NSTX-U

Supported by U.S. DEPARTMENT OF ENERGY Office of Science

Drift kinetic effects on 3D plasma response in high-beta tokamak resonant field amplification (RFA) experiments*

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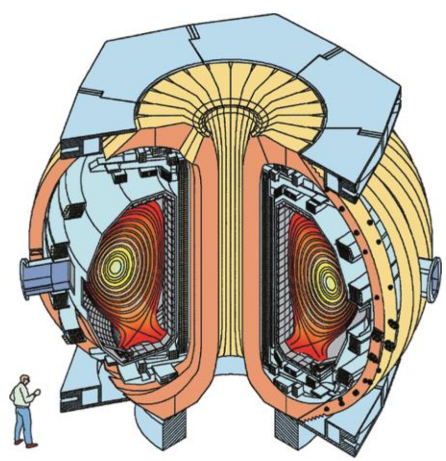
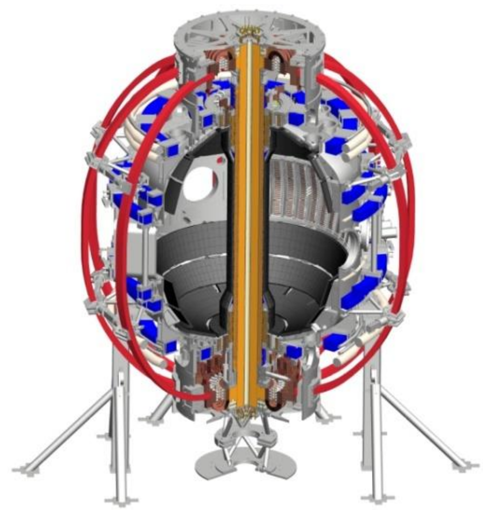
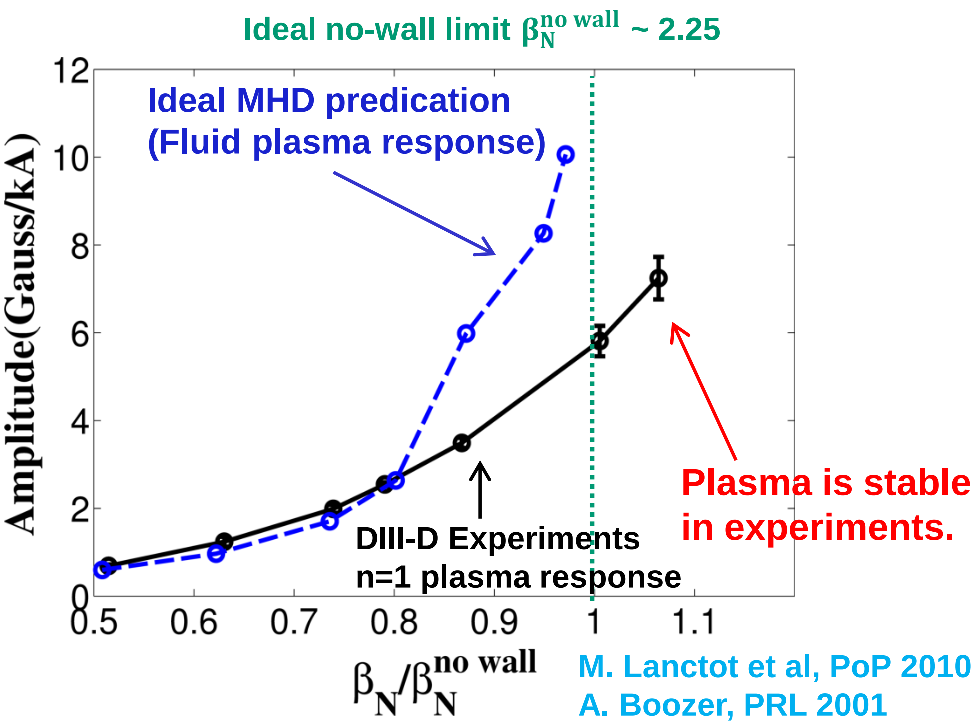




A Long Standing Issue is To Explain Linear Increase of Plasma Amplification Across and Beyond Ideal No-Wall Limit in Experiments.

- External non-axisymmetric (3D) magnetic perturbations
  - → strongly modify tokamak plasmas with perturbed plasma currents
  - → plasma response (include magnetic perturbation and plasma displacement etc.)
- Physics understanding and predictability for plasma response are not established yet.
- A long standing issue:



M. Lanctot et al, PoP 2010
A. Boozer, PRL 2001

Resolution of long standing issue is critical:

Neoclassical toroidal viscosity

Energetic particle losses

Error fields control

Resistive wall mode instability

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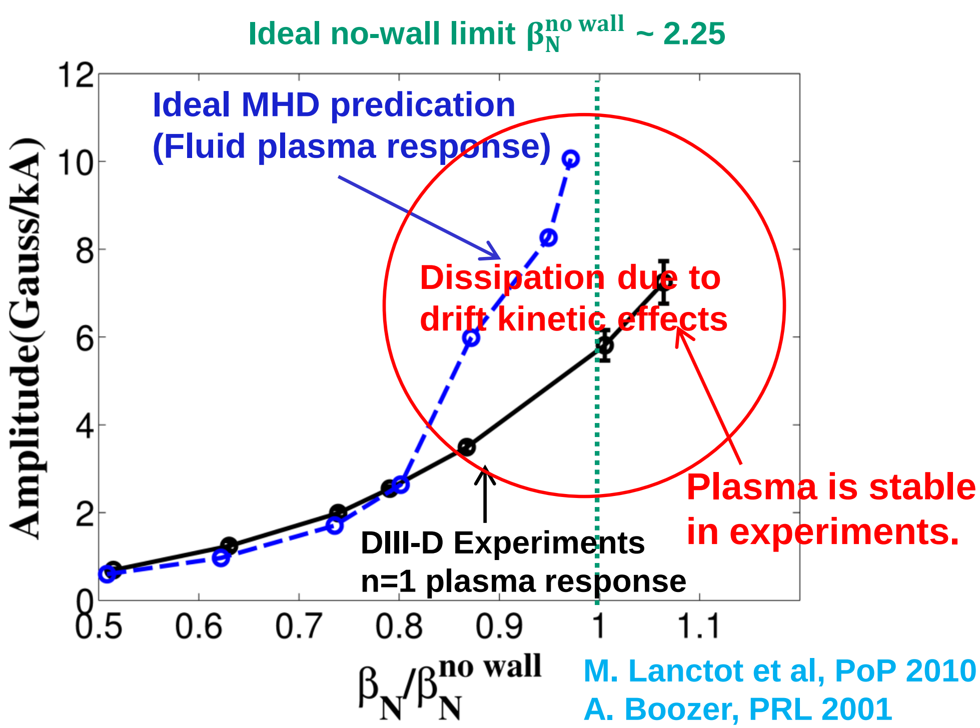

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Resolution of long standing issue is critical:

Neoclassical toroidal viscosity

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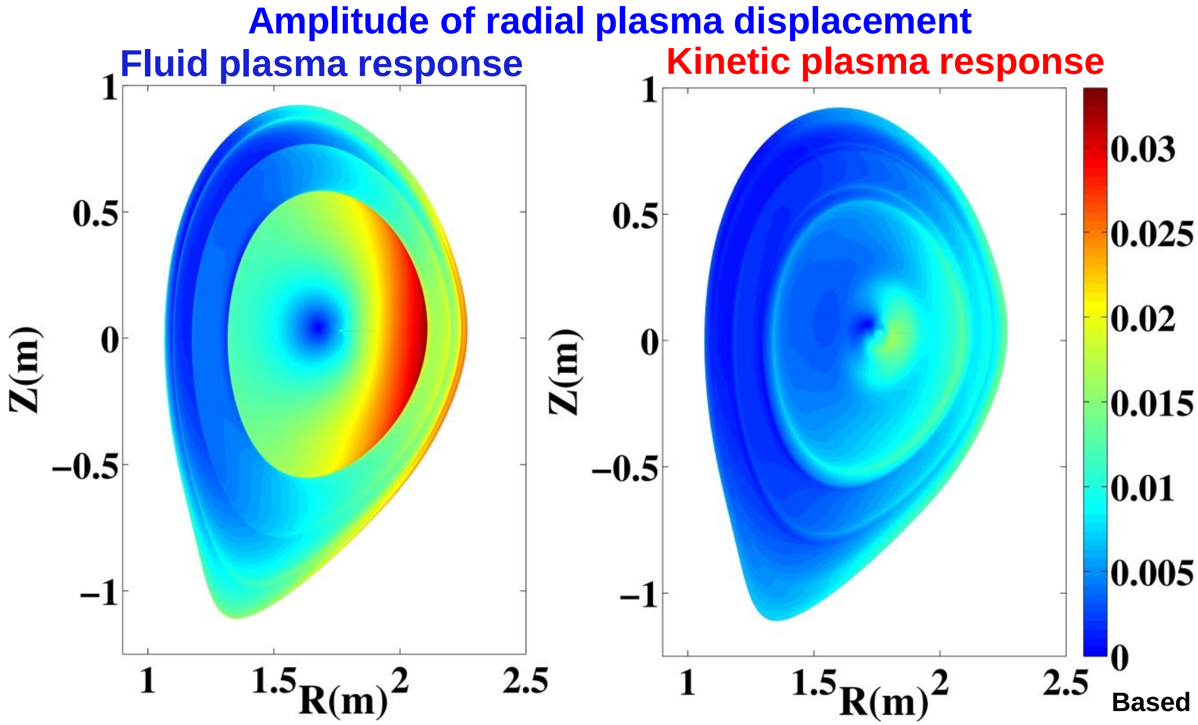
Kinetic Plasma Response Provides Resolution of Long Standing Issue Near/Above No-Wall Limit

Kinetic plasma response: Self-consistent calculation of MHD equation with drift kinetic effects (hybrid drift kinetic-MHD) → kinetic effects affect the plasma response



Based on DIII-D 135773

- Kinetic plasma response reproduces every aspect observed in external and internal measurements in DIII-D and NSTX experiments.
- Self-consistent calculation of hybrid kinetic-MHD is necessary to obtain quantitative modeling of kinetic plasma response.
- Thermal particles play a major role in modifying plasma response and keeps the finite amplification.
- Fluid rotation destabilizing effect and plasma-wall coupling effect are important in NSTX experiments.

Kinetic Plasma Response Modelling is Validated Through Quantitative Comparison with DIII-D and NSTX Experiments

- Kinetic plasma response modeled with hybrid-kinetic MHD equations using MARS-K

- Validation of kinetic plasma response modelling in DIII-D and NSTX experiments
  - Comparison of magnetic sensor measurements
  - Comparison of internal structure measured by Soft X ray
  - Characteristic of frequency scan in DIII-D and NSTX experiments

M. J. Lanctot et al, PoP 2010
J.-K. Park et al, PoP 2009

- Physical understanding of kinetic plasma response
  - Drift kinetic effects from thermal particles are crucial to obtaining the correct response.

    - precession resonance, bounce resonance and transit resonance of thermal ions
  - Fluid rotation is an important factor to determine the plasma response in NSTX.

Fluid Plasma Response Model is Based on Linearized MHD Equations (MARS-F)

MARS-F can solve linearized MHD equation with external coils, vacuum and resistive wall.

MHD equations developed in MARS-F:

$$i(\omega + n\Omega)\vec{\boldsymbol{\xi}} = \vec{\mathbf{v}} + (\vec{\boldsymbol{\xi}} \cdot \vec{\boldsymbol{\nabla}}\Omega)R^2\vec{\boldsymbol{\nabla}}\phi$$

$$i\rho(\omega + n\Omega)\vec{\mathbf{v}} = -\vec{\boldsymbol{\nabla}}p + \vec{\mathbf{j}} \times \vec{\mathbf{B}}_0 + \vec{\mathbf{J}}_0 \times \vec{\mathbf{b}} + \rho\left[2\Omega\hat{\mathbf{Z}} \times \vec{\mathbf{v}} - (\vec{\mathbf{v}} \cdot \vec{\boldsymbol{\nabla}}\Omega)R^2\vec{\boldsymbol{\nabla}}\phi\right] - \vec{\boldsymbol{\nabla}} \cdot (\rho\vec{\boldsymbol{\xi}})\Omega\hat{\mathbf{Z}} \times \vec{\mathbf{V}}_0$$

$$i(\omega + n\Omega)\vec{\mathbf{b}} = \vec{\boldsymbol{\nabla}} \times (\vec{\mathbf{v}} \times \vec{\mathbf{B}}_0) + (\vec{\mathbf{b}} \cdot \vec{\boldsymbol{\nabla}}\Omega)R^2\vec{\boldsymbol{\nabla}}\phi - \cancel{\vec{\boldsymbol{\nabla}} \times (\eta\vec{\mathbf{j}})}$$

$$i(\omega + n\Omega)p = -\vec{\mathbf{v}} \cdot \vec{\boldsymbol{\nabla}}P_0 - \Gamma P_0\vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{v}}$$

Ideal MHD
(no resistive instability is observed in experiments)

Applied field frequency

$$\vec{\mathbf{j}} = \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{b}}$$

Coil equations:

$$\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{b}} = \vec{\mathbf{j}}_{coil} \qquad \vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{j}}_{coil} = 0$$

vacuum and resistive wall are included.

Y.Q. Liu et al, PoP 2010

Linear Kinetic Plasma Response Model Uses a Hybrid Drift-Kinetic MHD Formulation (MARS-K)

MARS-K extends MARS-F and solves linearized MHD equations with perturbed kinetic pressure.

MHD equations:

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$$\vec{\mathbf{j}} = \vec{\boldsymbol{\nabla}} \times \vec{\mathbf{b}}$$

replaced by kinetic pressure

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$$\vec{\mathbf{p}} = p\overleftrightarrow{\mathbf{I}} + p_\parallel \vec{\mathbf{b}}\vec{\mathbf{b}} + p_\perp(\overleftrightarrow{\mathbf{I}} - \vec{\mathbf{b}}\vec{\mathbf{b}})$$

Kinetic pressure $p_\parallel$ and $p_\perp$ couple with MHD equations

$$p_\parallel e^{-i\omega t + in\phi} = \sum_{e,i} \int d\Gamma M v_\parallel^2 f_L^1$$

$$p_\perp e^{-i\omega t + in\phi} = \sum_{e,i} \int d\Gamma \frac{1}{2} M v_\perp^2 f_L^1$$

Drift-kinetic equation:

$$\frac{df_L^1}{dt} = f_\varepsilon^0 \frac{\partial H^1}{\partial t} - f_{P_\phi}^0 \frac{\partial H^1}{\partial \phi} - \nu_{eff} f_L^1$$

H^1: perturbed Lagrangian

Ignore finite orbit width effect.

Y.Q. Liu et al, PoP 2008

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Successful benchmarking among MARS-K, IPEC-PENT and MISK

Z.R. Wang et al. PoP 2014
J.W. Berkery et al, PoP 2014

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Resonant operator in f_L^1:

Diamagnetic drift

Applied field frequency

$$\lambda_{ml} = \frac{n[\omega_{*N} + (\hat{\varepsilon}_k - 3/2)\omega_{*T} + \omega_E] - \omega}{n\omega_d + [\alpha(m + nq) + l]\omega_b + n\omega_E - \omega - i\nu_{eff}}$$

Precession drift

Bounce/Transit

EXB

Crook Collisions

Y.Q. Liu et al, PoP 2008

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Precession motion of trapped particles ($\alpha=0$) is important when:

$$l=0 \text{ and } n\omega_d + n\omega_E - \omega \rightarrow 0$$

$$\lambda_{ml} = \frac{n[\omega_{*N} + (\hat{\varepsilon}_k - 3/2)\omega_{*T} + \omega_E] - \omega}{n\omega_d + n\omega_E - \omega - i\nu_{eff}}$$

$p_\parallel$ and $p_\perp$ ~ λ_{ml}

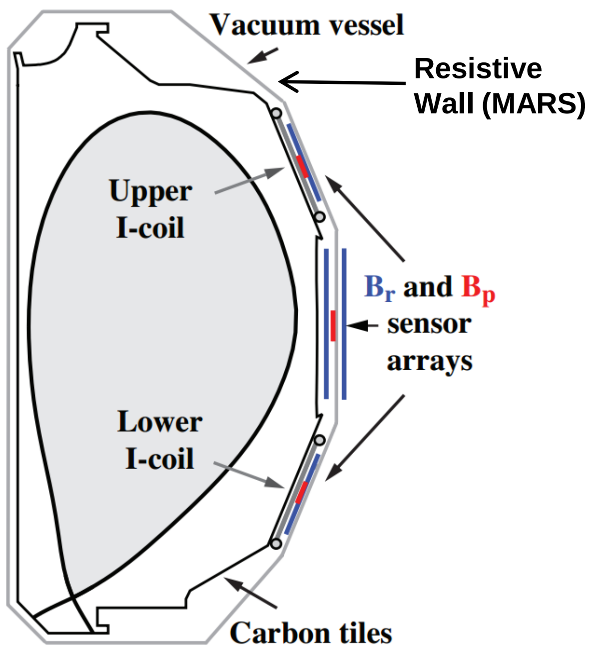
MARS-K in self-consistent approach:

Drift kinetic effects can modify plasma response.

Y.Q. Liu et al, PoP 2008

n=1 Plasma Response in DIII-D and NSTX is Studied with Different Coil Configurations

DIII-D configuration



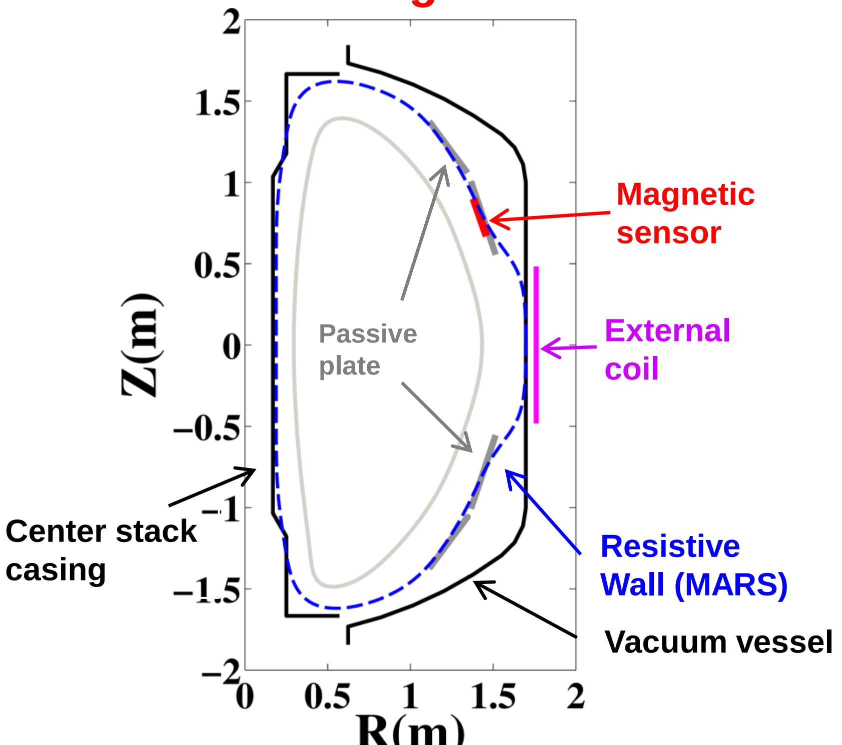
n=1 magnetic perturbation generated by I-coils. Phase difference of upper and lower I-coil current is $\Delta\phi$ =240 deg

The field rotation frequency is +10 Hz (Co-Current).

$$\frac{\delta B^{pla}}{I_{coil}} = \frac{\delta B^{tot} - \delta B^{vac}}{I_{coil}} \text{ (G/kA)}$$

M.J. Lanctot et al, PoP 2010

NSTX configuration



n=1 perturbation is applied by midplane external coils.

The field rotation frequencies are +30 Hz (Co-Current) and -30Hz (Counter-Current)

$$\frac{\delta B^{tot}}{\delta B^{vac}} = \frac{\delta B^{pla} + \delta B^{vac}}{\delta B^{vac}}$$

J.-K. Park et al, PoP 2009

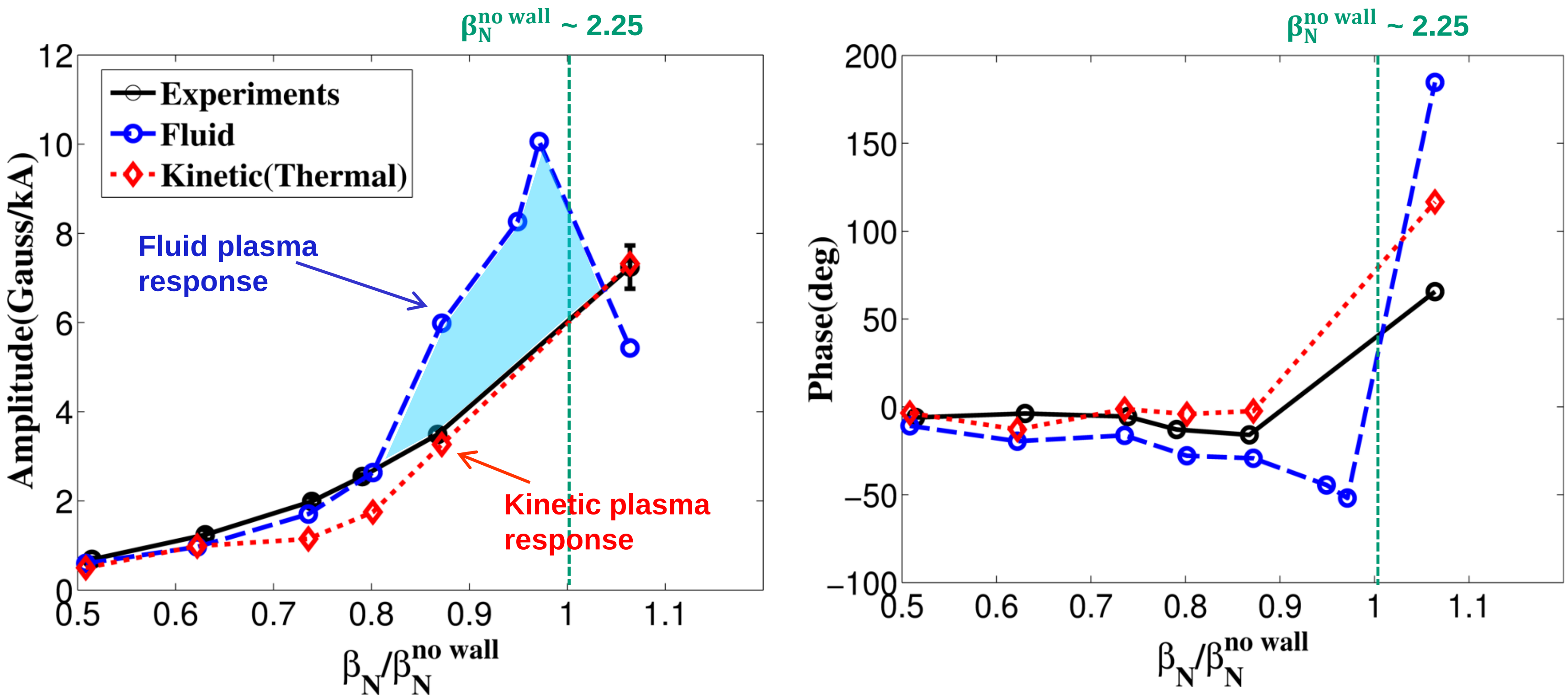
Validation of kinetic plasma response modelling

DIII-D: Comparison of Plasma Response on Magnetic Sensor Kinetic Plasma Response Agrees With Experiments

The simulated plasma response is compared with experimental measurements at <u>internal radial sensor on low field side of mid-plane</u>.

1) Fluid plasma response is solved by MARS-F
2) Kinetic plasma response is solved by MARS-K (Thermal case)

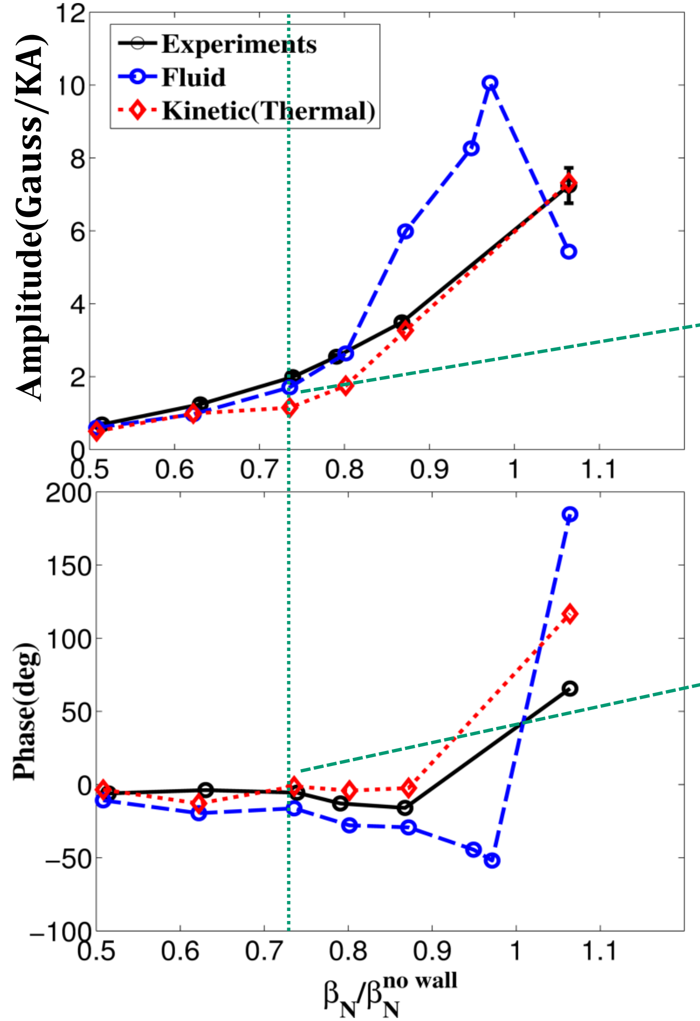
$P_{eq} = P_{th} + P_{EP}$, the kinetic $p_\perp, p_\parallel$ are contributed by thermal particles.



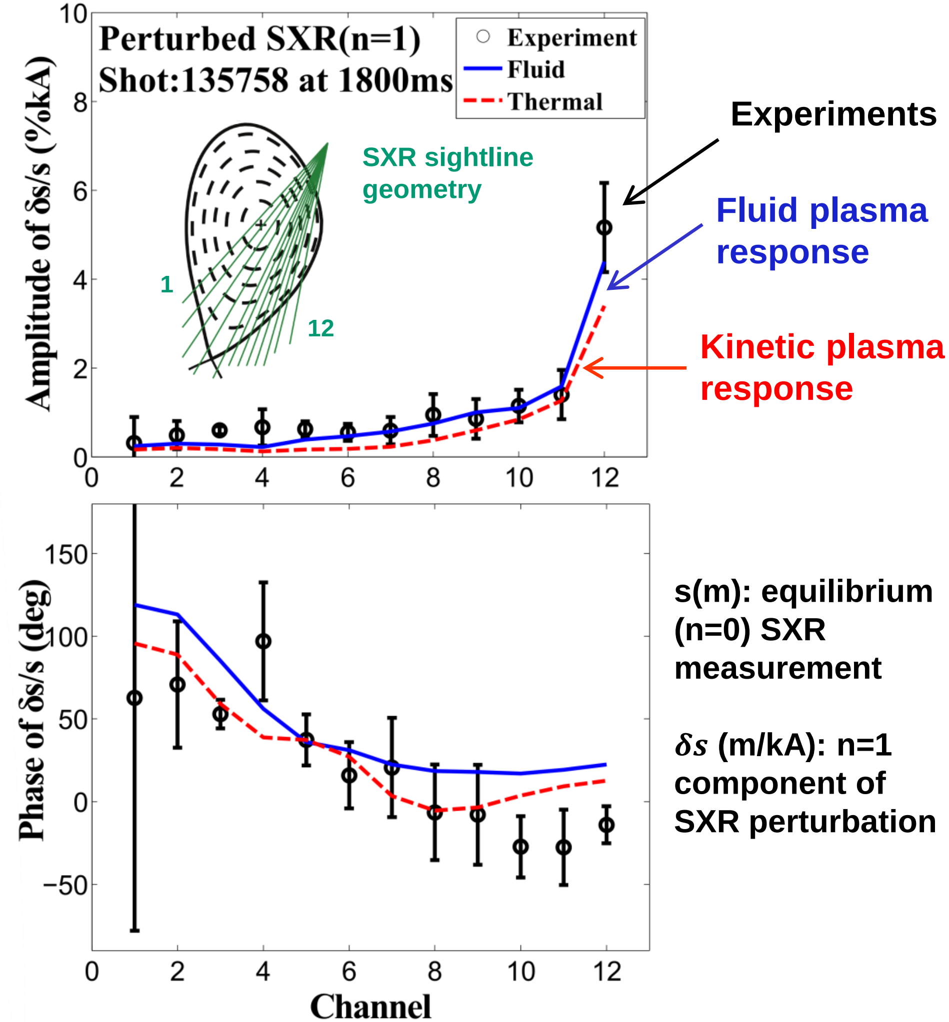
- The kinetic effect keeps the finite amplification of plasma response, as experiments, around the no-wall limit.
- Hybrid kinetic-MHD, agreeing with experiments, predicts the plasma is stable at the highest beta.

DIII-D: Internal Structure of Fluid/Kinetic Plasma Response Agrees with Soft X-Ray Measurements at Low Beta

Comparison of n=1 plasma response on ISL magnetic sensor



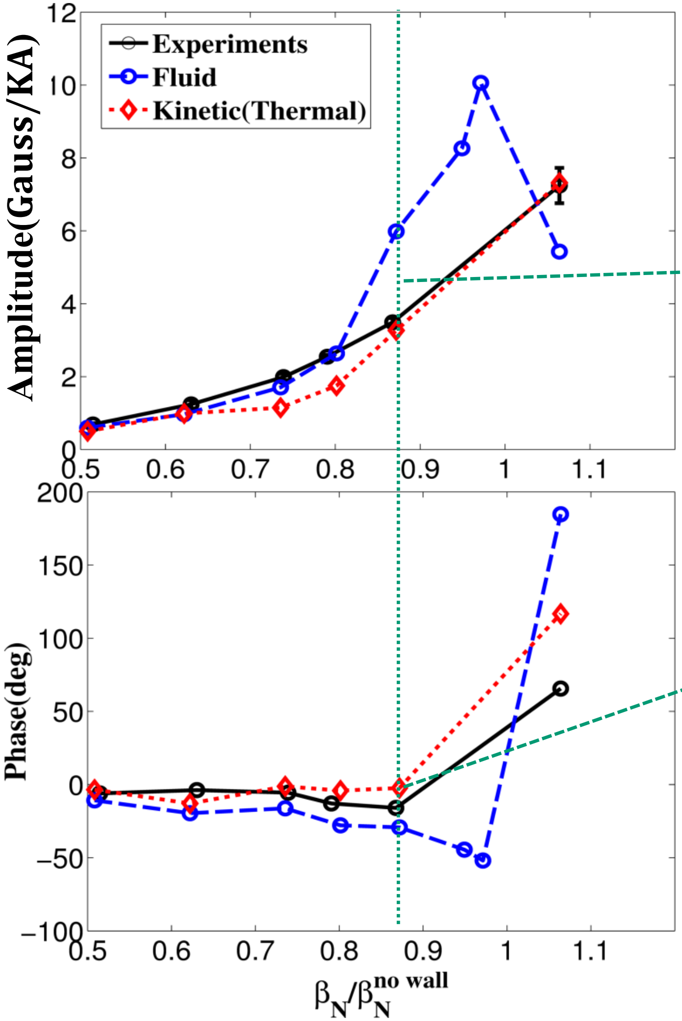
Internal response structure comparison



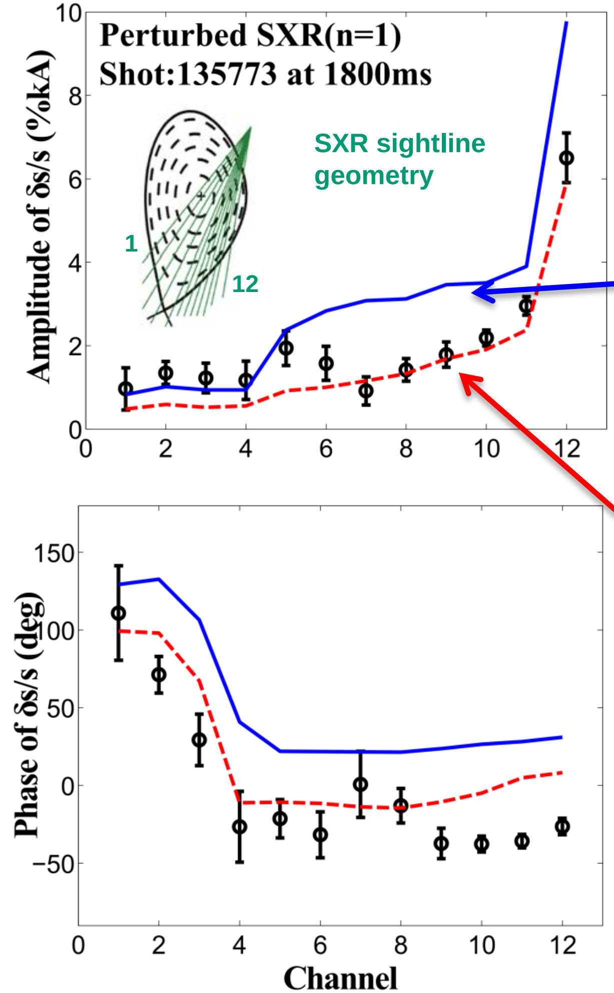
The quantitative agreement between experiments and theory for both fluid and kinetic cases.

DIII-D: Kinetic Plasma Response Prediction at High Beta Agrees with Internal Structure Measurement from Soft X-Ray

Comparison of n=1 plasma response on ISL magnetic sensor

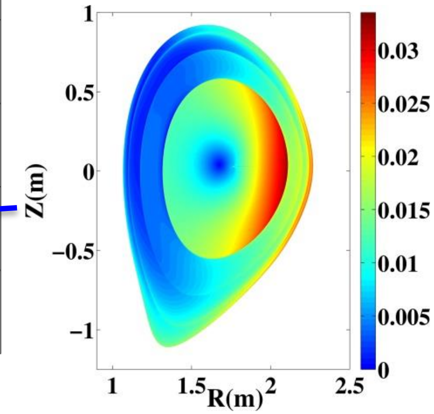


Internal response structure comparison

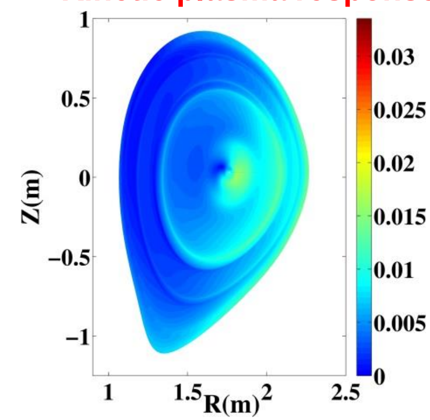


2D radial displacement

Fluid plasma response



Kinetic plasma response



The quantitative agreement between experiment and kinetic plasma response simulation.

NSTX: Kinetic Plasma Response With Thermal Particles Shows Quantitative Agreement with Magnetic Sensor Measurements

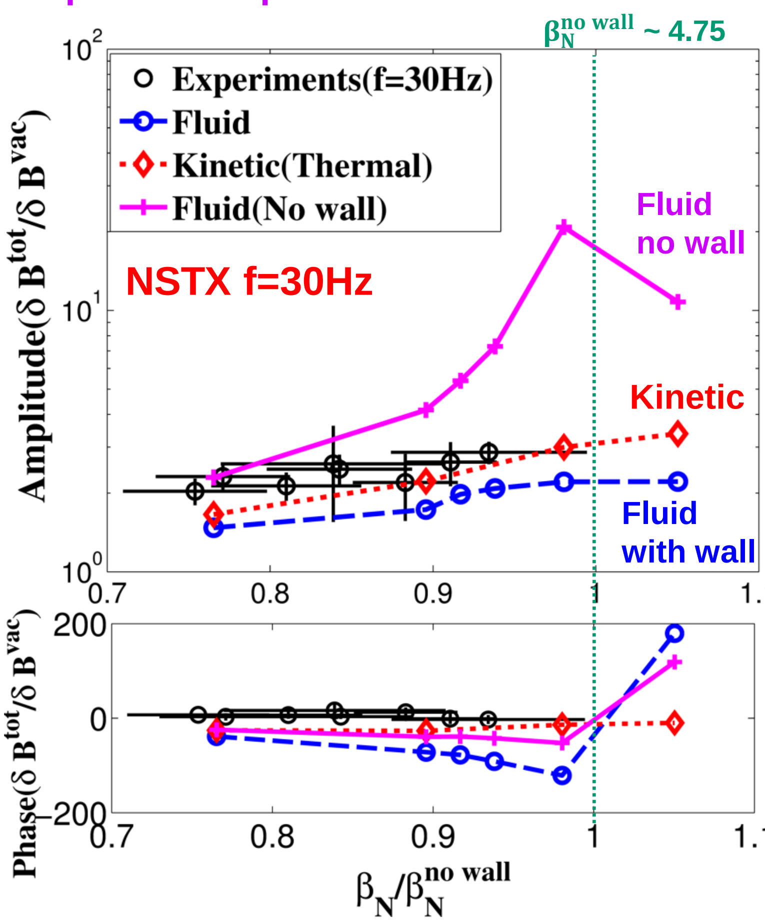
The simulated plasma response is compared with experiments at upper radial magnetic sensor.

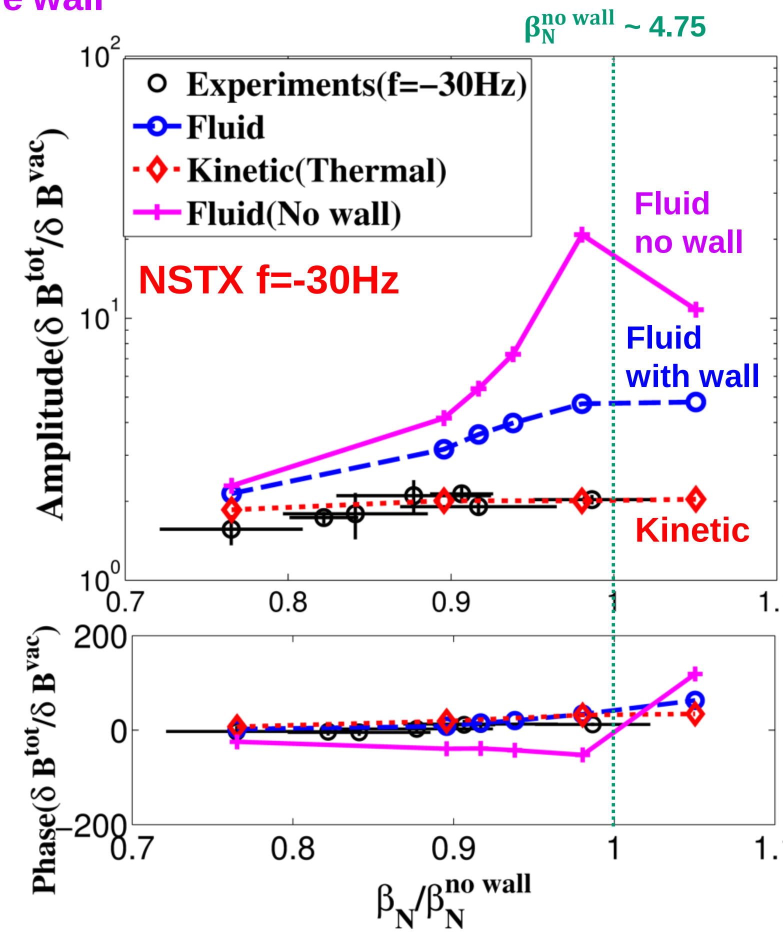
1) Fluid plasma response is solved by MARS-F

2) Kinetic plasma response is solved by MARS-K (Thermal case)

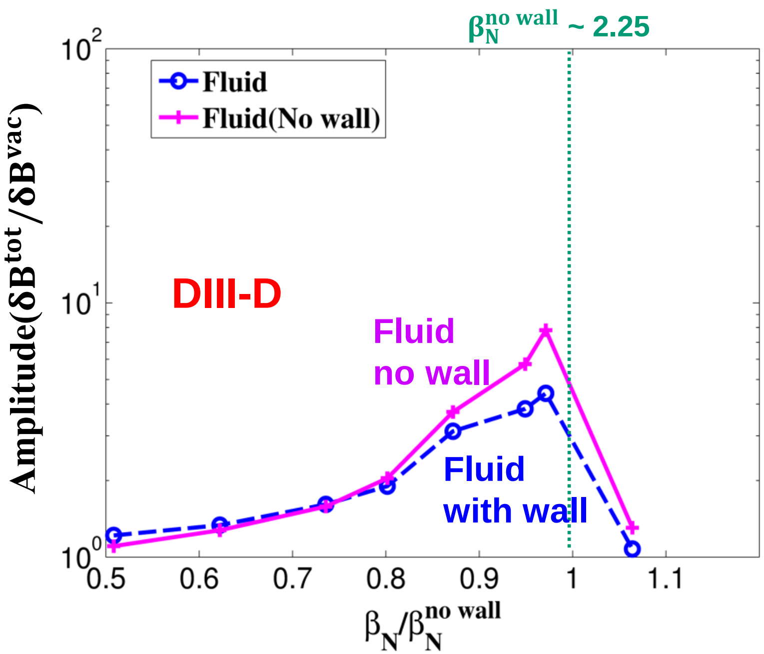
$P_{eq} = P_{th} + P_{EP}$, the kinetic $p_\perp, p_\parallel$ are contributed by thermal particles.

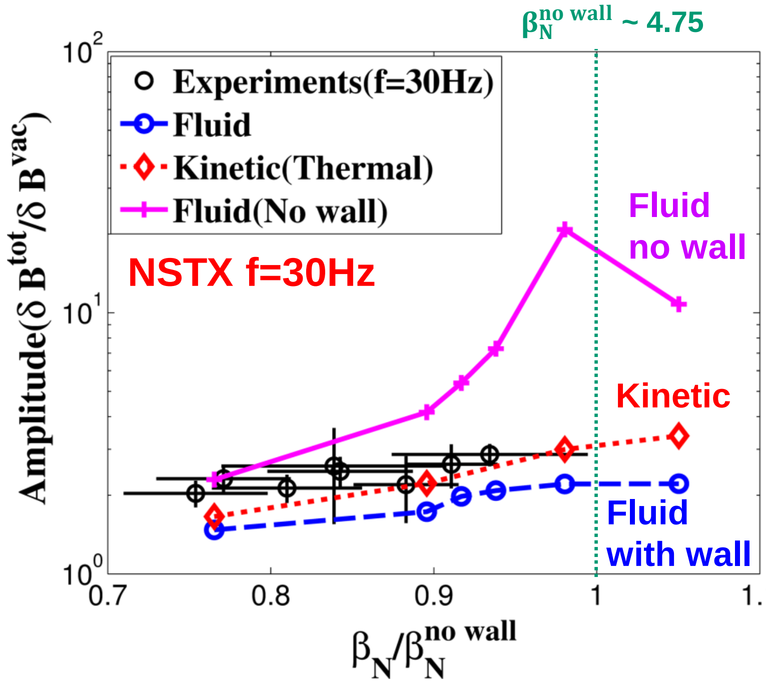
3) Fluid plasma response is solved without resistive wall





NSTX Shows Strong Plasma-Wall Coupling Effect in Plasma Response Experiments





A simple analysis based on (s, α) model in the approximation with a single dominant mode:

Park, Boozer et al, PoP 2009

At marginal stability: $\left|\frac{\delta B^{tot}}{\delta B^{vac}}\right| = \left|\frac{1}{2\pi n \tau_w f_{coil}}\right|$

No wall: $\tau_w \to 0 \;\Rightarrow\; \left|\frac{\delta B^{tot}}{\delta B^{vac}}\right| \to +\infty$

DIII-D experiments

τ_w=2.3ms, f_{coil}=10Hz , $\left|\frac{\delta B^{tot}}{\delta B^{vac}}\right|$=6.91

$2\pi f_{coil} < 1/\tau_w$ $\Rightarrow$ weak plasma-wall coupling

NSTX experiments

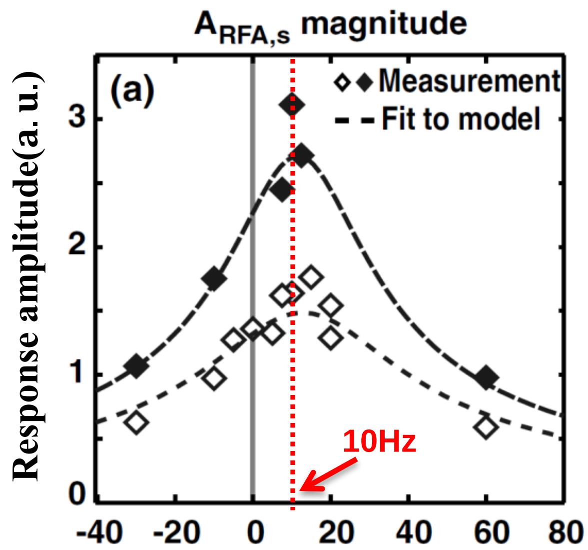
τ_w=3.5ms, f_{coil}=+/-30Hz, $\left|\frac{\delta B^{tot}}{\delta B^{vac}}\right|$=1.52

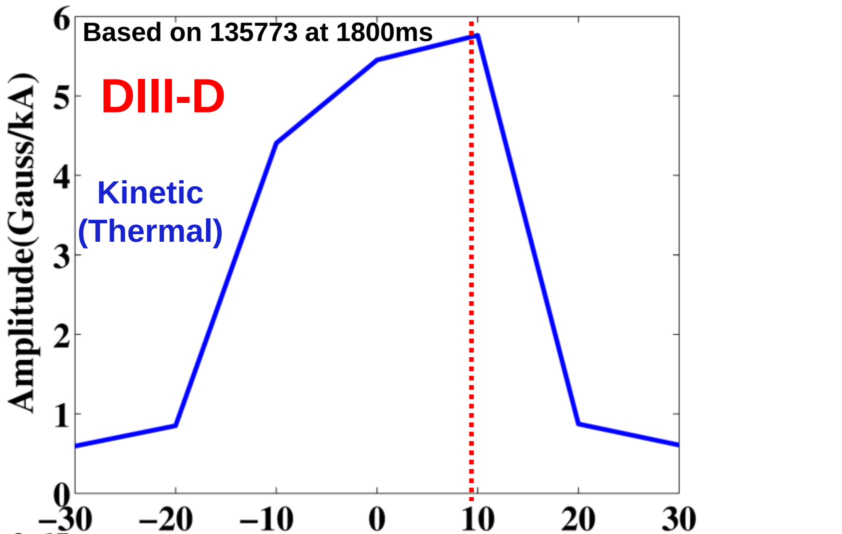
$2\pi|f_{coil}| \sim 1/\tau_w$ $\Rightarrow$strong plasma-wall coupling

Kinetic Plasma Response Reproduces Experimental Frequency Scan Characteristic (Indirect Comparison)

DIII-D

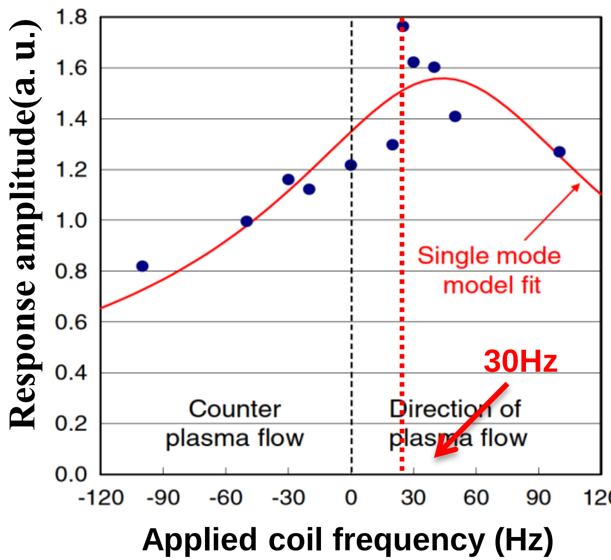
Reimerdes et al, PRL 2004

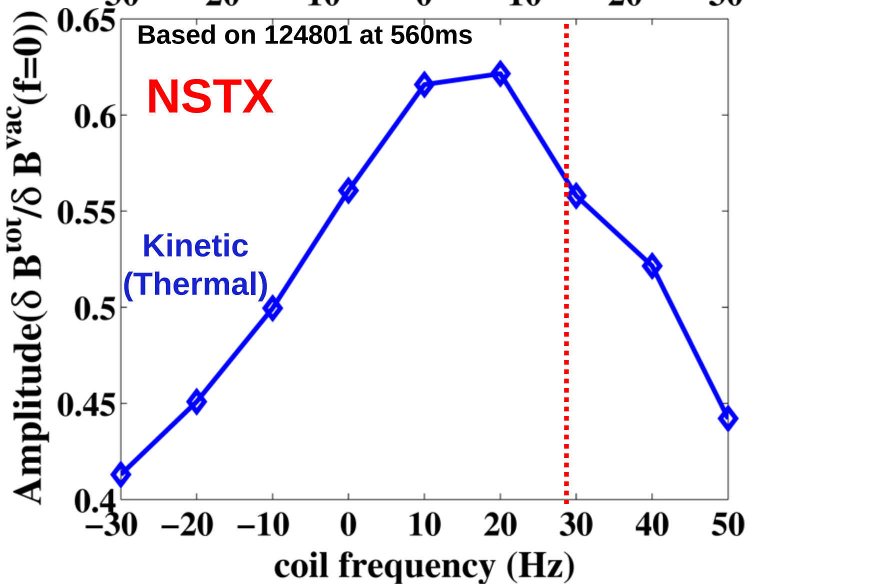




NSTX:

Sontag et al, NF 2007



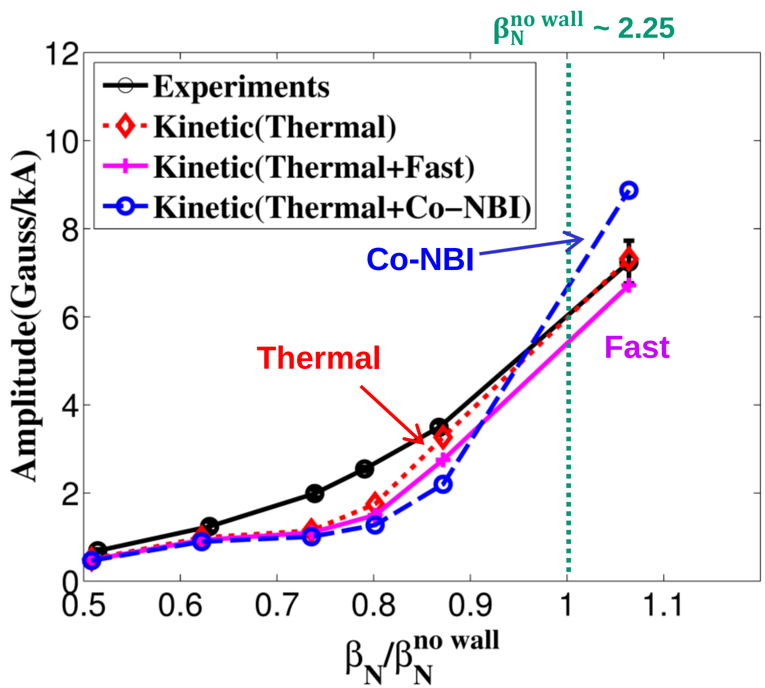


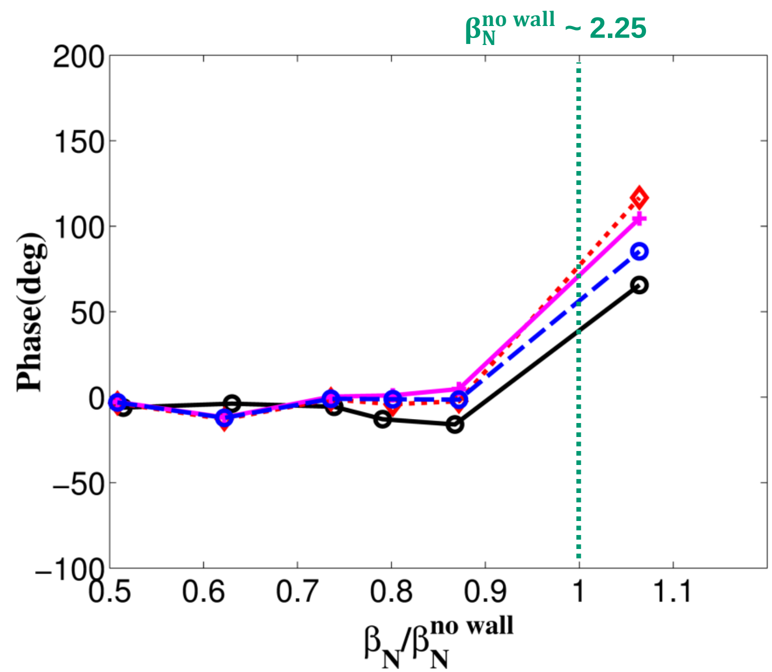
Physical understanding of kinetic plasma response

DIII-D: Thermal Particles Contribute Dominant Kinetic Effects to Kinetic Plasma Response

To verify the role of kinetic effects contributed by thermal particles in determining the kinetic plasma response, three cases are compared with the experiments:

1) 'Thermal' case: $P_{eq} = P_{th} + P_{EP}$, $p_\perp, p_\parallel$ are contributed by thermal particles.
2) 'Fast' case: $P_{eq} = P_{th} + P_{\mathrm{EP}}$, $p_\perp, p_\parallel$ include thermal particles + <u>isotropic slowing down</u> energetic particles.
3) 'Co-tangential NBI' case: $P_{eq} = P_{th} + P_{Co-NBI}$, $p_\perp, p_\parallel$ include thermal particles + <u>anisotropic Co-NBI</u> energetic particles.



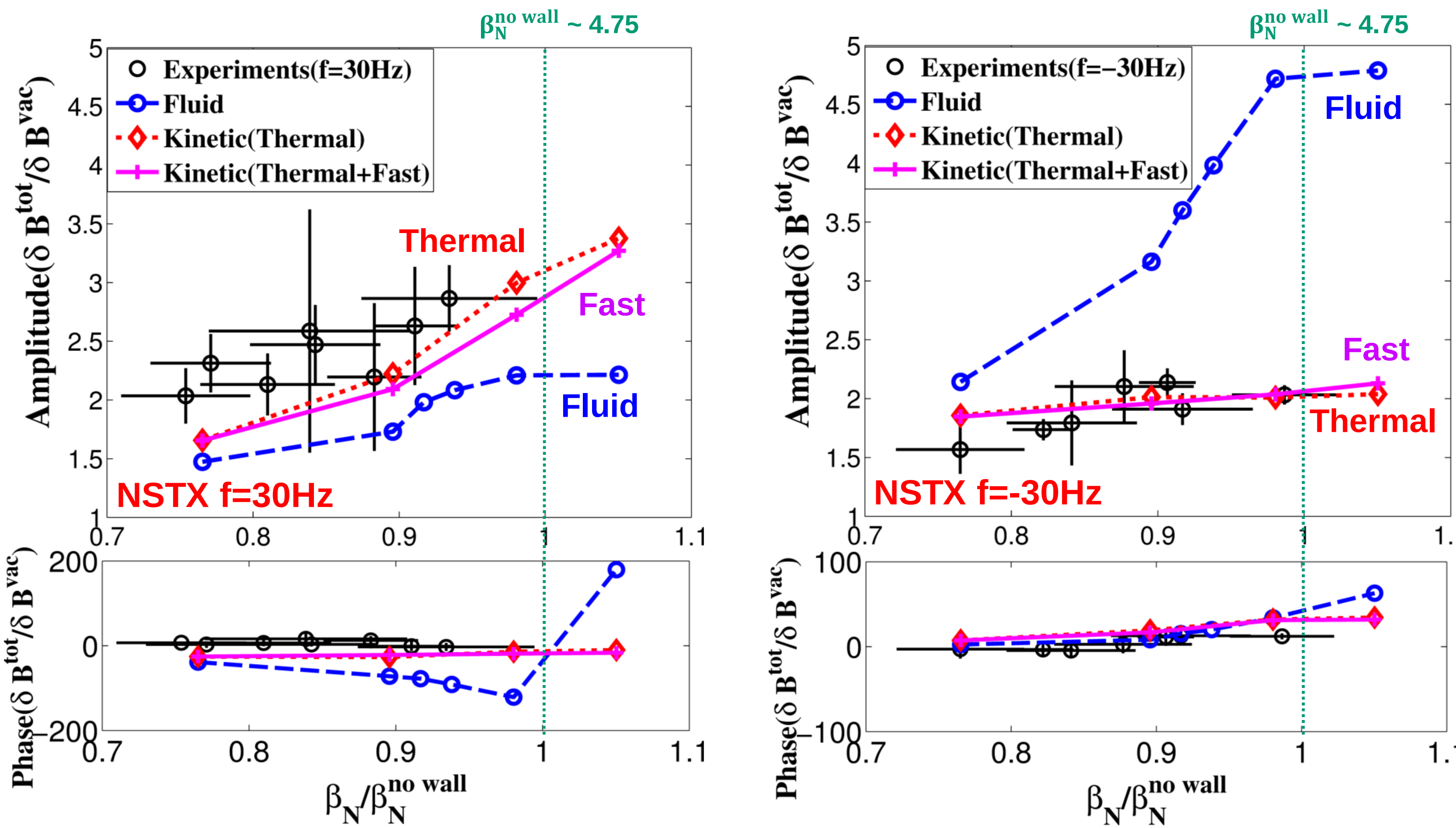


- **No important change of kinetic plasma response can be observed when adding energetic particles.**
- **Co-tangential NBI implies a more experimentally relevant distribution function should be modeled.**

NSTX: Thermal Particles Contributes Dominant Kinetic Effects to Kinetic Plasma Response

Two kinetic plasma responses cases are compared when f=30/-30Hz in NSTX experiments.

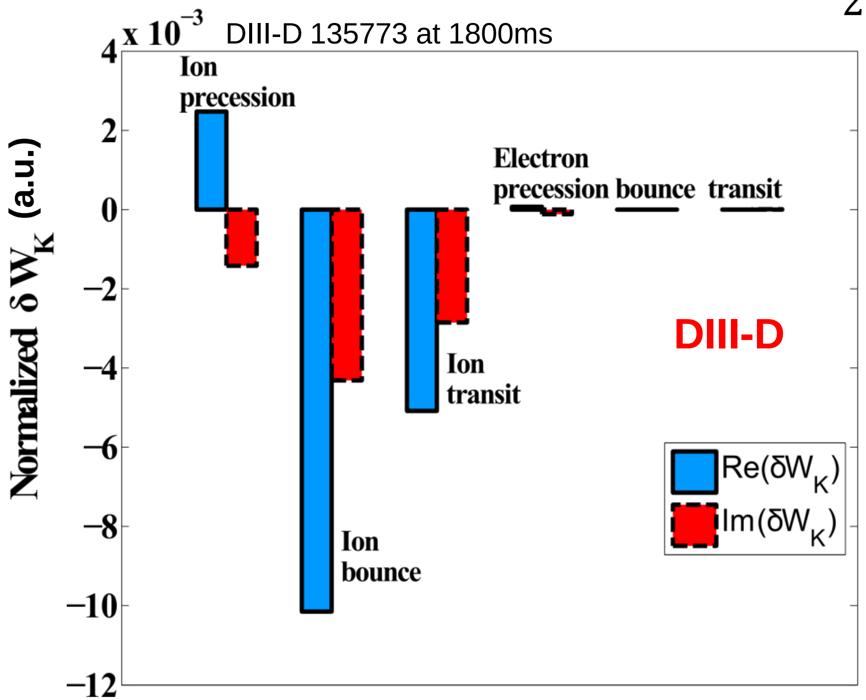
1) 'Thermal' case: $P_{eq} = P_{th} + P_{EP}$, $p_\perp, p_\parallel$ are contributed by thermal particles.
2) 'Fast' case: $P_{eq} = P_{th} + P_{\mathrm{EP}}$, $p_\perp, p_\parallel$ include thermal particles + isotropic slowing down energetic particles.

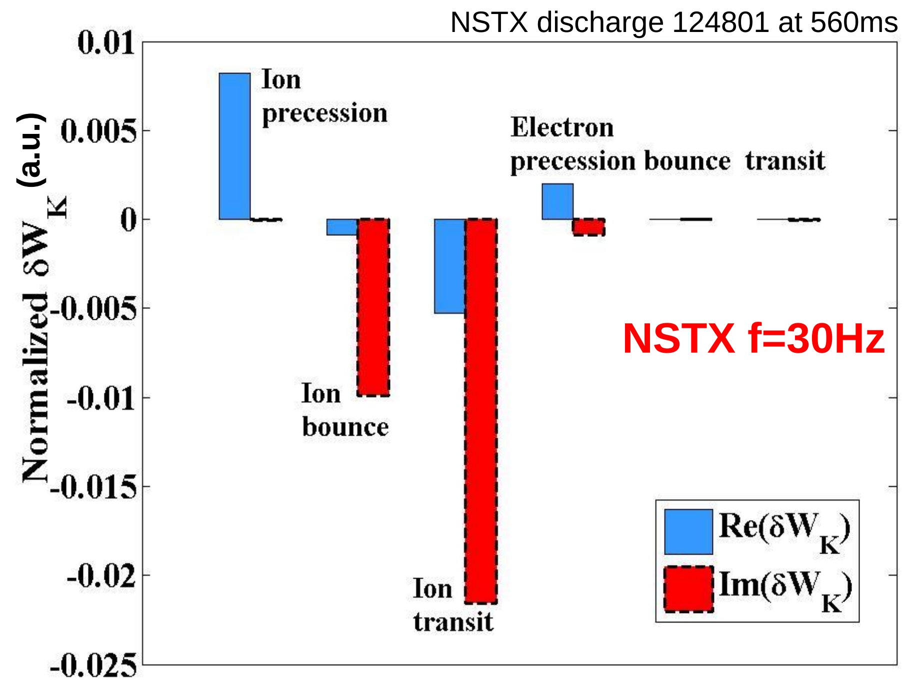


Precession, Bounce and Transit Resonances of Thermal Ions Contributes Dominant Kinetic Energy to Response

To understand which drift kinetic effect of thermal particles play a role to change the plasma response, the perturbed drift kinetic energy δW_K is analyzed near no-wall beta limit in DIIID and NSTX plasmas.

$$\delta \mathrm{W_K} = -\frac{1}{2}\int -\vec{\nabla}\cdot\vec{\boldsymbol{p}}^{kinetic}\cdot\vec{\boldsymbol{\xi}}_{\perp}^{*}$$

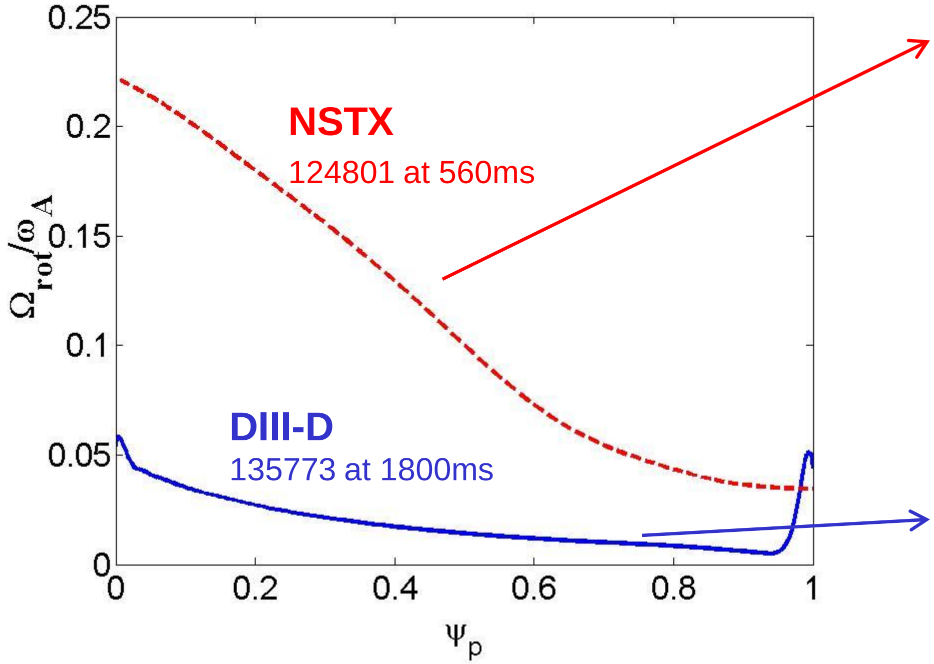


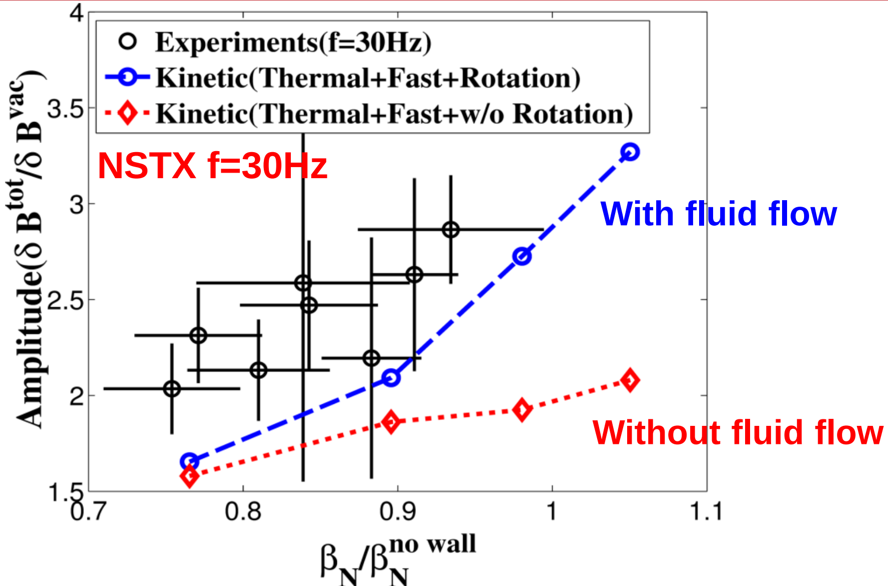


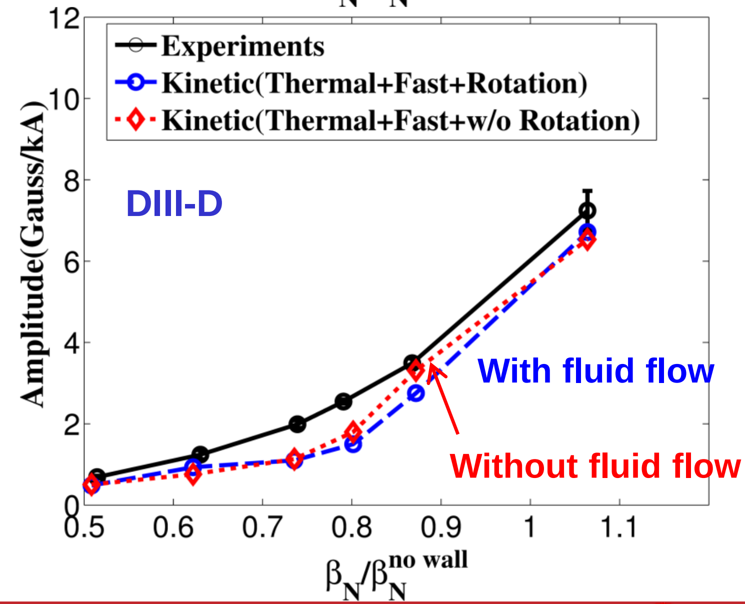
- Thermal electrons contribute much smaller δW_K due to high collision frequency, bounce frequency and transit frequency.
- The eventual response depends on the net contribution, after possible cancellations among all energy components.

Fluid Rotation Can Amplify Kinetic Plasma Response and Destabilize Plasma in NSTX

- NSTX experiments can have much higher fluid rotation than DIII-D.
- The plasma flow can significant affect kinetic plasma response and play a destabilizing role in NSTX plasmas.
- The results agree with Menard et al, BI2.00005, APS 2013







Inertial Energy Plays A Destabilizing Role in NSTX Plasmas

Momentum equation:

$$i\rho(\omega + n\Omega)\vec{\mathbf{v}} = \underline{-\vec{\boldsymbol{\nabla}}\cdot\vec{\mathbf{p}} + \vec{\mathbf{j}}\times\vec{\mathbf{B}}_0 + \vec{\mathbf{J}}_0\times\vec{\mathbf{b}}} + \rho\left[2\Omega\hat{\mathbf{Z}}\times\vec{\mathbf{v}} - \left(\vec{\mathbf{v}}\cdot\vec{\boldsymbol{\nabla}}\Omega\right)R^2\vec{\boldsymbol{\nabla}}\phi\right] - \vec{\boldsymbol{\nabla}}\cdot\left(\rho\vec{\boldsymbol{\xi}}\right)\Omega\hat{\mathbf{Z}}\times\vec{\mathbf{V}}_0$$

Inertial

Pressure Driven
Current Driven

Magnetic bending
Magnetic compressibility

Drift kinetic energy δW_K

Coriolis force

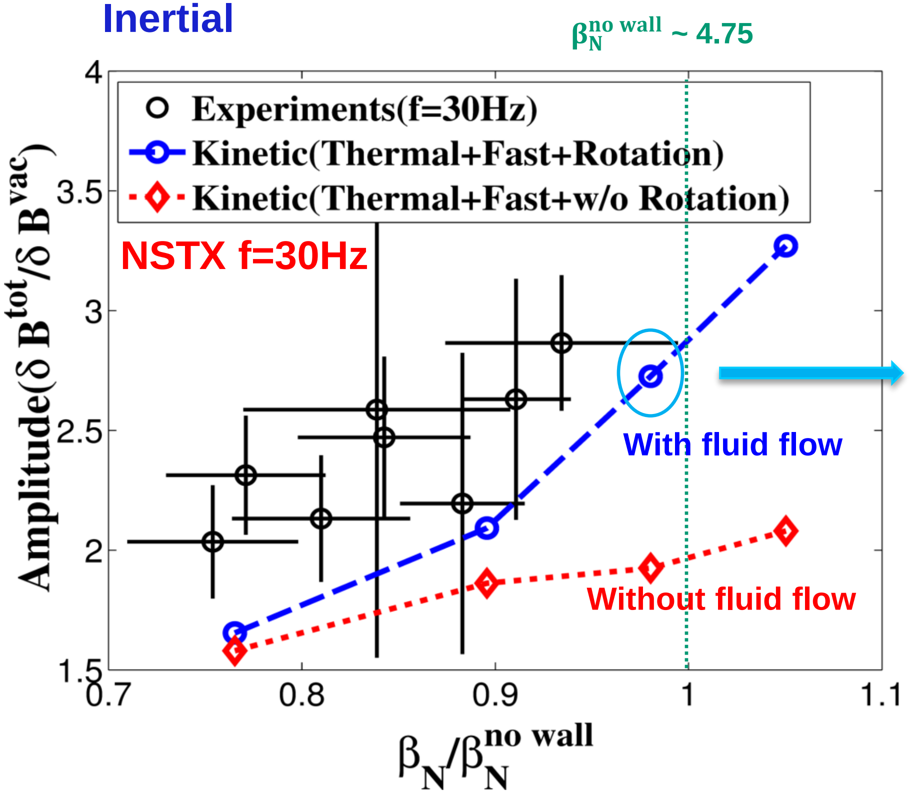
Centrifugal force

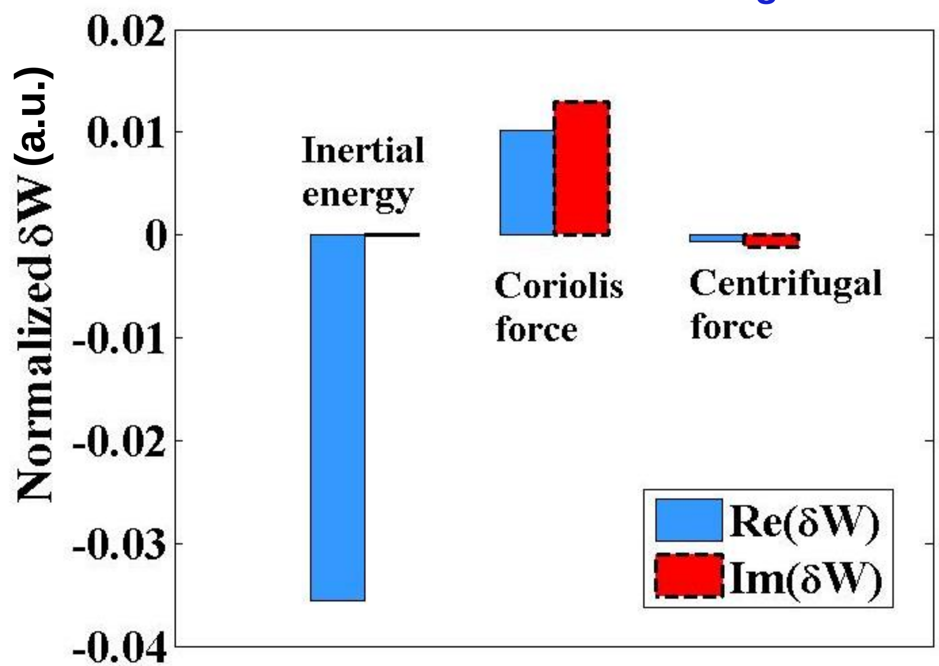
Inertial Energy Plays A Destabilizing Role in NSTX Plasmas

Momentum equation:

$$i\rho(\omega + n\Omega)\vec{\mathbf{v}} = -\vec{\boldsymbol{\nabla}}\cdot\vec{\mathbf{p}} + \vec{\mathbf{j}}\times\vec{\mathbf{B}}_0 + \vec{\mathbf{J}}_0\times\vec{\mathbf{b}} + \rho\left[2\Omega\hat{\mathbf{Z}}\times\vec{\mathbf{v}} - (\vec{\mathbf{v}}\cdot\vec{\boldsymbol{\nabla}}\Omega)R^2\vec{\boldsymbol{\nabla}}\phi\right] - \vec{\boldsymbol{\nabla}}\cdot(\rho\vec{\boldsymbol{\xi}})\Omega\hat{\mathbf{Z}}\times\vec{\mathbf{V}}_0$$

Inertial | Coriolis force | Centrifugal force





Inertial energy is negative and destabilizes the plasma which leads to larger amplification of plasma response.

Self-Consistently Solving Hybrid Kinetic-MHD Equations is Essential to Obtain Quantitatively Understanding of 3D Plasma Response in High Beta Tokamaks

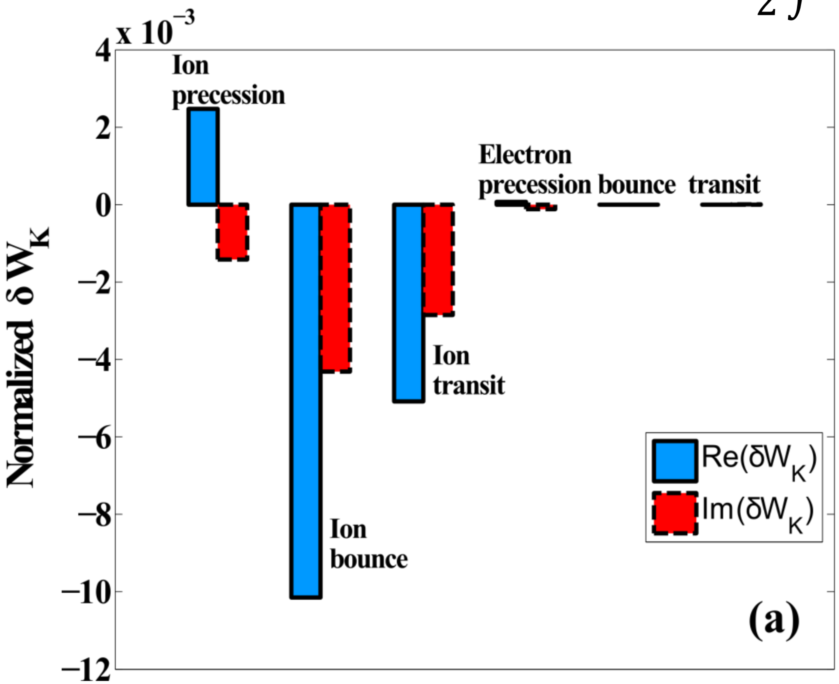
DIII-D	NSTX
Fluid/Kinetic plasma response agrees with experiments when β_N << ideal $\beta_N^{no\,wall}$.	
Only Kinetic plasma response agrees with experiments near/above $\beta_N^{no\,wall}$ limit.	
The kinetic effects from thermal ions plays a major role to determine 3D response. (precession, bounce and transit resonances of thermal ions)	
Different features in studied n=1 plasma response experiments	
Weak plasma-wall coupling	Strong plasma-wall coupling
Low fluid rotation (ω_{rot}~0.05ω_A) No significant impact on kinetic plasma response	Strong fluid rotation (ω_{rot}~0.2ω_A) Destabilize the plasma and amplify kinetic plasma response.

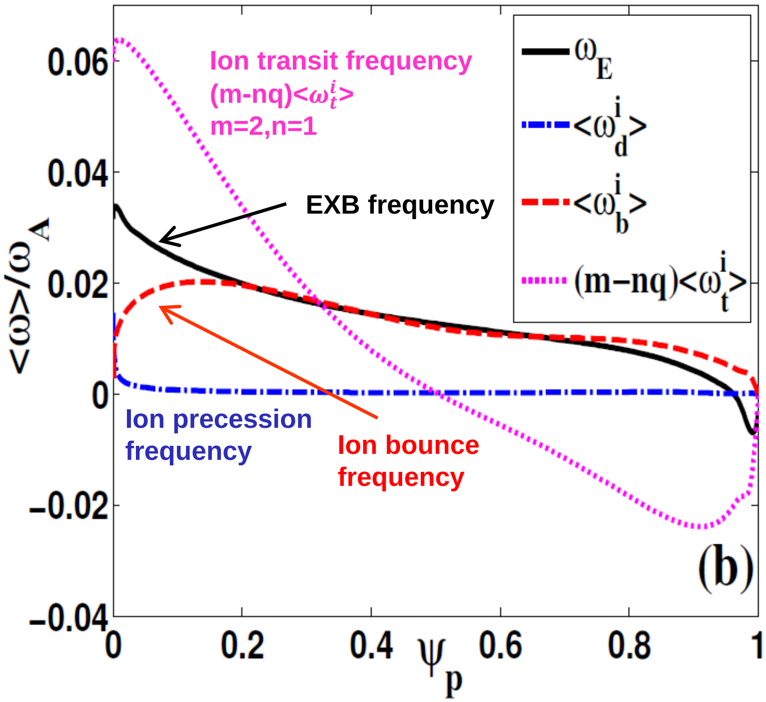
Backup Slides

DIII-D: Precession, Bounce and Transit Resonances of Thermal Ions Contributes Dominant Kinetic Energy to Response

To understand which drift kinetic effect of thermal particles play a role to change the plasma response, DIII-D discharge 135775 near no-wall beta limit is chosen for perturbed drift kinetic energy δW_K analysis.

$$\delta W_K = -\frac{1}{2}\int -\vec{\nabla}\cdot\vec{\boldsymbol{p}}^{kinetic}\cdot\vec{\boldsymbol{\xi}}_\perp^*$$



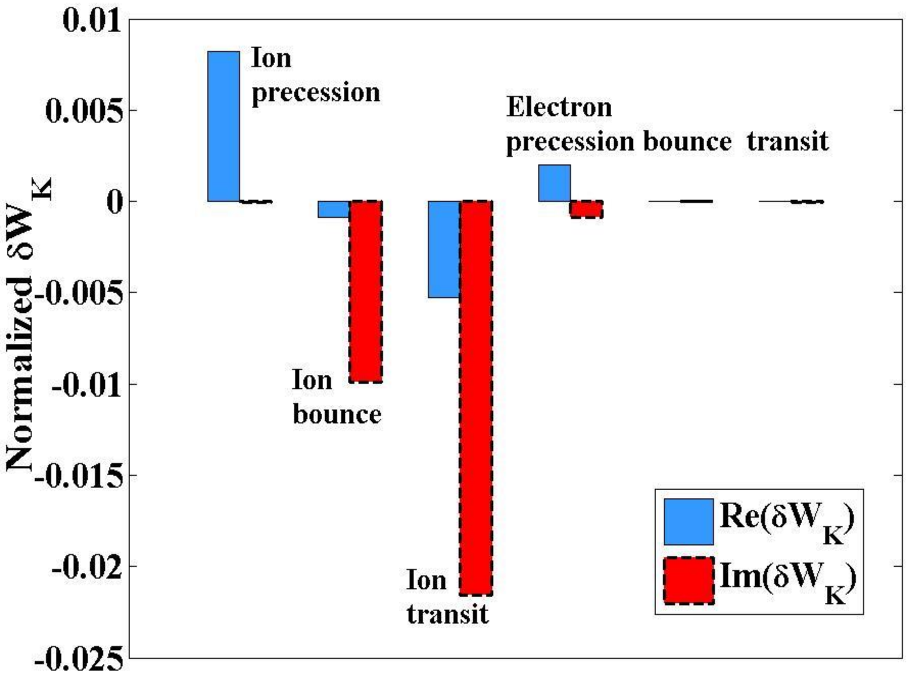


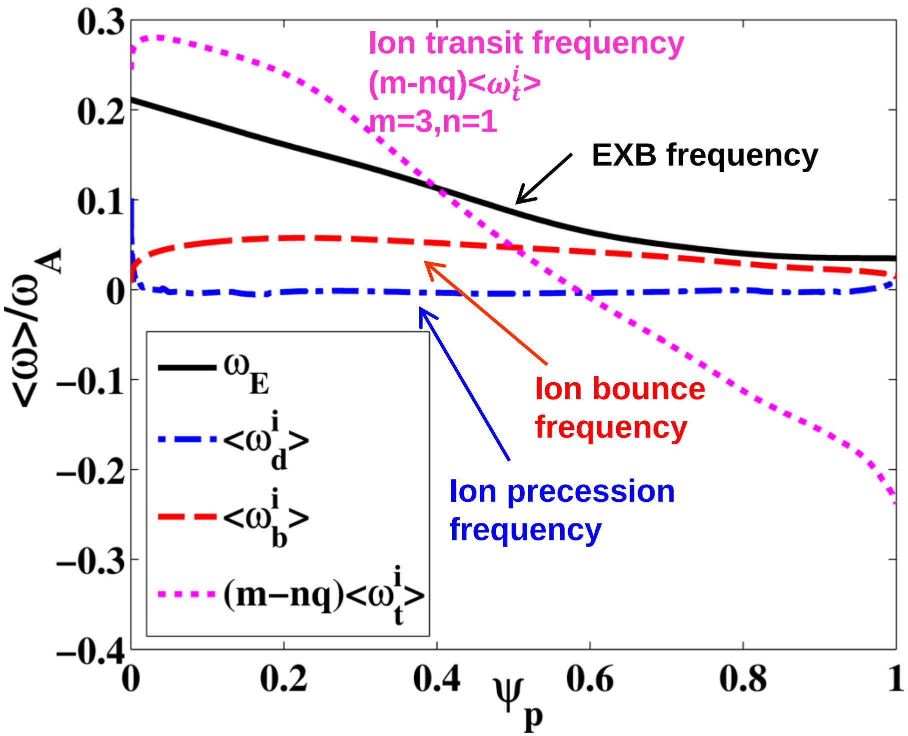
- Thermal electrons contribute much smaller δW_K due to high collision frequency, bounce frequency and transit frequency.
- The eventual response depends on the net contribution, after possible cancellations among all energy components.

NSTX: The Contribution of Precession, Bounce and Transit Resonances of Thermal Ions is Also Dominant

NSTX, f=30Hz case (Shot No. 124801 at 560ms) near the no-wall beta limit is chosen for δW_K analysis.

$$\delta W_K = -\frac{1}{2}\int -\vec{\nabla}\cdot\vec{\boldsymbol{p}}^{kinetic}\cdot\vec{\boldsymbol{\xi}}_{\perp}^{*}$$



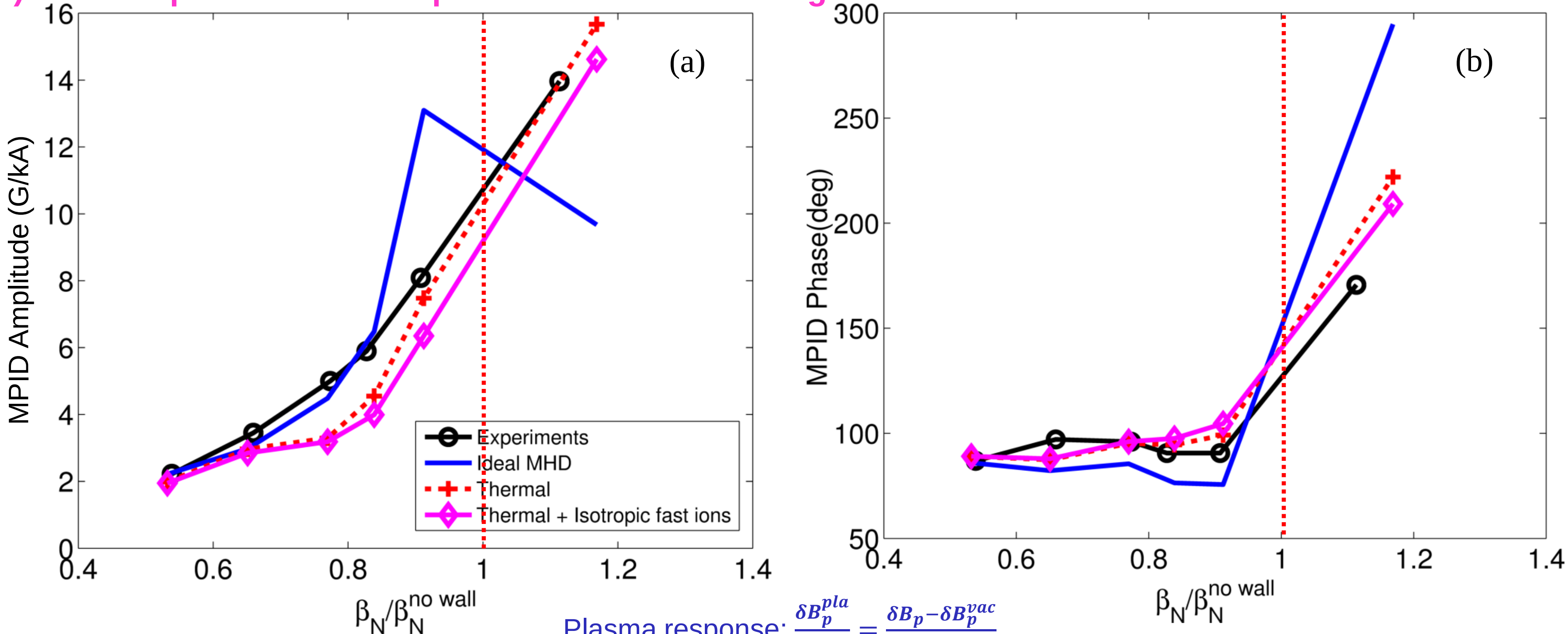


- **Similarly to DIII-D case, the precession, bounce and the transit resonances of thermal ions contributes the comparable δW_K to the kinetic plasma response in NSTX plasmas.**

DIII-D: Kinetic Plasma Response at MPID Sensor Agrees with Experiments

The plasma response solved by MARS-K is also compared with experimental measurements at MPID sensor.

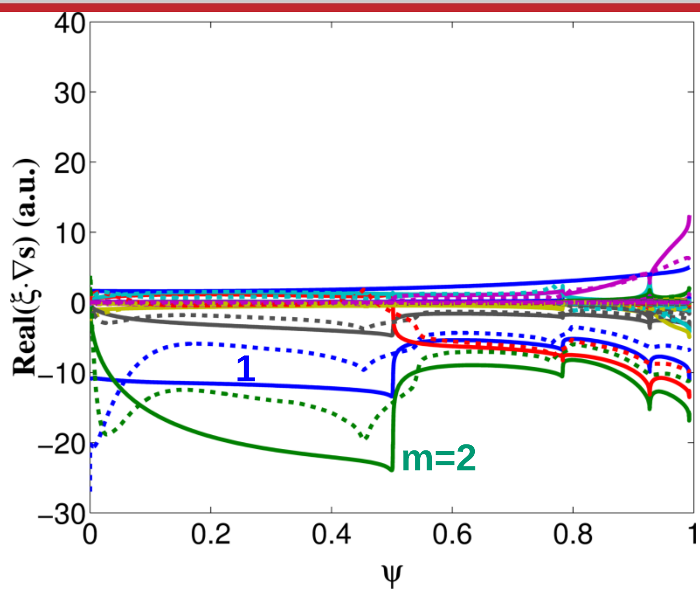
Two cases including kinetic effects due to different particles are considered:
1) Only thermal particles (ions and electrons)
2) Thermal particles + isotropic fast ions with slowing down distribution.

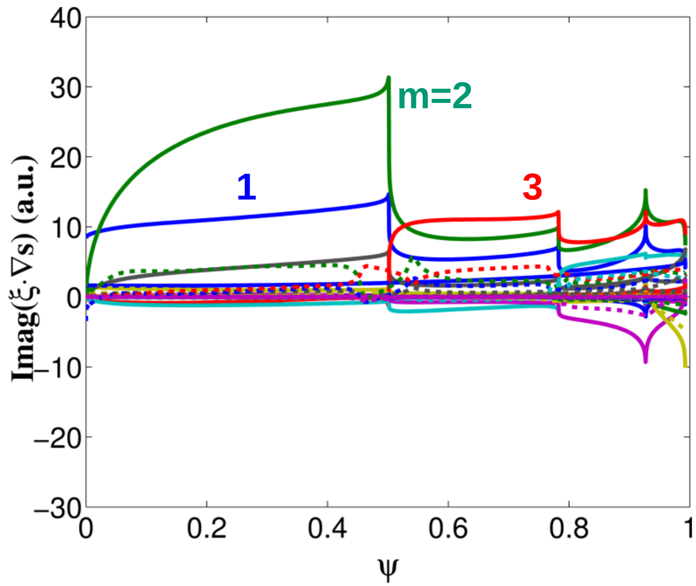


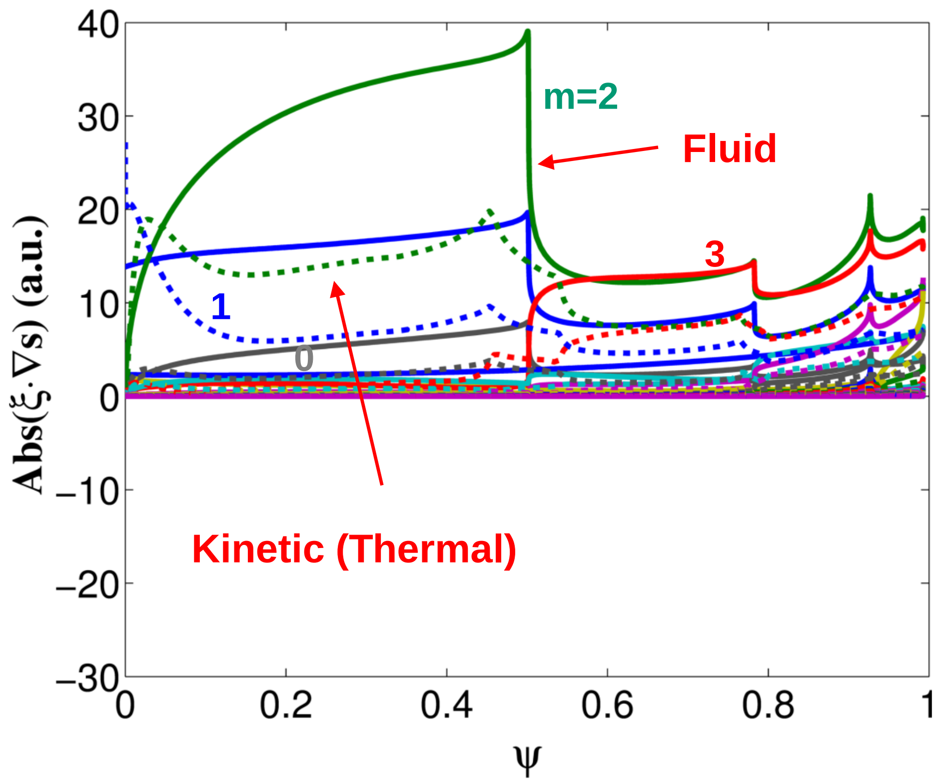
Plasma response: $\frac{\delta B_p^{pla}}{I_{coil}} = \frac{\delta B_p - \delta B_p^{vac}}{I_{coil}}$

- **The behavior of plasma response solved by MARS-K also shows a agreement with experimental measurements for both amplitude and phase at MPID sensor.**

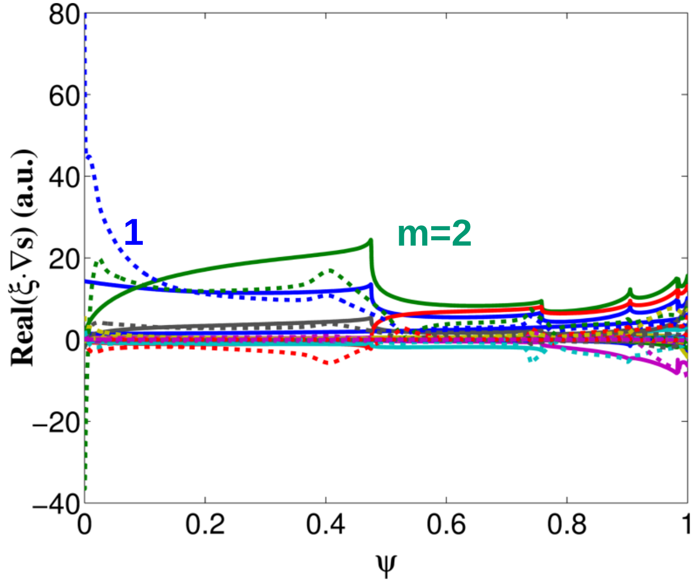
DIII-D: Kinetic Effects Strongly Modify Radial Plasma Displacement Near No-Wall Limit (Discharge 135773)

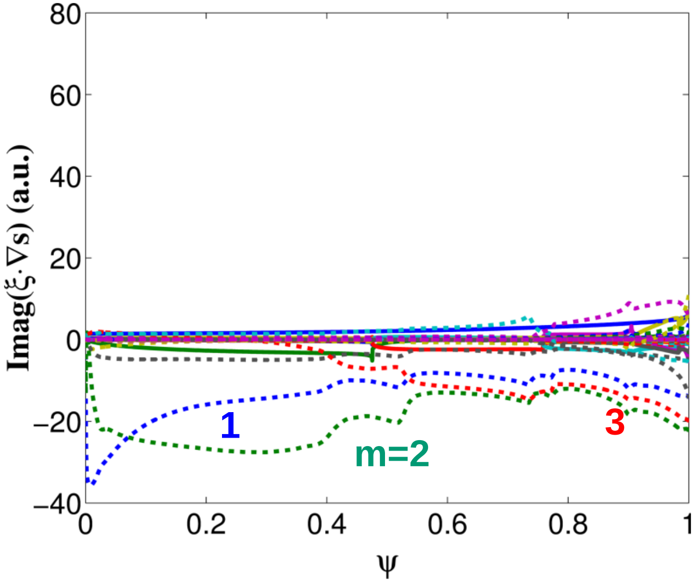


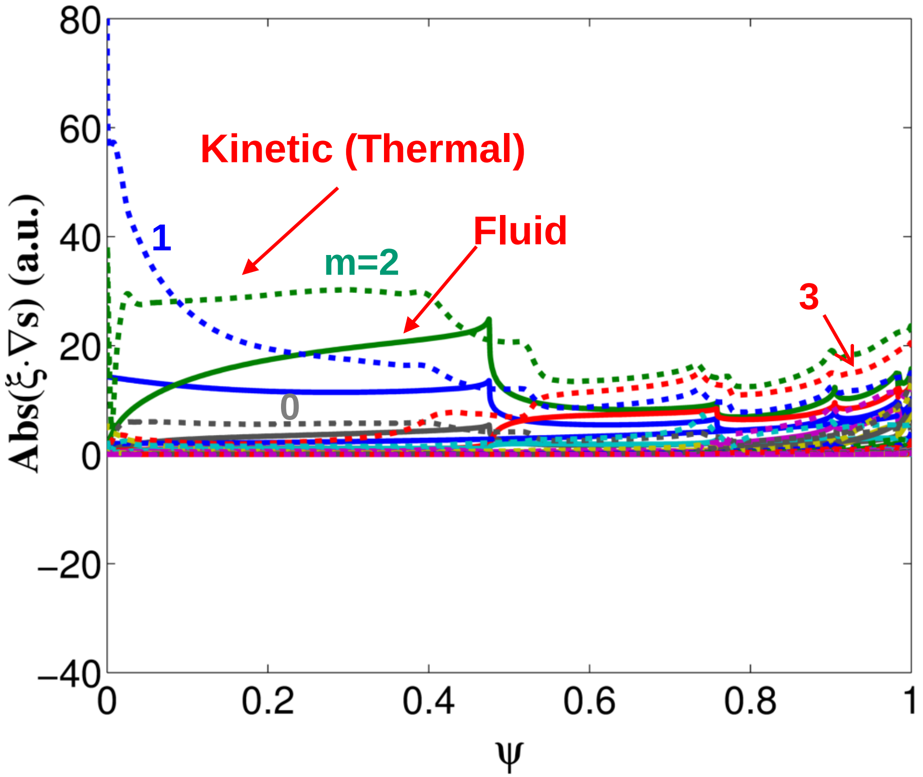




DIII-D: Kinetic Effects Strongly Modify Radial Plasma Displacement Above No-Wall Limit (Discharge 135759)







Fluid Plasma Response Model is Based on Linearized MHD Equations

MARS-F can solve the linearized MHD equation with external coils, vacuum and resistive wall.

MHD equations developed in MARS-F:

$$i(\omega+n\Omega)\boldsymbol{\xi}=\mathbf{v}+(\boldsymbol{\xi}\bullet\nabla\Omega)R^2\nabla\phi$$

$$i\rho(\omega+n\Omega)\mathbf{v}=-\nabla p+\mathbf{j}\times\mathbf{B}+\mathbf{J}\times\mathbf{Q}+\rho\left[2\Omega\hat{\mathbf{Z}}\times\mathbf{v}-(\mathbf{v}\bullet\nabla\Omega)R^2\nabla\phi\right]-\nabla\bullet(\rho\boldsymbol{\xi})\Omega\hat{\mathbf{Z}}\times\mathbf{V}_0$$

$$i(\omega+n\Omega)\mathbf{Q}=\nabla\times(\mathbf{v}\times\mathbf{B})+(\mathbf{Q}\bullet\nabla\Omega)R^2\nabla\phi-\nabla\times(\eta\mathbf{j})$$

$$i(\omega+n\Omega)p=-\mathbf{v}\bullet\nabla P-\Gamma P\nabla\bullet\mathbf{v}$$

$$\mathbf{j}=\nabla\times\mathbf{Q}$$

Coil equations:

$$\nabla\times\mathbf{Q}=\mathbf{j}_{coil}\qquad \nabla\cdot\mathbf{j}_{coil}=0$$

vacuum and resistive wall are included.

Y.Q. Liu et al, PoP 2010

Linear fluid plasma response model

(ignore plasma flow Ω and resistivity *η*=0)

$$i\omega\boldsymbol{\xi}=\mathbf{v}$$

$$i\rho\omega\mathbf{v}=-\nabla p+\mathbf{j}\times\mathbf{B}+\mathbf{J}\times\mathbf{Q}$$

$$i\omega\mathbf{Q}=\nabla\times(\mathbf{v}\times\mathbf{B})\cancel{-\nabla\times(\eta\mathbf{j})}$$

$$i\omega p=-\mathbf{v}\bullet\nabla P-\Gamma P\nabla\bullet\mathbf{v}$$

$$\mathbf{j}=\nabla\times\mathbf{Q}$$

Coil frequency ω → 0, fluid plasma response recovers 3D Ideal perturbed equilibrium.

J.-K. Park et al, PoP 2007

Linear Kinetic Plasma Response Model Uses a Hybrid Drift-Kinetic MHD Formulation (MARS-K)

MARS-K extends MARS-F and solves linearized MHD equations with perturbed kinetic pressure.

MHD equations:

$$i(\omega + n\Omega)\vec{\xi} = \vec{v} + \left(\vec{\xi}\cdot\vec{\nabla}\Omega\right)R^2\vec{\nabla}\phi$$

$$i\rho(\omega + n\Omega)\vec{v} = -\vec{\nabla}\cdot\vec{p} + \vec{j}\times\vec{B}_0 + \vec{J}_0\times\vec{b} + \rho\left[2\Omega\hat{Z}\times\vec{v} - \left(\vec{v}\cdot\vec{\nabla}\Omega\right)R^2\vec{\nabla}\phi\right] - \vec{\nabla}\cdot\left(\rho\vec{\xi}\right)\Omega\hat{Z}\times\vec{V}_0$$

$$i(\omega + n\Omega)\vec{b} = \vec{\nabla}\times\left(\vec{v}\times\vec{B}_0\right) + \left(\vec{b}\cdot\vec{\nabla}\Omega\right)R^2\vec{\nabla}\phi$$

$$i(\omega + n\Omega)p = -\vec{v}\cdot\vec{\nabla}P_0 - \cancel{\Gamma P_0\vec{\nabla}\cdot\vec{v}}$$

replaced by kinetic pressure

Applied field frequency

$$\vec{j} = \vec{\nabla}\times\vec{b}$$

Coil equations:

$$\vec{\nabla}\times\vec{b} = \vec{j}_{coil} \qquad \vec{\nabla}\cdot\vec{j}_{coil} = 0$$

Drift-kinetic equation:

$$\frac{df_L^1}{dt} = f_\varepsilon^0\frac{\partial H^1}{\partial t} - f_{P_\phi}^0\frac{\partial H^1}{\partial\phi} - \nu_{eff}f_L^1$$

Perturbed Lagrangian

$$H^1 = \frac{1}{\varepsilon_k}\left[Mv_\parallel^2\vec{\kappa}\cdot\vec{\xi}_\perp + \mu\left(b_\parallel + \vec{\nabla}B_0\cdot\vec{\xi}_\perp\right)\right]$$

Ignore finite orbit width effect.

Self-consistent approach:

Drift kinetic effects can modify plasma response

$$\vec{p} = p\overleftrightarrow{I} + p_\parallel\vec{b}\vec{b} + p_\perp\left(\overleftrightarrow{I} - \vec{b}\vec{b}\right)$$

Kinetic pressure $p_\parallel$ and $p_\perp$ couple with MHD equations

$$p_\parallel e^{-i\omega t + in\phi} = \sum_{e,i}\int d\Gamma M v_\parallel^2 f_L^1 \qquad p_\perp e^{-i\omega t + in\phi} = \sum_{e,i}\int d\Gamma\frac{1}{2}Mv_\perp^2 f_L^1$$

$$f_L^1 = -f_\varepsilon^0\varepsilon_k e^{-i\omega t + in\phi}\sum_{m,l}X_m H_{ml}\lambda_{ml}e^{-in\tilde{\phi}(t) + im\langle\dot{\chi}\rangle + il\omega_b t}$$

Resonant operator:

Diamagnetic drift

Applied field frequency

$$\lambda_{ml} = \frac{n[\omega_{*N} + (\hat{\varepsilon}_k - 3/2)\omega_{*T} + \omega_E] - \omega}{n\omega_d + [\alpha(m + nq) + l]\omega_b + n\omega_E - \omega - i\nu_{eff}}$$

Precession drift

Bounce/Transit

EXB

Crook Collisions

Y.Q. Liu et al, PoP 2008

Linear Kinetic Plasma Response Model Uses a Hybrid Drift-Kinetic MHD Formulation (MARS-K)

To carry out the computation of drift kinetic effects and kinetic energy

$$\delta W_K = -\frac{1}{2}\int -\vec{\nabla}\cdot\vec{\boldsymbol{p}}^{kinetic}\cdot\vec{\boldsymbol{\xi}}_\perp^* = -\frac{1}{2}\int d^3x\left[p_\perp\left(\vec{\nabla}\cdot\vec{\boldsymbol{\xi}}_\perp^* + \boldsymbol{\kappa}\cdot\vec{\boldsymbol{\xi}}_\perp^*\right) - p_\parallel\vec{\boldsymbol{\kappa}}\cdot\vec{\boldsymbol{\xi}}_\perp^*\right]$$

Kinetic pressure $p_\perp$ and $p_\parallel$ needs to be solved in MARS-K directly

$$(Jp_\parallel)_k = \frac{1}{\sqrt{\pi}}\sum_{e,i}\sum_{m,l,u}\frac{P_{e,i}}{B_0}\int d\Lambda I_{ml}H_{ml}^u G_{kml}^\parallel X_m^u, \qquad (Jp_\perp)_k = \frac{1}{\sqrt{\pi}}\sum_{e,i}\sum_{m,l,u}\frac{P_{e,i}}{B_0}\int d\Lambda I_{ml}H_{ml}^u G_{kml}^\perp X_m^u,$$

For given pinch angle Λ, resonant operator λ_{ml} in I_{ml} has energy dependence, G factor is about integration of particle motion between two turning points of trapped particles (the integration of poloidal angle with respect to passing particles are from 0 to 2π).

$$I_{ml} = \sum_\sigma\int_0^\infty d\hat{\epsilon}_k\hat{\epsilon}_k^{5/2}e^{-\hat{\epsilon}_k}\lambda_{ml}$$

$$G_{kml}^\parallel = \frac{1}{2\pi}\int_{\chi_L}^{\chi_U} JB\sqrt{1-\Lambda/h}\,e^{i[\alpha(m+nq)+l]\omega_b t(\chi)-in\phi(\chi)-ik\chi}d\chi$$

$$G_{kml}^\perp = \frac{1}{2\pi}\int_{\chi_L}^{\chi_U}\frac{JB\Lambda/(2h)}{\sqrt{1-\Lambda/h}}e^{i[\alpha(m+nq)+l]\omega_b t(\chi)-in\phi(\chi)-ik\chi}d\chi$$

General structure of integration at each flux surface is

$$\int_0^{h_{max}} d\Lambda\left(\int_0^\infty\cdots d\hat{\epsilon}_k\right)\left(\int_{\chi_L}^{\chi_U}\cdots d\chi\right)H_{ml}^\mu$$

Λ **is particle pinch angle,** $h = \frac{B_0}{B}$

χ *is poloidal angle*, $\hat{\epsilon}_k = \frac{\epsilon - Ze\phi}{T}$

Y.Q. Liu et al, PoP 2008
Y.Q. Liu et al. PoP 2014