linear β_e scaling
- ⇒ Nonlinear simulations in progress

- *Collisional* R-R model used by Wong et al.* for NSTX 116313 :

$$\chi_{\text{st}}^{\text{Wong}} = D_{\text{st}} \frac{\chi_{e,\parallel}}{L_c} = \left(\left| \frac{\delta B_r}{B} \right|^2 R \right) \frac{v_{\text{Te}} \lambda_{\text{mfp}}}{qR} = \left(\frac{\rho_e}{L_{\text{Te}}} \right)^2 \frac{v_{\text{Te}}^2}{v_{\text{ei}} q}$$

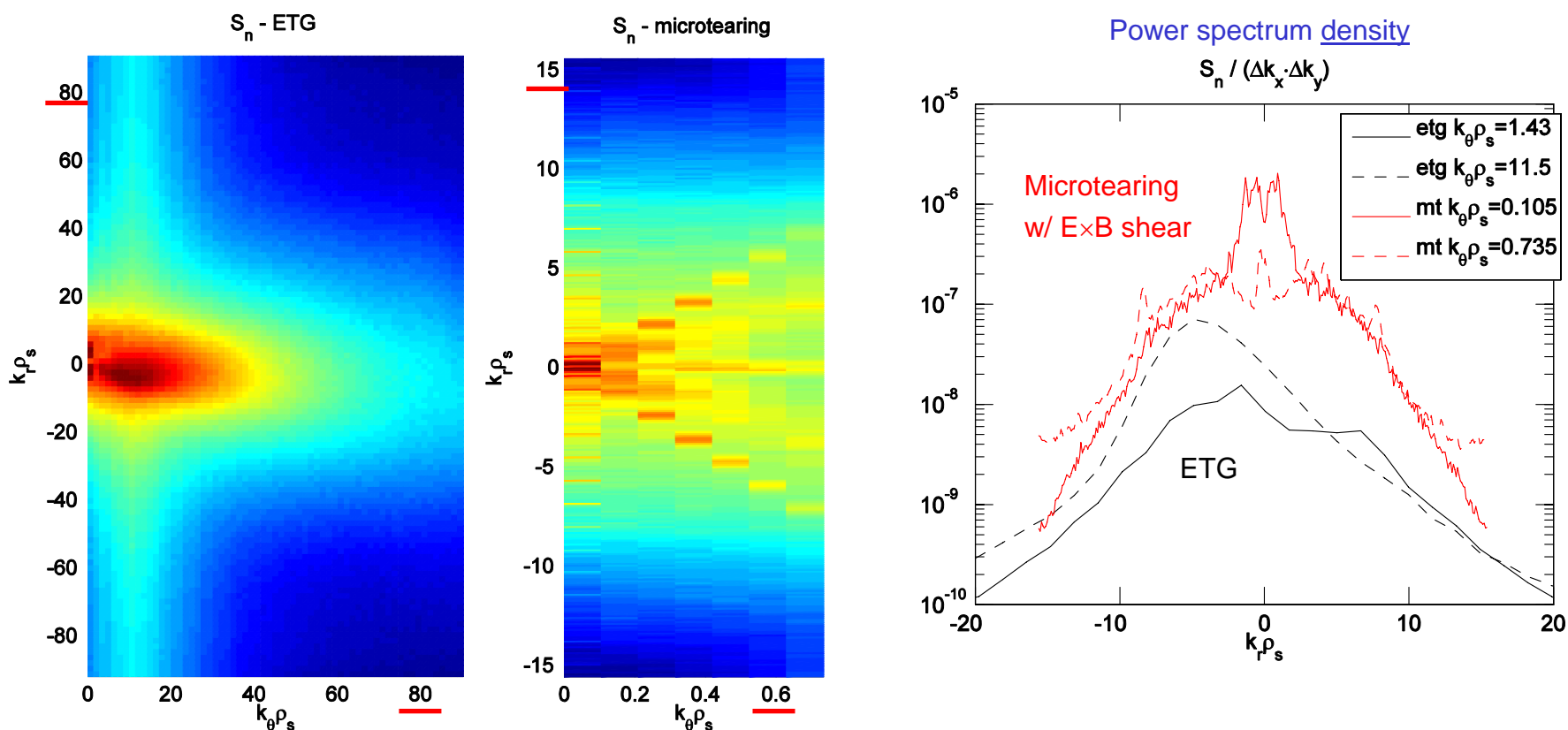
- *inversely* dependent on v_e
- no explicit dependence on β_e
- very sensitive to T_e and ∇T_e
- “knows” nothing about linear thresholds

$$\chi_{\text{st}}^{\text{Wong}} \propto T_e^{3/2} \cdot (\nabla T_e)^2 = T_e^{7/2} \cdot \left(\frac{1}{L_{\text{Te}}} \right)^2$$

*K.L. Wong et al., Phys. Rev. Lett **99**, 135003 (2007); Phys. Plasmas **15**, 056108 (2008)

May expect significant intensity in high-k scattering from microtearing

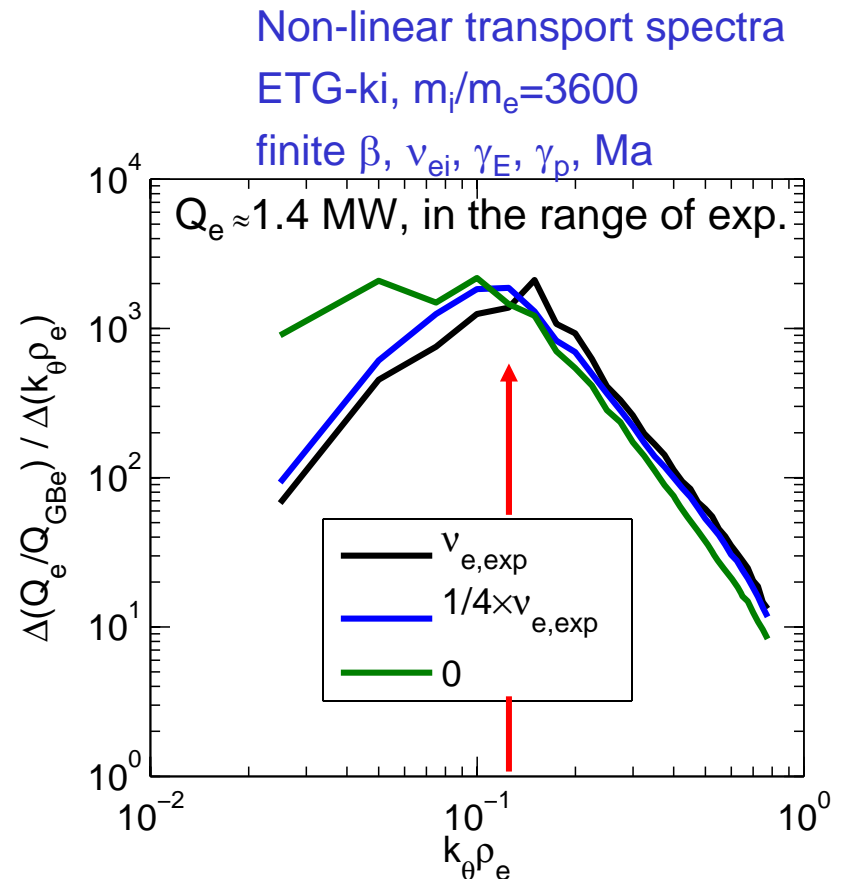
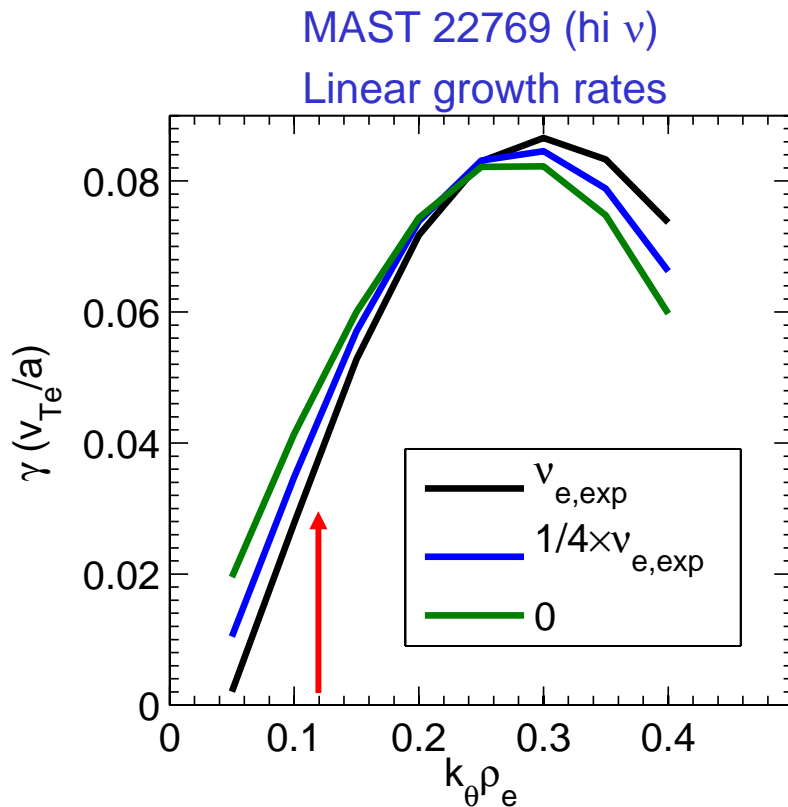
- Comparable $\delta n/n$ predicted for ETG (2.8×10^{-3}) and microtearing (1.7×10^{-3})
- But ETG spectrum much broader in $k_{\theta} \rho_s \rightarrow$ less intensity per unit $\Delta k_x \cdot \Delta k_y$
- Application of synthetic “high-k” diagnostic to simulations beginning (see adjacent posters by F.M. Poli & Y. Ren)



Nonlinear ETG simulations in MAST – v_* scaling

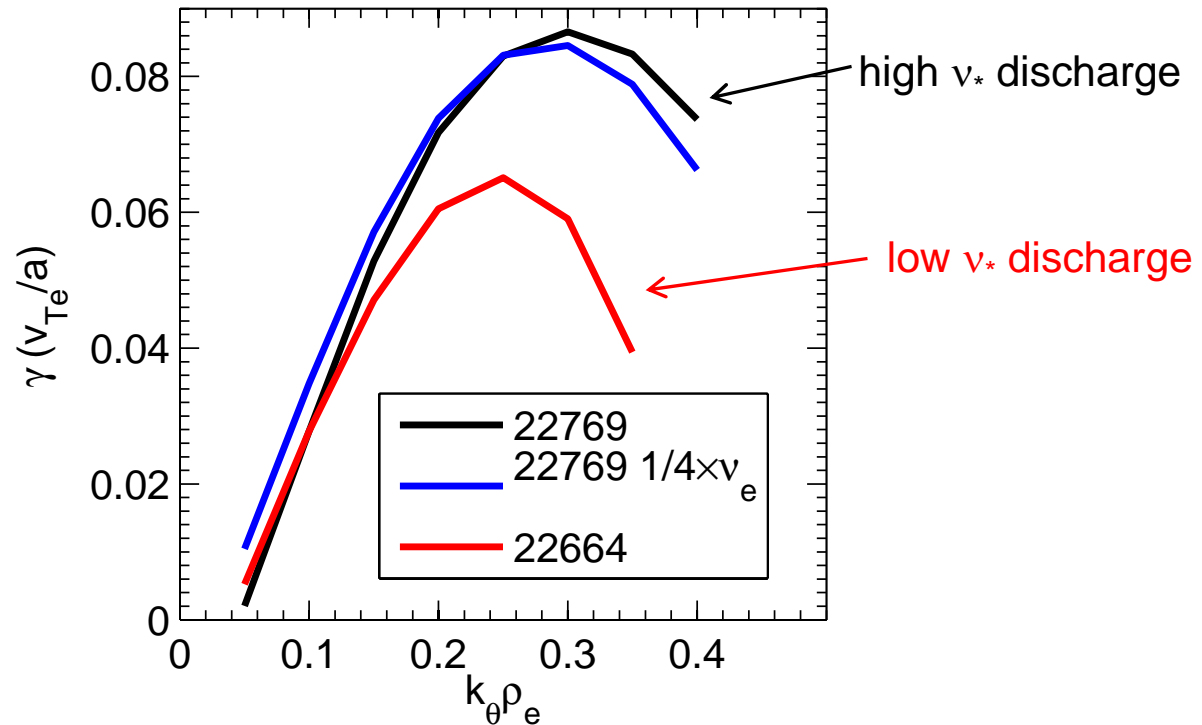
Decreasing collisionality destabilizes lower $k_{\theta}\rho_e$ ETG modes where transport dominates

- Little change in peak growth rates when scaling only v_e
- Lower $k_{\theta}\rho_e$ are destabilized (trapped electron contributions)
- Small increase in non-linear transport ($\sim 15\%$ at $v_{ei}=0$) \rightarrow **inconsistent with experimental scaling**



Peak growth rates reduced when directly comparing low v_* discharge

- Small changes in other dimensionless parameters add up



- Pursuing integrated transport predictions (e.g. with TGLF+NEO) including all experimental variations (see adjacent poster by J.L. Peterson)