The study of kinetic effect on the eigenfunction of resistive wall mode and the perturbed equilibrium

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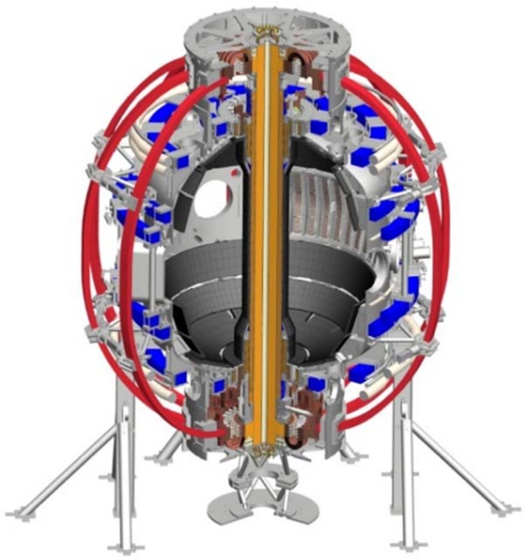

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Abstract

Kinetic effect plays an important role in determining Resistive Wall Modes (RWMs) stability and is also closely related to neoclassical toroidal torque in Tokamaks. Using the toroidal hybrid code Mars-K which self-consistently incorporates drift kinetic effect into magnetohydrodynamic (MHD) formulation, the previous study [1,2] shows that the kinetic resonance substantially changes the mode eigenfuntion particularly near the plasma edge.

- In this work, the Mars-K code in fluid limit is carefully benchmarked with IPEC code [3]. It shows an excellent agreement between the two codes in a simple tokamak case in the presence of the external magnetic perturbation and even in a challenging NSTX equilibrium with an extremely high q value at plasma edge.
- The equivalence between the neoclassical toroidal torque and the kinetic potential energy [4] is verified numerically.
- The code is applied to further study the modification of the RWM eigenfunction and the shielding effect of perturbed equilibrium due to kinetic effect preliminarily.

[1] J.E. Menard, Y. Liu, 39th EPS conference, P1.061(2012)
[2] Z.R. Wang et al, Phys. Plasmas 19, 072518 (2012)
[3] Jong-Kyu Park et al, Phys. Plasmas 14, 052110(2007)
[4] Jong-Kyu Park, Phys. Plasmas 18, 110702(2011)

MARS-K formulation (MHD-Kinetic hybrid code)

The linearized MHD equations including drift kinetic effects via perturbed pressure tensor

MHD equations:

$$(-i\omega+in\Omega)\xi=\mathbf{v}+(\xi\cdot\nabla\Omega)R^2\nabla\phi$$

$$\rho(-i\omega+in\Omega)\mathbf{v}=-\nabla\cdot\mathbf{p}+\mathbf{j}\times\mathbf{B}+\mathbf{J}\times\mathbf{Q}-\rho\left[2\Omega\hat{\mathbf{Z}}\times\mathbf{v}+(\mathbf{v}\cdot\nabla\Omega)R^2\nabla\phi\right]$$

$$(-i\omega+in\Omega)\mathbf{Q}=\nabla\times(\mathbf{v}\times\mathbf{B})+(\mathbf{Q}\cdot\nabla\Omega)R^2\nabla\phi$$

$$(-i\omega+in\Omega)p=-\vec{\mathbf{v}}\cdot\vec{\nabla}P-\cancel{\Gamma P\vec{\nabla}\cdot\vec{\mathbf{v}}}$$

replaced by kinetic pressure

$$\mathbf{j}=\nabla\times\mathbf{Q}$$

Solve MHD-Kinetic equations self-consistently

$$\mathbf{p}=p\mathbf{I}+p_{\parallel}\hat{\mathbf{b}}\hat{\mathbf{b}}+p_{\perp}\left(\mathbf{I}-\hat{\mathbf{b}}\hat{\mathbf{b}}\right)$$

$$p_{\parallel}e^{-i\omega t+in\phi}=\sum_{e,i}\int d\Gamma M v_{\parallel}^2 f_L^1 \qquad p_{\perp}e^{-i\omega t+in\phi}=\sum_{e,i}\int d\Gamma \frac{1}{2}M v_{\perp}^2 f_L^1$$

$$f_L^1=-f_{\varepsilon}^0\varepsilon_k e^{-i\omega t+in\phi}\sum_{m,l}X_m H_{ml}\lambda_{ml}e^{-in\tilde{\phi}(t)+im<\dot{\chi}>+il\omega_b t}$$

$$\text{Resonant operator: } \lambda_{ml}=\frac{n\left[\omega_{*N}+\left(\hat{\varepsilon}_k-3/2\right)\omega_{*T}+\omega_E\right]-\omega}{n\omega_d+\left[\alpha(m+nq)+l\right]\omega_b+n\omega_E-\omega-i\nu_{eff}}$$

- **ω_{*N} and ω_{*T} are the diamagnetic drift frequencies**
- **ω_E is the E cross B drift frequency**
- **ω_d and ω_b are the precession and bounce frequencies**
- **Trapped particle $\alpha=0$. Passing particle $\alpha=1$.**

Drift-kinetic equation:

$$\boxed{\frac{df_L^1}{dt}=f_{\varepsilon}^0\frac{\partial H^1}{\partial t}-f_{P_\phi}^0\frac{\partial H^1}{\partial\phi}-\nu_{eff}f_L^1}$$

$$H^1=\frac{1}{\varepsilon_k}\left[Mv_{\parallel}^2\vec{\kappa}\cdot\vec{\xi}_{\perp}+\mu\left(Q_{\parallel}+\vec{\nabla}B\cdot\vec{\xi}_{\perp}\right)\right]$$

Y.Q. Liu, et al Phys. Plasmas 15, 112503 2008

RWM Kinetic Energy Principle
The energy component analysis can also be used for NTV study

- The generalized dispersion relation for RWM (Neglecting inertia term)

$$(\gamma - i\omega_r)\tau_w^* = -\frac{\delta W_\infty + \delta W_k}{\delta W_b + \delta W_k}$$

$$\delta W_\infty = \delta W_F + \delta W_{v\infty}$$

$$\delta W_b = \delta W_F + \delta W_{vb}$$

Bo Hu and R. Betti PRL 2004

- potential energy components analyzed by Mars-F/K

Plasma fluid energy:

$$\delta W_F = \delta W_{mb} + \delta W_{mc} + \delta W_{pre} + \delta W_{cur}$$

Magnetic bending | Magnetic compressibility | Pressure driven | Current driven

$$\delta W_{mb} = \frac{1}{2}\int_{V^P} |\mathbf{Q}_\perp|^2 \, Jdsd\chi d\phi$$

$$\delta W_{mc} = \frac{1}{2}\int_{V^P} B^2 |\nabla \cdot \boldsymbol{\xi}_\perp + 2\boldsymbol{\xi}_\perp \cdot \boldsymbol{\kappa}|^2 \, Jdsd\chi d\phi$$

$$\delta W_{pre} = -\frac{1}{2}\int_{V^P} (\boldsymbol{\xi}_\perp \cdot \nabla P)(\boldsymbol{\kappa} \cdot \boldsymbol{\xi}_\perp^*) \, Jdsd\chi d\phi$$

$$\delta W_{cur} = -\frac{1}{2}\int_{V^P} J_\parallel (\boldsymbol{\xi}_\perp^* \times \mathbf{b}) \cdot \mathbf{Q}_\perp \, Jdsd\chi d\phi$$

Kinetic energy:

$$\delta W_K = \frac{1}{2}\int_{V^P} Jdsd\chi d\phi \left[p_\perp \frac{1}{B}\left(Q_\parallel^* + \nabla B \cdot \boldsymbol{\xi}_\perp^*\right) + p_\parallel \boldsymbol{\kappa} \cdot \boldsymbol{\xi}_\perp^* \right]$$

Vacuum energy:

$$\delta W_{v\infty} = \frac{1}{2}\int_{V^\infty} |\mathbf{Q}|^2 \, Jdsd\chi d\phi = -\frac{1}{2}\int_{S^P} b_1^n \widehat{V}_1^{*\infty} J_s d\chi d\phi$$

$$\delta W_{vb} = \frac{1}{2}\int_{V^b} |\mathbf{Q}|^2 \, Jdsd\chi d\phi = -\frac{1}{2}\int_{S^P} b_1^n \widehat{V}_1^{*b} J_s d\chi d\phi$$

Kinetic energy replaces the plasma compressibility term in fluid theory and can be complex number (mode – particle interaction) $\delta W_k = \delta W_k^{re} + i\, \delta W_k^{i}$

Vacuum energy terms when ideal wall is at infinity r=∞ and r=b

The new development of MARS-K for NTV calculation

- **In order to carry out the NTV calculation, the energy-dependent collisional frequency has been integrated into MARS-K for trapped particles.** The collisionality is the same as that used in IPEC [**Park, Phys. Rev. Lett. 102 065002 (2009)**].

Kinetic pressure

$$\left(Jp_{\parallel}\right)_k = \frac{1}{\sqrt{\pi}} \sum_{e,i} \sum_{m,l} \frac{P_{e,i}}{B_0} \int d\Lambda I_l H^u_{ml} G^{\parallel}_{kml} X^u_m \qquad \left(Jp_{\perp}\right)_k = \frac{1}{\sqrt{\pi}} \sum_{e,i} \sum_{m,l} \frac{P_{e,i}}{B_0} \int d\Lambda I_l H^u_{ml} G^{\perp}_{kml} X^u_m$$

Energy integral of resonant operator for trapped particles

$$I_{ml} = \sum_{\sigma} \int_0^{\infty} d\hat{\varepsilon}_k \hat{\varepsilon}_k^{5/2} e^{-\hat{\varepsilon}_k} \frac{n\left[\omega_{*N} + \left(\hat{\varepsilon}_k - 3/2\right)\omega_{*T} + \omega_E\right] - \omega}{n\omega_E + n\omega_D + l\omega_b - \omega + i\nu_{eff}}$$

Energy-dependent collisional frequency

$$\nu_{eff} = \nu_{Di} \hat{\varepsilon}_k^{-3/2} \left[1 + \left(l/2\right)^2\right] \qquad \nu_{Di} = \frac{n_d Z^p_{eff} \ln \Lambda_d}{3.5\times 10^{17} \sqrt{m_d / m_p} T_d^{3/2} \left(keV\right)} \; [1/s]$$

l is the bounce harmonic number of trapped particles, *d*=i,e

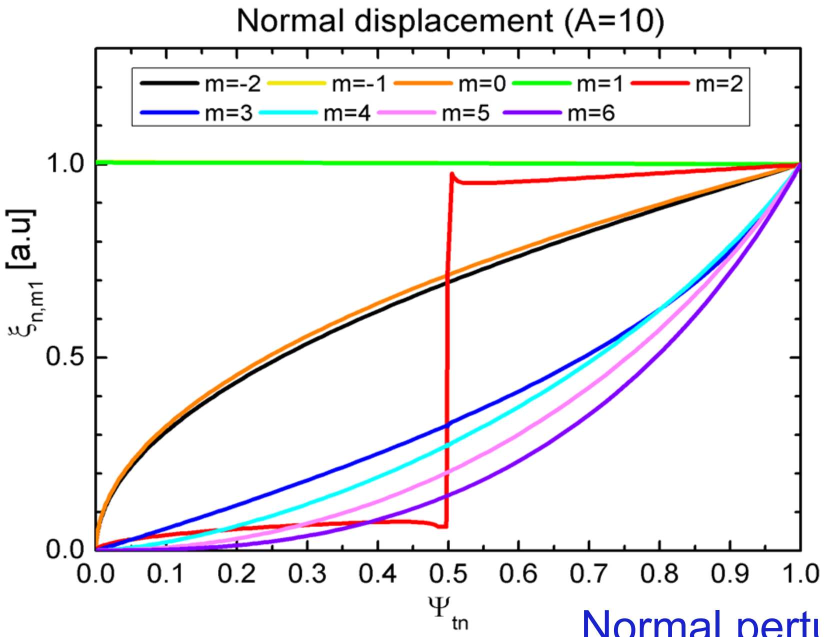
- **The Message Passing Interface (MPI)-based parallelization of kinetic calculation in MARS-K code has been completed.** Previously, the kinetic calculation in MARS-K consumed a long computation time particularly for the case requiring high numerical accuracy e.g. NSTX with high q edge value. The new parallelization allows each compute node independently calculates the kinetic terms on the assigned flux surface and sends back the result to master process for further solving the linearized MHD equations including the drift kinetic effect. This development can significantly improve the performance of the code and is important to the computational efficiency of energy integral including the energy-dependent collisional frequency.

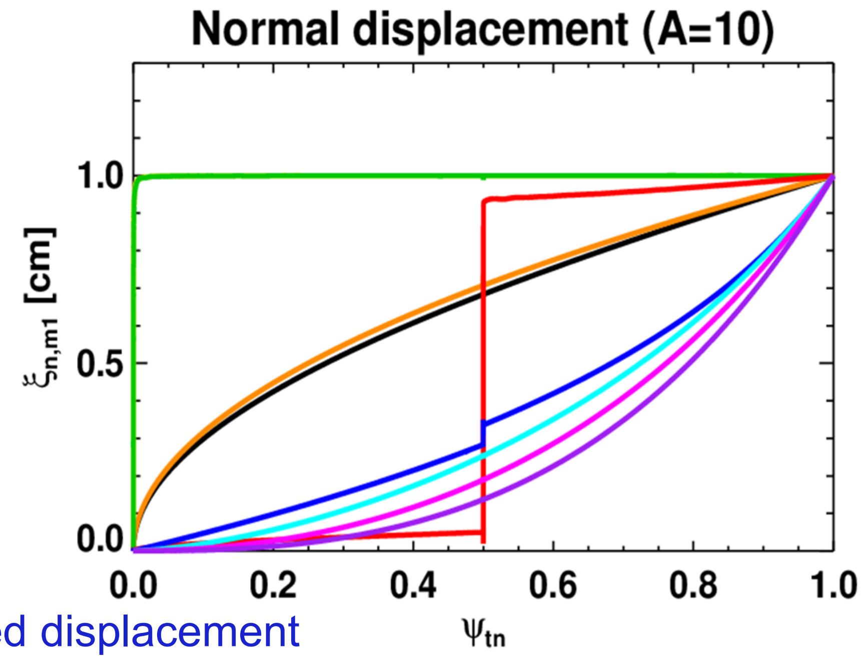
Ideal 3D perturbed tokamak equilibrium solutions have been compared between IPEC and MARS

IPEC and MARS were successfully benchmarked on ideal 3D equilibrium, using the perturbed A=10 larege aspect ratio tokamak case. 3D external coils were implemented in MARS-F to construct the perturbed equilibrium solutions. Agreements are very good.

Tokamak with A=10, q_0=1.67, q_a =2.50 β=0.0

***The applied external perturbation: m=-2 m=-1 m=0 m=1 m=2 m=3 m=4 m=5 m=6*, n=1**





Normal perturbed displacement

Mars-F result
in PEST coordinates

IPEC result
in cylindrical coordinates

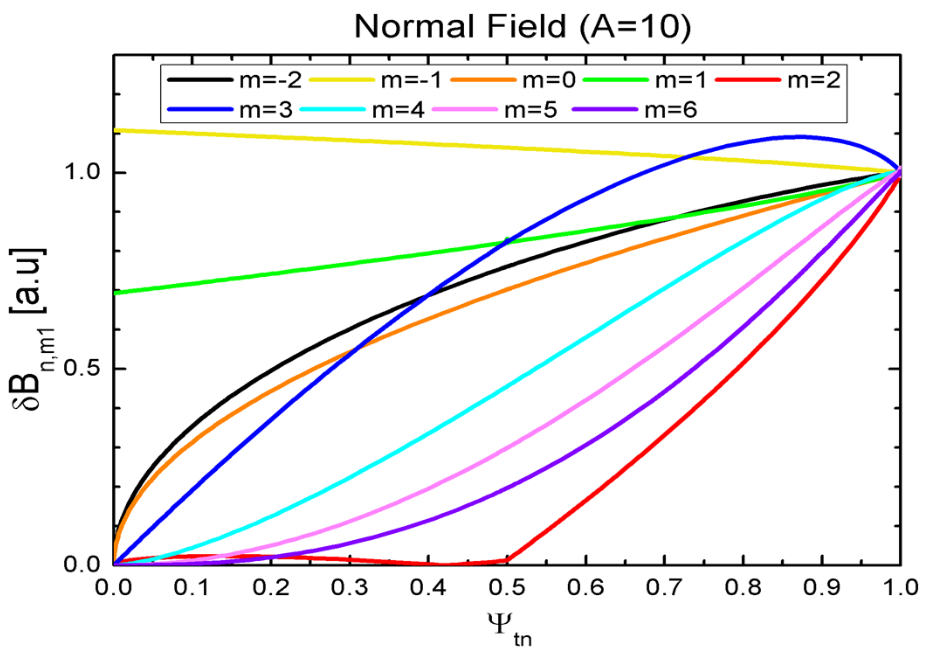
Very good agreement between IPEC and MARS-F

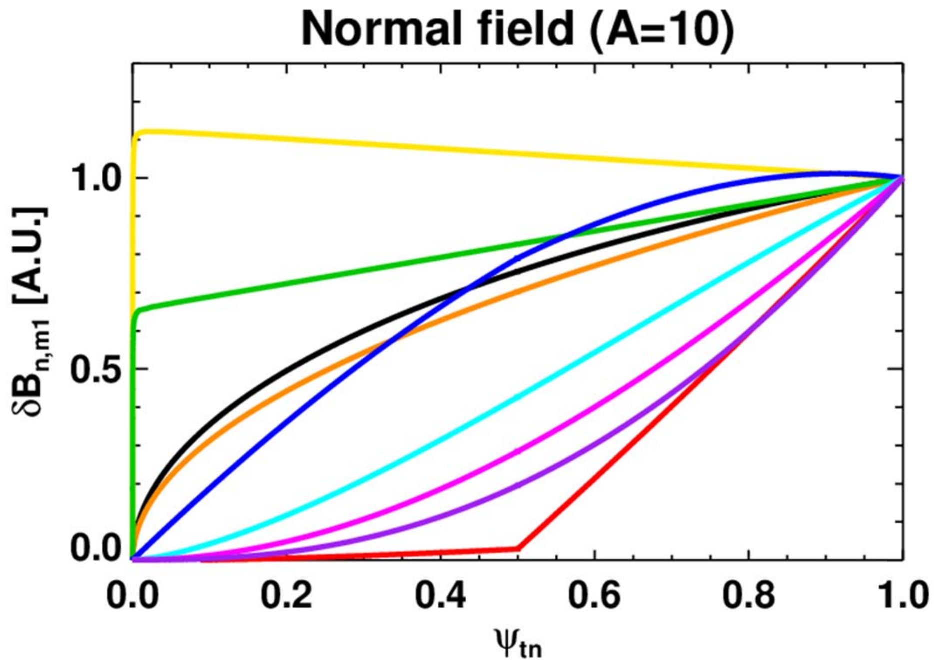
Ideal 3D perturbed tokamak equilibrium solutions have been compared between IPEC, MARS

In order to get the perturbed equilibrium solution, n=1 external perturbation with different m poloidal harmonic number is applied to plasma in both codes for the benchamrk.

Tokamak with A=10, q_0=1.67, q_a =2.50 β=0.0

***The applied external perturbation: m=-2 m=-1 m=0 m=1 m=2 m=3 m=4 m=5 m=6*, n=1**





Normal magnetic perturbation

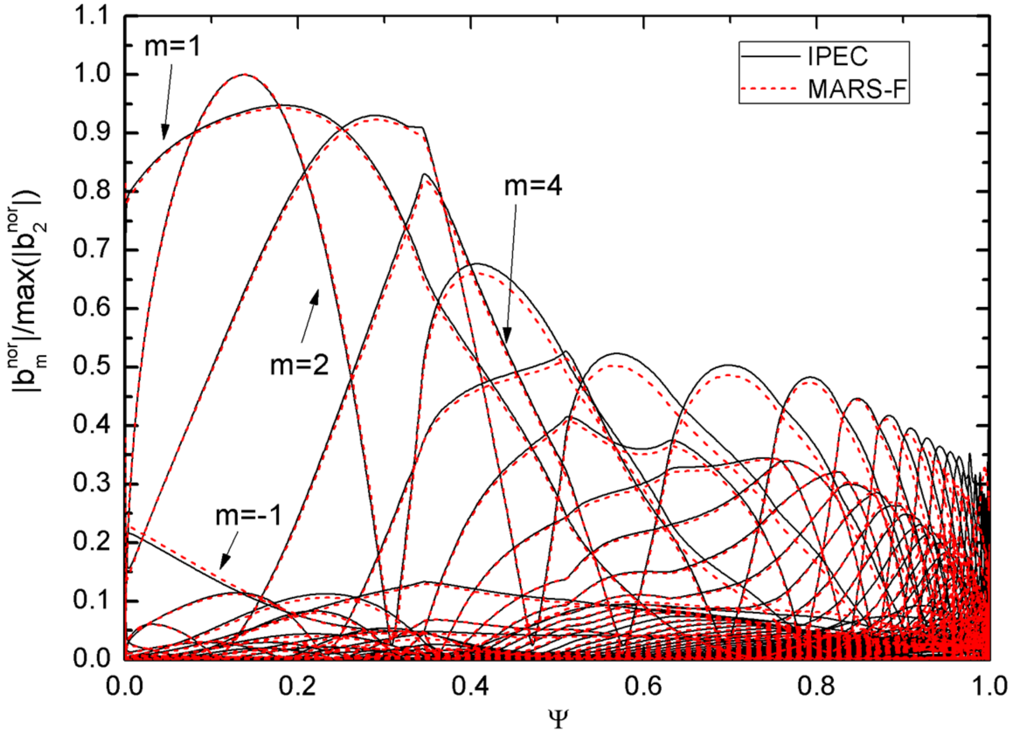
Mars-F result
in PEST coordinates

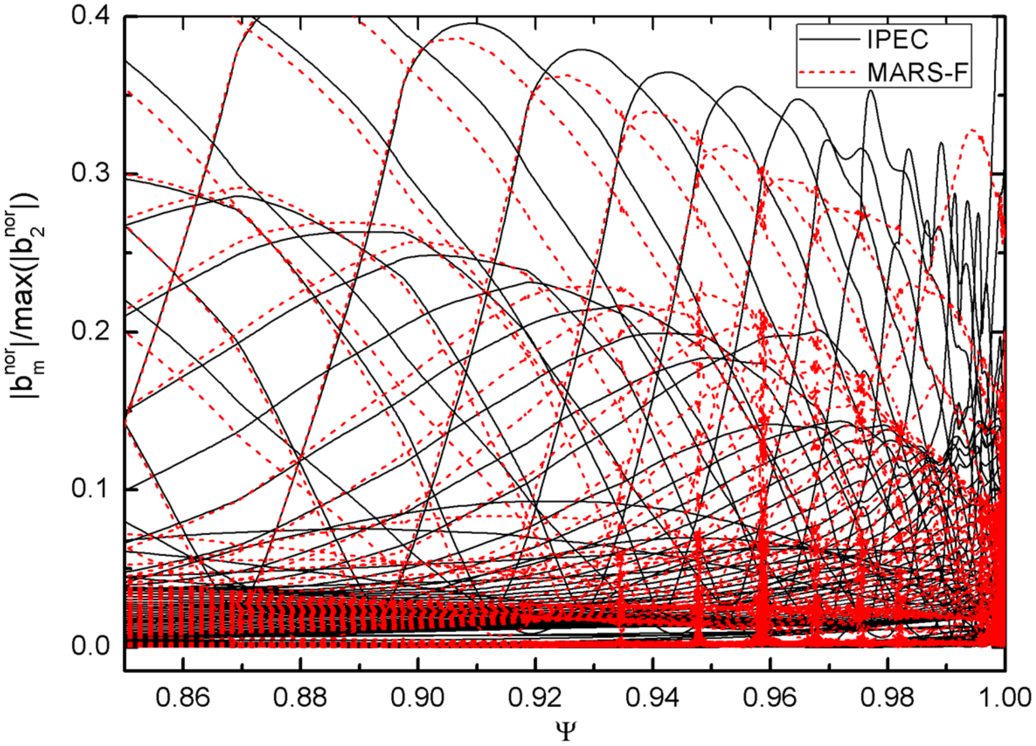
IPEC result
in cylindrical coordinates

Very good agreement between IPEC and MARS-F

IPEC and MARS ideal 3D equilibrium benchmark was also successful for a numerically challenging NSTX case

IPEC and MARS-F ideal solutions also agree very well for a challenging case, such as NSTX (Shot No.119621: q_0=1.40, q_a=19.2, β_N=5.08)

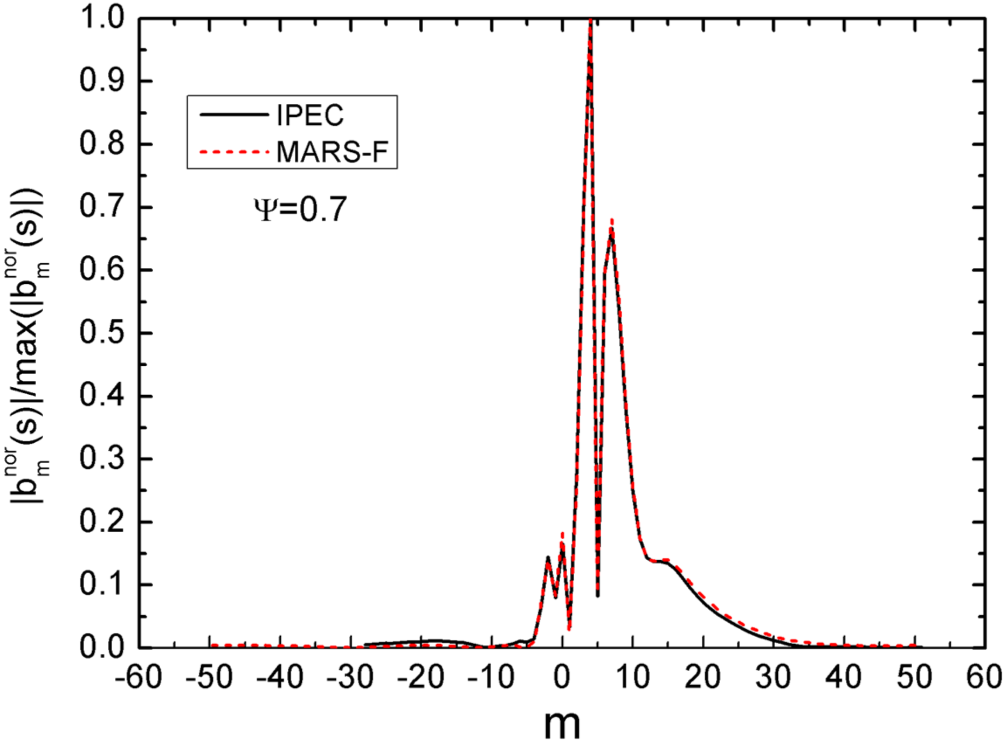


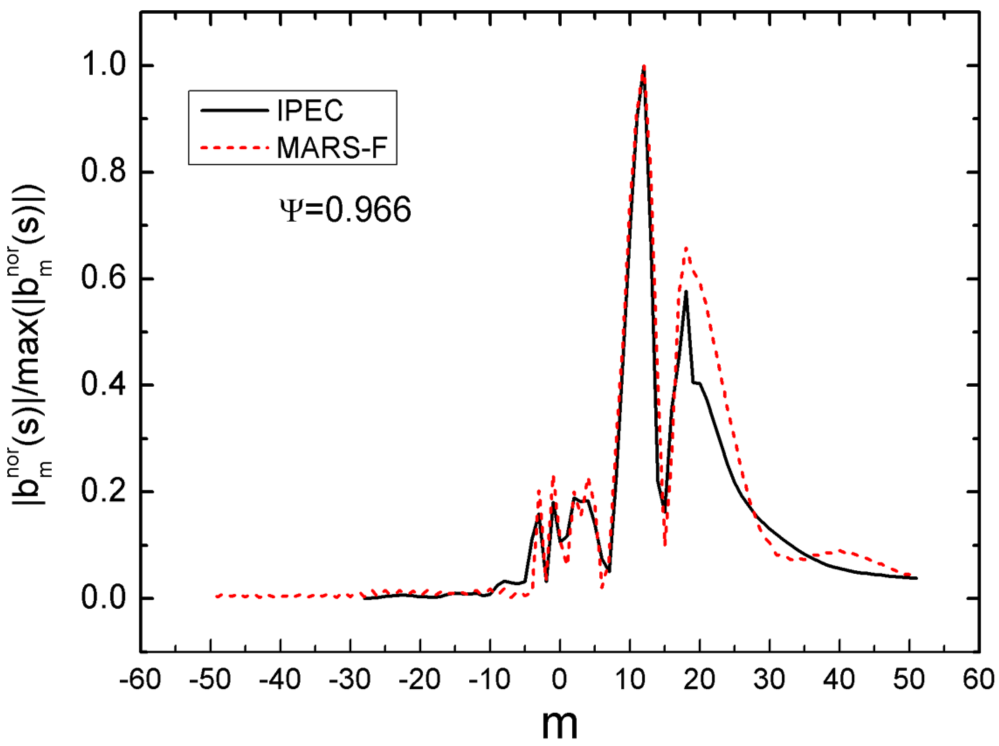


The normal magnetic perturbation shows a very good agreement between IPEC and MARS-F

IPEC and MARS ideal 3D equilibrium benchmark was also successful for a numerically challenging NSTX case

IPEC and MARS-F ideal solutions also agree very well for a challenging case, such as NSTX (Shot No.119621: q_0=1.40, q_a=19.2, β_N=5.08)





The spectrum of poloidal harmonic of normal magnetic perturbation shows a very good agreement between IPEC and MARS-F at different flux surface.

Our research aims at general perturbed equilibrium solver beyond ideal equilibrium, including kinetic modifications

- **3D field produces 3D transport and forces, which can change 3D equilibrium**

- **A well-known term is the anisotropic pressure created by 3D field**

$$\vec{\nabla}\bullet\delta\overleftrightarrow{p} = \delta\vec{j}\times\vec{B}_0 + \vec{j}_0\times\delta\vec{B}$$

$$\delta\overleftrightarrow{p} = p\overleftrightarrow{\mathbf{I}} + p_{\parallel}\hat{\mathbf{b}}\hat{\mathbf{b}} + p_{\perp}\left(\overleftrightarrow{\mathbf{I}} - \hat{\mathbf{b}}\hat{\mathbf{b}}\right) \qquad p_{\parallel} = \sum_{e,i}\int d\Gamma M v_{\parallel}^{2} f_{L}^{1} \qquad p_{\perp} = \sum_{e,i}\int d\Gamma \frac{1}{2} M v_{\perp}^{2} f_{L}^{1}$$

- **First, we are developing the capability to calculate this anisotropic pressure term (so the kinetic stabilizing energy and NTV torque) when δB is given by ideal equilibrium**

NTV Calculation: IPEC-NTV uses the analytical formula of drift kinetic equation.
POCA is local particle code and simulates the particle guiding center.

Kim,Park,POP 19, 082503(2012)

- **Next, we will include the anisotropic terms in the first place of equilibrium calculations and to obtain precise δB**

Park, PoP 18 110702 (2011)

$$T_{\varphi} = 2in\delta W_{K}$$

$$\delta W_{k} = \frac{1}{2M^{2}}\int d\psi_{p} d\varphi dE d\mu\left(\delta J f_{L}^{1}\right) \qquad T_{\varphi} = \frac{1}{M^{2}}\int d\psi_{p} d\varphi dE d\mu\left(\frac{\partial\delta J}{\partial\varphi} f_{L}^{1}\right)$$

MARS-K has the possibility to provide a general perturbed equilibrium solution including the NTV in a self-consistent way, as predicted by Park, PoP 18

- **Other effects by rotation (such as the inertia) will be also included**

$$\rho_0\left[\left(\vec{v}_0\bullet\vec{\nabla}\right)\delta\vec{v} + \left(\delta\vec{v}\bullet\nabla\right)\vec{v}_0\right] + \delta\rho\left(v_0\bullet\vec{\nabla}\right)\vec{v}_0 + \vec{\nabla}\bullet\delta\overleftrightarrow{p} = \delta\vec{j}\times\vec{B}_0 + \vec{j}_0\times\delta\vec{B}$$

Preliminary NTV benchmark shows qualitative agreement between IPEC and MARS, and the numerical evidence of T_φ=2inδW_K.

- **IPEC-NTV calculates NTV torque with simplified collisional operator.**

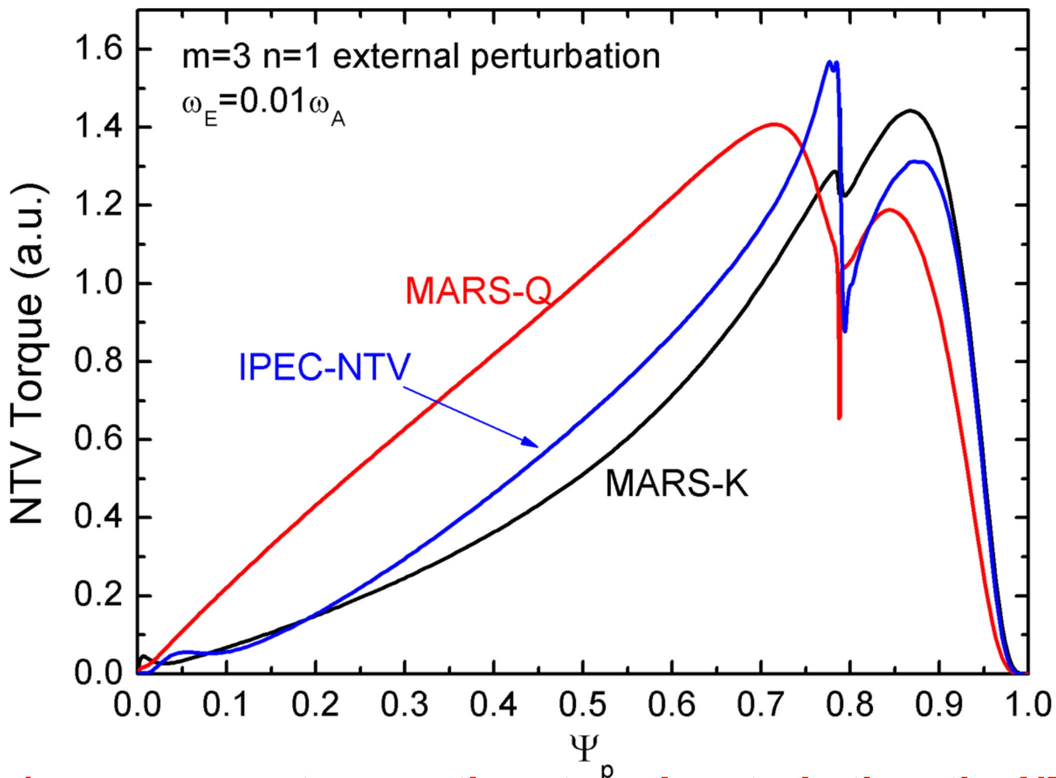
Park, PoP 18 110702 (2011)
Park, PRL 102, 065002 (2009)

- **MARS-Q calculates NTV torque based on the analytic formula that smoothly connect different collisionality regimes.**

Y.Q. Liu 2012 EPS conferencee
K. C. Shaing et al 2010 Nucl. Fusion 50 025022

- **MARS-K calculates δW_K and provides NTV torque based on the equivalence** $T_\varphi = 2in\delta W_K$

Large aspect ratio Tokamak with A=10, q_0=1.24, q_a =2.80 β_N=0.11



In the presence of m=3,n=1 non-resonant magnetic external perturbation, the NTV torque calculated by IPEC-NTV, MARS-Q and MARS-K shows a qualitative agreement, where the ideal perturbed equilibrium (no kinetic effect), and only l=0 bounce harmonic of trapped ions is considered in NTV calculation.

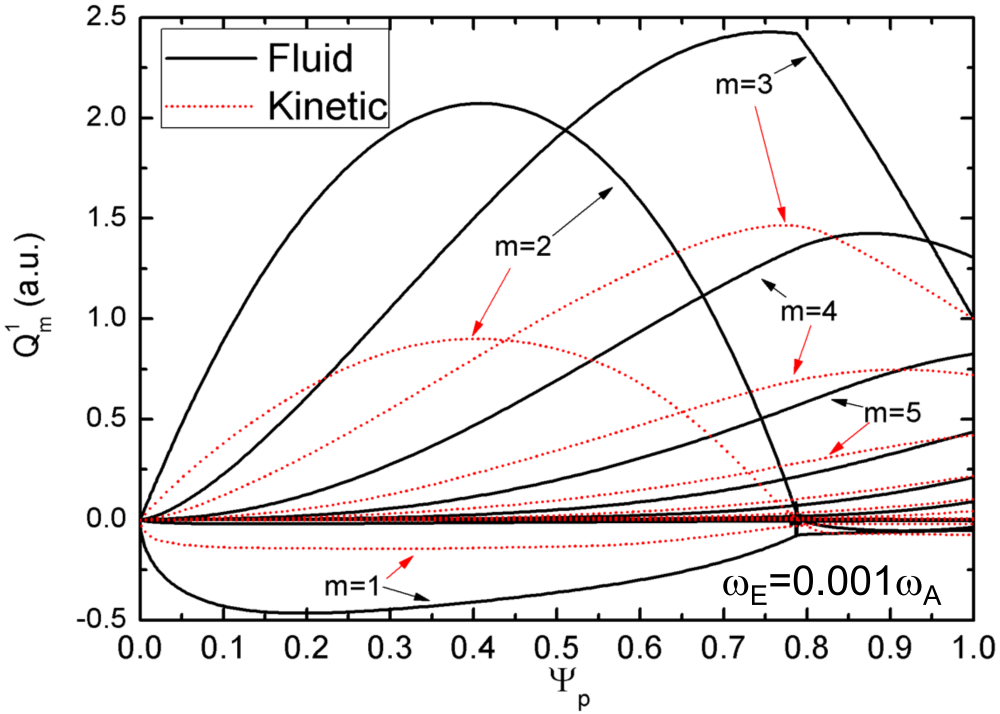
The shielding effect of perturbed equilibrium due to kinetic effect

With the fixed m=3,n=1 non-resonant external perturbation at plasma edge, compared with the solution of ideal perturbed equilibrium (Fluid), the general perturbed equilibrium including the kinetic effect self-consistently shows:

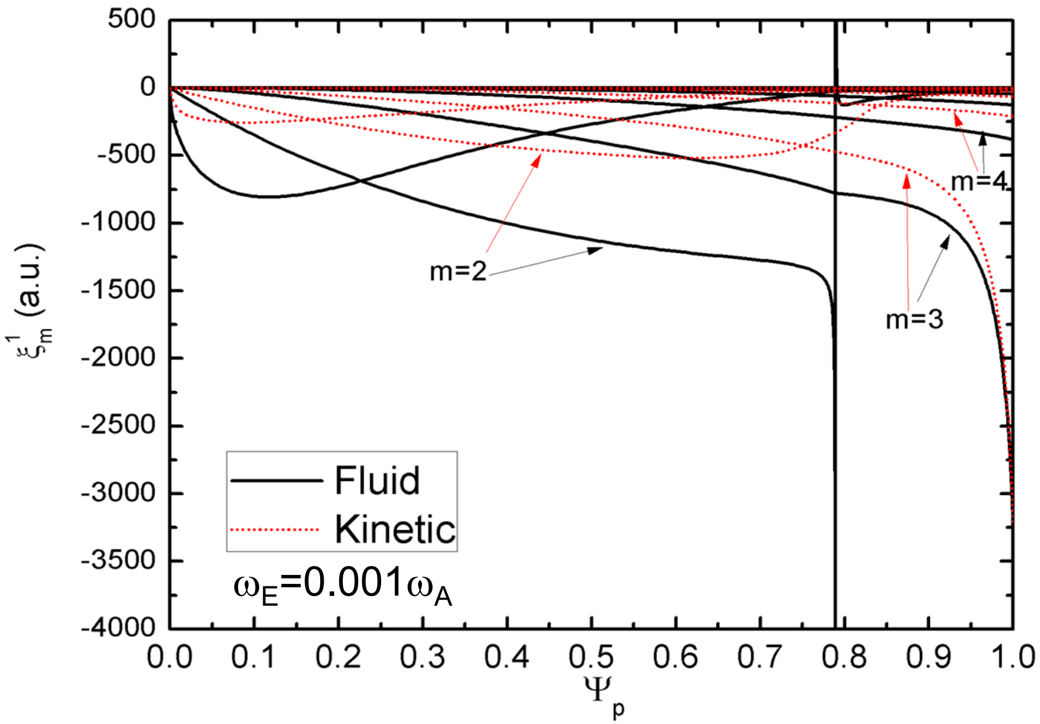
- **The magnetic perturbation inside plasma is shielded due to the kinetic effect.**
- **The plasma perturbed displacement is also suppressed.**

Large aspect ratio Tokamak with A=10, q_0=1.16, q_a =2.90 β=0.004, β_N=2.2

Magnetic perturbation in normal direction



Perturbed displacement in normal direction



l=0 and l≠0 bounce harmonics of trapped particles are included. The uniform ω_E is assumed.

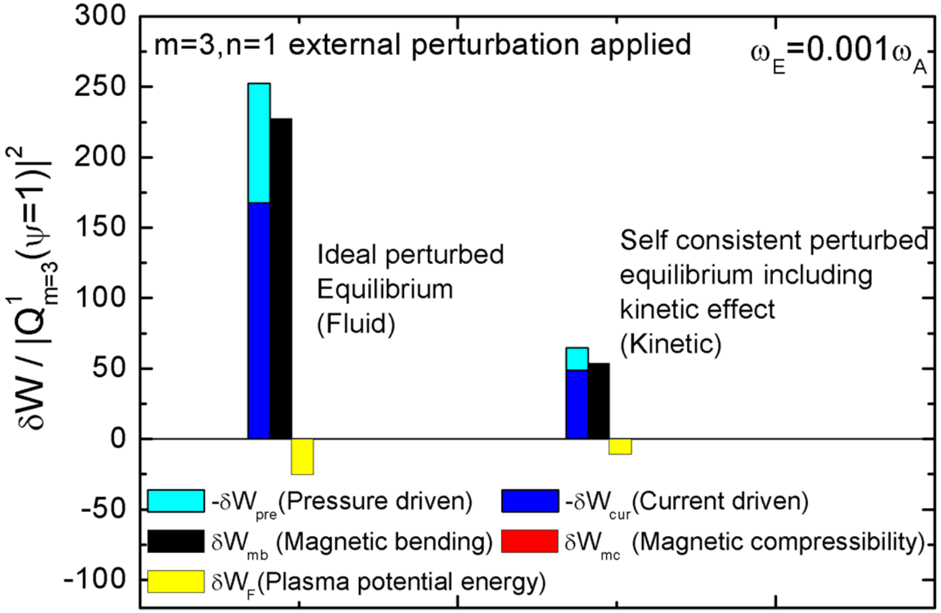
The plasma potential energy and NTV profile are strongly modified by shielding effect due to kinetic effect

With m=3, n=1 non-resonant external perturbation fixed at the plasma edge, the self-consistent calculation of perturbed equilibrium including the kinetic effect shows strongly modification on:

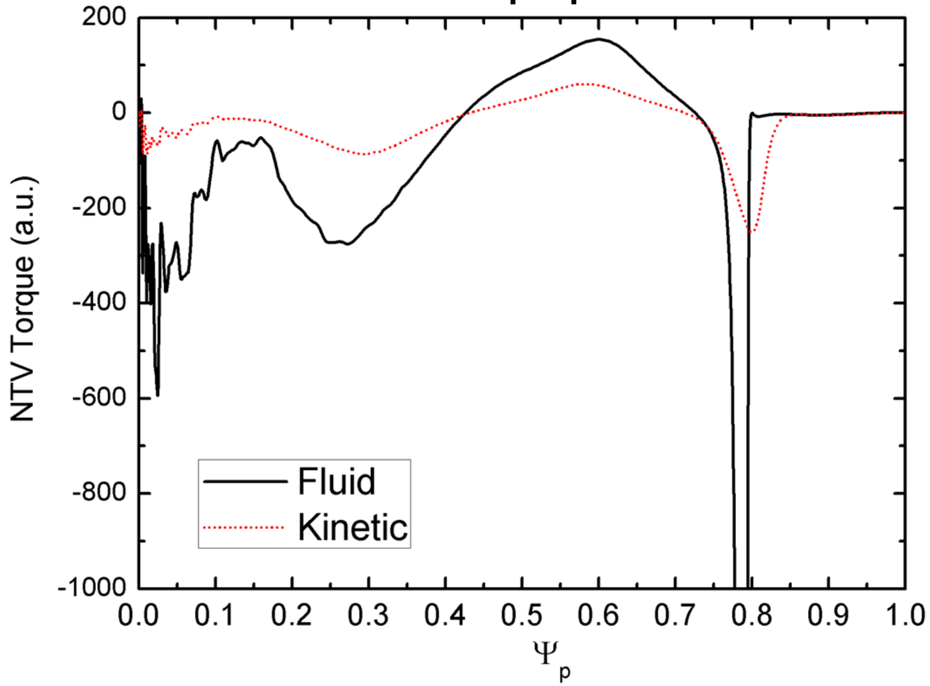
- **the fluid potential energy components and δW_F**
- **the NTV torque profile**

Large aspect ratio Tokamak with A=10, q_0=1.16, q_a =2.90 β=0.004, β_N=2.2

Potential energy analysis



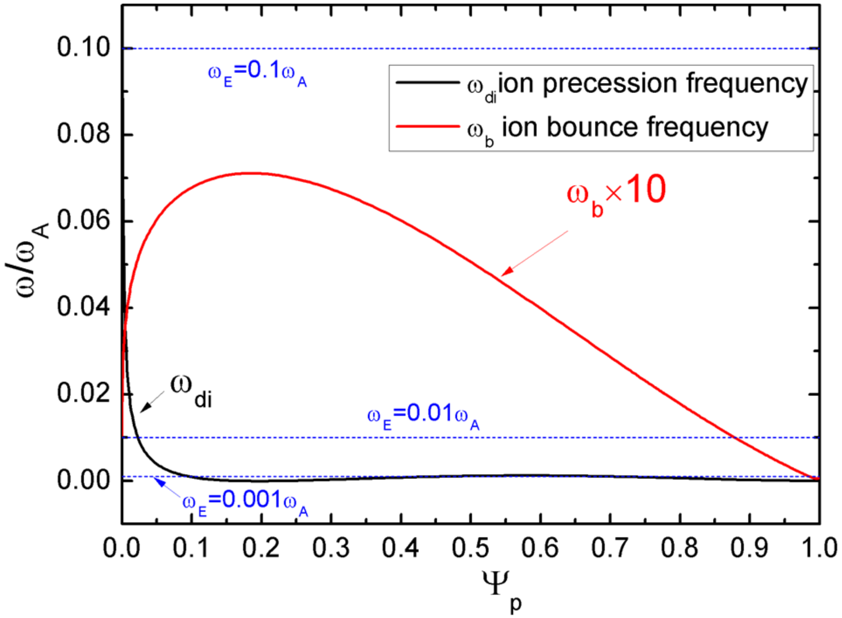
NTV torque profile

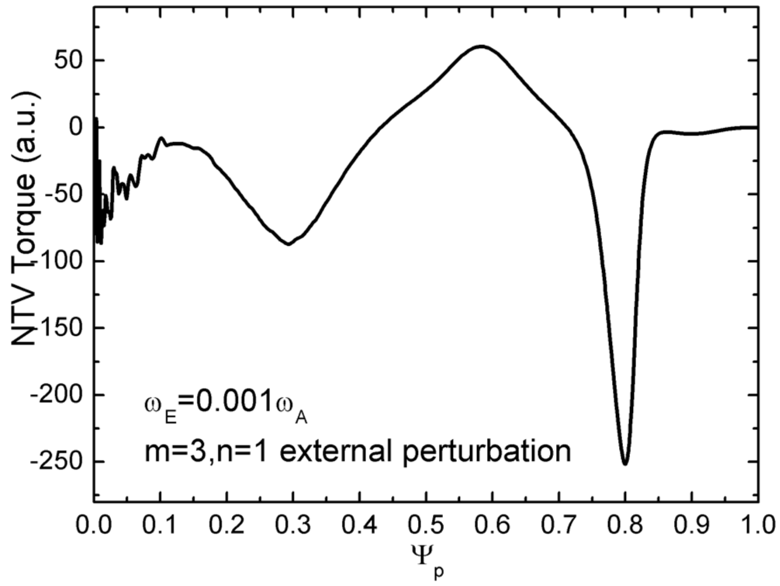


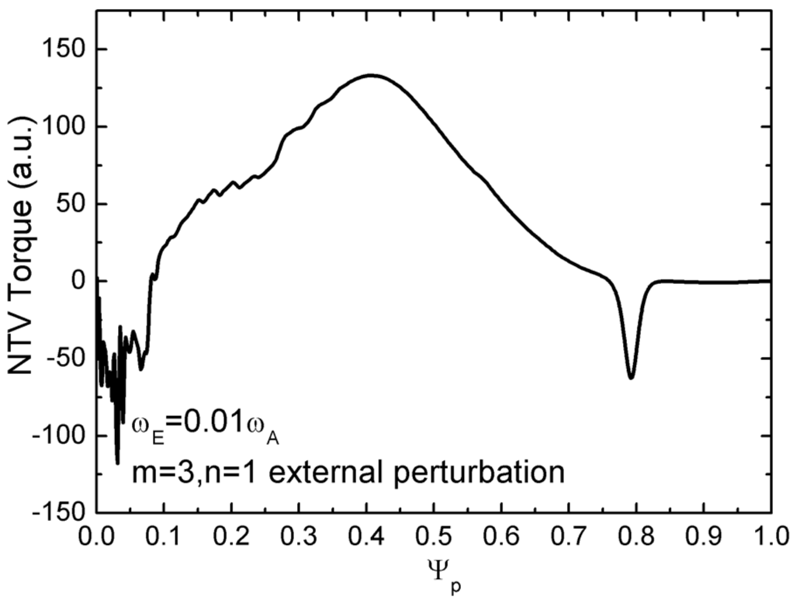
Plasma potential energy: $\delta W_F = \delta W_{mb} + \delta W_{mc} + \delta W_{pre} + \delta W_{cur}$

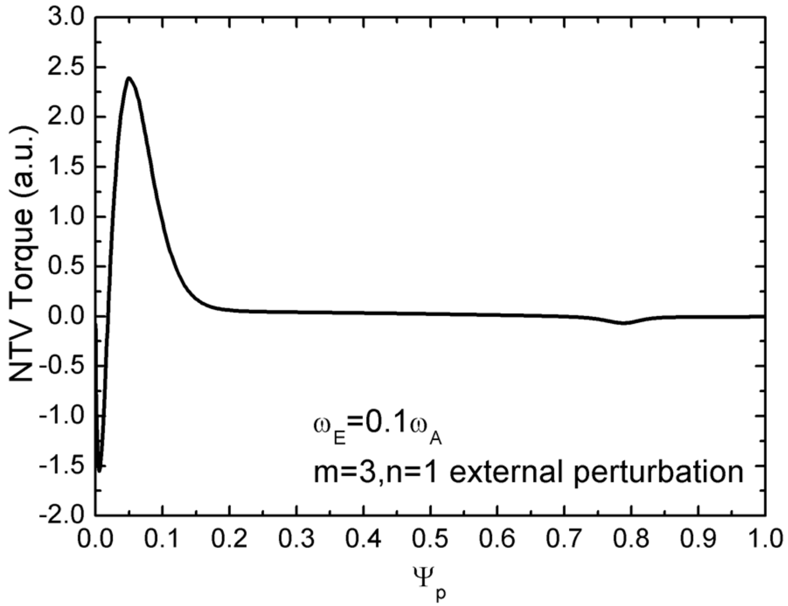
The variation of NTV torque profile with m=3,n=1 external perturbation while increasing ω_E frequency

Large aspect ratio Tokamak with A=10, q_0=1.16, q_a =2.90 β=0.004, β_N=2.2



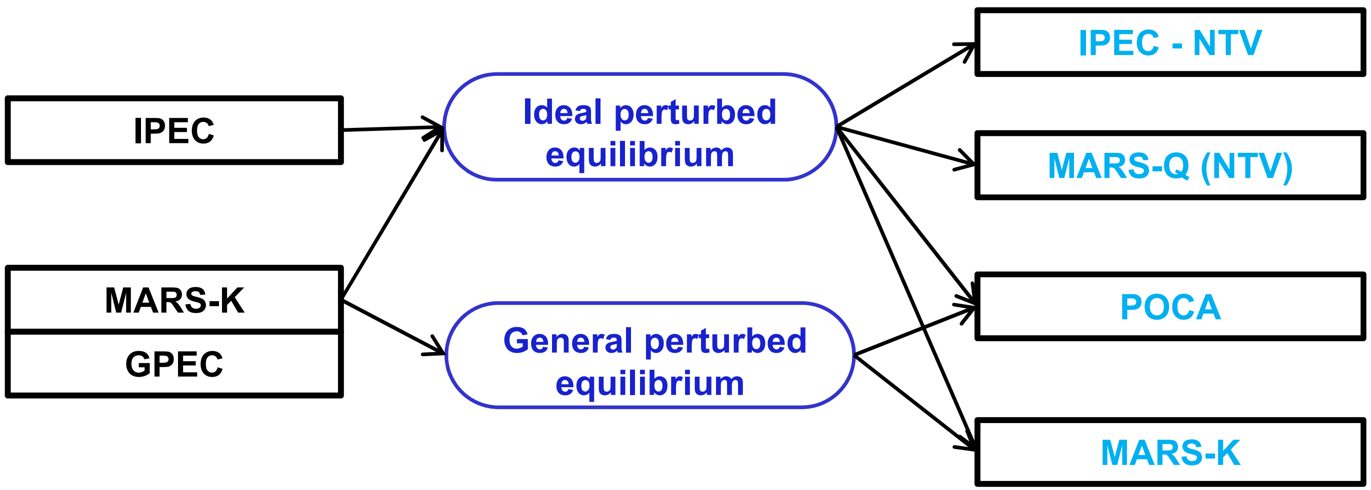






IPEC,MARS-K and POCA show multilevel capability on kinetic effect calculation

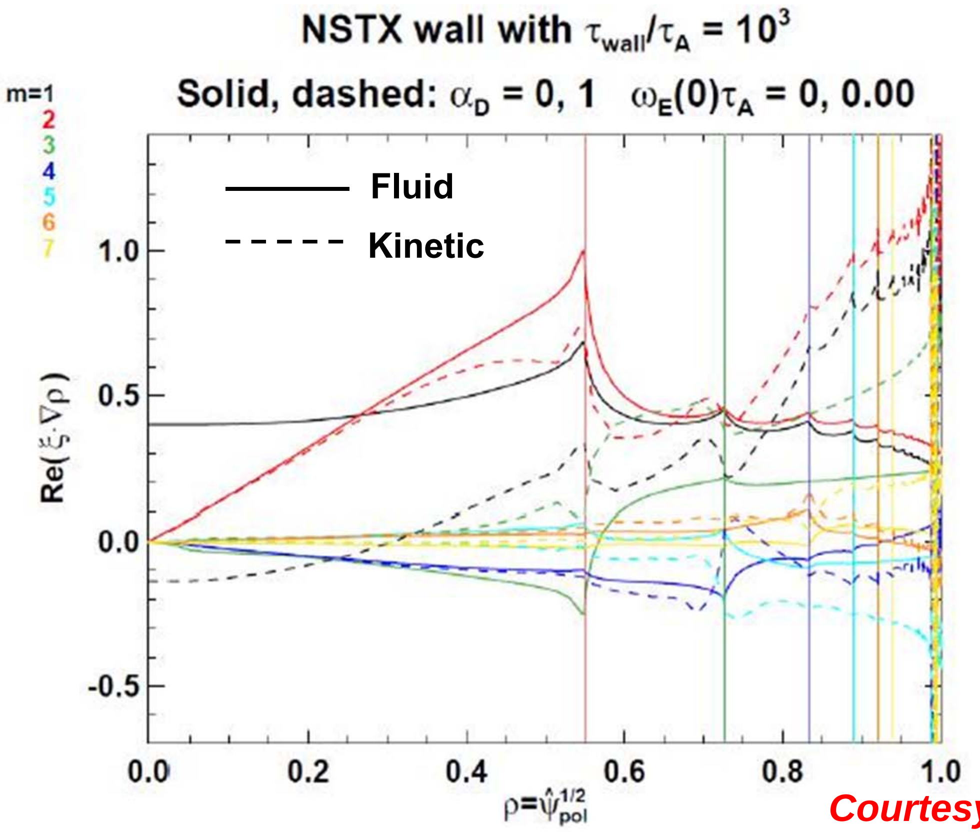
Perturbed equilibrium calculation **NTV calculation**

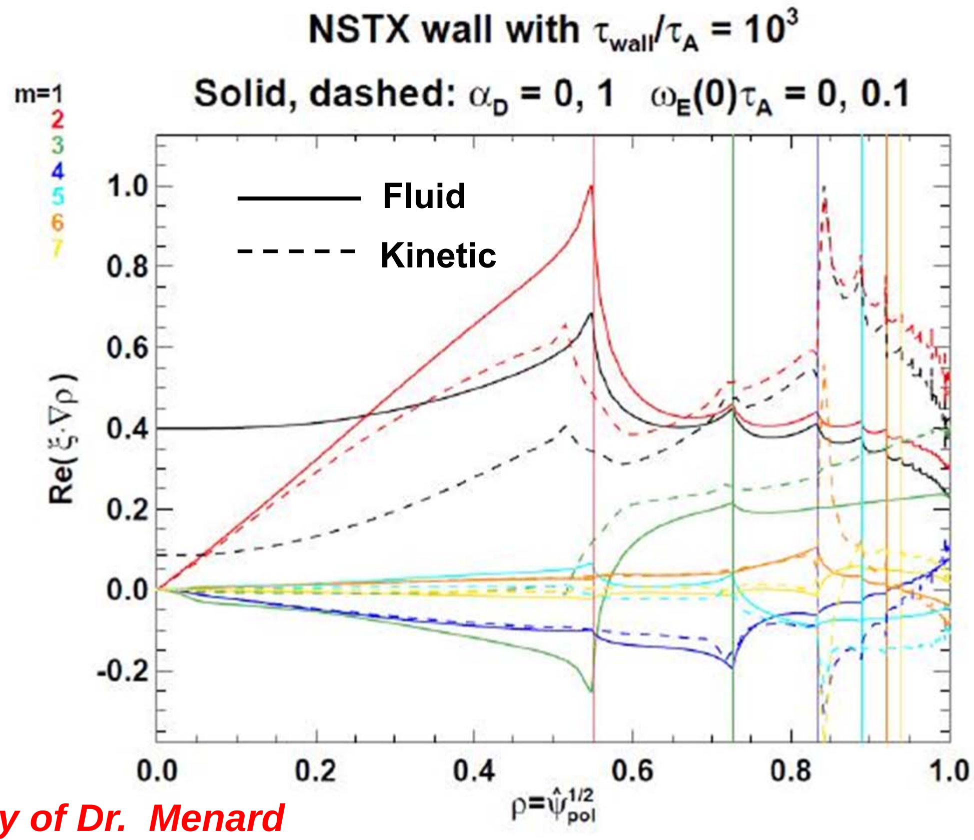


The feature of codes	IPEC-NTV	POCA	MARS-Q (NTV)	MARS-K
Ideal perturbed equilibrium	√	√	√	√
General perturbed equilibrium including anisotropic kinetic pressure and plasma rotation self-consistently				potentially
Collisional operator	Krook	Pitch-angle	Pitch-angle	Krook
Toroidal momentum conservation		√		
Finite banana orbit width		√		
l=0 harmonic of trapped particles	√	√	√	√
l≠0 harmonic of trapped particles	√	√		√
Passing particle contribution	potentially	√		potentially

Self-consistent calculation of RWM by MARS-K shows strong modification of eigenfunction by kinetic effect

- **Kinetic effect strongly modified RWM eigenfunction.**
- **Rotation added to full kinetic damping produces changes that deviate significantly from cases w/o rotation or dissipation.** *Menard, 2012 EPS Conference*



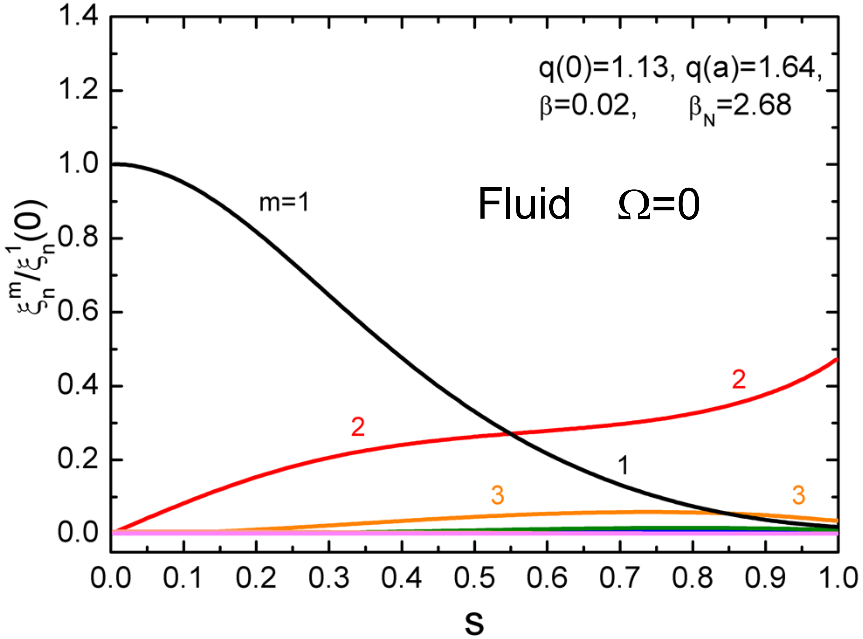


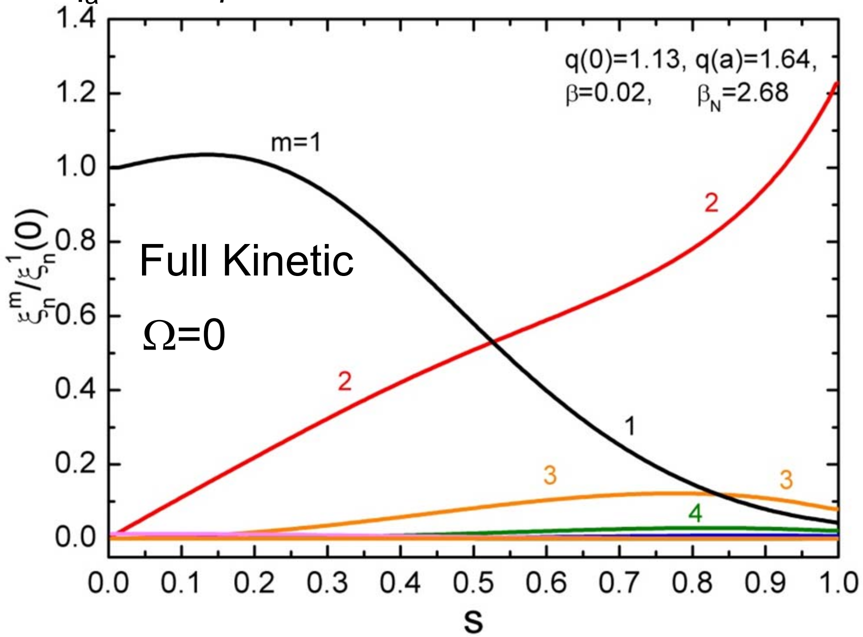
Courtesy of Dr. Menard

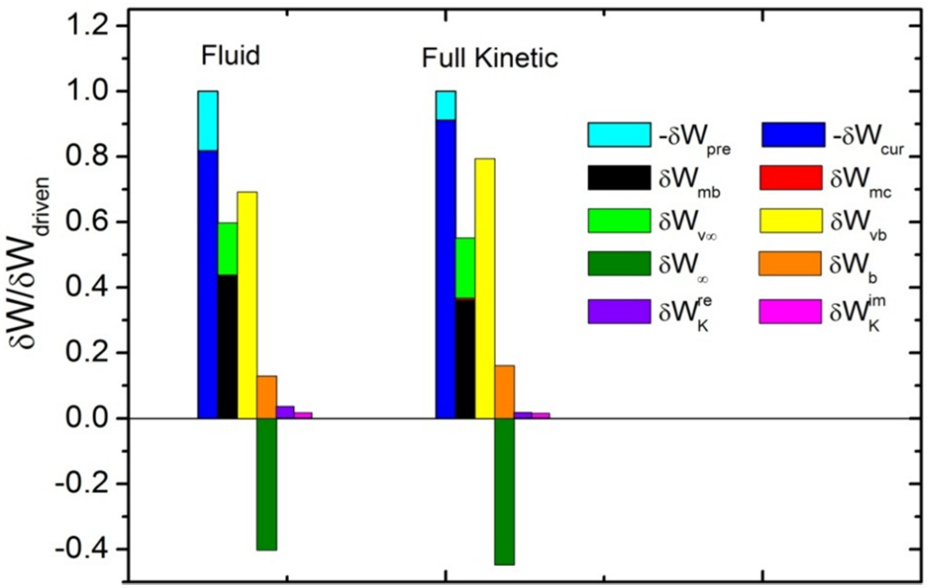
The result also implies that the general (self-consistent) perturbed equilibrium including drift kinetic effect can strongly modify the NTV torque due to the change of δB

Self-consistent calculation of RWM by MARS-K shows modification of potential energy components by kinetic effect

Tokamak current driven RWM case: q_0=1.13,q_a=1.64, β=0.02,a/R=0.2295







Very small plasma compressibility is neglected in fluid case

Comparing with fluid case, kinetic effects leads

- m=2 mode largely increases, which also cause $\delta W_{v\infty}$ and δW_{vb} to increase.
- Current driven term contributes larger fraction in the driven terms.
- δW_K has different value, but not changes RWM instability significantly due to large δW_∞ and δW_b

Perturbative approach: $(\gamma+i\omega_r)/\omega_A=2.51\cdot10^{-3} - 3.55\cdot10^{-4}i$;
Self-consistent approach: $(\gamma+i\omega_r)/\omega_A=2.74\cdot10^{-3} - 2.98\cdot10^{-4}i$

Two approaches may lead to rather large discrepancy in many other cases, sometimes qualitatively.

Summary

- IPEC and MARS-F have a very good agreement in the ideal 3D equilibrium benchmark.
- The numerical evidence of equivalence between NTV torque and Kinetic potential energy is preliminarily shown.
- The shielding effect of perturbed equilibrium due to the kinetic effect is preliminarily studied in the presence of non-resonant external perturbation in large aspect ratio tokamak. The result shows that the perturbed equilibrium is strongly modified by the kinetic effect. It implies the importance of calculating the general (self-consistent) perturbed equilibrium. The systematic study is undergoing.
- IPEC-NTV, POCA, MARS-K/Q shows the multiple level capability to perform NTV calculation.

  - IPEC-NTV for large amount of NTV analysis

  - POCA for a precise NTV analysis

  - MARS-K will be tested for the potential self-consistent δB

- Kinetic effect modifies the RWM eigenfunction and the fluid energy components. More studies should be done.