Nonambipolar Transport and Torque in Perturbed Equilibria

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Thesis

The Perturbed Equilibrium Nonambipolar Transport (PENT) code computes the Neoclassical Toroidal Viscosity (NTV) torque across kinetic regimes in generalized geometry

- PENT uses the "combined theory" [1] developed for calculating NTV across kinetic regimes
  - Emphasis: Bounce Harmonic Resonances & general geometry
  - Equivalence: Kinetic energy in equilibrium stability computation (MARS-K)

Here we will point out, by comparison with various models, where in the theory simplifying assumptions are or are not made and what effect those assumptions have on the final computation

[1] Park, Boozer, Menard, Phys. Rev. Lett. **102** (2009)

Computing NTV in perturbed equilibria

Torque from nonambipolar transport in non-axisymmetric fields

Perturbative approach valid when $|T_\varphi| \ll |2n\delta W|$ → linear MHD

General torque expression then written using the perturbed field, displacement, and perturbed (anisotropic) pressure

$$T_\varphi = \int d^3x \left(\frac{\partial \mathbf{x}}{\partial \varphi} \cdot \nabla \cdot \overleftrightarrow{\mathbf{\Pi}} \right) = -\int d^3x \left[\left(\delta p_\parallel - \delta p_\perp \right) \frac{1}{B} \frac{\partial \delta B}{\partial \varphi} + \delta p_\parallel \frac{\partial}{\partial \varphi} \left(\nabla \cdot \boldsymbol{\xi} \right) \right]$$

Computing NTV in perturbed equilibria

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$$\delta W_k = \frac{1}{2}\int d^3x \left[\delta p_\perp \frac{1}{B}\left(\delta B_{L\|} + \nabla B \cdot \boldsymbol{\xi}\right) + \delta p_\| \boldsymbol{\kappa} \cdot \boldsymbol{\xi}_\perp\right]$$

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Perturbed pressure found using first order Drift Kinetic Equation

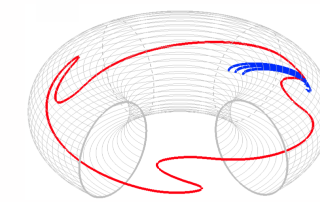
$$\delta p_{\|} = \int d^3v M v_{\|}^2 f_1, \quad \delta p_{\perp} = \int d^3v M v_{\perp}^2 f_1/2$$

$$\mathbf{v}_{\|} \cdot \nabla f_1 + \mathbf{v}_d^{\alpha} \frac{\partial f_1}{\partial \alpha} - C_1 f_1 = -\mathbf{v}_d^{\psi} \frac{\partial f_0}{\partial \psi}$$

Methods diverge when forming perturbed pressure from first order Drift Kinetic Equation

Semi-analytic methods (limiting regimes, combined, and kinetic energy) all seek integral solution

- Bounce average isolates non-axisymmetric effects closed orbits!
  - Thin banana approximation $\langle A \rangle \equiv \frac{\omega_b}{2\pi} \oint d\vartheta A \mathcal{J} B / v_{\|}$
    - Decoupled bounce harmonics $f_1 = \sum_{\ell} \delta f_\ell \mathcal{P}_\ell$
  - Krook collision operator $C\left[f_1\right] \equiv \nu f_1$



[1]

- Origin of kinetic resonances and offset rotation

$$\left\langle -i\ell\omega_b \frac{v_d^\alpha}{\langle v_d^\alpha \rangle} - 2\pi i n v_d^\alpha + \nu \right\rangle \delta f_\ell = \left\langle -{\mathcal{P}_\ell}^{-1} v_d^\psi \frac{\partial f_0}{\partial \psi} \right\rangle$$

[1] Kim, Park, Boozer, Phys. Rev. Let. **110** (2013)

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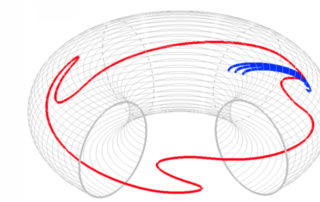

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$$\left\langle \nu \frac{v_\parallel}{B}\frac{\partial}{\partial\mu}\left(M v_\parallel \mu \frac{\partial \delta f_\ell}{\partial \mu}\right)\right\rangle = \left\langle -{\mathcal{P}_\ell}^{-1} v_d^\psi \frac{\partial f_0}{\partial \psi}\right\rangle$$

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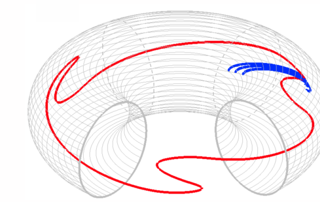

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$$\left\langle -i\ell\omega_b \frac{v_d^\alpha}{\langle v_d^\alpha \rangle} - 2\pi i n v_d^\alpha + \nu \right\rangle \delta f_\ell = \left\langle -\mathcal{P}_\ell{}^{-1} v_d^\psi \frac{\partial f_0}{\partial \psi} \right\rangle$$

$$\delta f_\ell = \frac{1}{4\pi^2 T} \frac{\omega_b \left[\omega_\varphi + \omega_{*T}\left(\frac{E}{T} - \frac{5}{2}\right)\right]}{i\left[\ell\omega_b + n\left(\omega_E + \omega_D\right)\right] - \nu} \frac{\partial \delta J_{-\ell}}{\partial \alpha} f_0$$

[1] Kim, Park, Boozer, Phys. Rev. Let. **110** (2013)

Despite divergence, final torque has a common form in all methods

Final integral solution has quadratic dependence on perturbed Action and complex resonance operator

$$T_\varphi \propto \int d\psi NT \int d\mu \bar{\omega}_b \left| \delta \bar{J}_\ell \right|^2 \int d\varepsilon \mathcal{R}_{T\ell}$$

Despite divergence, final torque has a common form in all methods

Final integral solution has quadratic dependence on perturbed Action and complex resonance operator

$$\delta W_k \propto \int d\psi \int d\mu d\varepsilon \frac{\partial f_0}{\partial \varepsilon} \tau_b \left| \langle \varepsilon H \mathcal{P}_\ell \rangle \right|^2 \lambda_\ell$$

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Final integral solution has quadratic dependence on perturbed Action and complex resonance operator

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Geometric factors in perturbed action can contribute significant torque

$$\delta J_\ell \equiv M \int d\vartheta \mathcal{P}_\ell \delta \left(v_\parallel \mathcal{J} B \right)$$

$$= \oint d\vartheta \frac{\mathcal{J} B}{v_\parallel} \mathcal{P}_\ell \left[(2E - 3\mu B) \frac{\delta B}{B} + (2E - 2\mu B) \frac{1}{\mathcal{J}} \nabla \cdot \boldsymbol{\xi} \right]$$

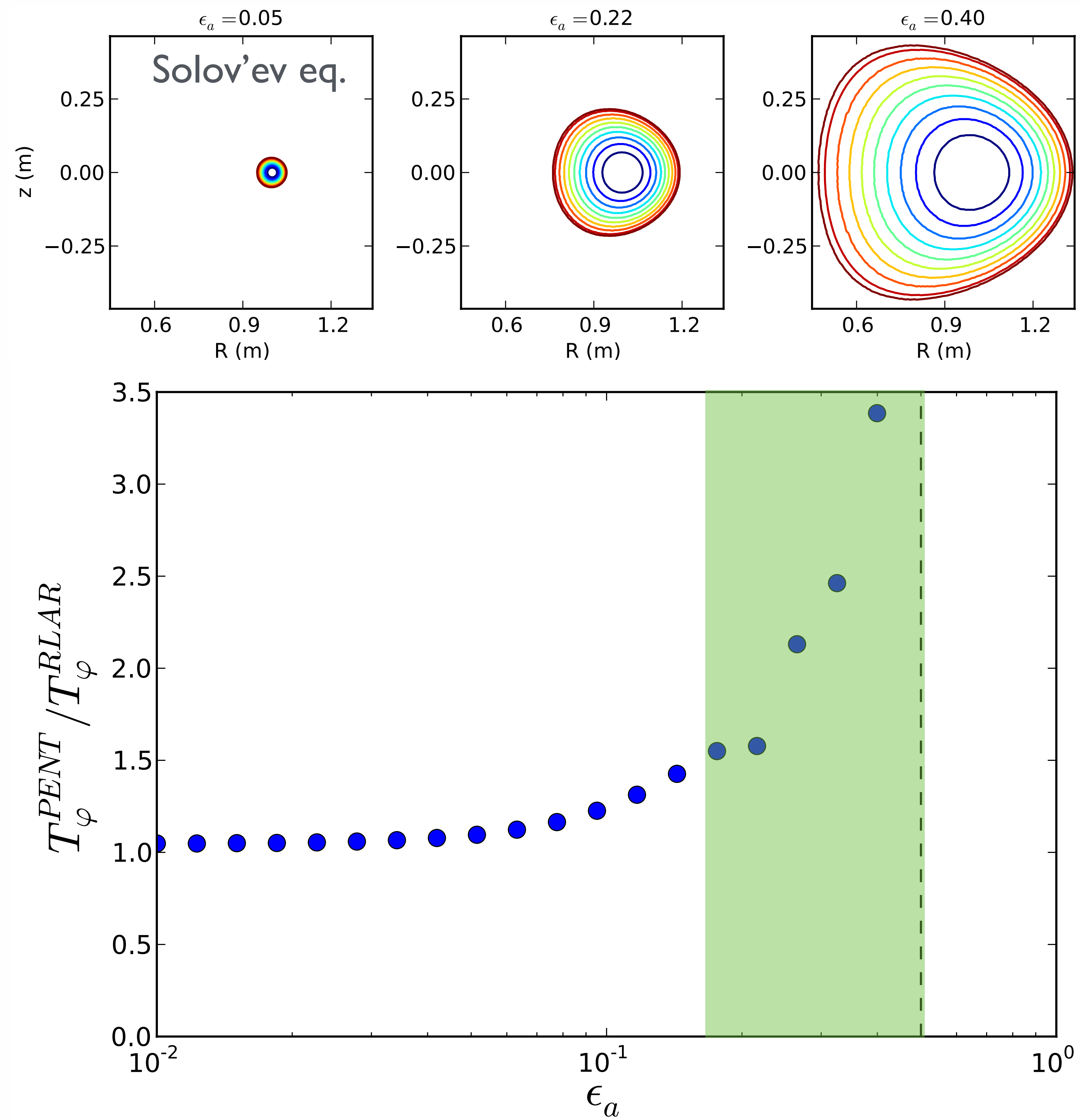
NTV is proportional to the square of the perturbed Action

- Often approximated as proportional to δB^2
- δJ_ℓ also contains a $\nabla \cdot \boldsymbol{\xi}$ term from change in arc-length contributing to the perturbed Jacobian
  - This term is often dropped under the assumption $E \approx \mu B$
  - The $\nabla \cdot \boldsymbol{\xi}$ can become very large with realistic aspect ratio and shaping

Geometric factors in perturbed action can contribute significant torque

Combined model can be continued analytically with Reduced Large-Aspect-Ratio (RLAR) geometric simplifications

$$\cancelto{0}{\frac{2}{\mathcal{J}}(E-\mu B)\nabla\cdot\boldsymbol{\xi}}$$



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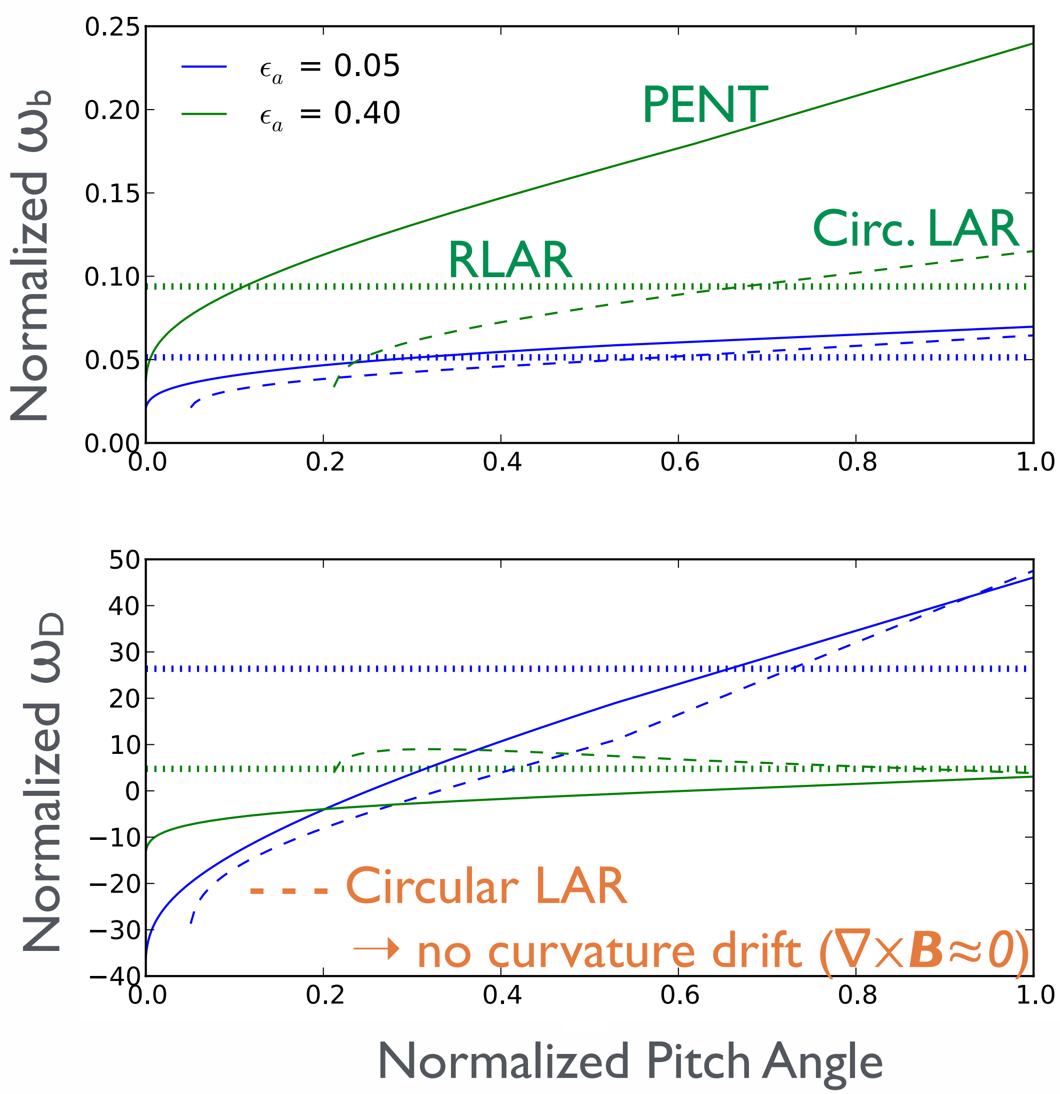
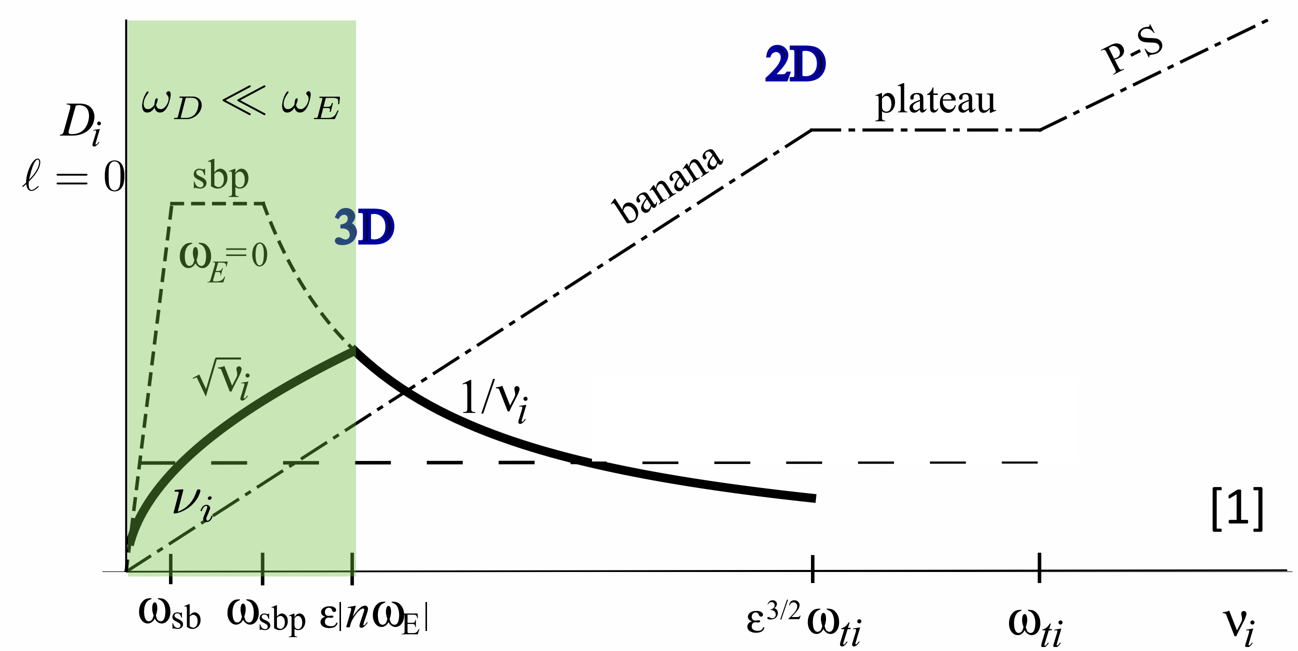
Geometry affects particle orbits, influencing sensitive resonance conditions

$$\mathcal{R}_{T\ell} \propto \frac{\left[\omega_\varphi + \omega_{*T}\left(\varepsilon - \frac{5}{2}\right)\right]}{i\left[\ell\omega_b + n\left(\omega_E + \omega_D\right)\right] - \nu}$$





[1] Callen, Nucl. Fus. **51** (2011)

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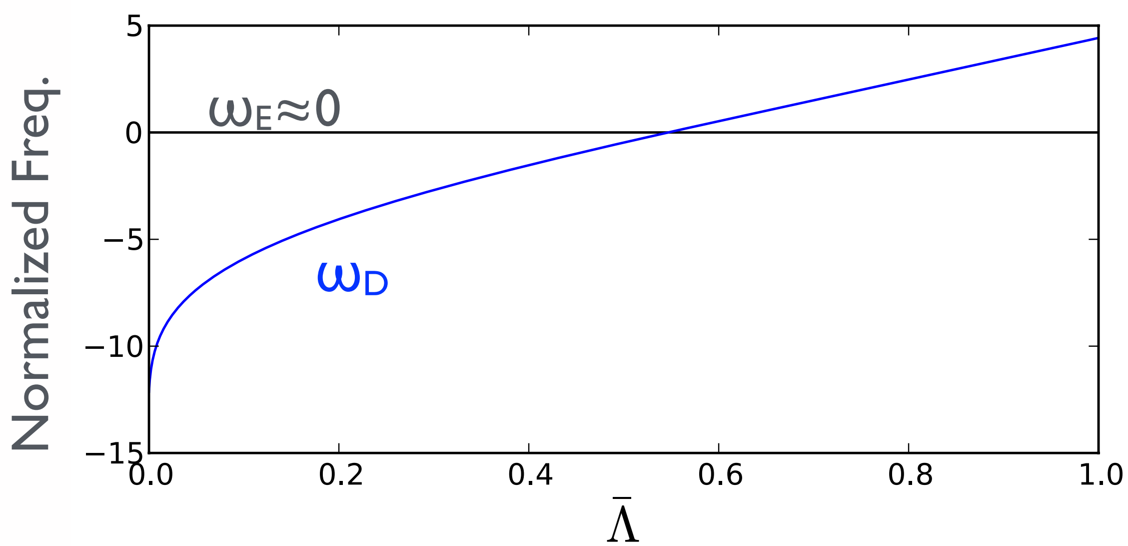
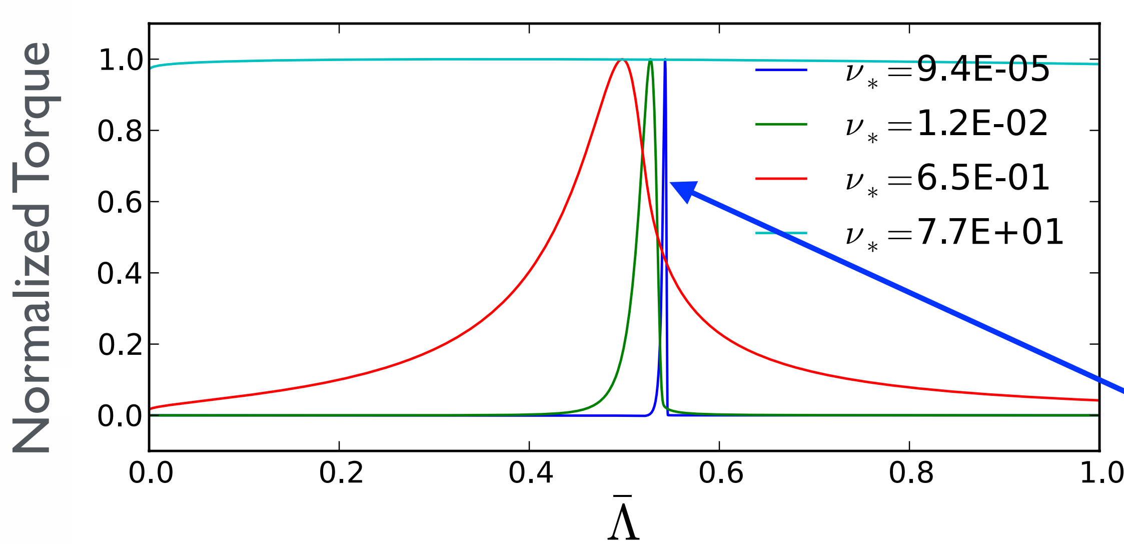
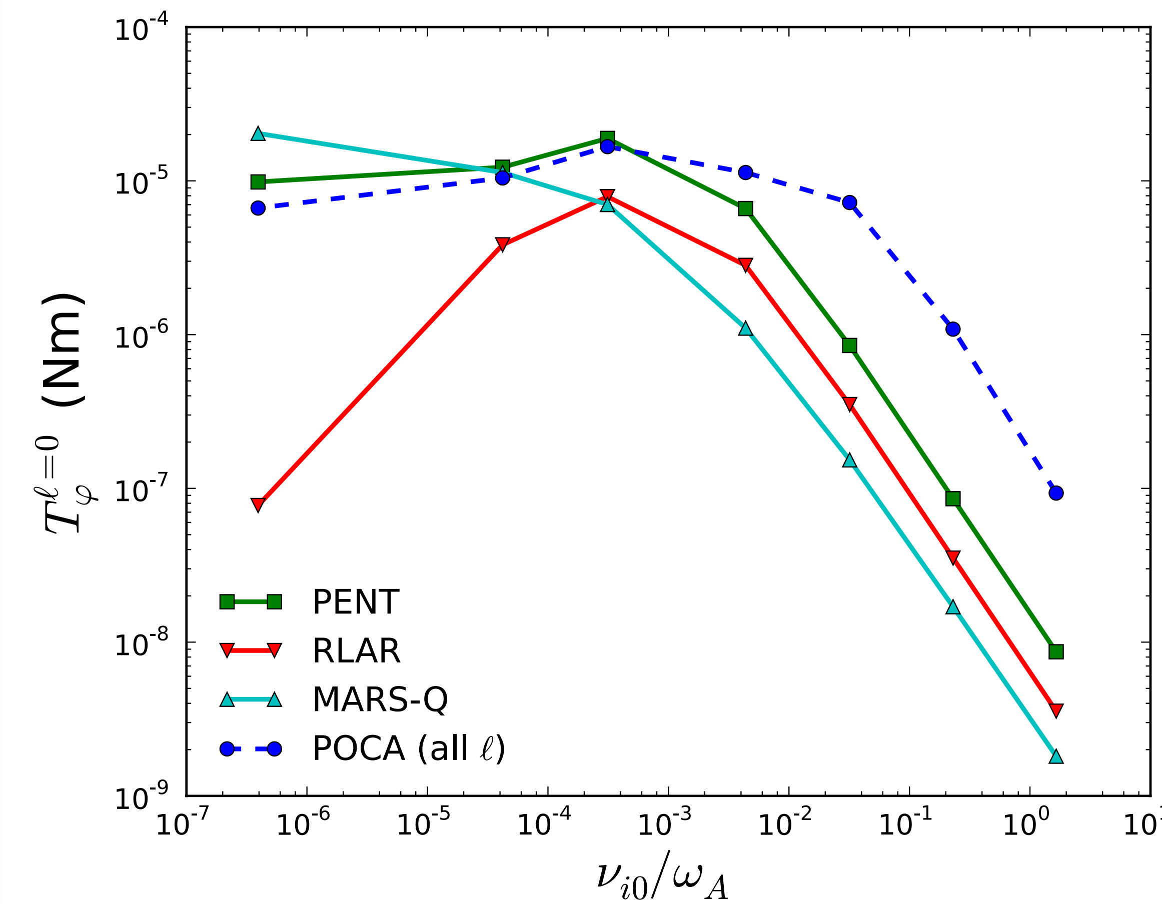




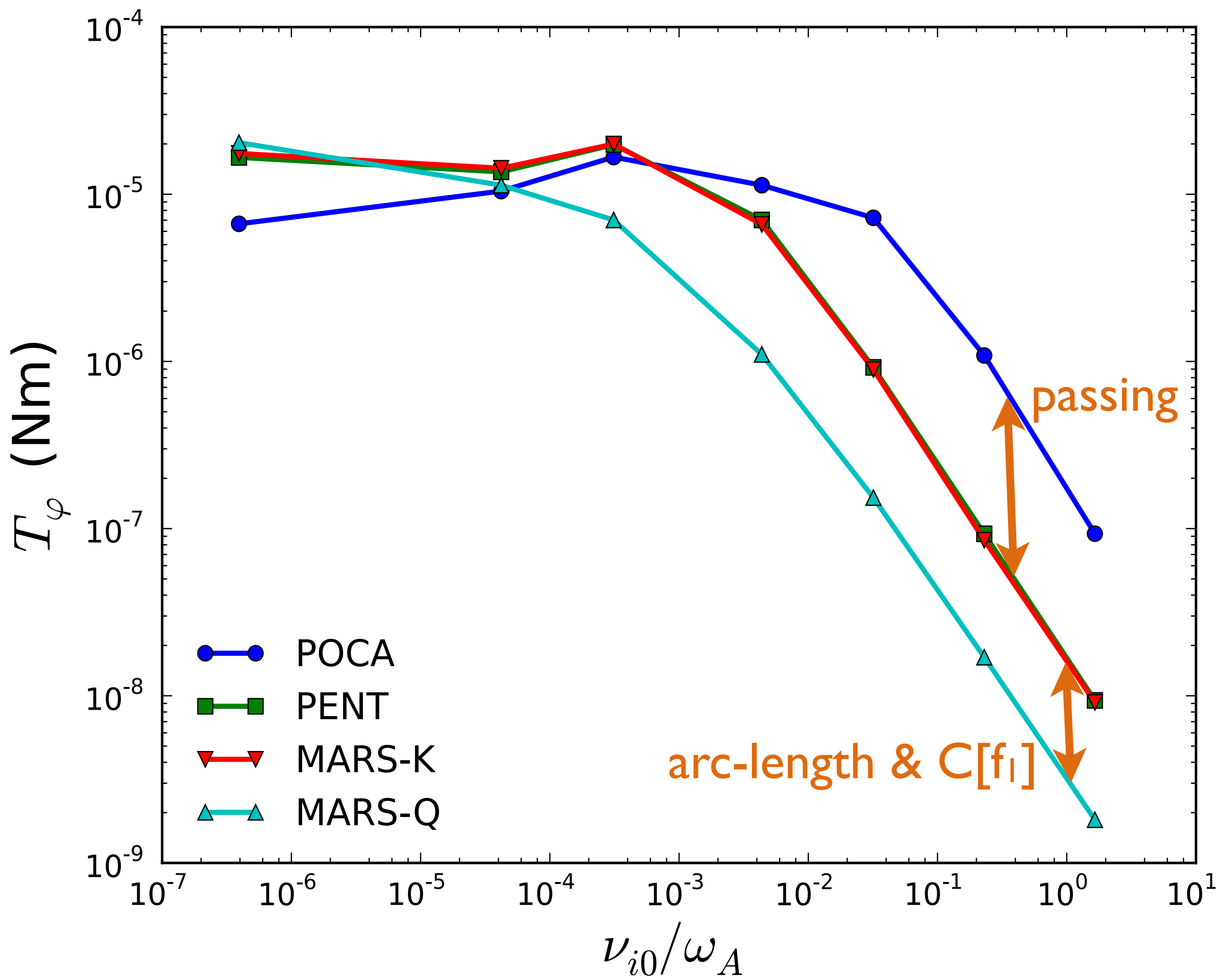


$$\mathcal{R}_{T0} \propto \frac{\nu}{\omega_D^2 + \nu^2} \xrightarrow{\nu \to 0} \delta(\omega_D)$$

Super banana plateau is a consequence of Λ-resonance singularity

- Exists “regardless of geometry”, but magnitude sensitive to geometry-dependent freqs.
    - Physics lost if reduce Λ dependent freqs.
- Important computational challenge [Satake, PRL **107** (2011), Logan, Phys. Plasma (submitted)]

Where PENT fits in: A NTV computation summary



To obtain semi-analytic integral solution...

- Focus on decoupled bounce harmonics (PENT,MARS-K)
- Focus on pitch-angle scattering collision operator (MARS-Q)

To obtain direct solution...

- Focus on full guiding center particle orbits (POCA by K. Kim NO6.00001)

Each method has its strengths...

- Now thoroughly benchmarked across kinetic regimes

Tomorrow's challenge in NTV torque modeling

$$\delta\mathbf{J}\times\mathbf{B}+\mathbf{J}\times\delta\mathbf{B}-\nabla\cdot\delta\mathbf{\Pi}=\rho\left[\left(\mathbf{v}\cdot\nabla\right)\delta\mathbf{v}+\left(\delta\mathbf{v}\cdot\nabla\right)\mathbf{v}\right]+\delta\rho\left(\mathbf{v}\cdot\nabla\right)\mathbf{v}$$

Next major step in NTV computation is self-consistent torque calculations

- Include anisotropic terms in initial perturbed equilibrium calculation
    - Include other rotation effects (inertial terms) as well
- "General Perturbed Equilibrium"
    - δW_k implementation in MARS-K [Liu, Phys. Plasma **15**, (2008)]
    - Under development for coupled IPEC-PENT-POCA solver