Abstract

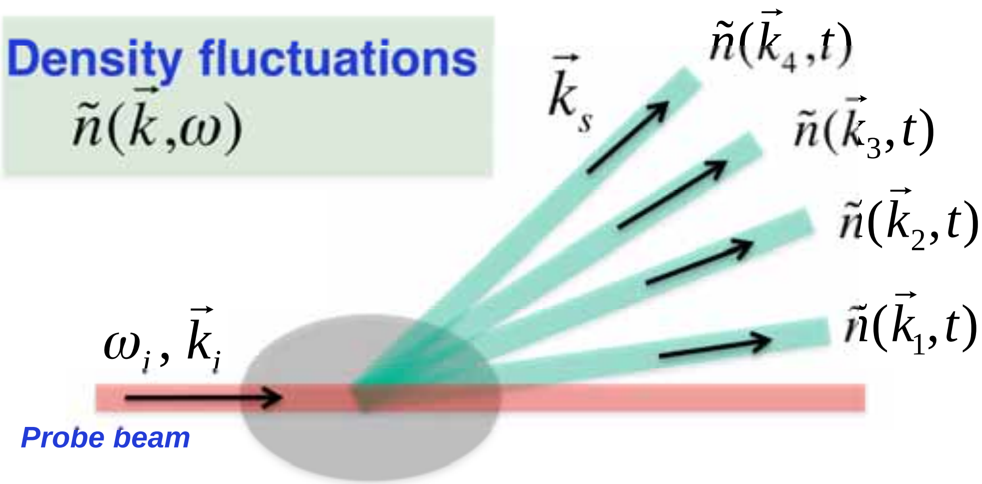
The comparison between density fluctuations measured with coherent scattering techniques and spectra from space resolved fluctuations computed from nonlinear gyro-kinetic codes are affected by a number of systematic errors and uncertainties. These include the scattering localization, the different wavenumber range covered, the simulation runtime, which mainly affect the slope of the k-spectrum. To bridge the gap between experiments and simulations, a synthetic diagnostic has been developed. Taking into account the beam propagation, the beam intensity profile, the instrument transfer function, the synthetic high-*k* predicts the collection efficiency in the (k_r,k_θ) space. When simulated spectra are filtered by the synthetic high-*k*, a closer agreement with experiments is found. Results from nonlinear simulations run in different plasma configurations, including L-mode and H-mode, will be presented.

Work supported by US DOE Contract DE-AC02-09CH11466 and the SciDAC GPS-TTBP project.

Outline

- Work motivated by
  - the observation of electrostatic turbulence on NSTX, in a wavenumber range consistent with ETG instabilities [*Mazzucato et al, PRL, **101**, 075001 (2008)*]
  - The increasing availability of nonlinear, global, ETG simulations
  - The need of cross-validating measurements and simulations
- What we have done
  - Developed a synthetic diagnostic for high-k scattering
  - Looked up the possible sources of systematic errors
- In this work:
  - Description of the synthetic high-k scattering diagnostics
  - Interface with two different global, gyrokinetic codes
    - GTS (*WX Wang et al, Phys. Plasmas **13** 092505 (2006)*)
    - GYRO (*J. Candy and R. Waltz, Journal Comp. Physics, 186-545 (2003)*)
  - Future development

High-k scattering measurements are local in space and limited in wavenumber range



High-k scattering measures $\{k_\perp\}$

$$\vec{k} = \vec{k}_s - \vec{k}_i$$

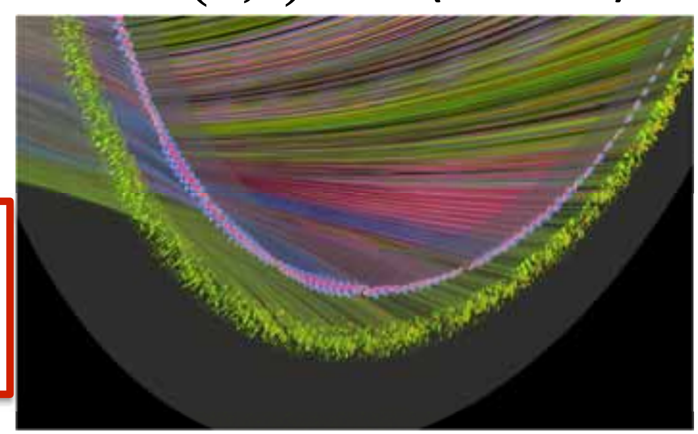
$\omega_i >> \omega$ => Small θ_s

$$k = 2k_i \sin(\theta_s/2)$$

simulations cover annulus of plasma

=> (k_r, k_θ) spectrum can be extracted

$\tilde{n}(\vec{x},t)$ **(from GTS)**



$P_{SIM}(k_r, k_\theta, \omega)$ **almost 2 orders of magnitude**

$P_{HK}(k_\perp, \omega)$ **finite range of $k_\perp$**

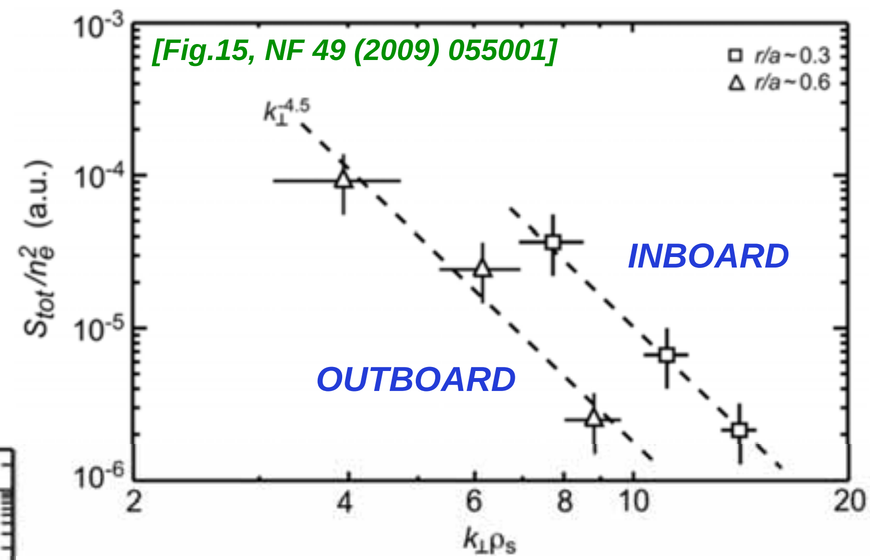
A direct comparison may lead to fortuitous agreement

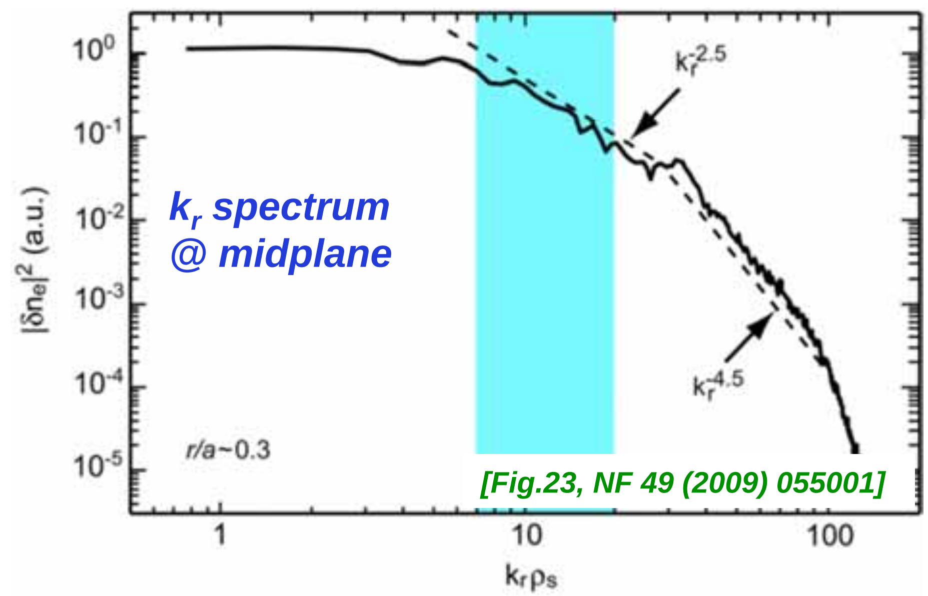
$P_{HK}(k_{\perp}^{j},\omega)$ ***Discrete in*** $k_{\perp}$

Good statistics in ω

$P_{SIM}(k_r,k_\theta,\omega)$ ***Wide range in*** (k_r,k_θ)

Poor statistics in ω





What has been overlooked ?

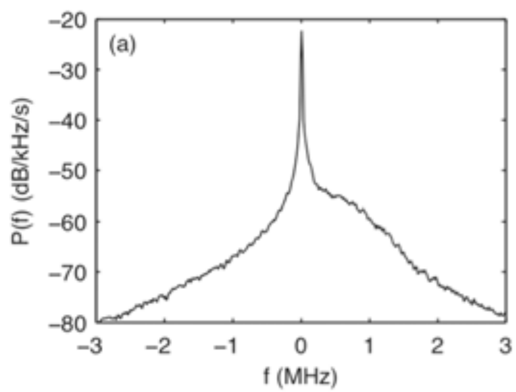
- ***selection of*** (k_r,k_θ)
- ***scattering volume***
- ***instrumental transfer function***

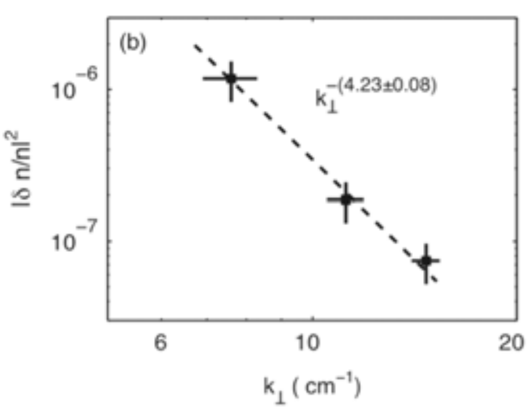
Two cases study: L-mode and H-mode plasma

Turbulence spectra from the high-k scattering diagnostic

#124901 **H-mode**

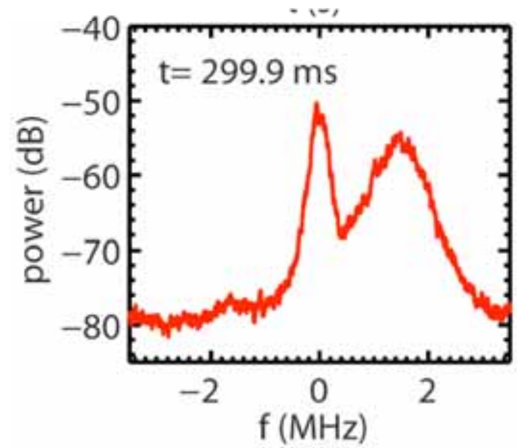
t = 300 ms, Δt=10ms

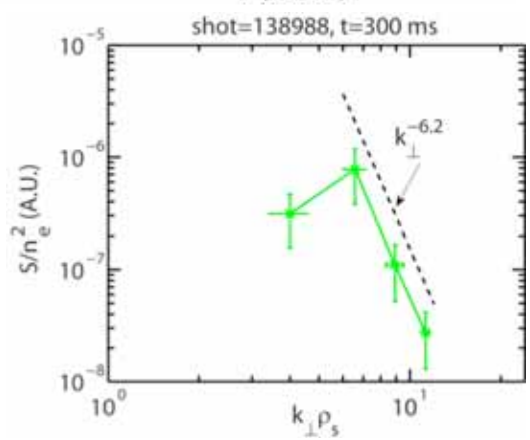




#138988 **L-mode**

t = 300 ms, Δt=1ms



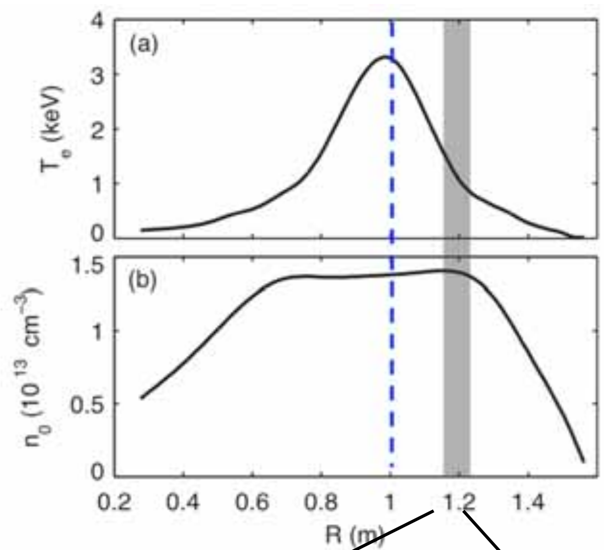


See poster BP9.67
Y. Ren *et al*

Level of fluctuations is similar, but spectra are steeper in L-mode

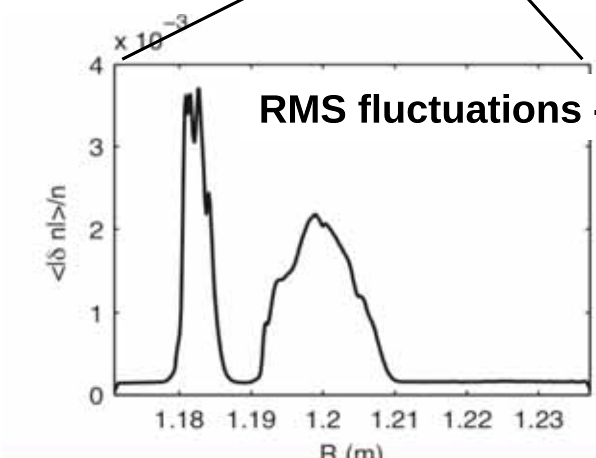
H-mode **#124901**

Measured profiles



R_m=0.19 m
R_0 =1.01 m
R_M=1.52 m

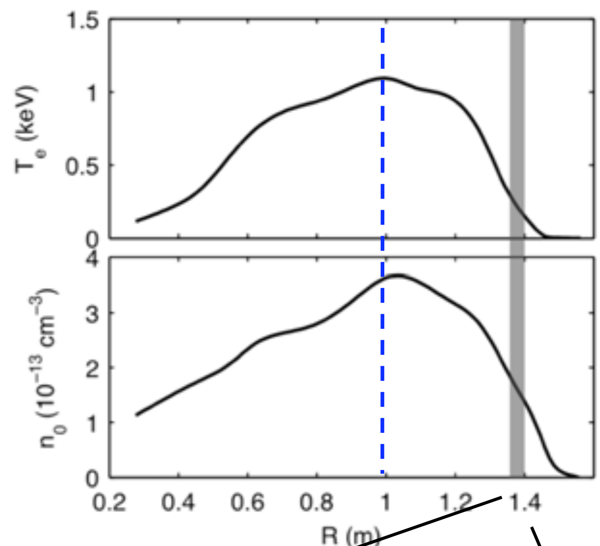
RMS fluctuations - GTS



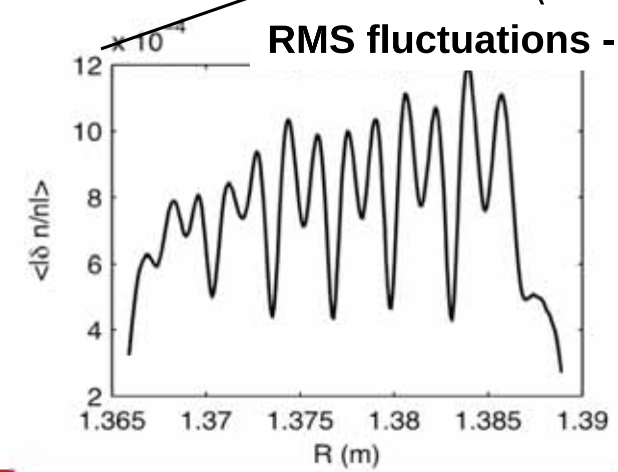
L-mode **#138988**

Measured profiles

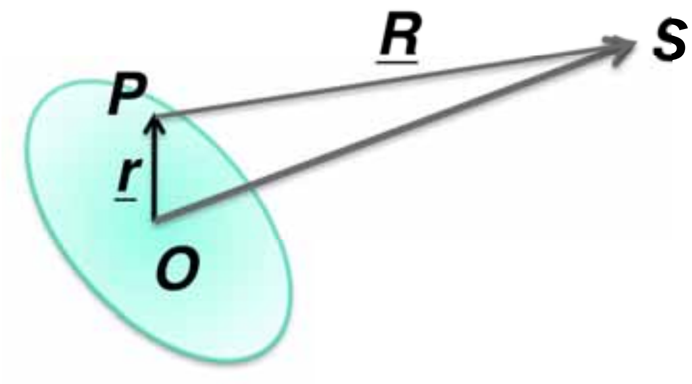
R_m=0.19 m
R_0 =0.99 m
R_M=1.46 m



RMS fluctuations - GYRO



The ingredients for the synthetic high-k are contained in the expression for the measured electric field



$$\mathbf{E}_s = \left[\frac{r_e}{R}\hat{s}\times\hat{s}\times\mathbf{E}_i\right]$$

Electric field scattered from one particle, calculated at retarded time: $t' = t - \frac{R}{c}$

$$\mathbf{E}_i(\mathbf{r},t) = \mathbf{E}_i(\mathbf{r}_\perp)\, e^{i(\mathbf{k}_i\cdot\mathbf{r}-\omega_i t)}$$

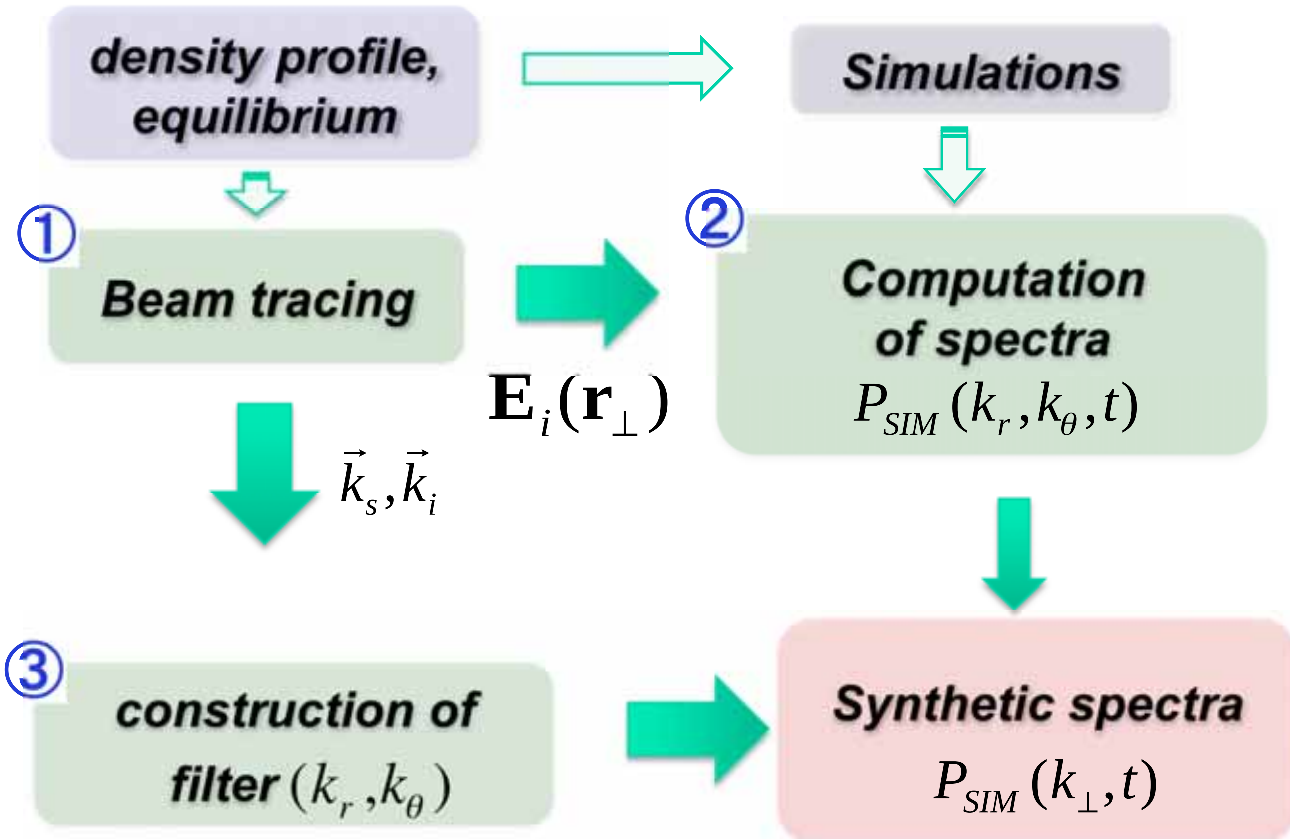
Fourier Transform of density fluctuations weighted by the beam intensity

$$\mathbf{E}_s(\nu_s) = \frac{r_e}{x} e^{i\mathbf{k}_s\cdot\mathbf{x}}\,(\hat{s}\hat{s}-\mathbf{1})\cdot\int_{T'} dt' \int_V d^3r'\, \mathbf{E}_i(\mathbf{r}_\perp)\, e^{i(\omega t'-\mathbf{k}\cdot\mathbf{r})}\,\tilde{n}(\mathbf{r}',t')$$

Direction & amplitude of k_s

Amplitude profile of beam (size of the scattering volume)

Three standalone blocks in the synthetic high-k



Block 2: computation of spectra

From beam tracing

INPUT: ***Scattering volume V centered @ (R_s,z_s)***

From simulations

$\tilde{n}(r,\theta,\varphi_i,t_j)$ $\quad \varphi_1 \cdots \varphi_M$ $\quad t_1 \cdots t_N$

Each poloidal plane is independent

Interface with the numerical code

Prepare density fluctuations data for computation of spectra

Interpolate scattering volume onto simulation grid

CODE DEPENDENT

$$\int_{T'} dt' \int_V d^3r' \, \mathbf{E}_i(\mathbf{r}_\perp) \, e^{i(\omega t' - \mathbf{k}\cdot\mathbf{r})} \, \tilde{n}(\mathbf{r}', t')$$

Computation of spectra

Weight ñ by V

↓

FT^θ — ***$N_r x N x M$***

↓

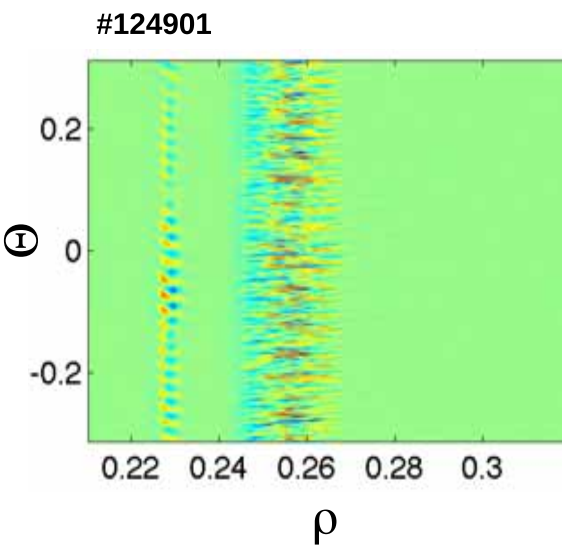
FT^r — ***$N_k^\theta x N x M$***

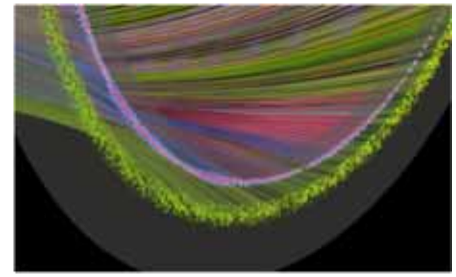
Same procedure for the two codes

OUTPUT FORMAT IS CODE DEPENDENT

Interface with GTS*: spectra are computed in real coordinates

[WX Wang et al, Phys. Plasmas **13** 092505 (2006)]*





<u>Simulation using:</u>

– numerical equilibrium
– experimental parameters
– electrostatic
– adiabatic ions

(ρ,θ) grid not regular

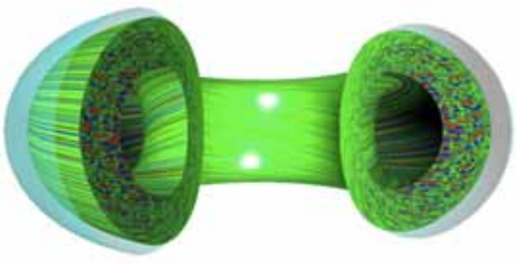
- $\Delta\rho$, $\Delta\theta$ are set by Larmor radius ρ_e
- $\Delta\theta$ is regular on each flux surface, it changes between flux surfaces

64 planes: toroidal separation comparable to scattering volume extension
=> each plane is dealt with independently

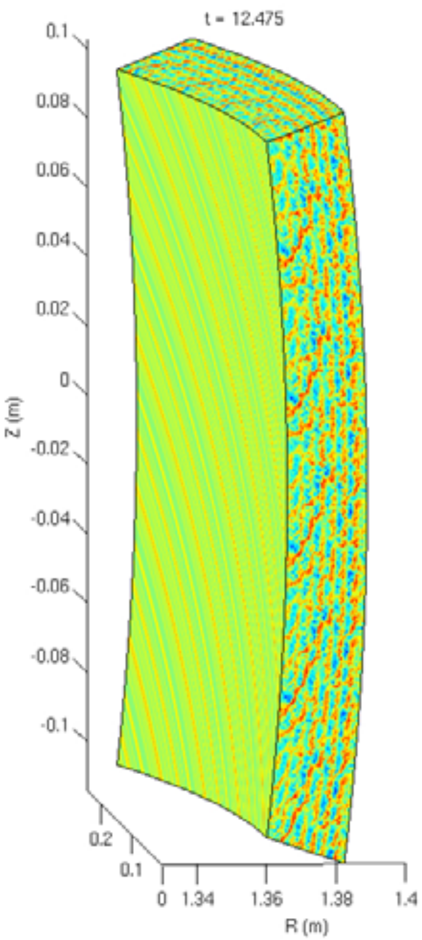
Compute spectra in real coordinates => *k* directly compared with exps

Interface with GYRO*: spectra are computed in flux coordinates

[J. Candy and R. Waltz, Journal Comp. Physics, 186-545 (2003)]*



#138988



<u>Local simulation using:</u>

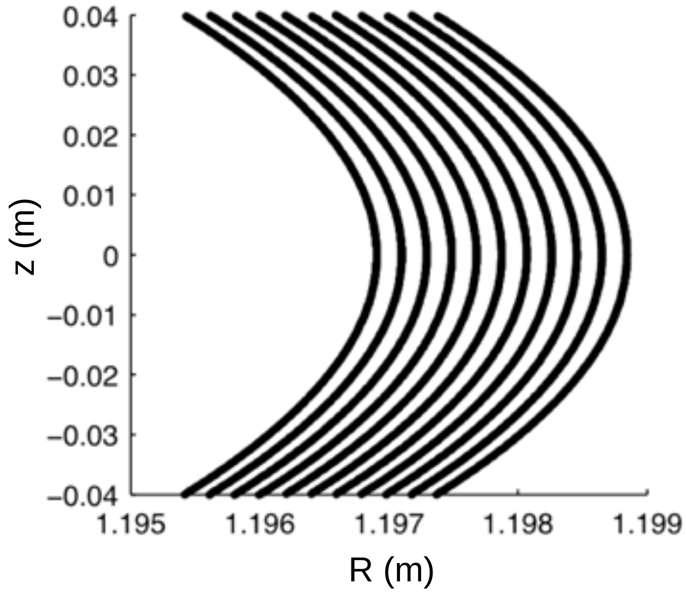
- numerical equilibrium
- experimental parameters
- finite collisionality
- toroidal flow and flow shear
- electrostatic (β_e is small)
- adiabatic ions (will ultimately use kinetic)

(ρ,θ) grid regular => compute spectra in flux coordinates

Periodicity along toroidal direction => need only a finite number of planes (50 for this simulation)

=> need to convert k to physical units to compare with experiments

Interface with GTS: k_θ spectra are computed in real space along a pseudo-polar direction



1. Along each flux surface in real space (R,z) we construct a trajectory :

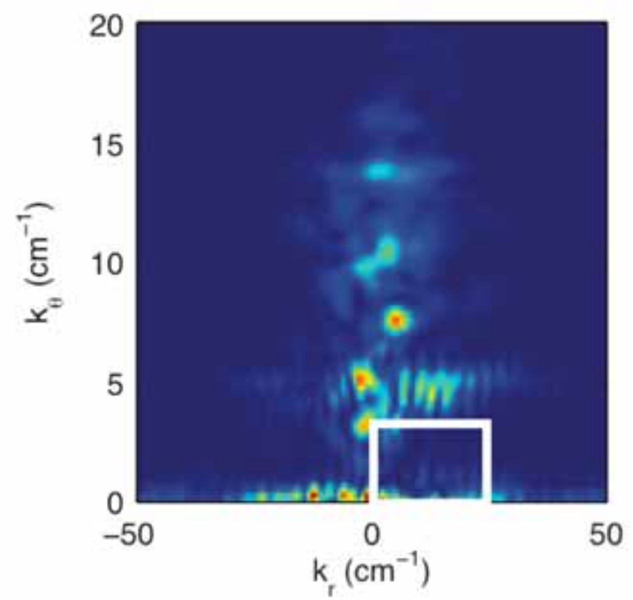
$$ds_j = \sqrt{(R_{j+1} - R_j)^2 + (z_{j+1} - z_j)^2}$$

2. Interpolate density along this trajectory using the same step for all flux surface (to have the same k_N)
3. Compute Fourier Transform using the same number of points (to have the same Δk_θ)

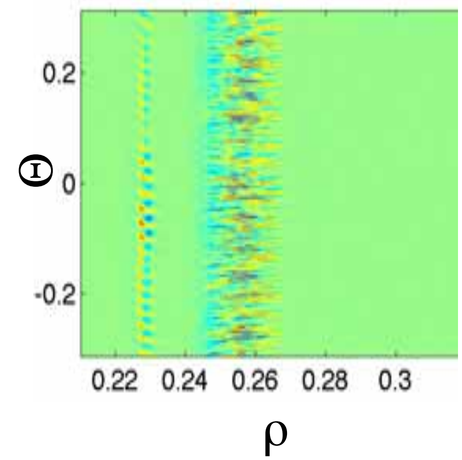
=> The Fourier components depend only on the value of R at midplane $\tilde{n}(R_{mid}, k_\theta, t_i)$

PROBLEM: in order to compute the transform along R, we need to interpolate amplitude and phase of Fourier components

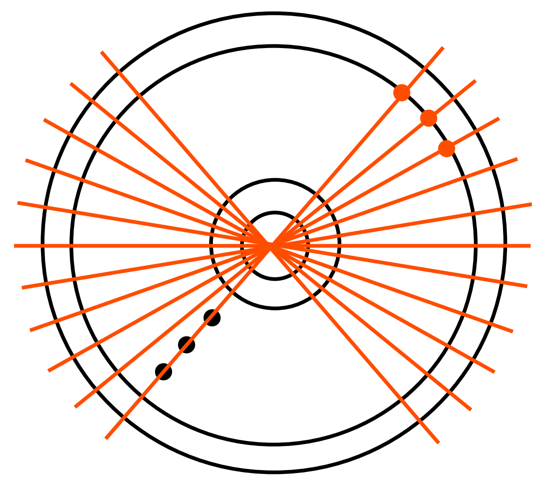
Interface with GTS: density fluctuations are interpolated in flux coordinates



phase interpolation generates artificial structures in k_r, due to phase jumps where density structures are localized.



NEXT STEP UPGRADE



This issue can be overcome by pre-processing density fluctuations in flux coordinates

- interpolate along ρ => uniform ΔR @ midplane
- redistribute data along θ

Block 1: Beam tracing*

* [Nowak and Orefice, Phys. Plasmas **1** 1242 (1994)]

Follow propagation of a Gaussian beam in an anisotropic plasma

$$\vec{E}_i(\mathbf{r},t) = \mathbf{E}_i(\mathbf{r}_\perp)\, e^{i[k_0 S(\mathbf{r}) - \omega_i t]} \qquad S = R + iI$$

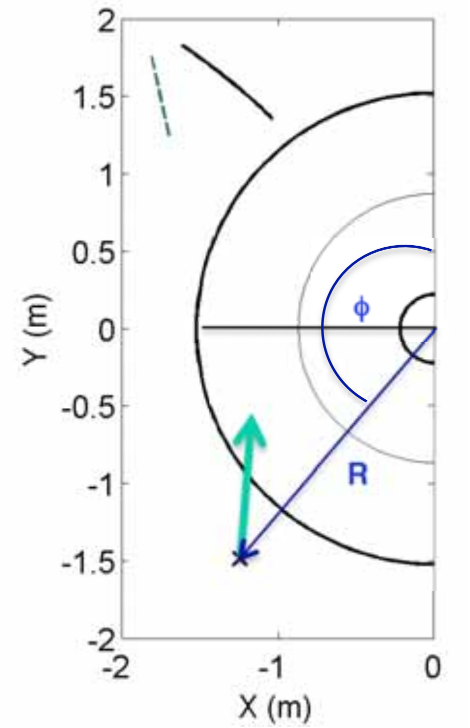
$$\begin{cases} R(x,y,z=0) = 0 \\ I(x,y,z=0) = -\dfrac{x^2+y^2}{k_0 a^2} \end{cases}$$

Initial conditions

$\Re: \quad (\nabla R)^2 - (\nabla I)^2 = N^2$ **=> Leads to** $D(\mathbf{x},\mathbf{k}',\omega)$

$\Im: \quad \nabla R \cdot \nabla I = 0$ **=> Amplitude is constant along rays**

$$\mathbf{k} = \mathbf{k}' + i\mathbf{k}'' = k_0(\nabla R + i\nabla I)$$



$$\begin{cases} \dfrac{d\mathbf{x}}{dt} = -\dfrac{\partial D/\partial \mathbf{k}'}{\partial D/\partial \omega} \\ \dfrac{d\mathbf{k}'}{dt} = \dfrac{\partial D/\partial \mathbf{x}}{\partial D/\partial \omega} \end{cases}$$

Ray tracing equations

Solved in cylindrical geometry (R,φ,z), assuming an equal (and small) time step for all rays

$$D(\mathbf{x},\mathbf{k}',\omega) \equiv (k')^2 - \left(\frac{\omega}{c}\right)^2 [n^2 + (\nabla I)^2] = 0$$

Dispersion relation (Hartree-Fock)

Block 1: Beam tracing*

** [Nowak and Orefice, Phys. Plasmas **1** 1242 (1994)]*

The term $(\nabla I)^2$ introduces a 'symmetry breaking' also in axisymmetric configurations => $\partial/\partial\varphi \neq 0$

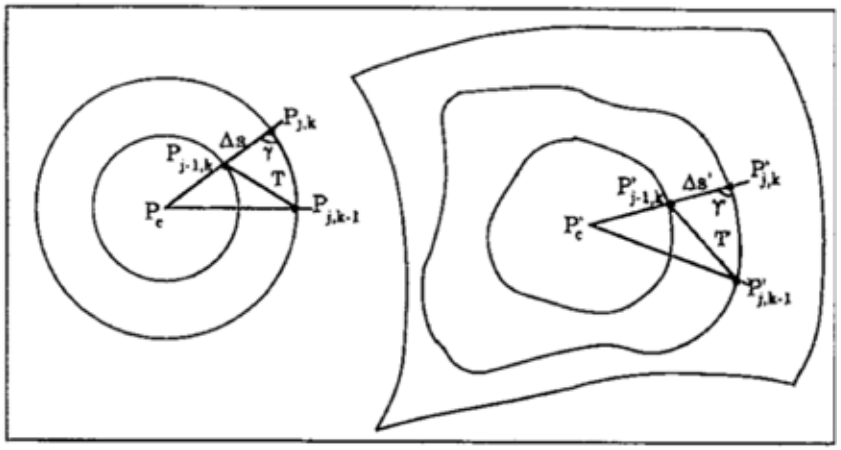
- Compute the components of $\nabla(\nabla I)^2$ using triangulation along directions s_1, s_2, s_3

  From the point $P(x,y,z)=P'_{j,k}$

  s_1, towards $P_1(x_1,y_1,z_1)=P_{j,k}$ (along the ray)

  s_2, towards $P_2(x_2,y_2,z_2)=P'_{j,k-1}$

  s_3, towards $P_3(x_3,y_3,z_3)=P'_{j-1,k}$

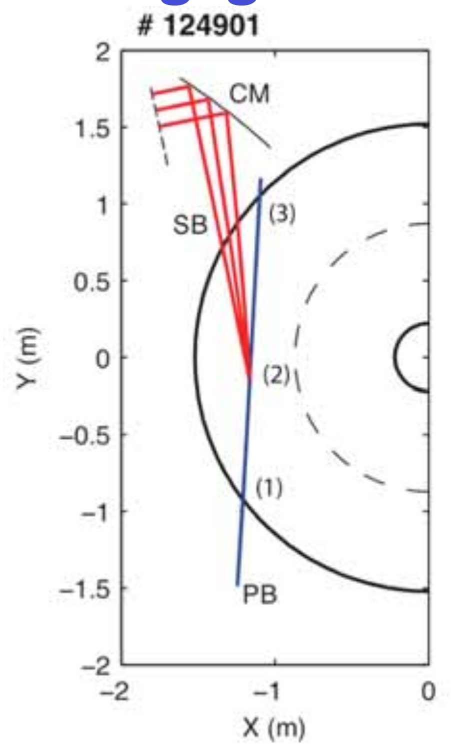


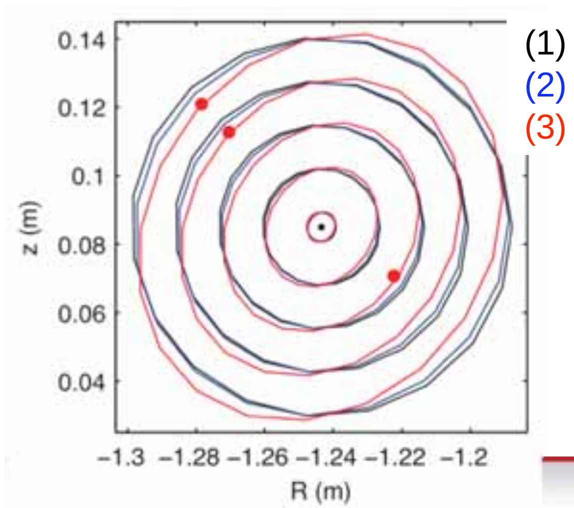
*[Fig.2 from Nowak and Orefice, Phys. Plasmas **1** 1242 (1994)]*

$$\frac{d(\nabla I)^2}{ds_i} \equiv \frac{\left[\nabla I(P_i)\right]^2 - \left[\nabla I(P)\right]^2}{ds_i} = \frac{1}{ds_i}\left(d\varphi_i \frac{\partial}{\partial\varphi} + dR_i \frac{\partial}{\partial R} + dz_i \frac{\partial}{\partial z}\right)(\nabla I)^2$$

$$\left|\nabla I(P'_{j,k})\right| = \left|\frac{1}{\sin\gamma(P_{j,k})} \frac{\partial I(P'_{j,k})}{\partial s'}\right| \qquad \frac{\partial I(P'_{j,k})}{\partial s'} = \frac{\Delta s}{\Delta s'} \frac{\partial I(P_{j,k})}{\partial s}$$

Negligible distortion of the wave front at the location of scattering





- No appreciable spreading of the beam at the location of scattering (2) (high frequency beam)

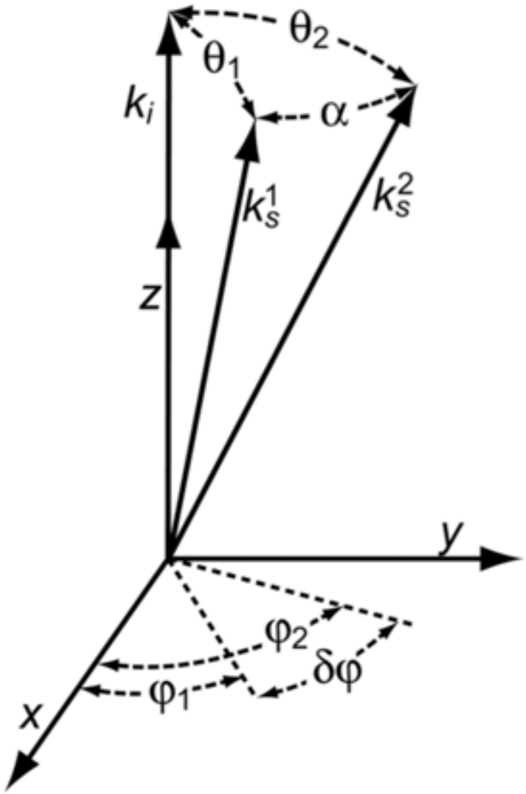
  => Gaussian function used as a weighting function for density fluctuations on the poloidal plane

$$\int_{T'} dt' \int_{V} d^3 r' \boxed{\mathbf{E}_i(\mathbf{r}_\perp)}\, e^{i(\omega t' - \mathbf{k}\cdot\mathbf{r})}\, \tilde{n}(\mathbf{r}', t')$$

⇒ full beam equations not necessary for the propagation and distortion of the beam, but important for the reconstruction of the 3D Instrumental Selectivity Function and of the filtering function (k_r,k_θ) for the simulated spectra

The collection efficiency is optimized at tangent injection

*[E. Mazzucato, Phys. Plasmas **10** 753 (2003)]*



$$F = \exp(-\alpha^2/\alpha_0^2) \qquad \alpha_0 = 2/k_i a$$

$$\alpha^2 \approx (\theta_2 - \theta_1)^2 + 4\theta_2\theta_1 \sin^2(\delta\varphi/2)$$

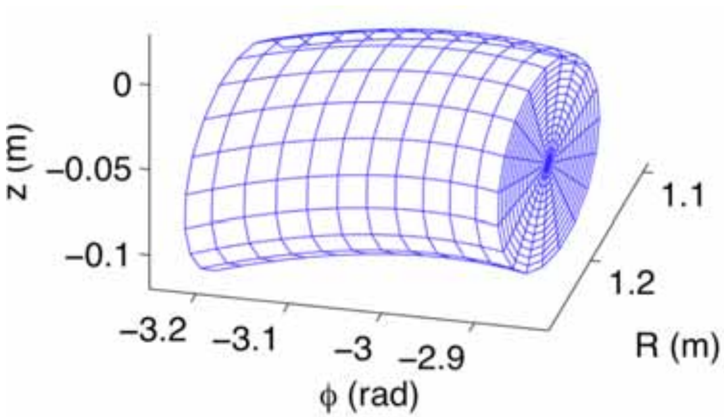
The collection efficiency depends on:

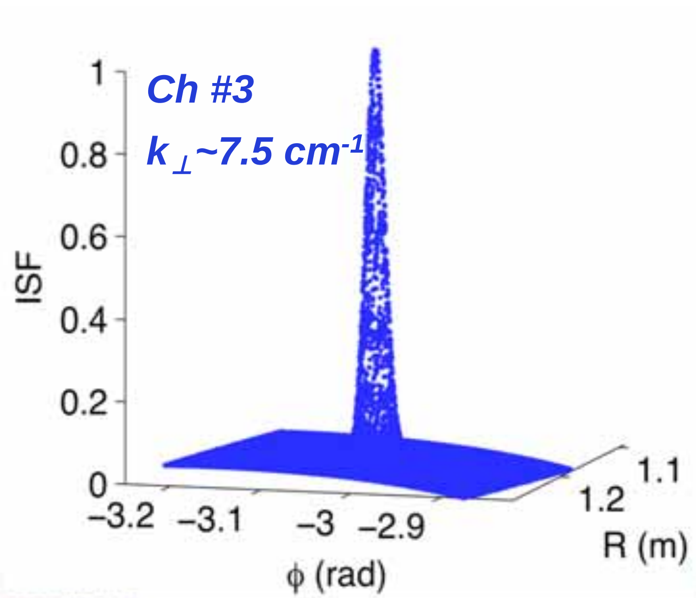
• the scattering angle

• direction of the magnetic field

Max efficiency for scattering along the detector line sight and for tangent injection.

The scattering volume is highly localized in the toroidal direction





1. take a toroidal length $L = \dfrac{2a}{\sin(\theta_s)}$
2. Compute the collection efficiency for all k_i ,θ_s within this volume

The Instrumental Selectivity Function (ISF) is highly localized in ϕ

The resolution in (R,z) is affected by the alignment of incident and scattered beam

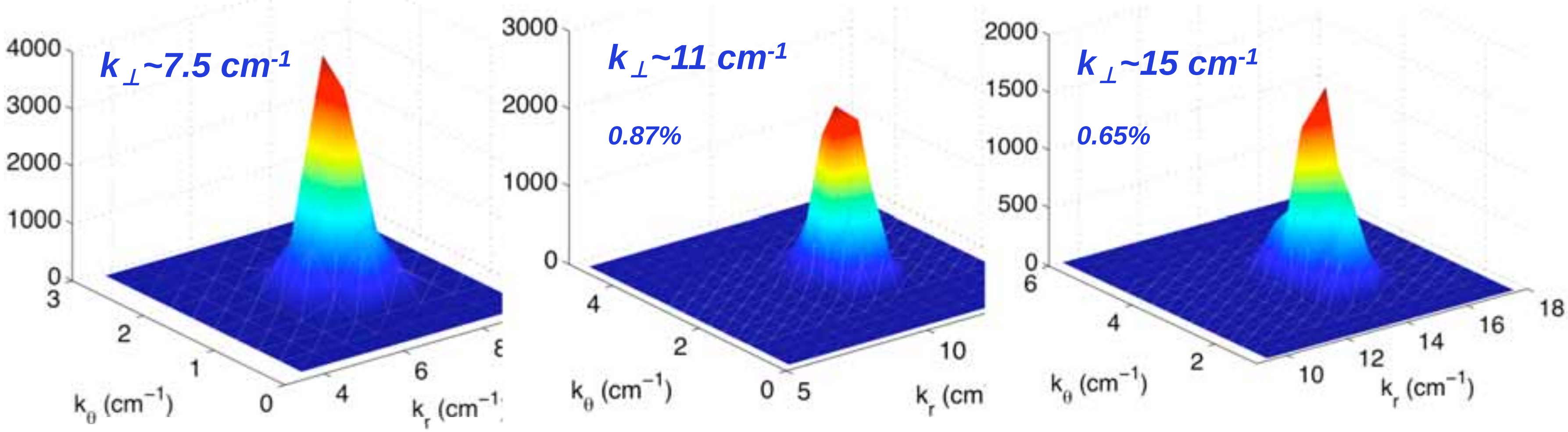
=> Use a function for the receiving window

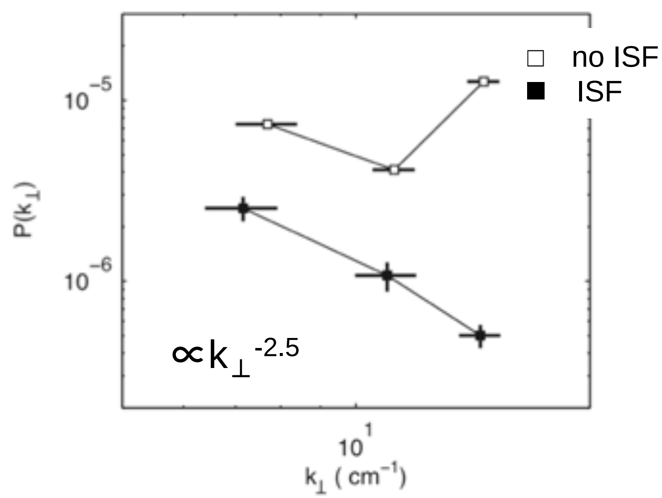
NEXT STEP UPGRADE :

use the 3D extension of the scattering volume for the computation of spectra. Include in the interface for both GTS and GYRO

The collection efficiency is used to reconstruct a (k_r, k_θ) filter for the simulation spectra

NSTX # 124901

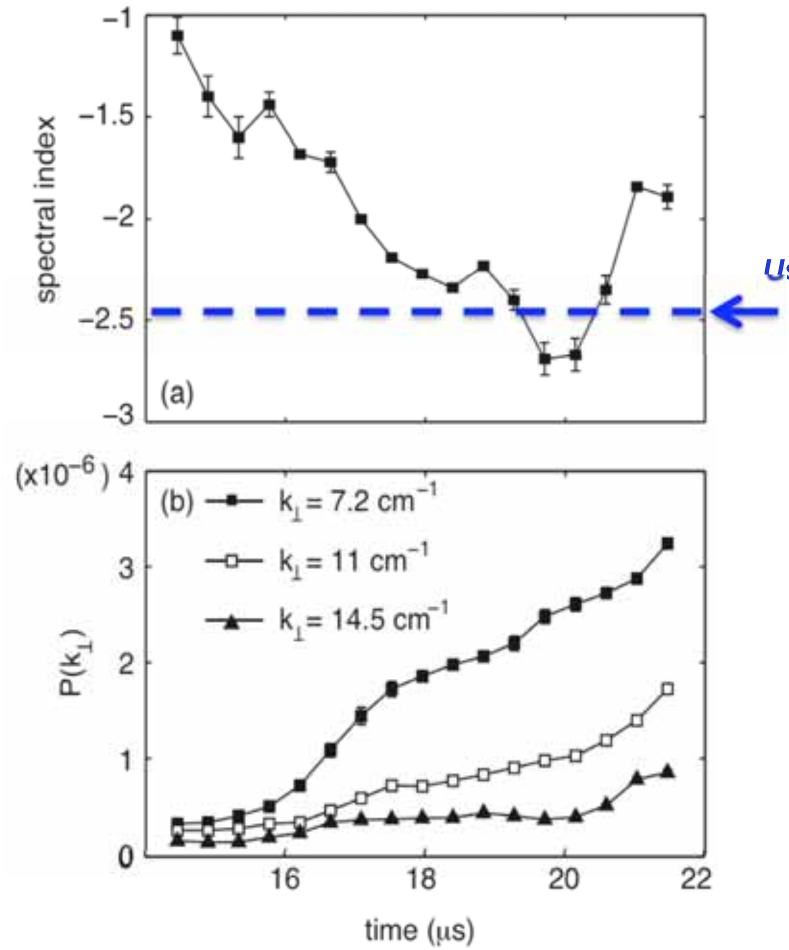




The reconstructed (k_r, k_θ) is important for amplitude correction of both simulated and measured spectra

Simulations must reach stationary phase for a meaningful comparison with experiments

From GTS simulations



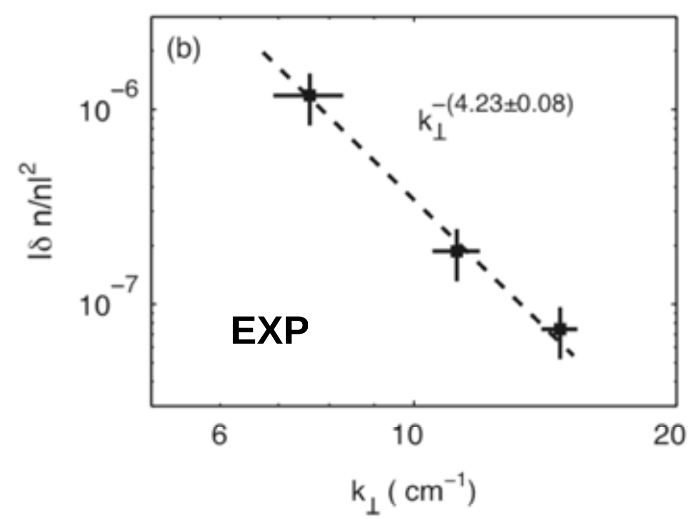
synthetic spectra become steeper (amplitude increases at the smallest $k_\perp$)

But still less steep than in experiments

Using $P(k_r)$ computed at midplane

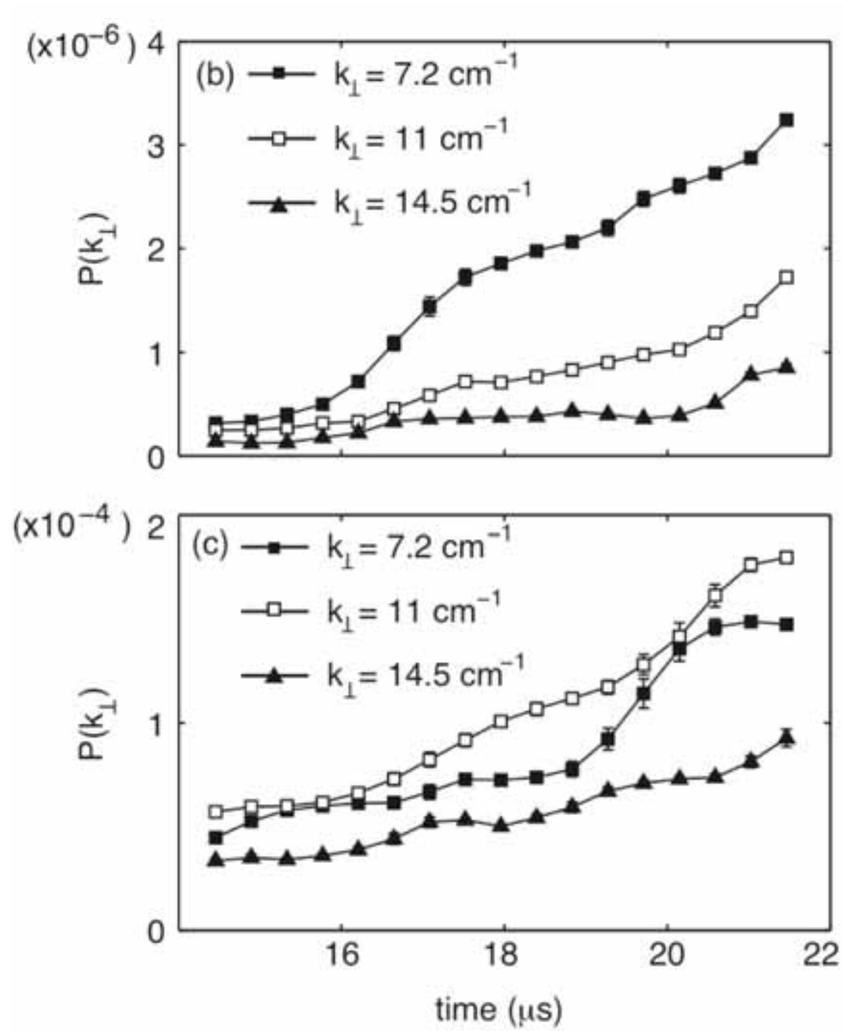
#124901

t = 300 ms, Δt=10ms



The position of the scattering volume does not affect significantly the synthetic spectra, but its size does

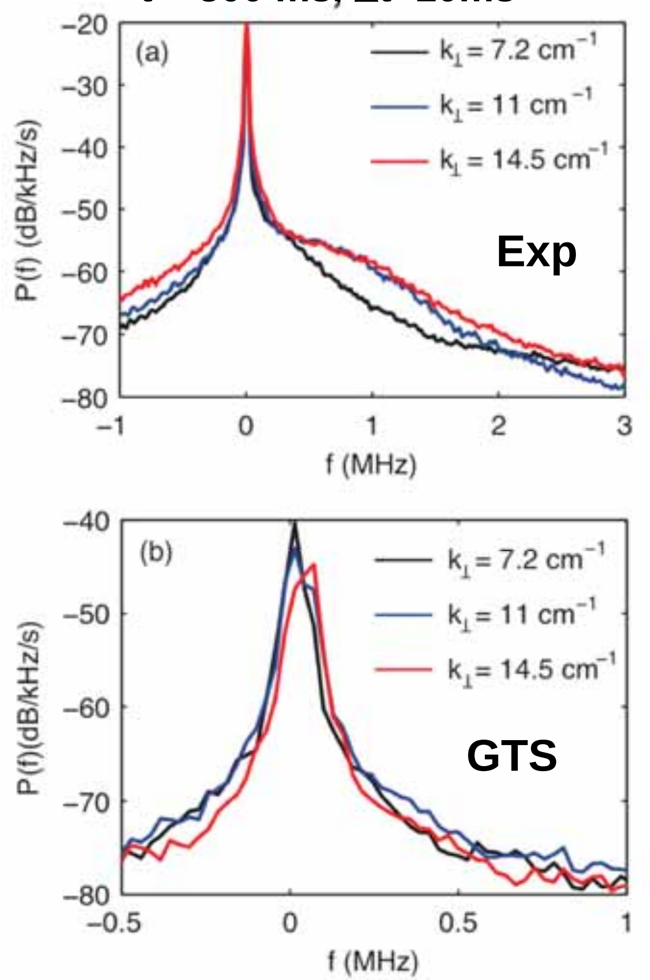
- Power-law dependence visible when the size of the scattering volume is taken into account (b)
- Simulated spectra do not exhibit a power-law dependence in the wavenumber range of experiments (spectra computed without weighting for the beam intensity profile)



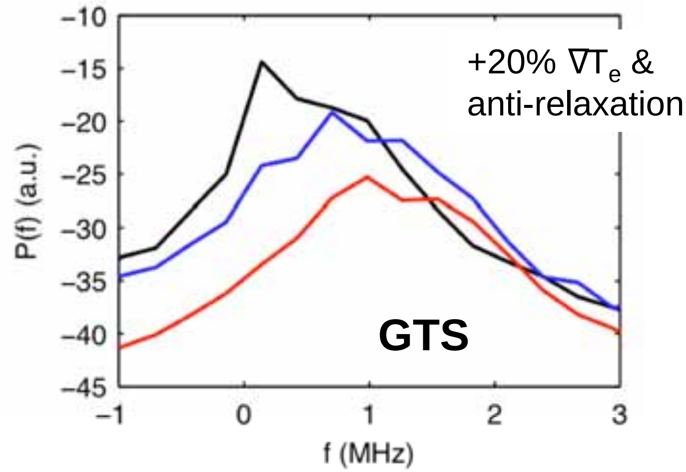
Frequency spectra are broader in experiments

#124901

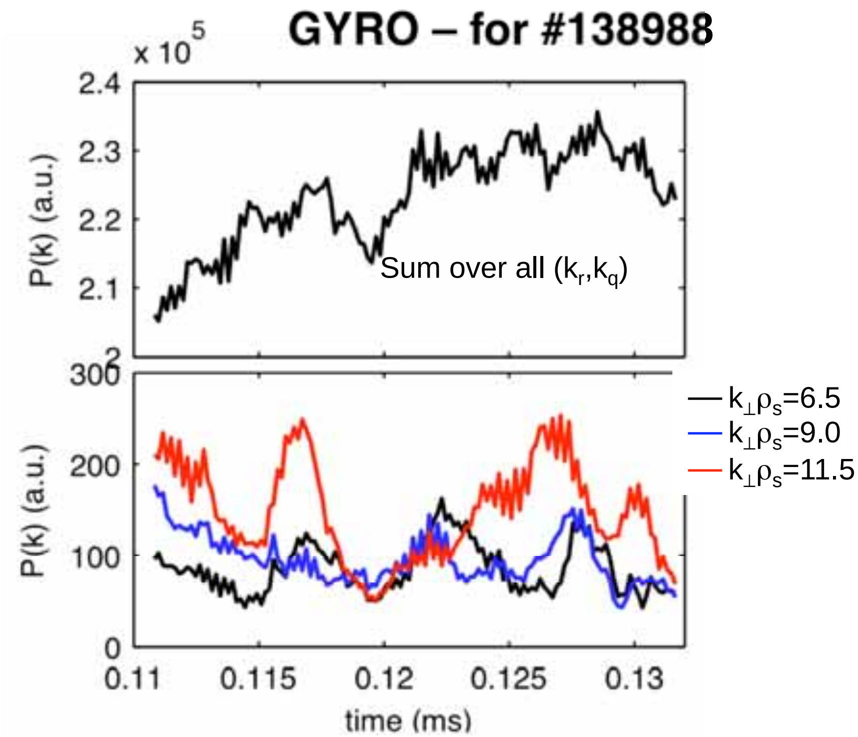
t = 300 ms, Δt=10ms

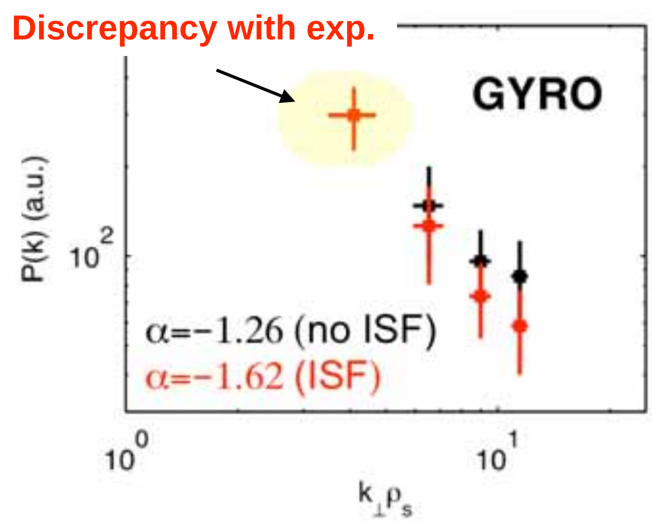


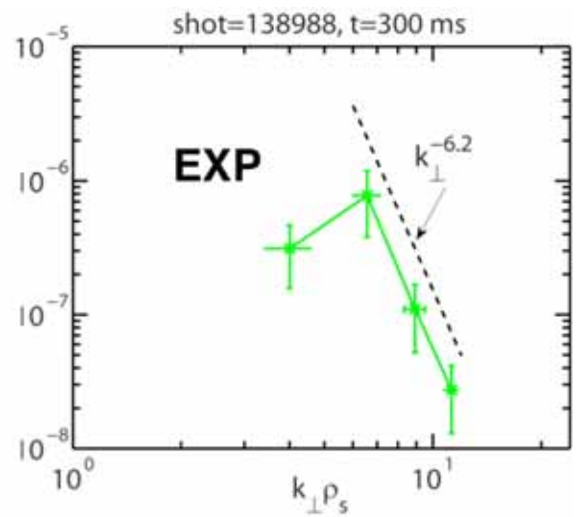
- Peaking frequency is comparable when a Doppler shift of 500 kHz is taken into account (from rotation measurements)
- Simulated spectra are much narrower than measured spectra (left)
- Better match when ∇T_e is increased by 20% and using an 'anti-relaxation' algorithm to maintain the gradient drive (below)



Spectral index less steep than in experiments also in L-mode







- Level of fluctuations appears to be statistically steady
- The ISF does not affect significantly the spectral slope
- the ISF cannot reproduce the dramatic decrease in amplitude at the lowest k
- Predicted ETG transport much smaller than experiment, may not be the dominant mechanism in this L-mode discharge

Summary

- The synthetic high-k scattering consists of standalone blocks (exportable to different codes)
- The computation of spectra uses only 1D interpolations and 1D FFTs

  $\Rightarrow$ to minimize interpolation errors and maximize efficiency

- Systematic errors in the synthetic spectra:
  - Spectral index is mainly affected by:
    - Length of simulations
    - Localization of the scattering volume
    - Collection efficiency
  - errors due to imperfect mapping of equilibrium and experimental profiles have negligible effect on the ray tracing compared to the above

Conclusions and future directions

There is still a long path to go for validating ETG simulations against measurements (and vice-versa)

- The high-k scattering measures in a range of k_θ well below values where simulated ETG turbulence peaks; other instabilities could contribute to the measured level of fluctuation in this range
  - Maybe with a different experimental layout?
- Simulated spectra are less steep than measured spectra
  - Both in L-mode and H-mode simulations
  - With both GTS and GYRO codes

  ⇒ There is still work to do on the simulations to reproduce the experiments
- Geometrical effects in the ISF are not sufficient to reproduce the measured spectral amplitude

  ⇒ Need to improve the Instrumental Selectivity Function, maybe including energy calibration as a function of k