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Slow wave excitation due to warm electrons in NSTX-U and tokamaks*

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Slow wave excitation due to warm electrons in NSTX-U and tokamaks*

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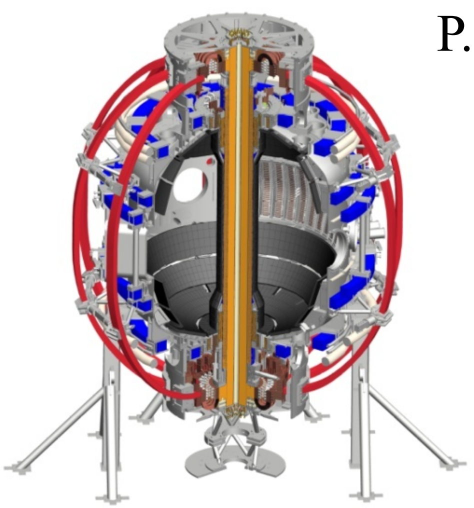

55th Annual Meeting of the APS Division of Plasma Physics
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§CSWPI: SciDAC Center for Simulation of Wave-Plasma Interactions

Poster Outline and Key Ideas

I. **Original Motivation and Background**
  - *Can the RF SciDAC full wave RF codes provide a complementary tool for understanding fast particle mode dynamics?*

II. **Summary of previous studies**
  - *"Slow modes" found in both the low frequency, ion cyclotron and high harmonic fast wave regimes, if sufficient resolution and hot electron effects are included*
  - *Behavior of slow mode consistent with theoretical model but code convergence issues were problematic*

III. **Results of most recent studies**
  - *Converged solutions with both the fast wave and slow wave present have been found in agreement with theoretical dispersion relations*
  - *Methods for dealing with convergence issues have been identified*

IV. **Future studies include:**
  - *simulations of rf-driven fast particle modes in NSTX-U*
  - *assessment of power absorbed by the slow mode in the HHFW regime*
  - *role of slow mode in ITER plasmas*

Fast ions excite a variety of modes in fusion plasmas:
Can Full Wave RF codes offer any insights?

Fast Particle Mode Simulation tools include:

For high frequency modes $\omega \cong \Omega_{ci}$:

Linear MHD NOVA code

Linear rf wave codes, TORIC and AORSA

Nonlinear HYM initial value, hybrid

For low frequency modes $\omega << \Omega_{ci}$:

Linear codes:

NOVA-K –hybrid MHD-kinetic code

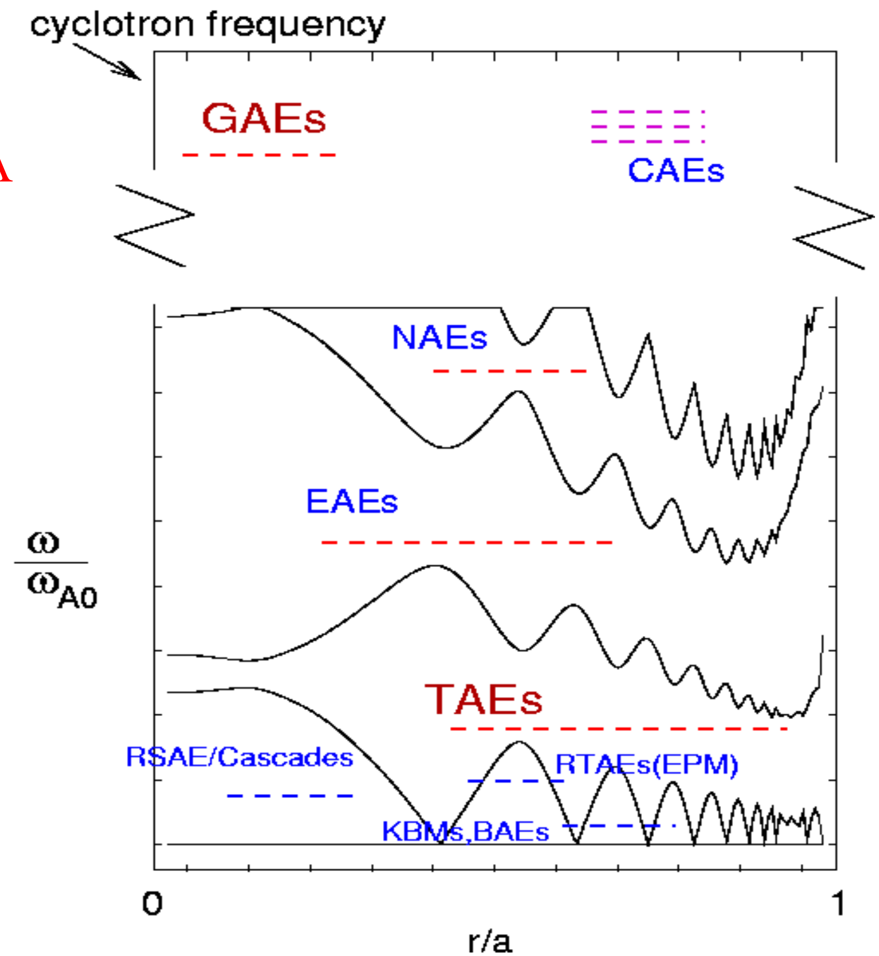
NOVA-KN – nonperturbative global hybrid

HINST – local nonperturbative ballooning

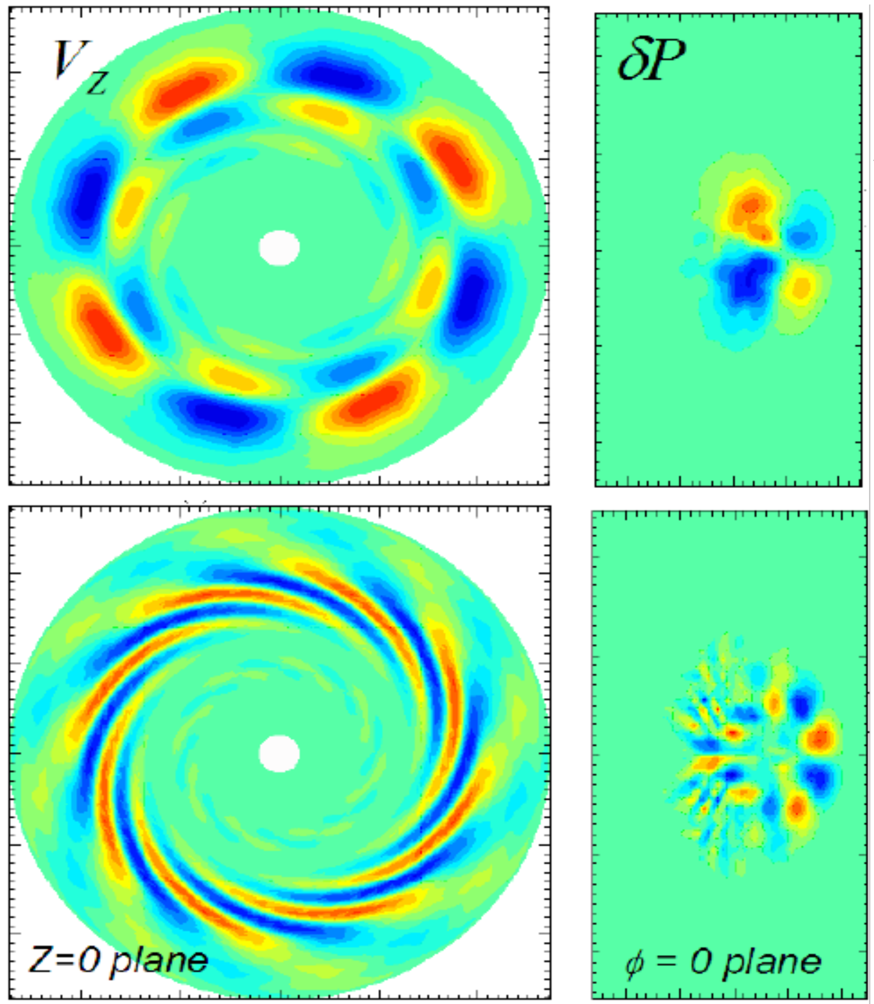
Nonlinear codes:

M3D – initial value hybrid MHD-kinetic

NOVA-K – reduced theoretical model for wave saturation and fast ion transport

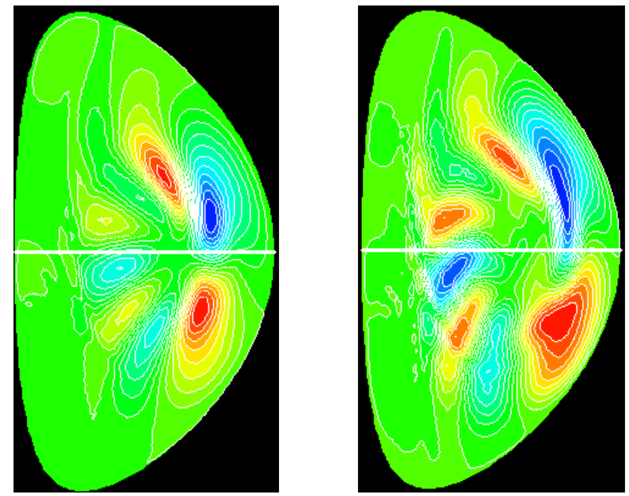


Predicted structure of fast particle modes is commensurate with TORIC and AORSA resolution capabilities



GAE

CAE



M3D nonlinear hybrid simulations of beam-driven TAE modes in NSTX (Fu)

[M3D has ≤ 200 radial nodes]

HYM code simulations of GAE and CAE in NSTX (Belova)

CAE's were first predicted by the HYM code and then observed in NSTX (2005)

TORIC and AORSA can address physics effects that are missing in other fast particle mode simulation tools

Most existing simulation tools for fast particle modes are based on linear or nonlinear MHD / hybrid models that neglect or treat approximately:

- FLR effects, finite ω/Ω_c effects, cyclotron resonances
- full toroidal geometry, fast ion distributions

⇒*TORIC* and *AORSA* retain these effects and, in addition, can provide much higher spatial resolution of modes, but in a linear or quasilinear treatment

⇒Approach: *Utilize TORIC and AORSA to study dynamics of driven modes in the linear regime*

- begin in the $\omega < \Omega_c$ regime
- modify code as needed for $\omega << \Omega_c$ regime
- validate code with results from driven mode experiments

Wave Fields modeled with linearized Wave Equation

Full wave electromagnetic field equation (inhomogeneous plasma):

$$\nabla \times \nabla \times \mathbf{E} - \frac{\omega^2}{c^2}\underbrace{\left(\mathbf{E} + \frac{i}{\omega\epsilon_0}\mathbf{J}_p\right)}_{\vec{\vec{K}}\cdot\vec{E}} = i\omega\mu_0\mathbf{J}_s$$

$\vec{\vec{K}} \cdot \vec{E}$, where $\vec{\vec{K}}$ is the dielectric tensor

and
$$\mathbf{J}_p(\mathbf{r}) = \int d\mathbf{r}' \overline{\overline{\boldsymbol{\sigma}}}(\mathbf{r}, \mathbf{r}') \cdot \mathbf{E}(r')$$

AORSA includes the complete non-local, integral operator for $\vec{\vec{K}}$ that is valid for “all orders” ($k_\perp \rho_s > 1$)

E.F. Jaeger et al, Phys. Plasmas 9 (2002) 1873

TORIC utilizes a quasi-local differential operator for $\vec{\vec{K}}$ that is valid in the HHFW regime but assumes that one dominant mode is propagating.

M. Brambilla, Pl. Phys. and Controlled Fus. Res. 44(2002)242

TORIC and AORSA Utilize Spectral Decomposition to Solve for the Wave Fields

AORSA utilizes a local Cartesian grid in the poloidal plane and a Fourier decomposition in the toroidal direction of symmetry, ϕ:

$$\vec{E}(x,y,\phi) = \sum_{n_\phi} \sum_{n,m} \vec{E}_{n_\phi,n,m} e^{in_\phi \phi} e^{i(k_n x + k_m y)}$$

where $x = R - R_0$ and y is distance from midplane

TORIC utilizes a poloidal mode expansion and radial finite elements in the poloidal plane and a Fourier decomposition in the toroidal direction.

$$E(\vec{r}) = \sum_{n_\phi} e^{in_\phi \phi} \sum_{m=-n\,\mathrm{mod}/2}^{n\,\mathrm{mod}/2} E_m(\psi, n_\phi) e^{im\theta}$$

and $E_m(\psi, n_\phi)$ is solved with nelm cubic Hermite polynomials

➢Both codes must be run on multi-processor supercomputers to be able to include sufficient modes in the expansions to achieve fine spatial resolution of short wavelength modes.

Hot electrons with $\omega < k_{//}\, v_{the}/2$ allow a second mode to propagate in the core plasma

Assuming cold ions (neglects IBW) and warm electrons, the simplest dispersion relation can be written as:

$$a\, n_{\perp}^{4} - b\, n_{\perp}^{2} + c = 0$$

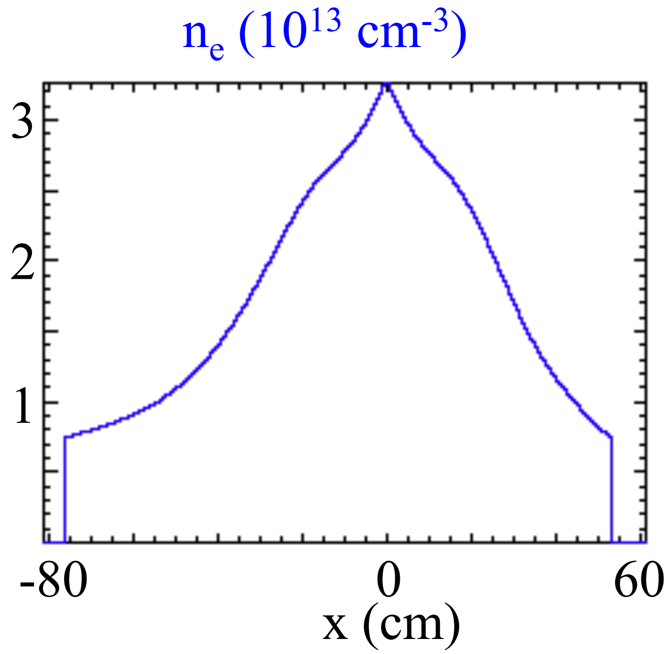
where $a \sim S$, $b \sim -K_{zz}\,(n_{//}^2 - S)$, and $c \sim K_{zz}\,(n_{//}^2 - R)(n_{//}^2 - L)$

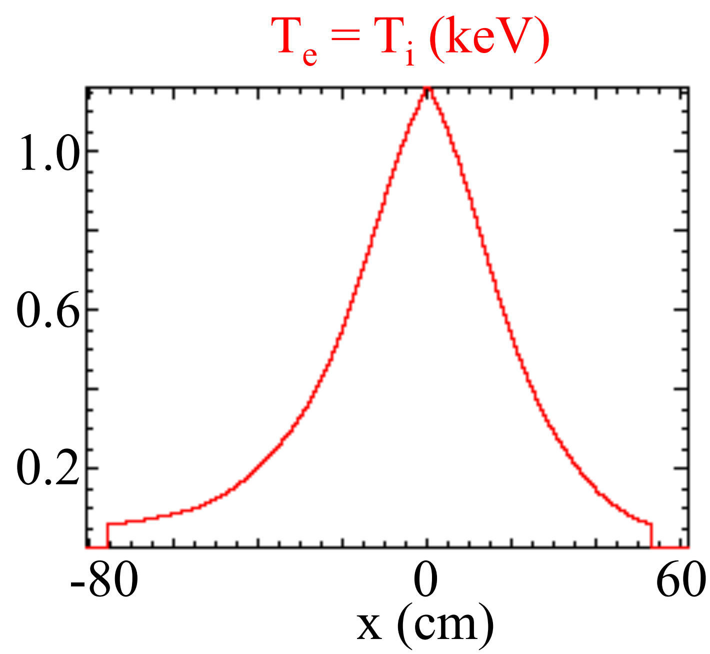
"fast root": $n_{\perp}^2 \sim c/b \sim (n_{//}^2 - R)(n_{//}^2 - L)/\,(S - n_{//}^2)$ *(the usual fast wave form)*

"slow root": $n_{\perp}^2 \sim b/a \sim -K_{zz}\,(n_{//}^2 - S)/S$ *(the usual slow wave form)*

With ***hot electrons*** such that $\omega < k_{//}\, v_{th}/2$, K_{zz} becomes positive and the slow mode can propagate, if $S<0$, or if $n_{//}^2 < S$ when $S>0$

Initial Simulations used EFIT equilibrium with measured n_e and T_e profiles from *NSTX 112705*



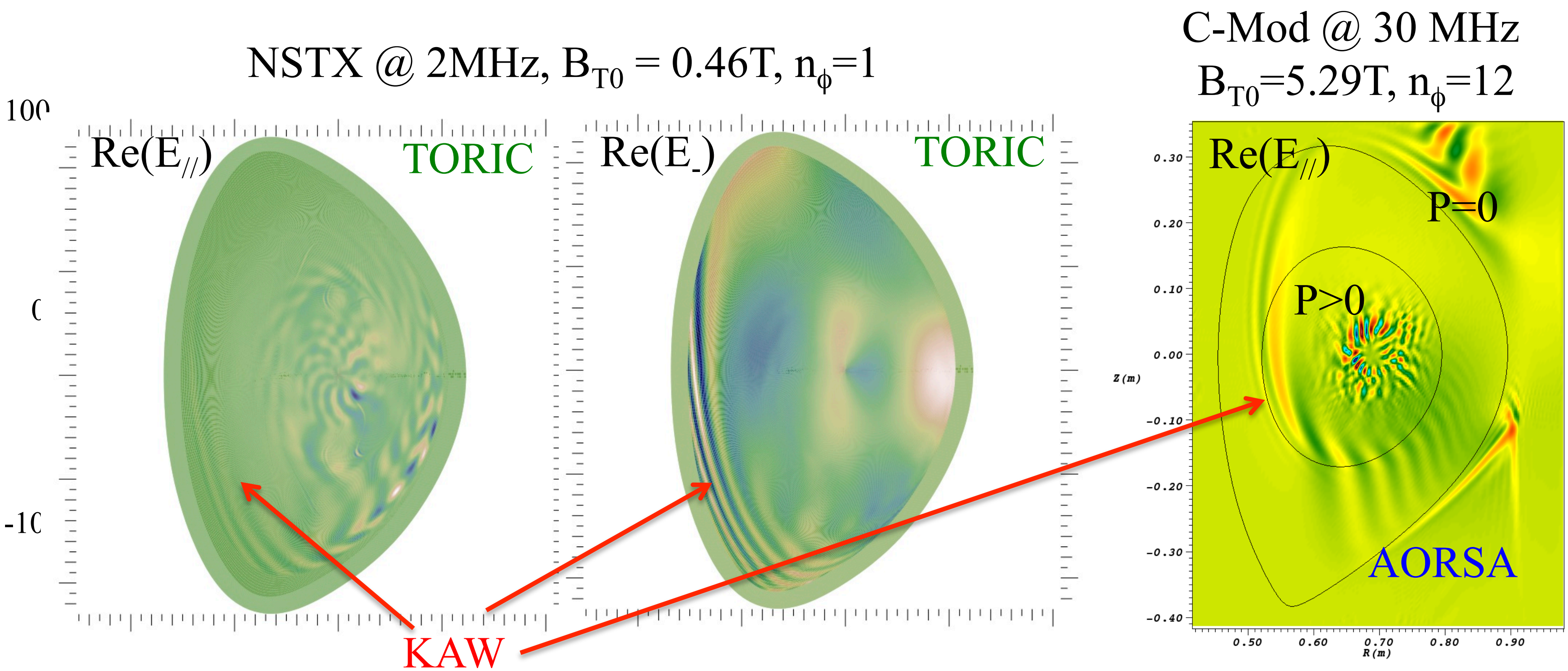


B_T(axis) = 0.426T I_p= 0.6 MA q_a=12

99% D, 1% H used in simulations

Note here that Ti ~ Te

Kinetic Alfvén Wave (KAW) found in initial simulations of rf-driven $\omega < \Omega_{ci}$ modes in NSTX and C-Mod



Driven RF field structures are consistent with GAE / CAE modes, but further analysis of the RF field polarizations is needed to identify modes

Note that $\omega / k_{//} v_{Te}$<1 , for "CAE" and KAW @ 2 MHz, n_ϕ=1, is about the same for HHFW @ 30 MHz, n_ϕ= -12 >> look for slow waves in the HHFW regime

What is the “slow mode” in the $\omega < \Omega_{ci}$ regime?

Slow Mode is a “kinetic Alfven wave” **[Stix**, *Waves in Plasmas* (AIP, NY, 1992) pg. 357 **]:**

In the $\omega < \Omega_{ci}$ regime, S is positive: $$S \sim \frac{-\omega_{pd}^2}{\omega^2 - \Omega_D^2} \sim \frac{\omega_{pd}^2}{\Omega_D^2} \sim \frac{c^2}{V_A^2}$$

But, for large enough poloidal modes, $k_{//} \sim mb_p/r + n_\phi b_T/R$ is large, so real(P) > 0

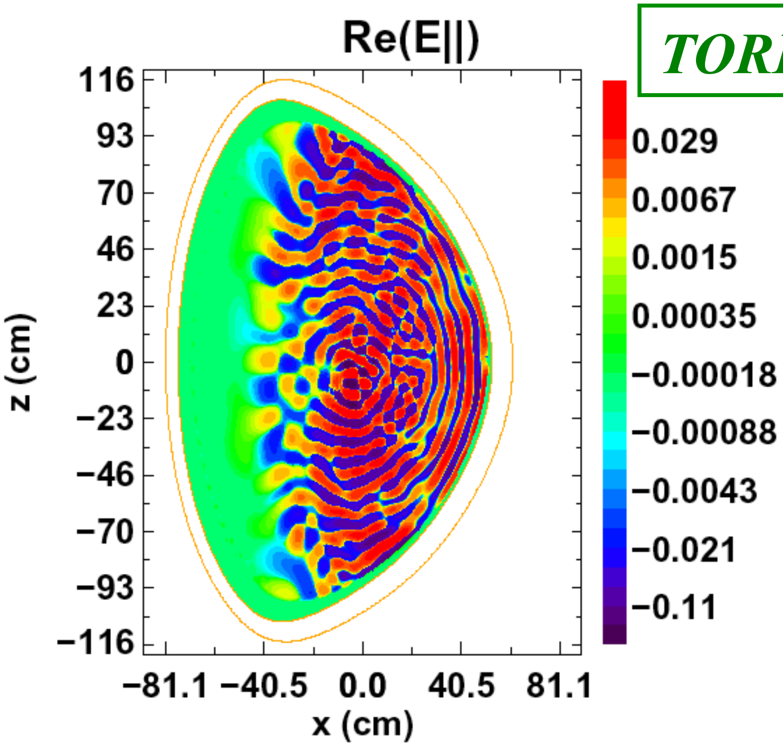
$$n_\perp^2 = \frac{-P}{S}(n_{//}^2 - S) \quad \Rightarrow$$

$$n_\perp^2 = -\frac{P}{|S|} n_{//}^2 \quad \text{if} \quad n_{//}^2 >> |S| \qquad \text{(evanescent)}$$

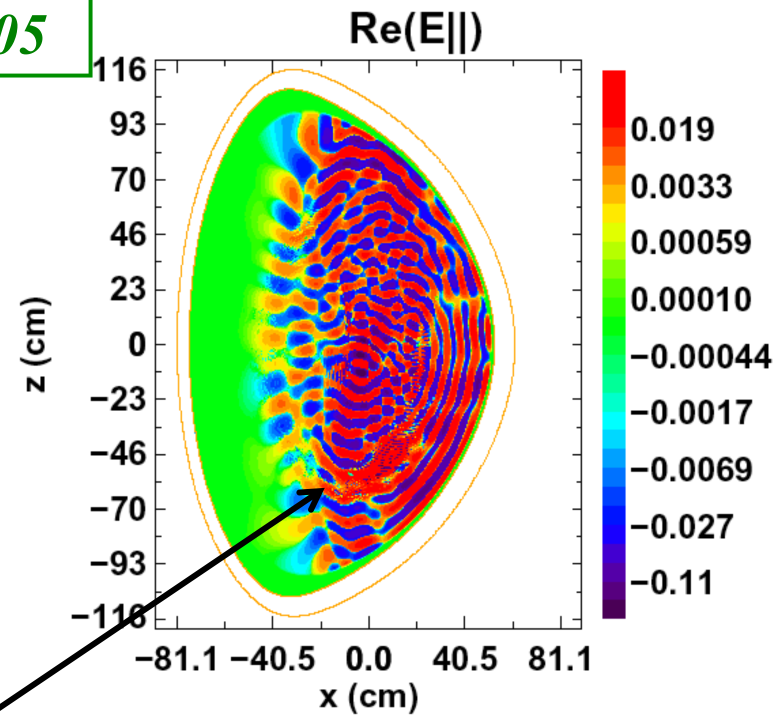
$$n_\perp^2 = P \quad \text{if} \quad n_{//}^2 << |S| \qquad \text{(KAW)}$$

"Slow Wave" found in the HHFW regime on NSTX with sufficiently high resolution and hot electrons

TORIC, circa 2005



n_{elm}=240 n_{mod}=127
Pabs = 2.71 / Prad = 2.42



slow mode
HHFW regime

n_{elm}=483 n_{mod}=255
Pabs = 1.83 / Prad = 1.82

Slow wave also found in AORSA simulation, though at even higher spatial resolutions, both the AORSA and TORIC solutions generally do not converge

What is the "slow mode" in the HHFW regime?

The slow mode is related to either the "electrostatic ion cyclotron mode" or the KAW:

[N. D'Angelo and R.W. Motley, Phys. Fluids ***5****, 633-634 (1962)]*

In the HHFW regime, S is negative: $S \sim \dfrac{-\omega_{pd}^2}{\omega^2 - \Omega_D^2} \sim \dfrac{-\omega_{pd}^2}{\omega^2} \sim \dfrac{-\omega_{pd}^2}{\Omega_{cd}^2}\,\dfrac{\Omega_{cd}^2}{\omega^2}$

But, for large enough poloidal modes, $k_{//} \sim mb_p/r + n_\phi b_T/R$ is large, so real(P) > 0

$$n_\perp^2 = \frac{-P}{S}\left(n_{//}^2 - S\right) \quad \Rightarrow$$

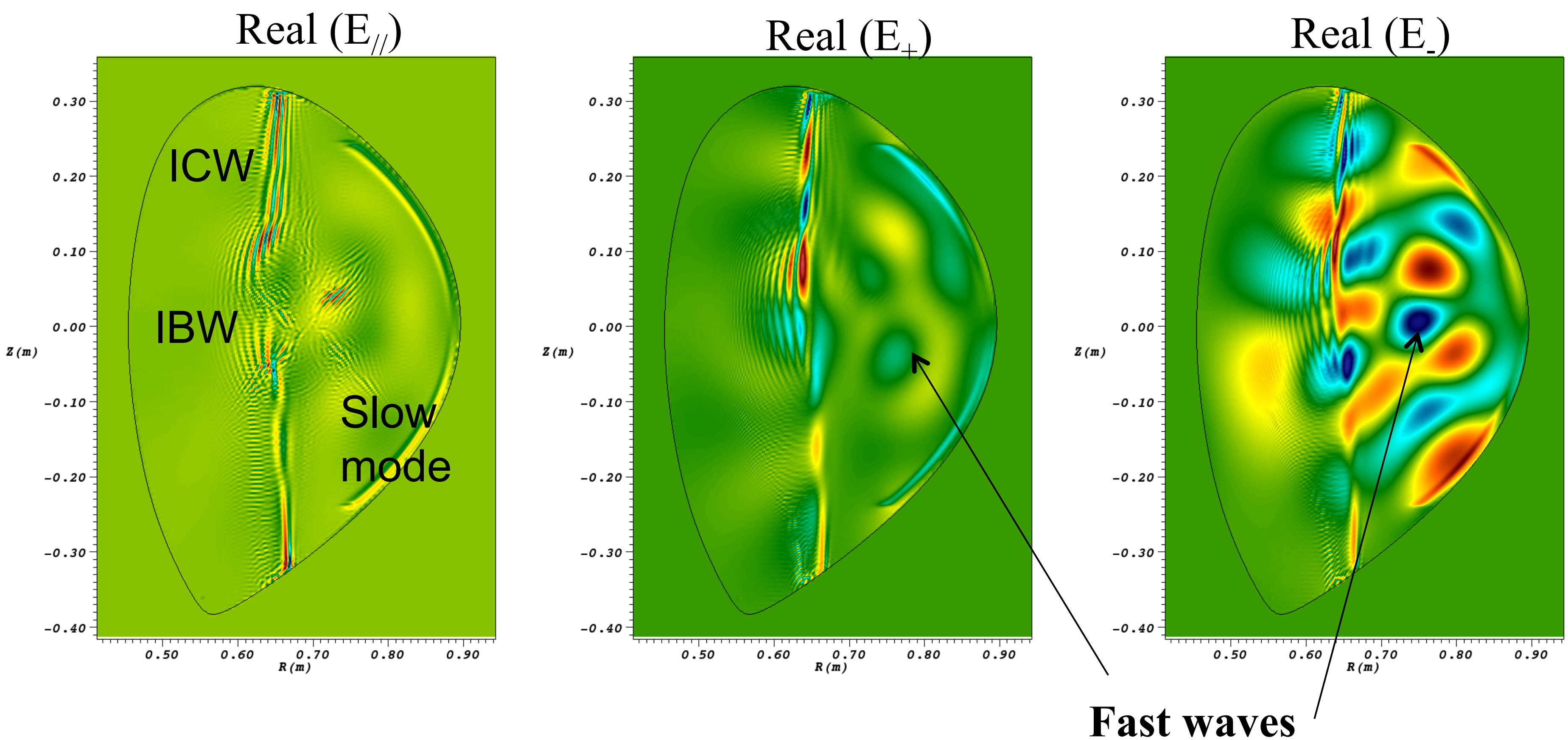
$$n_\perp^2 = \frac{P}{|S|}\, n_{//}^2 \quad \text{or} \quad \omega^2 = \Omega_D^2 + k_\perp^2 c_s^2 \qquad \text{if} \quad n_{//}^2 >> |S| \qquad \text{(ESICW)}$$

$$n_\perp^2 = P \quad \text{if} \quad n_{//}^2 << |S| \qquad \text{(KAW)}$$

note: requires $n_{//}^2 << c^2/N^2 V_A^2$ where N is the cyclotron harmonic, as well as $\omega^2 < k_{//}^2 v_{te}^2$

If hot ions are present, Ion Bernstein Waves may also be excited

Short Wavelength "Slow Mode" Excitation is also seen in the ICRF regime in C-Mod



AORSA simulation with 256X256 modes

91% D / 9% H minority heating in C-Mod with B_{T0}=5.29T, f_{RF}=80 MHz, n_ϕ=10

Slow mode also seen in TORIC simulations of this C-Mod discharge

Previous Slow Mode Simulations are Consistent with Theory

- Slow mode disappears if the electrons are "cold", or if the "l=0" component of $K_{zz,e}$ is omitted
- Slow mode remains if the ions are "cold".
- Slow mode has a significant component of $E_{//}$ polarization
- Slow mode appears off-midplane, as expected, if $k_{//}$ upshift due to B_p is required
  - Slow mode disappears if $k_{//}$ is held constant or very high values of $k_{//}$ are omitted or if B_p is ignored
- Slow mode appears in regions where $K_{xxc} = S$ is negative, for $\omega > \Omega_{ci}$,
  - i.e., in NSTX-U in the HHFW regime and in C-Mod on the low field side of the ion cyclotron layers
- ….and in regions where $K_{xxc} = S$ is positive, for $\omega < \Omega_{ci}$,
  - i.e., in NSTX-U and CMod on the high field side of the all cyclotron layers

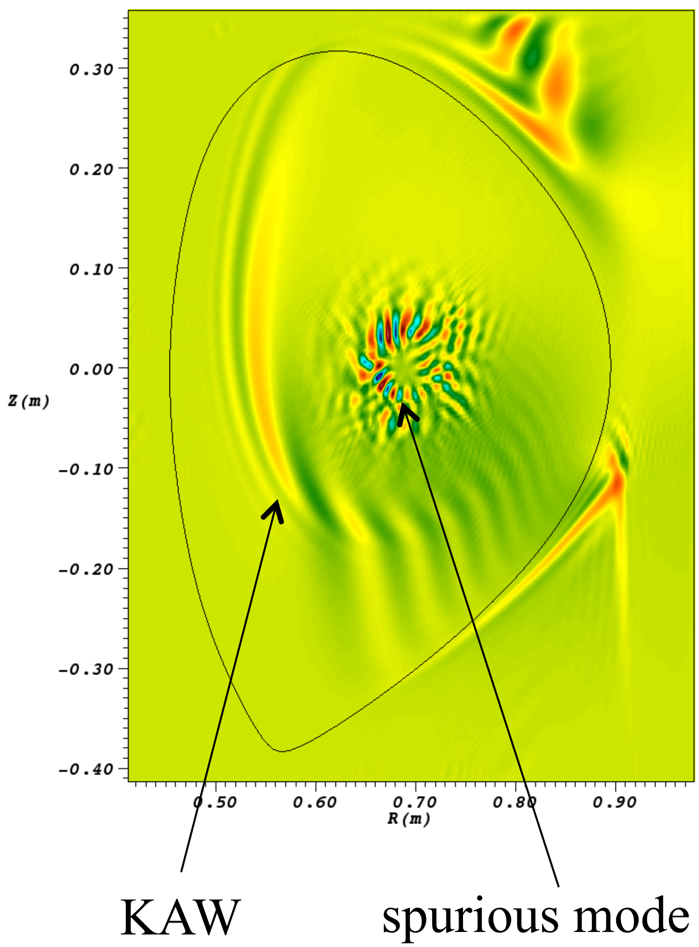
Some Difficulties were found in the Earlier Studies

- The slow mode wavelength in the simulations was often an order of magnitude longer than that predicted by the theoretical model
- The solutions often did not converge as the spatial grid was refined even further
- Spurious features would sometimes appear as the code convergence began to fail at higher resolutions
- AORSA solutions seems to include numerical pollution in the form of additional short wavelength structures that are associated with small $k_{||}$, *and these become worse at higher resolutions*
- The AORSA and TORIC codes have convergence problems with larger values of the toroidal wave number for the low frequency regime ($\omega < \Omega_{ci}$)

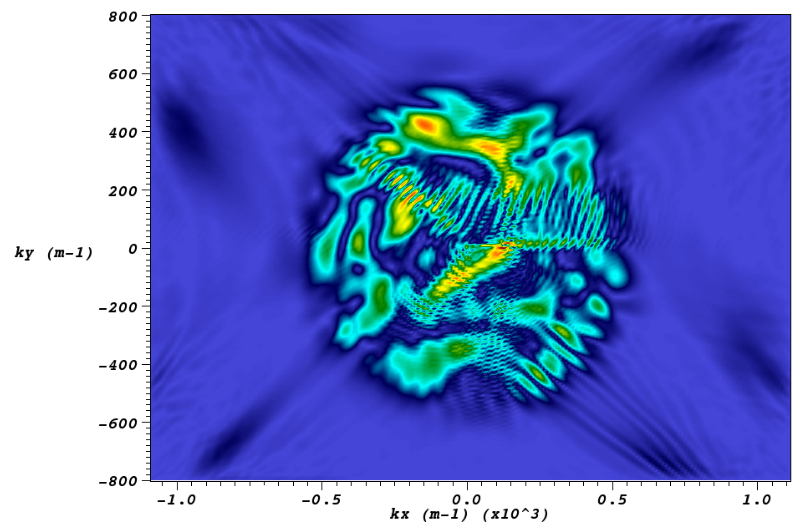
Initial AORSA simulations in the $\omega < \Omega_{ci}$ regime for a C-Mod discharge found a KAW plus a spurious mode

(AORSA2D: 200x200modes, f = 30 MHz, B_0 = 5.29 T, n_φ = 12, cold ions $\delta_0 = 1\text{x}10^{-4}$)

standard $Z^{(0)}$, $Z^{(1)}$, $Z^{(2)}$



$E_{||}$ spectrum



Convergence problems may be due to low $k_{//}$ modes present in the spectral representation with high spatial resolution that are undamped

Possible numerical problems associated with small k_{ll}'s in AORSA (and TORIC?)

Slow modes are excited when electrons are hot such $\{\omega < k_{//} v_{th}/2\}$, so that $Re(K_{zze})>0$

- requires finite T_e and B_p upshift of $k_{//} = \vec{k}\cdot\hat{b} = k_x b_x + k_y b_y + k_\phi b_T$

In the AORSA spectral expansion, one also get downshifts to smaller values of $k_{//}$ but only for high values of k_x, k_y, which are present in order to represent the slow mode

- these low $k_{//}$ terms that are present are undamped and can pollute the solution, giving rise to the spurious mode seen in the $E_{//}$ fields near the magnetic axis
- higher resolution exacerbates the problem since even more high k_x,k_y modes are added
- may be less problematic in TORIC since k// is more clearly defined:

$$k_{//} = \vec{k}\cdot\hat{b} = \frac{m}{r}b_p + \frac{n_\phi}{R}b_T$$

Very low values of $k_{//}$ seem unphysical, since there are many broadening mechanisms, such as the curvature of the equilibrium magnetic field lines, present in the plasma.

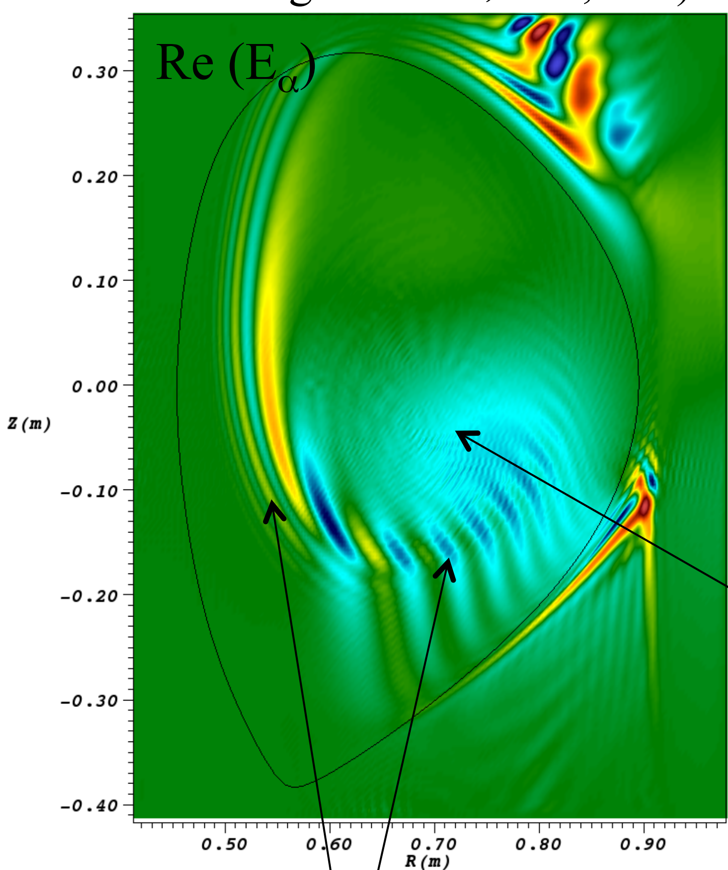
A modified version of the plasma dispersion function for electrons to address this issue is being developed and initial results, used here, are encouraging

- Details will be reported elsewhere [L.A. Berry, E.F. Jaeger, and D.L. Green et al]

Initial AORSA simulations, using the modified plasma dispersion function for electrons suppresses the spurious mode

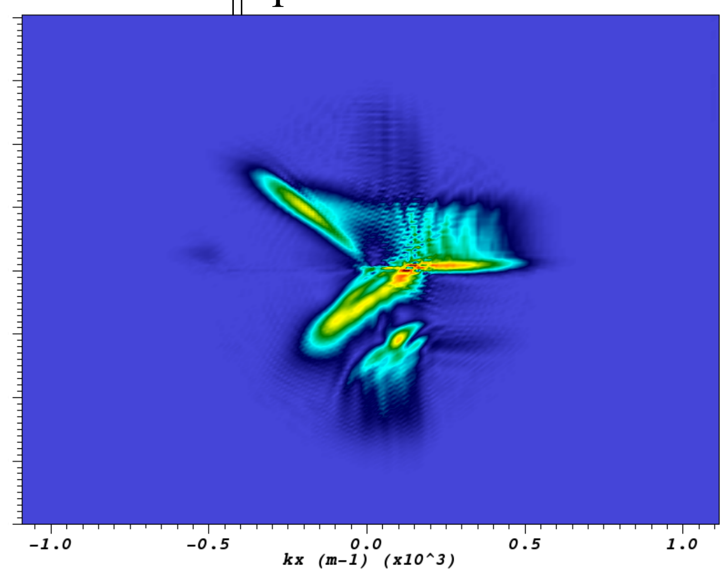
(AORSA2D: 200x200modes, f = 30 MHz, B_0 = 5.29 T, n_φ = 12, cold ions $\delta_0 = 1\mathrm{x}10^{-4}$)

Using new $Z^{(0)}$, $Z^{(1)}$, $Z^{(2)}$)



KAW

$E_{||}$ spectrum



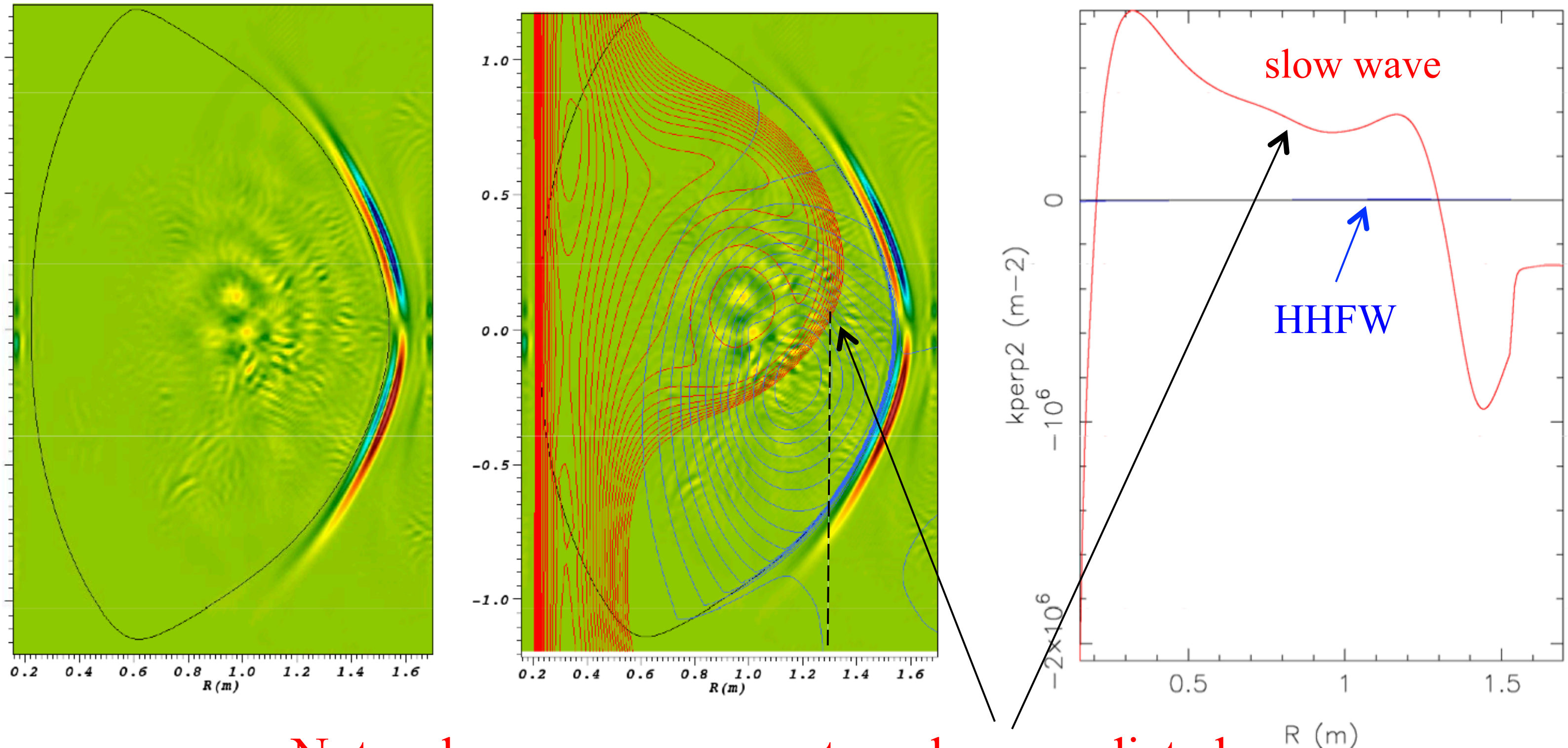
spurious mode largely suppressed and converged solutions are obtained

Similar initial results are found for the HHFW regime in NSTX and for the ICRF regime in C-Mod

AORSA now finds converged solutions with both fast and slow waves resolved at f=14MHz, n_ϕ=12 in NSTX test case

200 X 200 modes, $n_{min}=0.4\times10^{18}m^{-3}$, f=14 MHz, B_0=0.4509 T, n_ϕ=12, $\delta_0=1\times10^{-4}$

Real $E_{//}$ (with new $Z^{(0)}$, $Z^{(1)}$, $Z^{(2)}$) Midplane dispersion relation



Note: slow wave propagates where predicted

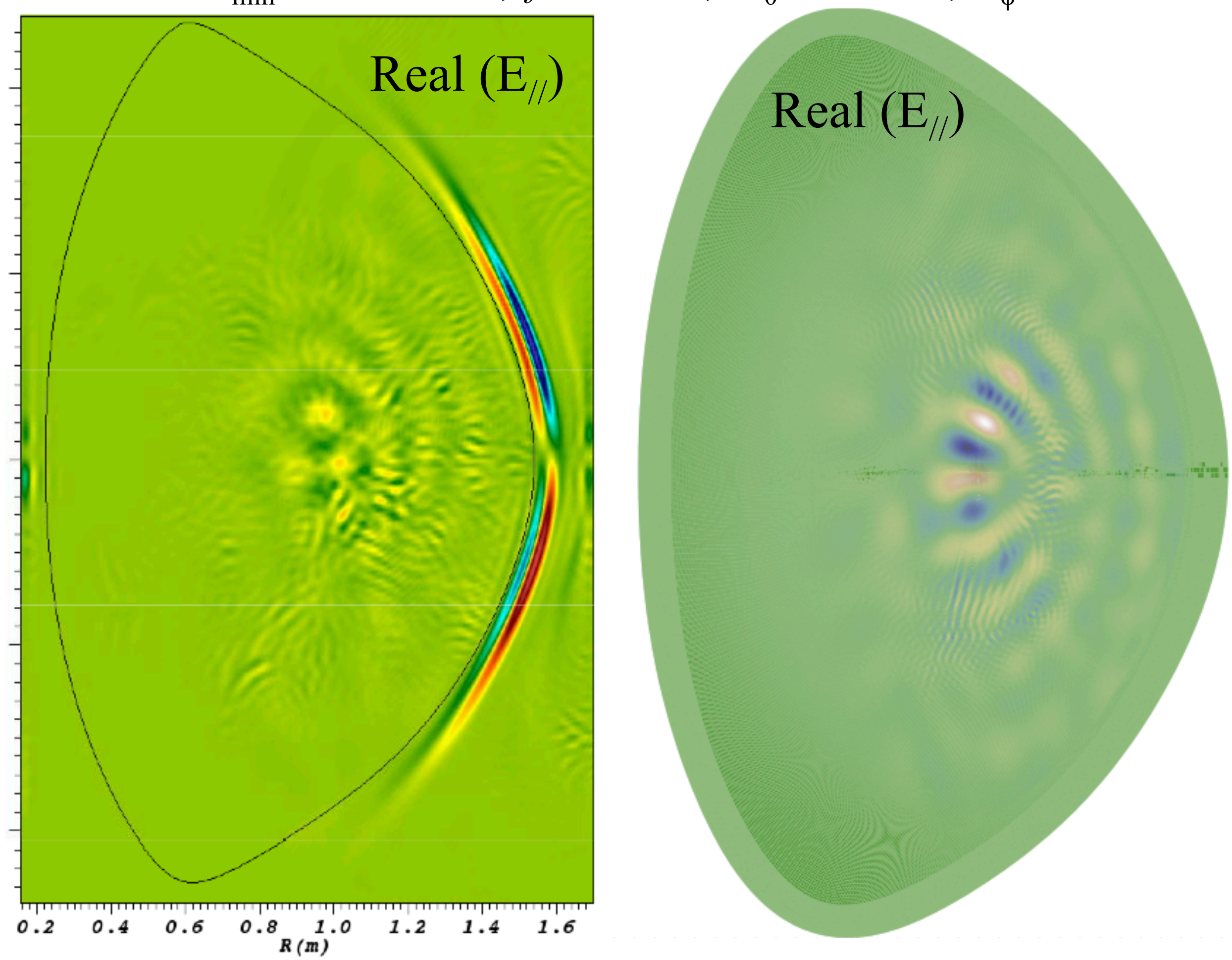
AORSA can find converged solutions with $f \geq$ 14 MHz with n_ϕ = 12

Converged solutions for f < 14 MHz and n_ϕ = 12 have not yet been obtained

TORIC and AORSA Wave Fields are comparable for the NSTX test case with *f* =14 MHz, n_ϕ=12

$n_{min}=0.4\times10^{18}m^{-3}$, f=14 MHz, B_0=0.4509 T, n_ϕ=12

AORSA
200 x 200 modes

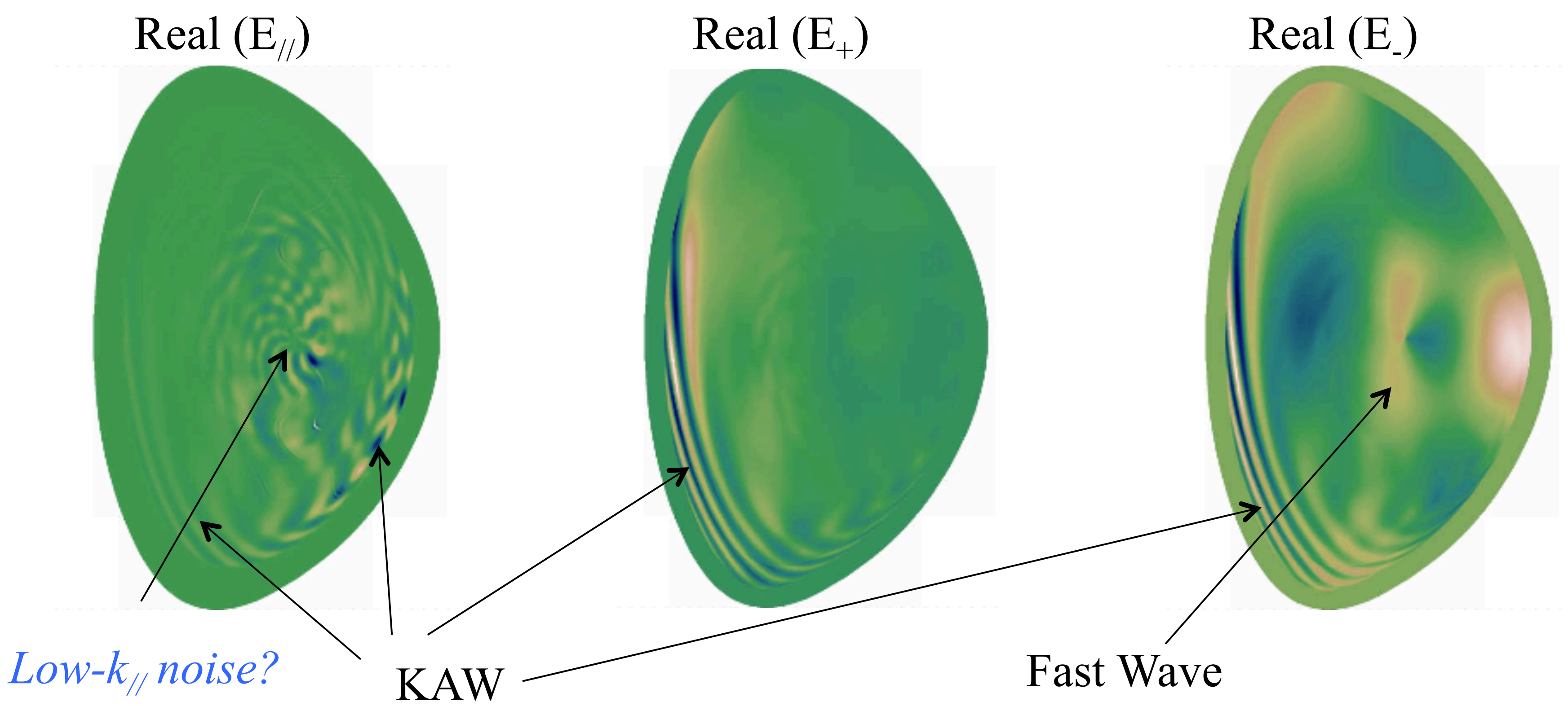


TORIC
n_{mod}=511
n_{elm}=960

TORIC can run with higher spatial resolution because of it's reduced model for the plasma dielectric response

Both the Fast Wave and the KAW are found in NSTX shot #112705 for $\omega < \Omega_{ci}$ with $n_\phi=1$ at high resolution

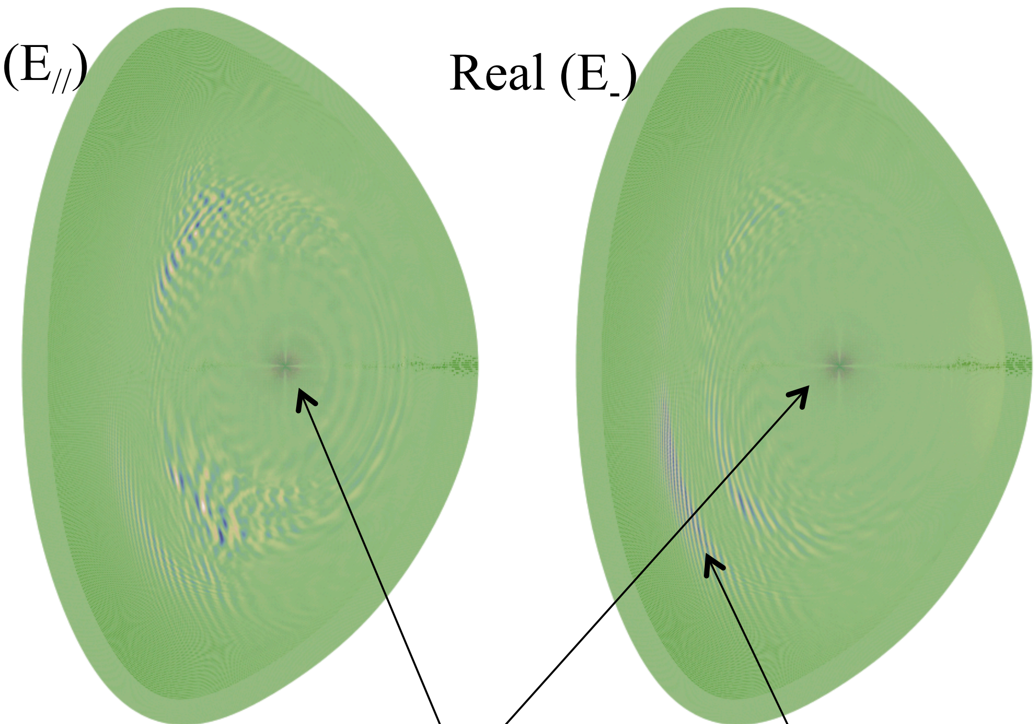
TORIC converged solution for f=2 MHZ with n_{mod}=1024 and n_{elm}=960 and n_ϕ=1



Some issues may remain with low $k_{//}$ noise but problem appears to be less severe

However, TORIC solutions for shot #112705 for $\omega < \Omega_{ci}$ do not converge with n_ϕ=12, even at high resolution

TORIC with f=2 MHZ with n_{mod}=511 and n_{elm}=960 and n_ϕ=12



no real visual evidence for fast wave propagation

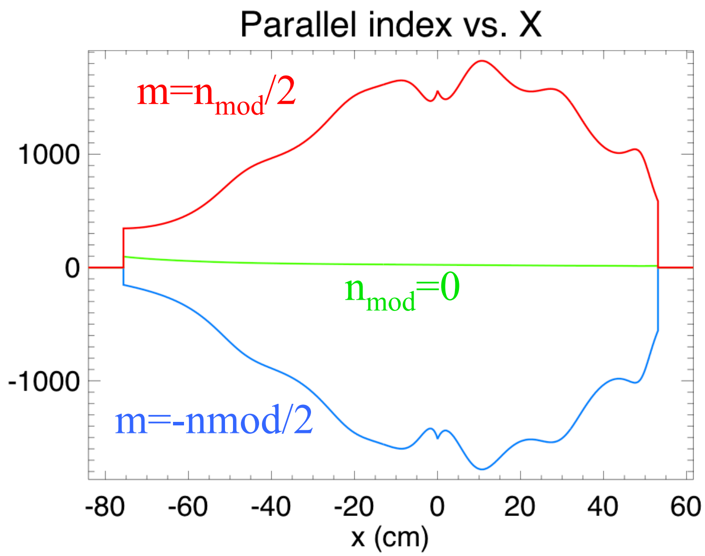
Internal power balance checks are off by a factor of about 670

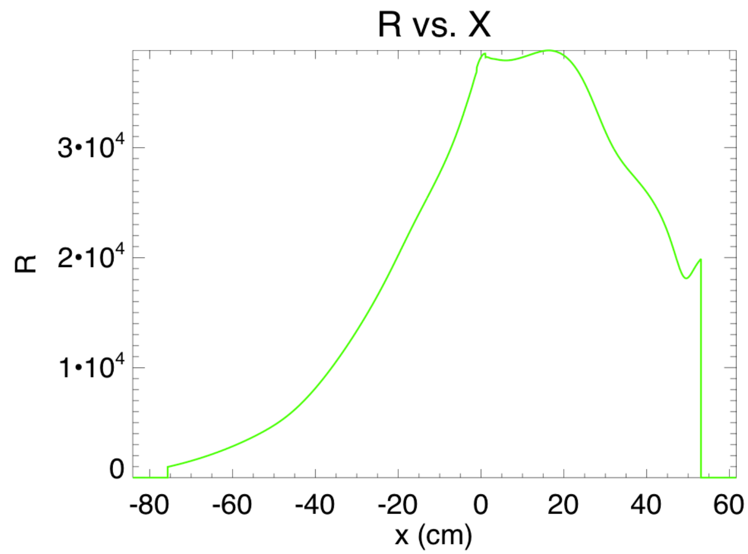
Wave Fields peak on axis and near high field side edge

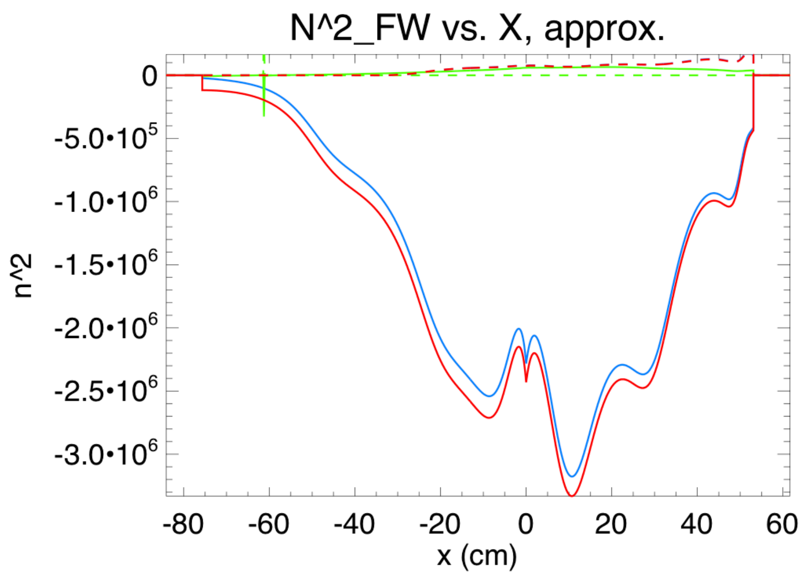
Convergence difficulties consistent with AORSA problems at low *f* and high n_ϕ

Problem may be related to wave evanescence at these parameters

For the NSTX 2 MHz test case, the fast wave appears to propagate with n_ϕ=1







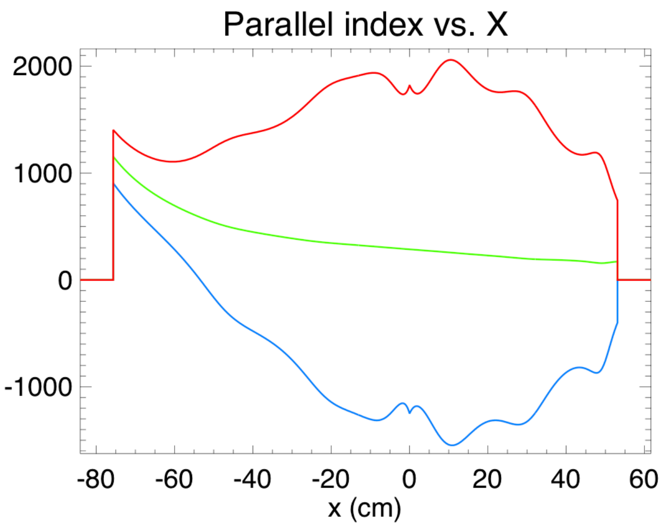
Near the axis with $n_{mod} \sim 0$

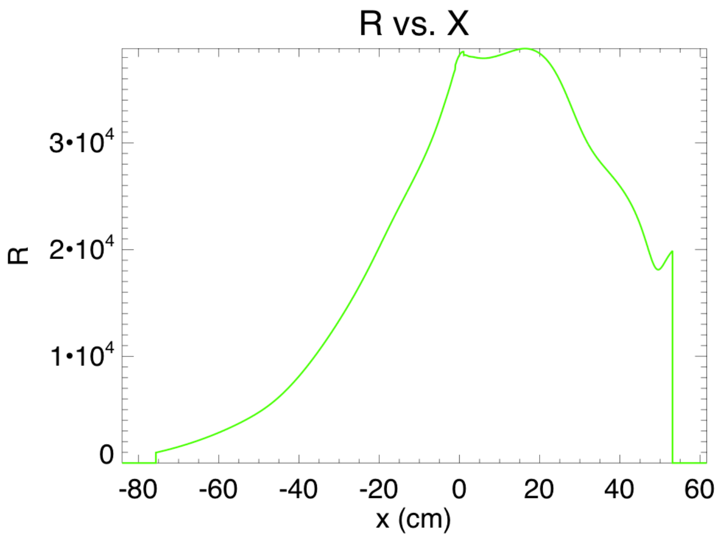
$n_{//}^2 \sim (50)^2 \sim 2.5 \times 10^3$

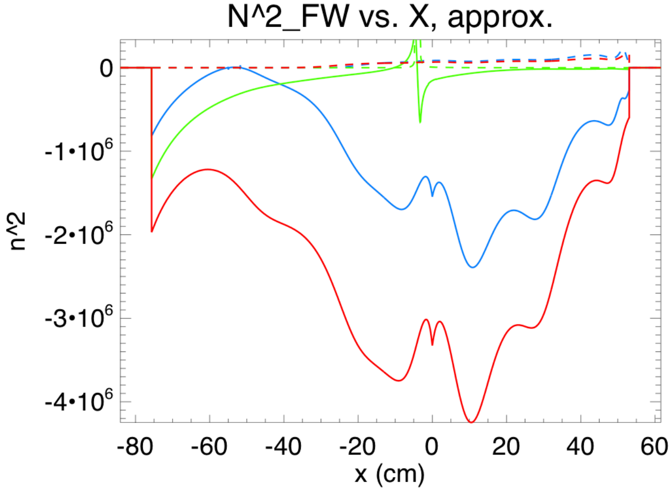
R ~ 4 x 104

so the fast wave propagates ($n_{//}^2$ < R), as seen in the dispersion relation

For the NSTX 2 MHz test case, the fast wave appears to be cutoff with n_ϕ=12







Near the axis with $n_{mod} \sim 0$

$n_{//}^2 \sim (400)^2 \sim 1.6 \times 10^5$

$R \sim 4 \times 10^4$

so the fast wave is evanescent ($n_{//}^2 > R$), as seen in the dispersion relation

- Suggests codes do not converge if the FW is cut off
- Will try sweeping the frequency to see if an eigenmode can be excited

Summary of key results

"Slow modes" have been found in both the low frequency, ion cyclotron and high harmonic fast wave regimes in NSTX and C-Mod, consistent with theory

- requires sufficient spatial resolution, hot electron effects, and $k_{//}$ upshift and downshift

Convergence difficulties in previous studies could be caused by:

- large difference between fast and slow mode wavelengths
- presence of down-shifted $k_{//}$ modes, needed in the spectral representations, that are undamped
- evanescence of the launched fast waves for certain parameters

Converged solutions with both the fast wave and slow wave present have been found in agreement with theoretical dispersion relations

Methods for dealing with convergence issues have been identified

- Work is underway to include $k_{//}$ broadening in the plasma dispersion function (L.A. Berry, E.F. Jaeger, and D.L. Green, to be reported elsewhere)

Future directions

- Continue studying methods to insure code convergence at high spatial resolution
- Assess effects of power absorption by the slow wave in NSTX-U, ITER, and other tokamaks
- Compare full wave field solutions against the GAE/CAE eigenmodes identified in the HYM code (Belova, 2010 APS-DPP meeting) for NSTX discharges
- Predict structure of rf-driven energetic particle modes in NSTX-U using the proposed dedicated launcher (E. Fredrickson et al)
- Explore frequency sweeping as a means of studying CAE/GAE eigenmode excitation
- Study effects of rf modifications of co-resonant fast particle distributions that excite the energetic particle modes (EPMs) in ITER and other plasmas
    - Determine if rf modification can help suppress these EPMs